Wageningen University - Department of Social Sciences

Operations Research and Logistics

MSc Thesis

Improving Lot-Sizing Models: From Theory to Practice

Abstract

Lot-sizing problems are important in many production areas. Yet, they are difficult to solve due to their large number of binary variables that arise in their mixed-integer linear models. For this reason, there is research on how these problems can be approximated in order to improve the computational performances of solvers. The study of Van Vyve and Wolsey (2006) is an example in which an approximation for lot-sizing problems is presented. However, it is unclear whether these theoretical improvements, that have been demonstrated on mainly artificial and simple models, suit well into more realistic instances as well. Therefore, in this research, the approximation technique of Van Vyve and Wolsey (2006) is tested and its model methodology is extended to account for perishability in lot-sizing.

The results show that the model formulations of Van Vyve and Wolsey (2006) have no computation problems. The simulations of the perishability model show that the software has to put more efforts in solving and the solving time increased drastically. The approximation technique is therefore applied on this model and subsequently shows that a related problem can be solved within a few seconds. However, the results of the approximation model contain some differences in the optimal values compared to the original model. The method of Van Vyve and Wolsey (2006) can be used in more practical situations and has the ability to decrease the computation complexity of lot-sizing models, but comes at a certain cost of less accurate results.

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Chapter 1

Introduction

Lot-sizing problems are found in many production areas and this issue of lot-sizing has existed since the start of industrialization (Hanafizadeh et al., 2019). Problems in production planning arise from setups between production batches of the same product. Due to the setups costs, it is frequently too expensive to make a specific product every time period. On the other hand, producing in bigger bulks will lead to high inventory holding costs to meet future demand (Suwondo, 2012). Lot-sizing tries to provide answers to two inventory control-related questions: when to produce and how much to produce? The second question is the size of the production lot, while the first question is also known as the reorder point (Hanafizadeh et al., 2019). The goal of a lot-sizing model is to identify the times when production should take place and the quantities that need to be produced in order to meet demand while reducing production, setup, and inventory holding costs.

Mixed Integer Linear Programming (MILP) is a technique frequently used to tackle lot-sizing problems. Even though the powerful computers and sophisticated solvers, MILP models can still be difficult to solve (Vanderbeck & Wolsey, 2008). There are model formulations which may not work properly under some circumstances, and then, the computational process will take too long. A common method for modeling set-up decisions in lot-sizing problems is the introduction of the binary variable. These binary variables can create situations that are far more challenging to solve than those that are entirely continuous linear. MILP models are sensitive to the size of the problem, which means that when there are many binary variables, the effectiveness of solvers can significantly reduce (Vanderbeck & Wolsey, 2008; Vielma & Pablo, 2015).

Nowadays, the literature could offer different reformulating and approximating approaches for improving the MILP model formulations. The potential of decreasing the size of the model by lowering the number of (binary) variables and constraints has already been investigated and verified (Alemany et al., 2018; Vielma & Pablo, 2015). However, these studies on improvement techniques for modelling are often more focused on the mathematical theory instead of putting this technique into existing applications. This makes it hard to predict if such improvement technique is applicable in practice. Therefore, one may experience two types of lot-sizing literature: studies that are written from a modelling point of view and lot-sizing studies that are written based on an application. The application studies do not focus on the most effective modelling formulation.

This research focuses on the approximation technique of Van Vyve and Wolsey (2006). The study of Van Vyve and Wolsey (2006) describes a approximation technique for lot-sizing models. They give some encouraging findings, however the majority of their study is conducted from a mathematical viewpoint. In actual applications, there are other things that need to be taken into consideration and it is not clear, whether their approximation technique still performs under these additional challenges. Therefore, their model methodology is extended.

The approximation technique of Van Vyve and Wolsey (2006) is discussed and examined in this research. Further, it was also investigated whether it is possible to apply the approximation technique into a more practical context. General lot-sizing and scheduling problems are well studied, but there has been less research on perishable or time-sensitive products (Alipour et al., 2020). Therefore, the lot sizing problem outlined by Van Vyve and Wolsey (2006) was transformed into a model that takes the perishability of the products into account. The aim of this research is to narrow the gap between the improvement theories for modelling and practical modelling.

To arrive at the conclusions, the following research objectives are addressed:

Objective: Verify the functioning of the approximation technique described by Van Vyve and Wolsey (2006).

Objective: Rewrite the model of Van Vyve and Wolsey (2006) to a more comprehensive application, for instance by implementing perishability into the model.

Objective: Apply the approximation technique on the rewritten model formulation and test its performances.

For the first objective, the computational performances of the models described by Van Vyve and Wolsey (2006) and the technique's effectiveness was tested by simulating scenarios in a solver. To fulfill the second objective, more scientific literature was found on perishability in lot-sizing problems and the model from Van Vyve and Wolsey (2006) was transformed to account for perishability. For the last research objective, the rewritten lot-sizing model was simulated and the approximation technique was applied to obtain its effect. By applying the approximation technique on the perishability lot-sizing model, it was shown how to reduce this gap between mathematical theories and practical modeling

This study does not consider all formulations of lot-sizing problems in the article of Van Vyve and Wolsey (2006). Only two MILP formulations were chosen and investigated. Specifically, this research only examin the constant capacity lot-sizing problem, the uncapacitaded lot-sizing problem with Wagner-Whitin costs and its approximated extended formulation.

This study consists of seven chapters and is organized as follows. A qualitative literature review on lot-sizing model is presented in Chapter 2 and the results show examples of the two types of lot-sizing studies. In Chapter 3, the methodology and research design is discussed. In the subsections, the lot-sizing model formulations of Van Vyve and Wolsey (2006) are presented and the reformulation technique is introduced. This chapter also contains subsections for the introduction of perishability in lot-sizing and the required model transformations. Chapter 4 outlines the design of the computational study and subsequently presents the results and its discussion. Chapter 5 contains the discussion of the approximation technique and this research. Chapter 6 consists of some concluding remarks, which is followed up by a chapter on recommendations for further research. Appendix A lists some additional information, which may be useful in understanding this research area and the used mathematical terminology.

Chapter 2

Literature review

In this chapter, a qualitative literature review is conducted. Section 2.1 describes how the review is conducted. Section 2.2 presents the found modelling studies and Section 2.3 consists of the application-oriented studies. In Section 2.4, the literature review is discussed.

2.1 Design of the review

This literature review concentrates on lot-sizing models. The objective is to demonstrate the two types of lot-sizing literature by giving some examples.

In order to obtain suitable and up-to-date information on lot-sizing, a literature review was conducted using keywords and phrases that are relevant to the topic. The review included both primary and secondary sources, such as original research studies and review studies. Several databases were employed (Scopus, WUR library research, Google Scholar, and ScienceDirect) to discover publications relevant to this research.

For the theoretical studies, the following key words were used and combined in the search queries to get some relevant hits:

Lot-sizing / Lot-size / Production Schedule / Mixed Integer Programming Model / Purpose / Formulations / Effective / Mathematics / Basic model / Binary variables / Reduction / Computational Performances / Complexity / Development / Effective / Solutions / Algorithm / Methods / Techniques / Strategies / Modeling

For the practical studies, the following key words were used and combined in the search queries:

Lot-sizing / Lot-size / Production Schedule / Mixed Integer Programming Model / Applications / Purpose / Formulations / Lot Streaming / Inventory Management / Simulations / Capacity Planning / Multi-product / Multi-period / Periodic-review / Advanced Models / Multi-item

Along with these key words, snowballing will be used. This means that a few keywords from the found studies will be used to find other literature.

