A VISCOUS FLUID MODEL FOR DEMONSTRATION OF GROUNDWATER FLOW TO PARALLEL DRAINS

A VISCOUS FLUID MODEL FOR DEMONSTRATION OF GROUNDWATER FLOW TO PARALLEL DRAINS

F. HOMMA

Research Associate, Institute for Land and Water Management Research
Wageningen, The Netherlands

International Institute for Land Reclamation and Improvement/ILRI
P.O. Box 45, 6700 AA Wageningen, The Netherlands 1983.

First published 1968
Second edition 1983

ISBN 90 70260 824

Printed in The Netherlands.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS

The author wrote this Bulletin at the request of the International Institute for Land Reclamation and Improvement. During his work he received advice and comments from various persons. He wants to express his gratitude in particular to Dr. L. F. Ernst, Dr. J. Wesseling of the Institute for Land and Water Management Research, Wageningen as well as to Messrs. J. G. van Alphen and J. M. van Staveren, both International Institute, for their valuable co-operation in this respect.

The model described in this Bulletin has been constructed by the Service Institute for Applied Mechanics and Technical Physics in Agriculture, Wageningen, by order of the International Institute.

1. INTRODUCTION

For the study of various problems related to groundwater flow analog models are often used. A model can be a valuable aid, particularly for more complicated problems, e.g. most non-stationary problems. Groundwater flow can be described by means of Darcy's law and the continuity equation. There is a wide choice in models since the following types are meeting the requirements (TODD, 1959; SANTING, 1963):

- heat models
- membrane models
- electrical models
- fluid models:
  - sand models or glassbead models
  - parallel plate models

Heat models are not much used because of insulation difficulties (KUIPERS, 1955).
Membrane models can be used, e.g. for the study of the phreatic surface when water is withdrawn from the soil by one or more wells (ZEE, PETERSEN and BOCK, 1955).
In comparison with other models, electrical models have the advantage that a great number of variations can be made in the course of the experiments without much extra time and cost (FELIUS, 1954; TODD and BEAR, 1961).
Viscous fluid models have the advantage that the flow near the drain and the influence of differences in specific weight can be simulated very easily. The latter is important in studying problems where a simultaneous flow of fresh and salt water occurs (SANTING, 1963; ERNST, 1962; BEAR, 1960).
Compared with the other models mentioned, a sand model can give the best approximation of the reality – it shows the influence of the unsaturated zone on the flow in the saturated zone (ZELLER, 1954).

For the visual demonstration of various properties of groundwater flow the parallel plate model may be preferred, having the following advantages:

a. The upper surface of the fluid in the model will automatically get its correct shape, just as the phreatic surface in the field. Electrical models and heat models need an additional adjusting.
b. The phreatic surface is immediately visible. The streamlines can easily be made visible.
c. By means of piezometric tubes the distribution of potentials can be found.
d. The stratification of the soil and the position of the drain in relation to these soil layers can be built in quite well.
e. Simultaneous flow of liquids with a different specific weight is possible.

The principle of the parallel plate model is based on the replacing of the network of pores in the soil by a continuous narrow slit. This system allows only investigation of two-dimensional flow. If the third dimension is of importance, another kind of model should be chosen (GROVER and KIRKHAM, 1961).
A parallel plate model is constructed from two flat sheets, at least one of these being transparent. Along the sides the plates are pressed together. Metal strips on these sides are used to maintain the required width of the interspace and for preventing leakage of fluid from the model. The width of the gap between the plates and the viscosity of the fluid applied must be such that only a laminar flow will be possible.

2. MODEL SCALES

When the field data are known (soil permeability, depth of the impermeable layer, precipitation, etc.), the dimensions of the model and the viscosity of the model liquid are chosen according to the available space and materials, the scales of the model can be determined. The magnitudes measured in the model can be converted into the situation in the field. This does not only apply for dimensions, hydraulic heads, discharges etc., but also for the time in which the process takes place.

The model scales can be derived from the differential equations governing the flow (BEAR, 1960; DIETZ, 1941; ERNST, 1962; SANTING, 1963). The presumption that the flow is horizontal throughout (Dupuit-Forchheimer theory) leads to the following formula:

$$k \frac{\delta}{\delta x}\left(h \frac{\delta h}{\delta x}\right) + N = \mu \frac{\delta h}{\delta t} \tag{1}$$

or

$$\tfrac{1}{2} k \frac{\delta^2 h^2}{\delta x^2} + N = \mu \frac{\delta h}{\delta t} \tag{2}$$

where:

k = hydraulic conductivity of the soil
x = horizontal coordinate
h = piezometric head
μ = effective porosity or drainable pore space of the soil
t = time
N = precipitation surplus

Indicating the corresponding magnitudes in the model by the index m, the following applies analogically for the model:

$$\tfrac{1}{2} k_m \frac{\delta^2 h_m}{\delta {x^2}_m} + N_m = \mu_m \frac{\delta h_m}{\delta t_m} \tag{3}$$

Multiplication of (2) by $\frac{k_m}{k}\left(\frac{x}{x_m}\right)^2\left(\frac{h_m}{h}\right)^2$ yields:

$$\tfrac{1}{2}\frac{k\,k_m}{k}\left(\frac{x}{x_m}\right)^2\left(\frac{h_m}{h}\right)^2\frac{\delta^2 h^2}{\delta x^2} + N\frac{k_m}{k}\left(\frac{x}{x_m}\right)^2\left(\frac{h_m}{h}\right)^2 = \mu\frac{k_m}{k}\left(\frac{x}{x_m}\right)^2\left(\frac{h_m}{h}\right)^2\frac{\delta h}{\delta t} \tag{4}$$

An h means the elevation of the fluid surface above the impermeable layer. The scales for h and the vertical coordinate z should be equal: $\{h\} = \{z\}$. As formulas (1) and (2) apply especially for almost horizontal flow, unequal horizontal and vertical scales – $\{x\} \neq \{z\}$ – need not be considered an objection. When the direction of the groundwater flow is at some places far from horizontal a very strong distortion of the model cannot be accepted. In cases with an anisotropic permeability of the soil a certain distortion is required by theory.

The requirement that the model must be reasonably manageable will determine the horizontal and vertical scales. There are many cases however where the variations in h are much smaller than the total vertical size of the area of groundwaterflow. This objection can be met by using different scales for h, and z. Replacing h in the above formulas by D + h, where D means the thickness of the aquifer below the level of the drains and h is the hydraulic head above this level, will cause only a slight change in these formulas. An exaggeration of the variations in h'_m in comparison to the chosen value for D_m can be effected in such a degree that a sufficient accuracy in the measurements of these variations will be obtained. It is obvious that in cases with an impermeable layer close to the draindepth, unequal scales for $\{h\}$ en $\{z\}$ should not be recommended.

