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The NRCS curve number equation derived from an instantaneous unit hydrograph: Some consequences

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ABSTRACT

The NCRS-curve number equation allows calculating the storm runoff from a rainfall event for specific types of land use. It was based on an analysis of direct runoff data using baseflow corrected hydrographs and rainfall. Given this basis, the curve number equation can be derived assuming a constant effective rainfall intensity and a cubic reciprocal function as the instantaneous unit hydrograph. The instantaneous unit hydrograph and the resulting curve number equation are further generalized by adding a lag time. The equation for a curve number related hydrograph is presented, allowing to fit this curve number-based hydrograph to event data. The curve number itself is shown be a function of a catchment response time and the average event rainfall intensity. As the catchment response time is linked to the time of concentration the curve number equation and the storage index can be linked to catchment- and flow type characteristics. First results suggest that including the rainfall intensity duration frequency function in the curve number equation may explain systematic deviations observed when fitting the NCRS curve number equation to measured data.

1. Introduction

The design of water management measures in catchments or other units of water management is generally based on national engineering practices, procedures and standards. One such engineering handbook for design in watershed management is the hydrology part of the NEH, developed for use in - and based on data from- the United States. For peak discharge analysis the NEH suggests using -or offers guidance on- a number of steps, one of which is converting rainfall to direct runoff, using the curve number equation (USDA-NRCS, 2004b). In a next step, the direct runoff, now regarded as effective (or excess) rainfall, is converted to a hydrograph for a specific catchment using a unit hydrograph (USDA-NRCS, 2007). The unit hydrograph is the catchment response function, and converts a unit excess rainfall over a finite period of time to the resulting partial catchment hydrograph. An extension of the catchment response function is the instantaneous unit hydrograph. It converts a unit excess rainfall which has fallen over an infinitely small period of time to the resulting partial catchment hydrograph. Summing the individual partial hydrographs over a rainfall event in a number of finite steps using the unit hydrograph yields the catchment hydrograph for that event. For the instantaneous unit hydrograph this process of generating and summing an infinite number of partial hydrographs is the so-called convolution integral. Given effective rainfall over time, both instantaneous unit hydrographs and unit hydrographs can be applied to generate a catchment hydrograph (for a detailed description of the above c.f. e.g. Brutsaert, 2013, or Bras, 1990). The curve number equation was initially developed analysing catchment scale rainfall -runoff events in gauged basins in the (continental) United States which were analyzed in terms of cumulative event rainfall, and the cumulative storm depth or -direct runoff, using the hydrograph corrected for base flow. It is characterized as a spatially lumped event-based rainfall-runoff response function (e.g. Bartlett et al., 2016). The equation is parameterized using land use, land management, and hydrological soil type in lookup tables, which is and was paid less attention to in other rainfall-runoff lumped models. In other locations with different soils, different land use types and a different climate its use raises validity issues, which is (or could be) addressed by site specific research (some examples: Korea: Shin et al., 2015; Mexico: Velásquez-Valle et al., 2017; India: Kadam et al., 2012). The starting point of this analysis is the reference version of the curve number equation (USDA-NRCS, 2004b):

$$Q = \frac{(P - I_a)^2}{P + S - I_a} \ if \ P \geqslant I_A, \ else \ Q = 0 \tag{1}$$

which can be rewritten as

$$Q = P - S - I_a + \frac{S^2}{(P + S - I_a)} if \ P \geqslant I_a, \ else \ Q = 0 \tag{2}$$

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Nomenclature

A	catchment area (m^2)
a	eq 28: positive empirical parameter
b	eq 28: positive empirical parameter
B	shape parameter of intensity duration frequency function
c	eq. 32: empirical parameter (–)
CN	(0–100) is the so-called curve number which is defined by land use and hydrological soil type
d	eq. 32: empirical parameter (–)
e	eq 32: empirical parameter (–)
f	auxiliary constant defining the maximum discharge (mm $hour^{-1}$), same symbol for both linear reservoir and curve number based discharge
f	eq 32: empirical parameter (–)
h(t)	the general designation for an instantaneous unit hydrograph ($hour^{-1}$)
h_f	matric head at the infiltration front for Green and Ampt equation (m)
I_a	the initial abstraction (mm)
k	inverse of the catchment response time for the instantaneous unit hydrograph for the curve number equation (=1/T*) ($hour^{-1}$); eq 33: unit conversion factor for Manning roughness factor)
K_e	eq 34: Effective hydraulic conductivity for subsurface flow, (m/s)
k_l	the inverse of the time constant for the linear reservoir ($hour^{-1}$)
K_s	eq 33: hydraulic conductivity for Green and Ampt infiltration (assumed to be the value at saturation), (m/s)
L	additional lag in instantaneous unit hydrograph for curve number equation (hour)
L	eq 28 main channel length (km); eq 29: overland flow path (m); eq 30: watershed length; equation; eq. 31,32,33: length overland flowpath (m); eq 34:length subsurface flowpath (m)
n	eq 31, 32, 33: Manning roughness coefficient (s $m^{-1/3}$)
P	the event rainfall depth (mm)
p	the intensity, (mm $hour^{-1}$)
p_1	function: intensity within event as a linearly increasing function of time
p_2	function: intensity within event as a linearly decreasing function of time
p_e	the constant excess (effective) intensity (mm $hour^{-1}$)
P_e	the effective event rainfall (mm)
p_l	a constant loss in intensity over time (smaller than p), (mm $hour^{-1}$)
$p_{s,r}$	extrapolated intensity (mm $hour^{-1}$) at zero event duration in intensity duration frequency function
Q	the event direct runoff (mm);
q	the event discharge (mm $hour^{-1}$)
s	eq 28 main channel slope (m/m); eq 29, 30 average catchment (watershed) slope (m/m); eq. 31,32 slope overland flow (m/m); 33: average slope in flow direction (m/m)
S	the storage index (mm)
S'	the ratio p/k_l for the linear reservoir (mm)
T	is the duration of the rainfall event (hour)
T*	catchment response time, eq 28, 32 (hour); eq 29, 30 (min), eq 31,33 (seconds)
t_a	the time effective precipitation starts; the event itself starts at t = 0 (hour)
T_c	time of concentration (hour)
T_{c1}	eq 28 empirical unit dependent parameter (hours km^{-1})
T_{c2}	eq 32: empirical parameter (hour)
t_d	duration of rising branch of the hydrograph (hour)
t_p	time to peak of the discharge (hour)
T_r	average recurrence interval (years)
T_s	scaling parameter duration (hour) in intensity duration frequency function
α	eq 5b: a constant reduction fraction (–)
α	eq 34: hillslope slope for subsurface flow (degrees)
β	fraction defined by extra lag in instantaneous unit hydrograph.
θ_e	storage coefficient (–)
θ_i	initial volumetric moisture content (–)
θ_s	volumetric moisture content at saturation (–)
λ	the proportionality factor between I_a and S: $I_a = \lambda S$.
τ	dummy integration variable for the convolution integral (hour)

where Q: the event direct runoff (mm),
P: the event rainfall depth (mm),
S: the storage index (mm),
I_a: the initial abstraction (mm).

The basic assumption leading to this equation is presented in USDA-NRCS (2004b) but is also provided by a.o. Mishra and Singh (2003) and Ritzema (2006).

The standard version of the curve number equation reduces the number of parameters to one by setting I_a to 0.2S:

$$Q = \frac{(P - 0.2S)^2}{P + 0.8S} \ \ if \ P \geqslant 0.2S, \ \ else \ Q = 0 \tag{3}$$

And the storage index is calculated using an auxiliary function

$$S = 254\left(\frac{100}{CN} - 1\right) \tag{4}$$

where CN (0–100) is the so-called curve number which is defined by land use and hydrological soil type. The justification of setting I_a to 0.2S for practical applications has been extensively analysed (ASCE (2009) and works quoted therein). More generally the initial abstraction I_a is assumed proportional to S: $I_a = \lambda S$. Currently a value of $\lambda = 0.05$ is suggested (ASCE, 2009).

The storage index S is determined from measured data of event rainfall P and associated direct runoff Q estimated from discharge measurements. The direct runoff rate Q consists of (combinations of) three components: direct precipitation on the channel, surface flow (or overland flow), and subsurface flow (e.g. Dingman, 2008). In overland flow a distinction is made between overland flow due to infiltration excess, and overland flow due to saturation excess. Direct runoff is assumed to dominate the hydrograph up to the point where the hydrograph recession starts, and base flow becomes dominant. Whereas in the development of the curve number equation the method used to determine direct runoff was never published, ASCE (2009, page 46) suggests as an aside that data from the report by Dalrymple (1965) presenting rainfall and direct runoff could have been used. Dalrymple (1965) defines baseflow by extending the trend of flow prior to the flood to the time of the peak and then drawing a line to a point on the recession limb a number of days after the peak, with the number depending on the size of the catchment. The discharge exceeding the baseflow rate is the direct runoff rate. This method of hydrograph separation was presented in earlier textbooks (e.g. Linsley et al., 1949), and is also presented by Dunne and Leopold (1978), and Dingman (2008). The curve number equation, its history, the factors which influence the parameters, the theoretical basis, and the implementation and use in models has been -and is being- extensively discussed (e.g. Ponce and Hawkins, 1996; Mishra and Singh, 2003; Garen and Moore, 2005; Bartlett et al., 2016; Chin, 2021; Hoeft, 2020). Reviews (e.g. Verma et al., 2017; Ormsbee et al., 2020) in addition provide an overview over the diversity of research areas and topics.

