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Agreeing on public goods or bads[☆]

Erik Ansink [a], Hans-Peter Weikard [b,*]

[a] *School of Business and Economics, Vrije Universiteit Amsterdam and Tinbergen Institute, Netherlands*
[b] *Department of Social Sciences, Economics Section, Wageningen University, Netherlands*

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ABSTRACT

Without regulation or agreement, public goods are underprovided and public bads are overprovided. Both problems are usually seen as flip sides of the same coin. In this paper we examine a situation where a public good is good for some agents but bad for others, depending on the provisioning level of the good. We allow agents to form a coalition to coordinate this provision. Our results show that, compared to games with only goods (or only bads), larger coalitions form in equilibrium. For a game specification with quadratic benefit- and cost functions, we find the grand coalition to be stable except when agents have identical or almost identical characteristics. The primary driver of coalition stability is the avoidance of a wasteful contest between agents pulling the provision level in opposing directions. In equilibrium, such wasteful contests are confined to a narrow range within the parameter space. This result connects the literatures on public goods and contests.

1. Introduction

A century after Lindahl (1919) published his groundbreaking book, the private provision of public goods remains a vibrant research field. In this paper we examine the private provision of a public good that is appreciated by some but disliked by others. We do so using a game in which agents can join a binding agreement to coordinate public good provisioning. Our model set-up follows a strand of literature that has emerged under the header of *international environmental agreements* and deals with international cooperation for the provision of global public goods such as mitigating climate change (Carraro and Siniscalco, 1993; Barrett, 1994). In a seminal paper analysing public goods that can be good for some while bad for others, Weitzman (2015) refers to such goods as '*gobs*' (goods or bads). We adopt Weitzman's terminology and provide a gobs model with three key features: (i) individual assessments of a gob as good or bad depend on the level of overall provision, (ii) contributions can be positive or negative, and (iii) agents can form a coalition to coordinate public gobs provision.

Our main motivating example are conservation efforts. To build intuition, consider the specific case of the return of wolves to some European countries such as Germany or the Netherlands. Conservationists appreciate it, but sheep farmers oppose it. The former, by protecting habitats, make costly efforts to help increase the wolf population. The latter, to the contrary, make costly efforts to keep wolves out, seek permission to hunt them or hunt them illegally. A round table where conservationists and farmers get together to negotiate a collectively preferred level of conservation could save efforts on both sides. Our paper explores the prospects of reaching agreement (or forming a stable coalition) in such settings. Generally, a coalition would internalise the externalities

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* Corresponding author.
E-mail address: hans-peter.weikard@wur.nl (H.-P. Weikard).

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associated with the provision of the public gob and avoid wasteful contest within the coalition. Our interest is to see whether such coalitions can be stable and would help to restrain wasteful contest within the coalition, but also between the coalition and agents that have not joined.

Besides conservation, many other examples of a similar structure can be found in the domains of environmental and public policy-making where attitudes with respect to local conditions, or preferences with respect to environmental standards differ. Weitzman (2015), for example, introduced gobs to discuss the potential role of geoengineering in climate policy-making. Geoengineering measures, like e.g., stratospheric aerosol injection, aim at global cooling. Such measures can be adopted by individual countries at relatively low cost but may be undesirable for other countries (Rickels et al., 2020). Hence, global cooling is perceived as a public good by some while it is perceived as a public bad by others who may have possibilities to make antagonistic efforts (Heyen et al., 2019).

The above examples reflect settings where players make contributions to the public gob privately. Our model also fits settings where players lobby for a publicly provided gob. For example, tax-financed public goods may be seen as overprovided by taxpayers facing a high tax rate or having a low preference for the public good. Conversely, taxpayers with low tax rates or strong preferences for the public good would see them as underprovided. Hence, increased provision would be a public good for the latter group and a public bad for the former. These opposing perceptions will often lead to opposing lobby activities and to rent-seeking contests where the prize is a public good with the associated tax burden (Katz et al., 1990). Similarly, legal standards that are too strict for some but too loose for others can be interpreted as gobs whenever agents on both sides make efforts to change them in their preferred direction.

In our modelling approach, to obtain sharp results, we use a specification where agents have quadratic benefits with respect to the public gob level and quadratic costs with respect to their own contribution (see Heyen and Tavoni, 2022, who use a similar specification but do not consider coalition formation). This specification allows us to derive a closed-form solution for coalition stability. It is also one of the few specifications for which analytical results are available in the literature that considers only public goods (Carraro and Siniscalco, 1993; Barrett, 1994). This allows us to compare public gobs with public goods. Moreover, it is the simplest specification that features non-orthogonal response functions (so that we do not presuppose dominant strategies in contributions) while remaining tractable in a setting with heterogeneous agents.

Our results suggest good prospects for cooperation on public gobs. We find larger coalitions compared to public goods games where everyone appreciates the good. Specifically, in Proposition 1 we find the grand coalition to be stable with the exception of situations where agents have identical or almost identical characteristics. The intuition for this result is that the formation of a coalition of heterogeneous agents creates opportunities for cost sharing. A coalition can obtain 'gains from trade' when members who, as singletons, would have little incentive to contribute would align their effort with other members. They will do so under an appropriate transfer rule. In addition, the possibility of negative contributions gives rise to increased incentives to cooperate. In our second result that we present in Proposition 2 we show that coalition formation avoids to a large extent the wasteful contest between agents pulling in opposing directions. The intuition here is that a singleton agent who makes efforts opposing the coalition's effort could be integrated into the coalition which increases overall payoffs (since wasteful effort is avoided) and stabilises the enlarged coalition.

2. Contribution to the literature

In this paper we connect four strands of literature: (i) The private provision of public goods, (ii) coalition formation for public goods provision, (iii) the provision of public gobs, and (iv) contests.

The analysis of the private provision of public goods is generally framed as a game between agents who derive utility from a public good and a composite numeraire good. Agents are budget constrained and the public good is available at constant prices. Warr (1983) and Bergstrom et al. (1986) show that the unique Nash-equilibrium provision level is not affected by a redistribution of income as long as the set of contributing agents is not affected. In other words, income redistribution is generally not conducive to improving public goods provision. More recent literature that has addressed private provision of public goods has been going in two main directions. The first broadens the assumptions on agents' preferences by considering, e.g., altruism (Bagnoli and Lipman, 1992) or social norms (Rege, 2004). The second explores the options to change incentives by changing the structure of the game, e.g., by introducing leadership (Buchholz et al., 1997) or specific mechanisms such as refund systems (Zubrickas, 2014; Kornek and Edenhofer, 2020; McEvoy and McGinty, 2023). Within this literature, we are not aware of studies that consider gobs, with the exception of Buchholz et al. (2018) and Heyen and Tavoni (2022), discussed below.