After performing the search queries, the quality of the retrieved articles were assessed. Its relevancy was determined by looking how closely the article relates to this research. Subsequently, the selected articles were included in the review using the inclusion criteria. These criteria are listed below.

- The studies must be written in English;
- The studies should be peer-reviewed publications;
- The studies must be published within the previous 25 years
- The content reviewed in the studies must only consider MILP models. Other models seen in the literature, such as the extensions of EOQ models were not included.
- Studies had to be primarily on models that contained the original lot sizing problem. These are the models that deal with the problem of balancing inventory and set-up costs. Studies dealing with supply chain design problems were not included in this review.

If the studies were considered to be relevant and fullfilled the criteria, they are discussed in one of the following sections. In addition to this review, the references of the obtained papers were checked to determine the type of studies they mainly refer to.

2.2 Improvements studies for modelling

In Table 2.1, studies on improving modelling and solving lot-sizing problems are included with their main results. These studies focus more on the efficiency of their lot-sizing models or on the capabilities of the solution processes.

The study of Van Vyve and Wolsey (2006) uses some rather basic lot-sizing problems and focus more on the mathematics of their approximation technique. Although, it is explained in the study of Miller and Wolsey (2003)

Table 2.1: Results of improvement studies on modelling

Study	Research aim	main result
Van Vyve and Wolsey (2006)	This study deals with lot-sizing and Travelling Salesman problems. The article presents the approximate extended formulations to create a formulation which is smaller, but still offers a good approximation.	Their approach is presented for different problems in lot-sizing and showed that smaller formulations are still able to obtain excellent bounds.
(Suwondo, 2012)	Presents a review on the model algorithms of dynamic lot-sizing problems. This research takes into account lot-sizing formulations with modern developments in problem and model formulation, and how the algorithms can be solved efficiently.	The difficulty of many extensions makes it hard to handle lot sizing issues. Many methods have been proposed to make the model formulas more precise. Some evidence suggests that the general algorithm of the Wagner-Whitin model has already produced a hopeful outcome and allowed for the continuation of some extensions.
(Jans & Degraeve, 2007)	The article covers the key elements of the different meta-heuristics and other solution approaches that have been specially created to address lot sizing problems.	A number of approaches to tighten the formulations are discussed to produce higher quality solutions. The use of these methods produces some encouraging outcomes, which should be exploited further.
(Absi & Kedad-Sidhoum, 2009)	This paper deals with a mathematical formulation of a lot-sizing problem with setup times. They create an algorithm to handle the induced as sub-problems and it applies a Lagrangian relaxation on the resource capacity constraints.	According to the numerical findings, the Lagrangian relaxation method produces better gaps and tighter lower bounds than a standard solver.
(Miller & Wolsey, 2003)	Formulas of discrete lot-sizing problem variations using mixed-integer programming are covered in this article.	Tight formulations are presented for the basic mixed-integer models that these problems contain. According to the article, these tight formulations are also applicable in industrial cases.
(Gutiérrez et al., 2021)	The economic lot-sizing issue with storage capacity is addressed with a set of effective MILP algorithms.	The results of the efficient algorithms are positive and the average running times are on average 100 times faster. By using a geometric technique, it is possible to accelerate the algorithm for a class of subproblems produced by dynamic programming.

that its research is motivated from a real-life problem, the study is still not recognised as an application-oriented study. The real-life problem is not further outlined and some basic discrete formulations are used to introduce the simple types of reformulations.

Another interesting literature review is conducted in the study of Glock (2012). It is focused on the so-called joint economic lot size (JELS) models. These models also determine the order, production, and shipment quantities while minimizing the costs. Glock (2012) experienced in its review that most studies have a theoretical focus. The review suggests to research the JELS models in more case studies and more empirical research.

When performing this literature review, it was also chosen to check the references used in the studies. It was seen that often the modelling studies refer to mainly other mathematical studies. Frequently, there are references dedicated to well-recognized mathematicians such as L. A. Wolsey and G.L. Nemhauser. For example, the study on heuristics for lot-sizing problems from Jans and Degraeve (2007) contains several references to studies from Wolsey and Nemhauser. The same holds for the studies of Absi and Kedad-Sidhoum (2009) and Miller and Wolsey (2003).

2.3 Application-oriented studies

Then, the studies on applications of lot-sizing are listed in Table 2.2. These studies focus more on the application instead of an effective formulation of the model.

Table 2.2: Results of practical studies

Study	Research aim	main result
(Kopanos et al., 2010)	This study presents a practical multi-product yoghurt lot-sizing case.	A new discrete/continuous mixed-integer linear programming model is given and applied successfully on the yoghurt case.
(Khaengkhan et al., 2021)	This study presents an application of a multi-level lot sizing problem with a comprehensive MILP model to address challenges with a real demand predicted from a sales history.	Two problems are addressed separately in a mathematical model. Further some experiments on crossover rates and mutation rates are presented for four cost scenarios.
(Paiva & Morabito, 2009)	This study provides a mixed integer programming model to optimize overall production planning decisions for sugar mills.	The outcomes of the model are encouraging. The decision-makers can better understand the variables and the related problems under consideration due to the model's insights.
(Doganis & Sarimveis, 2008)	A specific designed Mixed Integer Linear Programming model for yogurt packing lines with numerous parallel machinery is presented in this paper.	The results present a methodology for optimal scheduling of yogurt packaging lines. the model can significantly enhance operations by cutting setup and labor costs as well as the quantity of setups and production tasks.
(Andres et al., 2021)	For the production and scheduling of car plastic components, a mixed integer linear program model is researched in this article.	The results present a model that can satisfy the constraints of the related company. The model is also validated by a computational study.

In the last chapter of the study of Khaengkhan et al. (2021), it is suggested that the current solution process must be improved. It was experienced that the software used to find the solutions was ineffective and that it took a long time to find the solutions. The study of Paiva and Morabito (2009) presents a computational study. However, there is a difficulty in obtaining a optimal solution on some model formulations; prior simulations took twelve hours before achieving a proven optimum solution. Therefore, an specific optimality gap was accepted in order to solve the model more quickly. The study of Doganis and Sarimveis (2008) concludes that the model is only applicable for smaller sizes of time periods, as the computational process otherwise becomes too complex due to the binary variables. This suggests that the practical studies should consider more improvement studies for modelling to create their lot-sizing formulations.

When checking the references of these studies, it was seen that there is more diversity and more application-oriented references were found. For example, the study of Doganis and Sarimveis (2008) contain references on the research of milk products; its technology, chemistry, microbiology and its packaging. Paiva and Morabito (2009) refer to other case studies such as a scheduling problem for a rail system, electrofused grains and more applications in the sugar industry. Frequently, these practical studies are in collaboration with an respective company. Paiva and Morabito (2009) works on a case for sugar mills and Andres et al. (2021) collaborates with a car manufacturing company. It was also seen that some more modelling-oriented references were used in order to form their MILP model for the application. However, no focus was found on reformulating the model into a more effective formulation in terms of computational complexity.

2.4 Reflection of the review

The two types of lot-sizing literature are shown in a few examples with this literature review and in these examples there seem to be little interaction between improvement theories for modelling and the application-oriented studies in lot-sizing. To further outline and verify this interaction gap between the two types, a more comprehensive literature review must be conducted. It was experienced in this literature review that by using the suggested key words, a lot of hits (>100) were found in Scopus and Google Scholar. Due to the limited time, it was not feasible to read and scan all these studies, which makes this literature review less accurate and reliable.