If the scales x_m/x etc. are denoted by $\{x\}$, then (4) may be written as:

$$\tfrac{1}{2}\, k_m \frac{\{h\}^2}{\{x\}^2} \frac{\delta^2 h^2}{\delta x^2} + N \frac{\{k\}\,\{h\}^2}{\{x\}^2} \quad \mu \frac{\{k\}\,\{h\}^2}{\{x\}^2} \frac{\delta h}{\delta t} \tag{5}$$

or

$$\tfrac{1}{2}\, k_m \frac{\delta^2 h_m{}^2}{\delta x_m{}^2} + N \frac{\{k\}\,\{h\}^2}{\{x\}^2} = \mu \frac{\{k\}\,\{h\}\,\{t\}}{\{x\}^2} \frac{\delta h_m}{\delta t_m} \tag{6}$$

Equation (3) and (6) are equal if:

$$N_m = N \frac{\{k\}\,\{h\}^2}{\{x\}^2} \tag{7}$$

and

$$\mu_m = \mu \frac{\{k\}\,\{h\}\,\{t\}}{\{x\}^2} \tag{8}$$

From (7) the scale for the precipitation can be derived:

$$\{N\} = \frac{N_m}{N} = \frac{\{k\}\,\{h\}^2}{\{x\}^2} \tag{9}$$

and from (8) the time scale:

$$\{t\} = \frac{\{\mu\}\,\{x\}^2}{\{k\}\,\{h\}} \tag{10}$$

The scale for k depends on the width of the interspace, on the viscosity and on the specific weight of the model fluid. The fact that the choice of the fluid generally is free can be used in obtaining a favourable time-scale. For a laminar flow between two parallel plates one may deduce for the average velocity in an arbitrary point (ROUSE, 1961):

$$\bar{v} = \frac{1}{12} \frac{\rho g b^2}{\eta} \operatorname{grad} h \tag{11}$$

where:
$\bar{v}$ = average velocity in cm/sec.
g = acceleration of gravity in cm/sec^2.
b = interspace between the plates in cm
ρ = density in g/cm^3
η = viscosity in poise ($dyne/cm^2/sec$)
h = piezometric height in relation to a fixed horizontal level

Groundwater flow satisfies Darcy's law; the same applies for the model flow.
Both for field and model applies:

$$\bar{v} = - k \text{ grad } h \tag{12}$$

Comparison of formulas (11) and (12) gives for the permeability in the model:

$$k_m = \frac{1}{12} \rho g \frac{b^2}{\eta} \tag{13}$$

So for the scale for the permeability, it follows that:

$$\frac{k_m}{k} = \{k\} = \frac{\rho g b^2}{12 \eta k} \tag{14}$$

In comparison with the other scales not much variation can be obtained in the scale for the storage coefficient. This appears from the following derivation of the storage coefficient in the model.
Fluid drops falling free between the two plates move so fast that the drops take up relatively very little space, causing the storage coefficient to be only slightly below 1. But also in the case that the inflow is guided along one of the plates, resulting in a film of thickness *s*, to flow downwards in the unsaturated zone with a velocity whereby:

$$s = \frac{bN_m}{v'} \tag{15}$$

the storage coefficient in the model will not be much lower than 1, as can be deduced from:

$$\mu_m = 1 - \frac{s}{b} \tag{16}$$

For the flow of a fluid layer of thickness *s* across a plate, the average velocity is similar to (11):

$$v' = - \frac{s^2 \rho g}{3\eta} \text{ grad } h \tag{17}$$

From equations (15) and (17) and because in this case *grad, h* = —1, it follows:

$$b\, N_m = sv' = \frac{\rho g\, s^3}{3\eta} \tag{18}$$

Rewriting formula (13):

$$4bk_m = \frac{\rho g b^3}{3\eta} \tag{19}$$

shows very easily that the following expression is valid for the ratio between thickness of fluid film and width of the interspace:

$$\frac{s}{b} = \sqrt[3]{\frac{N_m}{4k_m}} \tag{20}$$

Substitution of (20) into (16) will give immediately:

$$\mu_m = 1 - \sqrt[3]{\frac{N_m}{4\ k_m}} \tag{21}$$

$$\mu_m = 1 - \sqrt[3]{\frac{1}{4}\frac{N}{k}\frac{\{h\}^2}{\{x\}^2}} \tag{22}$$

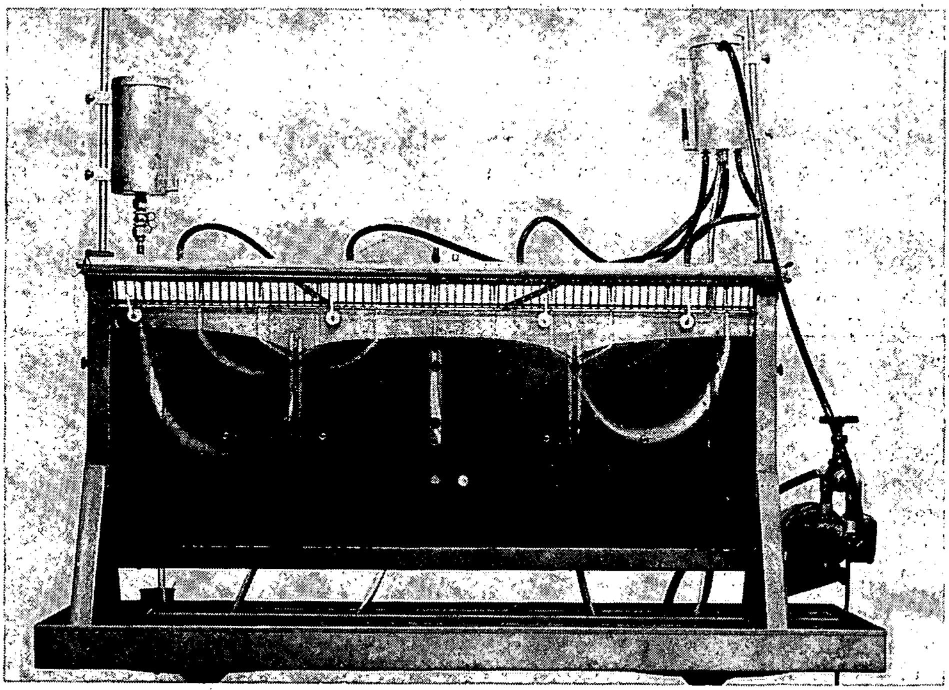
In practice N will be small as compared with k. The value of $\{h\}^2\ \{x\}^{-2}$ generally will be somewhere between 1 and 1000 and will be increasing while N/k will decrease. These facts together with the exponent 1/3 in formula (22) result in a value of μm usually between 0.5 and 0.9.

