European Review of Agricultural Economics Vol **50 (2)** (2023) pp. 771–809
doi:https://doi.org/10.1093/erae/jbac029
Advance Access Publication 28 December 2022

Robust nonparametric analysis of dynamic profits, prices and productivity: An application to French meat-processing firms

Frederic Ang[†,**,*] **and Pieter Jan Kerstens**[‡,§]

[†]*Business Economics Group, Wageningen University, P.O. Box 8130, 6700 EW Wageningen, The Netherlands;* [‡]*Centre for Economic Studies, KU Leuven, E. Sabbelaan 53, 8500 Kortrijk, Belgium;* [§]*Flemish Institute for Technological Research (VITO), Boeretang 200, 2400 Mol, Belgium*

Received April 2022; final version accepted November 2022

Abstract

Appropriately considering adjustment costs, this paper develops a robust nonparametric framework to analyse profits, prices and productivity in a dynamic context. Dynamic profit change is decomposed into a dynamic Bennet price indicator and a dynamic Bennet quantity indicator. The latter is decomposed into explanatory factors. It is shown to be a superlative indicator for the dynamic Luenberger indicator. The application focuses on 1,638 observations of French meat-processing firms for the years 2012–2019. Using *m*-out-of-*n* bootstrapped data envelopment analysis, we obtain robust estimates and confidence intervals. The components of dynamic productivity growth fluctuate substantially. However, these fluctuations are often statistically insignificant.

Keywords: profit, productivity, adjustment cost, data envelopment analysis, *m*-out-of-*n* bootstrap

JEL classification: D24, D25, Q13

1. Introduction

Maximising the wealth of shareholders is the primary long-term economic objective of businesses. Profits are frequently used by managers for benchmarking competitiveness, as they add to wealth. Changes in prices and

*Corresponding author: E-mail: frederic.ang@wur.nl

**Both authors have contributed equally to this paper.

productivity drive changes in profit. Selling at a higher price and buying at a lower price increases profits, *ceteris paribus*. Productivity gains indicate producing more output using less input, which increases profits, *ceteris paribus*. The fundamental limitation of the usual approach to assess changes in profits, prices and productivity is its *static* assumption that the level of all inputs and outputs can be changed instantaneously to the optimum. Investment in capital assets is necessary for business survival in the long run, for it increases future profits and productivity through expansion of production possibilities. Yet, in the short run, it decreases profits and productivity because of adjustment costs associated with sluggishly changing the level of the capital stock. Forward-looking companies push the competitive envelope through investment, at the temporary expense of profits and productivity. The static approach ignores adjustment costs and intertemporal linkages, which leads to a misalignment with the long-run objective of wealth maximisation. Addressing this problem, the current paper develops a framework to analyse profits, prices and productivity in a *dynamic* context.

The analytical links between profits, prices and productivity are well-established in the static context. Bennet (1920) shows that any value change can be additively decomposed into the sum of price change and quantity change in the context of consumption. By applying this framework to the production context, one can decompose static profit change into static price change and static productivity (quantity) change (Balk, 1998; Diewert, 2005). Earlier seminal contributions using such an approach include Kurosawa (1975), Eldor and Sudit (1981) and Miller and Rao (1989). There is a rich literature on decomposing productivity change into explanatory factors (see e.g. Ang and Kerstens, 2017; Balk, Barbero and Zofío, 2020; Brümmer, Glauben and Thijssen, 2002; Färe *et al.*, 1994; Chambers, Färe and Grosskopf, 1996; Plastina and Lence, 2018; Nishimizu and Page, 1982). As one of the components of profit change, productivity change has also been further decomposed in the context of profit change. Grifell-Tatjé and Lovell (1999, 2000) and Brea-Solís, Casadesus-Masanell and Grifell-Tatjé (2015) further decompose productivity change into technical change and various components of efficiency change using ratio-based distance functions and revenue or cost functions. Focusing on the Bennet quantity indicator, Balk (1998) does so by means of difference-based directional distance functions and profit functions. This decomposition holds if the price-normalised profit functions have a quadratic functional form with time-invariant second-order coefficients and allocative inefficiency is absent. Ang (2019) develops a general approach for decomposing all Bennet-type productivity indicators, which also builds on directional distance functions and profit functions, but does not require a quadratic functional form and allows for allocative inefficiency.