A second strand of literature is motivated by the general underprovision of transboundary or global public goods. This literature studies the stability of coalitions between countries that are characterised by their costs of provision and their benefits derived from the public good. A key concern of this literature is to spell out determinants of the size of stable coalitions and their effectiveness in terms of the provision of public goods. Coalitions are formed to overcome the inefficiently low provision level in a Nash equilibrium of a game with potentially many players. A coalition is stable when no member has an incentive to leave and no non-member has an incentive to join. In general, larger coalitions would provide more of the public good and a grand coalition would provide the efficient amount. In this literature, a two-stage game is a workhorse model. In stage 1, countries announce whether or not they join the coalition. In stage 2, the coalition members coordinate public good provisioning to maximise their joint net benefits in a game with non-members (Carraro and Siniscalco, 1993; Barrett, 1994). We contribute to this literature (see Benchekroun and Long (2012) and Hagen et al. (2020) for surveys) by generalising the domain of agents' preferences from public goods to public gobs and by generalising their strategy space from only positive contributions to positive and negative contributions.

Third, we contribute to a small recent literature that addresses the provision of public gobs. Theoretical work in this domain by Buchholz et al. (2018) extends the private provision of public goods literature, considering utility-maximising agents who face a given price of the public gob and thus constant marginal costs of provision. Their model considers two groups of agents; for one group more of the gob is always preferred, while the other always prefers a lower level of provision. Our model differs from this approach in two ways. First, we assume convex costs of provision and, second, our agents are not exogenously grouped into beneficiaries and victims of public gob provision. In our model, whether an agent prefers to have more or less of the gob is endogenous and depends on the level of provision. Similar to us, Heyen and Tavoni (2022) analyse the private provision of public gobs using a quadratic–quadratic specification. Their analysis differs in that they do not consider coalition formation, do not allow negative contributions, and they limit variation in bliss points to mean-preserving spreads. Close to our paper is some recent work on the implications of geoengineering options to combat climate change. In particular Weitzman (2015) considers asymmetric damage from a certain gob level where for any agent having too much may be more (or less) costly than having too little. Weitzman does not address the issue of private provision of the gob but suggests a voting mechanism that would implement an efficient gob level and avoid wasteful contest. Barrett (2008), Ricke et al. (2013), Heyen et al. (2019), Rickels et al. (2020), Ghidoni et al. (2023), Bakalova and Belaia (2023), and McEvoy et al. (2024) also analyze geoengineering, emphasising that measures taken by some countries to stabilise the climate could be opposed by others who differ in their assessment of the benefits and point to the potential dangers of the measures (for a recent review of this literature, see Heyen and Tavoni, 2024). With the exception of Heyen et al. (2019), who considers both geoengingeering and counter-geoengineering measures, this literature only considers positive contributions to the gobs level. Generally, the current geoengineering literature studies either numerically calibrated models or experimental set-ups. We add to this a closed-form solution of coalition formation in the public gobs game.

Finally, our paper connects the literature on public goods provisioning with the literature on contests. In standard contest models, a prize is allocated among agents who can make costly efforts to increase the probability of receiving the prize (Tullock, 1980; Rosen, 1986). Specific contest designs where effort contributes to the value of the prize, but does not affect other contestants' probabilities of winning, resemble the problem of public good provisioning (Konrad, 2009). Such similarities between public goods provisioning and contests have been noticed before (see e.g., Gradstein, 1993; Chung, 1996; Baik, 2016). Furthermore, contest games with alliance formation (Garfinkel, 2004) are close to our game-theoretic setup. When some agents prefer more while others prefer less of the public gob — choosing positive and negative contribution levels, respectively — our game can be interpreted as a contest with offensive vs. defensive activities as in Grossman and Kim (1995). Combining this game feature with alliance (or coalition) formation describes a mechanism to prevent dissipation of the prize, which is one of the main concerns in the literature on contests.

3. The gob model

Consider a set $N = \{1, \dots, n\}$ of agents who have single-peaked preferences with respect to a gob. There is a uniform level of the gob $G \in \mathbb{R}$ to which all agents are exposed. We label as $B_i \in \mathbb{R}$ agent i's preferred level (or bliss level) of the gob. If and as long as $B_i > G$, an increase of G is a public good for agent i. If, however, $B_i < G$, then it is a public bad for i. Let $(B_1, \dots, B_n)$ be the distribution of preferred gob levels and, without loss of generality, we order agents by their bliss levels such that $B_1 \leq \dots \leq B_n$. Each agent can contribute to the public gob. We denote agent i's contribution by $g_i \in \mathbb{R}$. Note that we allow for negative contributions. Positive (negative) contributions will increase (decrease) the public gob level and contributions are assumed to be additive. Let G_0 be the default level of the gob when no agent contributes, i.e., $g_i = 0$ for all i. The gob level obtained through individual contributions is

$$G = G_0 + \sum_{i \in N} g_i. \tag{1}$$

In what follows we normalise the game such that $G_0 = 0$. We denote aggregate contributions of any subset of agents $S \subseteq N$ as $g_S \equiv \sum_{i \in S} g_i$ and we use similar notation for aggregates of benefits, costs, and payoffs, defined below.

Agents derive benefits from the gob level and incur costs from their own (positive or negative) gob contributions:

$$b_i(G) = -\frac{1}{2}G^2 + \beta_i G, \tag{2}$$

$$c_i(g_i) = \frac{1}{2}g_i^2. \tag{3}$$

As discussed in the introduction, both functions are quadratic. Agents are heterogeneous in terms of their exogenous benefit function parameter β_i. Benefits peak at $B_i = \beta_i$ while costs have a unique minimum at $g_i = 0$ when contributing nothing. This model specification in which agents only differ in bliss points allows us to obtain sharp analytical results.

Payoff functions are given by the benefits from the gob level minus the costs of individual contributions:

$$\pi_i(g_i, G) = b_i(G) - c_i(g_i). \tag{4}$$

To build intuition, consider the following two-player example.