3. DESCRIPTION OF THE MODEL AND PREPARATION FOR ITS USE

The model (fig. 1) consists of two perspex plates each $\frac{1}{2}''$ thick. The plates are kept apart by 1.5 mm thick brass straps alongside and at the bottom of the model. An oil-proof gasket placed against these straps in the interspace ensures tight closing. When the model is filled, the pressure on the plates will be so great that they bulge strongly, unless countermeasures are taken.
By means of 4 small bolts with calibrated rings (diam. 6 mm, thickness 1.5 mm) at regular distances on a horizontal line halfway across the plates, this bulging out can be eliminated for the greater part. At the top of the plates, the distance is adjusted by means of screw-clamps, placed across the model.
The U-profile frame holding the parallel plates is provided with 4 legs so that the model may be mounted on a table without any further support. The frame also permits fastening a pump, sprinkler tubes and overflow tanks.

3.1. SUPPLY OF THE FLUID TO THE MODEL

The sprinkler device (fig. 1) consists of a wide, horizontally placed tube with 4 inflow junctions and, at the topside, a ventplug. At the lower side of this tube a large number of small pipes have been mounted in order to obtain a possibly equal distribution of the fluid entering the model at the upper side. These pipes have small diameters, so that the total resistance which the fluid will meet on its way from the tank to the model is mainly concentrated in these pipes. Therefore the quantity of fluid flowing through each of these pipes is mainly dependent on the pressure-height in the feeding tank and not on the length of the supply tubes.
Due to the narrow flow-opening (over a distance of 13.5 mm the bore is 1 mm ⌀) blockage is easily possible. For this reason, the pipes are connected to the supply pipe by screwthread so that they can be removed for cleaning.
At the top the plates have been bevelled so as to ensure an even oil film to flow along one of the two perspex plates. The sprinkler pipe has to be placed in such a way that the outlet pipes rest on the sloping plate side, leaving the openings completely free. This promotes an even distribution of fluid on the plate and prevents formation of drops. Drops have the disadvantage that they cause the supply to be irregularly distributed, giving an undulating phreatic surface. If in spite of this precaution the fluid is still not evenly distributed over the whole length of the model a thin steelwire may be stretched on the sloping side of the plate. The fluid is retained a little by the thread, and then flows in a layer of even thickness across the wire towards the interspace.
The feeding-tank is filled by means of a centrifugal pump suitable for heavy liquids. The pump capacity is established by means of an adjustable stop valve in the return pipe of the pump. The fluid coming out of the model is transported to a collector-tank. From there it is pumped back into the feeding tank again; an overflow pipe ensures a constant level in this latter tank.
At some places a coloured fluid can be brought into the model to make the flowlines visible. This takes place via a second sprinkler pipe, provided with a few discharge



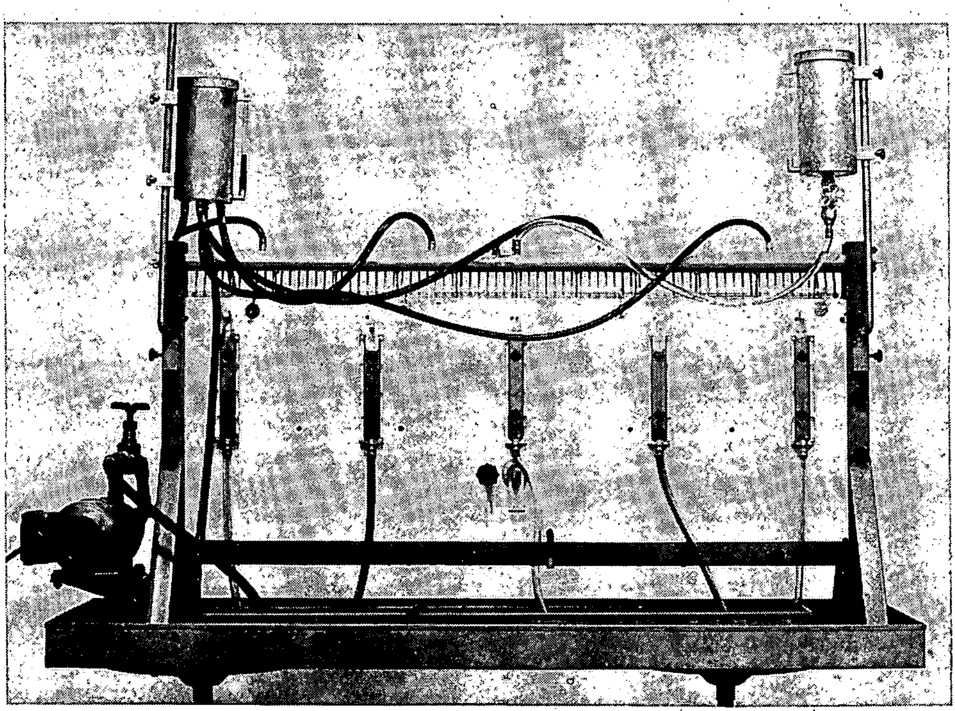


Fig. 1. Front and backside of the model

pipes, and connected with a separate tank. Only a little coloured fluid has to be used because in pumping round this fluid will get mixed with the uncoloured one. Nevertheless it will be necessary to replace the model fluid from time to time since decolorizing will mostly not be possible. The reservoir for the coloured fluid can be filled easily by hand instead of by pump considering the small quantities needed.

3.2. DISCHARGE OF THE FLUID FROM THE MODEL

Various possibilities for discharge of the supplied fluid have been built in the model with the purpose of demonstrating the influence of drain-distance, draindepth and drain diameter. At 10 cm from the topside five discharge openings have been made at distances of 25 cm; the same has been done at 20 cm from the top. By means of slides both sets may be optionally opened or closed.

At the back of each slide, there is a small reservoir open on the top in which the fluid is collected. Plastic tubes have to conduct the fluid from these tanks to the collector tank placed under the model. A direct connection of the discharge pipe to the outflow opening would entail the risk of air bubbles being sucked into the pipe, which would stagnate the discharge to an important degree.

If the two rows of discharge openings are closed then the discharge can take place via a centrally situated opening at 10 cm above the bottom. With the aid of a revolving disc, one can vary the size of this out-flow opening.

To empty the model, there is another outlet discharge at the lowest point. This outlet can be closed off with a cork.

3.3. PREPARATION FOR THE OPERATION OF THE MODEL

In principle the fluid used in the model has to comply in the first place with the requirement, that with the used interspace only laminar flow takes place. As a result there is a wide choice of model fluids, such as: glycerine, vegetable or mineral oil, sugar syrup, etc. With non-stationary experiments it is of importance to obtain such a time scale that the experiments proceed not too slowly and on the other hand not so fast that observations will become difficult. In order to achieve this with the chosen interspace of $1\frac{1}{2}$ mm, it is desirable to choose a fluid with a viscosity of about 20 to 40 centiPoise at room temperature (about 3 to 5° Engler; for motor oil SAE 10W).