The NRCS curve number equation can also be formulated as a soil moisture accounting procedure (e.g. Michel et al., 2005). In their analysis the initial abstraction is linked to the initial storage conditions at the start of the model calculations. In this context direct runoff is regarded as the difference between rainfall and infiltration. This has been and is a topic of research (a.o. Chin, 2017; Baiamonte, 2019; Kirkby and Cerdà, 2021); early papers are Aron et al. (1977); Hjelmfelt (1980). In their monograph on the curve number equation, Mishra and Singh (2003) dedicate a chapter to the relation between infiltration and direct runoff. Hoosyar and Wang (2016) show that in the case of infiltration excess and a constant rainfall intensity the resulting equation for the cumulative runoff yields the curve number equation for a specific retention curve and holds approximately for other soil types. Garen and Moore (2005) however stress that direct runoff need not only be infiltration excess overland flow, and that the difference between rainfall and direct runoff is not only infiltration. Another possible component of direct runoff is saturation excess overland flow from a source area varying in size (e.g. Dingman, 2008), which was used by Steenhuis et al. (1995) to theoretically derive the curve number equation. Whereas for some catchments it may be justified to focus either on infiltration excess, on saturation excess, on the kinematic wave equation for overland flow, or on soil moisture accounting and infiltration, this should not lead to neglecting other mechanisms which may contribute in varying proportions over varying fractions of the catchment area to the catchment scale direct runoff (Garen and Moore, 2005; Beven, 2012).

Beven (2012, p 206) summarizes research and discussions on the curve number equation by stating that at the small catchment scale, and in a simple functional form, the curve number equation incorporates some empirical knowledge of fast runoff generation by some combination of different flow mechanisms (infiltration excess, saturation excess, or processes referred to as subsurface stormflow by Brutsaert (2013)).

The question to be addressed is based on the history of the equation sketched above: if the curve number equation is or was based on hydrograph data for direct runoff, and the instantaneous unit hydrograph is in theory applicable, could an instantaneous unit hydrograph not also provide a basis for the curve number equation? And if it provides a basis what are consequences? Is this basis also sufficiently flexible to account for different mechanisms contributing to direct runoff?

2. Theory: Effective rainfall intensity and an instantaneous unit hydrograph

As described in the introduction the curve number equation allows to calculate cumulative direct runoff as a function of cumulative effective precipitation in a rainfall event. Cumulative direct runoff is the integral of the difference between the hydrograph and the estimated baseflow; the hydrograph -i.e. the direct runoff rate as a function of time- is the convolution integral of the effective intensity of the rainfall event and the instantaneous unit hydrograph. The instantaneous unit hydrograph describes the transformation of a unit of rainfall to discharge for the limit case of a time interval going to 0. To determine the relation between direct runoff and event precipitation assumptions regarding the effective rainfall intensity and the instantaneous unit hydrograph are then needed.

2.1. The effective rainfall intensity

To apply the instantaneous unit hydrograph (IUH) its input needs to be defined in terms of effective rainfall rates, or effective rainfall intensity. Effective rainfall intensity is the rainfall intensity corrected for losses. For approaches to rainfall losses, Brutsaert (2013) presents three options: a continuous constant loss rate, a constant proportional loss rate or an initial loss. The combination of these assumptions results in:

$$P_e = \int_{t_a}^{T} p\,dt \tag{5a}$$

$$p = \begin{cases} \alpha(p - p_l) & p \geq p_l \\ 0 & p < p_l \end{cases} \tag{5b}$$

where P_e is the effective event rainfall (mm),

t_a is the time effective precipitation starts; the event itself starts at $t = 0$ (hour),

T is the duration of the rainfall event (hour),

p is the rainfall intensity, (mm.hour^{-1}),

p_l is a constant loss over time (smaller than p), (mm.hour^{-1}),

α is a constant reduction fraction (–).

An assumption which would fit the context of the curve number equation is that of a constant intensity, with excess rainfall starting after the time required for the initial abstraction, i.e. P_e and P differ by a constant offset. This would suggest $\alpha = 1$, $p_l = 0$, and $t_a > 0$. A constant loss rate would require assuming $\alpha = 1$, $t_a = 0$, and $0 < p_l < p$, while a constant reduction fraction corresponds to the case $p_l = 0$, $t_a = 0$, and $0 < \alpha < 1$.

2.2. An instantaneous unit hydrograph

To illustrate the basic steps we will start from the instantaneous unit hydrograph for the linear reservoir (e.g. Bras, (1990, p. 443, eq. 9.89); also Dingman (2008, p. 453, eq. 9–52)), defined as

$$h(t) = k_l e^{-k_l t} \tag{6}$$

(h in 1/t). The parameter k_l (1/t) is the inverse of the catchment response time T* (cf. Dingman, 2008, p. 402).

2.2.1. Direct runoff for the linear reservoir

Given the assumption regarding effective rainfall, and the instantaneous unit hydrograph, the cumulative direct runoff can be calculated. First, convolution for a storm causing excess rainfall of constant intensity p_e (mm/hour) we find the rising limb of the hydrograph – a discharge q (mm/hour) for the linear reservoir, from t_a over time t (Bras, 1990, p. 444, eq. 9.91, Dingman, 2008, p. 403, box 9–2) as:

$$q(t) = p_e - p_e e^{-k_l(t-t_a)} \text{if } t \geqslant t_a, \text{ else } q = 0; t \leqslant T \tag{7}$$

where T is the duration of the event. For infinitely long events with constant effective rainfall intensity the rate of the discharge becomes equal to the direct runoff intensity p_e, and the system is at steady state. The infinite integral with constant (unit) rainfall intensity is also known as the S-hydrograph (cf. Dooge and O'Kane, 2003, p. 22). We will assume that direct runoff is generated during the duration of the rainfall event, and no other losses occur. Introducing Q, the cumulative (or total) direct runoff (mm), setting $q = dQ/dt$, and integrating q between $t = t_a$, and $t = T$ (time of rainfall duration) yields:

$$Q(T) = p_e T - p_e t_a - \frac{p_e}{k_l}\left(1 - e^{-k_l(T-t_a)}\right) \text{if } t \geq t_a, \text{ else } Q = 0; \ t \leq T \tag{8}$$

To transform this equation into a relationship between cumulative rainfall and cumulative direct runoff we use the following auxiliary equations, where *p* is the average rainfall intensity, and T the duration of the event:

$$T = \frac{P}{p} \tag{9a}$$

$$t_a = \frac{I_a}{p} \tag{9b}$$

Substitution yields:

$$Q(P) = \frac{p_e}{p}P - \frac{p_e}{p}I_a - \frac{p_e}{k_l}\left(1 - e^{-\frac{k_l}{p}(P-I_a)}\right) if\ P \geqslant I_a,\ else\ Q = 0 \tag{10}$$

Assuming $p_e = p$, i.e. no losses other than initial abstraction results in:

$$Q(P) = P - I_a - \frac{p}{k_l}\left(1 - e^{-\frac{k_l}{p}(P-I_a)}\right) if\ P \geqslant I_a,\ else\ Q = 0 \tag{11}$$

This result can now be compared to the curve number equation, which was written as (eq 2):

$$Q = P - I_a - S\left(1 - \frac{S}{(P + S - I_a)}\right)\ if\ P \geqslant I_a,\ else\ Q = 0 \tag{12}$$

We note that the ratio p/k_l has the unit mm, and that both the direct runoff equations and the curve number equation asymptotically become equal to $P - \left(\frac{p}{k_l}\ or\ S\right) - I_a$. Introducing a parameter S'(=p/k_l) illustrates the similarity:

$$Q(t) = P - S' - I_a + S' e^{-\frac{(P-I_a)}{S'}}\ if\ P \geqslant I_a,\ else\ Q = 0 \tag{13}$$

The rational function and the exponential behave differently so the shape will be different, when S' is set to S. The functions can be scaled (setting S' to S, dividing by S, and setting I_a to 0.2S) and are compared in Fig. 1. The scaled equations are:

$$\frac{Q}{S} = \frac{P}{S} - 1.2 + \frac{1}{\left(\frac{P}{S} + 0.8\right)}\ if\ \frac{P}{S} \geqslant 0.2,\ else\ Q = 0 \tag{14}$$

$$\frac{Q}{S} = \frac{P}{S} - 1.2 + e^{0.2 - \frac{P}{S}}\ if\ \frac{P}{S} \geqslant 0.2,\ else\ Q = 0 \tag{15}$$

2.3. A curve number equation based on an instantaneous unit hydrograph.

The close similarity between these two results begs the question whether an instantaneous unit hydrograph exists which after convolution with a constant intensity and a second integration of the rising limb of the hydrograph (from time required for initial abstraction to the event duration) yields the reference curve number equation. Assuming a constant effective rainfall intensity (p_e) equal to the observed rainfall intensity (p), it can be shown that if the product of this constant rainfall intensity p and the instantaneous unit hydrograph h(t) is given by

$$p(t)h(t) = p\frac{2k}{(kt+1)^3} \tag{16}$$

(where k is equal to 1/T*, and T* is the catchment response time) the result is indeed the curve number equation. This is shown in the following steps. The discharge function for the rising limb is derived using the convolution integral:

$$q = \int_{t_a}^{t} \frac{2kp}{(k(t-\tau)+1)^3} d\tau \tag{17}$$

and results in the discharge function:

$$q = p - \frac{p}{(1 + k(t - t_a))^2} \tag{18}$$

Integrating the discharge over the period between the start of runoff (t_a) and the end of the event T then yields the amount of water which becomes direct runoff:

$$Q = \int_{t_a}^{T} q dt = pT - pt_a + \left[\frac{p}{k + k^2(t - t_a)}\right]_{t_a}^{T} \tag{19}$$

or

$$Q = pT - pt_a + \frac{p}{k + k^2(T - t_a)} - \frac{p}{k} \tag{20}$$

which (using the assumptions formulated earlier, i.e. P = pT, $pt_a = I_a$, and in addition S = p/k) can be rewritten as:

$$Q = P - I_a - S\left(1 - \frac{S}{S + P - I_a}\right) \tag{21}$$

which is the curve number equation, subject to the condition $P \geq I_a$, as otherwise there is no direct runoff. The IUH of the linear reservoir and that leading to the curve number equation are compared in Fig. 2.