The dynamic theory of the firm appropriately considers adjustment costs.[1] The importance of adjustment costs in intertemporal decision-making has already been recognised for decades (see e.g. Treadway, 1969, 1970; Lucas Jr, 1967; Rothschild, 1971). The dynamic approach relies on characterising dynamic distance functions and dynamic economic optimising behaviour. The dynamic cost function is dual to the dynamic hyperbolic input distance function (Silva and Stefanou, 2007), dynamic radial input distance function (Ouellette and Yan, 2008; Rungsuriyawiboon and Stefanou, 2007) and dynamic input directional distance function (Serra, Oude Lansink and Stefanou, 2011; Silva, Oude Lansink and Stefanou, 2015). Adapting Chambers, Chung and Färe (1998)'s static framework to a dynamic context, Ang and Oude Lansink (2018) and Silva, Stefanou and Oude Lansink (2020) establish a dual relationship between the dynamic directional distance function and the dynamic profit function. Despite these developments, the analytical links between profits, prices and productivity have not been established from the dynamic perspective. In the current paper, we address this research gap.

The contributions of this paper are threefold. First, we introduce a comprehensive adjustment-cost framework for analysing the change in 'dynamic profit' (annual flow version of intertemporal profit in current-value terms) as components of dynamic price change and dynamic productivity change. The dynamic Bennet price indicator appears as a novel measure of dynamic price change that measures the extent to which changes in market prices of inputs are recovered by changes in market prices of outputs and shadow prices of capital (that is, the increase in the optimal value function when increasing the capital stock by one unit). The dynamic Bennet quantity indicator occurs as a novel measure of dynamic productivity change that simultaneously gauges output growth, net investment growth and input decline. Adapting Ang (2019)'s static framework to the dynamic context, it is further decomposed into dynamic technical change (that is, the shift of the adjustment-cost technology), dynamic technical efficiency change (that is, the catch-up with the front-runners) and dynamic mix efficiency change (that is, the change in ability to correctly allocate the mix of inputs, outputs and investments). Overall, this adjustment-cost framework provides a powerful tool to analyse economic performance and guide the resource reallocation required for increasing profits and productivity in the long run.

Second, we show that the introduced dynamic Bennet quantity indicator (with appropriate price normalisation) is theoretically grounded, in that it is a superlative indicator for the dynamic Luenberger indicator proposed by Oude Lansink, Stefanou and Serra (2015), in Diewert (1976)'s sense. If the dynamic directional distance function can be represented up to the second order by a quadratic functional form with time-invariant second-order

1 This paper uses adjustment-cost theory for modelling dynamic production in line with Silva, Stefanou and Oude Lansink (2020). See Fallah-Fini, Triantis and Johnson (2014) for other approaches to dynamic production.

coefficients and there is dynamic profit-maximising behaviour, then an appropriately price-normalised dynamic Bennet quantity indicator is equivalent to the dynamic Luenberger indicator. This theoretical result generalises the equivalence between the static Bennet quantity indicator and static Luenberger indicator shown by Chambers (1996, 2002) and Balk (1998), to the dynamic context. To the best of our knowledge, this is the first theoretical equivalence demonstrated in the dynamic context.

Third, we illustrate a statistically robust operationalisation of our proposed adjustment-cost framework by an application to 1,638 observations of large French meat-processing firms for the years 2012–2019. Competitiveness is a key issue in the European meat-processing sector. The French setting in particular is characterised by high labour costs, strict labour regulation and high taxation. The domestic market, which accounts for ca. 75 per cent of total revenues, is subjected to a decreasing demand for meat (Atradius, 2018). These factors underline the importance of dynamic economic analysis, which makes the French meat-processing sector a suitable candidate for a case study. The decomposition uses nonparametric data envelopment analysis (DEA), by which estimation does not require imposing a functional form. We obtain statistically robust estimates and confidence intervals for all components by means of the *m*-out-of-*n* subsampling bootstrap, recently developed by Simar and Wilson (2020). These robust estimates are based on a probabilistic formulation of DEA and ensure consistency. They asymptotically converge to the true values and are less susceptible to outliers, thereby overcoming the usual deterministic disadvantage of DEA.

