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RESEARCH ARTICLE

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Analysing determinate components of an approximated Luenberger–Hicks–Moorsteen productivity indicator: An application to German dairy-processing firms

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Abstract

The Luenberger–Hicks–Moorsteen (LHM) total factor productivity (TFP) indicator has sound theoretical properties, but its decomposition yields indeterminate components of technical change and scale efficiency change that can become infeasible. The current paper decomposes the approximating Bennet indicator, which results in determinate components of technical change, technical efficiency change, scale efficiency change and mix efficiency change that are always feasible. The application focuses on the German dairy-processing sector, an important postfarm supply chain actor. We compute 558 growth rates for the period 2011–2020. The results show that the LHM-approximating Bennet indicator decreases by on average 1.14% p.a., with substantial annual fluctuations. The underlying components of output- and input-oriented technical change also fluctuate substantially, and often conflict. Moreover, output- and input-oriented TFP efficiency change fluctuate moderately on average, which is mainly driven by scale efficiency change and mix efficiency

Abbreviations: CRS, constant-returns-to-scale; DEA, data envelopment analysis; EU, European Union; LHM, Luenberger–Hicks–Moorsteen; TFP, total factor productivity; US, United States; VRS, variable-returns-to-scale.

change. The components of technical efficiency change remain relatively stable on average. Indeterminateness is a relevant problem when decomposing the original LHM indicator for the current sample: depending on the specification, the proportion of infeasibilities when decomposing the original LHM indicator ranges between 6.09% and 15.95%. Our proposed determinate decomposition is thus a valuable complement. [EconLit Citations: D24, D25, Q13].

KEYWORDS

dairy-processing, data envelopment analysis, decomposition, productivity

1 | INTRODUCTION

Identifying the explanatory factors of economic performance is essential for managers and policy makers to guide decisions on resource allocation. Therefore, the production economics literature has shown interest not only in the appropriate definition of productivity change, but also its decomposition into various components of technical change and efficiency change. Solow (1957) fully attributes productivity change to technical change, which captures the shift of the production function. Nishimizu and Page (1982) enrich this framework by allowing for inefficient performance deviating from the technological frontier. Here, technical efficiency change indicates the catch-up to the frontier over time. Further refinements include the identification of scale efficiency change, which indicates the change in ability to exploit returns-to-scale, and mix efficiency change, which indicates the change in ability to allocate the correct mix of resources (O'Donnell, 2012).

From an agricultural perspective, there is a substantial literature documenting the global importance of public and private research and development in driving technical change and consequent productivity growth (e.g., Alston et al., 2009). Scale and mix efficiency change over time have also been found to be important determinants of agricultural productivity (O'Donnell, 2010). While bringing substantial benefits to society, through increased food supply and lower real prices, agricultural productivity growth has in effect constantly pressed farm managers to learn and adapt, through (i) adoption of new technology (both input and output related); (ii) reduction of technical inefficiency in relation to available production possibilities; (iii) substitution of capital for labour; (iv) specialisation; and (v) increasing business scale. Less well documented, particularly within specific product sectors, is the measurement and effect of productivity growth in the wider supply chain for foodstuffs. Furthermore, there is evidence that rates of farm level productivity growth seen in the 20th century may be less feasible in the 21st century (Pardey & Alston, 2021), and thus the relative contribution to overall productivity growth in the postfarm supply chain is of interest.

This paper focuses on productivity in the dairy-processing sector in Germany. To do so, we rely on a productivity framework based on the Luenberger–Hicks–Moorsteen (LHM) indicator introduced by Briec and Kerstens (2004). As mentioned by Ang and Kerstens (2017, 2020), the LHM indicator has several interesting theoretical properties. Traditionally, productivity change has been expressed as a ratio between productivity levels. However, ratio-based measures can become erroneous in the presence of negative, zero or very small values (Balk et al., 2003). As the LHM indicator can be expressed as a difference between productivity levels, it overcomes this issue. Moreover, being “additively complete” (O'Donnell, 2012), it can be expressed as the difference between an output indicator and an input indicator. This allows straightforward verification of the degree to which productivity change is driven by output change and input change. Finally, the LHM indicator is "determinate" (Briec & Kerstens, 2011). Determinateness guarantees that the LHM indicator cannot become indeterminate or infinity if any of its arguments becomes zero or infinity. In practice, this means that the LHM indicator can always be computed. This contrasts with the Malmquist index and the Luenberger indicator, which are prone to infeasibilities (Briec & Kerstens, 2009). Such infeasibilities arise if the observation considered cannot project to the reference technology. This may occur in distance functions where the period of the observation differs from that of the reference technology, particularly when using data envelopment analysis (DEA) or free disposal hull.

However, although the LHM indicator is determinate, its components of technical change and scale efficiency change are not. Hence, these components cannot always be computed. The reason is that they consist of intertemporal directional distance functions that are prone to infeasibilities. Evidently, indeterminate components thwart the analytical toolkit for understanding the explanatory factors of economic performance. The current paper addresses the problem of indeterminateness in decomposing the LHM indicator in an application to a comprehensive data set representing an important postfarm supply chain actor.

Our approach focuses on a decomposition of a price-normalised Bennet indicator that *approximates* the LHM indicator, instead of decomposing the LHM indicator itself. Ang and Kerstens (2020) show that the Bennet indicator, by which input (output) prices are *separately* normalised, is a superlative indicator for the LHM indicator in Diewert (1976)'s sense: that is, if there is profit-maximising behaviour and the directional distance function can be represented by a quadratic functional form up to the second order with time-invariant second-order coefficients. This price-normalised Bennet indicator coincides with the LHM indicator. This Bennet indicator only consists of a simple index formula of quantities and prices, not requiring any estimation of distance functions. It contrasts with the LHM indicator that does not require price information, but relies on the estimation of eight directional distance functions employing quantity information. The current paper introduces a decomposition of the LHM-approximating Bennet indicator into determinate, input- and output-oriented components of technical change, technical efficiency change, scale efficiency change and mix efficiency change.

The paper closest to ours is Ang (2019), who also develops a framework to decompose the Bennet indicator into components of technical change, technical efficiency change, scale efficiency change and mix efficiency change. However, Ang uses a *common* price normalisation for all inputs and outputs. Such a price-normalised Bennet indicator is a superlative indicator for the Luenberger indicator (Chambers, 2002). It is equivalent to the Luenberger indicator under the same behavioural and technological assumptions as in the equivalence to the LHM indicator. The difference between the two equivalences thus lies in the price normalisation of the inputs and outputs. Our proposed decomposition exploits the duality between the input (output) directional distance function and the cost (revenue) function, introduced by Chambers et al. (1996), whereas Ang (2019)'s decomposition of the Luenberger-approximating Bennet indicator exploits the duality between the directional distance function and the profit function in line with Chambers et al. (1998). The additive completeness of the LHM indicator carries over to the decomposition of its approximating Bennet indicator: all components have an input- or output-orientation. Therefore, our proposed decomposition allows detailed input- and output-specific advice on how to increase productivity to be given. The Bennet indicator approximating the additively *in*complete Luenberger indicator cannot yield such input- and output-oriented components, which precludes giving such detailed advice.

We illustrate our decomposition framework by an application to a sample of 694 observations of German dairy-processing firms for the years 2011–2020, employing DEA. The motivation for choosing Germany is primarily the relative richness of the Orbis data for this country. Further, while the economic performance of dairy *farms* has been considered extensively, both globally (e.g., Bravo-Ureta et al., 2020) and in the context of Germany more specifically (e.g., Abdulai & Tietje, 2007; Sauer & Latacz-Lohmann, 2015; Skevas et al., 2018), little work has been conducted on German dairy-*processing* firms. Exceptions are Soboh et al. (2014), where technical efficiency (as estimated by stochastic frontier analysis) is compared for co-operative and investor-owned dairy-processing firms in Europe; and Kapelko (2017), where the Luenberger productivity change indicator is used with a data set of large dairy-processing firms. Both studies include

Germany but do not disaggregate results for the country. A more recent paper of Čechura and Žáková Kroupová (2021) uses European dairy-processing industry data, including Germany, up to 2018. While concluding that overall, the industry was competitive and efficient, the authors note that it is lagging behind the best-practice technology.

2 | DECOMPOSING THE ORIGINAL LHM INDICATOR

Let $\mathbf{x} = (x_1, \ldots, x_N) \in \mathbb{R}^N_+$ and $\mathbf{y} = (y_1, \ldots, y_M) \in \mathbb{R}^M_+$ be the vector of respectively input quantities and output quantities. The actual technology set in time period t is described as:

$$T_t = \left\{(\mathbf{x}_t, \mathbf{y}_t) \in \mathbb{R}^{N+M}_+ : \mathbf{x}_t \text{ can produce } \mathbf{y}_t\right\}. \tag{1}$$

Following Chambers (2002), we make the conventional assumptions that T_t is closed and bounded, inputs and outputs are strongly disposable, and inaction is possible.

The directional distance function for (a, b, c), by which a, b and c can be defined in time t or $t + 1$, is (Chambers et al., 1998):

$$\overrightarrow{D}_c(\mathbf{x}_a, \mathbf{y}_b; \mathbf{g}^x, \mathbf{g}^y) = \sup\{\beta \in \mathbb{R} : (\mathbf{x}_a - \beta\mathbf{g}^x, \mathbf{y}_b + \beta\mathbf{g}^y) \in T_c\}, \tag{2}$$

if $(\mathbf{x}_a - \beta\mathbf{g}^x, \mathbf{y}_b + \beta\mathbf{g}^y) \in T_c$ for some β and $D_c(\mathbf{x}_a, \mathbf{y}_b; \mathbf{g}^x, \mathbf{g}^y) = -\infty$ otherwise. Here, $(\mathbf{g}^x, \mathbf{g}^y) \in \mathbb{R}^N_+ \times \mathbb{R}^M_+$ is a nonzero vector. We focus on two particular cases of $\overrightarrow{D}_c(\mathbf{x}_a, \mathbf{y}_b; \mathbf{g}^x, \mathbf{g}^y)$. First, we focus on the input directional distance function $\overrightarrow{D}_c(\mathbf{x}_a, \mathbf{y}_b; \mathbf{g}^x, \mathbf{0}^M)$, which contracts inputs along the input directional vector $\mathbf{g}^x$ as defined by Chambers et al. (1996), holding outputs constant, and is thus a measure of input technical inefficiency. Second, we focus on the output directional distance function $\overrightarrow{D}_c(\mathbf{x}_a, \mathbf{y}_b; \mathbf{0}^N, \mathbf{g}^y)$, which expands outputs along the output directional vector $\mathbf{g}^y$, holding inputs constant, and is thus a measure of output technical inefficiency.