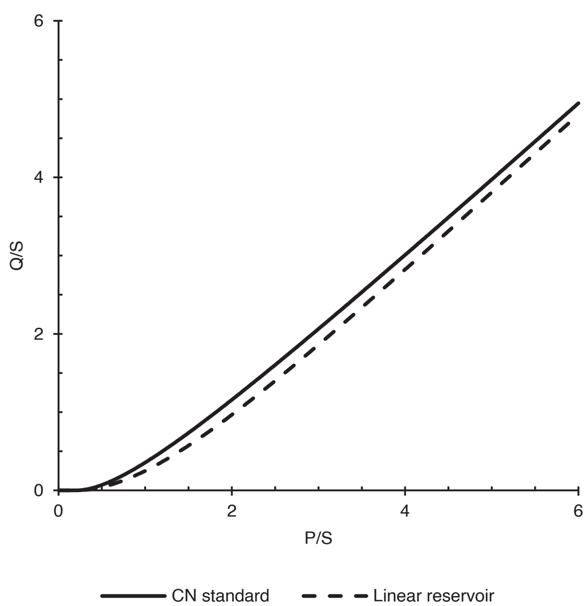

Fig. 1. Scaled rainfall- direct runoff curves for the standard curve number (solid line) and the linear reservoir (dashed line).

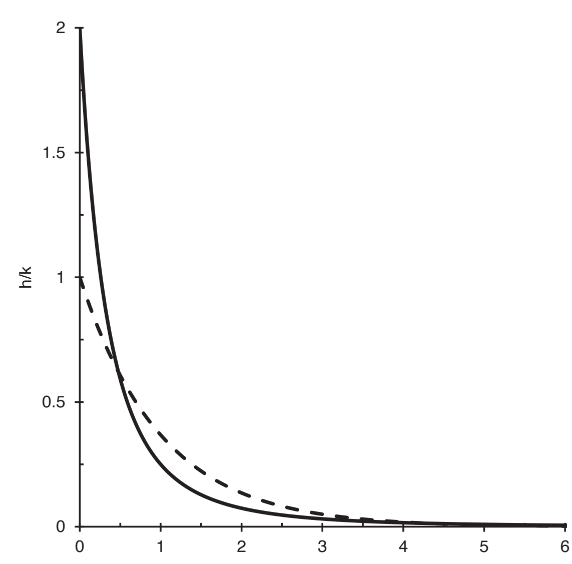

Fig. 2. The scaled instantaneous unit hydrographs for the curve number equation (solid line) and the linear reservoir (dashed line).

2.4. The direct runoff component of the hydrograph for a constant intensity event.

The hydrograph behaviour for the direct runoff component could be assumed to be symmetrical, in which case the parameters are identical for both the rising and falling limb of the hydrograph. The hydrograph can then be described by:

$$q = \begin{cases} p - \dfrac{p}{(1+k(t-t_a))^2}, & t_a \leqslant t < t_p \\ \dfrac{f}{(1+k(t-t_p))^2}, & t \geqslant t_p \end{cases} \tag{22a}$$

$$f = p - \frac{p}{(1+k(t_p-t_a))^2} \tag{22b}$$

and

$$q = 0,\ t < t_a. \tag{22c}$$

Expressing this equation in terms of S by either substituting k as p/S or by substituting p as kS is possible, but in the context of parameter estimation not desirable. In parameter estimation replacing t_p by $t_a + t_d$, where t_d is the length of the rise avoids convergence issues.

For comparison purposes the equations for the linear reservoir are the following:

$$q = \begin{cases} p - pe^{-k_l(t-t_a)}, & t_a \leqslant t < t_p \\ fe^{-k_l(t-t_p)}, & t \geqslant t_p \end{cases} \tag{23a}$$

where

$$f = p - pe^{-k_l(t_p-t_a)} \tag{23b}$$

and

$$q = 0,\ t < t_a \tag{23c}$$

The parameter p is the effective intensity, so rainfall intensity corrected for losses, but given the previous assumptions – only an initial loss – p_e and p are equal. The scaled hydrographs for the linear reservoir and the curve number reservoir are presented in Fig. 3a. Equation 22a-c also allow estimating parameters directly from base flow corrected hydrographs. To provide an example, we have used base flow corrected hydrograph data (Diskin and Boneh, 1975) to fit the hydrograph (assuming the same parameter values for the rising branch and for the recession, and using Eq. 22abc). The function was fitted to the data using the Excel solver to minimize the sum of squared differences for parameters, p, t_a, t_p and k. Fig. 3b shows the worst and best fit for illustrative purposes.

2.5. Introducing an additional lag.

Assuming an extra lag in the instantaneous unit hydrograph would reduce the conversion of rainfall to direct runoff. In that case the above steps can be repeated starting from the convolution integral of

$$p(t)h(t) = p\frac{2k}{(k(t+L)+1)^3} \tag{25}$$

where L is an extra lag (units of time) compared to the original IUH. The steps resulting in the lagged curve number equation are presented in Annex A for both the linear reservoir and the above IUH. The curve number equation can then be shown to be a specific case of the equation defined by

$$Q = \beta\frac{(\beta(P-I_a))^2}{S+\beta(P-I_a)}\ if\ P \geq I_a,\ else\ Q = 0 \tag{26}$$

where the extra lag is expressed as a parameter β (0 ≤ β ≤ 1). The parameter β strongly reduces the direct runoff. The equation reverts to the reference curve number equation for β = 1.

2.6. The relation between storage index S, the catchment response time 1/k and the time of concentration

The instantaneous unit hydrograph at the basis of the curve number equation (for a constant rainfall intensity, and β = 1) is given by

$$h(t) = \frac{2k}{(kt+1)^3} \tag{27}$$

and contains one parameter, a characteristic time (the catchment response time, T*=1/k). To determine the storage index S requires determining T* and multiplying by intensity p. The question is whether we can derive a relation between catchment time scales and the catchment response time T*, and whether the product pT*(=S) is constant. A central parameter in rainfall-runoff analysis is the time of concentration T_c, defined as the time base of the instantaneous hydrograph (Bras, 1990, p 432), which corresponds to the time at which the entire area considered contributes to the flow at the point of interest. As the curve number IUH (equation 27) decreases to zero asymptotically, the time base would be infinite. Assuming for practical purposes that the conversion to discharge is negligible if h(t) = 0.01 k, it can be shown that for the curve number equation $T_c = (\sqrt[3]{200}-1)T^* \approx 4.84T^*$. For the linear reservoir this assumption leads to the relation $T_c = T^*\ln(100) \approx 4.60T^*$

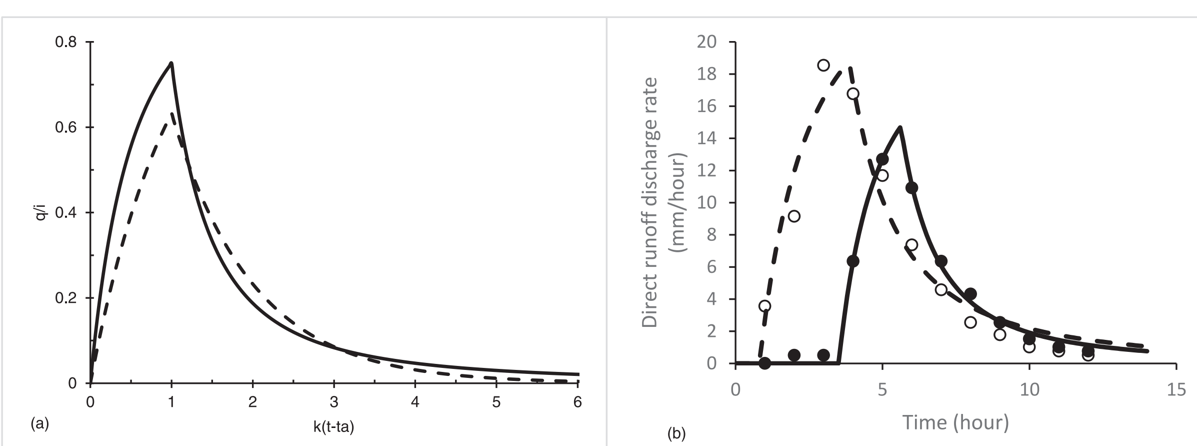

Fig. 3. A: Scaled hydrographs for linear reservoir and curve number equation; b: Example of a hydrograph fit -best (closed symbols, solid line) and worst (open symbols, dashed line)- using the standard curve number (data from Diskin and Boneh, 1975).

