BULLETIN

15

COMPUTING DRAIN SPACINGS

INTERNATIONAL INSTITUTE FOR LAND RECLAMATION AND IMPROVEMENT/ILRI

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Bulletin 15.

COMPUTING DRAIN SPACINGS

A generalized method
with special reference to sensitivity analysis
and geo-hydrological investigations

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In memory of

Dr S. B. Hooghoudt († 1953)

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Contents

1. Introduction

This Bulletin summarizes the latest developments that have taken place in The Netherlands on the subject of computing drain spacings using drainage equations, based on the assumption of steady-state conditions. Those based on non-steady state conditions will be handled in a separate bulletin.

It is assumed that the reader is familiar with drainage equations in general and with those developed in The Netherlands in particular (S.B.Hooghoudt en L.F.Ernst). These earlier contributions to the theory and practice of drainage equations have been summarized by Van Beers (1965), who dealt specifically with Dutch efforts in this field, and by Wesseling (1973), who also included methods developed in other countries.

To avoid the need to consult those earlier publications, the main principles of the Hooghoudt equation are given in Section 2 and those of the Ernst equation in Section 3.

When using equations based on steady-state conditions,one should realize that such conditions seldom occur in practice. Nevertheless the equations are extremely useful, because they make it possible:

- to design a drainage system which has the same intensity everywhere even though quite different hydrological conditions (transmissivity values) occur in the area
- to carry out a sensitivity analysis, which gives one a good idea of the relative importance of the various factors involved in the computations of drain spacings.

Drainage equations and nomographs: past and present

The equations and graphs that have been available up to now are useful for the "normal" drainage situation. By "normal", we mean that there is only one pervious layer below drain level and only a slight difference between the soil permeability above drain level (K_1) and that below drain level (K_2).

Most equations and graphs have their shortcomings. In the following drainage situations, for instance, there is only one possible equation that can be used:

$K_1 >> K_2$: a highly pervious soil layer above drain level and a poorly pervious soil layer below drain level: only Eq. Hooghoudt

$K_1 << K_2$: a heavy clay layer of varying thickness overlying a sandy substratum: only Eq. Ernst

$K_3 >> K_2$: the soil below drain level consists of two pervious layers, the lower layer being sand or gravel (aquifer): only Eq. Ernst

(This a common occurrence in drainage and is highly significant for the design.)

Aim of this bulletin

Because of these shortcomings and the inconvenience of working with different equations and graphs, the question was raised whether a simple equation with a single graph could be developed to replace the existing ones. The problem was solved by Ernst (1975), who combined the Hooghoudt equation and the Ernst equation for radial flow, resulting in a single expression which we shall call the Hooghoudt-Ernst equation.

Although the fundamentals of the equation have been published elsewhere by Ernst, it is the aim of this bulletin to focus attention on these recent developments and to illustrate the practical use of the equation and the corresponding graph which has been developed for this purpose (Graph I). The graph can be used for all the above drainage situations, although for the third one ($K_3 >> K_2$), an additional auxiliary graph will be needed.

It will be demonstrated that no graph at all is needed for most drainage situations, especially if one has available a Scientific Pocket Calculator (SPC).

Although not strictly necessary, a special graph has nevertheless been prepared for normal drainage situations and the use of pipe drains (Graph II). The reader will find that it gives a quick answer to many questions.

It may be noted that with the issue of this bulletin (No.15), *all graphs contained in Bulletin 8* are now out of date, although Graph 1 of Bulletin 8 (Hooghoudt, pipe drain) still remains useful for theoretically correct computations and for the $K_1 >> K_2$ situation; in all other cases Graph 2 of the present bulletin is preferable.

The reader will also note that in this new bulletin, *a revised nomenclature for various K- and D-values* (thickness of layer) has been introduced.

The modified meanings of these values are not only theoretically more correct but also promote an easier use of the K- and D-values.

Last but not least, the *importance of geo-hydrological investigations*, especially in irrigation projects, is emphasized because a drain spacing can be considerably influenced by layers beyond the reach of a soil auger.

Sensitivity analysis

The primary function of a drainage equation is the computation of drain spacings for drainage design. Since it summarizes in symbols all the factors that govern the drain spacing and the inter-relationship of these factors, it also allows a sensitivity analysis to be performed if there is a need to.

A sensitivity analysis reveals the relative influence of the various factors involved: the permeability and thickness of the soil layers through which groundwater flow can occur (depth of a barrier), wetted perimeter of drains, depth of drains, etc. This analysis will indicate whether approximate data will suffice under certain circumstances or whether there is a need for more detailed investigations. The drainage specialist will find the sensitivity analysis a useful tool in guiding the required soil and geohydrological investigations,which differ from project to project, and in working out alternative solutions regarding the use of pipe drains or ditches, drain depth, etc.

For a sensitivity analysis, however, it is a "conditio sine qua non" that the available equations and graphs should be such that the required calculations can be done easily and quickly.

In the opinion of the author, this condition has been fulfilled by the equations and graphs that will be presented in the following pages,especially if one has an SPC at his disposal.

2. Principles of the Hooghoudt equation

2.1 Ditches reaching an impervious floor

For flow of groundwater to horizontal parallel ditches reaching an impervious floor (Fig.1) horizontal flow only, both above and below drain level may be assumed, and the drain discharge, under steady-state conditions can be computed with a simple drainage equation:

$$q = \frac{8K_2D_2h}{L^2} + \frac{4K_1h^2}{L^2} \quad \text{or} \tag{1}$$

$$q = q_2 + q_1$$

where

- q = drain discharge rate per unit surface area per unit time (m^3 per day/m^2 or m/day)
- q_2 = discharge rate for the flow *below* drain level
- q_1 = discharge rate for the flow *above* drain level
- D_2 = thickness of the pervious soil layer *below* drain level (m) (depth to an impervious layer or depth of flow) or
  = cross-sectional area of flow at right angles to the direction of flow per unit length (metre) of drain (m^2/m)
- K_2 = hydraulic conductivity of the soil (flow region) *below* drain level (m/day)
- K_1 = hydraulic conductivity of the soil (flow region) *above* drain level (m/day); for homogeneous soils $K_1 = K_2$
- h = hydraulic head - the height of the water table above drain level mid-way between drains (m); note that the water table is defined as the locus of points at atmospheric pressure
- L = drain spacing (m)

If, for the flow above the drains, one wants to avoid the use of a certain notation (h) for two quite different factors, being a hydraulic head and an average cross-section of flow area ($\frac{1}{2}$ h), it is preferable to write Eq.(1) as

$$q = \frac{8K_2D_2h}{L^2} + \frac{8K_1D_1h}{L^2} \quad \text{(Fig.1)} \tag{1a}$$

where

- D_1 = average depth of flow region *above* drain level or average thickness of the soil layer through which the flow above the drains takes place.

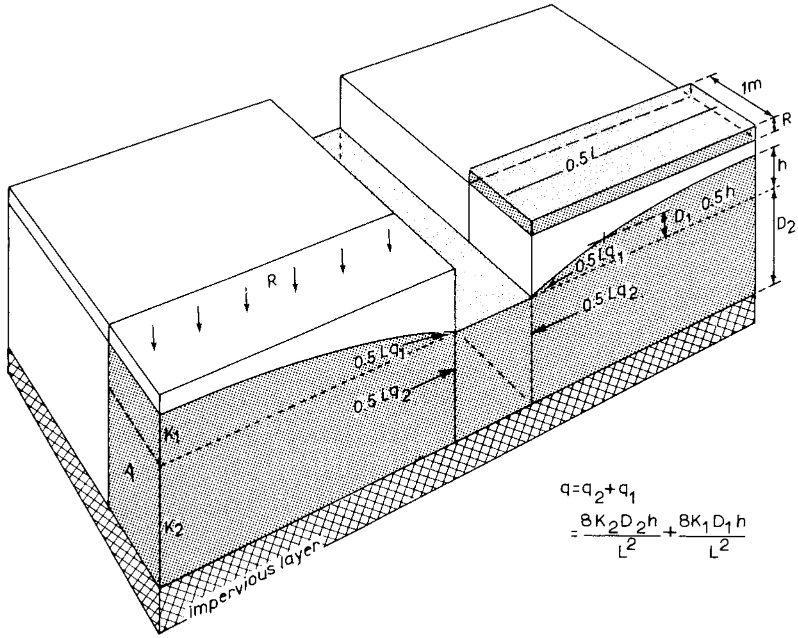


Fig.1. Cross-sections of flow area. Steady-state conditions: discharge (q) = recharge (R). Parallel spaced drains reaching an impervious floor.

Various discharge values: q = 0.005 m/day = 0.005 m^3 *per m^2 area drained* per day. When the drain spacing is 40 m, *the discharge per metre of drain* is qL = 0.005 × 40 = 0.2 m^3 per day; when this drain is 100 m long, *the discharge of the drain* will be 0.2 × 100 = 20 m^3 per day or 20,000/86,400 = 0.23 litres per sec., and in this case *the discharge per ha* will be 0.23 × 10,000/(40×100) = 0.58 lit.sec.ha.

Note that 1 lit.sec.ha = 8.64 mm per day or 1 mm per day = 0.116 lit.sec.ha.

In comparison with Eq.(1), Eq.(1a) shows more clearly that the discharge rate for the flow above and below the drains can be computed with the same horizontal flow equation; the only difference is the cross-section of flow area.

Eq.(1a) can be used for a drainage situation with two layers of different permeability (K_2 and K_1), drain level being at the interface of these layers. The equation can also be used for homogeneous soils ($K_2 = K_1$). This is possible because Hooghoudt distinguishes primarily not *soil layers* but groundwater *flow regions*, split up into a flow above drain level and a flow below drain level. These flow regions can coincide with soil layers but need not necessarily do so.

2.2 Ditches or pipe drains located above an impervious layer

When ditches or pipe drains are located above an impervious layer, the flow lines will be partly horizontal, and partly radial as they converge when approaching the drains. This causes a restriction to flow (resistance to radial flow) due to a decrease in the available cross-section of flow area. The smaller the wetted perimeter of the drain, the greater the resistance to radial flow.

In certain respects, this flow restriction can be compared to the traffic on a highway, where in a certain direction one of the lanes is blocked. In both cases, the available cross-sectional flow area has been reduced.