Example 1. Consider two agents $i = 1, 2$ with bliss points $B_1 = \beta_1 = -1$ and $B_2 = \beta_2 = 1$. Evaluated at the default gob level where $G_0 = 0$, the agents face similar marginal costs of contributing to the gob while their marginal benefits are diametrically opposed. Any contribution by agent 1 will be negative while any contribution by agent 2 will be positive, in order to pull the gob level in the direction of their respective bliss levels. In the Nash equilibrium the agents make contributions $g_1 = -1$ and $g_2 = 1$. The resulting gob level is $G = 0$ and the associated payoffs are $\pi_1 = \pi_2 = -\frac{1}{2}$. A cooperative agreement in which both agents would reduce their contributions to $g_1 = g_2 = 0$ would yield the same gob level while saving costs. With payoffs $\pi_1 = \pi_2 = 0$, cooperation would be advantageous for both agents.

Example 1 demonstrates that the public gobs game may have features of a contest. Such contest occurs whenever for two players i, j we have $B_i < g_{N\setminus\{i,j\}} < B_j$. In such situations, agent i would make a negative contribution while agent j would make a positive one. A coalition formed by agents i and j would avoid the wasteful efforts associated with the contest.

4. Coalition formation

We consider the formation of a single coalition $S \subseteq N$. Our game is based on the standard two-stage coalition formation game (see Carraro and Siniscalco, 1993; Hagen et al., 2020), often referred to as a cartel game. In the first stage, agents decide whether to join a single coalition $S \subseteq N$. We consider an open membership game, that is, all agents who decide to join will be coalition members $i \in S$ and act jointly in the second stage. Agents who do not join are singleton agents $i \in N \setminus S$. Denote this set of singleton agents by $\bar{S}$. In the second stage, the coalition and the singleton agents play a simultaneous-move game of public gobs provision. Since we assume quadratic cost- and benefit functions, the public gobs game has a unique equilibrium.

This equilibrium gob level when coalition S forms is implicitly given by the system of equations

$$\sum_{j\in S} b'_j(G) = c'_i(g_i), \text{ for all } i \in S; \tag{5a}$$

$$b'_i(G) = c'_i(g_i), \text{ for all } i \in \bar{S}. \tag{5b}$$

Full cooperation means that the grand coalition $S = N$ is formed, in which case Condition (5a) is the Samuelson condition for the efficient provision of public goods.

Equilibrium uniqueness allows us to define payoffs in terms of the coalition formed. That is, we write the payoff function as a cartel-partition function that uniquely defines a payoff $V_i(S)$ for every singleton agent $i \in \bar{S}$ and a coalition payoff $V_S(S)$ as a function of the coalition S that may form. We allow for transfers between coalition members and assume that transfers are arranged to stabilise a coalition if possible (see e.g. Carraro et al., 2006; Weikard, 2009), such that[1]

$$V_i(S) \geq V_i(S_{-i}) \text{ if and only if } V_S(S) \geq \sum_{i\in S} V_i(S_{-i}) \text{ for all } i \in S. \tag{6}$$

Condition (6) implies that if coalition S does not earn enough to cover the outside-option payoffs $V_i(S_{-i})$ of its members, no member will receive her outside-option payoff in that coalition. It is thus not advantageous for any agent to join a coalition that would not earn at least the sum of the outside-option payoffs. We can now define the concept of coalition stability.

Definition 1. A coalition is stable if it is both internally and externally stable. Assuming that transfers are arranged according to Condition (6):

1. A coalition S is internally stable if and only if $V_S(S) \geq \sum_{i\in S} V_i(S_{-i})$.
2. A coalition S is externally stable if there is no agent $j \notin S$ such that $V_j(S_{+j}) \geq V_j(S)$.

Definition 1 says that a coalition is internally stable if it can guarantee that each member receives at least her outside-option payoff. It is externally stable if no singleton agent has an incentive to join as she would earn less than her outside-option payoff. The following general result follows from Definition 1 and the assumption of stabilising transfers (6). The result links internal and external stability and we will use it when discussing Example 2 below, as well as in the proof of Proposition 2.

Lemma 1 (*Weikard, 2009*). *Coalition S is externally unstable, i.e., there is an agent $j \notin S$ who prefers to join S over being a singleton agent, if and only if the enlarged coalition S_{+j} is internally stable.*

We can describe the coalitional preferences by the sum of the benefits of the members. A coalition can be characterised by the gob level B_S that maximises $\sum_{i\in S} b_i(G)$. Because we use quadratic benefit functions, the coalitional benefit function is also quadratic. It follows that B_S is unique and lies strictly between the bliss levels of the members with the lowest and the highest index numbers in S. In our specification we simply have $B_S = \frac{1}{s}\sum_{i\in S} \beta_i$.

In general, coalition formation will change the gob level provided in equilibrium. A case like Example 1, where coalition formation prevents wasteful contest but does not change the gob level, is a special case. If the formation of coalition S changes the gob level to the advantage (disadvantage) of agent $k \notin S$, we will say that the formation of S has a positive (negative) spillover effect on agent k. Intuitively, negative spillovers are conducive to the formation of larger coalitions. An agent, by joining the coalition, can impact the coalition's provision level to her advantage or benefit from the transfers provided to stabilise the coalition. For symmetric public goods games with negative spillovers, Yi (1997, Proposition 4.1) finds the grand coalition to be the unique stable coalition. Positive spillovers, by contrast, hamper coalition formation as they generate free-rider incentives. In our game, however, spillovers can be positive for some agents and negative for others.

The following example illustrates differences in coalition stability between public goods and public gobs, as well as the role of positive and negative spillovers and of transfers, which will be helpful in interpreting our main results.

[1] In the following we use the shorthand notation S_{-i} and S_{+i} for $S \setminus \{i\}$ and $S \cup \{i\}$, respectively.

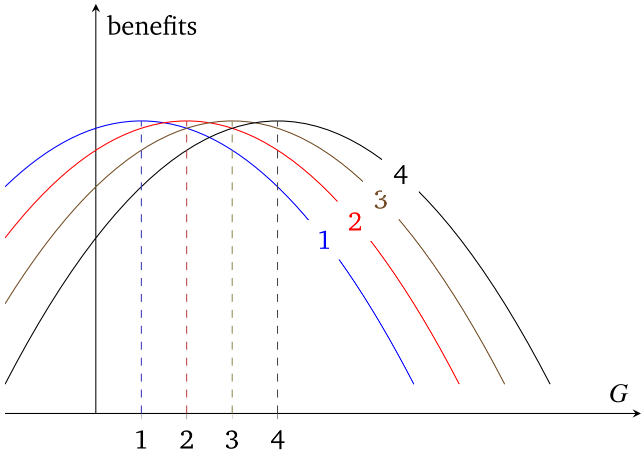

Fig. 1. Benefit functions for four agents with bliss points at $G = 1, 2, 3, 4$.