Chapter 3

Research design and methodology

This chapter consists of three main sections. Section 3.1 introduces the article of Van Vyve and Wolsey (2006) in more detail. Section 3.2 presents the model formulations of Van Vyve and Wolsey (2006) that were taken into consideration. In Section 3.3, perishability in lot-sizing is discussed and how it can be implemented into the model. Subsection 3.3.2 shows how to apply the approximation technique on the perishability lot-sizing model.

3.1 Complex problems

Van Vyve and Wolsey (2006) state that problems that can be solved in an acceptable time consist of a specific type of MILP formulations, which can be operated algorithmically. However, the negative result of some larger models is that the problem becomes harder to solve and becomes therefore more complex.

To express the complexity of a model, mathematicians use the concept of time complexity. The time complexity can be seen as a measure of how frequently a statement is executed in an algorithm. For clarification, it does not represent the actual time it takes for the model to find a solution, since that is also affected by other factors such as the programming software and its computational power (Lopez Yse, 2020). The big $\mathcal{O}$ notation is used in mathematical language to demonstrate time complexity and it defines growth rates of execution time. In this way, multiple algorithms with the same growth rate can be represented by the same expression (Faust et al., 2015). The size of input is often described by a natural number n (Habala, 2020). For example, if the execution time it takes to run an algorithm grows exponential to the size of the input, time complexity is shown as $\mathcal{O}(2^n)$. Van Vyve and Wolsey (2006) expresses the complexity of their models in a different way. They use the big $\mathcal{O}$ notation, but they express the complexity for their models in: *complexity variables* $\times$ *complexity constraints*. Since the execution time typically increases with the number of variables and the number of constraints, this notation may give a better ability to identify the bottleneck.

Polynomially solvable problems are of polynomial size $\mathcal{O}(n^c)$. According to the article of Van Vyve and Wolsey (2006), formulations with polynomial sizes of $\mathcal{O}(n^3)$ variables and/or constraints are already too big to be applied in practice. Van Vyve and Wolsey (2006) state that these higher orders have no chance of success in actual problems with large sizes of n, especially if the intention is to employ such formulations for Branch-&-Cut operations.

Relaxations make it possible to reduce the computational complexity of models. The use of relaxations could result in solutions that are more efficient than those obtained through traditional methods (Bazaraa et al., 2006). This, however, gains solutions to a slightly different problem, and it is often unclear whether this more relaxed problem is still appropriate for the application. Linear programming relaxation is a technique used to handle a MILP problem as a linear programming problem (Cai, 2017). This technique is used to solve problems that have a large number of (binary) variables and constraints for example by ignoring the integrality constraints. However, this integrality relaxation appears to be case-dependent, as there are numerous situations which resulted in worsening of the computational performances (Alemany et al., 2018). The article of Van Vyve and Wolsey (2006) suggests that even dropping integrality constraints might not be sufficient to be able to solve the computational problems due to its complexity.

The goal of Van Vyve and Wolsey (2006) is to develop a relaxation of the formulation that is noticeably smaller, while yet offering a decent approximation of the original problem. The article discusses how to construct the approximate extended formulation for a wide range of computational problems.

3.2 Lot-sizing model formulations

In the following three sections, the constant capacity lot-sizing model and the uncapacitated lot-sizing model of Van Vyve and Wolsey (2006) are presented and discussed. Also, the approximate extended formulation of the uncapacitated lot-sizing model is presented in Section 3.2.3.

3.2.1 The constant capacity lot-sizing model

First, the study of Van Vyve and Wolsey (2006) introduces a basic lot-sizing model. They explore variants of the single item lot-sizing problem. To do this, the authors use a standard MILP formulation. This formulation includes the demand for each period (d_t), the unit production costs (p_t), the unit storage costs (h_t), and the fixed set-up cost (f_t) which is incurred if there is production in the period. The constant production capacity is represented by C. Additionally, the formulation has variables for the production amount in each period (x_t), the stock at the end of each period (s_t), and a binary variable (y_t) which is set to one if there is a set-up in that period. The number of time periods is given by n. The basic version of the lot-sizing model (LSP) reads:

$$\text{LSP: minimize } \left(\sum_{t=1}^{n} p_t x_t + \sum_{t=0}^{n} h_t s_t + \sum_{t=1}^{n} f_t y_t\right) \tag{3.1}$$

subject to

$$s_{t-1} + x_t = d_t + s_t \qquad \text{for } t = 1, \ldots, n, \tag{3.2}$$

$$x_t \leq C y_t \qquad \text{for } t = 1, \ldots, n, \tag{3.3}$$

$$x_t, s_t \geq 0, y \in \{0, 1\}^n, \qquad \text{for } t = 1, \ldots, n, \tag{3.4}$$

The objective (3.1) is the minimization of the summation of the costs during production. Constraints (3.2) are the inventory balance equations that also make sure that demand in every period t is met. Constraints (3.3) make sure that if there is any production in period t, the produced amount must be lower or equal to the production capacity. Constraints (3.4) are the non-negativity and binary constraints, or in other words the range constraints. This formulation is known as the constant capacity lot-sizing problem (LSP).

Without the use of a integrality relaxation, this model is difficult to solve, as it is a NP-Hard problem due to the binary variable (y_t). This makes the model hard to solve as the problem size increases (Taslaman, 2013). Fortunately, there are linear programming relaxations that are able to transfer the NP-hard problem to a related polynomial solvable problem (Zhou et al., 2016). This makes it possible to learn more about the optimal solution of the original problem by using the solution of the relaxed linear problem. But nevertheless, it should be realized that by using the linear programming relaxation, less precise results should be anticipated because a slightly different problem is solved. When integrating the integrality relaxation in this case, LSP gains a size of $\mathcal{O}(n) \times \mathcal{O}(n)$ variables and constraints, which was expected to give no computation problems. Therefore, no approximation is formulated for this model.

3.2.2 The uncapacitated model with Wagner-Whitin costs

Another formulation of a lot-sizing model is described in this section. This model is referred as the uncapacitated model with Wagner-Whitin costs (ULSP). If the production costs follow a specific structure, then the lot-sizing problem is denoted by the authors as having Wagner-Whitin costs. The condition is expressed by Van Vyve and Wolsey (2006) by means of the following inequality: $p_t + h_t \geq p_{t+1}$ for every t in the time horizon.

Wagner-Whitin costs are those where the production unit costs plus the cost of holding inventory for the current period is equal or larger than the production unit costs in the next period. This would mean that it would be beneficial to produce only the demand every time period and to have no holding costs, since the production unit costs cannot increase with each t. However, it should be noted that fixed set-up costs should also be considered before making this assertion.

The goal of the objective function of ULSP is similar to the objective function of LSP: minimizing the costs during production. However, the constraints are formulated differently in the ULSP and the production quantity variable x is left out in the formulation. Nevertheless, the production quantity x can be determined by understanding your setup decisions and stock levels over the time periods. However, Van Vyve and Wolsey (2006) choose not to include the variable x in this formulation when considering the Wagner-Whitin cost structure. The ULSP is formulated by Van Vyve and Wolsey (2006) in the following way:

$$\text{ULSP: minimize } \left(\sum_{t=0}^{n} h_t s_t + \sum_{t=1}^{n} f_t y_t \right) \tag{3.5}$$

subject to

$$s_{t-1} \geq \sum_{l=t}^{k} d_l \left(1 - \sum_{i=t}^{l} y_i \right) \qquad \text{for } t = 1, \ldots, n, \quad k = t, \ldots, n, \tag{3.6}$$

$$s_t \geq 0, y \in \{0,1\}^n. \qquad \text{for } t = 1, \ldots, n, \tag{3.7}$$

The objective function (3.5) minimizes the holding and fixed setup costs. According to inequality (3.6), the stock at the end of period $(t-1)$ must contain all the demand d_l if there are no setups throughout the time period from t to k. If there is an set-up planned during the interval from t to k, the value of $\sum_{i=t}^{l} y_i$ becomes 1 and the right-side of the inequality becomes 0. Note that the capacity constraint is not included in this model anymore, since the problem is characterized as uncapacitated. Constraints (3.7) are the range constraints.