For the drainage situation described above, the Hooghoudt equation expressed in terms of Eq.(1a) reads

$$q = \frac{8K_2dh}{L^2} + \frac{8K_1D_1h}{L^2} \tag{2}$$

where

d = the thickness of the so-called "equivalent layer" which takes into account the convergence of flow below the drain (q_2) (radial flow) by reducing the pervious layer below the drain (D_2) to such an extent that the horizontal resistance (R_h) plus the radial resistance (R_r) of the layer with a thickness D_2 equals the horizontal resistance of the layer with a thickness d. This d-value is a function of the drain radius (r), L- and D_2-value (Hooghoudt's d-tables).

If we compare Eq.(1a) with Eq.(2), it can be seen that both are horizontal flow equations. The only difference being that the D_2-value of Eq.(1a) (horizontal flow only) has been changed into a d-value in Eq.(2) (horizontal and radial flow).

Summarizing, the main principles of the approach of Hooghoudt are:

(1) Primarily, he distinguishes groundwater *flow regions*, split up into flow *above* and *below* drain level, and only secondarily does he distinguish soil layers;

(2) for the flow region above drain level, only horizontal flow need be considered (transmissivity K_1D_1), whereas for the flow region below drain level, both horizontal flow (transmissivity K_2D_2) and radial flow have to be taken into account;

(3) the radial flow is accounted for reducing the depth D_2 to a smaller depth d, the so-called equivalent layer.

The adoption of the first principle resulted in a uniform nomenclature for the Hooghoudt and Ernst equations and the use of the same equations for homogeneous soil and a soil with the drains at the interface of two layers.

The adoption of the second principle resulted in a change in the original Ernst equation, with respect to the magnitude of the radial flow.

The third principle, dealing with the mathematical solution of the problem of radial flow, has been changed. Instead of changing the D_2-value into a smaller d-value - the radial flw has been taken into account by changing a L_o-value (drain spacing based on horizontal flow only, $L_o^2 = 8KDh/q$) into a smaller L-value (actual drain spacing based on horizontal and radial flow).

This alternative solution results in one basic drainage equation and only one general graph that can be used for all drainage situations, pipe drains as well as ditches, and all drain spacings, without having to use d-tables or a trial-and-error method or several graphs.

3. Principles of the Ernst equation

3.1 The original drain spacing equation of Ernst

The general principle underlying Ernst's basic equation (1962) is that the flow of groundwater to parallel drains, and consequently the corresponding available total hydraulic head (h), can be divided into three components: a vertical (v), a horizontal (h), and a radial (r) component or

$$h = h_c + h_h + h_r = qR_v + qLR_h + qLR_r$$

where q is the flow rate and R is the resistance.

Working out various resistance terms, we can write the Ernst equation as

$$h = q\,\frac{D_v}{K_v} + q\,\frac{L^2}{8KD} + q\,\frac{L}{\pi K_2}\ln\frac{aD_2}{u} \qquad (3)$$

where

h, q, K_2, D_2, L = notation Hooghoudt's equation (Section 2.1)

D_v = thickness of the layer over which vertical flow is considered; in most cases this component is small and may be ignored (m)

K_v = hydraulic conductivity for vertical flow (m/day)

KD = the sum of the product of the permeability (K) and thickness (D) of the various layers for the horizontal flow component according to the hydraulic situation:
one pervious layer below drain depth: $KD = K_1D_1 + K_2D_2$ (Fig.2a)
two pervious layers below drain depth: $KD = K_1D_1 + K_2D_2 + K_3D_3$ (Fig.2b)

a = geometry factor for radial flow depending on the hydraulic situation:
$KD = K_1D_1 + K_2D_2$, $\alpha = 1$
$KD = K_1D_1 + K_2D_2 + K_3D_3$, the α-value depends on the K_2/K_3 and D_2/D_3 ratios (see the auxiliary graph Ia)

u = wetted section of the drain (m); for pipe drains $u = \pi r$.

Eq.(3) shows that the radial flow is taken into account for the total flow (q), whereas the Hooghoudt equation considers radial flow only for the layer below drain level (q_2).

It should be noted that Eq.(3) has been developed for a drainage situation where $K_1 \ll K_2$ (a clay layer on a sandy substratum) and can therefore only be used where the flow above drain level is relatively small. However, if one uses this equation for the case that $K_1 >> K_2$ **(a sandy layer on a clay layer, a not** uncommon situation), the result is a considerable underestimate of drain spacing compared with the result obtained when the Hooghoudt equation is used.

According to Ernst (1962) and Van Beers (1965), no acceptable formula has been found for the special case that $K_1 >> K_2$, and it was formerly recommended that the Hooghoudt equation be used for this drainage situation. Since that time, as we shall describe below, a generalized equation has been developed, which covers all K_1/K_2 ratios.

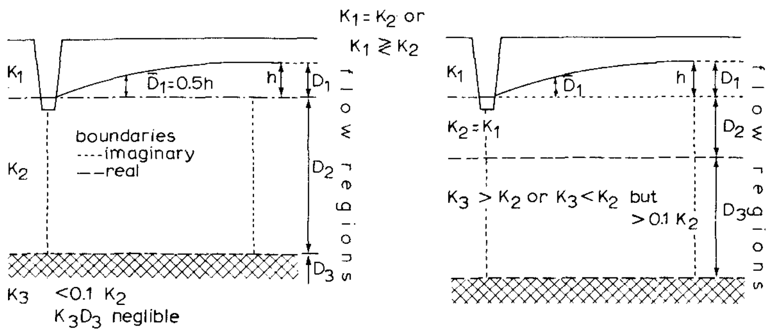


Fig.2a. Soil below the drain: only one pervious layer (D_2) ($KD = K_1D_1 + K_2D_2$)

Fig.2b. Soil below the drain: two pervious layers (D_2, D_3) ($KD = K_1D_1 + K_2D_2 + K_3D_3$)

. . . . imaginary boundary
- - - - real boundary

Fig.2. Geometry of the Ernst equation if the vertical resistance may be ignored.

3.2 The generalized or the Hooghoudt-Ernst equation

This new equation is based on a combination of the approach of Hooghoudt (radial flow only for the flow below the drains, q_2) and the equation of Ernst for the radial flow component.

Neglecting the resistance to vertical flow and rewriting Eq.(3), we obtain

$$q = \frac{8KDh}{L^2 + \frac{8KD}{\pi K_2} L \ln \frac{aD_2}{u}} \qquad (3a)$$

If we consider only horizontal flow above the drains (q_1) and both horizontal and radial flow below the drains (q_2), we may write

$$q = \frac{8K_1D_1h}{L^2} + \frac{8K_2D_2h}{L^2 + \frac{8}{\pi} D \quad L \ln \frac{aD_2}{u}} \qquad (4)$$

Introducing an equivalent drain spacing (L_o), i.e. a drain spacing that would be found if horizontal flow only is considered, we get

$$L_o^2 = \frac{8KDh}{q} \qquad (5)$$

Substituting Eq.(5) into Eq.(4) yields

$$\frac{8KDh}{L_o^2} = \frac{8K_1D_1h}{L^2} + \frac{8K_2D_2h}{L^2(1 + \frac{8D_2}{\pi L} \ln \frac{aD_2}{u})} \qquad (6)$$

After a somewhat complicated re-arrangement (see Appendix A) the generalized equation reads

$$\left|\frac{L}{L_o}\right|^3 + \frac{8c}{\pi L_o} \left|\frac{L}{L_o}\right|^2 - \frac{L}{L_o} - B \frac{8c}{\pi L_o} = 0 \qquad (7)$$

where

L = drain spacing based on both horizontal and radial flow (m)

L_o = drain spacing based on horizontal flow only (m)

c = $D_2 \ln \frac{aD_2}{u}$, a radial resistance factor (m)

B = $\frac{K_1D_1}{KD}$ = the flow above the drain as a fraction of the total horizontal flow

GRAPH I

To avoid a complicated trial-and-error method as required by the Hooghoudt approach, Graph I has been prepared. For different c/L_o and B-values, it gives the corresponding L/L_o-value which, multiplied by L_o, gives the required drain spacing (L-value). However, as will appear furtheron, this graph is usually only needed for the following specific situations:

- $K_1 >> K_2$ or $B > 0.1$;
- there are two pervious layers below drain level, $K_3D_3 > K_2D_2$, and no Scientific Pocket Calculator (SPC) is available;
- one wants to compare the generalized equation with other drainage equations or one wants to prepare a specific graph (see Section 4.1 and Appendix C 1).

NOTE: Comparing the Hooghoudt equation with the Ernst equation and using $u = 4r$ instead of $u = \pi r$, a greater similarity is obtained.

3.3 The modified Hooghoudt-Ernst equation

In most actual situations the B-factor (K_1D_1/KD ratio) will be small and therefore the last term in Eq.(7) has little influence on the computed drain spacing. Neglecting B, and rewriting Eq.(7) (multiplying by L_o^3/L and substituting $8KDh/q$ for L_o^2), we obtain the equation for the B = 0 line of Graph I:

$$L^2 + \frac{8L}{\pi} D_2 \ln \frac{D_2}{u} - 8KDh/q = 0 \qquad (8)$$

If we compare Eq.(8) with the original Ernst equation (3a), it can be seen that the factor $\frac{KD}{K_2}$ of Eq.(3a) has changed into $\frac{K_2D_2}{K_2}$, which equals D_2. However, for a drainage situation with two pervious layers below drain level, $\frac{K_2D_2}{K_2}$ becomes $\frac{K_2D_2 + K_3D_3}{K_2}$ and the equation for this situation reads

$$L^2 + \frac{8L}{\pi} \frac{K_2D_2 + K_3D_3}{K_2} \ln \frac{aD_2}{u} - 8\ KD\ h/q = 0 \qquad (9)$$

As regards the use of Eq.(9) it can be said that, in practice, Eq.(9) is very useful if an SPC is available; if not, Graph I has to be used.

Eq.(8), on the other hand, will seldom be used. The simple reason for this is that for most drainage situations with a barrier (only K_2D_2), a simplified equation can be used.