The three studies that are the closest to our proposed framework are Ang and Oude Lansink (2018), Silva, Stefanou and Oude Lansink (2020: p. 125–128, Section 4.3) and Ang (2019). Ang and Oude Lansink (2018) and Silva, Stefanou and Oude Lansink (2020) decompose dynamic profit inefficiency into components of dynamic technical inefficiency and dynamic mix inefficiency. However, they only focus on contemporaneous dynamic efficiency analysis, whereas our paper uses these components as building blocks for intertemporal dynamic productivity analysis. Ang (2019) develops a decomposition of the static Bennet quantity indicator into various components. Our paper goes beyond Ang's static framework by using a dynamic perspective that considers adjustment costs and conceptualising dynamic productivity change as a component of dynamic profit change.

2. Theoretical framework

2.1. Preliminaries

Following the exposition and notation of Ang and Oude Lansink (2018) and Silva, Stefanou and Oude Lansink (2020: p. 125–128, Section 4.3), we start with the preliminaries. Let $\mathbf{x}_t \in \mathbb{R}^u_+$ represent the vector of variable input quantities and $\mathbf{y}_t \in \mathbb{R}^v_+$ the vector of output quantities and capital stock vector

$\mathbf{K}_t \in \mathbb{R}_+^f$ with the corresponding vector of investment quantities $\mathbf{I}_t \in \mathbb{R}^f$ in time period *t*. The adjustment-cost technology set is defined by:

$$\mathcal{T}_t(\mathbf{K}_t) = \{(\mathbf{x}_t, \mathbf{I}_t, \mathbf{y}_t, \mathbf{K}_t) | (\mathbf{x}_t, \mathbf{I}_t) \text{ produces } \mathbf{y}_t \text{ given } \mathbf{K}_t\}. \quad (1)$$

We assume that $\mathcal{T}_t(\mathbf{K}_t)$ is closed and convex, inputs and outputs are freely disposable, investments are negatively monotonic, the capital stock is reverse nested and investment inaction is possible. Investment augments the capital stock, which increases future production possibilities. It also comes at the expense of production for a given level of input use and requires a higher input use to maintain production, which implies that investment is associated with contemporaneous adjustment costs. These axiomatic properties also imply that the adjustment costs convexly increase with investment. Appendix A provides a formal treatment of these axiomatic properties.

Let $\boldsymbol{\delta}$ be the vector of time-invariant depreciation rates for $\mathbf{K}_t$. Define the corresponding vector of net investments as $\mathbf{NI}_t = \mathbf{I}_t - \boldsymbol{\delta}\mathbf{K}_t$. An equivalent representation of $\mathcal{T}_t(\mathbf{K}_t)$ is the dynamic directional distance function:

$$D_t(\mathbf{x}_t, \mathbf{NI}_t, \mathbf{y}_t; \mathbf{g}) = \sup \left\{ \beta \in \mathbb{R} : (\mathbf{x}_t - \beta\mathbf{g}^x, \mathbf{NI}_t + \beta\mathbf{g}^I, \mathbf{y}_t + \beta\mathbf{g}^y) \in \mathcal{T}_t(\mathbf{K}_t) \right\}, \quad (2)$$

if $(\mathbf{x}_t - \beta\mathbf{g}^x, \mathbf{NI}_t + \beta\mathbf{g}^I, \mathbf{y}_t + \beta\mathbf{g}^y) \in \mathcal{T}_t(\mathbf{K}_t)$ for some β and $D_t(\mathbf{x}_t, \mathbf{NI}_t, \mathbf{y}_t; \mathbf{g}) = -\infty$ otherwise. Here, $\mathbf{g} = (\mathbf{g}^x, \mathbf{g}^I, \mathbf{g}^y) \in \mathbb{R}_+^{u+f+v}$ represents the directional vector. Equation (2) generalises Chambers, Chung and Färe (1998)'s formulation of the static directional distance function to the dynamic context. It is a measure of dynamic technical inefficiency that simultaneously contracts input use and expands output production and net investments along $\mathbf{g}$.