Another requirement with which the fluid must comply is that the adhesion between the fluid and the model has to be strong enough to obtain even distribution of the fluid over the whole length of the model. It was found from various experiments that the most suitable model fluid is pure mineral oil. Oil that has been doped should be avoided because dust or oxides may give a formation of films which could very easily lead to blockage of the model. An oil applied successfully for a viscous fluid model with perspex plates and an interspace width of $1\frac{1}{2}$ mm, is Shell Ondina oil 17 (4° Engler), an odourless and tasteless oil, which has not been doped and which is also being used in pharmaceutical industries.

In the separate construction design (APPENDIX) dotted lines schematically indicate how the various connections with hoses should be brought about. The supply-tank

is connected with 4 tubes of equal length to the sprinkling pipe. The supply connection is coupled with the pressure section of the pump whereas the overflow has an outlet via a hose in the collecting tank.
This collecting tank is coupled directly or via a supply tank with the suction part of the pump. Hoses are fastened to the hose columns of the five discharge tanks of the model; they debouch into the central collecting tank. The second feeding tank which serves for the addition of dyed fluid, is connected with the appropriate sprinkling pipe. When all these connections are properly fixed the supply tank or the collecting tank is filled with fluid. In preparing for the first operation the pump must also be filled. The motor can now be started. By means of the tap on the return pipe the quantity of oil is regulated in such a way that the overflow pipe of the feeding tank discharges only a small quantity of fluid. The supply should never be regulated through pinching off hoses, because this may result in a loosening of these due to the increased pressure. The ventilation taps on the sprinkling pipe are opened and should not be closed until all air bubbles have been removed from the hoses and until oil is flowing out of these taps. All discharge openings are then closed until the model is full to the brim of the parallel plates. This is necessary in order to obtain good flowing of the oil up to the top of the plates. Now the outlets representing the drains can be opened according to the situation which has to be demonstrated. An adequate oil supply can be realized by setting the feeding tank at a corresponding level.
Care must be taken to keep the model clean, because of the small dimensions of the flow-outlets of the sprinkling pipes. When the model is not in use, it may be recommended to cover it with a plastic sheet as dust and oil remains may form deposits which are hard to remove. If dirt has nevertheless settled in the model it is necessary to clean it thoroughly with petrol. Even when using so-called 'oil-proof' plastic hoses, these turn hard through long contact with oil; they must then be replaced.

4. DRAINAGE FORMULAS

The influence of drain distance, drain depth and drain diameter on the groundwater level may be demonstrated by some experiments with the model. The known drainage formulas for stationary flow can be compared with the results of the model.

HOOGHOUDT, among others, developed formulas which afford the calculation of drain distances in relation to the precipitation surplus, the permeability of the soil, the depth of the drain and the impermeable layer. The drain depth is usually determined by the prevailing soil condition and agricultural situation. The mean drainage intensity depends on climatological circumstances.

The allowed maximum elevation of the phreatic surface is determined by the requirements set in connection with crop growing, soil conditions and tillage. The hydrologic conditions corresponding to these requirements make it possible to start with a determination of the drain distance and depth. Prior to this, the permeability of the soil and the depth of a possible impermeable layer, has to be determined by measurements in the field (VAN BEERS, 1958; WESSELING, 1957).

Summarizing, we find that the height difference between the level in the drains h_o and the level midway between the drains h_m is dependent on the permeability factor k – in a layered soil several k values may be of importance –; furthermore, on the depth D of the impermeable layer or on the thickness of the various permeable layers, on the drain distance L and the diameter of the drain tile $2r_o$ or – for open drains or good permeable trench fillings – on the wet perimeter of the drains.

This relation can, in general sense, be represented as follows:

$$\frac{h_m - h_o}{L} = f\left\{\frac{k_1}{q}, \frac{k_2}{q}, \ldots \frac{D_1}{L}, \frac{D_2}{L}, \ldots \frac{r_o}{L}\right\} \tag{23}$$

However, the above equation is unsuitable for practical application because the value of the function (f) is not defined.

For practical use, however, several formulas have been derived, partly with the aid of the Dupuit-Forchheimer theory (horizontal flow). By introduction of a special factor HOOGHOUDT (1940) made this formula also suitable for cases with radial flow.

With respect to the depth of the impermeable layer three cases of homegeneous soil may be mentioned here for which different formulae are available.

a. *The soil is permeable to infinite depth*

This theoretical supposition will give a sufficient approximation in practice as soon as the requirement $D/L > 0.25$ is satisfied, where D is the thickness of the permeable layer and L the drain distance (fig. 2 section 5).

The relation between discharge and the height of the groundwater level midway between the drains is found by the formula:

$$q = \frac{8kdm}{L^2} \tag{24}$$

where:

q = discharge in m/day

k = hydraulic conductivity of the soil in m/day

m = height of the groundwater level midway between the drains measured from the drain level in m

L = distance between the drains in m

d = factor representing the thickness of an equivalent layer in meters. This factor, d, is dependent on the drain distance and the wet perimeter of the drain, and is given by HOOGHOUDT in tables (see also VAN BEERS, 1965).

b. *An impermeable layer at a certain depth*

If the impermeable layer is at such a depth below the level of the drains that it may not be considered to be infinite i.e. $D/L < 0.25$, the formula reads as follows:

$$q = \frac{kdm + 4km^2}{L^2} \tag{25}$$

In this formula, the magnitude d is dependent on the depth of the impermeable layer, the drain distance and the wet perimeter of the drain. The values of d may be found from the tables given by HOOGHOUDT (see a).

c. *Drains are lying on an impermeable layer*

If the drains are placed on the impermeable layer above, the discharge formula reads:

$$q = \frac{4km^2}{L^2} \tag{26}$$

In practice it often occurs that the soil consists of layers of varying permeability or that the soil around the drains has a strongly deviating permeability. Also for these cases a number of adequate formulas have been developed; however, we shall not deal with those in the present paper. Reference may be made among others to ERNST (1962), VAN BEERS (1965) and ROTHE (1924).

5. MODEL EXPERIMENTS

In the model the influence of the various variables from formula (23) may be demonstrated.

a. *A specific drainage situation* (D/L > 0.25)

If the upper five discharge outlets are in open position and the quantity of precipitation is set at a certain value, after a short time an equilibrium will be reached which represents a specific combination of the factors L, h_o, D, r_o, q, k (see fig. 2). Departing from this situation one or more magnitudes can be changed either one by one or simultaneously.