Table 1
Provision levels of all agents and payoffs (before transfers) to agent 2 for selected coalitions based on Example 2.

S	g_1	g_2	g_3	g_4	G	π_2	Internal stability	External stability
∅	−1	0	1	2	2	1	✓	–
34	−1.43	−0.43	2.14	2.14	2.43	0.82	–	–
134	0.91	−0.36	0.91	0.91	2.36	0.87	✓	–
1234	0.59	0.59	0.59	0.59	2.35	0.76	✓	✓

Example 2. Consider a game with four players and quadratic benefit- and cost functions. Let their bliss levels be at $B_1 = 1, B_2 = 2, B_3 = 3, B_4 = 4$ and normalise their benefit functions such that the peaks are equal at $b_i(G = B_i) = 1$. The example is illustrated in Fig. 1. We solve the system of Eqs. (5) and calculate equilibria for all possible coalitions. We report selected results in Table 1.

Example 2 reveals some interesting differences between public gobs and public goods. Table 1 zooms in on selected coalitions and their implications for only one of the agents in the example: agent 2. The first row shows equilibrium contributions for all four agents in case no coalition forms. Total contributions sum up to $G = 2$, which is identical to agent 2's bliss point and he enjoys a payoff of $\pi_2 = 1$ without any own contribution. By Lemma 1, this 'empty' coalition is not externally stable since there exists an enlarged coalition that is internally stable. For example, agents 1 and 2 could form an internally stable coalition. In the second row of the table we consider the case that agents 3 and 4 would team up in a coalition. If coalition $\{3, 4\}$ forms, members will increase the gobs level and induce negative spillovers on agents 1 and 2. In response, agent 1 increases and agent 2 adopts countermeasures by providing negative contributions, thus exacerbating the wasteful contest. Next, since coalition $\{3, 4\}$ is externally unstable, by Lemma 1, at least one of the remaining singleton agents would prefer to be a member. If, say, agent 1 joins to form $\{1, 3, 4\}$, some of that wasteful effort is reduced, but still, the coalition induces negative spillovers on agent 2 who makes a negative contribution. Again, external instability of $\{1, 3, 4\}$ implies that agent 2 will also join to form the stable GC. There are three effects that help stabilising the GC. First, notice that contribution levels are equal across agents. As we assume cost symmetry, it is efficient to share contributions equally between all members. Second, wasteful effort is reduced as all agents' contributions work in the same direction. Third, transfers satisfying (6) will ensure that neither agent 2 nor any other agent has an incentive to deviate. In Table 1 we report payoffs before transfers. Note, however, that internal stability and condition (6) guarantee that transfers will be arranged such that all agents receive at least their outside option payoff.

A well-known result from the literature on coalitions with public goods is that in a specification with symmetric agents and quadratic benefit- and cost functions (and hence positive spillovers), the equilibrium coalition size will not be larger than 2 (Finus, 2001). Table 1 shows that when agents differ in bliss points, such that the game becomes a gobs game, this result does not generalise. In our example the grand coalition emerges as the unique stable coalition. The following section provides a general analysis, demonstrating that this outcome is broadly representative of gobs games, albeit with some exceptions.

5. Analysis

In this section, we derive analytical results on coalition stability for the public gobs game as well as associated levels of wasteful contest. We do so employing replacement functions that have been introduced in the literature on aggregative games (Cornes and Hartley, 2007; Cornes, 2016).[2] The key idea of this approach is to write agent i's contribution to the public good not as a response

[2] The aggregative game approach was first developed by Selten (1970, Chapter 9).

function — i.e., a function of other agents' contributions — but as a function of the total contribution of all agents including i. For our game such replacement functions for singletons and members can be obtained from the FOCs when maximising (4). We obtain

$$g_i = \sum_{j\in S} \beta_j - sG \quad \text{for all} \quad i \in S, \tag{7a}$$

$$g_i = \beta_i - G \quad \text{for all} \quad i \in \bar{S}. \tag{7b}$$

Aggregating all contributions given by (7) and solving for G we obtain the equilibrium provision level

$$G(S) = \frac{s\sum_{i\in S}\beta_i + \sum_{i\in\bar{S}}\beta_i}{s^2 - s + n + 1}. \tag{8}$$

Because of the aggregative structure of the game, the provision level, the coalition payoff, and the sum of the outside-option payoffs only depend on aggregates of the benefit parameters of the agents. We will exploit this feature of the game in the analysis of coalition stability below.

To identify internally stable coalitions we need to check whether the coalition payoff $V_S(S)$ is sufficient to cover the sum of the outside-option payoffs $V_o(S) \equiv \sum_{i\in S} V_i(S_{-i})$. We call $\Phi(S) \equiv V_S(S) - V_o(S)$ the stability function which, if weakly positive, indicates the internal stability of S; see Definition 1. To construct the stability function, notice that a deviation of agent $i \in S$ such that coalition S_{-i} is formed will change the equilibrium provision level from $G(S)$ to $G(S_{-i})$, which we construct using (8):

$$G(S_{-i}) = \frac{(s-1)\sum_{j\in S_{-i}}\beta_j + \sum_{j\in\bar{S}_{+i}}\beta_j}{s^2 - 3s + n + 3}. \tag{9}$$

Using the equilibrium provision levels (8) and (9), we can derive both terms of the stability function. We obtain

$$\begin{aligned} V_S(S) = & \left(\sum_{i\in S}\beta_i\right)^2 \left(\frac{2ns^2 - sn^2 + s}{2(s^2 - s + n + 1)^2}\right) \\ & + \sum_{i\in S}\beta_i \sum_{j\in\bar{S}}\beta_j \left(\frac{(s^2+1)(n-s+1)}{(s^2 - s + n + 1)^2}\right) \\ & - \left(\sum_{j\in\bar{S}}\beta_j\right)^2 \left(\frac{(s^3+s)}{2(s^2 - s + n + 1)^2}\right) \end{aligned} \tag{10}$$

and

$$\begin{aligned} V_o(S) = & \left(\sum_{i\in S}\beta_i\right)^2 \left(\frac{(s-1)(s^2 - 3s + 2n + 2)}{(s^2 - 3s + n + 3)^2}\right) \\ & + \sum_{i\in S}\beta_i \sum_{j\in\bar{S}}\beta_j \left(\frac{2(n-s+1)}{(s^2 - 3s + n + 3)^2}\right) \\ & - \left(\sum_{j\in\bar{S}}\beta_j\right)^2 \left(\frac{s}{(s^2 - 3s + n + 3)^2}\right) \\ & - \sum_{i\in S}\beta_i^2 \left(\frac{-7 + n^2 + 10s - 3s^2 - 2s^3 + s^4 + 2n(-1 - s + s^2)}{2(s^2 - 3s + n + 3)^2}\right). \end{aligned} \tag{11}$$

Our first main result, stated in Proposition 1, assesses the internal stability of the grand coalition (GC). Since the GC cannot be enlarged, external stability is satisfied and, therefore, internal stability implies stability. The proposition demonstrates that the grand coalition is stable except in situations where agents have identical or almost identical characteristics. A stable GC implies an efficient provision level of the public gob. After proving and illustrating the proposition we derive a number of additional insights.