(cf. Dingman, 2008, Box 9–2). Equations to calculate the time of concentration based on catchment characteristics can now be used to estimate the catchment response time. The literature and discussion on this topic are extensive. A review (Kaufmann-Almeida et al., 2022) lists 125 equations to estimate the time of concentration. A subset of the empirical equations (n = 27, Kaufmann-Almeida et al. (2022), adapted here to yield the catchment response time) have the form:

$$T^* = \frac{T_{c1}}{4.84} L^a s^{-b} \tag{28}$$

where L and s are often main channel length (km) and the main channel slope (m/m), and the parameters a and b are a positive empirical value ($a = 0.72 \pm 0.19$; $b = 0.32 \pm 0.15$). The constant 4.84 is the conversion constant defined by the time base of the curve number instant unit hydrograph. T_{c1} is an empirical unit dependent parameter, which combines the units used when establishing the equation ($T_{c1} = 0.16$ (68% range is 0.06–0.75 h per km^{-a}, lognormal), and T* in hour. Two approaches include the curve number. The first, the NRCS lag equation (USDA-NRCS, 2010), also equation 2 in Gericke and Smithers (2014), metric units) is:

$$T^* = \frac{1}{4.84} \frac{L^{0.8} [S + 25.4]^{0.7}}{706.9 s^{0.5}} \tag{29}$$

when replacing (25400/CN-228.6) in their equation by S + 25.4. L is the overland flow path (m); s is the average catchment slope ($m.m^{-1}$) and S is the curve number-based storage (mm). T* is in minutes. 706.9 is both a unit conversion constant (in this case with units $m^{0.8}.mm^{0.7}.min^{-1}$) and an empirical parameter. For this equation, McCuen et al. (1984) find an average error of 0.3 h at a mean time of concentration of 1.81 h, for catchment areas up to 1600 ha (16 km^2); the maximum error is 6 h. The second equation (USDA-NRCS, 2010), equation 15A-6; also equation A32 in Gericke and Smithers, 2014) includes catchment area A (m^2) as a regressor (metric units):

$$T^* = \frac{1}{4.84} \frac{[S]^{0.313}}{3.13 s^{0.150}} \left(\frac{A}{L}\right)^{0.594} \tag{30}$$

L is the watershed length (m); s is the average watershed slope (m.m-1) and S is the curve number-based storage (mm). T* is in minutes. Note that the ratio A/L is again a length, the average watershed width. The ratio A/L can also be interpreted as twice the flow distance towards the main flow axis. The empirical coefficient 3.13 is unit dependent (in this case it has units $m^{0.594}.mm^{0.313}.min^{-1}$). The equation was derived for watersheds in the USA (n = 78, $r^2 = 0.58$, area between 0.1 and 1412 ha). The USDA-NRCS (2010) focusing on travel times for overland flow suggests calculating the time of concentration as the sum of travel times for sheet flow, shallow concentrated flow, and open channel flow along the most distant flow path. In the cases in which overland flow is the dominant mechanism, and given that the CN method characterizes catchment surface properties rather than channel dimensions, the relation between sheet flow parameters and the time of concentration seems especially relevant. Brutsaert (2013, page 479, eq. 12.22) presents an equation for the time to equilibrium in the case of turbulent overland flow, based on the kinematic wave equation, assuming no infiltration:

$$T^* = \frac{1}{9} T_c = \frac{1}{9} \left(\frac{1}{p^2} \left(\frac{nL}{s^{\frac{1}{2}}} \right)^3 \right)^{\frac{1}{5}} ; \frac{nL}{\sqrt{s}} < 30m \tag{31}$$

In this equation n is the Manning roughness coefficient ($s.m^{-1/3}$), L is the length of the overland flowpath (m), p is the rainfall intensity ($m.s^{-1}$), and s is the slope as fraction (–), T* is in seconds. The factor 9 is calculated assuming that q = 0.99p (i.e. situation sufficiently close to equilibrium), and that the time of concentration is the time to equilibrium. We can then derive $t_{0.99} - t_a = T_c = 9T^*$. The equation can be rewritten as:

$$T^* = \frac{1}{9} T_c = \frac{T_{c2}}{9} p^c n^d L^e s^f \tag{32}$$

where c = -0.4, d = 0.6, e = 0.6 and f = -0.3. Functions of this type (sheet flow) reviewed by Almeida- Kaufman et al. (2022) have also been used to describe catchment behaviour. For 14 cases quoted, we find the median T_{c2} as 0.12 h (50% between 0.09 and 0.16, in hour). The power function parameters from this set are close to the theoretical values (c = -0.40 ± 0.04, d = 0.62 ± 0.06, e = 0.60 ± 0.07, and f = -0.33 ± 0.04). A fifth equation, for infiltrating basins combining overland flow based on the kinematic wave, and the Green and Ampt infiltration equation (Akan and Houghalen, 2003, p. 103, eq. 5.12):

$$T^* = \frac{1}{9} T_c = \frac{1}{9} \left(\frac{Ln}{k\sqrt{s}} \right)^{\frac{3}{5}} \frac{1}{(p - K_s)^{\frac{2}{5}}} + \frac{3.1}{9} \left(\frac{K_s}{p^2} \right)^{\frac{4}{3}} (\theta_s - \theta_i) |h_f| \quad if\ K_s < 0.4p \tag{33}$$

where L is the flow length (m), n/k is Manning roughness factor (k is a unit conversion factor, ($m^{1/3}s^{-1}$) for SI units), s is the average slope of the catchment in the flow direction (m/m), p is the effective intensity (assumed constant, m/s); K_s is the hydraulic conductivity for Green and Ampt infiltration (assumed to be equal to hydraulic conductivity at saturation; m/s) and h_f the matric head (m) at the infiltration front depending on the initial moisture content θ_i and θ_s (the moisture content at saturation). For an impermeable soil ($K_s = 0$), this equation reduces to the equation of sheet flow above quoted by Brutsaert (2013, page 479, eq. 12.22), and Akan and Houghalen (2003, p. 101, eq. 5.5); T* is again in seconds. Finally Brutsaert (2013, p. 479, eq. 12.23) also presents an equation for the time of concentration for subsurface flow on hillslopes with permeable soils. This allows to calculate the catchment response time in this situation as:

$$T^* = \frac{1}{9} T_c = \frac{\theta_e L}{9 K_e sin\alpha} \tag{34}$$

where L is the hillslope length (m); α is the hillslope slope; θ_e is the storage coefficient; K_e is the effective conductivity ($m.s^{-1}$), and T* is in seconds.

The six equations for T* selected here can be multiplied by the intensity p, and allow to estimate S. The limitation of the current review however is that the dominant flow mechanism is either not specified (the equation is empirical); is based on (partial) infiltration excess, or on flow in permeable soils. The equations so far suggest that S is a function of slope (cf also (Muhammad et al., 2020)) and a catchment length scale, often the main channel length, and that based on the approaches presented, the product S = pT* is not a constant. The decrease of T* with slope, leading to a decrease in S and a corresponding increase of CN values with slope is supported (though by limited data) by Sprenger (1978, quoted as table 4.3 in Ritzema, 2006), more recent by Ajmal et al. (2020). The dependence of CN on slope, using the table in Ritzema (2006), is presented in Fig. 4a. Variability due to land use type and soil type is large. ASCE (2009) also quotes results that suggest a decrease of CN with slope. When focusing on sheet flow, a Manning roughness coefficient is required. Qualitatively matching the values for the Manning roughness coefficient for sheet flow with CN values (USDA-NRCS, 2010) results in Fig. 4b, which offers some support that S-values increase (and CN values decrease) with Manning roughness. If rainfall is infiltrating, soil physical properties become important. As the saturated conductivity increases the catchment response time becomes longer; and S increases (CN decreases). The CN values associated with hydrological soils classes (A-D) qualitatively match this response (ASCE, 2009), as shown in Fig. 4c. As to the relation between storage and a length scale, water harvesting literature for microcatchments (e.g. Sharma, 1986, 252 to 432 m^2 and Li et al., 2006, 5-50 m^2) shows that runoff efficiency (mm runoff/mm rain), or the runoff coefficient) decreases with area, suggesting storage index S increases with area. Kirkby and Cerda (2021)