3.4 The simplified Hooghoudt-Ernst equation

After Graph I had been prepared on linear paper, it was found that for c/L_o-values < 0.3 and B-values < 0.1, the following relations hold

$$L/L_o = 1 - c/L_o \qquad \text{or} \qquad L = L_o - c \quad \text{(see Graph I)} \tag{10}$$

The question then arose whether these conditions were normal or whether they were rather exceptional.In practice,the simplified equation proved to be almost always applicable.In addition,it was found from the calculations needed for the preparation of Graph II($r = 0.10$ m, all K- and D-values) that, except for some uncommon situations ($K = 0.25$ m/day, $D > 5$ m), the equation $L = L_o - c$ is a reliable one, where $C = D_2 \ln \frac{D_2}{u}$.

Note: Many years ago W.T.Moody, an engineer with the U.S.Bureau of Reclamation proposed a similar correction ($D \ln \frac{D}{4r}$) to be subtracted from the calculated spacing (Maasland 1956, Dumm 1960). The only difference between the correction proposed by Moody and that in this bulletin is that now the conditions under which the correction may be applied are precisely defined.

4. Application of the generalized equation and corresponding graphs

There are many different drainage situations, five of which will be handled in this section.

SITUATION 1: $K_1 = K_2$	Homogeneous soil; $D < \frac{1}{4}L$; pipe drainage
SITUATION 2: $K_1 \lesseqgtr K_2$	Slight differences between soil permeability above and below drain level; differences in depth to barrier ($D \lesseqgtr \frac{1}{4}L$); pipe and ditch drainage
SITUATION 3: $K_1 >> K_2$	A highly pervious layer above drain level and a poorly pervious layer below drain level
SITUATION 4: $K_1 << K_2$	A heavy clay layer of varying thickness overlying a sandy substratum; the vertical resistance has to be taken in account
SITUATION 5: $K_3 \lesseqgtr K_2$	Soil below drain level consists of two pervious layers (K_2D_2, K_3D_3); the occurrence of an aquifer ($K_3 >> K_2$) at various depths below drain level.

4.1 Drainage situations

SITUATION 1: Homogeneous soil; $D < \frac{1}{4}L$; pipe drains

$K_1 = K_2$ *The use of the simplified equation and Graph II.*

This is the most simple drainage situation; the required preparatory calculations are limited and a graph is usually not needed. For comparison with other drain spacing equations, we shall use the example given by Wesseling (1973).

Note that - for reasons of convenience - in this and the other examples the units in which the various values are expressed have been omitted with the exception of the L-value. However, for values of $\underline{h}$, $\underline{D}$, and $\underline{r}$, read metres; for $\underline{q}$ and $\underline{K}$, read m/day and for KD, read m^2/day.

DATA GIVEN	PREPARATORY CALCULATIONS	COMPUTATION DRAIN SPACING (L)	
		L_o^2 = KD 8h/q	
h = 0.600	h/q = 300	L_o = 100.9	$L = L_o - c$ = 87 m
q = 0.002	8h/q = 2400	c = 13.8	
K_1=0.8 D_1=0.30	K_1D_1 = 0.24	c < 0.3 L	Eq.Hooghoudt: L = 87 m
K_2=0.8 D_2=5.0	K_2D_2 = 4.0	B < 0.10	Eq.Ernst: L = 84 m
			Eq.Kirkham: L = 85 m
r = 0.10	KD = 4.24	(Eq.10 may be	Eq.Dagan: L = 88 m
u = πr	B = 0.06	used)	(Wesseling 1973)

NOTE: writing h = 0.600 instead of 0.6, is not meant to suggest accuracy, but is only for convenience in determining the h/q value.

If no SPC is available, the c-value ($D_2 \ln \frac{D_2}{u}$) can be obtained from Graph III.

The simplified formula is very convenient if we want to know the influence that different u-values will have on the drain spacing. For example

r = 0.05 m, then c = 17.3 and L = 84 m
u = 1.50 m, then c = 6.0 and L = 95 m

The influence of different K or D-values is also easy to find. However, if the u-value is fixed, it is better to use Graph II or a similar graph, which gives a very quick answer to many questions.

GRAPH II

This graph is extremely useful for the following purposes and where the following conditions prevail:

Purposes

- A great number of drain spacing computations have to be made, for instance, for averaging the L-values in a project area instead of performing one calculation of the drain spacing L with the average K- or KD-value;

- One wants to find out quickly the influence of a possible error in the K-value or the influence of the depth of a barrier (D-value);
- One wants to demonstrate to non-drainage specialists the need for borings deeper than 2.10 m below ground level because a boring to a depth of 2.10 m results in a D_2-value of about 1 m, being 2.10 m minus drain depth.

Conditions

- Homogeneous soil below the drains (only K_2D_2);
- The wetted perimeter of the drains has a fixed value, say pipe drains with r = 0.10 m or ditches that have a certain u-value;
- An error of 3 to 5% in the computed drain spacing is allowable.

Considering these purposes and conditions, it will be clear that the availability of Graph II or similar graph is highly desirable, except when:

- exact theoretical computations are required
- $K_1 >> K_2$
- there are two permeable layers below drain level instead of one (K_2D_2,K_3D_3).

For these three conditions Graph II cannot be used and one must resort to Graph I.

Other q-, h- or K_1/K_2 values than those given on the graph

Graph II has been prepared for the following conditions:

h = 0.6 m, q = 0.006 m/day or h/q = 100, K_1=K_2 m/day, and r = 0.10 m

For the specific purposes and conditions for which this graph has been prepared (approximate L-values suffice) an adjustment is only required for different h/q values, through a change in the K_2-values to be used. For instance, in the example given for Situation 1 we have a h/q value of 300. Therefore

$$K_2' = K_2 \times \frac{(h/q)'}{100} = 0.8 \times 3 = 2.4$$

and we read for D = 5 m, L = 87 m.

However, if many computations have to be done, it is preferable to prepare a graph or graphs for the prevailing specific situation. For instance, for the conditions prevailing in The Netherlands, three graphs for pipe drains would be desirable: q = 7 mm/day and h = 0.3, 0.5, and 0.7 m.

For irrigation projects a q-value of 2 mm/day is usually applied.

If one wants to know the magnitude of an introduced error ($h \neq 0.6$, $K_1 \neq K_2$), the extra correction factor (f) for K_2' can be approximated with the formula

$$f = \frac{D_2 + K_1/K_2\ D_1'}{D_2 + D_1}$$

where

$$D_1 = 0.30 \quad \text{and} \quad D_1' = 0.5\ h' .$$

For example:

$$h' = 0.90 \quad D_1' = 0.45$$

$$K_1/K_2 = 0.5,\ D_2 = 5 \qquad f = \frac{5 + (0.5 \times 0.45)}{5 + 0.30} = 0.986 \quad \text{or}$$

about 1% difference in the L-value

When the assumption of a homogeneous aquifer contains a rather large error (e.g. K_1=2K_2), and moreover a larger hydraulic head being available (k'=0.9 m), we get f=1.11 or 5% difference in the L-value.

Preparation

The preparation of a speficic graph is very simple, and takes only a few hours. There are two methods of preparation.

Appendix C 1 gives an example of how it is done if the d-tables of Hooghoudt are available. This is the easiest way.

Appendix C 2 shows a preparation based on the generalized equation in combination with Graph I. This method gives the same result, but requires more calculations.

Finally note that Graph II demonstrates clearly that if, in a drainage project, augerholes of only 2 m depth are made ($D_2 \lesseqgtr 1$ m), considerable errors in the required drain spacing can result.

For example:

Given:

$h/q = 300$ (irrigation project), $K_2 = 0.8$, then $K_2' = 2.4$;

drain depth = 1.5 m

Depth to barrier	Flow depth (D_2)	Spacing (L)	$h/q = 100$, $K_2 = 0.5$
2.50 m	1 m	50 m	L = 22 m
3.50 m	2 m	63 m	28 m
6.50 m	5 m	87 m	34 m
11.50 m	10 m	105 m	37 m
∞	∞	130 m	37 m

This example may show that:

- Graph II is very suited to carry out a sensitivity analysis on the influence of the depth of a barrier, etc.
- The need for drilling deeper than 2 m. Note that here only the value D_2 has been considered. However, there can also be a considerable change in the K_2-value.
- The relative influence of the D_2-value changes with the spacing obtained.

SITUATION 2: Slight differences between soil permeability above and below
$K_1 \lesseqgtr K_2$ drain level ($K_1 \lesseqgtr K_2$);
Differences in depth to a barrier ($D \lesseqgtr 1/4L$);
pipe and ditch drainage

For the situation $D < \frac{1}{4}L$ and drainage by ditches, the computation of the drain spacing, as well as the computation sheet is the same as have been given in Situation 1. Only if $K_1 > K_2$ and D_2 is small should special attention be paid to the question whether $B < 0.10$.

For the situation $D > \frac{1}{4}L$, the following equation (Ernst 1962) can be used:

$$L \ln \frac{L}{u} = \pi K_2 \ h/q \qquad (11)$$

The use of this equation will be demonstrated below and will be followed by a sensitivity analysis for the u and D_2 factor.

GIVEN		COMPUTATION
h = 0.800	h/q = 400	$L \ln \frac{L}{u} = \pi \times 0.8 \times 400 = 1005$
q = 0.002		Graph III, for u = 1.50 → L = 205 m
K_1 = 0.40		If a SPC is available, the L-value can also
K_2 = 0.80	u = 1.50	be obtained by a simple trial- and error-method
		error method

Equation (11) does not take the horizontal resistance into account because it is negligible compared with the radial resistance; nor is the flow above the drain considered. Only if the computed L-value is small, say about 40 m or less, is a small error introduced.

If Graph I only is available, or one wants to check the computed L-value by using the generalized equation, the following procedure can be followed:

Estimate the drain spacing and assume a value for D_2 between $\frac{1}{4}$L and $\frac{1}{2}$L (beyond this limit, the computed spacing would be too small); compare the two L-values (control method) or check whether the assumed D_2-value > $\frac{1}{4}$L and < 0.5L (computation method).

For the above example we get:

Given:	D_2 = 80			
Assume	K_2 = 0.8	KD = 64	L_o = 452	c/L_o = 0.70
Assume	u = 1.5	8 h/q=3200	c = 318	Graph I: L/L_o = 0.45 → L = 204 m
		Using a SPC and Eq.(8)	→	L = 202 m

Note: It may be useful - by way of exercise - to try other D_2 values and to compare the resulting L-values.