When ignoring the integrality constraints, the model is transferred to a polynomial solvable problem again. The ULSP has a formulation of $\mathcal{O}(n) \times \mathcal{O}(n^2)$ variables and constraints. This means for the variables that the complexity grows in direct proportion to the size of the input. For the constraints, the time is proportional to the squared size of the input data set. This quadratic function is also a type of polynomial complexity. Because of this difference in formulation complexity compared to the LSP, the ULSP was expected to be more difficult to solve.

3.2.3 The approximate extended formulation

From a computational point of view, it is preferable to drop some constraints for the ULSP of size $\mathcal{O}(n) \times \mathcal{O}(n^2)$ in order to obtain a smaller and less complex formulation. Therefore, Van Vyve and Wolsey invent a time interval relaxation for the model to obtain an approximate extended formulation.

For the relaxation, Van Vyve and Wolsey add a condition with control parameter K, which is a value chosen by the producer. The time interval in the constraints of (3.6) are defined by its t and k, of which k represents the end of the interval. All the constraints of (3.6) that satisfy the next condition for the time interval are left out by the relaxation: $k \geq t + K$. Thus, if k is smaller than t plus control parameter K, the constraint is included when solving the problem.

When K is set to its maximum value ($K = n$), the formulation is equivalent to the complete formulation, meaning that the model is able to to achieve the strongest possible formulation. This formulation is larger and more complex than other formulations with lower K values. When K is set to 0, the approximate formulation is blank. By adjusting the value of K, the strength of the formulation and the size of the formulation is contrasted. Subsequently, the goal is to find the optimum trade-off for the particular application.

The notation of the constraints of (3.6) in this approximate extended formulation becomes:

$$s_{t-1} \geq \sum_{l=t}^{k} d_l \left(1 - \sum_{i=t}^{l} y_i\right) \quad \text{for } t = 1, \ldots, n, \ k = t, \ldots, n, \ | k < t + K$$

The introduction of control parameter K makes it possible to construct formulations of a smaller complexity. When applying the condition to ULSP, the formulation has a size of $\mathcal{O}(n) \times \mathcal{O}(Kn)$ variables and constraints. In such manner, both the variables and constraints obtain a linear time complexity. It was expected that by choosing an accurate K, the complexity of the model could be lowered. This might be able to solve the computational problems and reduce the solving time of large problems. However, since constraints are left out, the computed optimal point of this approximation might be different for the original problem and, thus, it is in question whether the computed solution can be implmented in practice.

3.3 Perishability in lot-sizing

Product deterioration is common in many production systems, and they manifest as physical exhaustion, loss of functionality and volume, or obsolescence. A type of deterioration is also known as perishability, and once a product is beyond its expiration date, it cannot be sold anymore (Acevedo-Ojeda et al., 2019). Businesses may suffer large losses as a result and therefore perishability should be taken into account when determining the optimal lot-sizes.

The food industry is one of the sectors where order quantities are crucial, as it includes perishable goods. Food supply chains are extremely complex. They must cope with limited shelf-life, product degradation, and price variability in addition to standard supply chain management challenges (Trienekens et al., 2012). Excessive orders and production volumes could lead to products being disposed; once its quality has fallen to a particular point and it is no longer marketable. This food waste has a negative effect on the supply chain performance. On the whole, the growing amount of food waste belongs currently to one of the biggest environmental issues in the world (EPA, 2022).

To attack this problem, it is crucial to effectively manage and control production resources along the whole production chain, from raw material to manufacturing to product storage and distribution. Production planning also plays an important role in this effective management. The production quantity should be as good as possible to prevent food waste or lost sales. The fact that the food products have a limited shelf life is the main element that makes production planning challenging. A product with a longer shelf-life is a large competitive advantage (Yalçiner, 2021). Therefore, it is essential to take shelf-life into account when applying lot-sizing models in the food industry.

3.3.1 The perishability model formulation

Van Vyve and Wolsey (2006) focus in their research mainly on the most effective notation for their basic lot-sizing models. They do not really incorporate real-life challenges of lot-sizing problems in their formulations. In this section, LSP is rewritten to account for perishable products which makes the model more applicable to a realistic situation. The LSP model of Van Vyve and Wolsey (2006) was adapted.

For the formulation, a planning horizon of n periods is considered and the products have a shelf-life of m periods. The index j represents the age in periods of a product. The holding costs of a product of age j in period t are h_{jt}. When the product reaches its final age m, the costs of disposing are included in the holding costs for h_{mt}. The production unit and fixed costs are respectively denoted by p_{jt} and f_t. The demand for period t is symbolized by d_t.

The following decision variables are present in the formulation. The amount of product of age j produced/purchased in period t is indicated by x_{jt}. The stock of age j at the end of each period is denoted by s_{jt}. Then, q_{jt} is the quantity of product of age j which fulfills the demand in time period t. The binary variable y_t is used to decide whether a set up of production is required or not. The perishability model reads as follows:

$$\text{minimize} \quad \left(\sum_{t=1}^{n} \sum_{j=1}^{m} p_{jt} x_{jt} + \sum_{t=1}^{n} \sum_{j=1}^{m} h_{jt} s_{jt} + \sum_{t=1}^{n} f_t y_t \right) \tag{3.8}$$

subject to

$$s_{j-1,t-1} + x_{jt} = q_{jt} + s_{jt} \qquad \text{for } j = 1, \ldots, m, \quad t = 1, \ldots, n, \tag{3.9}$$

$$x_{jt} \leq C y_t \qquad \text{for } j = 1, \ldots, m, \quad t = 1, \ldots, n, \tag{3.10}$$

$$\sum_{j=1}^{m} q_{jt} = d_t \qquad \text{for } t = 1, \ldots, n, \tag{3.11}$$

$$s_{1t} = 0 \qquad \text{for } t = 0, \ldots, n, \tag{3.12}$$

$$s_{j0} = 0 \qquad \text{for } j = 1, \ldots, m, \tag{3.13}$$

$$q_{jt}, x_{jt}, s_{jt} \geq 0, y \in \{0,1\}^n, \qquad \text{for } j = 1, \ldots, m, \qquad t = 1, \ldots, n, \tag{3.14}$$

The objective function (3.8) is to minimize the costs during production. The inventory balance constraints (3.9) ensure that any stock ages at the end of period t. Constraints (3.10) are the production capacity constraints. Constraints (3.11) makes sure that the demand is met by summing the age j of product quantity q_{jt}. Constraints (3.12) and (3.13) set the stock levels for age ($j = 1$) and time ($t = 0$) at zero. At the start of the time period, it is assumed for this model that there is no stock available. Additionally, no stock is available of age one, since only products of age one are produced. If not all the produced products are used for fulfilling the demand, they become stock worth of age two ($j = 2$). Constraints (3.14) are the range constraints.