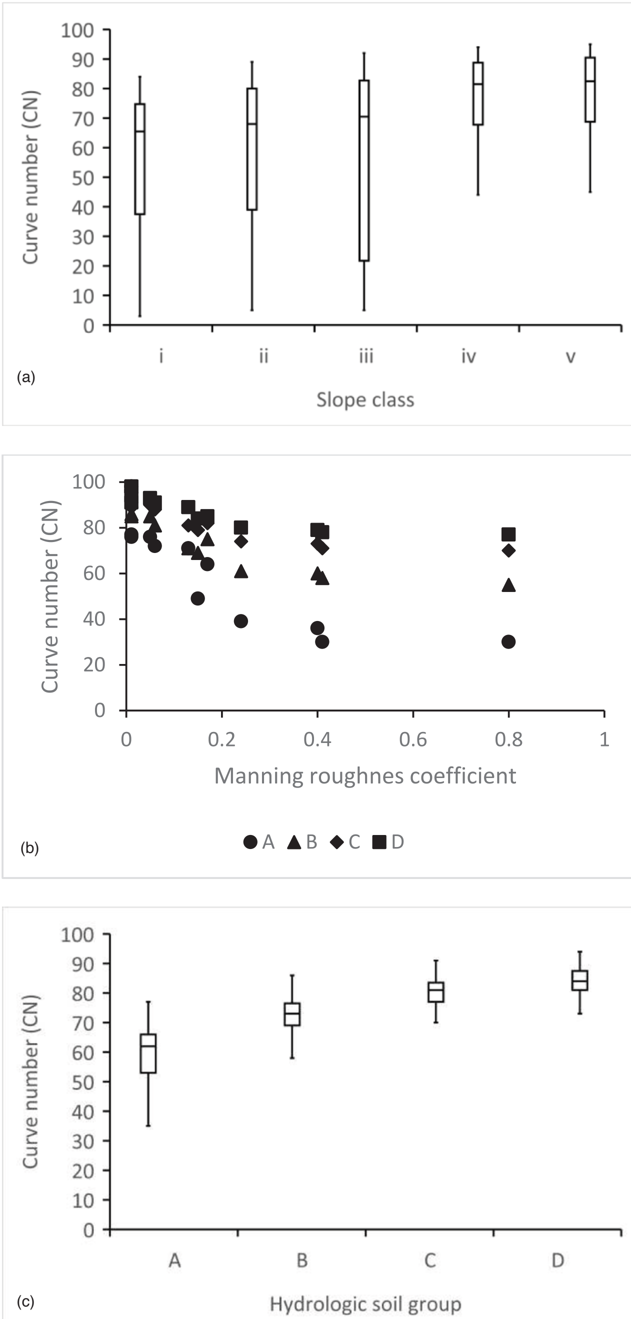

Fig. 4. a. Box and whisker plot of curve numbers as a function of slope class (i < 1%; ii 1–5%, iii 5–10%, iv 10–20%, v > 20%; based on Sprenger, 1978, quoted in Ritzema, 2006). Curve numbers tend to increase as a function of slope, suggesting a decrease in storage index S. b: Curve numbers as a function of Manning roughness coefficient for overland flow, based on descriptions and values in USDA-NRCS (2010) (Manning) and USDA-NRCS (2004a) (CN). Symbols indicate different hydrological classes (A,B,C,D). Curve numbers decrease as a function of roughness, indicating an increase in storage index S. c: Box and whisker plot of curve numbers as a function of hydrologic soil group (A,B,C,D) based on USDA-NRCS (2004a) taken as best approximation of saturated hydraulic conductivity. Curve numbers increase in the order A,B,C, D, i.e. the highest conductivity has the lowest curve number and the highest storage index.

show that threshold storage above which runoff occurs increases as a power function of plot length (plot lengths up to 16 m). For larger areas (up to 800 ha) the study by Simanton et al. (1996) showed a linear decrease in CN with area (increase of S), attributed to the decrease of area averaged rainfall with area, and losses in ephemeral channels. There is currently no tabulated effect of scale on CN or S. This cursory overview suggests that the equations for T_c and consequently T* do not contradict trends in the tabulated CN-values, but that effect of a specific variable (e.g. slope) may be small compared to the effects of other variables (hydrology class, and land use type).

2.7. The relation between storage index S, the average event intensity p, and the total event rainfall P

The effect of rainfall intensity p requires a separate analysis. The curve number equation with an additional lag β (eq. 26) can also be written as ((re)substituting $S = p/k$, using $I_a = \lambda S$, and setting $P = pT$)

$$Q = \beta^2 \frac{\left(pT - \lambda \frac{p}{k}\right)^2}{pT + \left(\frac{1}{\beta} - \lambda\right)\frac{p}{k}}\ if\ T \geq \frac{\lambda}{k},\ else\ Q = 0 \tag{35}$$

with the catchment response time expressed as 1/k. The equation can also be expressed in terms of T* (=1/k) as:

$$Q = \beta^2 p \frac{\left(T - \lambda T^*\right)^2}{T + \left(\frac{1}{\beta} - \lambda\right)T^*}\ if\ T \geq \lambda T^*,\ else\ Q = 0 \tag{36}$$

In this form the equation shows that cumulative direct runoff is a linear function of intensity for rainfall events of equal duration T, in the case β, T* and λ are constants. Rainfall intensity p is -however- not only a function of duration T, but also of the average recurrence interval T_r, and is described by an intensity duration frequency function (IDF; $p(T,T_R)$). A first analysis of the effect of including intensity in the curve number equation can be based on an empirical IDF equation modified from Dingman (2008, eq. 4-28b):

$$p = \frac{p_{s,r}}{\left(\frac{T}{T_s}\right)^B + 1} \tag{37}$$

where B and T_s are empirical coefficients. The parameter $p_{s,r}$ is the (extrapolated) intensity at duration T = 0 for a specific recurrence interval T_r. It is practical to write the curve number equation in terms of total event rainfall P, which can be done by calculating p(T,Tr), calculating Q(p,T), calculating P as pT and then plotting Q as a function of P. Alternatively, for large relative durations, the intensity can be approximated as:

$$p \approx \frac{p_{s,r}}{\left(\frac{T}{T_s}\right)^B} \tag{38}$$

In this case the intensity can also be written as a function of the event rainfall:

$$p \approx p_{s,r}\left(\frac{P}{P_{s,r}}\right)^{\frac{-B}{1-B}} \tag{39}$$

Substitution equation 39 in equation 36 results in a curve number equation which includes the dependency of event rainfall on intensity and duration, and (not yet explicitly formulated) the average recurrence interval:

$$Q = \beta^2 \frac{\left(P - \lambda \frac{p_{s,r}}{k}\left(\frac{P}{P_{s,r}}\right)^{\frac{-B}{1-B}}\right)^2}{P + \left(\frac{1}{\beta} - \lambda\right)\frac{p_{s,r}}{k}\left(\frac{P}{P_{s,r}}\right)^{\frac{-B}{1-B}}}\ if\ P \geq \lambda \frac{p_{s,r}}{k}\left(\frac{P}{P_{s,r}}\right)^{\frac{-B}{1-B}},\ else\ Q = 0 \tag{40}$$

Defining a recurrence interval dependent storage index:

$$S_{s,r} = \frac{p_{s,r}}{k} \tag{41}$$

Allows to (re-)write the equation as:

$$Q = \beta^2 \frac{\left(P - \lambda S_{s,r}\left(\frac{P}{P_{s,r}}\right)^{\frac{-B}{1-B}}\right)^2}{P + \left(\frac{1}{\beta} - \lambda\right) S_{s,r}\left(\frac{P}{P_{s,r}}\right)^{\frac{-B}{1-B}}} \; if \; P \geq \lambda S_{s,r}\left(\frac{P}{P_{s,r}}\right)^{\frac{-B}{1-B}}, \; else \; Q = 0 \tag{42}$$

This equation will be used to analyze the effects of the different parameters on the shape of the curve number equation.

The results so far depend on the assumption of an intensity which is constant during an event, and an initial loss (the initial abstraction). In addition a third parameter (β), allowing for an additional lag time in the catchment was introduced. Two additional parameters (B and $P_{s,r}$) were necessary to include the effect of the relation between intensity, duration and event rainfall. The re-interpretation of the curve number equation and the added parameter will affect the shape of the function Q (P). To analyze deviations from the standard curve number equation, ASCE (2009) calculate CN-values for each data pair and plot these as a function of rainfall P. The central equation in this approach is the following:

$$S = 5\left[P + 2Q - \sqrt{4Q^2 + 5PQ}\right] \tag{43}$$

where S is calculated for all data pairs P,Q. CN is calculated using

$$CN = \frac{100}{1 + \frac{S}{254}} \tag{44}$$

This equation is found solving the standard curve number for S and selecting the negative root. If the data pairs exactly match the standard curve number equation the result will be described by the equation

$$CN = \begin{cases} \dfrac{100}{1 + \dfrac{5P}{254}} & Q = 0 \\ CN & Q > 0 \end{cases} \tag{45}$$

This procedure defines a check for possible bias in terms of the parameter of interest (CN). Whereas in residual analysis one would expect residuals with zero mean and no discernable trend when plotted as a function of the independent variable, in this analysis – once runoff occurs (Q > 0) – one would expect the calculated CN values to be distributed with a constant mean (the fitted CN) with randomly distributed errors with zero mean, again when plotted against the independent variable P. To give an example we have plotted the result for a CN value of 70 for the standard curve number and that for a modified standard with a λ of 0.05, and for a λ of 0.35 as a function of rainfall P. The results (cf. Fig. 5) show a systematic change in calculated CN values,

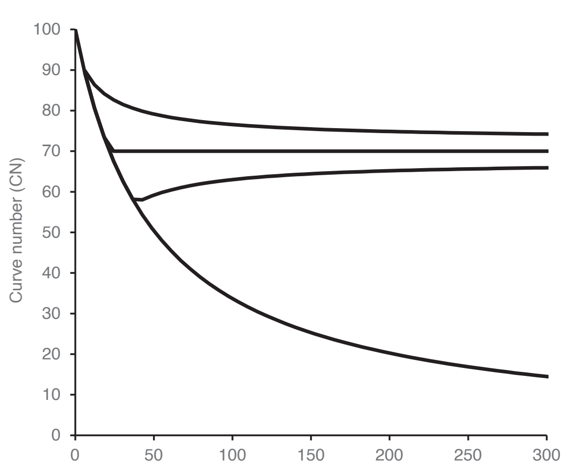

Fig. 5. Bias as reflected in the CN(P) plot for λ (bottom 0.1 middle 0.2, top 0.3). No bias, and constant curve number for λ = 0.2. Lower line: the point at which runoff is exactly 0, slightly larger rainfall leads to runoff for that curve number.

which in this case is explained by a value of λ other than 0.2. When analysing measured data using a CN(P)-plot, the result could then suggest changing the value of λ, to remove bias. The question is whether these CN(P) bias plots are sufficiently different for the different parameters (i.e. p, β, $P_{s,r}$, B, and λ) to pinpoint a likely cause.