Sensitivity analysis for u-value and depth to a barrier (D-value)

$D_2 = \infty$	$D_2 = 5$ m KD = 0.16 + 4.0 = 4.16 8 h/q = 3200 L_o=115 m	
u = 1 m → L = 192 m	u = 1 m → c = 8 m → L = 107 m	
u = 1.5 m → L = 205 m	u = 1.5 m c = 6 m → L = 109 m	
u = 2 m → L = 215 m	u = 2 m c = 4.5 m → L = 110 m	
u = 10 m → L = 300 m		
u = 0.30 m → L = 162 m	u = 0.30 m → L = 101 m;	u = 0.20 → L = 101 m
		u = 0.40 → L = 103 m

u-value

This sensitivity analysis shows that the influence of difference in the u-value increases as L increases. However, differences of 50% or more are generally of little importance. Therefore the u-values of pipe drains can be approximated by taking r = 0.10 m (u = 0.30) and the u-values of ditches approximated by taking the width of the ditch and two times the water depth (usually 2 × 0.30 m).

The slope of the ditch need not be taken into account because of the reasons mentioned. Moreover, neither the water level in the ditch nor the drain width are constant factors.

D-value

The large error made by assuming $D_2 = 5$ m instead of $D_2 = \infty$, as in the above example (L = 109 m instead of 205 m), is easily made if, in an irrigation project (q = 0.002, h/q and L-value very large), no hydro-geological investigations are carried out.

SITUATION 3: *A highly pervious layer above drain level and a poorly pervious layer below drain level*
$K_1 >> K_2$

The use of Graph I

This particular situation is frequently found. It may be of interest to use the data given below to compute drain spacings with other drainage equations and then to compare the results with those obtained with the equation of Hooghoudt or the generalized Hooghoudt-Ernst equation.

DATA GIVEN		PREPARATORY CALCULATIONS	COMPUTATION	
$h = 1.000$		$h/q = 200$	$L_o = 53.7$ m	
$q = 0.005$		$8h/q = 3200$	$c = 12.6$ m	
$K_1 = 1.6$	$D_1 = 0.50$	$K_1D_1 = 0.8$	$c/L_o = 0.235$	$L/L_o = 0.88$ (Graph I)
$K_2 = 0.2$	$D_2 = 5.0$	$K_2D_2 = 1.0$	$B = 0.44$	$L = 0.88 \times 53.7 = 47.3$ m
	$r = 0.10$	$KD = 1.8$		
	$u = 0.40$*	$B = 0.44$		

Note: The computation sheet used here is the same as that used for Situation 1, except that no c/L_o-value is needed in Situation 1.

**For theoretical comparisons with the results of the equation of Hooghoudt, it is preferable to use $u = 4\ r$ instead of $u = \pi r$*

Comparison of the Ernst equations with the equation of Hooghoudt

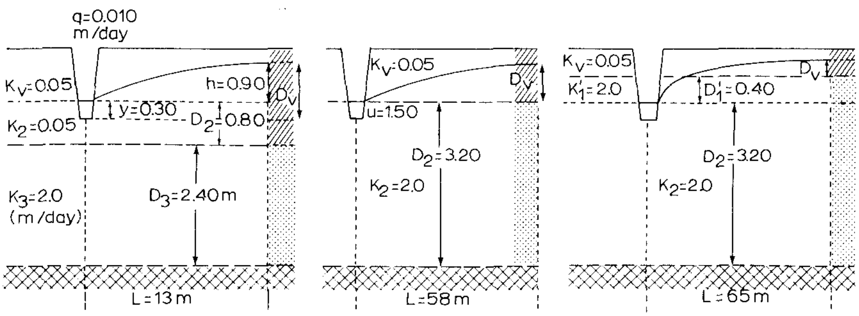
Original equation of Ernst (Eq.3a)	$L = 32$ m
Modified equation of Ernst (Eq.8)	$L = 39.9$ m
Generalized equation of Ernst (Eq.7)	$L = 47.2$ m
Equation Hooghoudt (Eq.2)	$L = 47.2$ m
Graph II: $K_2' = 0.2 \times 2 \times 1.7 = 0.68$,	$D_2 = 5$ m $\rightarrow L + 41$ m

It should be born in mind that the original equation of Ernst never has been recommended for the considered situation with a major part of the flow through the upper part of the soil above drain level ($K_2 << K_1$). Therefore it is not surprising that the unjustified use of this formula will result in a pronounced underestimating of the drain spacing; the modified equation is somewhat better, while the generalized equation gives the same results as the equation of Hooghoudt. In addition, it is demonstrated that the use of Graph II for the $K_1 >> K_2$ situation also results in an underestimate of the drain spacing.

SITUATION 4: *A heavy clay layer of varying thickness overlying a sandy*
$K_1 << K_2$ *substratum; the vertical resistance has to be taken into account*

This is another drainage situation that occurs frequently. Because the thickness of the clay layer can vary, three different drainage situations can result (see Fig.3). In this example, it is assumed that the maximum drain depth is -1.40 m, in view of outlet conditions, and that the land is used for arable farming (h = drain depth - 0.50 m = 0.90 m).

The computation of the vertical component ($h_v = q \frac{D_v}{K_v}$, see Section 3.1) is somewhat complicated because the D_v-values varies with the location of the drain with respect to the more permeable layer. However, Fig.3 and the corresponding calculations may illustrate sufficiently clearly how to handle the specific drainage situations. It may be noted that as far as the author is aware the solution given by Ernst for this drainage situation is the only existing one.



Ex.4a — Drain level above the boundary

$D_v = h + y$

$h_v = \frac{q}{K_v} D_v$

$h' = h - h_v$

$KD = K_2D_2 + K_3D_3$

$c = \frac{KD}{K_2} \ln \frac{aD_2}{u}$

Ex.4b — Drain level coincides with the boundary

$D_v = h$

$h_v = \frac{q}{K_v} h$

$h' = h - h_v$

$KD = K_2D_2$

$c = D_2 \ln \frac{D_2}{u}$

Ex.4c — Drain level below the boundary of the two soil layers

$D_v = h - D_v'$

$h_v = \frac{q}{K_v} D_v$

$h' = h - h_v$

$KD = K_1'D_1' + K_2D_2$

$c = D_2 \ln \frac{D_2}{u}$

Fig.3. Geometry of the Ernst equation if vertical resistance has to be taken into account ($K_1 << K_2$).

Procedure

- Determine D_v according to the specific situation;
- Calculate h_v, the loss of hydraulic head due to the vertical resistance, by using $h_v = \frac{D}{K_1}$;
- Calculate h' from $h' = h - h_v$ where h' is the remaining available hydraulic head for the horizontal and radial flow
- Calculate the h'/q and KD-value. Note that the horizontal flow in the upper layer with low permeability may be ignored;
- Compute L; for Example 1, Graph Ia is required in addition to Graph I or an SPC and Eq.(9); for Examples 2 and 3, use $L = L_o - c$.

The following examples are intended to illustrate the procedure and the layout of a computation sheet. (The data used have been taken from Fig.3).

Example 4a

$D_v = h+y = 0.90 + 0.30 = 1.20$						$h/q = 66$
$h_v = q/K_1 \times D_v = 0.2 \times 1.2 = 0.24$			$h' = h - h_v = 0.90 - 0.24 = 0.66$			$8\ h/q = 528$
$K_v = 0.05$	$D_2 = 0.80$	$K_2D_2 = 0.04$	$L_o = 50.6$ m	$c/L_o = 1.45$		$L/L_o = 0.25$
$K_3 = 2.0$	$D_3 = 2.40$	$K_3D_3 = 4.80 \rightarrow$	c = 73.3 m	B = 0		L = 12.6
$K_3/K_2 = 20$	$D_3/D_2 = 3$	KD = 484	$\left[c = \frac{KD}{K_2} \ln \frac{aD_2}{u}\right]$			
	a = 4.0 (see Ex.5)					

Alternatives

- Pipe drains (u=0.30) → L = 5 m
- Ditch bottom in the more permeable layer (ditch depth at 2.20) →
  u = 0.90 and h = 1.70 → L_o = 80 m; c = 2m → L = 78 m
- Pipe drains at -2.20 m → L_o = 80 m; c = 5 m → L = 75 m

Note: The last two alternatives mean that the drainage water will have to be discharged by pumping.

Example 4b

$D_v = h = 0.90$				$h/q = 72$
$h_v = 0.2 \times 0.90 = 0.18$		$h' = 0.90 - 0.18 = 0.72$		$8\ h/q = 576$
$K_2 = 2.0$	$D_2 = 3.20$	$KD = 6.40$	$L_o = 60.7$ m	$L = 58.3$ m
			$c = 2.4$ m	
			$\left[c = D_2 \ln \frac{D_2}{u}\right]$	

Example 4c

$D_v = h - D_1' = 0.90 - 0.40 = 0.50$			$h' = 0.80$	$8\ h/q = 640$
$h_v = 0.2 \times 0.50 = 0.10$				
$K_1' = 2.0$	$D_1' = 0.40$*	$K_1'D_1' = 0.80$	$L_o = 67.9$ m	$L = 65.5$ m
$K_2 = 2.0$	$D_2 = 3.20$	$K_2D_2 = 6.40$	$c = 2.40$ m	
		$KD = 7.20$		

**The available cross-section for horizontal flow ≈ thickness of the more permeable layer above drain depth.*

Remarks

If we compare the computed drain spacing for Example 4b (L=58 m) with that of Example 4c (L=65 m), we can conclude that for a given drain depth the exact thickness of the heavy clay layer is of minor importance as long as the bottom of the drain is located in the more permeable layer.

If, however, the clay layer continues below drain depth, as in Example 4a (L=13 m), drain spacings would have to be very narrow indeed and the area will scarcely be drainable (for pipe drains, L=5 m, for a ditch, L=13 m).

The only way out here is to use deep ditches (L = 78 m) or pipe drains (L = 75 m) that reach into the permeable layer, and to discharge the drainage water by pumping.

SITUATION 5: *Soil below drain depth consists of two pervious layers*
$K_3 \lesseqgtr K_2$ *(K_2D_2, K_3D_3).*

Graph Ia.
The occurrence of an aquifer ($K_3 >> K_2$) at various depths below drain level

Hydrologically speaking, this drainage situation is very complicated. Up to now the problem could only be solved by using an additional graph (Ia) (Ernst 1962) or by the construction of various graphs for various drainage situations (Toksöz and Kirkham, 1971).