Our behavioural assumption is dynamic profit maximisation. We assume that a firm is maximising its current and discounted stream of future profits at any base period $t \in [0, +\infty[$. Let $\mathbf{p}_t \in \mathbb{R}_{++}^v$ represent the vector of output prices, $\mathbf{K}_{t_0} \in \mathbb{R}_+^f$ the initial capital stock vector, $\mathbf{w}_t \in \mathbb{R}_{++}^u$ the vector of variable input prices, $\mathbf{c}_t \in \mathbb{R}_{++}^f$ the vector of capital input prices and $r \geq 0$ the discount rate. We assume that there is perfect competition in homogeneous factor markets and output markets, in which the firm is a price taker and updates the expectations as the base period changes. All firms are assumed to have identical and static expectations on $\boldsymbol{\delta}$ and *r*. We formulate the dynamic profit maximisation problem as:

$$\mathcal{J}_t(\mathbf{p}_t, \mathbf{K}_t, \mathbf{w}_t, \mathbf{c}_t) = \sup_{\mathbf{y}_t, \mathbf{x}_t, \mathbf{I}_t} \int_t^{+\infty} [\mathbf{p}_t\mathbf{y}_t - \mathbf{w}_t\mathbf{x}_t - \mathbf{c}_t\mathbf{K}_t] e^{-rt} dt \quad (3a)$$

s.t.

$$\dot{\mathbf{K}}_t = \mathbf{NI}_t \qquad \text{with } \mathbf{K}_t = \mathbf{K}_{t_0} \quad (3b)$$

$$D_t(\mathbf{x}_t, \mathbf{I}_t, \mathbf{y}_t, \mathbf{K}_t; \mathbf{g}) \geq 0 \qquad \text{with } t \in [0, +\infty[\quad (3c)$$

We can express the flow version of the dynamic profit function as the product between the discount rate and the maximised sum of all future discounted profits, $r\mathcal{J}_t(\mathbf{p}_t, \mathbf{K}_t, \mathbf{w}_t, \mathbf{c}_t)$. As the level of $\mathbf{K}_t$ is assumed to be fixed in time period *t*, it is convenient to define it as the difference between the flow version of the *restricted* dynamic profit function and the capital cost in current-value terms,

$$r\mathcal{J}_t(\mathbf{p}_t, \mathbf{K}_t, \mathbf{w}_t, \mathbf{c}_t) = r\bar{\mathcal{J}}_t(\mathbf{p}_t, \mathbf{K}_t, \mathbf{w}_t, \mathbf{c}_t) - \mathbf{c}_t\mathbf{K}_t, \tag{4}$$

where $r\bar{\mathcal{J}}_t(\mathbf{p}_t, \mathbf{K}_t, \mathbf{w}_t, \mathbf{c}_t)$ is the product between the discount rate and the maximised sum of all future discounted, *restricted* profits. Additionally, $r\bar{\mathcal{J}}_t(\mathbf{p}_t, \mathbf{K}_t, \mathbf{w}_t, \mathbf{c}_t)$ can be written without loss of generality as:

$$r\bar{\mathcal{J}}_t(\mathbf{p}_t, \mathbf{K}_t, \mathbf{w}_t, \mathbf{c}_t) = \sup_{\mathbf{y}_t, \mathbf{x}_t, \mathbf{I}_t} \{\mathbf{p}_t\mathbf{y}_t - \mathbf{w}_t\mathbf{x}_t + \nabla_K \mathcal{J}_t(\mathbf{p}_t, \mathbf{K}_t, \mathbf{w}_t, \mathbf{c}_t)\mathbf{NI}_t\} \tag{5a}$$

$$\text{s.t. } D_t(\mathbf{x}_t, \mathbf{NI}_t, \mathbf{y}_t; \mathbf{g}) \geq 0, \tag{5b}$$

whenever it exists. Here, $\nabla_K \mathcal{J}_t(\mathbf{p}_t, \mathbf{K}_t, \mathbf{w}_t, \mathbf{c}_t)$ represents the partial derivative of $\mathcal{J}_t(.)$ with respect to capital. Being the shadow price of capital, it indicates how much the optimal value function increases by an incremental increase in the capital stock.[2] The restricted dynamic profit function $r\bar{\mathcal{J}}_t(.)$ indicates the maximum restricted dynamic profit (that is, the flow version of maximum intertemporal, restricted profit in current-value terms). Our maximisation problem in equation (3a) is expressed in continuous time, which is the most general formulation. The current-value formulation in equation (5a) is expressed at time *t*.[3]