Proposition 1 (*Stability of the Grand Coalition*). *In a quadratic public gobs cartel formation game the stability condition for the grand coalition is*

$$\Phi(N) \geq 0 \iff \sum_{i\in N}\beta_i^2 \geq \frac{1}{n}\left(\sum_{i\in N}\beta_i\right)^2 \left(\frac{n(n^5 - 2n^3 + 4n^2 - 3n - 4)}{(1+n^2)(n^4 - 4n^2 + 8n - 7)}\right).$$

The grand coalition is stable for a large range of distributions of bliss points and is only unstable when agents are (almost) symmetric.

Proof. For the grand coalition we have $s = n$ and, since there are no remaining singletons, $\sum_{j\in\bar{S}}\beta_j = 0$. Therefore the second and third terms of both (10) and (11) cancel. This simplifies the stability condition to

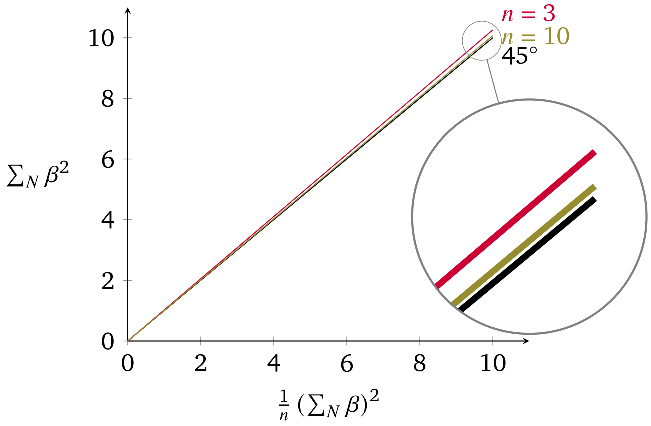

Fig. 2. The area of instability of the grand coalition is a wedge between the 45° line and the stability line for some given n, here depicted for $n = 3$ and $n = 10$.

$$\begin{aligned}\Phi(N) = &\left(\sum_{i\in N}\beta_i\right)^2\left(\frac{n}{2} - \frac{2(n-1)}{3-2n+n^2} + \frac{-4+7n-4n^2+n^3}{(3-2n+n^2)^2} - \frac{n^3+n^5}{2(1+n^2)^2}\right)\\ &+ \sum_{i\in N}\beta_i^2\left(\frac{1}{2} - \frac{2(2-n)}{3-2n+n^2} + \frac{(n-2)^2}{(3-2n+n^2)^2}\right) \geq 0\\ &\iff \sum_{i\in N}\beta_i^2 \geq \frac{1}{n}\left(\sum_{i\in N}\beta_i\right)^2\left(\frac{n(n^5-2n^3+4n^2-3n-4)}{(1+n^2)(n^4-4n^2+8n-7)}\right). \end{aligned} \tag{12}$$

The term on the LHS and the first two terms on the RHS give Chebyshev's inequality, which always holds. The remaining term on the RHS depends only on n and is always positive; it is larger than 1 for all $n \geq 3$, has a maximum value for $n = 3$ at $\frac{159}{155} \approx 1.026$ and is approaching 1 for increasing n. Therefore the stability condition is violated only if Chebyshev's inequality holds (approximately) with equality, i.e., when agents are (almost) symmetric. □

Proposition 1 is illustrated in Fig. 2. Any possible distribution of agents' bliss points is characterised by $\sum_{i\in N}\beta_i^2$ and $\left(\sum_{i\in N}\beta_i\right)^2$. The area below the 45° line is not feasible because Chebyshev's inequality must hold. On the 45° line Chebyshev's inequality holds with equality. This represents the case of symmetric agents. The area of instability is a narrow wedge between the 45° line and the stability line for some given n, here depicted for $n = 3$ and $n = 10$. Notice that the wedge gets narrower for larger n.

In public gobs games, like in public goods games, player heterogeneity helps stabilise larger coalitions. For public goods games this feature has been discussed by Weikard (2009), Pavlova and de Zeeuw (2013) and Finus and McGinty (2019). For public gobs games, however, Proposition 1 shows that the stabilising effect of agent heterogeneity is much stronger, leading to stable grand coalitions in a large part of the parameter space. Only when agents are (almost) symmetric, which makes the public gobs game a public goods game, the GC is unstable. Indeed, it is well-known that such games have no stable grand coalition whenever $n > 2$.

To see Proposition 1 at work consider the following example.

Example 3. Generalising Example 2, consider a game with n players having their bliss points at $B_1 = k+1$, $B_2 = k+2$, …, $B_n = k+n$, where $k \geq 0$ is a positive constant. Example 2 is obtained for $n = 4$ and $k = 0$. This uniform distribution allows us to obtain closed-form expressions for $\sum_{i\in N}\beta_i^2$ and $\left(\sum_{i\in N}\beta_i\right)^2$:

$$\sum_{i\in N}\beta_i^2 = \frac{1}{6}\left(n + 6kn + 6k^2n + 3n^2 + 6kn^2 + 2n^3\right), \tag{13}$$

$$\left(\sum_{i\in N}\beta_i\right)^2 = \frac{1}{2}\left(n + 2kn + n^2\right). \tag{14}$$

An increase in k shifts all bliss levels away from the default gob level $G = 0$. As a result, coalition members become 'more similar'. The gob becomes a conventional public good whenever the smallest bliss level becomes sufficiently large: $B_1 = k+1 > G(N)$. Substituting (13) and (14) into stability condition (12), we find a stable GC for any n if $k = 0$. For larger k, however, GC stability breaks down for sufficiently small n. For example, when $n = 3$ ($n = 5$) the threshold value for k where the GC is still just stable is 3.08 (7.33). If $k > 6.49$ ($k > 11.64$) only a trivial coalition $s = 1$ is stable and the game is a conventional public goods game.[3]

Corollaries of Proposition 1 follow for two special cases: two-player games (Corollary 1) and games where agents have opposed preferences (Corollary 2).