The graph of Ernst that can be used for all situations (various K_3/K_2 and D_3/D_2 ratios) gives the results he obtained by applying the *relaxation method.* A somewhat modified form of this graph has been published by Van Beers (1965).

The reliability and importance of Graph Ia

Considering the method by which this graph has been constructed, the question arises as to how reliable it is. The correctness of an equation can easily be checked, but not the product of the relaxation method.

Fortunately, the results obtained with this graph could be compared with the results obtained with 36 special graphs, each one constructed for a specific drainage situation (Toksöz and Kirkham, 1971). It appeared that both methods gave the same results (Appendix B). Thus the conclusion can be drawn that the generalized graph of Ernst is both a reliable and an important contribution to the theory and practice of drainage investigations. It is particularly useful in drainage situations where there is an aquifer (highly pervious layer) at some depth (1 to 10 m or more) below drain level, a situation often found in irrigation projects.

When there are two pervious soil layers below drain level, the two most common drainage situations will be: $K_3 << K_2$, and $K_3 >> K_2$.

Situation $K_3 << K_2$

The availability of Graph Ia enables us to investigate whether we are correct in assuming that, if $K_3 < 0.1\ K_2$, we can regard the second layer below drain depth (K_3D_3) as being impervious. If we consider the L-values for this situation, as given in Appendix B, we can conclude that although the layer K_3D_3 for $K_3<0.1\ K_2$ has some influence on the computed drain spacing, it is generally so small that it can be neglected. However, if one is not sure whether the second layer below drain level can be regarded as impervious, the means (equation and graph) are now available to check it.

Situation $K_3 >> K_2$

This situation is of more importance than the previous one because it occurs more frequently than is generally realized and has much more influence on the required drain spacing.The examples will therefore be confined to this situation.

Examples (see Fig.4)

Given: The soil or an irrigation area consists of a loess deposit (K = 0.50 m/day) of varying thickness. In certain parts of the area an aquifer occurs (sand and gravel, K = 10 m/day, thickness 5 m).

In the first set of examples (A) the loess deposit is underlain by an impervious layer at a certain depth, varying from 3 to 40 m. In the second set of examples (B), instead of an impervious layer, an aquifer is found at a depth of 3 m and 8 m, whereas Examples C give alternative solutions in relation to drain depth and the use of pipe drains instead of ditches.

It is intended to drain the area by means of ditches (drain level = 1.80 m, wetted perimeter (u = 2 m). The maximum allowable height of the water table is 1 m below surface (h = 0.80 m). The design discharge is 0.002 m/day (h/q = 400).

A. Influence of the location of an impervious layer. Homogeneous soil

For this simple drainage situation, only the result of the drain spacing computations will be given.

Example	Depth barrier	Drain spacing	
		Ditch (u=2 m)	Pipe drain (u=0.30 m)
A 1	3 m	50 m	49 m
A 2	8 m	96 m	84 m
A 3	40 m	146 m	107 m

These results show that the depth of a barrier has a great influence on the drain spacing and that the influence of the wetted perimeter of the drains (u) can vary from very small to considerable, depending on the depth of the barrier.

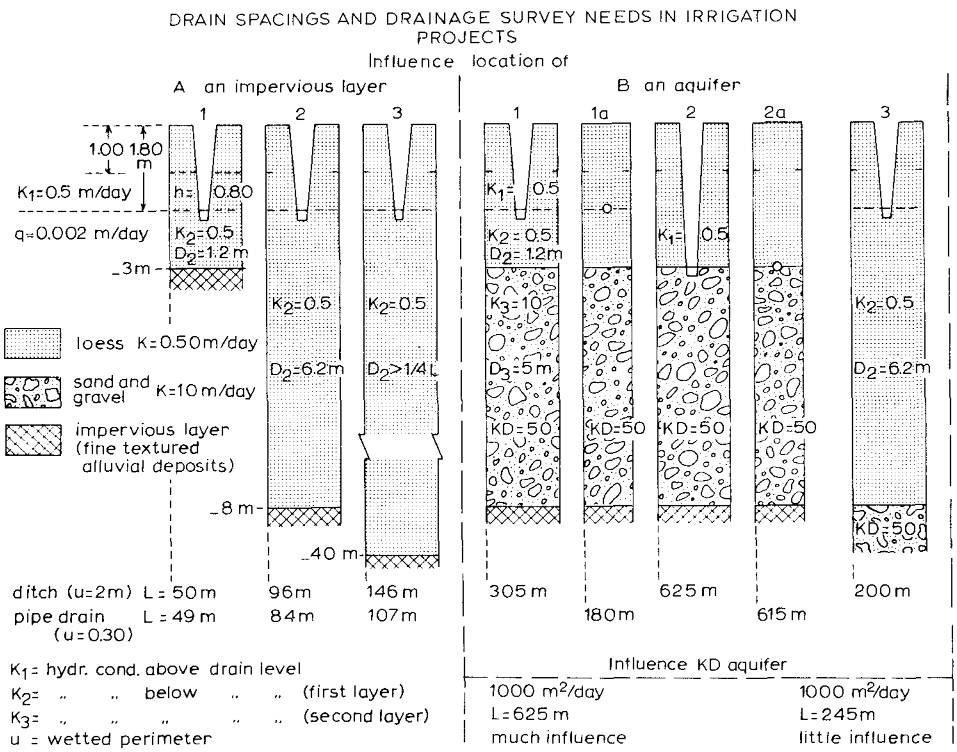


Fig.4. Drain spacings and drainage survey needs in irrigation projects.

B. Influence of the location of an aquifer

The various situations that will be handled here are:

Example	Depth aquifer	Drain level	Ditches	Pipe drains	Spacing
B 1	- 3 m	- 1.80 m	+		305 m
B 1a	- 3 m	- 1.80 m		+	180 m
B 2	- 3 m	- 3 m	+		625 m
B 2a	- 3 m	- 3 m		+	615 m
B 3	- 8 m	- 1.80 m	+		200 m

Example B 1

Aquifer at -3 m (1.20 m below drain level)

h = 0.800		h/q = 400		
q = 0.002		8h/q = 3200		
K_2 = 0.50	D_2 = 1.20	K_2D_2=0.60	L_o = 402 m	$L = L_o - c$ = 313 m
K_3 = 10	D_3 = 10	K_3d_3 = 50	c = 89 m	SPC: L = 305 m
K_3/K_2 = 20	D_3/D_2=4	KD = 50.6	c < 0.3 L_o	
	a = 4.0		(c/L_o=0.22)	

Note: The flow above the drains can be neglected in this drainage situation $K_3 \gg K_2$). *Therefore,* $KD = K_2D_2 + K_3D_3$, *whereas* $c = \frac{KD}{K_2} \ln \frac{aD_2}{u}$

If KD-values are high (here KD = 50 m^3/day), the flow in the K_2D_2 layer can also be neglected and KD = K_3D_3 , the more so because the KD-value of the aquifer is a very approximate value.

The L-value can be determined in two ways: either by using Graph I or by using Eq.(9c) in combination with an SPC. It is recommended that both methods be used to allow a check on any calculation errors. Small differences may occur in the results of the two methods, but this is of no practical importance.

Example B 1a

Drainage by pipe drains (u=0.30 m), instead of ditches

L_o = 402 m (see Ex.B 1)	c/L_o = 0.69
c = 277 m	L/L_o = 0.45 → L = 180 m

Note that in this situation the use of pipe drains instead of ditches has a great influence on the resulting drain spacing.

Example B 2

Ditch bottom in the aquifer (u=2 m); drain level -3 m.

h = 2.000	8 h/q = 8.000	L_o = 632 m	$L = L_o - c$ = 627 m
q = 0.002			
	KD = 50	c = 5 m	

Example B 2a

Pipe drain in the aquifer (u=0.30 m), drain level -3 m.

L_o = 632 m (see Ex.B 2)	$L = L_o - c$ = 618 m
c = 14 m	

Note that in this situation the use of pipe drains instead of ditches has very little influence on the drain spacing, because here the radial resistance is very small.

Example B 3

The aquifer at - 8 m (6.20 m below drain depth); KD = 50 m^2/day; ditch (u = 2 m); drain level -1.80 m.

h = 0.800		h/q = 400		
q = 0.002		8q/h =3200		
K_2 = 0.50	D_2=6.20	K_2D_2 = 3.10	L_o = 412 m	c/L_o=0.61
K_3 = 10.0	D_3 = 5.0	K_3D_3 = 50.0	c = 253 m	L/L_o=0.49 → L=202 m
K_3K_2=20	D_3/D_2=0.8	KD = 53.1		
	a = 3.5			

C. Influence of the KD-value of an aquifer

Example C 1

In Example B 1 (aquifer at -3 m, KD-value = 50 m^2/day → L = 305 m, the KD-value has been estimated from borings to be at least 50 m^2/day. Now the question arises whether it is worthwhile to carry out pumping tests to obtain a better estimate.

If, in a certain drainage situation, one wants to analyse the influence of the magnitude of the KD-value on the spacing, it is convenient to calculate firstly, $\frac{1}{K_2} \ln \frac{aD_2}{u}$, which in this case equals 1.75.

Assume KD = 100

$8\ h/q = 3{,}200$	$L_o = 566$	$c/L_o = 0.31$
$KD = 100$	$c = 100 \times 1.75 = 175$	$L = L_o - c = 300$ m
	$KD = 500 \rightarrow L = 570$ m	
	$KD = 1000 \rightarrow L = 625$ m	

These computations show that in this case it will indeed be worthwhile to carry out pumping tests.

Example C 2

In Example B 3, with the depth of the aquifer at -8 m and KD = 50, L = 200 m. Making the same computations as for Example C 1, we get:

$KD = 100$; $KD \rightarrow a = 4$; $\frac{1}{K_2} \ln \frac{aD_2}{u} = 5.0$

$8\ h/q = 3200$	$L_o = 566$ m	$c/L_o = 0.89$	
$KD = 100$	$c = 504$ m	$L/L_o = 0.38$	$L = 215$ m

KD = 1000

$8\ h/q = 3200$	$L_o = 1789$ m	$c/L_o = 2.82$	SPC and Eq.9
$KD = 1000$	$c = 5040$ m	$L = \frac{\pi L_o^2}{8c} = 250$ m	$L = 245$ m

These results show that here an estimate of the KD-values will suffice and therefore - in contrast to Example B 1 - no pumping tests are required.