The directional distance function defined in Equation (2) serves as the building block of the LHM indicator introduced by Briec and Kerstens (2004):

$$\begin{aligned}
&LHM(\mathbf{x}_t, \mathbf{y}_t, \mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{g}^x, \mathbf{g}^y)\\
&= \frac{1}{2}[(\overrightarrow{D}_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{0}^N, \mathbf{g}^y) - \overrightarrow{D}_t(\mathbf{x}_t, \mathbf{y}_{t+1}; \mathbf{0}^N, \mathbf{g}^y))\\
&+(\overrightarrow{D}_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_t; \mathbf{0}^N, \mathbf{g}^y) - \overrightarrow{D}_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{0}^N, \mathbf{g}^y))]\\
&-\frac{1}{2}[(\overrightarrow{D}_t(\mathbf{x}_{t+1}, \mathbf{y}_t; \mathbf{g}^x, \mathbf{0}^M) - \overrightarrow{D}_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}^x, \mathbf{0}^M))\\
&+(\overrightarrow{D}_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{g}^x, \mathbf{0}^M) - \overrightarrow{D}_{t+1}(\mathbf{x}_t, \mathbf{y}_{t+1}; \mathbf{g}^x, \mathbf{0}^M))].
\end{aligned} \tag{3}$$

The LHM indicator $LHM(\cdot)$ is an additively complete total factor productivity (TFP) indicator that measures the sum of output quantity growth, $\frac{1}{2}[(\overrightarrow{D}_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{0}^N, \mathbf{g}^y) - \overrightarrow{D}_t(\mathbf{x}_t, \mathbf{y}_{t+1}; \mathbf{0}^N, \mathbf{g}^y)) + (\overrightarrow{D}_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_t; \mathbf{0}^N, \mathbf{g}^y) - \overrightarrow{D}_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{0}^N, \mathbf{g}^y))]$, and input quantity decline, $-\frac{1}{2}[(\overrightarrow{D}_t(\mathbf{x}_{t+1}, \mathbf{y}_t; \mathbf{g}^x, \mathbf{0}^M) - \overrightarrow{D}_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}^x, \mathbf{0}^M)) + (\overrightarrow{D}_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{g}^x, \mathbf{0}^M) - \overrightarrow{D}_{t+1}(\mathbf{x}_t, \mathbf{y}_{t+1}; \mathbf{g}^x, \mathbf{0}^M))]$. A positive $LHM(\cdot)$ indicates that the firm has improved its conversion from input quantities to output quantities between period t and $t + 1$. As none of the eight directional distance functions can yield infeasibilities, $LHM(\cdot)$ fulfils the determinateness axiom (Briec & Kerstens, 2011).

Ang and Kerstens (2017), employing data for US agriculture, show how to decompose $LHM(\cdot)$ into components of technical change, technical efficiency change and scale efficiency change, using an output orientation or an input orientation. Using the output orientation, we have the following components of technical change OTC_{LHM}, technical efficiency change $OTEC_{LHM}$ and scale efficiency change $OSEC_{LHM}$:

$$LHM(\cdot) = OTC_{LHM} + OTEC_{LHM} + OSEC_{LHM}, \text{where} \tag{4a}$$

$$\begin{aligned} OTC_{LHM} \equiv \frac{1}{2}\{&[\vec{D}_{t+1}(\mathbf{x}_t, \mathbf{y}_t; \mathbf{0}^N, \mathbf{g}^y) - \vec{D}_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{0}^N, \mathbf{g}^y)] \\ &+[\vec{D}_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{0}^N, \mathbf{g}^y) - \vec{D}_t(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{0}^N, \mathbf{g}^y)]\}, \end{aligned} \tag{4b}$$

$$OTEC_{LHM} \equiv \vec{D}_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{0}^N, \mathbf{g}^y) - \vec{D}_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{0}^N, \mathbf{g}^y), \text{ and} \tag{4c}$$

$$\begin{aligned} OSEC_{LHM} \equiv \frac{1}{2}\{&[\vec{D}_t(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{0}^N, \mathbf{g}^y) - \vec{D}_t(\mathbf{x}_t, \mathbf{y}_{t+1}; \mathbf{0}^N, \mathbf{g}^y)] \\ &+[\vec{D}_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_t; \mathbf{0}^N, \mathbf{g}^y) - \vec{D}_{t+1}(\mathbf{x}_t, \mathbf{y}_t; \mathbf{0}^N, \mathbf{g}^y)]\} \\ &-\frac{1}{2}\{[\vec{D}_t(\mathbf{x}_{t+1}, \mathbf{y}_t; \mathbf{g}^x, \mathbf{0}^M) - \vec{D}_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}^x, \mathbf{0}^M)] \\ &+[\vec{D}_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{g}^x, \mathbf{0}^M) - \vec{D}_{t+1}(\mathbf{x}_t, \mathbf{y}_{t+1}; \mathbf{g}^x, \mathbf{0}^M)]\}. \end{aligned} \tag{4d}$$

Here, OTC_{LHM} measures the shift of the technological frontier in the output direction between t and $t + 1$. One assesses this shift with respect to $(\mathbf{x}_t, \mathbf{y}_t)$ and $(\mathbf{x}_{t+1}, \mathbf{y}_{t+1})$ and taking the arithmetic average hereof to avoid an arbitrary choice of base period. A positive OTC_{LHM} indicates an expansion of production possibilities in the output direction. Additionally, $OTEC_{LHM}$ measures the extent output technical inefficiency decreases. A positive $OTEC_{LHM}$ indicates a decrease in output technical inefficiency and hence a catch-up with the frontier in the output direction. Finally, $OSEC_{LHM}$ indicates the extent to which the employed scale changes, as measured by a change in the frontier's gradient. In particular, the frontier's gradient is assessed by finite, difference-based approximations at t and $t + 1$. As in OTC_{LHM}, $OSEC_{LHM}$ is computed by the arithmetic average of the values of both periods to avoid an arbitrary choice of base period. A positive $OSEC_{LHM}$ indicates an improvement in scale economies. In practice, one may residually assess $OSEC_{LHM} \equiv LHM(\cdot) - OTC_{LHM} - OTEC_{LHM}$ for computational facility.

This decomposition requires the computation of two additional intertemporal directional distance functions: $D_{t+1}(\mathbf{x}_t, \mathbf{y}_t; \mathbf{0}^N, \mathbf{g}^y)$ and $D_t(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{0}^N, \mathbf{g}^y)$. It reveals that the LHM indicator can be regarded as the sum of Chambers et al. (1996)'s output-oriented Luenberger indicator and $OSEC_{LHM}$. The decomposition of the output-oriented Luenberger indicator yields the same components, OTC_{LHM} and $OTEC_{LHM}$. In the context of the computation of the Luenberger indicator, Briec and Kerstens (2009) prove that $D_{t+1}(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}^x, \mathbf{g}^y)$ and $D_t(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{g}^x, \mathbf{g}^y)$ can become infeasible and are thus indeterminate. Entailing these particular components with $\mathbf{g}^x = \mathbf{0}^M$, OTC_{LHM} and $OSEC_{LHM}$ as a result violate the determinatess axiom. The above decomposition can also be executed for the input direction with $\mathbf{g}^y = \mathbf{0}^N$, yielding analogous components. The same argumentation holds: input-oriented technical change and scale efficiency change violate the determinateness axiom. Shen et al. (2019) propose to perform both decompositions and take the arithmetic averages of all respective components. This may worsen the indeterminateness problem, as the total number of infeasibilities is determined by the union of the infeasibilities from the output-oriented decomposition, on the one hand, and the infeasibilities from the input-oriented decomposition, on the other hand. This total number equals at least the number of infeasibilities in the separate output (input)-oriented decomposition.

3 | DECOMPOSING THE LUENBERGER-HICKS-MOORSTEEN-APPROXIMATING BENNET INDICATOR

Let $\mathbf{w} = (w_1, ..., w_N) \in \mathbb{R}^N_{++}$ and $\mathbf{p} = (p_1, ..., p_M) \in \mathbb{R}^M_{++}$ be the vector of respectively input prices and output prices. Denoting "·" as the dot product and without imposing axiomatic assumptions on $(\mathbf{x}, \mathbf{y})$, the Bennet (quantity) indicator that assesses the difference in TFP between period t and $t + 1$ is defined as (Balk, 1998; Chambers, 2002):

$$
\begin{aligned}
&B(\mathbf{x}_t, \mathbf{y}_t, \mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{w}_t, \mathbf{w}_{t+1}, \mathbf{p}_t, \mathbf{p}_{t+1}) \\
&= \frac{1}{2}(\mathbf{p}_t + \mathbf{p}_{t+1}) \cdot (\mathbf{y}_{t+1} - \mathbf{y}_t) - \frac{1}{2}(\mathbf{w}_t + \mathbf{w}_{t+1}) \cdot (\mathbf{x}_{t+1} - \mathbf{x}_t).
\end{aligned} \tag{5}
$$

The Bennet indicator $B(\cdot)$ is an additively complete TFP indicator that measures the difference between output quantity growth, $\frac{1}{2}(\mathbf{p}_t + \mathbf{p}_{t+1}) \cdot (\mathbf{y}_{t+1} - \mathbf{y}_t)$, and input quantity growth, $\frac{1}{2}(\mathbf{w}_t + \mathbf{w}_{t+1}) \cdot (\mathbf{x}_{t+1} - \mathbf{x}_t)$. A positive $B(\cdot)$ indicates that the firm has improved its conversion from input quantities to output quantities between period t and $t + 1$. It is an *empirical* productivity measure that does not necessitate estimation of distance functions, but requires the availability of prices and quantities of all inputs and outputs. This contrasts with the *theoretical* LHM indicator, which only requires information on quantities of inputs and outputs, but requires the estimation of eight directional distance functions (Lovell, 2016). Consisting of a simple index number formula that always can be computed, $B(\cdot)$ fulfils the determinateness axiom like the LHM indicator.

Let us, without loss of generality, normalise the vector of input and output prices in period t as $(\check{\mathbf{w}}_t, \hat{\mathbf{p}}_t) \equiv \left(\frac{\mathbf{w}_t}{\mathbf{w}_t \cdot \mathbf{g}^x}, \frac{\mathbf{p}_t}{\mathbf{p}_t \cdot \mathbf{g}^y}\right)$. Ang and Kerstens (2020) show that the following Bennet indicator approximates the LHM indicator developed by Briec and Kerstens (2004):

$$
BLHM \equiv B(\mathbf{x}_t, \mathbf{y}_t, \mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \check{\mathbf{w}}_t, \check{\mathbf{w}}_{t+1}, \hat{\mathbf{p}}_t, \hat{\mathbf{p}}_{t+1}). \tag{6}
$$

In particular, Ang and Kerstens (2020) prove that $BLHM$ is a superlative indicator for $LHM(\cdot)$ in Diewert (1976)'s sense. If the input and output directional distance functions can be represented by a quadratic functional form up to the second order with time-invariant second-order coefficients and there is profit-maximising behaviour, then $BLHM$ is equivalent to $LHM(\cdot)$.

