Integrating time-temperature dependent deterioration in the economic order quantity model for perishable products in multi-echelon supply chains

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\textbf{ABSTRACT}

This paper focuses on a novel approach for managing cold chains of highly perishable products. We extend the classical and frequently applied economic order quantity model (EOQ) for its application in multi-echelon supply chains for highly perishable products. Literature shows that order-level inventory systems for perishable items are commonly modelled by parametric approaches e.g., a time-dependent (Weibull) deterioration rate. However, the deterioration of perishable fresh products is a complex process, heavily influenced by product dependent characteristics and, crucially, depending on environmental conditions. We take both time-temperature-dependent deterioration, and the multi-echelon aspect of cold chains into account and integrate both aspects into a generically applicable EOQ-based methodology. We demonstrate the concept from a real-life case study for cold chain management in floriculture. Two commonly used multi-echelon supply chains are considered and compared. The results show that the optimal order levels in different echelons of the supply chains are substantially different. In addition to the sojourn time, the temperature has a major impact on order levels, the total supply chain costs, and the remaining shelf-life at the retail level. We demonstrate that the proposed concept for extending the EOQ-based model to time- and temperature-dependent deterioration can be generalized and applied to other specific causes of product-dependent deterioration.

1. Introduction

Since Harris [23] introduced the Economic Order Quantity model (EOQ) in 1913, the development of inventory control received great interest in both academia and practice. Even a century after its creation, Andriolo et al. [3] conclude in their survey and analysis of EOQ literature that EOQ inventory control models are still attractive and widely accepted by industries, particularly due to their succinctness, simplicity, elegance, and effectiveness. Meanwhile, a vast amount of literature has been published on inventory lot sizing, foremost driven on either addressing the underlying assumptions or, more recently, on extending the valuable concept to new aspects and circumstances in practice like for instance introducing a freshness-dependent selling price [15], introducing compound interest [8], the effects of discount policies [21, 48], EOQ-based inventory control of two products with demand substitution [47], or considering an EOQ model with customer’s income-dependent demand and price-dependent supply [28].

One of its important extensions refers to the control and maintenance of inventories for deteriorating items. Goyal and Giri [19] emphasize the distinction between decaying and deteriorating products. The main distinction between both categories refers to the usable lifetime. Decaying products lose their value over time and may become obsolete, but they are not destroyed (e.g., products from the discrete part industry like mobile phones, or style goods in fashion merchandizing) while deteriorating products have a maximum useable lifetime or “shelf-life”. Agro products like fish, meat, milk, green vegetables, fruits, or cut flowers typically belong to the class of deteriorating products that physically decay and are irreversibly destroyed over time.

Fresh products need to be kept in specific atmospheric conditions along the entire supply chain in which time and temperature are the most important quality parameters [35,36]. Consequently, management systems for fresh products must be based on time-temperature...
measurements and require a holistic approach in which decisions of stakeholders at different echelons are carefully coordinated [16,36,46]. Mercier et al. [36] state that management systems based on time-temperature measurements are very limited, partly due to the inaccuracy of shelf-life estimates from temperature measurements.

Research has shown that integrated and collaborative decisions across subsequent links in supply chains translate into greater joint benefits for all supply chain actors compared to scenarios where each echelon makes individual decisions [11,40]. However, several review papers conclude that holistic approaches for the design and management of fresh product supply chains are still scarce [1,36,46,50]. Duong et al. [16] focus specifically on inventory management for perishable products and confirm the lack of holistic approaches. The authors conclude that single-echelon inventory papers are reaching a saturation point and research attention should increasingly focus on multi-echelon models.

Contemporary consumers expect a wide variety of high-quality and "ripe on arrival" fresh products with a long shelf-life to be available all year round [45,46,55]. This trend in (foremost) developed countries has led to an ever-widening array of fresh produce, increased the demand for those products, encouraged global competition, and induced the transition from locally to globally oriented, long-distance, multi-echelon supply chains [1,46,55]. As a consequence, the design and configuration of the related supply chain networks [17,18] make controlled storage conditions including the efficient management of transportation, distribution, and inventory management critical [46].

This study focuses specifically on EOQ-based inventory control for highly perishable fresh products in multi-echelon supply chains. In general, these chains may comprise several supply chain actors like growers, auctions, wholesalers, importers and exporters, retailers, and shops. Particularly for perishable products, the post-harvest sojourn time and temperature conditions during (intermediate) storage and transportation are extremely important for product deterioration. Many highly perishable products are still alive after harvesting and respiration continues in a supply chain (e.g., fresh vegetables, fruits, or flowers). Other products foremost suffer from fungal, microbial and/or enzymatic product deterioration. Either way, time-temperature controlled storage and transportation conditions during the entire supply chain have a significant impact on the rate of deterioration for highly perishable products.