Importance of geohydrological investigations

If we compare Situation B 3 with that of A 2 (Fig.4), we get:

A 2 : impervious layer at -8 m $\rightarrow L \simeq 100$ m

B 2 : instead of an impervious layer, an aquifer at -8 m $\rightarrow L \simeq 200$ m

This comparison of drain spacings shows clearly that a drain spacing can be considerably influenced by layers beyond the reach of a soil auger.

From Fig.4 it will be clear that if in the given situation drainage investigations are only conducted to a depth of 2 m and a barrier at 3 m is assumed, the recommending drain spacing will be ≃ 50 m.

If, however, geo-hydrological investigations are conducted, they will reveal that parts of the area can be drained with spacings of 300 m (drain level -1.8 m) or 600 m if the drain level is -3 m.

4.2 Summary of graphs and equations

Graphs

G.I : c/L_o-, B- and L/L_o-values → L-values (drain spacing)

G.Ia : K_3/K_2- and D_3/D_2-values → a-value (auxiliary graph for radial resistance)

G.II : Homogeneous soil and pipe drains → L-value (for all D_2- and K_2-values)

G.III: $D_2 \ln \frac{D_2}{u}$ or $L \ln \frac{L}{u}$ (auxiliary graph if a SPC is not available)

Equations

Only one pervious layer below drain depth

$D_2 < \frac{1}{4} L$			USE
Eq.(3a)	$L^2 + \frac{8KD}{\pi K_2} L \ln \frac{D_2}{u} - 8\ KD\ h/q = 0$	original equation	out of use
Eq.(7)	$\left(\frac{L}{L_o}\right)^3 + \left(\frac{8c}{\pi L_o}\right)\left(\frac{L}{L_o}\right)^2 - \frac{L}{L_o} - B\left(\frac{8c}{\pi L_o}\right) = 0$	generalized eq.	G.I
Eq.(8)	$L^2 + \frac{8}{\pi} L\ D_2 \ln \frac{D_2}{U} - 8\ KD\ h/q = 0$	modified equation	SPC
Eq.(10)	$L = L_o - c$	simplified eq.	for c <0.3 B <0.1
	where $L_o = 8\ KD\ h/q$ $c = D_2 \ln \frac{D_2}{u}$		
$D_2 > \frac{1}{4} L$			
Eq.(11)	$L \ln \frac{L}{u} = \pi\ K_2\ h/q$		G.III or SPC

Two pervious layers below drain depth

Eq.(9)	$L^2 + \frac{8}{\pi} L \frac{KD}{K_2} \ln \frac{aD_2}{u} - 8\ KD\ h/q = 0$		G.I or SPC
	where $KD = K_1D_1 + K_2D_2 + K_3D_3$	or for $K_3 >> K_2$:	G.Ia
	$KD = K_2D_2 + K_3D_3$		
	$a = f\ (K_3/K_2,\ D_3/D_2)$		

4.3 Programmes Scientific Pocket Calculator (SPC)

Note: These programmes should be adjusted if necessary, to suite the specific SPC used

$$L^2 + bL - c = 0 \rightarrow L = \frac{-b + \sqrt{b^2 + 4c}}{2} = \frac{-b + \sqrt{4\left[(\frac{1}{2}b)^2 + c\right]}}{2} \quad \text{or}$$

$$L = -\tfrac{1}{2}b + \sqrt{(\tfrac{1}{2}b)^2 + c}$$

Eq.(8): $L^2 + \frac{8}{\pi} LD_2 \ln \frac{D_2}{u} - 8KD\ h/q = 0$

$KD = K_1D_1 + K_2D_2$

$\frac{1}{2}b = \frac{4}{\pi} D \ \ln \frac{D_2}{u}$

Eq.(9): $L^2 + \frac{8}{\pi} L \frac{KD}{K_2} \ln \frac{aD_2}{u} - 8KD\ h/q = 0$

$KD = K_2D_2 + K_3D_3$

$\frac{1}{2}b = \frac{u}{\pi}\frac{KD}{K_2} \ln \frac{aD_2}{u}$

Programme examples

KD ENT 8 h/q (×)	KD ENT STO 8 h/q (×)
D_2 ENT u (÷) (ln) or π(÷)u(÷)(ln)	a ENT D_2 (×) u (÷) (ln)
D_2 (×) 4 (×) π (÷)	RCL (×) K_2 (÷) 4 (×) π (÷)
STO ENT (×) (+) (√x) RCL (−)	STO ENT (×) (+) (√x) RCL (−)

or

D_2 (÷) u (=) (ln)	a (×) D_2 (÷) π (÷) 0.10 (÷) (ln)
(×) D_2 (×) 4 (÷) π (=)	(×) KD (÷) K_2 (×) 4 (÷) (=)
STO RCL (x^2) (+) KD (×) 8 h/q (=)	STO RCL (x^2) (+) KD (×) 8 h/q (=)
(√x) (−) RCL (=)	(√x) (−) RCL (=)

D_2 = 5	KD = 4.24	L=87.07		a = 4.6	KD = 17.3	
u = a4	8 h/q = 2.400			D_2 = 1.6	K_2 = 1.2	L = 73.22
				u = πr	8 h/q = 800	
				r = 0.10		

Appendix A.
Derivation of the generalized equation of Ernst

The basic equation (Eq.6, Section 3.2) reads

$$\frac{8KDh}{L_o^2} - \frac{8K_1D_1h}{L^2} = \frac{8\ K_2D_2h}{L^2\left(1 + \frac{8D_2}{\pi L}\ \ln\frac{aD_2}{u}\right)}$$

Multiplying all terms by $\frac{L^2}{8\ K_2D_2h}$ gives

$$\frac{KD}{K_2D_2}\frac{L^2}{L_o^2} - \frac{K_1D_1}{K_2D_2} = \frac{1}{1 + \frac{8D_2}{\pi L}\ \ln\frac{aD_2}{u}} \qquad \text{or}$$

$$\left[\frac{KD}{K_2D_2}\frac{L^2}{L_o^2} - \frac{K_1D_1}{K_2D_2}\right]\left[1 + \frac{8D_2}{\pi L}\ \ln\frac{aD_2}{u}\right] = 1$$

Multiplying by $\qquad \frac{L}{L_o} \qquad \frac{L}{L_o}$

and setting $\frac{L}{L_o} = x$, we get

$$\left[\frac{KD}{K_2D_2}x^2 - \frac{K_1D_1}{K_2D_2}\right]\left[x + \frac{8D_2}{\pi L_o}\ \ln\frac{aD_2}{u}\right] = x \qquad \text{or}$$

$$\frac{KD}{K_2D_2}x^3 + \left[\frac{8KD}{\pi K_2L_o}\ \ln\frac{aD_2}{u}\right]x^2 - \left[\frac{K_1D_1}{K_2D_2} + 1\right]x - \frac{8K_1D_1}{\pi K_2L_o}\ \ln\frac{aD_2}{u} = 0$$

writing $\frac{K_1D_1}{K_2D_2} + 1 = \frac{K_1D_1 + K_2D_2}{K_2D_2} = \frac{KD}{K_2D_2}$; $D_2\ \ln\frac{aD_2}{u} = c$ and

multiplying all terms with $\frac{K_2D_2}{KD}$ yields the final equation

$$x^3 + \left[\frac{8c}{\pi L_o}\right]x^2 - x - B\left[\frac{8c}{\pi L_o}\right] = 0$$

where $\qquad x = \frac{L}{L_o}$, $B = \frac{K_1D_1}{KD}$ and $c = D_2\ \ln\frac{aD_2}{u}$

If we compare the above basic equation with the Hooghoudt equation and we assume that both equations yield the same result, then

$$d = \frac{D_2}{1 + \frac{8D_2}{\pi L} \ln \frac{D_2}{u}}$$

where $u = \pi r$

From a comparison of the d-value obtained by using this equation and Hooghoudt's d-table for r = 0.10 m, it appeared that in most cases the greatest similarity was obtained by using

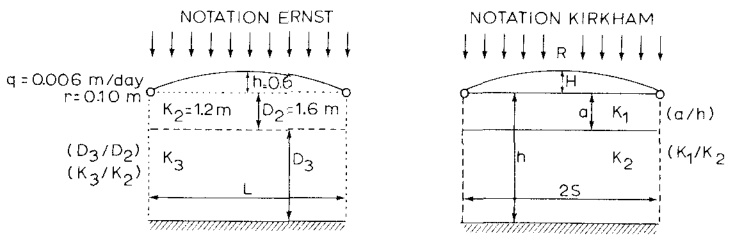
$$\frac{8D_2}{\pi^3 r} = \frac{D_2}{3.88\ r}$$

instead of $\frac{D_2}{\pi r}$, where also the use of $u = 4\ r$ gave better results than $u = \pi r$.

It should be noted that this is only of theoretical importance. For reasons of convenience the author prefers the use of $u = 4\ r$.

Appendix B.
Layered soil below drains

Fig.5. Comparison of calculated drain spacings based on the equation and graph of Ernst and on 36 graphs prepared by Toksöz and Kirkham (1971)



CALCULATED DRAIN SPACINGS ACCORDING TO ERNST AND KIRKHAM

(the latters' drain spacings in italics)

		K_3/K_2	K_3	a/h =0.8		0.4		0.2		0	
				D_3/D_2=0.25		1.5		4		∞	
				D_3 → 0.40		2.4		6.4		∞	
K_3 < 0.1 K_2	$K_3 < K_2$	0.02	.02	36.0	*36.0*	36.4	*36.5*	36.8	*36.8*	36.9	*36.8*
		.10	.12	36.4	*36.8*	37.9	*38.0*	39.9	*39.0*	41.0	*42.0*
K_3 > 0.1 K_2		.20	.24	36.8	*36.8*	39.7	*40.0*	43.4	*42.0*	45.7	*46.0*
		.50	.60	37.7	*36.8*	44.4	*45.0*	51.3	*50.0*	55.7	*56.0*

$K_3 > K_2$	2	2.4	42.4	*43.0*	59.7	*59.0*	73.2	*72.0*	85.6	*83.0*
	5	6	49.4	*48.0*	75.7	*74.0*	91.3	*90.0*	103.9	*101.0*
	10	12	57.7	*56.0*	89.8	*90.0*	103.1	*101.0*	112.7	*112.0*
	50	60	84.0	*83.0*	112.5	*112.0*	119.3	*118.0*	121.6	*122.0*
		∞	125.0	*123.2*	125.0	*123.2*	125.0	*123.2*	125.0	*123.0*

Procedure

For the type of calculations given above (many values: some variable, some fixed) the following procedure is recommended:

1) determine the fixed values, which are here:

$$K_2D_2 = 1.92; \qquad \ln \frac{aD_2}{u} = \ln a + \ln \frac{D_2}{\pi r} = \ln a + 1.63$$

$$4/\pi K_2 = 1.06; \qquad 8\ h/q = 800$$

2) calculate the various KD-values ($KD = K_3D_3 + 1.92$) and determine the a-values (Eq.1a); write down these values and use the required consistency in the rows of figures as a control for their correctness.