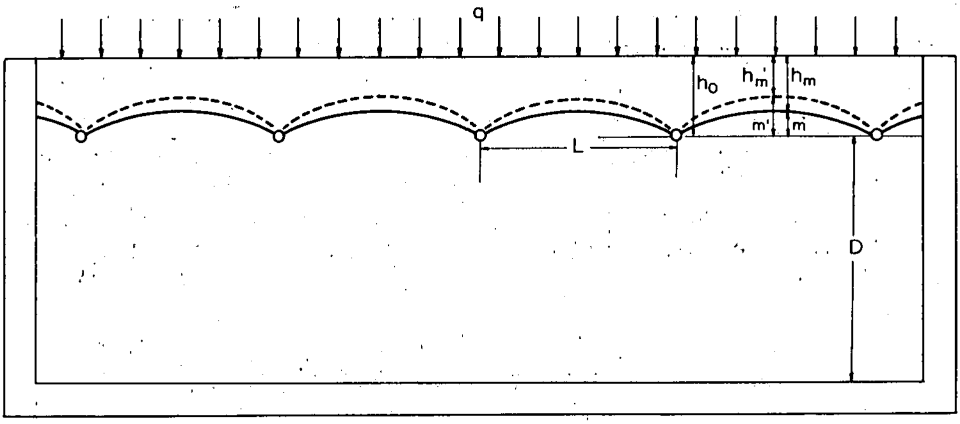


Fig. 2. Effect of increased rainfall intensity

b. *Variation in precipitation intensity*

An enlargement of the quantity of precipitation to be discharged will cause a raise of the groundwater level midway between the drains. In spite of the slight increase of the wet perimeter of the drain the radial resistance will not notably decrease. As a final result, when comparing with situation *a*, a bulging up of each part of the curved phreatic surface occurs (fig. 2).

c. *Variation in drain depth*

By opening the lower row of discharge outlets the drain depth is changed and at the same time the relative depth of the impermeable layer. As D/L > 0.25 also applies for this row if all discharge outlets are used, the bulging between two drains (the height m) will be the same as in situation a. In this case, the depth of the (curved) phreatic level h_m is increased to the same amount as the draindepth (fig. 3).
If the depth to the impermeable layer is decreased even further, by making use of the lowest discharge outlet, the change in the ratio D/L will be of influence.
The ratio D/L is now < 0.25, which means that the factor d from the formula by Hooghoudt decreases also. The phreatic surface then has another form and bulge (fig. 4). Compare also formulas (24) and (25).

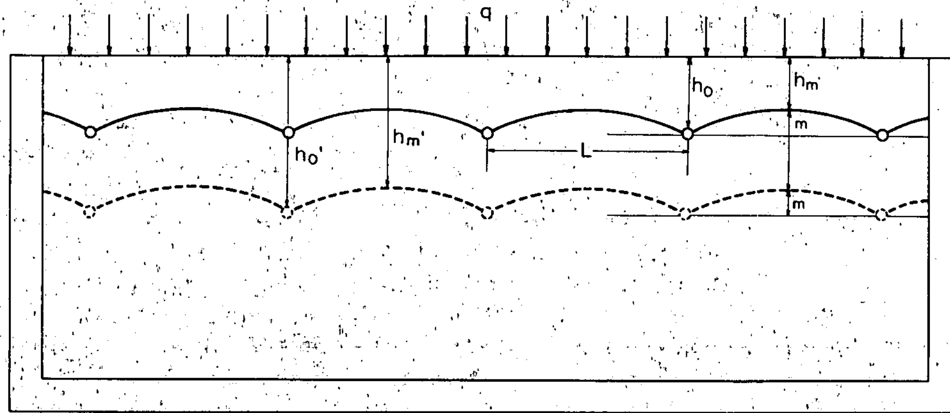


Fig. 3. Effect of deeper drainage system when D/L > 0.25

d. *Variation in drain spacing*

When two or more openings are closed in such a way that the symmetry in the model is maintained, the drainage formulas like (24) and (25) remain valid, but for the drain distance L another value has to be substituted. With increasing drain distance the hydraulic head m increases, corresponding to a decrease of the depth of the phreatic surface (see fig. 5).

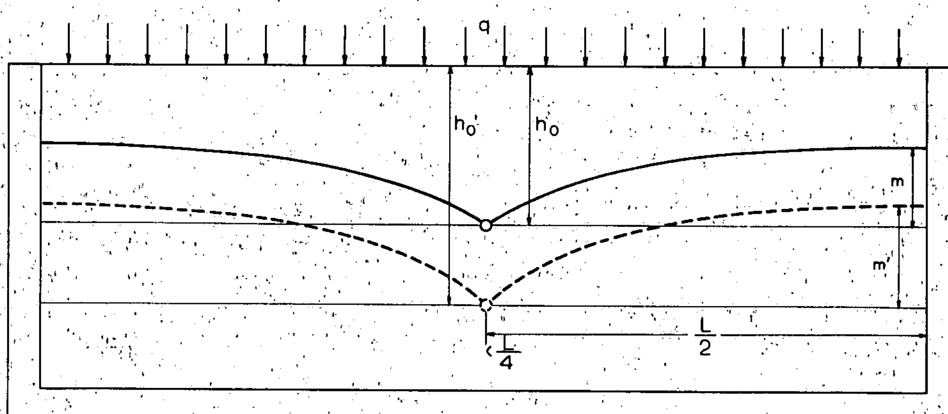


Fig. 4. Effect of deeper drainage system when D/L < 0.25

e. *Variation in drain diameter*

If both upper rows of discharge outlets are closed, discharge takes place via the only central discharge outlet at the lower level. As mentioned already under c, the relation D/L changes in such a way that the situation corresponds rather to formula (25).

By varying the radius of the outflow opening by means of the disc built in for this purpose, the influence of the wet perimeter of drains (tile drains, ditches, etc.) can be shown. A larger drain-diameter will give a marked descent of the fluid surface and also a slight change in shape near the discharge outlet (fig. 6).