[3] Mathematica code for this example is available upon request.

Corollary 1. *If and only if $n = 2$, the GC is stable regardless of agents' bliss points.*

Proof. In the proof of Proposition 1 we have established that there is a 'region' of instability if $n \geq 3$. Evaluating the stability function (12) for $n = 2$ we obtain $\sum_{i \in N} \beta_i^2 \geq \frac{1}{2}\left(\sum_{i \in N} \beta_i\right)^2 \frac{44}{45}$. This inequality always holds since Chebyshev's inequality requires $\sum_{i \in N} \beta_i^2 \geq \frac{1}{2}\left(\sum_{i \in N} \beta_i\right)^2$ and the factor $\frac{44}{45} < 1$ makes the RHS even smaller. □

Corollary 2. *If agents' bliss levels are distributed such that $\sum_{i \in N} \beta_i = 0$, then the GC is stable for all n.*

Proof. The LHS of the stability function (12) is always (weakly) positive because the bliss level parameters are squared, while the RHS is equal to zero. □

We now turn to examining the small area of instability as illustrated by Fig. 2. In this wedge, we do not observe a stable GC, but stable partial coalitions of any size can occur. We will show that, in equilibrium, wasteful contest is largely absent. We formally define 'no wasteful effort' and then characterise the parameter space where such wasteful effort is avoided both within the coalition and between the coalition and singletons.

Definition 2. There is no wasteful effort if and only if

1. $g_S \geq 0$ and $g_j \geq 0$ for all $j \notin S$, or
2. $g_S \leq 0$ and $g_j \leq 0$ for all $j \notin S$.

The definition says that all agents are exerting effort in the *same* direction and thus contest is avoided.

Proposition 2 (*No Wasteful Effort*). *In a quadratic public gobs cartel formation game wasteful effort does not occur in equilibrium. There are two exceptions. Wasteful effort will occur in equilibrium only if:*

(i) $n = 3$, $s = 2$, $G(S) < B_S$ ($G(S) > B_S$), and the remaining singleton agent's bliss level is weakly smaller (larger) than and close to $G(S)$.

(ii) $n > 3$, $G(S) < B_S$ ($G(S) > B_S$), at least one singleton agent's bliss level is weakly smaller (larger) than and close to $G(S)$ and the coalition is sufficiently small, i.e., a size-s coalition in a game with n agents lies below the contour in the (n,s)-space that solves the polynomial $\Omega(n,s) = 0$ (given in the Appendix and illustrated by Fig. 3).

Proof. The result is proven in the Appendix. □

Proposition 2 says that, generally, in equilibrium all agents exert effort in the same direction. Exceptions can occur for small coalitions (relative to the number of agents) consisting of fairly homogeneous agents while any counteracting singleton agent's bliss level would be close to the equilibrium provision level and thus this agent spends little counteracting effort. Hence, even in cases where wasteful effort occurs, the wasted effort is limited, as can be seen from our proof in the Appendix. For example, for $n = 4$ and $n = 5$, no wasteful effort can occur in equilibrium. For $6 \leq n \leq 14$ wasteful effort can only occur when $s = 1$, the case of a trivial coalition.

The intuition for this result is as follows. Suppose a coalition exerts positive effort, pulling up the level of G, then a singleton agent pulling down could be integrated into the coalition which increases overall payoffs (since wasteful effort is avoided) and stabilises the enlarged coalition. By Lemma 1, a stable enlargement implies that the initial coalition is externally unstable and, therefore, not an equilibrium. This intuition only fails in cases where agents are sufficiently similar such that the cost savings of integrating a counteracting agent are insufficient to stabilise the enlarged coalition.

6. Conclusion

In this paper, we analyze the private provision of a public good when additional provision is good for some agents and bad for others. Such goods have been called *gobs* in the recent literature. Our model includes two additional features. One is that we allow for both positive and negative contributions to the gob. The other is that agents can form coalitions to coordinate the provision of the public gob. We establish two main results. First, we characterise the class of games where the grand coalition is stable (i.e., an equilibrium outcome) and the provision level is efficient. We find a stable and efficient grand coalition if agents are sufficiently heterogeneous (Proposition 1). Second, we find that even in cases where the grand coalition is not an equilibrium, a potential contest between agents exerting effort in different directions is generally avoided in equilibrium. In short, if agents are (sufficiently) heterogeneous, they can form a stable coalition which avoids a wasteful contest. Conversely, if agents are (sufficiently) homogeneous, the similarity of agents implies that all agents exert effort in the same direction, even though coalition stability breaks down and agents do not coordinate their efforts. The case where moderate wasteful effort may occur lies in between these polar cases and is characterised in Proposition 2.

We obtain our results in a setting with quadratic benefits from public gobs and quadratic costs of provision. We argue in Section 1 that a generalised version of our model does not yield closed-form solutions. Without functional specifications of costs and benefits the stability function would not allow to draw any conclusions about size and composition of equilibrium coalitions. The quadratic–quadratic specification captures relevant and commonly used features of social situations such as increasing marginal cost and

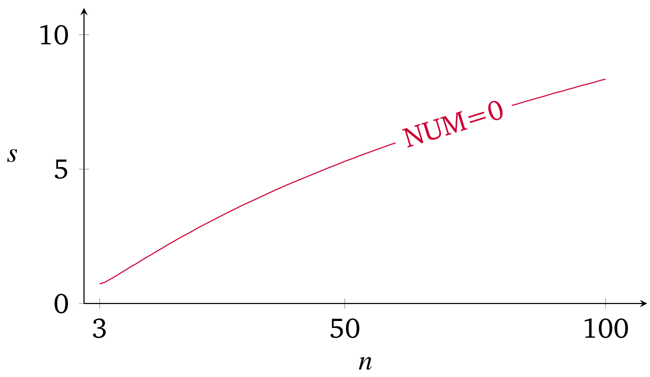

Fig. 3. Contour for NUM = 0 in the (n, s)-space for $n > 3$. Points below the contour satisfy NUM < 0, i.e., we observe stable coalitions where wasteful effort may occur.

decreasing marginal benefits as an agent comes closer to her bliss point. Seemingly simpler specifications where either costs or benefits are linear will give kinks in the response and replacement functions which require tedious analysis.