Commonly, there are two ways (including all relevant combinations) to manage the quality of fresh produce from origin to destination:

1. Speeding up the goods flow in the chain [5] e.g., by choosing smaller ordering and transportation lot sizes, and/or using different modes of transport e.g., air freight instead of sea freight.

2. Postponing or accelerating the aging, i.e., ripening during the sojourn time in a supply chain to ensure peak quality on arrival e.g., by using time-temperature integrating technologies in intelligent reefer containers. Intelligent containers for fresh products in sea freight are often equipped with additional control units to make autonomous decisions on climate control of its transported cargo.

Mercier et al. [36] state that time-temperature dependent deterioration is highly product dependent. According to Janssen et al. [27] and Mirabelli and Solina [37], product-dependent deterioration functions are barely addressed in fresh product multi-echelon models. The aim of this study is to develop a generalizable concept for the integration of product-specific deterioration derived from biological sciences in EOQ-based inventory models for multi-echelon supply chains. To the best of our knowledge, such an integrated approach has not been discussed in existing literature. The approach will be demonstrated from a real-life case study for cold chain management in cut flower supply chains.

The remainder of this paper is structured as follows. To highlight the research gap, Section 2 gives an overview of (EOQ-based) literature related to product-dependent deteriorating functions in multi-echelon supply chains. Section 3 focuses on generalizable model development for extending the classical EOQ model to time-temperature dependent product deterioration, in multi-echelon supply chains. A tested cut flower vase life prediction model from literature will be integrated to demonstrate the approach. In Section 4, the extended EOQ model will be applied to two different multi-echelon cold chains for cut roses i.e., common temperature-controlled air freight versus intelligent reefer containers for sea freight. Section 5 provides the numerical results for both supply chains. Conclusions follow in Section 6.

2. An overview of existing literature

Especially for inventory replenishment, the basic EOQ model is extended in various directions. With respect to deterioration, Andriolo et al. [3] show in their comprehensive survey that a century of EOQ developments have either focus on using constant and time-varying deterioration rates, or on assuming general (time-dependent) Weibull distributions with different shape and scale parameters.

At a more general level of inventory models for perishable products, several interleaved reviews have been published in the past 25 years. Mercier et al. [33,34] confirm in their review on continuously deteriorating inventory models (42), Goyal and Giri [19] review the advances in this field from 1990 to the year 2000. Bakker et al. [4] provide an update, succeeding the work of Goyal and Giri [19] from 2001 to 2011, and Janssen et al. [27] continue the work of Bakker et.al. by providing a review from 2012 to 2015. A remarkable observation is that Rafaat already called in 1991 for the development of models based on more sophisticated functions which can better map real-life deterioration processes. However, after 25 years of research, Janssen et al. [27] conclude that sample functions still prevail in literature and more realistic deterioration functions are lacking.

Mirabelli and Solina [37] provide a review from 2005 to 2020 and notice a growing interest for optimization strategies of integrated management for perishable supply chains. However, most of the papers concern at most 2-level supply chains and the proposed optimization models are mainly validated theoretically, and case studies are lacking. Marchi et al. [34] confirm in their review on supply chain finance for deteriorating products that 71 % of the studies focus on a single actor perspective, 23 % on a two-echelon supply chain, and only a 5 % considers a three-echelon supply chain. We take the underlying biological principles of product deterioration as a starting point for highly perishable material flows in multi-echelon supply chains. Product deterioration for perishable products starts immediately after harvest. Product quality continuously decreases irreversibly in, and between, consecutive echelons. The only options to manage the quality of fresh product flows between harvest and consumer delivery are either to reduce the total sojourn time, and/or to slow down product decay by changing the environmental conditions. From all post-harvest environmental conditions, temperature regimes in consecutive stages of the supply chains are crucial. However, its impact has not been considered in EOQ models, yet.

Fresh products may differ in types of post-harvest deterioration due to microbial, enzymatic, fungal, and/or respiration activity for product deterioration. Next to (lead) time, cooling is important to inhibit respiration, suppress enzyme activity, reduce the production of ripening agents, and slow down the growth of micro-organisms which make both temperature and sojourn time critical aspects for fresh produce in multi-echelon supply chains [35,34,36].