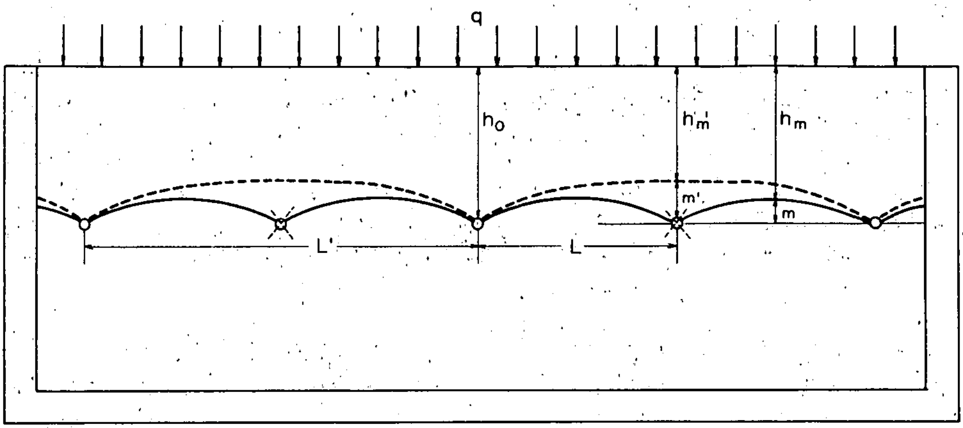


Fig. 5. Effect of a larger spacing (L′ = 2L)

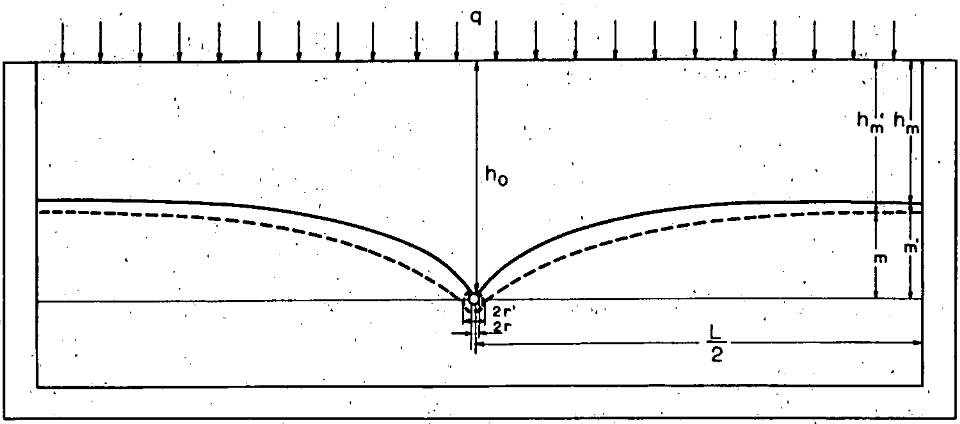


Fig. 6. Effect of increase in wet perimeter of the drain (larger ditch or larger diameter of a tile drain)

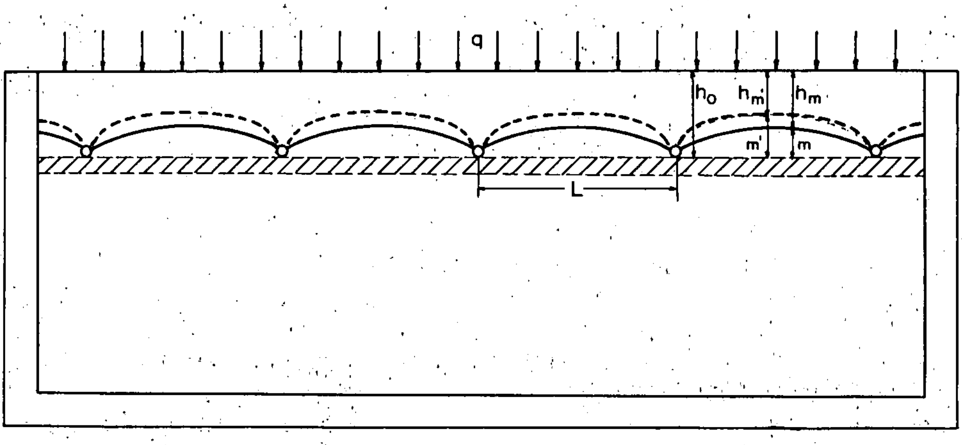


Fig. 7. Effect of an impermeable layer under the drains

f. *Variations in the thickness of the permeable layer*

A very shallow impermeable layer may be obtained by inserting a strip just below the upper row of outflow openings. For this purpose the terminal screws in the frame must first be loosened.

An impermeable layer directly under the drains causes a strong bulging up of each curved level between two drains; compared to situation a, the hydraulic head m must be much larger as to maintain the same discharge q. Also the shape of this curved level is changed from a parabolic form to an elliptic form (fig. 7). Compare formulas (24) and (26)!

g. *Variation in stratification*

By attaching a strip to one of the plates, with a thickness of about $^1/_2$ to $^3/_4$ of the interspace width, a smaller permeability is simulated than in the rest of the model. It should be noted that the resulting smaller width can only be kept within the required tolerancies by very accurate work. By bringing in a strip of wire-netting of 1.5 mm thickness and the desired breadth, and a suitable mesh width, one may simulate the influence of stratified layers in the soil profile. This stratification causes a small difference in bulging and shape of the phreatic surface (fig. 8). The shape of the streamlines also changes.

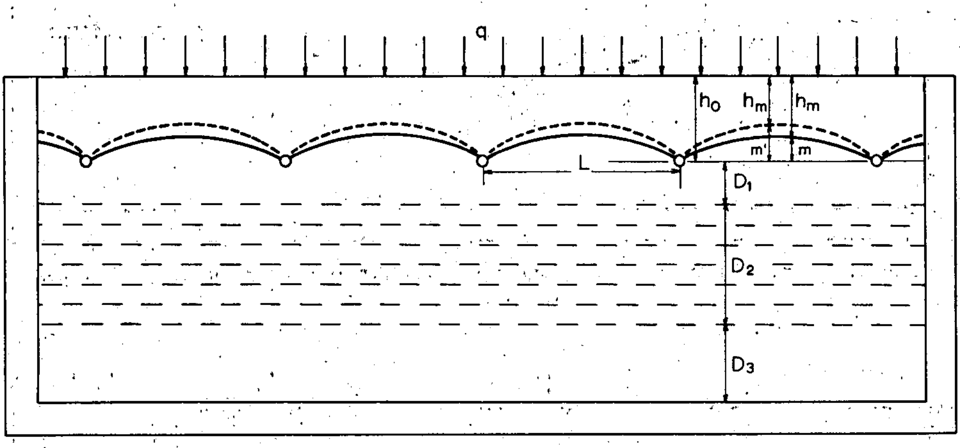


Fig. 8. Effect of stratified layers in the soil profile under the drains

h. *Variation in open water level above the level of tile drains*

Through partial pinching off the discharge hoses of the collecting tank or by strongly enlarging the supply, it is possible to obtain a situation whereby the fluid surface remains above the drains, which corresponds to the situation when tile drains are discharging under submerged condition.

Apart from a smaller groundwater depth, this causes a flatter shape of the phreatic surface (fig. 9).