Our strategy is to decompose $BLHM$ by adapting the framework developed by Ang (2019) to the current context. Ang shows how to decompose $B(\mathbf{x}_t, \mathbf{y}_t, \mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \bar{\mathbf{w}}_t, \bar{\mathbf{w}}_{t+1}, \bar{\mathbf{p}}_t, \bar{\mathbf{p}}_{t+1})$, where $(\bar{\mathbf{w}}_t, \bar{\mathbf{p}}_t) \equiv \left(\frac{\mathbf{w}_t}{\mathbf{w}_t \cdot \mathbf{g}^x + \mathbf{p}_t \cdot \mathbf{g}^y}, \frac{\mathbf{p}_t}{\mathbf{w}_t \cdot \mathbf{g}^x + \mathbf{p}_t \cdot \mathbf{g}^y}\right)$. The decomposition proposed in the current paper involves a separate consideration of output- and input-oriented components.

Our decomposition framework requires making assumptions on the technology. Henceforth, we maintain the same assumptions on the technology as in case of the LHM indicator. The technology set T_t is defined by Equation (1). We assume that T_t is closed and bounded, inputs and outputs are strongly disposable, and inaction is possible. At time t, T_t is characterised by the contemporaneous directional distance function $\vec{D}_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}^x, \mathbf{g}^y)$ (Chambers et al., 1998):

$$
\vec{D}_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}^x, \mathbf{g}^y) = \max\{\beta \geq 0 : (\mathbf{x}_t - \beta \mathbf{g}^x, \mathbf{y}_t + \beta \mathbf{g}^y) \in T_t\}, \tag{7}
$$

where, as in Equation (2), $(\mathbf{g}^x, \mathbf{g}^y) \in \mathbb{R}_+^N \times \mathbb{R}_+^M$ is a nonzero vector. Since $(\mathbf{x}_t, \mathbf{y}_t) \in T_t$, Equation (7) is simpler than Equation (2). Following Chambers et al. (1998), a nonnegative $\vec{D}_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}^x, \mathbf{g}^y)$ exists if and only if $(\mathbf{x}_t, \mathbf{y}_t) \in T_t$. Therefore, we can replace the *supremum* operator by the *maximum* operator and state $\beta \geq 0$. As before, we focus on $\vec{D}_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{0}^N, \mathbf{g}^y)$ and $\vec{D}_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}^x, \mathbf{0}^M)$, which are both nonnegative in this setting.

We assume profit-maximising behaviour, which we separately disentangle into revenue-maximising and cost-minimising behaviour. Let us consider input and output prices at time s, $(\mathbf{w}_s, \mathbf{p}_s)$. The revenue and cost functions for T_t are respectively (Chambers et al., 1996):

$$
R_t(\mathbf{x}_t, \mathbf{p}_s) = \max_{\mathbf{y}}\{\mathbf{p}_s \cdot \mathbf{y} \text{ s.t. } \vec{D}_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{0}^N, \mathbf{g}^y) \geq 0\} \tag{8a}
$$

and

$$C_t(\mathbf{y}_t, \mathbf{w}_s) = \min_{\mathbf{x}}\{\mathbf{w}_s \cdot \mathbf{x} \text{ s.t. } \vec{D}_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}^x, \mathbf{0}^M) \geq 0\}, \tag{8b}$$

which both exist under the maintained assumptions on T_t and any strictly positive vector of input and output prices. Following the duality result of Chambers et al. (1996), we can decompose nonnegative revenue inefficiency, $RI_t(\mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{p}}_s, \mathbf{g}^y)$, and nonnegative cost inefficiency, $CI_t(\mathbf{x}_t, \mathbf{y}_t, \check{\mathbf{w}}_s, \mathbf{g}^x)$, as follows:

$$RI_t(\mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{p}}_s, \mathbf{g}^y) \equiv R_t(\mathbf{x}_t, \hat{\mathbf{p}}_s) - \hat{\mathbf{p}}_s \cdot \mathbf{y}_t = \vec{D}_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{0}^N, \mathbf{g}^y) + OMI_t(\mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{p}}_s, \mathbf{g}^y) \tag{9a}$$

and

$$CI_t(\mathbf{x}_t, \mathbf{y}_t, \check{\mathbf{w}}_s, \mathbf{g}^x) \equiv \check{\mathbf{w}}_s \cdot \mathbf{x}_t - C_t(\mathbf{y}_t, \check{\mathbf{w}}_s) = \vec{D}_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}^x, \mathbf{0}^M) + IMI_t(\mathbf{x}_t, \mathbf{y}_t, \check{\mathbf{w}}_s, \mathbf{g}^x), \tag{9b}$$

where $OMI_t(\mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{p}}_s, \mathbf{g}^y)$ and $IMI_t(\mathbf{x}_t, \mathbf{y}_t, \check{\mathbf{w}}_s, \mathbf{g}^x)$ are nonnegative. Here, $OMI_t(\mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{p}}_s, \mathbf{g}^y)$ is the output mix inefficiency and indicates the deviation from the revenue-maximising point because of an incorrect mix of outputs that cannot be explained by technical output inefficiency; $IMI_t(\mathbf{x}_t, \mathbf{y}_t, \check{\mathbf{w}}_s, \mathbf{g}^x)$ is the input mix inefficiency and indicates the deviation from the cost-minimising point because of an incorrect mix of inputs that cannot be explained by technical input inefficiency. Output mix inefficiency and input mix inefficiency thus indicate deviations from respectively revenue-maximising behaviour and cost-minimising behaviour. Observe that these components of mix inefficiency in our framework coincide with those of allocative inefficiency in Chambers et al. (1998).

Let us consider the benchmark technology set that exhibits *constant returns to scale* (CRS) at time t, T_t^{CRS}:

$$T_t^{CRS} = \{\delta(\mathbf{x}_t, \mathbf{y}_t) \in T_t \text{ for all } \delta > 0\}. \tag{10}$$

Here, T_t^{CRS} is the conical closure of T_t, which implies $T_t^{CRS} \supseteq T_t$. The relationships in Equations (7), (8a)–(8b), and (9a)–(9b) can then also be established for CRS. Denoting "CRS" in the superscript, this yields $\vec{D}_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t; \mathbf{0}^N, \mathbf{g}^y)$, $\vec{D}_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}^x, \mathbf{0}^M)$, $R_t^{CRS}(\mathbf{x}_t, \hat{\mathbf{p}}_s)$, $C_t^{CRS}(\mathbf{y}_t, \check{\mathbf{w}}_s)$, $RI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{p}}_s, \mathbf{g}^y)$, $OMI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{p}}_s, \mathbf{g}^y)$, $CI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \check{\mathbf{w}}_s, \mathbf{g}^x)$, and $IMI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \check{\mathbf{w}}_s, \mathbf{g}^x)$.

We define primal output scale inefficiency, $POSI_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{0}^N, \mathbf{g}^y)$, and primal input scale inefficiency, $PISI_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}^x, \mathbf{0}^M)$, at time t as:

$$POSI_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{0}^N, \mathbf{g}^y) = \vec{D}_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t; \mathbf{0}^N, \mathbf{g}^y) - \vec{D}_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{0}^N, \mathbf{g}^y) \tag{11a}$$

and

$$PISI_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}^x, \mathbf{0}^M) = \vec{D}_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}^x, \mathbf{0}^M) - \vec{D}_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}^x, \mathbf{0}^M), \tag{11b}$$

where $POSI_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{0}^N, \mathbf{g}^y)$ and $PISI_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}^x, \mathbf{0}^M)$ are nonnegative. Here, $POSI_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{0}^N, \mathbf{g}^y)$ indicates the deviation from the optimal scale in terms of output quantities; $PISI_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}^x, \mathbf{0}^M)$ indicates the deviation from the optimal scale in terms of input quantities.

We define dual output scale inefficiency, $DOSI_t(\mathbf{x}_t, \hat{\mathbf{p}}_s)$, and dual input scale inefficiency, $DISI_t(\mathbf{y}_t, \check{\mathbf{w}}_s)$, for the technology at time t and prices at time s as:

$$DOSI_t(\mathbf{x}_t, \hat{\mathbf{p}}_s) = R_t^{CRS}(\mathbf{x}_t, \hat{\mathbf{p}}_s) - R_t(\mathbf{x}_t, \hat{\mathbf{p}}_s) \tag{12a}$$

and

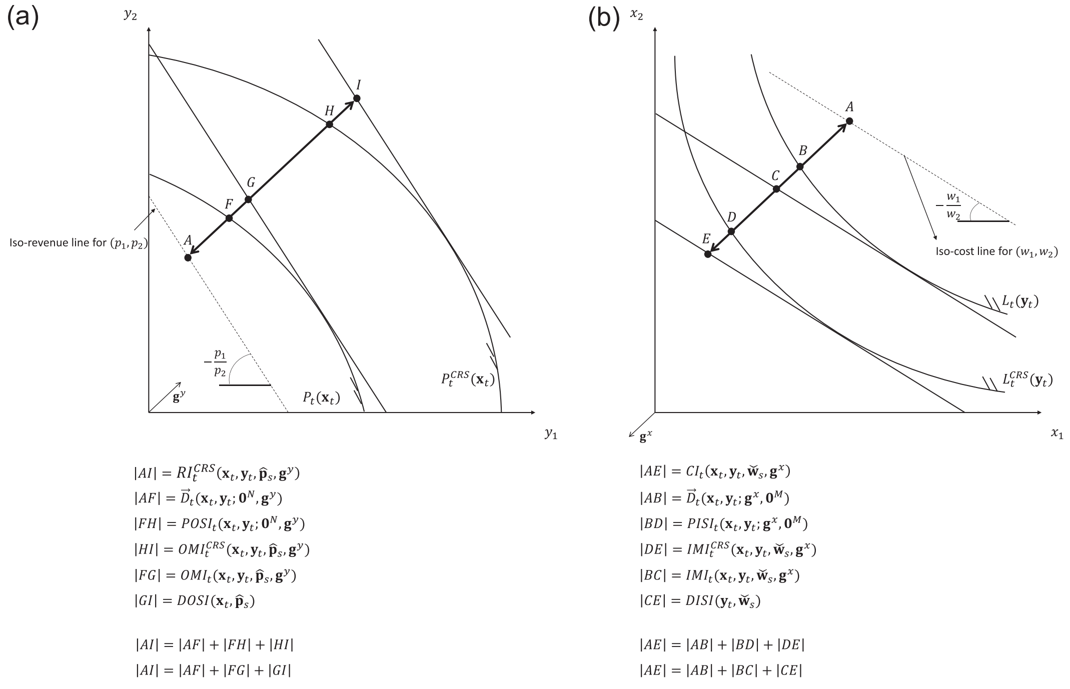

FIGURE 1 Components of revenue inefficiency and cost inefficiency. (a) Two decompositions of revenue inefficiency, (b) two decompositions of cost inefficiency.

$$DISI_t(\mathbf{y}_t, \check{\mathbf{w}}_s) = C_t(\mathbf{y}_t, \check{\mathbf{w}}_s) - C_t^{CRS}(\mathbf{y}_t, \check{\mathbf{w}}_s), \tag{12b}$$

where $DOSI_t(\mathbf{x}_t, \hat{\mathbf{p}}_s)$ and $DISI_t(\mathbf{y}_t, \check{\mathbf{w}}_s)$ are non-negative. Here, $DOSI_t(\mathbf{x}_t, \hat{\mathbf{p}}_s)$ indicates the deviation from the optimal scale in quantities and prices of outputs; $DISI_t(\mathbf{y}_t, \check{\mathbf{w}}_s)$ indicates the deviation from the optimal scale in terms of quantities and prices of inputs.