Sensor-based, temperature-controlled storage, information has been used for inventory management to estimate the quality of perishables in supply chains [13,20,29,31,41,56]. Dada and Thiesse [13] develop a time-temperature-based simulation model to study the quality of perishable goods at a retailer under (seven) different issuing policies at the distributor’s level. The supply chain comprises a manufacturer, a distribution centre, and a retailer’s store. Products deteriorate according
to a linear model with a certain percentage of daily quality decay. Grunow and Piramuthu [20] model three independent scenarios i.e., for distributors, retailers, and consumers, and study the benefits of using sensor-enabled RFID (Radio-Frequency Identification) technology in a highly perishable food supply chain. The authors develop conditions under which the implementation of RFID technology could be beneficial for retailers and distributors and derive expressions for the premium a customer would be willing to pay for such information. The expression for quality degradation is based on the sojourn time between the links in the supply chain and a general product decay parameter $\lambda$. Ketzenberg et al. [29] specifically focus on determining the value of information for integrating upstream Time-Temperature History (TTH) in a supply chain (e.g., enabled by RFID technologies) into inventory replenishment decisions at the retail level. It is assumed that if the complete TTH at retail entrance level is known, the remaining shelf-life can be calculated. The authors formulate the perishable inventory problem as a Markov Decision Process and evaluate the value of (TTH) information by simulation.

Zanoni and Zavanella [56], Piramuthu and Zhou [41], and Ketzenberg et al. [29] focus on two subsequent echelons in a supply chain. Broekmeulen and Van Donkelaar [5] and Piramuthu and Zhou [41] consider variable holding and purchasing costs, but no fixed ordering costs and energy costs for cooling between retail and consumers while Dada and Thiess [13], Kouki and Jouini [31], and Ketzenberg et al. [29] consider the distributor and retail echelons in a supply chain. Kouki and Jouini [31] consider fixed ordering costs, variable holding, and purchasing costs. Zanoni and Zavanella [56] study a system that consists of a distributor that procures, and stocks chilled or frozen products from a producer, and transports the products directly to retailers considering both fixed and variable costs at producer’s and distributor’s level.

Specifically for EOQ-based modelling approaches, product decay is often based on time-dependent (Weibull) deterioration [2,24,32,38,39,43,51]. Moreover, to highlight the impact of parameters on optimal policies, the validation of models is mostly based on numerical illustrations instead of case-studies. Most recently, [7,9,10] studies the EOQ model for exponentially deteriorating items. In these studies, items deteriorate at a constant rate $\delta$, proportionally to the inventory level per unit time which leads to an exponentially decreasing inventory level function over time. Deteriorated items are immediately removed from the stock level, and replenishment is assumed to take place instantaneously [9,10].

However, product decay for perishable items is usually more complicated i.e., product dependent, and foremost determined by post-harvest environmental conditions in supply chains. We follow biological sciences, in which both sojourn time and sojourn temperature have been identified as the main reasons leading to natural deteriorating processes which makes temperature-controlled logistics extremely important in cold chain management [30,36].

In biological sciences, time-temperature effects in product-dependent deterioration models have been studied in the literature. Tromp et al. [49] combine for instance time and temperature to study the remaining vase life for cut roses. Gelik and Reid [12] study the impact of temperature on postharvest performance for roses and gypsophila flowers. Zhang et al. [57] study the effect of storage time and temperature on the nutritional quality of walnut male inlorescences while lao et al. [25] focus on temperature monitoring for quality prediction and inventory control in the cold chain of ready-to-eat food during logistics flows. So far, no research is known in which both time and temperature aspects are integrated into multi-echelon EOQ-based modelling approaches.

Since cut flowers are still alive after harvesting, the respiration rate increases exponentially at rising temperatures. This process can be slowed down by setting the right storage temperature [12,22]. For example, roses stored at 2.5 °C have a respiration rate at 20 ml CO2/(kg h)$^{-1}$ and 8.1 days of remaining vase life while roses stored at 10 °C generate slightly more than doubled respiration rate and reduce the remaining vase life to 6.5 days [12]. In addition to temperature, the sojourn time for cut flowers in the supply chain is important. The longer the sojourn time, the more quality losses are expected at the same temperature. Due to this reason, many companies try to shorten their supply chains by reducing the sojourn time.

Several studies have been published to measure quality losses and predict the remaining vase life (RVL) of flowers after harvesting [26,49,52,53]. Tromp et al. [49] and Verdouw et al. [54] use the concept of the time-temperature sum. A comparable approach of Accumulated Heat Units (AHU) has been described for fungal, time-temperature-based product deterioration in Mango’s [44]. The indicator in Tromp et al. [49] is based on a cultivar-dependent fixed vase life (VL) of non-stored flowers and an historical database of storage times and temperatures. Tromp et al. [49] use the so-called degree-days model for quality decay of cut roses and develop a scientific basis for using the time-temperature sum as a predictor for the remaining VL after storage and transportation in the distribution chain, both via theoretical calculations and fitting experimental VL data, comprising both dry storage (in a box) and wet storage (in a bucket filled with water). According to this model, the remaining VL can be found by subtracting the recorded time-temperature sum divided by the room temperature from the VL of non-stored flowers. For example, if the VL of non-stored flowers is 10 days at 20 °C (room temperature), while during the distribution chain the flowers are kept at 5 °C for 8 days, the recorded time–temperature sum equals 40 °C days, such that the model predicts a remaining VL of 8 days at 20 °C. The model implies that when storing the flowers during 12 days instead of 8 days at 5 °C, the remaining VL only decreases by another extra day [49]. This model has been used in the case-study of Section 4 for the integration of product specific deterioration in EOQ-based inventory models in multi-echelon supply chains.