This formulation is with the integrality relaxation a model of $\mathcal{O}(mn)$ constraints and $\mathcal{O}(mn)$ variables. It holds for both the constraints and the variables that their time complexity is proportional to the size of the input data set. This means that the model formulation is expected to be less complex to solve than the ULSP.

3.3.2 The approximate perishability model

Van Vyve and Wolsey (2006) present also a general approach for any lot-sizing model to relax a formulation, which can be applied for the perishability model. With this approximation, they define a time interval relaxation for each $[a, b]$ such that $1 \leq a \leq b \leq n$. In this time interval, a represents the initial number and b represents the final number of the interval relaxation. Thus, a range of $a = 1$ and $b = n$ means that the entire formulation is used and the problem is similar to the original problem. Once more, the control parameter K is introduced to define the time interval: $b = a + K - 1$. The time interval relaxation can also be applied on the perishability model:

$$\text{minimize} \left(\sum_{t=a}^{b} \sum_{j=1}^{m} p_{jt} x_{jt} + \sum_{t=a}^{b} \sum_{j=1}^{m} h_{jt} s_{jt} + \sum_{t=a}^{b} f_t y_t \right) \tag{3.15}$$

subject to

$$s_{j-1,t-1} + x_{jt} = q_{jt} + s_{jt} \qquad \text{for } j = 1, \ldots, m, \quad t = a+1, \ldots, b, \tag{3.16}$$

$$x_{jt} \leq C y_t \qquad \text{for } j = 1, \ldots, m, \quad t = a, \ldots, b, \tag{3.17}$$

$$\sum_{j=1}^{m} q_{jt} = d_t \qquad \text{for } t = a, \ldots, b, \tag{3.18}$$

$$s_{1t} = 0 \qquad \text{for } t = a, \ldots, b, \tag{3.19}$$

$$s_{j,a-1} = 0 \qquad \text{for } j = 1, \ldots, m, \tag{3.20}$$

$$q_{jt}, x_{jt}, s_{jt} \geq 0, y \in \{0,1\}^{b-a} \qquad \text{for } j = 1, \ldots, m, \quad t = a, \ldots, b, \tag{3.21}$$

For the formulation of the objective function and constraints, the time interval of $[a, b]$ is implemented. It should be noted that constraints (3.20) hold for $t = a$, as no stock can be incorporated from periods before the time interval. Overall, this will result in a formulation that uses $\mathcal{O}(mK) \times \mathcal{O}(mK)$ variables and constraints.

Van Vyve and Wolsey (2006) formulate an efficient way to approximate every possible time intervals K in a longer time horizon. With this method, a full time horizon will be split into multiple time boxes. These time boxes are the time frames that are optimized in the solver and they make it possible to approximate the time interval of size K. Van Vyve and Wolsey (2006) state that having an accurate amount of overlapping time boxes is essential, since then better performances of the branch-and-bound method will be obtained for the approximation. In order to approximate any interval of exactly size K, $\lceil \frac{n}{K-1} \rceil - 1$ overlapping time boxes of size $\min(2(K-1), n)$ are needed. For example, when considering 12 time periods and $K = 4$, there are $\frac{12}{4-1} - 1 = 3$ overlapping time boxes needed. The size of formulated time boxes will be $(2(4-1) = 6$ periods. This example is represented in Figure 3.1.

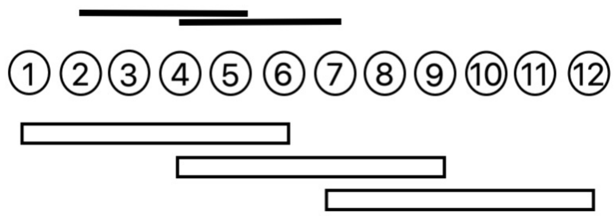

Figure 3.1: An example of determining time boxes. The black lines show interval $K = 4$ and the white boxes correspond to the formulated time boxes of 6.

The three formulated time boxes of Figure 3.1 are optimized seperately. This means that the optimized lot-sizing planning for time frames [1,6], [4,9] and [7,12] are obtained via a solver. Then, Van Vyve and Wolsey (2006) argue that every possible interval of ($K = 4$) is included within the results of one time box; the approximation of [1,4] can be found in the solutions of the first time box, the approximation of [3,6] in the first box and the approximation of [7,10] in the third time box. This corresponds to that every black stripe is completely contained within the results of one white time box, which can be seen from the figure. By solving the three time boxes, an approximation for every interval of exactly 4 in the time horizon will be known. This, however, means that the solutions should be only read and expressed in time intervals of exactly 4. Longer time intervals are not always completely included in one white box. For example a interval size of [6,10] is not included within the results of one white box, so no approximation can be read. Despite that the shorter intervals are completely contained in one white box, they do not correspond with the amount of overlapping white boxes. For example, the interval [4,6] can be found in the solutions of the first white box, but also in the second white box. It might be that production decisions are different between the two boxes and that causes confusing. Therefore, when using this method, one should only read and present the solutions in intervals of the chosen K.

With this time interval relaxation, the amount of constraints and variables that needs to be processed by the solver can be decreased until it is adequate. This would make it possible to decrease the complexity of a model. However, applying the time interval relaxation will lead to solving a different problem and there may be differences expected when comparing results to the original problem.

Chapter 4

Computational results and discussion

The first section in this chapter explains how the computational study was designed. Section 4.2 presents the computational performances of the model formulations of Van Vyve and Wolsey (2006) and is followed up by a discussion of its results. Subsequently, in Section 4.3, the results of the perishability model are presented and discussed.

4.1 Computational study

A computational study is conducted to compare the performances of the models. Three simplified situations are tested for each formulation of the model. By simplifying the situation, the computation performances are measured more efficient, which is critical for determining the effectiveness of the different formulations.

Three situations are described with planning horizons 10, 50 and 100. A demand is assigned to every period, which is a randomly chosen amount between 10 and 30. The same holds for the fixed costs, which is randomly chosen between 10 and 25, and holding costs, which is randomly chosen between 0.4 and 1.0 for each period t. These parameters differ per period to create the complex cost situation, which can also be expected in realistic applications. The different parameters were generated with the function RANDBETWEEN in Microsoft Excel. In order to fulfill the Wagner-Whitin cost structure ($p_t + h_t \geq p_{t+1}$), the production unit costs is a constant value over the planning horizons. The production capacity is also an constant value. The parameters of the situations are expressed in Table 4.1.

This computation study is also conducted for the perishability lot-sizing model and its approximated model. Situations with different time periods (n) and shelf life (m) were tested to get to know the computational performances of these models. These parameters are also shown in Table 4.1. The dataset needed minor adjustments compared to the previous situations, as it needs to deal with perishable products. For every situation, a maximal shelf life (m) was assigned and the disposal cost was added to the holding cost of product age m. The m represents the maximal shelf life of the products in the production facility. If the product reaches this age, the products needs to be dismissed. This resulted in constant holding cost for products at age m of 1.8.

Table 4.1: Parameters of the simplified situations

Time periods (n)	10, 50, 100
Demand (d_t)	between 10 and 30
Fixed costs (f_t)	between 10 and 25
Holding costs(h_t)	between 0.4 and 1.0
Production unit cost (p)	2
Production capacity (C)	45
Shelf life (m)	5, 10, 15
Disposal costs (h_{mt})	1.8

The generated dataset in Microsoft Excel was used for all simulations. The mathematical programming package software Fico Xpress IVE was used to run the solutions. The optimized results showed the solving time in seconds, number of nodes needed in branch-and-bound and the optimality gap in percentages. The optimality gap displays the difference between the optimal value and the current best bound. Further, a solving time limit of 1800 seconds was established, which means that the simulation will be forced to stop at that time point. Regardless of whether the solution is already optimal or not.