Combining Equations (9a), (11a), and (12a), $RI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{p}}_s, \mathbf{g}^y)$ can be decomposed in two ways (see Figure 1a):

$$RI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{p}}_s, \mathbf{g}^y) = \vec{D}_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}^x, \mathbf{0}^M) + POSI_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{0}^N, \mathbf{g}^y) + OMI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{p}}_s, \mathbf{g}^y) \tag{13a}$$

and

$$RI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{p}}_s, \mathbf{g}^y) = \vec{D}_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{0}^N, \mathbf{g}^y) + DOSI_t(\mathbf{x}_t, \hat{\mathbf{p}}_s) + OMI_t(\mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{p}}_s, \mathbf{g}^y). \tag{13b}$$

Combining Equations (9b), (11b), and (12b), $CI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \check{\mathbf{w}}_s, \mathbf{g}^x)$ can be decomposed in two ways (see Figure 1b):

$$CI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \check{\mathbf{w}}_s, \mathbf{g}^x) = \vec{D}_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}^x, \mathbf{0}^M) + PISI_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}^x, \mathbf{0}^M) + IMI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \check{\mathbf{w}}_s, \mathbf{g}^x) \tag{14a}$$

and

$$CI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \check{\mathbf{w}}_s, \mathbf{g}^x) = \vec{D}_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}^x, \mathbf{0}^M) + DISI_t(\mathbf{y}_t, \check{\mathbf{w}}_s) + IMI_t(\mathbf{x}_t, \mathbf{y}_t, \check{\mathbf{w}}_s, \mathbf{g}^x). \tag{14b}$$

These components provide the building blocks required for the decomposition of *BLHM*. They are determinate. The primal components $POSI_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{0}^N, \mathbf{g}^y)$ and $PISI_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}^x, \mathbf{0}^M)$, and are nonnegative, as all observations have been defined as elements of the feasible technology set (thus avoiding intertemporal comparisons). The dual components $OMI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{p}}_s, \mathbf{g}^y)$, $OMI_t(\mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{p}}_s, \mathbf{g}^y)$, $IMI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \check{\mathbf{w}}_s, \mathbf{g}^x)$, $IMI_t(\mathbf{x}_t, \mathbf{y}_t, \check{\mathbf{w}}_s, \mathbf{g}^x)$, $DOSI_t(\mathbf{x}_t, \hat{\mathbf{p}}_s)$, and $DISI_t(\mathbf{y}_t, \check{\mathbf{w}}_s)$ are also all nonnegative, as revenue and cost functions are always well-defined for any vector of strictly positive output and input prices (Färe & Primont, 1995).

We are now ready to decompose $BLHM$ into output technical change, OTC, and output TFP efficiency change, $OTFPEC$, on the one hand, and input technical change, ITC and input TFP efficiency change, $OTFPEC$, on the other hand:

$$\begin{aligned} BLHM &= \left[\frac{1}{2}(\hat{\mathbf{p}}_t + \hat{\mathbf{p}}_{t+1}) \cdot (\mathbf{y}_{t+1} - \mathbf{y}_t)\right] + \left[-\frac{1}{2}(\hat{\mathbf{w}}_t + \hat{\mathbf{w}}_{t+1}) \cdot (\mathbf{x}_{t+1} - \mathbf{x}_t)\right] \\ &\equiv OC + IC \\ &\equiv [OTC + OTFPEC] + [ITC + ITFPEC], \end{aligned} \tag{15a}$$

where

$$OTC \equiv \frac{1}{2}\left\{\left[R_{t+1}^{CRS}(\mathbf{x}_{t+1}, \hat{\mathbf{p}}_t) - R_t(\mathbf{x}_t, \hat{\mathbf{p}}_t)\right] + \left[R_{t+1}^{CRS}(\mathbf{x}_{t+1}, \hat{\mathbf{p}}_{t+1}) - R_t^{CRS}(\mathbf{x}_t, \hat{\mathbf{p}}_{t+1})\right]\right\}, \tag{15b}$$

$$\begin{aligned} OTFPEC \equiv \frac{1}{2}\Big\{&\left[RI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{p}}_t, \mathbf{g}^y) - RI_{t+1}^{CRS}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \hat{\mathbf{p}}_t, \mathbf{g}^y)\right] \\ &+\left[RI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{p}}_{t+1}, \mathbf{g}^y) - RI_{t+1}^{CRS}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \hat{\mathbf{p}}_{t+1}, \mathbf{g}^y)\right]\Big\}, \end{aligned} \tag{15c}$$

$$ITC \equiv \frac{1}{2}\left\{\left[C_t^{CRS}(\mathbf{y}_t, \check{\mathbf{w}}_t) - C_{t+1}^{CRS}(\mathbf{y}_{t+1}, \check{\mathbf{w}}_t)\right] + \left[C_t^{CRS}(\mathbf{y}_t, \check{\mathbf{w}}_{t+1}) - C_{t+1}^{CRS}(\mathbf{y}_{t+1}, \check{\mathbf{w}}_{t+1})\right]\right\}, \text{ and} \tag{15d}$$

$$\begin{aligned} ITFPEC \equiv \frac{1}{2}\Big\{&\left[CI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \check{\mathbf{w}}_t, \mathbf{g}^x) - CI_{t+1}^{CRS}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \check{\mathbf{w}}_t, \mathbf{g}^x)\right] \\ &+\left[CI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \check{\mathbf{w}}_{t+1}, \mathbf{g}^x) - CI_{t+1}^{CRS}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \check{\mathbf{w}}_{t+1}, \mathbf{g}^x)\right]\Big\}. \end{aligned} \tag{15e}$$

Here, $BLHM$ is shown as the sum of output growth, $OC \equiv \frac{1}{2}(\hat{\mathbf{p}}_t + \hat{\mathbf{p}}_{t+1}) \cdot (\mathbf{y}_{t+1} - \mathbf{y}_t)$, and input decline, $IC \equiv -\frac{1}{2}(\hat{\mathbf{w}}_t + \hat{\mathbf{w}}_{t+1}) \cdot (\mathbf{x}_{t+1} - \mathbf{x}_t)$, in which we decompose OC and IC separately. We closely follow the exposition of Ang and Kerstens (2023, pp. 26–29) in taking Laspeyres and Paasche perspectives for further decomposition. Technical change indicates the shift of the CRS frontier between periods t and $t + 1$. It is assessed dually with regard to the revenue function in OTC, and the cost function in ITC. The change in the maximum revenue for $\hat{\mathbf{p}}_t$ defines the Laspeyres-type measure of output-oriented technical change, $R_{t+1}^{CRS}(\mathbf{x}_{t+1}, \hat{\mathbf{p}}_t) - R_t^{CRS}(\mathbf{x}_t, \hat{\mathbf{p}}_t)$; for $\hat{\mathbf{p}}_{t+1}$, it defines the Paasche-type measure of output-oriented technical change, $R_{t+1}^{CRS}(\mathbf{x}_{t+1}, \hat{\mathbf{p}}_{t+1}) - R_t^{CRS}(\mathbf{x}_t, \hat{\mathbf{p}}_{t+1})$. Figure 2a shows that OTC is the arithmetic average of these two components. The change in the minimum cost for $\check{\mathbf{w}}_t$ defines the Laspeyres-type measure of input-oriented technical change, $C_t^{CRS}(\mathbf{y}_t, \check{\mathbf{w}}_t) - C_{t+1}^{CRS}(\mathbf{y}_{t+1}, \check{\mathbf{w}}_t)$; for $\check{\mathbf{w}}_{t+1}$ it defines the Paasche-type measure of input-oriented technical change, $C_t^{CRS}(\mathbf{y}_t, \check{\mathbf{w}}_{t+1}) - C_{t+1}^{CRS}(\mathbf{y}_{t+1}, \check{\mathbf{w}}_{t+1})$. Figure 2b shows that ITC is the arithmetic average of these two components. If $OTC\,(ITC) > 0$, the production possibilities (input requirement) set, benchmarked by the maximum revenue (minimum cost), expands (contracts), which indicates technical progress in the output (input) direction.

Further, $OTFPEC$ and $ITFPEC$, respectively, represent the change in revenue inefficiency and cost inefficiency between periods t and $t + 1$. The change in revenue inefficiency for $\hat{\mathbf{p}}_t$ defines the Laspeyres-type measure of revenue efficiency change, $RI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{p}}_t, \mathbf{g}^y) - RI_{t+1}^{CRS}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \hat{\mathbf{p}}_t, \mathbf{g}^y)$; for $\hat{\mathbf{p}}_{t+1}$ it defines the Paasche-type measure of revenue efficiency change, $RI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{p}}_{t+1}, \mathbf{g}^y) - RI_{t+1}^{CRS}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \hat{\mathbf{p}}_{t+1}, \mathbf{g}^y)$). Figure 1a shows that $OTEC$ is the arithmetic average of these two components. The change in cost inefficiency for $\check{\mathbf{w}}_t$ defines the Laspeyres-type measure of cost efficiency change, $CI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \check{\mathbf{w}}_t, \mathbf{g}^x) - CI_{t+1}^{CRS}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \check{\mathbf{w}}_t, \mathbf{g}^x)$; for $\check{\mathbf{w}}_{t+1}$, it defines