3. Model development

To demonstrate the generalizability of the approach, we distinguish two product-dependent deterioration functions. For the sake of illustration, this section starts following the commonly used Arrhenius equation for time-temperature indicator systems [35] and takes a continuous exponential function for microbial product deterioration (e.g., in milk and meat) as a starting point. Next, we follow a comparable approach for another cause of product deterioration i.e., respiration in long-distance supply chains e.g., for cut flowers.

3.1. EOQ-based modelling for microbial deterioration

The total cost function in the classical, single-echelon EOQ model is defined as:

$$C_{\text{tot}}(Q) = \frac{1}{2} hQ + FD + pD$$

(1)

Where $h$ is the constant holding cost per period per unit, $Q$ the order size, $F$ the fixed ordering costs, $D$ the demand per period and $p$ the fixed purchase price per product. Since the term $pD$ does not affect the optimal order quantity, the EOQ problem has been described as:

$$\min_{Q} \left\{ \frac{1}{2} hQ + \frac{FD}{Q} \right\}$$

(2)

The optimal value $Q^*$ can be derived from $\frac{dC_{\text{tot}}}{dQ} = 0$ which implies that $Q^*$ can be obtained by the classical formula:

$$Q^* = \sqrt{\frac{2FD}{h}}$$

If the underlying product is perishable (e.g., milk), its deterioration depends on both the sojourn time ($\tau$) and the sojourn temperature ($\theta$). Suppose the process of product deterioration is induced by micro-organisms. Their growth can be described by (3):
\[ N_i = N_0 e^{\rho(\tau)} \]

where

\[ N_i := \text{the density of micro-organisms per unit at time } t \]
\[ \rho(\tau) := \text{temperature-dependent growth coefficient of the micro-organisms} \]
\[ s := \text{the sojourn time} \]
\[ \tau := \text{temperature} \]

If \( G(\tau) \) is defined as the costs for keeping the products at \( \tau \) degrees during the sojourn time, the constrained optimization problem (4) can be formulated comprising inventory costs, ordering costs and costs related to cooling:

\[
\min_{Q^*} \left\{ \frac{1}{2} hQ + \frac{FD}{Q} + G(\tau) \right\} \\
\text{s.t.} \\
N_0 e^{\rho(\tau)} \leq N_{\text{max}} \tag{4}
\]

where \( N_{\text{max}} \) is a predefined upper bound for the density of micro-organisms to guarantee the shelf-life. The sojourn time \( s \) in (3) can be substituted by the maximum time on stock i.e., \( \frac{Q}{2} \). The remainder of this article is based on the proposition that in the optimum, the constraint in problem (4) is binding. To prove this proposition, we assume that the statements i) and ii) in Assumption 1 hold:

Assumption 1.

i) The function \( G \) is strictly monotonically decreasing in \( \tau \). In other words, as the cooling temperature \( \tau \) gets closer to ambient levels, the corresponding cooling costs \( G(\tau) \) decrease monotonically.

ii) The function \( \rho(\tau) \) is strictly monotonically increasing in \( \tau \). In other words, as the cooling temperature \( \tau \) gets closer to the ambient temperature, the growth of micro-organisms increases monotonically.

Proposition 1. Let us assume that there is an optimal solution \((Q^*, \tau^*)\) for problem (4) and Assumption 1 holds. Then, the inequality constraint \( N_0 e^{\rho(\tau^*)} \leq N_{\text{max}} \) is binding in \((Q^*, \tau^*)\).

Proof. We assume that the assertion does not hold and derive a contradiction. To achieve this, we consider the point \((Q^*, \tau^* + \epsilon)\) that is non-binding for the constraint in (4) and assume it is optimal for problem (4). Since the constraint is assumed to be non-binding, we have the strict inequality:

\[ N_0 e^{\rho(\tau^*)} > N_{\text{max}} \]

Then, however, we may consider the point \((Q^*, \tau^* + \epsilon)\) for \( \epsilon > 0 \) and sufficiently close to zero. Due to continuity of the constraint, this point is also feasible for problem (4) and, moreover, in view of the fact that \( G \) is strictly monotonically decreasing in \( \tau \) we have

\[
\frac{1}{2} hQ^* + \frac{FD}{Q^*} + G(\tau^*) > \frac{1}{2} hQ^* + \frac{FD}{Q^*} + G(\tau^* + \epsilon)
\]

This shows that the point \((Q^*, \tau^*)\) is not optimal in contrast to our assumption.