4.2 Results and discussion of LSP and ULSP

The described situations were optimized for the different formulations of Van Vyve and Wolsey (2006). The computational results of LSP from Section 3.2.1 are listed in Table 4.2

Table 4.2: Computational results of LSP

Periods (n)	Optimality Gap (%)	Nodes	Time (s)
10	8.9e-05	1	0.1
50	0	1	0.1
100	9.98e-5	1	0.1

This model does not have computational problems for the three different sizes of input. The optimality gaps are of negligible value and only a single node in branch-and-bound is necessary. Then, in Table 4.3, the computational performances of ULSP from Section 3.2.2 are presented.

Table 4.3: Computional results of the ULSP

Periods (n)	Optimality Gap (%)	Nodes	Time (s)
10	0	1	0.1
50	0	1	0.1
100	0	1	0.1

The same holds for this model, no computational problems are detected and all the simulations are immediately solved. Therefore, it was not needed to apply the approximate extended formulation on this model. As can be seen in Table 4.3, there is no room for improvement on the computational performances. The approximation model would only decrease the accuracy and the completeness of the results.

The LSP model, also known as the constant capacity lot-sizing problem, is known to be a NP-hard problem. Florian et al. (1980) and Bitran and Yanasse (1982) confirm that constant capacity problem is a NP-hard problem for different cost functions. Due to the arrival of linear programming relaxation, the problem can be transferred from NP-hard to a related polynomially solvable problem. However, again, it should be noted that the solutions to this related problem might not be optimal for the original problem. The complexity of LSP gains a size of $\mathcal{O}(n) \times \mathcal{O}(n)$ variables and constraints, which means it was also not considered as a problem by Van Vyve and Wolsey. Therefore, the results of this model met the expectations. It is noticed that the original LSP model with the integrality relaxation can be solved without computational problems for the three different sizes of input.

ULSP with the integrality relaxation has a larger size of time complexity with the size of $\mathcal{O}(n)\times\mathcal{O}(n^2)$ and was expected to be harder to solve. Nevertheless, the results have shown that there are no computational problems for this model as well. For both LSP and ULSP, no interval relaxation is required, so the approximation technique from Van Vyve and Wolsey (2006) is not applied.

4.3 Results and discussion perishability model

Then, the perishability model and its approximation model were tested. For each scenario, a maximum shelf life (m) was established in such way to obtain a clear overview of its effect. In Table 4.4 the chosen m and the computational results of the perishability lot-sizing model are listed.

Table 4.4: Computational results of the perishability lot-sizing model

Period (n)	Shelf life (m)	Optimality Gap (%)	Nodes	Time (s)
15	2	8.24e-5	1	0.1
15	3	0	1	0.1
15	4	7.85e-5	3	0.1
50	5	7.39e-5	182	1.4
50	10	9.77e-3	5524	34.0
50	15	7.51e-5	1830	45.0
100	5	6.97e-3	6698	492.6
100	10	3.48	207371	1800.0
100	15	4.25	196669	1800.0

As can be detected from the table, there are no computational problems with the smaller situation of 15 time periods. Regardless of the differences in the shelf life of the products, the model can be solved immediately and no optimality gap is obtained. This means that the integrality relaxations seems to be sufficient for these cases. For the situation with 50 time periods, the software already has more difficulty in solving, but still succeeds within a few seconds. The optimality gaps are of negligible values. However, it could be noticed that the increase in shelf life has a considerable effect on the solving time. The longer the shelf life, the more efforts the software has to put into solving and more nodes in branch-and-bound need to be explored. The larger situation of 100 time periods becomes more difficult to solve. It was managed to solve the smaller shelf life of 5 periods in some minutes. However, the longer shelf life considerations could not be solved to optimality within the time limit of 1800 seconds. Their optimality gap is higher compared to the other situations, since the simulation was forced to stop before reaching its optimality. With this larger application, it becomes too complex for the solver and too many variables and constraints need to be processed.

As a result of this observation, it was chosen to apply the approximate perishability formulation for the situation with 100 time periods. The other situations with less time periods were considered to be able to solve within an acceptable time. There is no need to approximate in those circumstances, as doing so would affect the outcome's accuracy.

For the approximation, a time interval of ($K = 26$) was chosen in the time horizon of 100 periods. This was considered as an useful interval and it gains integer number in further calculations. There are $\frac{100}{26-1} - 1 = 3$ overlapping time boxes needed of size $(2(26-1) = 50$. This means that the situation is solved in three time boxes of [1,50], [25,75], [50,100] for all shelf life options. The computational results are shown in Table 4.5.

Table 4.5: Computational results of the approximation model

Period (n)	Shelf life (m)	Time box	Optimality Gap (%)	Nodes	Time (s)
100	5	[1,50]	7.98e-5	3	0.4
100	5	[25,75]	8.15e-5	3	0.3
100	5	[50,100]	8.10e-5	1030	0.5
		Total	2.42e-4	1036	1.2
100	10	[1,50]	7.83e-5	1040	1.7
100	10	[25,75]	8.03e-5	14	1.2
100	10	[50,100]	7.98e-5	9	1.2
		Total	2.38e-4	1063	4.1
100	15	[1,50]	7.78e-5	1082	2.9
100	15	[25,75]	7.98e-5	11	1.9
100	15	[50,100]	7.93e-5	31	1.8
		Total	2.37e-4	1124	6.4

Over the entire 100 period time horizon, each interval of $K = 26$ is approximated with this method. To clarify with an example, an approximation of the optimized production planning for interval [68,92] can be found in the solutions of the last time box [50,100]. The results show that the total solving time is drastically reduced to a more acceptable time. The larger shelf life considerations can now be solved within a few seconds. However, with this approach, the approximations can only be expressed using intervals of 25 periods. The results are not really applicable for intervals of 30 periods or larger, as then the approximation may be less accurate.

The larger situations on the perishability model are much harder to solve compared to LSP and ULSP. Implementing perishability to the model increases the problem complexity. In this situation, more constraints and variables needed to be processed which increased the computational time. The expectations for the approximation model were met. The time interval relaxation was able to reduce the complexity and therefore lower the solving time.

Comparison of objective values

To estimate the impact on the accuracy of the results, the smaller situation of 50 time periods is tested with the approximation model and compared with the original model on their objective values. The approximation model only gives the costs for intervals, but a rough estimation is calculated to determine the total costs over the full time horizon. As mentioned before, the results should be expressed in only ranges of the intervals, but for research purposes it was decided to compare the objective values of the full time horizons. For the approximation of this 50-time-period situation, three different K are tested and compared on their objective values. For example, with $K = 11$, 4 time boxes of size 20 are implemented. For the estimation of the total costs in the full time horizon, the costs of the 4 time boxes are summed and subsequently divided by the ratio $\frac{4\times 20}{50}$. It is possible to estimate the costs by using this ratio. The detailed calculations of these estimations are shown in Appendix B. The results are listed in Table 4.6.