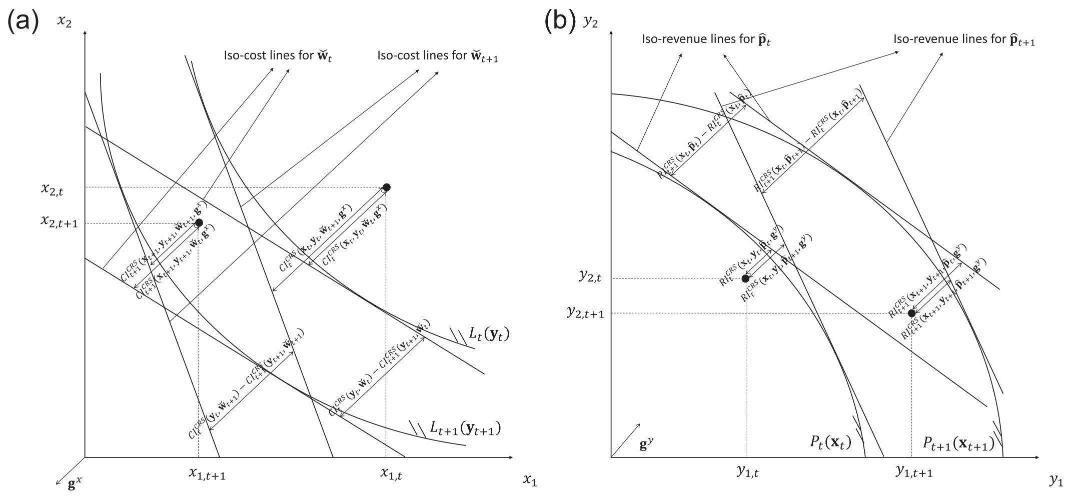

FIGURE 2 Output- and input-oriented components of *BLHM*. (a) Decomposing *ITC* and *ITFPEC*, (b) Decomposing *OTC* and *OTFPEC*.

the Paasche-type measure of cost efficiency change, $CI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \check{\mathbf{w}}_{t+1}, \mathbf{g}^x) - CI_{t+1}^{CRS}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \check{\mathbf{w}}_{t+1}, \mathbf{g}^x)$. Figure 1b shows that *ITEC* is the arithmetic average of these two components. If *OTFPEC* (*ITFPEC*) > 0, the revenue (cost) inefficiency decreases, which indicates an increase in TFP efficiency in the output (input) direction.

We further decompose *OTFPEC* in two ways. First, assessing Equation (13a) for periods t and $t + 1$, we decompose *OTFPEC* into output-oriented components of technical efficiency change *OTEC*, primal scale efficiency change *POSEC*, and mix efficiency change for CRS $OMEC^{CRS}$:

$$OTFPEC = OTEC + POSEC + OMEC^{CRS}, \tag{16a}$$

where

$$OTEC \equiv \overrightarrow{D}_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{0}^N, \mathbf{g}^y) - \overrightarrow{D}_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{0}^N, \mathbf{g}^y), \tag{16b}$$

$$POSEC \equiv POSI_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{0}^N, \mathbf{g}^y) - POSI_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{0}^N, \mathbf{g}^y), \text{and} \tag{16c}$$

$$\begin{aligned} OMEC^{CRS} \equiv \frac{1}{2}\Big\{&\Big[OMI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{p}}_t, \mathbf{g}^y) - OMI_{t+1}^{CRS}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \hat{\mathbf{p}}_t, \mathbf{g}^y)\Big] \\ &+\Big[OMI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{p}}_{t+1}, \mathbf{g}^y) - OMI_{t+1}^{CRS}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \hat{\mathbf{p}}_{t+1}, \mathbf{g}^y)\Big]\Big\}. \end{aligned} \tag{16d}$$

Here, *OTEC* assesses output-oriented technical efficiency change, which assesses the extent to which output technical inefficiency changes, and thus catches up with frontier in the output direction. Additionally, *POSEC* evaluates the change in scale operation in terms of output quantities. Finally, $OMEC^{CRS}$, indicates the change in ability to use the correct mix of outputs, with regard to the revenue-maximising point on the CRS technologies. Its change for $\hat{\mathbf{p}}_t$ defines the Laspeyres-type measure of output mix efficiency change for CRS, $OMI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{p}}_t, \mathbf{g}^y) - OMI_{t+1}^{CRS}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \hat{\mathbf{p}}_t, \mathbf{g}^y)$; for $\hat{\mathbf{p}}_{t+1}$, it defines the Paasche-type measure of output mix efficiency change for CRS, $OMI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{p}}_{t+1}, \mathbf{g}^y) - OMI_{t+1}^{CRS}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \hat{\mathbf{p}}_{t+1}, \mathbf{g}^y)$. Taking the arithmetic average of these two components yields $OMEC^{CRS}$.

Second, assessing (13b) for periods t and $t+1$, we decompose $OTFPEC$ into output-oriented components of technical efficiency change $OTEC$, dual scale efficiency change $DOSEC$, and actual mix efficiency change $OMEC$:

$$OTFPEC = OTEC + DOSEC + OMEC, \tag{17a}$$

where

$$OTEC \equiv \overrightarrow{D}_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{0}^N, \mathbf{g}^y) - \overrightarrow{D}_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{0}^N, \mathbf{g}^y), \tag{17b}$$

$$\begin{aligned} DOSEC \equiv \frac{1}{2}\{&[DOSI_t(\mathbf{x}_t, \hat{\mathbf{p}}_t) - DOSI_{t+1}(\mathbf{x}_{t+1}, \hat{\mathbf{p}}_t)] \\ &+[DOSI_t(\mathbf{x}_t, \hat{\mathbf{p}}_{t+1}) - DOSI_{t+1}(\mathbf{x}_{t+1}, \hat{\mathbf{p}}_{t+1})]\}, \text{and} \end{aligned} \tag{17c}$$

$$\begin{aligned} OMEC \equiv \frac{1}{2}\{&[OMI_t(\mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{p}}_t, \mathbf{g}^y) - OMI_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \hat{\mathbf{p}}_t, \mathbf{g}^y)] \\ &+[OMI_t(\mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{p}}_{t+1}, \mathbf{g}^y) - OMI_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \hat{\mathbf{p}}_{t+1}, \mathbf{g}^y)]\}. \end{aligned} \tag{17d}$$

Again, $OTEC$ assesses output-oriented technical efficiency change, with the same interpretation as in the first output-oriented decomposition. In addition, $DOSEC$ evaluates the change in scale operation in the terms of prices and quantities of output. Its change for $\hat{\mathbf{p}}_t$ defines the Laspeyres-type measure of dual output scale efficiency change, $DOSI_t(\mathbf{x}_t, \hat{\mathbf{p}}_t) - DOSI_{t+1}(\mathbf{x}_{t+1}, \hat{\mathbf{p}}_t)$; for $\hat{\mathbf{p}}_{t+1}$, it defines the Paasche-type measure of dual output scale efficiency change, $DOSI_t(\mathbf{x}_t, \hat{\mathbf{p}}_{t+1}) - DOSI_{t+1}(\mathbf{x}_{t+1}, \hat{\mathbf{p}}_{t+1})$. Taking the arithmetic average of these two components yields $DOSEC$. Finally, $OMEC$ indicates the change in ability to use the correct mix of outputs, with regard to the revenue-maximising point on the actual technologies. Its change for $\hat{\mathbf{p}}_t$ defines the Laspeyres-type measure of output mix efficiency change, $OMI_t(\mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{p}}_t, \mathbf{g}^y) - OMI_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \hat{\mathbf{p}}_t, \mathbf{g}^y)$; for $\hat{\mathbf{p}}_{t+1}$, it defines the Paasche-type measure of output mix efficiency change, $OMI_t(\mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{p}}_{t+1}, \mathbf{g}^y) - OMI_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \hat{\mathbf{p}}_{t+1}, \mathbf{g}^y)$. Taking the arithmetic average of these two components yields $OMEC$.

In analogy to $OTFPEC$, we further decompose $ITFPEC$ in two ways. First, assessing Equation (14a) for periods t and $t+1$, we decompose $ITFPEC$ into input-oriented components of technical efficiency change $ITEC$, primal scale efficiency change $PISEC$, and mix efficiency change for CRS $IMEC^{CRS}$:

$$ITFPEC = ITEC + PISEC + IMEC^{CRS}, \tag{18a}$$

where

$$ITEC \equiv \overrightarrow{D}_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}^x, \mathbf{0}^M) - \overrightarrow{D}_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{g}^x, \mathbf{0}^M), \tag{18b}$$

$$PISEC \equiv PISI_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}^x, \mathbf{0}^M) - PISI_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{g}^x, \mathbf{0}^M), \text{and} \tag{18c}$$

$$\begin{aligned} IMEC^{CRS} \equiv \frac{1}{2}\Big\{&\Big[IMI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \check{\mathbf{w}}_t, \mathbf{g}^x) - IMI_{t+1}^{CRS}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \check{\mathbf{w}}_t, \mathbf{g}^x)\Big] \\ &+\Big[IMI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \check{\mathbf{w}}_{t+1}, \mathbf{g}^x) - IMI_{t+1}^{CRS}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \check{\mathbf{w}}_{t+1}, \mathbf{g}^x)\Big]\Big\}. \end{aligned} \tag{18d}$$

Here, $ITEC$ assesses the extent to which input technical inefficiency changes, and thus catches up with frontier in the input direction. Additionally, $PISEC$ evaluates the change in scale operation in the terms of input quantities. Finally, $IMEC^{CRS}$ indicates the change in ability to use the correct mix of inputs, with regard to the cost-minimising point on the CRS technologies. Its change for $\check{\mathbf{w}}_t$ defines the Laspeyres-type measure of input mix efficiency change for CRS, $IMI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \check{\mathbf{w}}_t, \mathbf{g}^x) - IMI_{t+1}^{CRS}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \check{\mathbf{w}}_t, \mathbf{g}^x)$; for $\check{\mathbf{w}}_{t+1}$, it defines the Paasche-type measure of input mix efficiency change for CRS, $IMI_t^{CRS}(\mathbf{x}_t, \mathbf{y}_t, \check{\mathbf{w}}_{t+1}, \mathbf{g}^x) - IMI_{t+1}^{CRS}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \check{\mathbf{w}}_{t+1}, \mathbf{g}^x)$. Taking the arithmetic average of these two components yields $IMEC^{CRS}$.