From the above it follows that, in the optimum, the constraint in problem (4) is binding i.e., an equality. Now, problem (4) can be reduced to (5) by rewriting the order size \( Q \) for the equality constraint in (4) as a function of the temperature:

\[
Q = \frac{D}{\mu(\tau)} \ln \left( \frac{N_{\text{max}}}{N_0} \right)
\]

Now, problem (4) can be reformulated and simplified as a function of the decision variable \( \tau \) in (5)

\[
\min_{\tau} \left\{ \frac{1}{2} hQ^* + \frac{FD}{Q^*} + \frac{G(\tau^*)}{\mu(\tau)} \right\}
\]

The unconstrained problem (5) refers to a single-echelon supply chain. Next, the approach is projected on a multi-echelon supply chain. (Fig. 1)

For the sake of simplicity, we focus on \( m = 2 \) echelons in Fig. 2. If the inventory level of supplier 1 drops to zero after \( Q_1/D \) periods, replenishment is assumed to take place instantaneously. The supply quantity \( Q_2 \) for supplier 1 implies a decrease in stock for supplier 2. In Fig. 2 we assume, solely as an illustration, that \( Q_2 = n_2 Q_1 \) with \( n_2 = 3 \).

Order quantities for fresh products at retail level are commonly expressed in integer units e.g., bunches of flowers, pre-packed trays of fruits, cartons of milk, or integer numbers of pre-packed pieces of meat in single packages. This principle, also referred to as the "integrality property", together with the common assumptions of the classical EOQ-model e.g., a steady state demand rate and no capacity limitations, enforces that the order quantity at upper echelons is an integer multiplier of the order quantity at lower echelons. This alignment of order levels between consecutive echelons enforces less inventory fragmentation regarding shelf-lifes, it reduces undesirable "customer order picking" for expiration dates at retail level, reduces order frequencies, exploits economies of scale, and fosters collaborations between different stakeholders in multi-echelon supply chains.

From Fig. 2 it can be concluded that the maximum sojourn times
If we assume, for the time being, that the transportation time between the two echelons is negligible, then the total sojourn time $s_{\text{tot}} = s_1 + s_2 = n_2 \frac{Q_2}{D}$. Generally, for an $m$-echelon supply chain holds $s_{\text{tot}} = \prod_{i=2}^{m} n_i$.

We can impose similar assumptions on the cost functions and costs related to cooling in both echelons: Let $h_1, n_2$ be the variable inventory cost coefficients for supplier 1 and 2 respectively, and $F_1, F_2$ the corresponding fixed setup costs, then the optimization model for the two echelons case is:

$$
\min_{Q_1, n_2} \left\{ \frac{1}{2} \sum_{i=1}^{2} h_i Q_i + \frac{F_1 D}{Q_1} + \frac{1}{2} h_2 (n_2 - 1) Q_1 + \frac{F_2 D}{n_2 Q_2} \right\}
$$

Like in the single echelon case, we assume that deterioration of the product depends on time and temperature. Again, using (3) for calculating the growth of micro-organisms, the constrained optimization problem (7) can be formulated comprising inventory costs, ordering costs and costs related to cooling in both echelons:

$$
\min_{Q_1, n_2, \tau_1, \tau_2} \left\{ \frac{1}{2} \sum_{i=1}^{2} h_i Q_i + \frac{F_1 D}{Q_1} + G_1(\tau_1) + \frac{1}{2} h_2 (n_2 - 1) Q_1 + \frac{F_2 D}{n_2 Q_2} + G_2(\tau_2) \right\}
$$

s.t.

$$
\frac{\alpha_i}{n_i} \exp\left(\frac{\tau_i}{T_i} - \frac{\tau_j}{T_j}\right) \leq N_{\text{max}}
$$

Solving (7) is a constrained optimization problem in the variables $Q_1, n_2, \tau_1,$ and $\tau_2$. Again, to prove that the constraint in (7) is binding in the optimum, we impose similar assumptions on the cost functions $G_1(\tau_1)$ and $G_2(\tau_2)$ for keeping products in both echelons at $\tau_1$ respectively $\tau_2$ degrees.