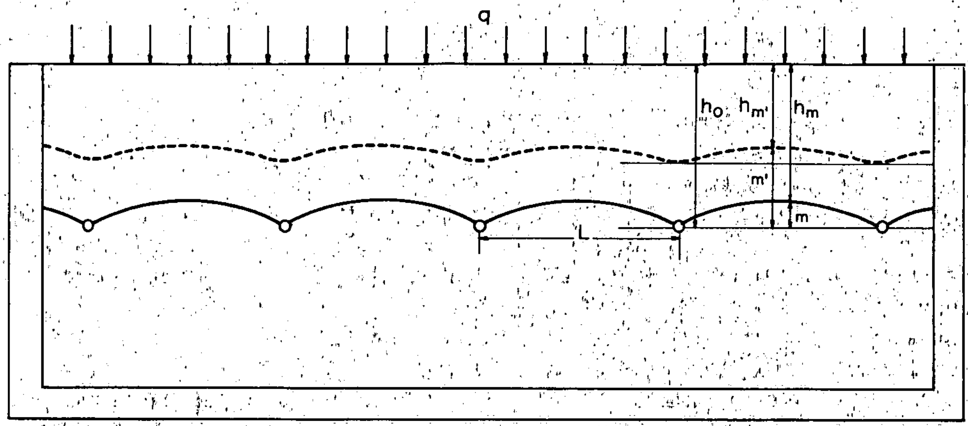


Fig. 9. Phreatic level when the tile drain outlets are submerged

i. *Variation in permeability*

If a change in the permeability of the whole model is to be simulated, this can be affected by choosing an oil of another viscosity. The scale for the permeability k is inversely proportional to the viscosity of the model fluid.

6. CONSTRUCTION OF THE MODEL

The construction drawing[1]) of the model is annexed in the back cover.
The viscous fluid model consists of two perspex plates (15) and (16) (1219.2 × 450 × 12 mm) kept separate by 1.5 mm thick brass strips (46) and (48).
An 'oil proof' plastic cord closes the model along the sides and the bottom. The model is fixed into a metal U-profile frame (42) (40 × 40 × 40 × 3 mm) with bolts pressing against the protecting strip (44). The supports (39) with ground plates (40) make it possible to place the model on a table.
Sockets are fastened to the frame (45). In these sockets the supports for suspending the tanks (49) and (60) for the oil supply are placed.
The upper side of the frame has on both sides a slot for taking up the supply pipe (32). The same is done in the wings (41) welded to the frame on behalf of the other supply pipe.
These brass pipes (28/26 mm) are closed off at the ends by a soldered attachment (29) fitting in the appropriate slot and fastened with M10 winged nuts (71).
At the bottom of the brass pipes there are strips (33) with holes, in which M4 screw-thread has been tapped for fastening the pipes (35) and (36). After these have been screwed in to the right depth, they are fixed with a lock-nut. On the upperside, there are hole columns (37) and a drainplug (39) for venting the air from the supply tubes.
Both brass tanks (200 × 100 mm ⌀) are provided with clamps (56) making it possible to attach them to appropriate supports (17). The feeding tank is provided with four

[1]) A complete construction drawing has been made by the "Technische en Physische Dienst voor de Landbouw", (Service Institute for Applied Mechanics and Technical Physics in Agriculture), 12 Mansholtlaan, Wageningen, the Netherlands. This construction drawing with details on a larger scale as well as the list of parts and materials may be obtained from this service upon request.

hose columns for connection to the sprinkling pipe. An overflow pipe (58) serves for maintaining a constant level whereas a gauge glass (57) makes it possible to check this. The feeding of the tank takes place via a downward bended pipe (52). The tank (60) for the dyed fluid is also provided with a gauge glass (57) but only with one tap with hose column. As the quantity of dyed fluid supplied to the model is small in relation to the un-dyed fluid a constant level is therefore less necessary.

Behind the two rows of outlets, collecting tanks (4) have been built in to make free outflow possible. In doing so, the risk of stagnations in the discharge pipes by confined air, is largely prevented. In these tanks, valves (1) are made, which can be placed in three positions with handles (2) Each of three positions can be maintained with the aid of small steel balls at the back of which compression springs have been placed. Packing rings around the outlet openings close off the space between the model and the valve. The collecting tanks have been screwed against the model after insertion of packing. At the bottom of the tanks, hose columns serve for fastening hoses to the collecting tank (21).
A central discharge opening of 20 mm ⌀ may be closed off with a perspex disc (22). With the aid of a knob (27) with compression spring (28) this disc can revolve so that an optional choice can be made for an opening of 3, 5, 10 or 20 mm ⌀ for the discharge pipe (18). Here again, a rubber ring around the central opening ensures a good closure. When using the smaller outlets (3 and 5 mm ⌀) of this disc and with a

relatively large discharge, it may be desirable to lengthen the discharge pipe with a wide, completely fluid filled hose. This causes greater suction and a lower fluid height above the drain outlet.
Close to the bottom of the model, a discharge pipe (19) is installed for emptying it.
When using mineral oil as model liquid, one should use a quality of rubber or plastic qualified in commerce as being 'oil proof'. The plastic hoses used must be of the same quality as the closing material because otherwise the hoses will turn hard very quickly and consequently become unfit for further use.
As a result of the hardening of hoses, there is a real chance that they will become loose. Hose columns can easily break when made from plastic. As perspex has a very great temperature sensitiveness, and expansion of the plates is not equal to that of the metal frame, the model should not be exposed to excessive temperature fluctuations. This could result in leakage through warping or deviation in the plate separation, the latter being difficult to correct.

SUMMARY

Notwithstanding large progress in the investigation of groundwater flow by mathematical analysis, the use of analog models – especially sand models, parallel plate models and electrical models – has maintained a valuable place in research methods. As regards the viscous fluid models, it may be mentioned that this model is very suitable for demonstration purposes and has still other advantages in comparison with other models (Section 1). After a description of the principles and the construction, a summing up has been given of the possibilities for demonstration of general features of groundwater flow to parallel drains. By simple tests of short duration the influence can be shown of precipitation, drain distance, drain depth, drain diameter and thickness of the watercarrying layer on drainage (Section 5).
Items deserving special attention, when bringing the model into operation are discussed (Section 3).
Comparison of field data and results of the model tests are conveniently done by making use of the various model scales (Section 2).
In Section 4 some well known drainage formulas are discussed for shallow (ROTHE) and deep impermeable layers (HOOGHOUDT). In Section 5 it is described how the various magnitudes incorporated in Hooghoudt's formulas may be varied in the model. During model tests the surface of the fluid will very quickly become adjusted to the new boundary conditions, showing a complete similitude to what can be expected under real field conditions (figs. 2 to 9).

REFERENCES

BEAR, J. 1960. Scales of viscous analogy models for groundwater studies. Journ. of the Hydr. Div. Proc. A.S.C.E. Vol. 86, no HY 2.

BEERS, W. F. J. VAN, 1958. The auger hole method. Bull. 1. Int. Inst. for Land Recl. and Improvement.

BEERS, W. F. J. VAN, 1965. Some nomographs for the calculation of drain spacings. Bull. 8. ILRI.