Second, assessing Equation (14b) for periods t and $t + 1$, we decompose $ITFPEC$ into components of input-oriented technical efficiency change $ITEC$, dual scale efficiency change $DISEC$, and actual mix efficiency change $IMEC$:

$$ITFPEC = ITEC + DISEC + IMEC, \tag{19a}$$

where

$$ITEC \equiv \overrightarrow{D}_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}^x, \mathbf{0}^M) - \overrightarrow{D}_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{g}^x, \mathbf{0}^M), \tag{19b}$$

$$\begin{aligned} DISEC \equiv \frac{1}{2}[\{[DISI_t(\mathbf{y}_t, \check{\mathbf{w}}_t) - DISI_{t+1}(\mathbf{y}_{t+1}, \check{\mathbf{w}}_t)] \\ +[DISI_t(\mathbf{y}_t, \check{\mathbf{w}}_{t+1}) - DISI_{t+1}(\mathbf{y}_{t+1}, \check{\mathbf{w}}_{t+1})]\}, \text{and} \end{aligned} \tag{19c}$$

$$\begin{aligned} IMEC \equiv \frac{1}{2}\{[IMI_t(\mathbf{x}_t, \mathbf{y}_t, \check{\mathbf{w}}_t, \mathbf{g}^x) - IMI_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \check{\mathbf{w}}_t, \mathbf{g}^x)] \\ +[IMI_t(\mathbf{x}_t, \mathbf{y}_t, \check{\mathbf{w}}_{t+1}, \mathbf{g}^x) - IMI_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \check{\mathbf{w}}_{t+1}, \mathbf{g}^x)]\}. \end{aligned} \tag{19d}$$

Again, $ITEC$ assesses input-oriented technical efficiency change, with the same interpretation as in the first input-oriented decomposition. In addition, $DISEC$ evaluates the change in scale operation in the terms of prices and quantities of input. Its change for $\check{\mathbf{w}}_t$ defines yields the Laspeyres-type measure of dual input scale efficiency change, $DISI_t(\mathbf{y}_t, \check{\mathbf{w}}_t) - DISI_{t+1}(\mathbf{y}_{t+1}, \check{\mathbf{w}}_t)$; for $\check{\mathbf{w}}_{t+1}$, it defines the Paasche-type measure of dual input scale efficiency change, $DISI_t(\mathbf{y}_t, \check{\mathbf{w}}_{t+1}) - DISI_{t+1}(\mathbf{y}_{t+1}, \check{\mathbf{w}}_{t+1})$. Taking the arithmetic average of these two components yields $DISEC$. Finally, $IMEC$ indicates the change in ability to use the correct mix of inputs, with regard to the cost-minimising point on the actual technologies. Its change for $\check{\mathbf{w}}_t$ yields the Laspeyres-type measure of input mix efficiency change, $IMI_t(\mathbf{x}_t, \mathbf{y}_t, \check{\mathbf{w}}_t, \mathbf{g}^x) - IMI_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \check{\mathbf{w}}_t, \mathbf{g}^x)$; for $\check{\mathbf{w}}_{t+1}$, it defines the Paasche-type measure of input mix efficiency change, $IMI_t(\mathbf{x}_t, \mathbf{y}_t, \check{\mathbf{w}}_{t+1}, \mathbf{g}^x) - IMI_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \check{\mathbf{w}}_{t+1}, \mathbf{g}^x)$. Taking the arithmetic average of these two components yields $IMEC$.

Let us now compare the decomposition of $BLHM$ to that of $LHM(\cdot)$ as in Section 2. First, the decomposition of $BLHM$ yields determinate components, while this is not the case for the decomposition of $LHM(\cdot)$. Second, the use of revenue functions and cost functions assumes profit-maximising behaviour when decomposing $BLHM$. No such behavioural assumption is required in the decomposition of $LHM(\cdot)$. Third, as a result, several components differ in definition and interpretation, although both decompositions contain the same component of technical efficiency change. Technical change is defined dually in terms of prices and quantities, relying on cost-minimising and revenue-maximising behaviour in the decomposition of $BLHM$. It is defined primally in terms of quantities without behavioural assumptions in the decomposition of $LHM(\cdot)$. Scale efficiency change can be defined both primally and dually in the decomposition of $BLHM$, indicating the wedge between CRS and VRS technologies. By contrast, scale efficiency change is only defined primally in the decomposition of $LHM(\cdot)$. It assesses the change in the frontier's gradient. Components of mix efficiency change occur in the decomposition of $BLHM$, but not in that of $LHM(\cdot)$. Fourth, the decomposition of $BLHM$ yields separate output- and input-oriented components, which together add up to $BLHM$. The output (input)-oriented decomposition of $LHM(\cdot)$ does not yield any input (output)-oriented components. Overall, this means that all components but the common component of technical efficiency change cannot be completely compared between the two decompositions. It also means that the output-oriented and input-oriented counterparts may conflict, as they underlie aggregate output change and aggregate input change, respectively, rather than TFP change as a whole. For instance, in our application, we observe that output-oriented components of technical change and scale efficiency change often differ substantially from their input-oriented counterparts.

4 | EMPIRICAL APPROACH

We operationalise our decomposition framework using convex DEA. Following Banker et al. (1984), the actual technology set is approximated by variable-returns-to-scale (VRS). Supposing there are I firms in period t, the approximated VRS technology, $\hat{T}_t$, is:

$$\hat{T}_t = \left\{ (\mathbf{x}_t, \mathbf{y}_t) \middle| \sum_{i=1}^{I} \lambda_{i,t}\mathbf{x}_{i,t} \leq \mathbf{x}_t, \sum_{i=1}^{I} \lambda_{i,t}\,\mathbf{y}_{i,t} \geq \mathbf{y}_t, \sum_{i=1}^{I} \lambda_{i,t} = 1 \right\}, \tag{20}$$

where $\lambda_{i,t}$ are the intensity weights indicating the peers. The constraints respectively impose strong disposability of inputs, strong disposability of outputs, and VRS. The approximated technology set for CRS, $\hat{T}_t^{CRS}$, is computed by omitting the last constraint:

$$\hat{T}_t^{CRS} = \left\{ (\mathbf{x}_t, \mathbf{y}_t) \middle| \sum_{i=1}^{I} \lambda_{i,t}\mathbf{x}_{i,t} \leq \mathbf{x}_t, \sum_{i=1}^{I} \lambda_{i,t}\,\mathbf{y}_{i,t} \geq \mathbf{y}_t \right\}. \tag{21}$$

We compute all required components by using these constraints for the objective functions defined in Equations (7) and (8a)–(8b).

5 | DATA

The empirical application focuses on German dairy-processing firms for the years 2011–2020. We combine company-level accountancy data from Orbis (2022) and price index data from Eurostat (2022). The inputs are labour, materials and fixed assets. The output is turnover. These variables are expressed in monetary terms. We obtain implicit quantities by deflating them by the respective price indices. The material price index and fixed asset price index are producer price indices for "intermediate goods" and "capital goods" in industry, respectively. The labour price index is the labour cost index in manufacturing. As the ORBIS database distinguishes between firms that are involved in the "operation of dairy and cheese making," "manufacture of ice cream," and both, we use the respective price indices. We use 2011 as the base year for all price indices. The price indices vary across periods, but are common for all observations within a given year. Following Cox and Wohlgenant (1986), this means that price differences between observations result in quantity differences. Implicitly, it is assumed that a higher quality is reflected by a higher price, and eventually, by applying the product rule, a higher quantity. This procedure avoids conflation of cost inefficiency and technical input inefficiency. We exclude implausible values below €1, 000. The final dataset contains 694 observations, from which we retrieve 558 growth rates. Table 1 shows the descriptive statistics of the undeflated variables and the price indices. In line with Ang (2019), our Bennet indicator has a "variable" formulation, in that it excludes fixed inputs, while the decomposition using DEA includes fixed inputs in the production technology. We employ the average value of the variables as the directional vector. Assuming that the level of fixed assets remains unchanged within the year, we assign a zero value for the corresponding directional subvector. We thus assume that firms cannot increase technical efficiency by decreasing the level of fixed assets. Nonetheless, our technological specification ensures that the performance of the observation considered is only compared to that of the (linear combination of) peers that have at most the same level of fixed assets.

TABLE 1 Descriptive statistics.

Statistic	Mean	SD	Min	Max
Revenue (in €1000)	259,968	662,785	193	5,943,773
Material cost (in €1000)	203,599	527,440	74	4,847,575
Labour cost (in €1000)	18,458	49,713	1	448,276
Fixed asset cost (in €1000)	44,006	113,653	4	1,020,418
Material price index (dimensionless)	0.946	0.096	0.756	1.061
Labour price index (dimensionless)	1.117	0.077	1.000	1.224
Fixed asset price index (dimensionless)	1.038	0.024	1.000	1.075
Output price index (dimensionless)	1.040	0.056	0.953	1.099
Output price index of operation of dairy and cheese making (dimensionless)	1.039	0.058	0.948	1.099
Output price index of manufacture of ice cream (dimensionless)	1.065	0.030	1.000	1.094

6 | RESULTS

6.1 | The LHM-approximating Bennet indicator

Figure 3 shows the LHM-approximating Bennet indicator *BLHM* and its components of output growth *OC* and input decline *IC*, computed by employing Equation (6). The values are expressed as average percentage changes between two adjacent years, covering the period 2011–2020.[1] The annual average *BLHM* is −1.14% per annum (p.a.) for the whole period, which indicates an average TFP decline. Additionally, *BLHM* fluctuates between on average −11.46% in 2014–2015 and +12.60% in 2016–2017. Although the average *OC* is positive for six out of nine periods, this is often offset by *IC* being on average negative for six out of nine periods. The offsetting effect can for instance be seen in the period 2015–2016, in which an average output growth of 3.64% is offset by an average input growth of 9.62%, resulting in substantial TFP decline on average, *BLHM* = −5.98%.

6.2 | Decomposing the LHM-approximating Bennet indicator

Using Equations (15a)–(15e), Table 2 shows *BLHM* and the components of *OTC*, *OTFPEC*, *ITC*, and *ITFPEC*. While somewhat different in magnitude, the TFP efficiency change components are qualitatively similar. Here, *OTFPEC* and *ITFPEC* show an average annual decline of 0.13% and 2.01% in output- and input-oriented TFP efficiency, respectively. In both cases, the signs of the annual averages fluctuate over time, but are the same per given period. By contrast, the average *OTC* and *ITC* show opposite signs in the periods 2011–2012, 2012–2013, 2013–2014, 2015–2016, and 2018–2019. Overall, *OTC* and *ITC* are respectively on average +2.58% and −1.58% p.a. for the whole period. These results suggest that the frontier improves when benchmarked with regard to the revenue-maximising point, but deteriorates when benchmarked with regard to the cost-minimising point. Overall, the output

[1]One may also opt to present the results using a cumulative growth rate with 2011 as the base year. However, *BLHM* is intransitive, which means that when directly comparing two observations, the resulting value may differ from the value obtained by indirectly comparing them through a third observation (O'Donnell, 2012). In practice, this implies that the cumulative TFP change through chaining differs from the one through fixing the base year 2011. In line with for example, Ang and Kerstens (2023), we therefore show the annual TFP change *between* adjacent years rather than the change compared to the base year 2011. Overall, there is no ideal presentation, as intransitivity complicates multitemporal comparison.