**Assumption 2.** We assume that the following statements hold:

i) The functions $G_1$ and $G_2$ are strictly monotonically decreasing in $\tau_1$, respectively $\tau_2$.

ii) Again, the function $\mu$ is strictly monotonically increasing in $\tau$.

**Proposition 2.** If Assumption 2 holds and, moreover, if there is an optimal point $(Q_1, n_2, \tau_1, \tau_2)$ for problem (7), then the inequality constraint in (7) is binding for $(Q_1, n_2, \tau_1, \tau_2)$.

**Proof.** The proof follows the same line of arguments as the proof of Proposition 1. Let us assume that there is an optimal point $(Q_1, n_2, \tau_1, \tau_2)$ for which the constraint in (7) is non-binding. Then we have the strict inequality:

$$
\frac{\alpha_i}{n_i} \exp\left(\frac{\tau_i}{T_i} - \frac{\tau_j}{T_j}\right) < N_{\text{max}}
$$

Then, due to continuity of the constraint, the point $(Q_1, n_2, \tau_1 + \varepsilon, \tau_2)$ for some $\varepsilon > 0$ and sufficiently close to zero, also fulfills the inequality and is, thus, feasible for problem (7). Again, using strict monotonicity of the functions it becomes clear that this point also provides an improved objective value since $G_1$ is strictly monotonically decreasing in $\tau_1$. Thus, the point $(Q_1, n_2, \tau_1, \tau_2)$ is not optimal in contrast to our assumption. □

Analogous to the transition between (4) and (5) previously, problem (7) can also be simplified by exploiting the fact that in an optimum the constraint in (7) is binding, which follows immediately from Proposition 2. Again, we emphasize that Assumption 2 is a rather natural condition that is commonly fulfilled in practice.

Reformulating the (equality) constraint gives the following expression for $Q_1$:

$$
Q_1 = \frac{D}{(n_2 - 1) \mu(\tau_1) + \mu(\tau_2)} \ln \left( \frac{N_{\text{max}}}{N_{\text{eq}}} \right)
$$

By substituting (8) into the objective function of (7), a simplified unconstrained optimization problem (comparable to (5)) in the variables $n_2, \tau_1,$ and $\tau_2$ results for the two echelons case. This problem can be solved by minimizing the simplified cost function for different (integer) values of $n_2, \tau_1,$ and $\tau_2$.

3.2. Deterioration by respiration in long-distance supply chains

Besides intermediate storage, long-distance multi-echelon chains for perishable products require time and temperature control between echelons e.g., for different modes of transport and/or handling. In this section we apply the concept to a typical long-distance multi-echelon supply chain in floriculture.

Consider a supply chain that consists of $i = 1, ..., m$ echelons with different storage temperatures $\tau_i$ and sojourn times $s_i$. Between the echelons different phases $j = 1, ... J$ for transport (modes), or handling are distinguished. Each phase is associated with its own temperature and sojourn time, denoted by $\tau_p$ and $s_p$, respectively. Fig. 3 shows an illustration for $m = 3$ and $J = 2$.

Tromp et al. [49] showed that the remaining vase life (RVL) of cut roses can be predicted effectively by the degree days model in which the recorded time-temperature sum divided by the room temperature is subtracted from the vase life of non-stored flowers:

$$
RVL = VL_0 - \frac{1}{20} \sum_{k=1}^{m} (t_k - 273.15)\theta_k
$$

where $VL_0$ refers to the vase life of freshly harvested (non-stored) cut flowers at room temperature ($20$ °C), $t_k$ to the temperature in Kelvin (K), and $\theta_k$ to the storage time (days) in stage $k$.

Applying (9) to the example in Fig. 3 means that the remaining vase life at the end of the chain equals $VL_0$ minus the total losses of vase life during storage in all echelons $i$ ($LVL_i$) and intermediate phases $j$ for handling and transport ($LVL_j$):

$$
RVL = VL_0 - \sum_{i=1}^{m} LVL_i - \sum_{j=1}^{J} LVL_j
$$

If the parameter $VL_{\text{min}}$ denotes a predefined lower bound for the vase life at retail level, then:

$$
VL_0 - \frac{1}{20} \sum_{k=1}^{m} (t_k \epsilon_i + \sum_{j=1}^{J} \tau_p \epsilon_j) \geq VL_{\text{min}}
$$

Correspondingly to the assumptions in the problems (4) and (7), the remaining vase life (RVL) equals the predefined lower bound $VL_{\text{min}}$ in the optimum. Formally, this can be shown along the same lines of arguments as the proof for the Propositions 1 and 2. From a practical point of view, this can also be explained. If the constraint would be non-binding i.e., the remaining vase life (RVL) at retail level is larger than the predefined lower bound $VL_{\text{min}}$. In such cases, the costs can still be reduced while ensuring a sufficient quality level $VL_{\text{min}}$. Note that we again assume strict
Supply chain data for air freight.