DIETZ, D. N., 1941. Een modelproef ter bestudering van niet-stationaire bewegingen van het grondwater. Verslagen en Mededelingen betreffende de Volksgezondheid (Aug./Sept.).

ERNST, L. F., 1962. Grondwaterstromingen in de verzadigde zône en hun berekeningen bij aanwezigheid van horizontale evenwijdige open leidingen. Verslagen van Landbouwk. Onderz., no. 67.1 5.

FELIUS, G. P., 1954. Recherches hydrologiques par des modèles électriques. Ass. Int. Hydr. Sci. Gen. de Rome. II, pp. 162–169.

GROVER, B. L. and KIRKHAM, D., 1961. A glassbead glycerol model for non-steady state tile drainage. Soil Sci. Soc. of Amer. Proc. Vol. 25, no. 2.

GROVER, B. L. and KIRKHAM, D., 1964. Solving tile drainage problems by using model data. Research Bull. 523. Agric. and Home Economics Experiment Station. Iowa State University.

HOOGHOUDT, S. B., 1940. Bijdrage tot de kennis van enige natuurkundige grootheden van de grond. Deel 7. Verslagen van Landbouwkundige Onderzoekingen, no. 46 (14): B515–704.

KUIPERS, H., 1955. Een warmtemodel voor grondwaterstromingen. Versl. en Med. Comm. Hydr. Onderz. TNO, no. 2.

ROTHE, J., 1924. Die Strangentfernung bei Dränungen. Landw. Jahrb. vol. 59, pp. 453–490.

ROUSE, H., 1961. Fluid mechanics for hydraulic engineers. Dover Public. Inc., New York.

SANTING, G., 1963. Modelonderzoek. Versl. en Mededelingen Comm. Hydr. Onderzoek TNO No. 8.

TODD, D. K., 1959. Groundwaterhydrology. Wiley and Sons, New York.

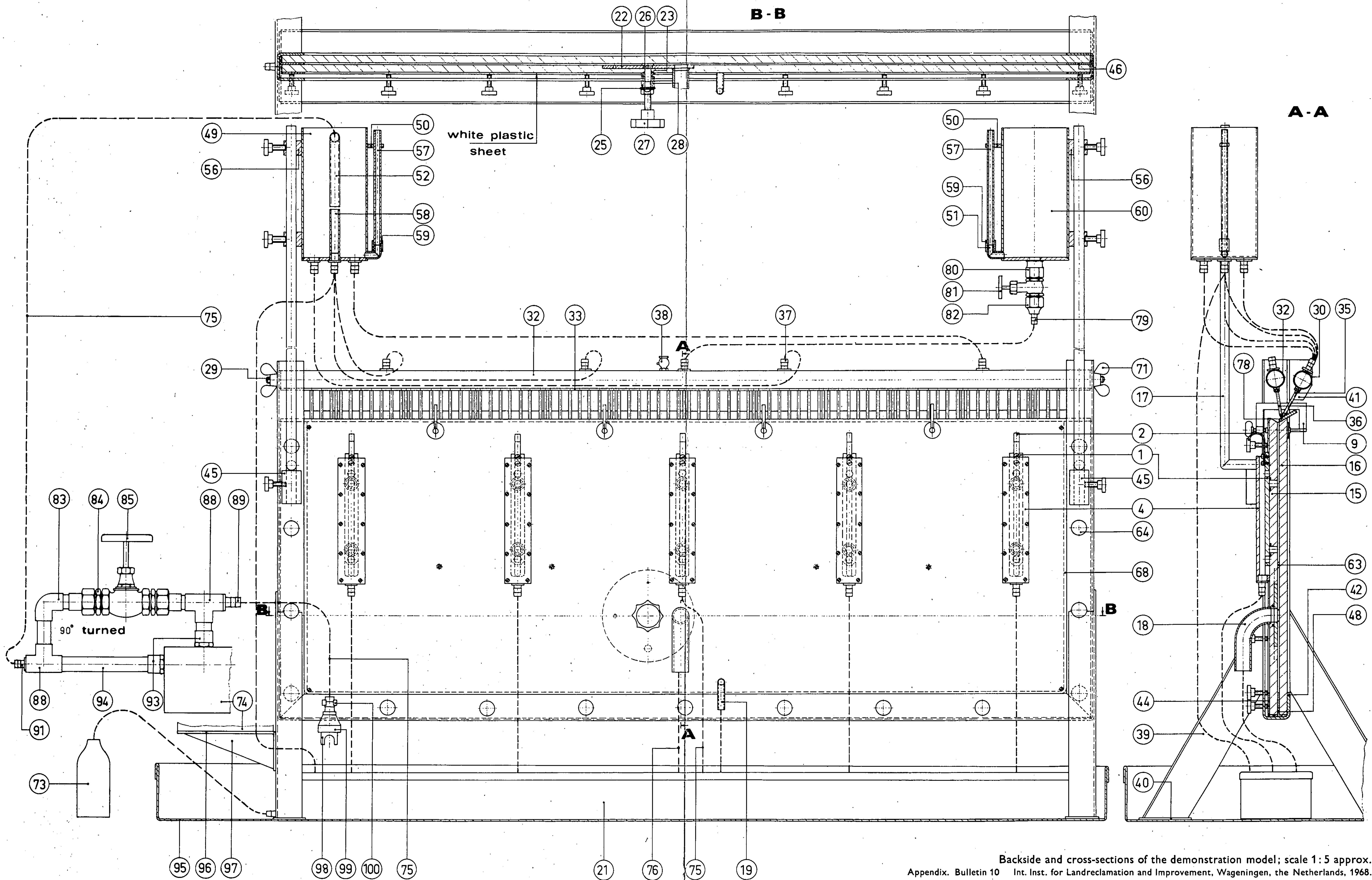
TODD, D. K. and BEAR, J., 1961. Seepage through layered anisotropic porous media. Journ. of the Hydr. Div. Proc. A.S.C.E. Vol. No. 87, HY 3.

WALTON, W. C. and PRICKETT, T. A., 1963. Hydrogeologic electric computer. Journal of the Hydraulics Div. Proc. A.S.C.E. Vol. 89.

WESSELING, J., 1957. Enige aspecten van de waterbeheersing in landbouwgronden. Versl. Landbouwk. Onderz. no. 63 (5).

ZEE, C. H., PETERSON, D. F. and BOCK, R. O., 1955. Flow into a well by electric and membrane analogy. Proc. A.S.C.E., vol. 81, Sept. 817.

ZELLER, J., 1954. Comparison de l'efficacité des drains et des tranchées à l'aide d'essais sur modèles. Ass. Int. Hydr. Sci. Gen. de Rome, II, pp. 478–493.



Backside and cross-sections of the demonstration model; scale 1 : 5 approx.