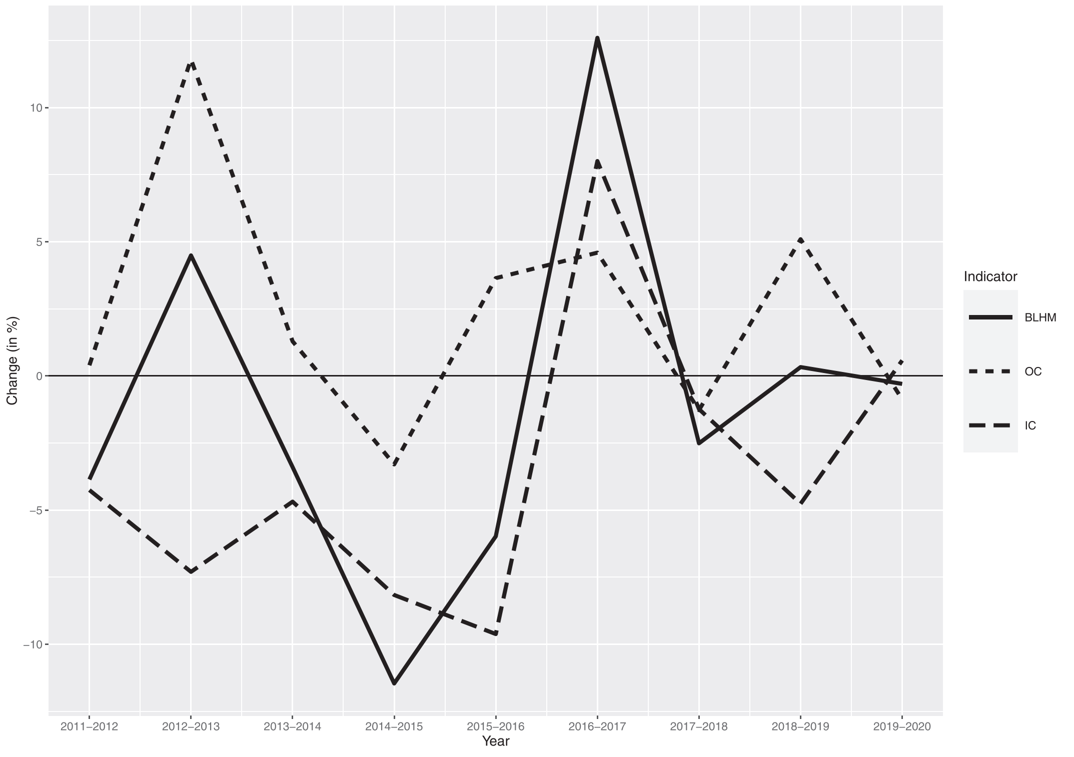

FIGURE 3 LHM-approximating Bennet indicator, output growth and input decline.

TABLE 2 *BLHM* and the components of *OTC*, *OTFPEC*, *ITC*, and *ITFPEC*.

Period	*BLHM* (%)	*OTC* (%)	*OTFPEC* (%)	*ITC* (%)	*ITFPEC* (%)
2011–2012	−3.86	+1.48	−1.08	−0.73	−3.52
2012–2013	+4.49	+13.24	−1.45	−0.68	−6.62
2013–2014	−3.40	+4.31	−3.02	−0.12	−4.57
2014–2015	−11.46	−6.17	+2.87	−8.32	+0.15
2015–2016	−5.98	+4.09	−0.44	−7.89	−1.73
2016–2017	+12.60	+6.07	−1.48	+12.56	−4.56
2017–2018	−2.51	−2.02	+0.74	−1.50	+0.27
2018–2019	+0.33	+3.58	+1.51	−5.79	+1.02
2019–2020	−0.30	−1.67	+0.79	−0.38	+0.96
Overall	−1.14	+2.58	−0.13	−1.58	−2.01

growth is largely driven by an increase of production possibilities given input use. However, it is offset by input growth, which can be attributed to decreases in TFP efficiency and a higher input requirement for production.

Table 3 shows two output-oriented decompositions of *OTFPEC*. Employing Equations (16a)–(16d), $OTFPEC = OTEC + POSEC + OMEC^{CRS}$, and employing Equations (17a)–(17d), $OTFPEC = OTEC + DOSEC + OMEC$. However, since the empirical application only considers one output, there is no output mix inefficiency. As a result,

TABLE 3 Decomposition of *OTFPEC*.

Year	*OTFPEC* (%)	*OTEC* (%)	*POSEC* (%)	$OMEC^{CRS}$ (%)	*DOSEC* (%)	*OMEC* (%)
2011	+1.08	+0.65	+0.43	0.00	+0.43	0.00
2012	+1.45	−1.40	+2.84	0.0	+2.84	0.00
2013	+3.02	+0.33	+2.69	0.00	+2.69	0.00
2014	−2.87	−0.09	−2.78	0.00	−2.78	0.00
2015	+0.44	−0.32	+0.76	0.00	+0.76	0.00
2016	+1.48	+0.02	+1.45	0.00	+1.45	0.00
2017	−0.74	+0.75	−1.49	0.00	−1.49	0.00
2018	−1.51	−0.13	−1.38	0.00	−1.38	0.00
2019	−0.79	−0.90	+0.11	0.00	+0.11	0.00
Overall	+0.13	−0.14	+0.28	0.00	+0.28	0.00

TABLE 4 Decomposition of *ITFPEC*.

Year	*ITFPEC* (%)	*ITEC* (%)	*PISEC* (%)	$IMEC^{CRS}$ (%)	*DISEC* (%)	*IMEC* (%)
2011	−3.52	−0.75	−0.49	−2.27	−2.22	−0.55
2012	−6.62	+1.30	−2.12	−5.80	−8.61	+0.69
2013	−4.57	−0.08	−2.47	−2.02	−4.52	+0.03
2014	+0.15	−0.61	+0.89	−0.12	+2.17	−1.40
2015	−1.73	+0.19	−1.36	−0.56	+0.92	−2.84
2016	−4.56	+0.26	+0.10	−4.91	−5.09	+0.27
2017	+0.27	−0.85	+1.08	+0.04	+0.94	+0.18
2018	+1.02	+0.14	+0.97	−0.09	+0.56	+0.32
2019	+0.96	+0.73	−0.62	+0.85	−0.69	+0.92
Overall	−2.01	+0.05	−0.44	−1.62	−1.84	−0.22

$OMEC^{CRS} = OMEC = 0$ and the components of output scale efficiency change are the same in primal and dual terms, that is, $POSEC = DOSEC$. The average distance to the technical frontier measured in the output direction does not change much throughout, as evidenced by the small magnitudes in *OTEC*. Averaged over the whole period, $POSEC = DOSEC = +0.28\%$ p.a. The annual average ranges from −2.78% in 2014–2015 to +2.84% in 2012–2013. Overall, *OTFPEC* follows the trend of $POSEC = DOSEC$ more closely than that of *OTEC*.

Table 4 shows two input-oriented decompositions of *ITFPEC*. Using Equations (18a)–(18d), $ITFPEC = ITEC + PISEC + IMEC^{CRS}$, and using Equations (19a)–(19d), $ITFPEC = ITEC + DISEC + IMEC$. As in the output-oriented decomposition, the average distance to the technical frontier measured in the input direction does not change much throughout, as evidenced by the small magnitudes in *ITEC*. Additionally, *PISEC*, $IMEC^{CRS}$, *DISEC* and *IMEC* are on average all negative considering the whole studied period. Average annual fluctuations are prominent for $IMEC^{CRS}$ (ranging between −5.80% in 2012–2013 and +0.85% in 2019–2020) and *DISEC* (ranging between −8.61% in 2012–2013 and +2.17% in 2014–2015) than for *PISEC* (ranging between −2.47% in 2013–2014 and +1.08% in 2016–2017) and *IMEC* (ranging between −2.84% in 2015–2016 and +0.92% in 2019–2020).

Moreover, *PISEC* and *DISEC*, on the one hand, and $IMEC^{CRS}$ and *IMEC*, on the other hand, differ substantially in magnitude. In the latter comparison, the signs of the annual changes are often even conflicting. This is interesting, as *PISEC* and *DISEC* both capture change in scale operation, whereas $IMEC^{CRS}$ and *IMEC* both capture the change in ability to employ the correct mix of inputs. Further research is needed to understand why this happens. Finally, we note that the separate decomposition of *OTFPEC* and *ITFPEC* can yield counterparts that may contradict each other. For instance, we observe that the components of scale efficiency change often differ substantially depending on whether one uses the output orientation or the input orientation. Nevertheless, this is consistent with the often conflicting *OTFPEC* and *ITFPEC*, which are in turn partly determining *OC* and *IC*.

6.3 | Comparison with original LHM indicator

Figure 4 compares *BLHM* to $LHM(\cdot)$. As expected, *BLHM* and $LHM(\cdot)$ are both determinate and are as a result always feasible. For the studied German dairy-processing firms, *BLHM* approximates $LHM(\cdot)$ well, as evidenced by the high