Compared to the literature on international environmental agreements that analyses public goods provisioning, our results are much more positive. Agreements on public gobs do not appear to suffer from small coalition sizes that are typically found for public goods. We find large coalitions. More specifically, we find a stable grand coalition except in situations where agents have identical or almost identical characteristics. The main driver of the difference in coalition size between public goods and public gobs is the avoidance of wasteful contest. Absent a coalition, such contests always occur in the case of public gobs (ignoring some trivial games), but never in the case of public goods. The additional benefit from avoided contest enhances the incentives to join the coalition.[4]

With public goods a coalition is more successful if more public goods are provided. In the case of public gobs, where some prefer less while others prefer more, we cannot apply such simple metric. Yet, one tangible metric of success can be identified for the case of public gobs. It is best illustrated using the application of our model to the case of geoengineering, briefly discussed in Section 1. A significant concern in the discussion on geoengineering is the potential for countries to act as free-drivers, employing geoengineering technologies unilaterally without considering potentially negative impacts on other countries. Our results show that we may not be too concerned about free-driving. The possibility of counter-geoengineering measures implies that it is in the interest of all countries to cooperate and coordinate on the employment of such technologies in order to prevent a wasteful contest (Heyen et al., 2019). This example illustrates one of the key insights of our model. In general, our results add to understanding the incentives for cooperation on gobs in cases where positive and negative contributions are possible.

CRediT authorship contribution statement

Erik Ansink: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Project administration, Methodology, Investigation, Formal analysis, Conceptualization. **Hans-Peter Weikard:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Project administration, Methodology, Investigation, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix. Proof of Proposition 2

To prove Proposition 2, we first establish the following lemma on the instability of coalitions consisting of symmetric agents.

Lemma 2. *In a quadratic public gobs cartel formation game no coalition of agents with identical bliss levels can be stable, except in the specific case where members do not make a contribution in equilibrium.*

[4] Our results should not be understood as saying that wasteful contest cannot occur, since they are derived in a static setting with perfect information while we know from the contest literature that there are many situations where contest may still be rational (see e.g. Skaperdas and Syropoulos, 1996).

Proof. We construct the general stability function $\Phi(S)$ from (10) and (11) that allows for $s < n$ and where every member's bliss level is identical and denoted by β_m. That is, $\beta_i = \beta_m \ \forall i \in S$. The stability function can be rewritten as

$$\Phi(S) = \left(\sum_{j\in\bar{S}} \beta_j + \beta_m(-1-n+s) \right)^2 \left(\frac{(s-s^2)\left(-7+7s-11s^2+9s^3-5s^4+s^5+n^2(1+s)+2n(-1-2s^2+s^3)\right)}{2(3+n-3s+s^2)^2(1+n-s+s^2)^2} \right).$$

The first factor contains the bliss level parameters. Because it is squared, it is always (weakly) positive. It can be zero and hence, a symmetric coalition is just stable in the special case that $\beta_m = \frac{1}{n-s+1}\sum_{j\in\bar{S}}\beta_j$. This occurs when the equilibrium provision level $G(S) = \beta_m$ and members are indifferent between membership and being a singleton. If the first factor is (strictly) positive, the second factor determines the sign of the stability function. We find that it is negative for all $n \geq 3$ and all $s \geq 2$. That is, the stability condition is violated for all non-trivial coalitions, except when members' contribution is zero. □

The Lemma says that no non-trivial ($s \geq 2$) coalition consisting of symmetric agents can be stable regardless of the bliss levels of members and singletons and for any number of agents and any size of the coalition.[5]

We proceed with the proof of Proposition 2.

Proof. To prove Proposition 2 we identify two main cases. (i) When the grand coalition is stable, all agents coordinate their actions, wasteful contest is avoided, and the proposition holds. (ii) When the grand coalition is unstable, we need to show whether in equilibrium members and singletons exert effort in the same direction. If the GC is unstable, agents are (almost) symmetric as we know from Proposition 1. We identify two sub-cases. (iia) First consider that all agents are fully symmetric. Then, by Lemma 2, we have no stable coalition. However, since agents have the same bliss level, they would exert the same effort and the proposition holds. (iib) The remaining case is when agents have different bliss levels, but not so different that the GC would be stable, i.e., we are inside the wedge depicted in Fig. 2. The remainder of the proof deals with this case.

Since it is not generally true that wasteful effort cannot occur in equilibrium, we determine a no-wasteful-effort condition. We do so for the first part of Definition 2 where members exert positive effort, i.e., we assume $g_S \geq 0$ and therefore $B_S \geq G(S)$. Wasteful effort implies that there exists some singleton $j \notin S$ with $g_j < 0$ and therefore $B_j = \beta_j < G(S)$. The proof for the case $g_S \leq 0$ works in the same way and can be skipped.

The strategy to characterise the no-wasteful-effort condition is as follows. We first obtain the internal stability function from Eqs. (10) and (11). Next, we consider constraints on the distribution of bliss levels (β_i) that need to be satisfied for a coalition $S \subset N$ to be stable. Finally, we identify the classes of equilibria (i.e., stable coalitions) in the (n, s)-space for which wasteful effort can and cannot occur. We employ this strategy first for part (i) of the proposition where $n = 3$ and subsequently for part (ii) where $n > 3$.

Combining (10) and (11), the stability function is given by

$$\begin{aligned}\Phi(S) = &\left(\sum_{i\in S}\beta_i\right)^2 \left(\frac{(1-s)(2+2n-3s+s^2)}{(3-3s+s^2+n)^2} + \frac{s(1-n^2+2ns)}{2(1+n-s+s^2)^2} \right) \\ &+ \sum_{i\in S}\beta_i \sum_{j\in\bar{S}}\beta_j \left(-\frac{2(1+n-s)}{(3-3s+s^2+n)^2} + \frac{(1+n-s)(1+s^2)}{(1+n-s+s^2)^2} \right) \\ &+ \left(\sum_{j\in\bar{S}}\beta_j\right)^2 \left(\frac{s}{(3-3s+s^2+n)^2} - \frac{s(1+s^2)}{2(1+n-s+s^2)^2} \right) \\ &+ \sum_{i\in S}\beta_i^2 \left(\frac{-7+n^2+2ns^2-2ns-2n+s^4-2s^3-3s^2+10s}{2(3-3s+s^2+n)^2} \right). \end{aligned} \quad (15)$$