Table 4.6: Objective value comparisons of the original and approximation model

Control parameter K	Original model	Approximation model	Difference (%)
13.5	2351	2391	1.7
11	2351	2450	4.2
6	2351	2641	12.3

It can be seen in the table that there is a noticeable difference between the objective value of the original model and the approximation model, which is as expected. The production planning of the original model has less costs than those obtained from the approximation model. The larger the control parameter K, the more accurate the results of the approximation model are. However, the larger the K, the bigger the time intervals are and more effort is required from the solver. Thus, lowering the computation complexity with the approximation model comes with the cost of less accurate results.

Chapter 5

Discussion

First, the approximation technique of Van Vyve and Wolsey (2006) on ULSP should be discussed. The binary variables in the lot-sizing formulations may create situations that are more challenging to solve. They could be the cause of poorer computational performances. Van Vyve and Wolsey (2006) did not focus on the binary variables. Their technique on ULSP implies a relaxation by dropping some constraints by using a time interval relaxation. It is also known that reducing the amount of (binary) variables has an impact as relaxation. With fewer variables, the algorithm has fewer solutions to consider, which saves time and reduces the computational complexity of the problem. On the other hand, lowering the amount of constraints can also relax the model, but does not have the same impact as the reduction of variables. However, this impact of lowering the amount of constraints on the ULSP cannot be confirmed in this research, as the computational performances by the original model cannot be improved.

For the general approach of Van Vyve and Wolsey (2006), it is tried to ease the situation by another interval relaxation. They are redesigning the time horizon rather than rewriting the model or adding additional restrictions. By enabling a reduction in the amount of constraints and variables that needs to be processed, the problem complexity can be decreased. This general approach will not be useful for smaller applications, as then the integrality relaxation seems sufficient. However, the approach is applicable for the larger applications that can be split into multiple time intervals. Only intervals of that particular size are covered by the optimized results, not the full time horizon. As a result, the actual optimization of the original problem may differ in some decisions. Therefore, when utilizing the approximation procedure, it has an impact on the accuracy and completeness of the results.

Reducing the amount of variables and constraints will result in a loss of information. Thus, the producer should decide whether to sacrifice result's accuracy in order to shorten the solving time.

Furthermore, it is arguable whether the input data and other structures of the problem contributed to a relatively simple computational complexity for the lot-sizing models. Therefore, three causes are described in the following sections: The data set, Problem complexity and Sophisticated solvers. These sections can explain why the LSP and ULSP models are easily solved without any computation issues.

The data set

With the data set, different situations for the simulations were designed. However, it should also be said that these situations contain assumptions which are not very representative in a practical applications. The costs range could be oversimplified and therefore the costs may not accurately capture the true complexity of the lot-sizing problem. Van Vyve and Wolsey (2006) use the Wagner-Whitin costs, which is specific type of cost structure where $p_t + h_t \geq p_{t+1}$ for every t. The study of Pochet and Wolsey (1994) has shown that by using this cost structure, a lot-sizing model can be implemented of size $\mathcal{O}(n)$. The data set has been made to fulfill this cost structure. There was a constant unit production cost of 2. A consequence is that the solver only needs to optimize the fixed setup costs and the holding costs planning. The solver does not need to take the production unit costs into account. This data structure may be able to lower the problem complexity and the reasoning suggests that the data set was not quite accurate for lot-sizing problems in reality.

Problem complexity

Besides the effect of the data set, other structures of the problem simplified the lot-sizing model. According to the study of Karimi et al. (2003), the complexity of lot-sizing model depends on some characteristics and features of the lot-sizing problem. These characteristics can prove the simplicity of LSP and ULSP. The characteristics are listed in Table 5.1.

Table 5.1: Complexity characteristics for lot-sizing problems identified by Karimi et al. (2003)

Characteristic	Explanation
Planning horizon	In terms of time terminologically, there are two types of lot sizing issues: *big bucket* issues and *small bucket* issues. Big bucket problems have time periods that are long enough to create multiple items, whereas small bucket problems have time periods that are too short to produce more than one item at a time. Big bucket problems have a larger problem complexity.
Number of products	In a single-item production problem, a planning activity must be organized for only one end item, but there are multiple end items in multi-item production planning. A multi-item production is harder to solve and increases complexity.
Number of levels	There exist single-level and multi-level production methods in a lot-sizing problem. The single-level systems is typically straightforward; the finished product is directly made from raw materials. For multi-level production there are more steps in the production line. Single-level problems are easier to solve.
Capacity restriction	The presence of restrictions on resources or capacities will increase the problem complexity
Product deterioration	There are more limitations in the inventory holding duration when products could deteriorate. This would increase the problem complexity.
Demand	Demand is said to be deterministic if the value is known in advance (static or dynamic), but it is said to be probabilistic if the exact value is unknown and the demand values occurring are dependent on some probability. Problems with deterministic demands are easier to solve.
Setup structure	A simple setup structure is one in which the setup time and cost for a period are independent of the decisions made in earlier periods. In a complex setup structure more factors are dependent.
Inventory shortage	If shortage is permitted, it implies that the present demand can be supplied in future periods (backlogging situation) or that demand may not even be permitted to be satisfied (lost sales situation). These situations will make the problem more complex.

A feature that characterizes the complexity is the planning horizon. The models of Van Vyve and Wolsey (2006) consider that more products can be produced in one time period, thus the models are considered as a big bucket issue. Another characteristic that is recognized, are the the capacity constraints. The basic lot-sizing model (LSP) of Van Vyve and Wolsey (2006) considers the capacity constraints, but the uncapacitated model (ULSP) does not. By defining the problem as uncapacitated for the second model, the problem complexity is reduced again. Besides the capacity constraint and the big bucket issue, other complexity characteristics from Table 5.1 cannot be recognised in LSP and ULSP. Therefore, the models from Van Vyve and Wolsey (2006) can be regarded as relatively simple in terms of problem complexity. This further compounds the research gap. Van Vyve and Wolsey (2006) specifically focus on approximating a lot-sizing problem, which is rather simple and will not be seen in reality. Although, it should be stated that Van Vyve and Wolsey (2006) consider more problems in their study such as Traveling Salesman Problem and the Fixed-Charged Network Problem, but those were not considered in this research.

By implementing perishability or product deterioration in the model, more complexity is recognised in the problem. According to the identified characteristics from Karimi et al. (2003), the additional limitations for holding inventory makes solving the situation more difficult. This is also verified with the results of the perishability model in the computational study. The solver needs to put more effort in solving this model compared to LSP and ULSP.

Sophisticated solvers

Van Vyve and Wolsey (2006) use Xpress MP version 14.05 for their computational study, which is released in 2004. For this research, Xpress Mosel version 5.2.0 is used, which is released in 2020. Fico Xpress continuously improves its products and has released a lot of versions (FICO Xpress, n.d.). The study of Luppold et al. (2018) has demonstrated that there are significant variations in the computational performances between different solvers. The solvers of today are superior than the problem solvers of the past, which could have created differences in the expectations.

Chapter 6

Conclusion

Objectives of this research are to verify the approximation of Van Vyve and Wolsey (2006) and to place their model in a more practical setting by rewriting it to a more realistic application.

The approximate extended formulation from Van Vyve and Wolsey (2006) can be applied on different formulations of lot-sizing models. For that, a time interval relaxation is indicated by establishing control parameter K.

The considered models of Van Vyve and Wolsey (2006) are tested on their computational performances. Both LSP and ULSP show no computational problems and are solved immediately. The approximate formulation of ULSP is not needed, as there is no room for improvement regarding the computational performances.