2.8. Analysis of the additional parameters in terms of bias in the CN(P) plot.

The above describes a method to analyze differences between the standard curve number equation and measured data or the results from a modified version of the equation, such as equation 42.

The effect of the 4 parameters (β, λ, B, and $P_{s,r}$) is illustrated by the rainfall-runoff plot Q(P), and the bias plot CN(P), both for multiple values of the parameter, and a given CN value (CN = 50, which determines $S_{s,r}$). Whereas the parameters β and λ have a reference value for which the equation reverts to the standard curve number equation (β = 1, $0 < \beta \leq 1$ and λ = 0.2, $0 \leq \lambda \leq 1$), the reference value for B is 0 ($0 < B < 1$), in which case $P_{s,r}$ has no effect. The effect of $P_{s,r}$ is therefore analyzed for B = 0.5, and the effect of B is analyzed for $P_{s,r} = \lambda S_{s,r}$. The results are presented in Fig. 6a-h. When the CN(P) plots are calculated using real data, they could provide a first idea which parameter (or parameters) in the equation cause the mismatch. Analyzing rainfall runoff data in terms of CN(P) plots i.e. the bias of the standard curve number equation with respect to the data, ASCE (2009) distinguishes three different types of responses in catchments over the available rainfall range: complacent, standard, and violent. Complacent catchments are best described by a linear relation between direct runoff and rainfall; standard catchments show CN-values decreasing with rainfall in a way that suggests an asymptotic CN value, whereas violent catchments show CN-values asymptotically increasing with rainfall, in a way that suggests a constant value for large rainfall amounts. Examples of the three types are given by Hawkins (1993), and ASCE (2009), whereas Tedela et al. (2012) presents examples for standard behaviour. In that context the bias due to $P_{s,r}$ and B and to some extent λ < 0.2 could be classified as violent, whereas β < 1 and λ > 0.2 could be classified as either standard, or complacent.

2.9. Bias analysis including the effect of the average recurrence interval.

The previous analysis using eq. 42 did not include the effect of the average recurrence interval. Intensity-duration-frequency information is often published by national services. For the United States e.g. intensity duration frequency data have been or are being published and updated by NWS (2022). To illustrate the effect of including the recurrence interval on the calculated runoff the IDF- tabulated values for Coweeta (Coweeta experimental station, NC; NWS, 2022) were retrieved. Using the standard curve number equation and the full range of the IDF, direct runoff was calculated for a specific curve number. Minimizing a sum of squares criterion using the Solver add-in of Microsoft Excel, equation 36 was fitted to the data to estimate T* and λ; β was set to 1. The associated CN -value was calculated using equation 43–45 for each Q-P pair. The CN values were also calculated using independently sorted rainfall and direct runoff (so-called frequency matching or rank re-ordered data e.g. Tedela et al., 2012, or Hawkins, 1993). Examples of the Q(P) and CN(P) plots are presented in Fig. 7a-d for eq. 36 fitted to CN = 50, and to CN = 75; (λ = 0.2). Comparing the CN(P) plots for the rank ordered data show that the behaviour of direct runoff changes -the result for CN = 75 would be characterized as violent, whereas the result for CN = 50 would be characterized as standard. Interestingly the fitted values of λ are 0 for both cases.

2.10. Effect of a varying intensity within an event.

A simplifying assumption leading to the above results is that of a constant (effective) intensity during the rainfall event. To check the

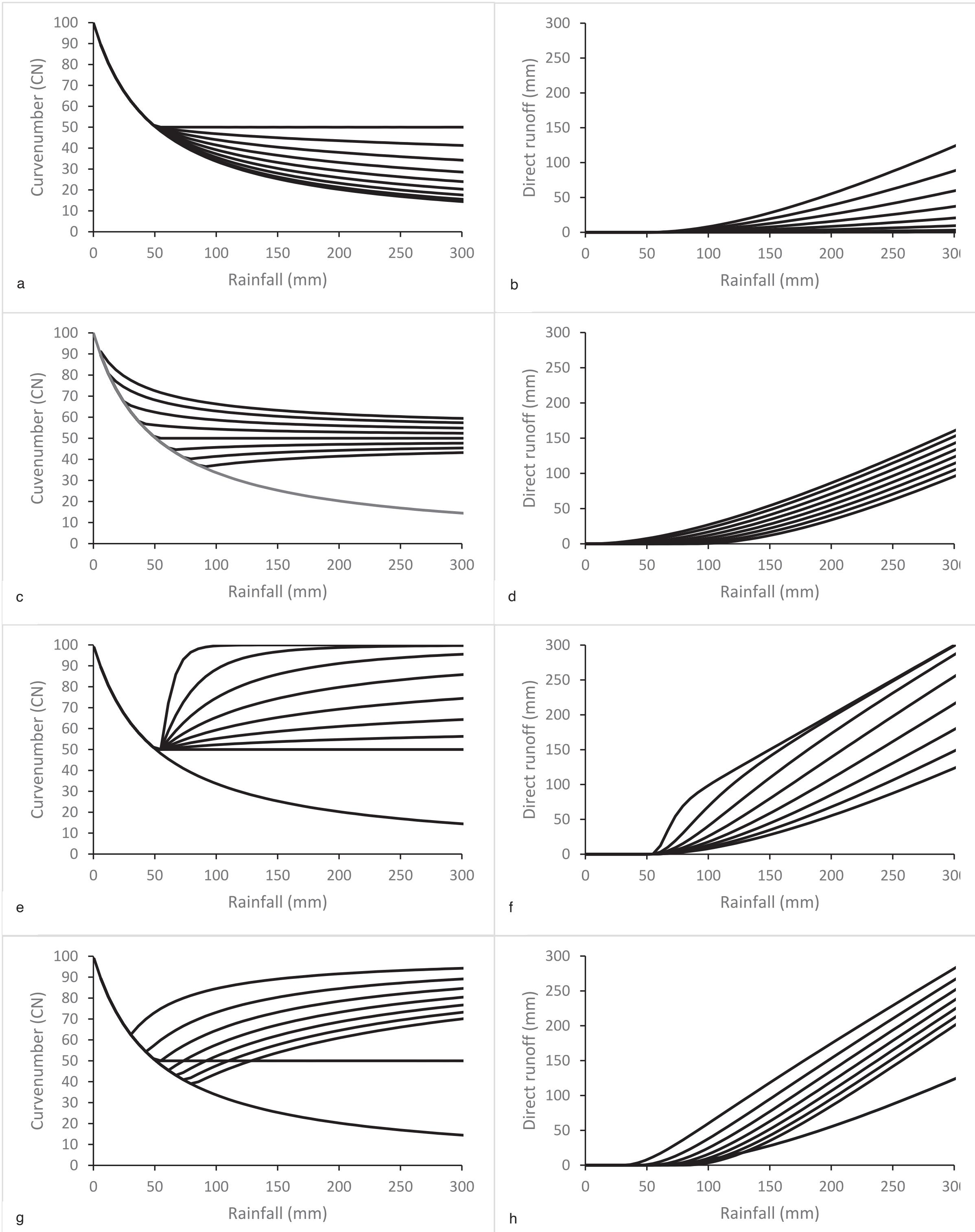

Fig. 6. a,c,e g (left column): Curve number rainfall curve CN(P) for different values of β (Fig. 6a),) λ (c), B (e), $P_{s,r}$ (g) Parameter β increases from the lower (β = 0.125) to the upper curve, and has its' standard value (β = 1) at the highest curve. Parameter λ decreases from the lower (λ = 0.35) to the upper curve (λ = 0.01) in steps of 0.05 and has the standard value (0.2). Parameter B increases from its standard value (B = 0.0) to the upper curve (B = 0.9) in steps of 0.13. Parameter $P_{s,r}$ increases from the lower ($P_{s,r}$ = 0.0) to the upper curve ($P_{s,r}$ = 0.9) in steps of 18. The reference value for $P_{s,r}$ is chosen as the value at which runoff is exactly zero. Fig. 6 b, d, f, h (right column): Rainfall-runoff curve Q(P) for different values of β (Fig. 6b), λ (d), B (f), $P_{s,r}$ (h). When one of these curves is observed this would suggest to modify the specific parameter in the rainfall-runoff equation. Parameters B and Ps,r would need to be modified jointly. For β the response would be characterized as complacent. Behavior for values of λ < 0.2 would be characterized as violent; for values of λ>0.2 behavior could be characterized as standard. Behavior for values of B and $P_{s,r}$ would always be characterized as violent.