Part (i) (n = 3) First, consider $s = 1$ such that the coalition is trivially internally stable. Without loss of generality, we normalise $\sum_{i\in S}\beta_i = s$ such that $B_S = 1$. By (8) this implies that $G(S) = \frac{1+\sum_{j\in\bar{S}}\beta_j}{4}$. Since we assume $B_S \geq G(S)$, we obtain $\sum_{j\in\bar{S}}\beta_j \leq 3$. Next, assume we have wasteful effort. Then there is one singleton j, with $\beta_j < \frac{1+\sum_{i\in\bar{S}}\beta_i}{4}$. Denote the other singleton agent by k and combine $\sum_{i\in\bar{S}}\beta_i = \beta_j + \beta_k$ with the previous inequality to obtain $\beta_k > 3\beta_j - 1$. We use (15) to calculate stability of the enlarged coalition $S \cup \{j\}$ and write it in the (β_k, β_j)-space: $\beta_k \geq 1 + \beta_j - 3\sqrt{2}\sqrt{1-2\beta_j+\beta_j^2}$. This last condition implies $\beta_k > 3\beta_j - 1$ so that if there is wasteful effort for $s = 1$, then there is also an internally stable enlargement of S. By Lemma 1, this implies that we do not find an equilibrium with wasteful effort when $s = 1$. Next, consider $s = 2$ with only one singleton agent k. Again, we normalise $\sum_{i\in S}\beta_i = s = 2$. By (8) this implies that $G(S) = \frac{4+\beta_k}{6}$. For wasteful effort to occur we require $\beta_k < G(S)$, which implies $\beta_k < \frac{4}{5}$. Assuming internal stability of S we use (15) to calculate a minimum value for $\sum_{i\in S}\beta_i^2$. When the bliss level of agent k gets closer to $\frac{4}{5}$, we find that $\sum_{i\in S}\beta_i^2$ tends to $\frac{51}{25}$ when S is just minimally stable. We find a small range of β parameter values for the

[5] The Lemma holds in our current setting where we do not consider different cost parameters. In a more general model where we introduce a cost parameter we find stable coalitions of size $s = 2$ for sufficiently high costs. We report more details in Ansink and Weikard (2023, Section 6).

two coalition members in the interval $\left(1-\frac{\sqrt{2}}{10}, 1+\frac{\sqrt{2}}{10}\right)$ where a two-player coalition is stable and exerts positive effort, while the remaining singleton exerts negative effort and so we have wasteful contest. Integrating the contesting singleton leads to an unstable GC, and hence, by Lemma 1, we find an equilibrium with wasteful effort when $s = 2$.

Part (ii) (**n** > **3**) As before, without loss of generality we normalise $\sum_{i\in S}\beta_i = s$ so that the average β of members is $B_S = 1$. Next, we fix $\sum_{i\in S}\beta_i^2$ (which captures the degree of heterogeneity of members) at a value such that S is just internally stable, i.e., $\Phi(S) = 0$. This leaves us with an internal stability condition $\Phi(S)$ that depends only on n, s, and $\sum_{j\in\bar{S}}\beta_j$, i.e., the sum of singleton agents' bliss levels.[6]

Now, we need to determine for which combination of parameters n, s, and β_j, if any, wasteful effort can occur in equilibrium. Note that stability of S is equivalent to the internal stability of S and, by Lemma 1, the external instability of the enlarged coalition $S\cup\{j\}$ for any singleton $j \notin S$. In light of the latter condition, notice that S is most likely to be stable if S is internally stable and any enlarged coalition $S\cup\{j\}$ is likely to be unstable. The latter is true if agents in $S\cup\{j\}$ and therefore in S are more homogeneous in terms of bliss levels. This is the reason why we can fix $\Phi(S) = 0$ as we did before.[7] For any wasteful effort to occur in equilibrium there must exist a singleton j such that $\beta_j < G(S)$ and at the same time β_j must be sufficiently close to B_S since, when a relatively 'similar' agent joins the coalition, the enlarged coalition is least likely to be internally stable and thus S is most likely to be stable.

Formally, then, for any internally stable coalition S with $B_S \geq G(S)$ and $j \notin S$ with $B_j = \beta_j < G(S)$ we assess the internal stability of $S\cup\{j\}$ (which is equivalent to the external instability of S). We find

$$\Phi(S\cup\{j\}) = \frac{\text{NUM}}{\text{DEN}}s^2\left(\sum_{j\in\bar{S}}\beta_j - n + s - 1\right)^2, \tag{16}$$

where numerator NUM and denominator DEN are higher-degree polynomials containing only parameters n and s. Since the second and third terms of the stability function are always positive, we are left with examining the fraction $\frac{\text{NUM}}{\text{DEN}}$. It can be proven that DEN > 0 for all $n \geq 3$ and $s \geq 1$. For NUM we have

$$\begin{aligned}\text{NUM} = &-3 - n^6 + 2s - 4s^2 + 7s^3 - 10s^4 - 6s^5 + 48s^6 - 49s^7 + 22s^8 - 22s^9 + 8s^{10}\\ &-5s^{11} + s^{12} - 2n^5(1-5s+2s^2) + n^4(7-10s-12s^2+35s^3-5s^4)\\ &+4n^3(3-17s+27s^2-22s^3-4s^4+10s^5) + n^2(-3-48s+136s^2-206s^3\\ &+211s^4-168s^5+4s^6+10s^7+5s^8) + 2n(-5+s+4s^2-28s^3+77s^4-89s^5\\ &+66s^6-56s^7+9s^8-5s^9+2s^{10}).\end{aligned} \tag{17}$$

The pairs (n, s) such that NUM ≥ 0 indicate internal stability of the enlarged coalition $S\cup\{j\}$ and thus external instability of coalition S. Such coalitions are not equilibria. Hence, equilibria with wasteful effort can only occur when NUM < 0. As a result, NUM ≥ 0 constitutes our no-wasteful-effort condition. Fig. 3 gives the contour for NUM = 0 in the (n, s)-space. Points below the contour satisfy NUM < 0, that is we find no stable enlargement of S, thus S is stable while agent $j \notin S$ exerts negative effort. Conversely, at points above the contour wasteful effort cannot occur. □

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[6] For details and subsequent calculations in the proof, our Mathematica script is available upon request.

[7] If S were more than minimally internally stable, then an enlarged coalition $S\cup\{j\}$ is more likely to be stable which implies that S is more likely to be externally unstable and, thus, not an equilibrium coalition.

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