Perishability can be implemented for the LSP model. The simulations of this model show that the software has to put more efforts in solving and was not able to optimize the larger application within 1800 seconds. The larger situations are therefore tested on the approximate perishability formulation. In this way, the larger situations can be solved in time intervals within a few seconds.

Overall, the approximation of Van Vyve and Wolsey (2006) is able to reduce the complexity of lot-sizing models, but comes at a certain cost of less accurate results. The approach can be applied in practical modelling. Nonetheless, the simple lot-sizing problems seem not to have any computational problems. In such situations, it is not needed to apply an approximation. The producer must choose whether to compromise the accuracy of the results in order to fasten the solution process.

Chapter 7

Recommendations

The findings of this study should be utilized as a guide for the computational performances of lot-sizing problems in the current sophisticated solvers. The results limitations have been acknowledged in the discussions, and some of them could be decreased by further research. Some recommendations for this research are listed below.

- In further research, the problem complexity of the lot-sizing models should be increased with the characteristics identified by Karimi et al. (2003), see Table 5.1. The two lot-sizing models of Van Vyve and Wolsey (2006) seem to be relatively simple and therefore have no issues when solving. Perishability is added to the model as the first move toward practical modeling. However, real-world situations face more difficulties than just perishability, so the model can be even further expanded to a more practical situation.

- Another recommendation could be to better analyse the effect of the data set on the solving process. As stated in the discussion of the results, the structure of the data set could have implied already some relaxations for the solving process. To analyze the impact of the data set and its structures, further research could be conducted.

- Further, the quality of the results is mainly grounded on the computational time in this research. It is also interesting to see whether there are many differences in set up decisions when applying the interval relaxation of Van Vyve and Wolsey (2006). For example, to compare the decisions in the overlapping intervals with the decisions of the original model could be useful to analyze. In this way, it can become clearer for the producer how to interpret the obtained results, what the impact will be on the results accuracy and how to choose a right interval relaxation.

- Finally, other improvement techniques for modelling could be explored to compare the outcomes. For example, Another promising technique is disjunctive programming from Balas (1998). This approach seems to be significant for zero-one programming and can transfer the integer models into disjunctive models. Further research could be conducted to analyse whether this approach is applicable for practical lot-sizing problems.

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Appendix A

Terminology and definitions

This appendix could provide some additional information to better understand this paper and the Van Vyve and Wolsey (2006) article. This appendix offers additional background information and mathematical terminology that are applied in the studies. The references are shown in the Bibliography.

History of lot-sizing problems

Over the past fifty years, the lot-sizing problems have been intensively researched (Suwondo, 2012). It is one of the most difficult problems in production planning, since a lot of conditions should be taken into account (Karimi et al., 2003). The Economic Order Quantity, or EOQ, was created by Harris as the first solution to this issue. Demands in the EOQ model are constant across an indefinite planning horizon. The model was able to give the optimal size, but did not consider dynamic demands. In real-life applications, it rarely happens that demands stay constant throughout an indefinite time period. As pioneers, Wagner and Whitin (1958) took on this lot-sizing issue. They provided the ideal lot-sizing solution by their method using dynamic programming. For issues with deterministic and dynamic demands and costs, the Wagner-Whitin model is helpful (Hanafizadeh et al., 2019). Later, some improvements were made on the Wagner-Whitin algorithms and other solving methods were discovered.

Combinatorial problems

In the article of Van Vyve and Wolsey (2006) they mention a lot about combinatorial problems. These are problems which consists of a finite set of constraints and a collection of objects. The goal is to find an optimal object from the collection that meets every constraint.

Mixed Integer Linear Programming

Combinatorial optimization problems can be solved by Mixed Integer Linear Programming (MILP). MILP is an extension of linear programming. MILP are mathematical models that cope with optimizing a linear objective function under linear constraints, but with some restrictions on some components. Both discrete and continuous variables are used in MILP. Hence, some decision variables are required to be integers while others are permitted to be continuous (Balaman, 2019). Often a flow of product or material is represented by a continuous variable and the design or configuration of the production chain is presented by an integer variable. There is also a special type of integer variable in MILP, which is called a binary variable. These variables can only take the possible values 1 or 0 and this can decide whether an action should be performed or not (Sioshansi & Conejo, 2017). For example, in a lot-sizing model, it may represent whether the product should be produced in a specific period or not.

Branch-and-bound

MILP models are usually solved by using the branch-and-bound method. The key concept is to successively subdivide the solution space into different sets, which are called branches. For maximization, upper bounds for the objective function over the subsets are calculated in order to eliminate some of these subsets and narrow the solution space. A subset is eliminated if its upper bound is strictly smaller than a lower bound at the maximum value. This is obtained by evaluating the objective function at an feasible point (Theurich et al., 2021). This process of eliminating continues until the gap between the upper and lower bound becomes sufficiently small.

Branch and Cut

The Branch and Cut method is a more efficient method for solving integer programming models. It combines the branch-and-bound and the cutting plane technique. The principle is to add constraints to produce whole solutions. Cutting planes (constraints) are added to an optimal solution such that the optimal integer solution is preserved, but the optimal solution violates at least one of the new constraints. Practically speaking, the optimization challenge of branch-and-bound is simplified by the addition of a few cutting planes (Naud et al., 2020).

Convex hull

The article of Van Vyve and Wolsey (2006) use terminology of computational geometry, for instance the convex hull. The convex hull is known as the smallest shape that encompasses the entire collection of points (Qi et al., 2022). The convex hull is essential for convex optimization, which is a technique used to discover the optimal solutions for a problem with a matching algorithm.

Valid inequalities

Valid inequalities are the constraints that reduce the feasible space without removing the integer solutions (Frias, 2019). The valid linear inequalities can be used to define boundaries of the convex hull (Belotti et al., 2010).

Heuristics

A heuristic is a problem-solving method used in computer science that is intended to find an acceptable solution more quickly, but is not guaranteed to find the optimal solution. In optimization problems such as for lot-sizing problems, heuristics are frequently used.

Appendix B

The estimation of the total costs with the approximation model

In this appendix, more detailed tables are given on how the estimations of the total costs for the approximation model are calculated. The estimations are based on the scenario with 50 time periods and 15 shelf life periods. Every table implements a different K in the scenario. First, the objective values of the intervals are summed. Subsequently, the ratio $\frac{\text{amount of intervals} \times \text{size of intervals}}{50}$ is used to divide the summed costs to get an estimation for the 50-time-period scenario. The results are listed below in the tables.

Table B.1: Estimation of total costs with $K = 13.5$

Time box	Objective value
[1,25]	1200
[13,38]	1154
[25,50]	1233
Total	3587
Estimation for 50 periods	2391

The total costs of the intervals is divided by $\frac{3 \times 25}{50} = 1.5$ to get the estimation for the total costs of the 50-time-period scenario.

Table B.2: Estimation of total costs with $K = 11$

Time box	Objective value
[1,20]	963
[10,30]	965
[20,40]	960
[30,50]	1033
Total	3921
Estimation for 50 periods	2450

The total costs of the intervals is divided by $\frac{4\times20}{50} = 1.6$ to get the estimation for the total costs of the 50-time-period scenario.

Table B.3: Estimation of total costs with $K = 5$

Time box	Objective value
[1,10]	489
[5,15]	598
[10,20]	540
[15,25]	463
[20,30]	468
[25,35]	542
[30,40]	556
[35,45]	554
[40,50]	548
Total	4758
Estimation for 50 periods	2641

The total costs of the intervals is divided by $\frac{9\times10}{50} = 1.8$ to get the estimation for the total costs of the 50-time-period scenario.