3) make a program for the available SPC, based on Eq.9 and the constant values

Appendix C 1. Construction of Graph II, based on Hooghoudt's table for r = 0.10 m

$$L^2 = \frac{8K_2dh + 8K_1d_1h}{q} \quad \text{or for } K_1 = K_2,\ L = 8Kd'\ h/q \quad K = \frac{L^2}{8d'h/q} \quad K = \frac{0.125L^2\times10^{-2}}{d'}$$

$$d' = d + D_1 = d + 0.5h \quad h/q = 100$$

	L=	10	15	20	30	40	50	75	100	150	200 (m)
	$0.125L^2\times10^{-2}$=	.125	.281	.50	1.125	2.0	3.125	7.03	12.5	28.125	50.0
D_2 = 0.5	d=	.49	.49	.49	.50	.50	.50	.50			
	d'=	.79	.79	.79	.80	.80	.80	.80			
	K=	.158	.356	.633	1.141	2.50	3.91	8.79			
1.	d=	.80	.86	.89	.93	.96	.96	.97	.98		
	d'=	1.10	1.16	1.19	1.23	1.26	1.26	1.27	1.28		
	K=	.114	.242	.420	.915	1.59	2.48	5.58	9.76		
2.	d=	1.08	1.28	1.41	1.57	1.66	1.72	1.80	1.85		
	d'=	1.38	1.58	1.71	1.87	1.96	2.02	2.10	2.15		
	K=	.091	.178	.292	.602	1.02	1.55	3.35	5.81		
3.	d=	1.13	1.45	1.67	1.97	2.16	2.29	2.49	2.60	2.71	
	d'=	1.43	1.75	1.97	2.27	2.46	2.59	2.79	2.50	3.02	
	K=	.087	.160	.254	.496	.813	1.21	2.52	4.31	9.31	
5.	d=			1.88	2.38	2.75	3.02	3.49	3.78	4.12	
	d'=			2.18	2.68	3.05	3.32	3.79	4.08	4.42	
	K=			.229	.420	.656	.941	1.86	3.06	6.36	
10.	d=				2.57	3.23	3.74	4.74	5.47	6.45	7.09
	d'=				2.87	3.53	4.04	5.04	5.77	6.75	7.39
	K=				.392	.567	.774	1.39	2.17	4.17	6.76
∞	d=				2.58	3.24	3.88	5.38	6.82	9.55	12.20
	d'=				2.88	3.54	4.18	5.68	7.12	9.85	12.50
	K=				.391	.565	.748	1.24	1.76	2.86	4.0

Appendix C 2. Construction of Graph II, based on the generalized equation of Ernst for $K_1 = K_2$

$h = 0.600$ m; $q = 0.006$ m/day; $h/q = 100$; $8\,h/q = 800$; $r = 0.10$ m; $u = 42 = 0.40$ m; $D' = D_2 + 0.5h = D_2 + 0.30$; $KD = KD'$; $B = \frac{KD_1}{KD} = \frac{0.5h}{D'} = \frac{0.30}{D'}$

$L_o^2 = KD\,8h/q$; $c = D_2 \log \frac{D_2}{u}$; L *(italics)* $= L_o - c$; ( ) difference > 5%

D_2	D'	B	c		K=0.1	0.25	0.5	1.0	2.5	5.0
0.5	0.80	.38	.11	KD	.08	.20	.40	.80	2.0	4.0
				L_o	8.0	12.6	17.9	25.3	40.0	56.6
				c/L_o	.014	.008				
				L/L_o	.99	1.0	1.0	1.0	1.0	1.0
				L	7.9 *7.9*	12.5 *12.5*	17.9 *17.8*	25.3 *25.1*	40.0 *39.9*	56.6 *56.6*
1.0	1.30	.23	.92	KD	.13	.325	.65	1.30	3.25	6.50
				L_o	10.2	16.1	22.8	32.2	51.0	72.1
				c/L_o	.09	.06	.04	.03	.02	.01
				L/L_o	.92	.94	.96	.97	.98	.99
				L	9.4 *9.3*	14.8 *15.2*	21.9 *21.9*	31.3 *31.3*	50.0 *50.1*	71.4 *71.2*
2.0	2.30	.13	3.22	KD	.23	.575	1.15	2.30	4.75	11.50
				L_o	13.6	21.4	30.3	42.9	67.8	95.9
				c/L_o	.237	.150	.106	.075	.047	.033
				L/L_o	,79	.855	.895	.92	.95	.965
				L	10.7 *10.4*	18.3 *18.2*	27.3 *27.1*	39.5 *39.7*	64.4 *64.6*	92.6 *92.7*

D_2	D'	B	c		K=0.1	0.25	0.5	1.0	2.5	5.0
	3.30			KD	.330	.825	1.65	3.30	8.25	16.50
		.09		L_o	16.2	25.7	36.3	51.4	81.2	114.9
3.0			6.05	c/L_o	.373	.235	.166	.117	.075	.053
				L/L_o	.68	.775	.83	.88	.92	.94
				L	11.0 *(10.2)*	19.9 *19.7*	30.2 *30.3*	45.2 *45.4*	74.7 *75.2*	108.0 *108.9*
	5.30			KD	.53	1.325	2.65	5.30	13.25	26.50
		.06		L_o	20.6	32.6	46.0	65.1	103.0	145.6
5.0			12.63	c/L_o	.613	.388	.274	.194	.123	.087
				L/L_o	.55	.66	.735	.80	.865	.90
				L	11.3 *(8.0)*	21.5 *(20.0)*	33.8 *33.4*	52.1 *52.5*	89.1 *90.4*	131.0 *133.0*
	10.30			KD	1.03	2.575	5.15	10.30	25.75	51.50
		.03		L_o	28.7	45.4	64.2	90.8	143.5	203.0
10.0			32.19	c/L_o	1.12	.71	.50	.355	.224	.159
				L/L_o	.38	.49	.58	.66	.765	.825
				L	11.3 *(10.9)*	22.3 *(53.2)*	37.2 *(32.0)*	59.9 *58.6*	109.8 *111.3*	167.5 *170.8*

∞ : $L \ln \frac{L}{u} = \pi k_2 \, h/q$ or $K_2 = \frac{L \ln L/u}{100\,\pi}$

L =	40	50	75	100	150	200	250
K_2 =	0.59	0.77	1.25	1.76	2.83	3.96	5.12

List of symbols

Symbol	Description	Dimension
a	geometry factor for radial flow depending on the hydraulic situation	dimensionless
B	the flow above the drain as a fraction of the total horizontal flow = K_1D_1/KD	dimensionless
c	radial resistance factor	m (meters)
d	thickness of the equivalent layer of Hooghoudt	
D_1	average depth of flow region above drain level	
D_2	thickness of the pervious soil layer below drain level = cross-sectional area of flow at right angles to the direction of flow per unit length (m) of drain	(m^2/m) m
D_3	thickness of the pervious layer, if any, below layer D_2	m
D_v	thickness of layer over which vertical flow is considered	m
h	hydraulic head = the height of the water table above drain level midway between the drains	m
K_1	hydraulic conductivity (h.c.) of the soil (flow region) above drain level	m/day
K_2	h.c. below drain level (layer D_2)	m/day
K_3	h.c. of layer D_3	m/day
K_v	h.c. for vertical flow	m/day
KD	the sum of the product of the permeability (K) and thickness (D) of the various layers for the horizontal flow component according to the hydraulic situation	m^2/day
L	drain spacing	m
q	drain discharge rate per unit surface area per unit time	(m^3 per day/m^2) m/day
q_1	discharge rate of the flow above drain level	m/day
q_2	discharge rate of the flow below drain level	m/day
r	radius of the drain	m
u	wetted perimeter of the drain	m

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GRAPH I Determination of drainspacing with the generalized Ernst equation