Table 1

<table>
<thead>
<tr>
<th>Stage</th>
<th>Order quantity Q_i</th>
<th>Sojourn time (days)</th>
<th>Storage temperature (°C)</th>
<th>Fixed ordering costs (€/order)</th>
<th>Holding costs (€/bunch/week)</th>
<th>Transport costs (€/bunch)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - Grower</td>
<td>Q_i</td>
<td>3</td>
<td>1-5</td>
<td>20</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>2 - Handling</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 - (Grower - Nairobi)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 - Transport to Schiphol</td>
<td></td>
<td>3/8</td>
<td>1</td>
<td>500</td>
<td>0.035</td>
<td></td>
</tr>
<tr>
<td>5 - Handling</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 - (Airport - Wholesaler)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 - Wholesaler</td>
<td>Q_i</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 - Transport to Schiphol</td>
<td></td>
<td>1</td>
<td>1-5</td>
<td>50</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>4 - Handling</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 - (Wholesaler - Retailer)</td>
<td>Q_i</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 - Truck (Wholesaler - Retailer)</td>
<td></td>
<td>0.5</td>
<td>1</td>
<td>50</td>
<td>0.002</td>
<td></td>
</tr>
<tr>
<td>7 - Retailer</td>
<td>Q_i</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 - Customers</td>
<td></td>
<td>D=1000 weekly</td>
<td></td>
<td>50</td>
<td>0.13</td>
<td></td>
</tr>
</tbody>
</table>

4. Application to the Kenya-Dutch cut rose supply chain

The cut-flower market has become highly international in recent years and it is led by the main trading hubs located in Europe [14]. With 25% of annual growth rate, floriculture is considered as a high-value agricultural sector in which cut flowers account for 60% of the world trade, worth about US$ 11 billion annually.

Kenya is one of the countries that produces and exports cut roses to the European market. Most of the cut roses exported to Europe are from large to medium-scale growers. At the beginning of the chain, roses are harvested at the tight-bud stage and moved into cold storage facilities on farms. Currently, cut roses are either transported by air or sea freight with cooling systems and sold to wholesalers and retailers (flower shops) and finally reach customers with at least a 7-days vase life.

Fig. 4 shows current practice for the Kenyan-Dutch cold chain by air freight consisting of m = 3 echelons (grower, wholesaler, retailer), and J = 4 intermediates phases for transport and handling [22]. Based on the observations in practice, the temperature is commonly set to 1°C in the upstream chain while temperatures from the wholesaler to the retailer are commonly set to 10°C. The associated lead times for transport and handling in the different phases j = 1, .., 4 are presented in brackets for each phase.

For air transport, Table 1 provides the estimated costs.
Transportation costs are based on the value chain analysis in Hortiwise (2012). Ordering and inventory holding costs are based on expert knowledge from practice.

Table 2 provides an overview of the energy performance for all stages and phases in the cold chain. The parameters $p_{r,i}$ and $p_{f,j}$ are computed according to (11) in which $t_{r,\text{ref}}$ (Kenya) is 298.15 K; $t_{r,\text{ref}}$ (NL) is 293.15 K; $t_{\text{f,ref}}$ (Kenya) is 299.15 K; $t_{\text{f,ref}}$ (NL) is 294.15 K. Both the (sojourn) time and temperatures in the intermediate phases $j$ are fixed. In the phases $j = 3$ and 4, the lead-times are substantially longer than for airfreight. All other parameter values are equal for both cold chains.

Table 3 provides the costs for the sea freight chain. The values for the energy performance of the cooling system are the same as in the airfreight chain (see Table 2).

5. Results

The optimal order quantities for the retailer ($Q_{r,\text{ref}} = Q_{r}$), wholesaler ($Q_{w,\text{ref}} = Q_{w}$) and grower ($Q_{g,\text{ref}} = Q_{g}$) and total costs TC in (12) are shown in Table 4. For a weekly demand of 1000 bunches and a minimum vase life requirement at retail level of $VL_{\text{min}} = 7$ days in (10), the optimal order quantities for the Dutch retailer are 301 (air freight) or 206 bunches (sea freight).

After the third replenishment for the retailer, the wholesaler is out of stock and receives supply via the handling agents from the grower in Kenya (903 respectively 618 bunches) for the next cycle of three replenishments to the retailer. The supply chain network by air freight is substantially more expensive (22%) than its alternative by sea freight. Fig. 6 shows how the costs are divided over all echelons $i$ including the intermediate phases $j$ in both supply chains. The difference in total costs between both supply chains is foremost due to the distinction in transport modes for phase $j = 3$ (Fig. 4 and 5).