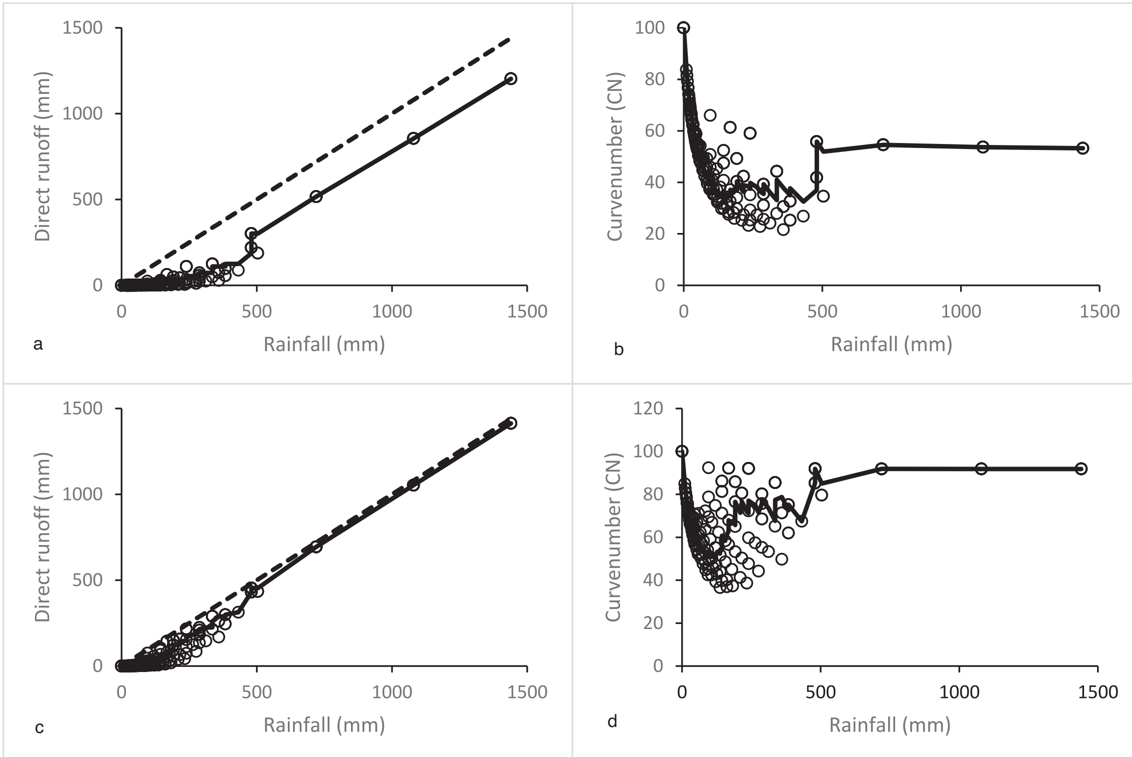

Fig. 7. a, c (left column): Rainfall-runoff plot Q(P) using tabulated IDF data of Coweeta (NC). Parameters λ and T* (eq. 34) chosen such that behavior best corresponds to a CN of 50 (7a) resp 75 (7c) Calculated datapairs (symbols) and the ordered (ranked, frequency matched) datapairs (solid line) are plotted. Fig. 7 b and d (right column): Curve number rainfall plot CN(P). Calculated datapairs (symbols) and the ordered datapairs (solid line) are plotted. At a CN value of roughly 50 (7b), response to rainfall would be characterized as standard, as CN-values would seem to decrease towards a constant value for large values of event rainfall. At a CN value of roughly 75 (7d), response to rainfall would be characterized as violent, as it would seem to increase towards a constant value for large values of event rainfall.

effect of this assumption we assumed a linearly increasing or decreasing intensity with a total rainfall equal to P. After evaluating the convolution integral, and integrating the hydrograph over the event duration, the bias in the function CN(P) was analysed.

The two intensity functions are

$$p_1 = \frac{2P}{T^2}t \tag{46a}$$

$$p_2 = \frac{2P}{T} - \frac{2P}{T^2}t \tag{46b}$$

The convolution integrals are

$$q_1 = \int_{t_a}^{t} \frac{2P}{T^2}\tau \frac{2pS^2}{(p(t-\tau)+S)^3} d\tau \tag{47a}$$

$$q_2 = \int_{t_a}^{t} \left(\frac{2P}{T} - \frac{2P}{T^2}\tau\right) \frac{2pS^2}{(p(t-\tau\ )+S)^3} d\tau \tag{47b}$$

where we have now formulated the instantaneous unit hydrograph in terms of the mean intensity. For the first case, after integrating twice, the cumulative discharge up to the peak discharge at time T is

$$Q_1 = P\left(1 - \frac{I_a^2}{P^2}\right) - 2S + \frac{2S^2}{P - I_a + S}\frac{I_a}{P} + \frac{2S^2}{P} ln\left[\frac{P + S - I_a}{S}\right] \tag{48a}$$

The second case is a combination of the curve number hydrograph with an intensity 2p (=2P/T) from which the cumulative discharge Q_1 is then subtracted:

$$Q_2 = 2P - 2I_a - 2S + \frac{2S^2}{S + P - I_a} - Q_1 \tag{48b}$$

Using the example of CN = 70, with rainfall between 0 and 350 mm in steps of 10 mm (36 pairs), the first case, an intensity increasing with time leads to slightly lower curve numbers (68 ± 2), whereas the second case (decreasing intensity) leads to slightly higher curve numbers (72 ± 2). Fig. 8 shows the CN(P) plot, and the Q(P) plot for both cases.

3. Discussion

We set out to investigate a possible link between an instantaneous unit hydrograph and the curve number equation. This is shown to be possible, and convolution of an instantaneous unit hydrograph with a constant intensity and a further integration over the event duration results in an analogue of the curve number equation, as the example of the linear reservoir shows. The assumption of a constant intensity was tested, and showed that the resulting equation is not very sensitive to linear variation of the within-event intensity. To retain simplicity a one parameter instantaneous unit hydrograph would be preferable, but not essential. Adding an additional parameter, a lag time L, resulted in a curve number equation which for the same storage index has much lower discharge. As shown in an analysis of the behaviour of the equation for different parameter values, this additional parameter allows for so-called complacent behaviour. Another possible two-parameter IUH model is a series of n linear reservoirs with time constant k, with a gamma probability density function as the IUH (the Nash model, Bras, 1990, p. 446). The convolution of the gamma probability function with a constant intensity would yield a cumulative gamma distribution describing the direct runoff rate for the rising limb. The definite integral

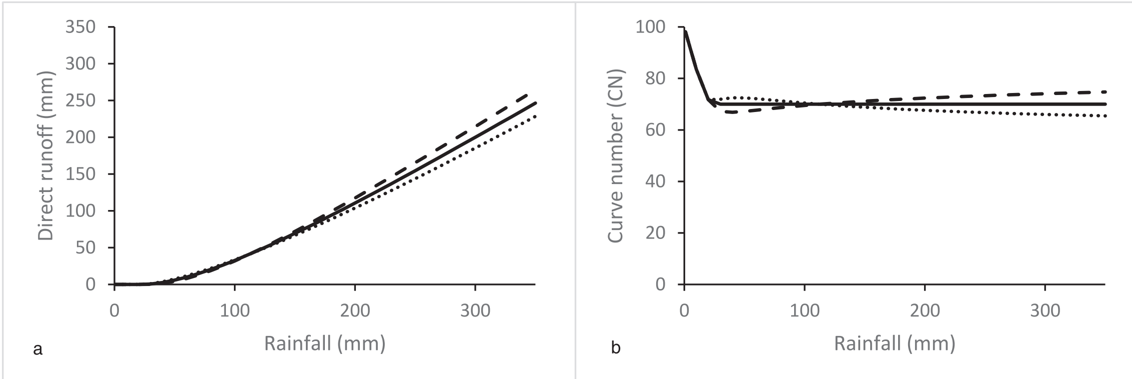

Fig. 8. Rainfall runoff curves Q(P) (8a) and curve number as a function of rainfall CN(P) (8b), for linearly increasing (dotted line), constant (solid line), and linearly decreasing intensity (dashed line) during a rainfall event. Average intensity is equal in the three cases. Both runoff and curve numbers are slightly different, which can be explained by the time required to exceed the initial abstraction, which is shorter in case of the decreasing intensity.

of the cumulative gamma distribution would then become the Nash analogue of the curve number equation. From the point of presenting concise and simple formulations this is not an attractive perspective. Starting from the relation between Q and P, a function with an oblique asymptote could allow to derive the q(p) and the IUH function (first derivative: rising limb of the hydrograph; second derivative: the instantaneous unit hydrograph for a constant intensity) which could serve to describe rainfall-runoff behaviour. A generalization of the curve number equation (as the ratio of two rational functions) where the numerator is exactly a power 1 larger than the denominator would meet this requirement. Another function with a linear asymptote is the expolinear equation presented as an alternative for the curve number equation by Paz-Pellat (2009), referring to Goudriaan and Van Laar (2012).

An interesting consequence of the presented equations is that both fitting a hydrograph to an event and fitting a curve number equation to a series of events is possible using the same parameters. Comparisons between the series based parameter values and the event based parameters, but also changes from event to event could provide insight in causes of parameter variability, such as the variation in vegetation cover. Seasonal variation in curve numbers was analysed by e.g. d'Asaro et al. (2018).