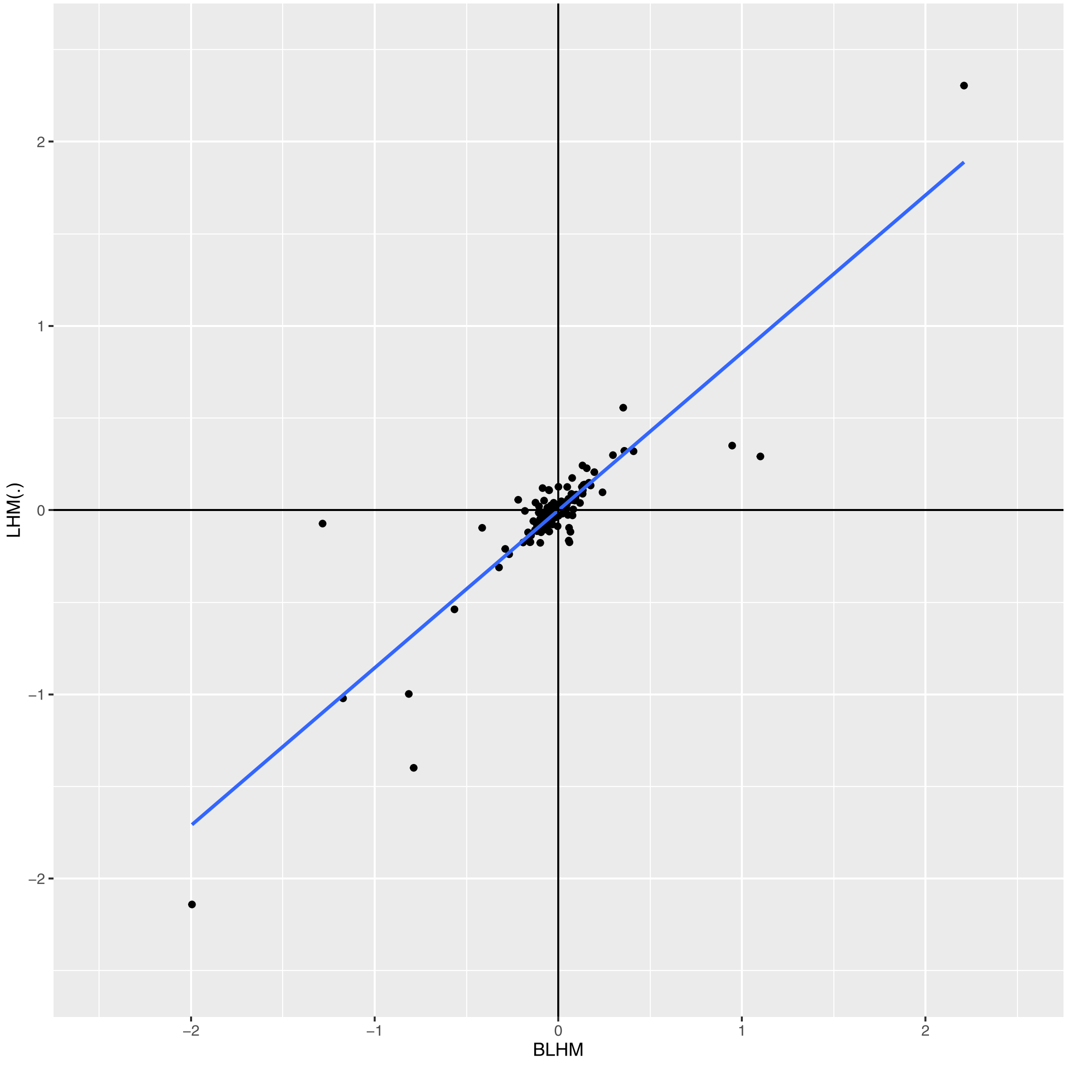

FIGURE 4 LHM-approximating Bennet indicator versus original LHM indicator.

Pearson correlation of 0.89. Ang and Kerstens (2020) also find *BLHM* estimates similar to the $LHM(\cdot)$ estimates for a sample of Italian food and beverage companies for the period of 1995–2007.

Table 5 shows the proportion of infeasibilities when decomposing the original LHM indicator. As expected, the decomposition yields indeterminate, and hence possibly infeasible, components of technical change and scale efficiency change. The Y-oriented decomposition is given by Equations (4a)–(4d). For the X-oriented and combined $Y-X$-oriented decompositions, we respectively refer to Ang and Kerstens (2017) and Shen et al. (2019). In the Y-oriented decomposition, 64 out of 558 computations (11.47%) are infeasible. In the X-oriented decomposition, 34 out of 558 computations (6.09%) are infeasible. The $Y-X$-oriented decomposition, which uses an arithmetic average of the output- and input-oriented decompositions, results in 89 infeasibilities out of 558 computations (15.95%).

Table 6 evaluates the Pearson correlation between several components of *BLHM* (technical change, primal scale efficiency change and dual scale efficiency change) and the respective components of $LHM(\cdot)$ (technical change and twice scale efficiency change). For the Y-oriented decomposition, we compare (i) OTC in Equation (15b) to OTC_{LHM} in Equation (4b), (ii) $POSEC$ in Equation (16c) to $OSEC_{LHM}$ in Equation (4d), and (iii) $DOSEC$ in Equation (17c) to OTC_{LHM} in Equation (4d). The component of technical efficiency change is the same for *BLHM* and $LHM(\cdot)$. As this would result in a perfect correlation of one, this is noninformative and hence omitted. Overall, these correlations are low, which suggest substantial differences between the analogous components. Although the Pearson correlation is moderately high when evaluating technical change in the Y-oriented decomposition (0.40) and $Y-X$-oriented decomposition (0.40), it is very low and even negative for the X-oriented decomposition (−0.02). The Pearson correlation is very low for the Y-, X- and $Y-X$-oriented decomposition with regard to primal scale efficiency change (0.02, 0.05 and 0.07, respectively). Finally, the Pearson correlation is very low when evaluating dual scale efficiency change in the Y-oriented decomposition, and somewhat higher in the X- and $Y-X$-oriented decompositions (0.27 and 0.21, respectively). Nevertheless, as explained in Section 3, these components cannot be interpreted in exactly the same way, which compromises the interpretation of a direct comparison.

6.4 | Implications for management and policy

TFP, as indicated by a negative *BLHM*, declines over time, but with considerable variability in each year. The variability may indicate that firms allocating resources in response to a preceding year's prices rather than (unknown

TABLE 5 Proportion of infeasibilities in the decomposition of the original Luenberger–Hicks–Moorsteen indicator.

Decomposition	Infeasibilities
Y-oriented	64/558 (11.47%)
X-oriented	34/558 (6.09%)
$Y-X$-oriented	89/558 (15.95%)

TABLE 6 Pearson correlation between components of *BLHM* and respective components of $LHM(\cdot)$.

Component of *BLHM*	Y-oriented decomp. of $LHM(\cdot)$	X-oriented decomp. of $LHM(\cdot)$	$Y-X$-oriented decomp. of $LHM(\cdot)$
Technical change	0.40	−0.02	0.40
Primal scale efficiency change	0.02	0.05	0.07
Dual scale efficiency change	0.02	0.27	0.21

Note: The components of *BLHM* are expressed in the orientation of the decomposition of $LHM(\cdot)$.

when planning) a particular year's prices. However, such a variability may also partly be attributed to sampling bias, to which DEA is susceptible. The decline is largely driven by a decline in input-related technical change and an increase in input-related inefficiency; these combine to offset the increase in output-related technical change. The predominantly negative *ITC* show that innovation in cost-saving technologies are critical to stimulate productivity growth. This holds especially given the perishable nature of dairy products. One can achieve this by investing in public and/or private Research and Development. In the context of the dairy-processing sector, this may materialise through enhanced automation and robotics, which streamlines milking, packaging, and sorting.

As a country that tended to fill its national milk quota, Germany, particularly in the north, was potentially well placed to take advantage of the final abolition of the EU milk quota scheme in 2015 (Jansik & Irz, 2015). Germany's share of EU milk production is still dominant, but has declined slightly from 23% in 2015 to 22% in 2021. Evidence from the farm management literature suggests that there is some indication that the 2015 changes to the quota system led to an improvement in farm TFP in many European countries, but not Belgium, the Czech Republic, United Kingdom, or Germany (Čechura et al., 2021). Dairy farms in eastern Germany in particular, despite improved technical efficiency, exhibited reduced scale efficiency and reduced technological change: indeed, postquota technological change in eastern Germany fell by more than any other region reported in Čechura et al. (2021). The increase and subsequent decline in the BLHM measure of TFP reported here in Figure 3 may perhaps be associated with this: an initial increase in production immediately after quotas were removed may have been offset later by farm-level supply problems as structural adjustment occurred, particularly on farms in former East Germany.

7 | CONCLUSIONS

The LHM indicator developed by Briec and Kerstens (2004) has sound theoretical properties, but its decomposition yields indeterminate components of technical change and scale efficiency change that can become infeasible. The current paper addresses this problem by focusing on a Bennet indicator that is shown by Ang and Kerstens (2020) to approximate the LHM indicator. The decomposition of the LHM-approximating Bennet indicator yields determinate components of technical change, technical efficiency change, scale efficiency change and mix efficiency change that are always feasible. The application focuses on 694 observations of German dairy-processing companies, for which we compute 558 growth rates for the period 2011–2020.

The results show that *BLHM* decreases by on average −1.14% p.a., with substantial annual fluctuations. The underlying components of *OTC* and *ITC* also fluctuate substantially, and often conflict. Moreover, *OTFPEC* and *ITFPEC* fluctuate moderately on average, which is mainly driven by scale efficiency change and mix efficiency change. The components of technical efficiency change remain relatively stable on average. Nevertheless, we cannot rule out the possibility that the variability is in part caused by sampling bias, to which DEA is known to be prone to. Depending on the specification, the proportion of infeasibilities when decomposing the original LHM indicator ranges from 6.09% to 15.95%. Indeterminateness is thus a relevant problem when decomposing the original LHM indicator for the current sample. Our proposed determinate decomposition is valuable from this perspective. Nonetheless, we should remain prudent, as the determinateness is obtained at the cost of making the behavioural assumption of profit maximisation. If such economic behaviour is violated, the components relying on value functions should be further scrutinised. In this light, our methodological advance should be seen as an analytical tool that complements, rather than substitutes, the incomplete results provided by the decomposition of the original LHM indicator.

We have several recommendations for future research. First, we recommend to complement our decomposition framework with an econometric analysis of the determinants. In this way, we are able to provide more insights on how to enhance business performance.

Second, it would be interesting to spell out the theoretical conditions under which the two proposed quadripartite decompositions for input decline (and output growth) coincide. Focusing on the ratio-based input distance function and its dual cost function, Zelenyuk (2014) articulates the theoretical conditions under which primal and dual scale efficiencies coincide. One may adapt these insights to the difference-based context of input (output) directional distance functions and cost (revenue) functions. In the present application, the decomposition of input decline reveals that *PISEC* (*IMEC*^{CRS}) and *DISEC* (*IMEC*) differ substantially, even though they are supposed to capture similar phenomena. Having an understanding why this happens would allow the empirical analyst to give better, less ambiguous managerial advice.

Third, the decomposition framework could be approximated in a nonconvex fashion using free disposal hull (Deprins et al., 1984; Tulkens, 1993). The current paper employs DEA, which convexly approximates the production technology. While convexity is a common assumption in the field of economics, nonconvex estimates may be more realistic in some settings and differ. In the contemporaneous setting of efficiency analysis, such differences have been empirically observed for value functions (Ang et al., 2018; Kerstens & Van de Woestyne, 2021) and distance functions (Kerstens, 1996). In the intertemporal setting of productivity change, several studies report differences (Ang & Kerstens, 2017; Kerstens et al., 2022, 2018), although there can be special cases of equivalence (Ang et al., 2023). The differences between convex and nonconvex estimates for the *components* of *BLHM* remain to be investigated.

Fourth, the proposed decompositions could be computed using a statistically robust approach. Employing DEA, our approach cannot distinguish between statistical noise and inefficiency. One can alleviate the problem of sampling bias with a bootstrapping procedure (Simar & Wilson, 1998, 2020). However, as observed in the study of Ang and Kerstens (2023), such a procedure may be computationally intensive. Another route is the use of stochastic frontier analysis (Aigner et al., 1977; Meeusen & van Den Broeck, 1977), which may overcome such computational hurdles. However, the adaptation of the decomposition framework to the parametric context is hitherto unknown. Addressing this knowledge gap is left for future research.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from ORBIS database. Restrictions apply to the availability of these data, which were used under license for this study. Data are available from https://orbis.bvdinfo.com/ip with the permission of ORBIS database.

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