Sojourn times in each echelon $i$ are calculated according to Table 2 and 3 and shown in Table 5. The temperatures in the intermediate phases $j$ of the network, and at retail level $i = 1$ are set to common practice. The difference in total lead times between both supply networks is (again) mainly due to the shipping in phase $j = 3$. However, in contrast to the total costs, now at the benefit of air freight. The significant difference in total lead times between both supply chains may explain why transport by air freight still dominates in current practice, despite the substantial difference in total costs between both networks.

Fig. 7 shows the impact of both time and temperature on the losses of Vase Life (VL) for air- and sea freight. Despite the significantly larger losses in phase $j = 3$ for air freight (23%), Fig. 7 shows that the main losses of VL are due to the common lack of cooling regimes at retail level.

Given the calculated order levels, the introduction of cooling facilities at retail level may have a significant effect on the remaining vase life (RVL) at consumer level. Reversely, alternative cooling regimes and order levels can be recalculated assuming cooling facilities at retail level which gives insights in the development of the total costs in the cold chains. Although the latter integrated approach may translate into greater joint benefits for multi-echelon supply chains, it may not justify the additional investments for (most) retailers without considering the (dis) advantages for stakeholders upstream in the supply chains.
6. Concluding remarks

Literature shows that order-level inventory systems for perishable items are commonly modelled by parametric approaches based on probability distributions e.g., a time-dependent (Weibull) deterioration rate. In practice, deterioration processes of perishable fresh products are highly intricate, characterized by a strong dependence on specific product attributes. Crucially, these processes are foremost influenced by the interplay between both the (sojourn) time and the prevailing environmental conditions throughout the supply chain. Among all environmental conditions, temperature is the most critical factor for the deterioration of perishable, fresh products. Furthermore, addressing product deterioration in inventory management systems should prioritize holistic approaches, such as employing multi-echelon models that encompass entire cold chains.

This study aims to address the aforementioned limitations. Instead of employing a constant (time-dependent) deterioration rate, we propose a generalizable concept to integrate knowledge from biological sciences on product-specific deterioration into EOQ-based inventory management models. For the integration of product dependent deterioration in an EOQ-type modelling approach, general properties for constrained optimization problems are exploited for a single echelon.
case to derive an unconstrained optimization problem. Next, we extend the approach for its use in multi-echelon supply chains. To demonstrate the generalizability of the approach, product decay based on two fundamentally different deterioration processes i.e., microbial and respiration activities, are considered. Other time-temperature dependent deterioration models e.g., due to enzymatic degradation, and/or the breakdown by fungi can be integrated accordingly.

We demonstrate the applicability of extending the classical EOQ-model to product dependent deterioration in a multi-echelon supply chain, from a real-life case study in floriculture. Two commonly used multi-echelon supply chains for cut roses are considered and compared for a pre-defined minimum product quality at retail level. The results clearly show the combined impact of sojourn time and temperature regimes on the order quantity levels, the total costs, and the losses of product quality in all echelons and intermediate phases of transportation and handling in the supply chains. Case-specific analyses can provide new insights to initiate discussions between different supply chain actors and contribute to the substantiation of what is feasible in practice and what is not, which may lead to new collaborative guidelines between different actors in (long) supply chains.

We study the impact of the (sojourn) time- and variable temperature regimes on the lifetime of perishable fresh products in multi-echelon supply chains. Future research may integrate other environmental conditions to develop models, specifically tailored for real-life product-dependent deterioration processes. Furthermore, this study takes the classical EOQ model, including its fundamental assumptions, as a starting point. Meanwhile, numerous studies have scrutinized and addressed the underlying assumptions, or even extended the EOQ-concept to new aspects and circumstances in practice. Future research may be inspired by incorporating previous accomplishments into our approach, to develop novel EOQ-based models for perishable (fresh) products in multi-echelon supply chains.

CRediT authorship contribution statement

G.D.H. Claassen: Conceptualization, Formal analysis, Investigation, Methodology, Software, Writing – original draft, Writing – review & editing, Validation, Supervision. P. Kirst: Conceptualization, Methodology, Validation, Writing – review & editing. A. Thai Thi Van: Data curation, Formal analysis, Investigation, Software, Visualization. J.C.M. A. Snels: Data curation, Formal analysis, Resources, Writing – review & editing. X. Guo: Data curation, Formal analysis, Resources, Validation, Writing – review & editing. P. van Beek: Conceptualization, Methodology, Supervision, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests nor personal relationships that could have any impact on the work as reported in the manuscript.

Data availability

Data will be made available on request.

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