A strong point of the curve number equation (Ponce and Hawkins, 1996) is that it captures the effect of land use changes and interventions in a single parameter, the storage index S. It was possible to keep the equation in a one to two parameter form assuming the initial abstraction I_a to a constant (λ) times the storage index S. The presented theoretical basis suggests to rethink catchment land use changes and interventions in terms of the catchment response time T*, instead of in terms of the storage index S. Based on the equations for T* reviewed so far, S as the product pT* is not constant, but using T* theoretically links different site factors to S. The initial abstraction (now as λpT*) is retained. Other parameters added, those of the IDF-function, are catchment independent. The IDF-function used in the presented analysis is empirical, but allows to illustrate effect of including rainfall duration. The use of tabulated values for the IDF reflects another level of available information, and shows the effect of adding recurrence time as a variable. This analysis could be extended by using continuous IDF functions. An IDF function based on a statistical distribution, the GEV distribution, was proposed by Koutsoyiannis (2004), and used by e.g. Overeem et al. (2010). It has been parameterized at a global scale by Courty et al. (2019). Using these IDF functions would allow a more general analysis than the one presented here. In terms of including intensity or event duration in the curve number equation, Jain et al (2006) conclude that including a power function dependency of I_a on rainfall, and a power function correction of the rainfall for the event duration improves the performance compared to the standard curve number equation. The effect could have been similar to the inclusion of intensity in the theory presented here, but further mathematical analysis is needed to show to which extent the outcome of the equations corresponds. Also, an analysis of the equation performance based on available rainfall-runoff data has not been executed in this paper.

Two interesting observations can be made on the basis of the detailed IDF table retrieved for Coweeta: including the local IDF function in combination with the catchment response time (eq 34) gives rise to both standard and violent behaviour. In addition for both examples presented here λ is estimated to be 0. An extensive analysis estimating the value of λ quoted in ASCE (2009) supports a value of $\lambda = 0.05$, smaller than the standard value of 0.2. It would be interesting to see if a similar extended analysis using the rainfall intensity and duration would support a value for λ of 0, effectively eliminating the parameter.

For practical analyses, and in addition to the local IDF function, values for β, λ and T* would still be required. Whereas the literature offers empirical and theoretical support for the calculation of T*, the results neither offer theoretical support regarding the value of the initial abstraction, nor do they provide an approach for the time lag L, which may also be affected by land use change. Based on the instantaneous unit hydrograph the initial abstraction (I_a) arises from the assumption of a period (t_a) between the start of the rainfall event and the start of direct runoff in which there is no direct runoff. A question not addressed here is how available equations for the time of concentration and consequently T* -notably those based on the kinematic wave equation including infiltration rate, rainfall intensity, and moisture content - would affect runoff, and whether this also sheds light on the initial abstraction and the introduction of an additional lag (the parameter β) in catchments showing complacent behaviour. At this point this has to remain a speculative consequence of this analysis.

If the proposed theory is supported by further analysis, an application oriented question is whether tabulated values for the storage index S can be converted to the catchment response time T*. Given the suggested dependence of S on rainfall intensity, duration and recurrence interval, minimally the rainfall dataset used to estimate S would be required for any reanalysis. If this dataset is not or no longer available, estimating T* becomes speculative, and the procedure difficult to generalize, as there seem to be several dataset selection criteria on the basis of which S is determined. This is discussed by ASCE (2009, p 45 ff.) who distinguish datasets based on annual peak flows, and extended datasets based on multiple yearly events. As an example, Tedela et al (2012) uses both annual flood maxima, and partial rainfall series to determine curve numbers. From a different perspective this topic was analyzed by Stewart et al. (2012) who compared storage index values S estimated using different rainfall datasets and different methods to handbook values. Differences could not (or no longer) be explained – the shift from daily total rainfall measured once every day to using a tipping bucket may have been a watershed moment.

4. Conclusion

Given the objective it was shown that starting from an instantaneous unit hydrograph and assuming a constant intensity during the event allows to derive the curve number equation. The storage index S in the curve number equation is then shown to be a combination of rainfall intensity p, and the catchment response time 1/k. A cursory review does not contradict the possibility that factors quantifying the catchment response time (a.o. slope, soil type, a catchment length scale, and a Manning roughness coefficient for sheet flow) similarly affect values of S. The instantaneous unit hydrograph allows to derive a hydrograph which can be directly fitted to the direct component of a runoff event. Introducing an additional lag in the instantaneous unit hydrograph reduces direct runoff reduces direct runoff as a function of event rainfall. Furthermore, the use of a generalized curve number equation rewritten in terms of intensity and duration allows to describe catchment behaviour characterized as complacent, standard, and violent for which the standard curve number equation shows systematic bias. To check the effect of varying intensity within an event, alternative equations are derived and analyzed assuming a linear decrease or increase in the intensity during the event. The effect of this modification on the CN value is not large (2 units difference between the CN-values, at a CN of 70). However, a linear increase or decrease of intensity within an rainfall event could be too mild for specific climates, in which case more complex additional analyses could be required.

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Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Annex A

The combination of a linear channel and a linear reservoir is described by an instantaneous hydrograph modified from the linear reservoir as:

$$h(t) = ke^{-k(t+L)} \tag{A1}$$

where L (units of time) is the lag due to a linear channel. Compared to the linear reservoir (zero lag), the higher the lag, the less water is converted to direct runoff.

To calculate the direct runoff rate, the convolution integral with a constant intensity but an initial abstraction time t_a is written as

$$q = \int_{t_a}^{t} pke^{-k(t-\tau+L)} d\tau \tag{A2}$$

This integral results in

$$q = pe^{-kL} - pe^{-k(t-t_a+L)} \tag{A3}$$

Integrating the direct runoff rate over the duration of the rainfall and the start of the runoff yields the cumulative direct runoff;

$$Q = \int_{t_a}^{T} pe^{-kL} - pe^{-k(t-t_a+L)} dt \tag{A4}$$

Which results in

$$Q = \left[pe^{-kL}t + \frac{p}{k}e^{-k(t-t_a+L)} \right]_{t_a}^{T} \tag{A5}$$

Evaluating this equation:

$$Q = pe^{-kL}T + \frac{p}{k}e^{-k(T-t_a+L)} - pe^{-kL}t_a - \frac{p}{k}e^{-kL} \tag{A6}$$

which is the equation for the linear reservoir with an extra exponential term. After some rewriting, using $P = pT$, $I_a = pt_a$, and setting p/k to S' results in

$$Q = e^{-kL}P - e^{-kL}I_a - S' e^{-kL}\left(1 - e^{-\frac{(P-I_a)}{S'}}\right) \tag{A7}$$

For $L = 0$ this is just the equation derived for the linear reservoir. Increasing the lag decreases the total cumulative runoff. In conclusion: including a lag in the IUH is equivalent to multiplying the original cumulative direct runoff by a fraction, which depends on the catchment response time (1/k) and the additional lag L.

Assuming an extra lag in the reciprocal cubic instantaneous unit hydrograph (cf. eq. 16) would also reduce the conversion of rainfall to runoff. This can be achieved by increasing the time coordinate. So

$$h(t) = \frac{2k}{(k(t+L)+1)^3} \tag{A8}$$

which can also be written as:

$$h(t) = \frac{2k}{(kt+b)^3} \tag{A9}$$

where b is 1 + kL. The convolution integral to derive the direct runoff rate is

$$q = \int_{t_a}^{t} \frac{2pk}{(k(t-\tau)+b)^3} d\tau \tag{A10}$$

Which results in the equation for the direct runoff rate:

$$q = \frac{p}{b^2} - \frac{p}{(b+k(t-t_a))^2} \tag{A11}$$

The next step is the integration of the discharge over the duration of the event to derive the cumulative event runoff:

$$Q = \int_{t_a}^{T} \frac{p}{b^2} - \frac{p}{(b+k(t-t_a))^2} dt \tag{A12}$$

Which is

$$Q = \left(\frac{p}{b^2} t + \frac{p}{k} \left(\frac{1}{b+k(t-t_a)} \right) \right)_{t_a}^{T} \tag{A13}$$

Evaluating the integration boundaries results in:

$$Q = \frac{p}{b^2} T - \frac{p}{b^2} t_a + \frac{p}{k} \left(\frac{1}{b+k(T-t_a)} \right) - \frac{p}{bk} \tag{A14}$$

Substituting P = pT, $I_a = pt_a$, and S = p/k:

$$Q = \frac{P}{b^2} - \frac{I_a}{b^2} + S\left(\frac{S}{bS+P-I_a} \right) - \frac{S}{b} \tag{A15}$$

Which can be rewritten as:

$$Q = \frac{1}{b^2} \frac{(P-I_a)^2}{bS+P-I_a} \tag{A16}$$

and in a more regular pattern:

$$Q = \frac{1}{b} \frac{\left(\frac{P-I_a}{b}\right)^2}{S+\frac{P-I_a}{b}} \tag{A17}$$

Introducing $\beta = \frac{1}{b} = \frac{1}{1+kL}$ yields

$$Q = \beta \frac{(\beta(P-I_a))^2}{S+\beta(P-I_a)} \tag{A18}$$

The result differs from that of the linear reservoir in that the effective rainfall ($P\text{-}I_a$) is also reduced by the factor β. Note: b should be larger than one in order to have direct runoff smaller than $P\text{-}I_a$. We see b = 1 + kL, i.e. lag should be positive.

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