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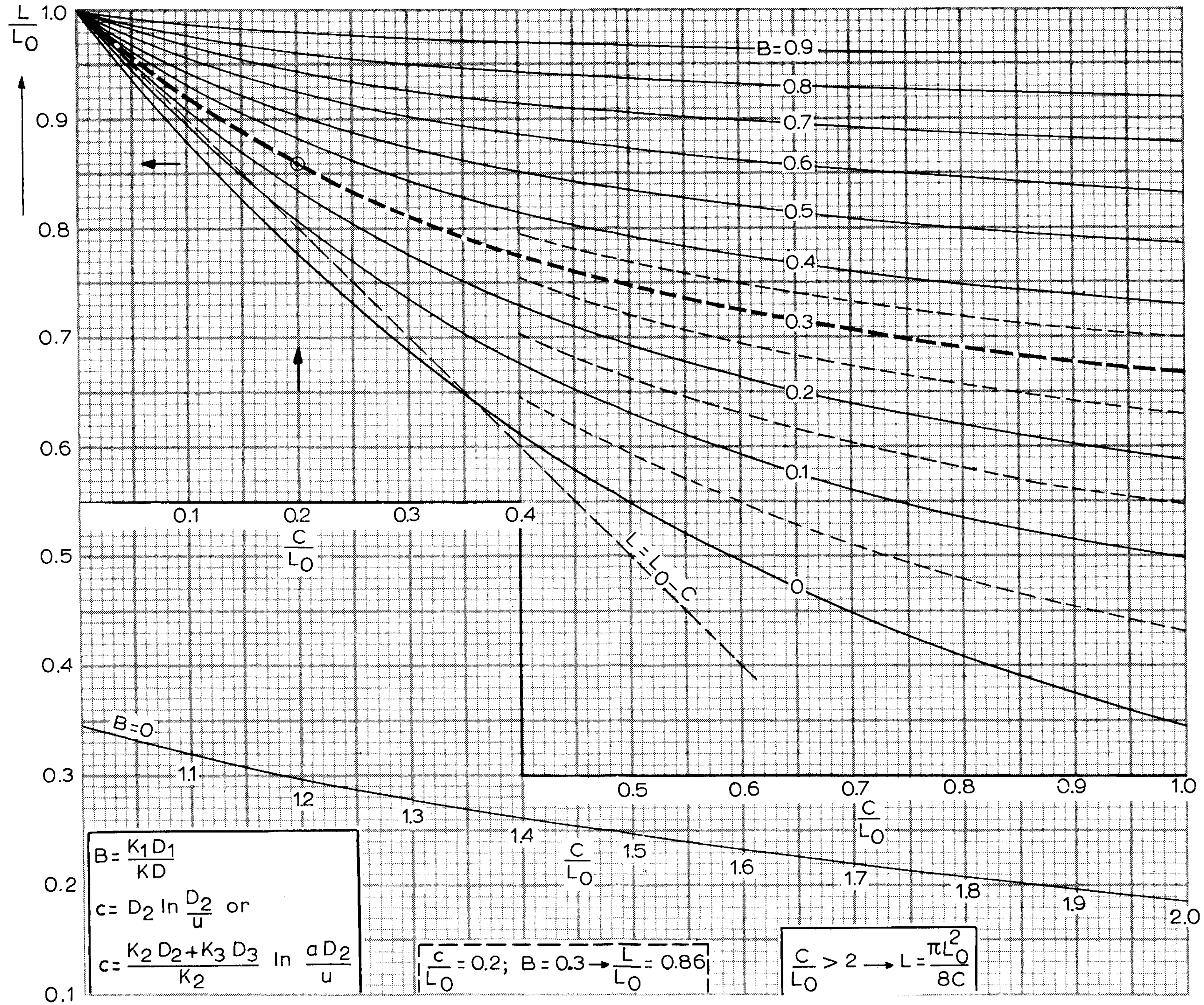
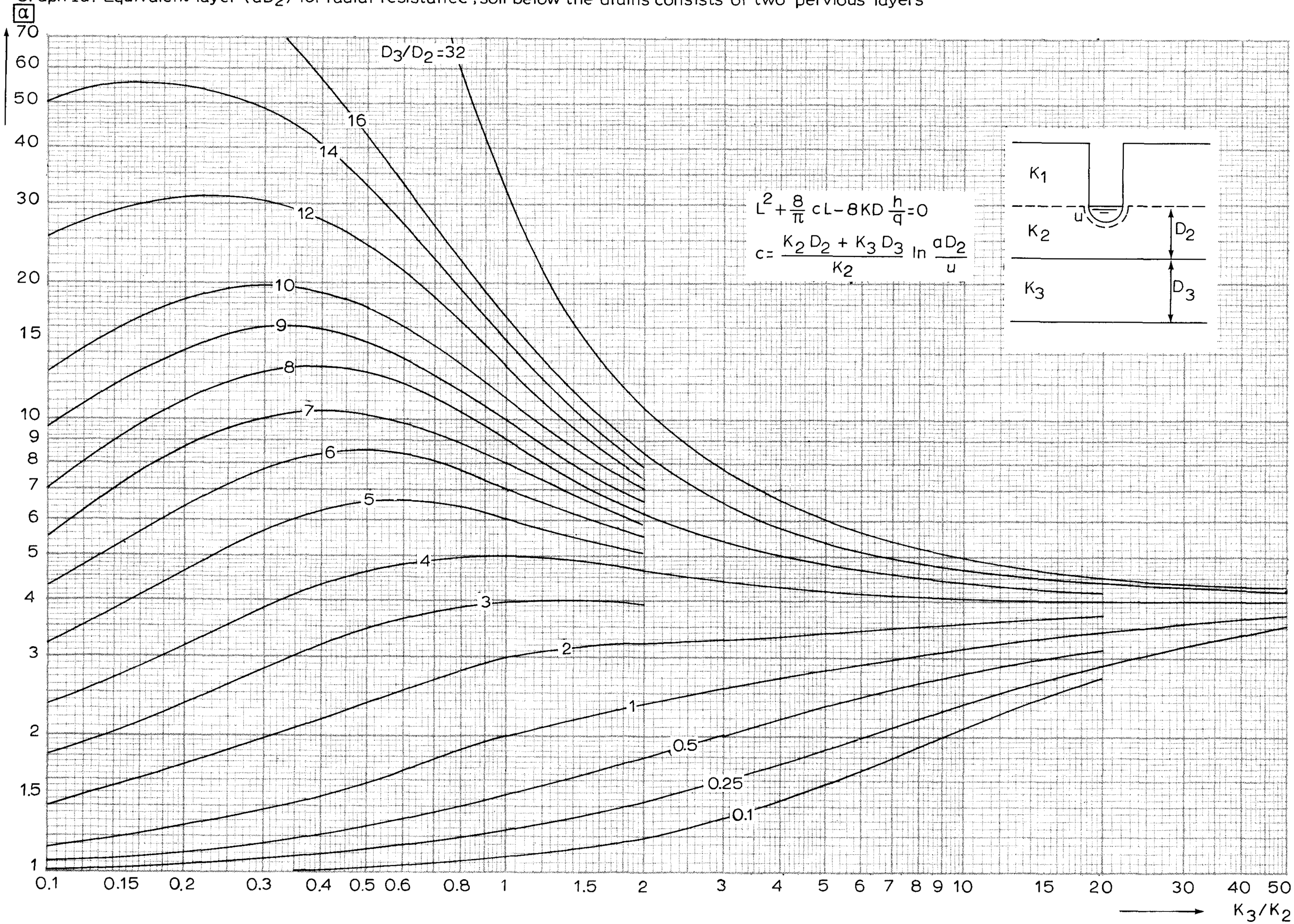
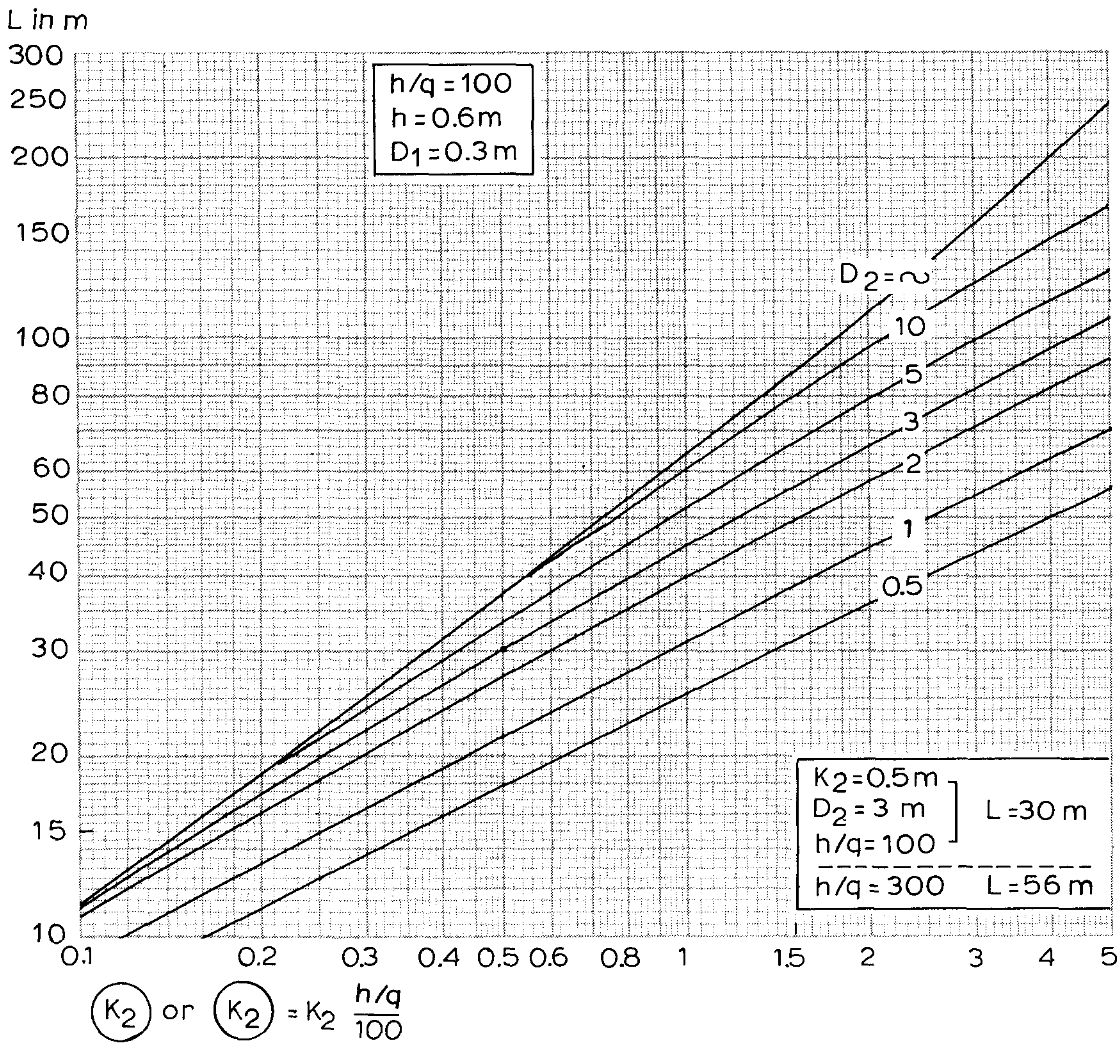


Graph Ia: Equivalent layer (aD_2) for radial resistance; soil below the drains consists of two pervious layers

Graph Ia: Equivalent layer (aD_2) for radial resistance; soil below the drains consists of two pervious layers



GRAPH II Homogeneous soil and pipe drains (r=0.10 m)



Graph III : Auxiliary graph for $D \ln \frac{D}{u}$ or $L \ln \frac{L}{u}$

Graph III : Auxiliary graph for $D \ln \frac{D}{u}$ or $L \ln \frac{L}{u}$