Equations (5a)–(5b) can be written as an unconstrained maximisation problem with Lagrange multiplier λ_t:

$$\begin{aligned} &r\bar{\mathcal{J}}_t(\mathbf{p}_t, \mathbf{K}_t, \mathbf{w}_t, \mathbf{c}_t) \\ &= \sup_{\mathbf{y}_t, \mathbf{x}_t, \mathbf{I}_t} \{\mathbf{p}_t\mathbf{y}_t - \mathbf{w}_t\mathbf{x}_t + \nabla_K \mathcal{J}_t(.)\mathbf{NI}_t + \lambda_t D_t(\mathbf{x}_t, \mathbf{NI}_t, \mathbf{y}_t; \mathbf{g})\} \end{aligned} \tag{6}$$

where $\lambda_t = \mathbf{p}_t\mathbf{g}^y + \mathbf{w}_t\mathbf{g}^x + \nabla_K \mathcal{J}_t(\cdot)\mathbf{g}^I$.

The first-order conditions (FOCs) are:

$$\mathbf{w}_t = \lambda_t \nabla_x D_t(\cdot), \tag{7a}$$

$$\nabla_K \mathcal{J}_t(\cdot) = -\lambda_t \nabla_I D_t(\cdot), \tag{7b}$$

2 The adjustment costs are reflected by $D_t(\mathbf{x}_t, \mathbf{NI}_t, \mathbf{y}_t; \mathbf{g})$ in the constraints. This approach differs from approaches in which adjustment costs are modelled as a function of investment in the objective function (see for example Oude Lansink and Stefanou, 1997).

3 Formulating the infinite-horizon maximisation problem is also possible in discrete time, which would likewise result in a current-value formulation expressed at time *t* (Nemoto and Goto, 1999; Nemoto and Goto, 2003).

$$\mathbf{p}_t = -\lambda_t \nabla_y D_t(\cdot). \tag{7c}$$

In what follows, we normalise (shadow) prices as $(\tilde{\mathbf{w}}_t, \nabla_K \tilde{\mathcal{J}}_t, \tilde{\mathbf{p}}_t) \equiv \left(\frac{\mathbf{w}_t}{\lambda_t}, \frac{\nabla_K \mathcal{J}_t}{\lambda_t}, \frac{\mathbf{p}_t}{\lambda_t}\right)$, stack quantities for period s as $\mathbf{Q}_s \equiv (\mathbf{x}_s, \mathbf{NI}_s, \mathbf{y}_s)$ and normalised (shadow) prices for period t as $\tilde{\mathbf{P}}_t \equiv (\tilde{\mathbf{w}}_t, \nabla_K \tilde{\mathcal{J}}_t, \tilde{\mathbf{p}}_t)$ and denote dynamic technical inefficiency by $D_t(\mathbf{Q}_t; \mathbf{g})$.

Let us now consider the *actual* (instead of maximum) price-normalised, restricted dynamic profit of the firm, in which quantities and (shadow) prices are measured in periods *s* and *t*, respectively:

$$\Pi(\mathbf{Q}_s, \tilde{\mathbf{P}}_t) = \tilde{\mathbf{p}}_t \mathbf{y}_s - \tilde{\mathbf{w}}_t \mathbf{x}_s + \nabla_K \tilde{\mathcal{J}}_t \mathbf{NI}_s. \tag{8}$$

Let $\Pi(\mathbf{Q}_s^*, \tilde{\mathbf{P}}_t) = \sup_{\mathbf{Q}} \left\{\Pi(\mathbf{Q}, \tilde{\mathbf{P}}_t) \text{s.t. } D_s(\mathbf{Q}; \mathbf{g}) \geq 0\right\}$ with $\mathbf{Q}_s^*$ the associated optimal quantities.

There is a dual relationship between $\Pi(\mathbf{Q}_s^*, \tilde{\mathbf{P}}_t)$ and $D_s(\mathbf{Q}_s; \mathbf{g})$. This allows analysis of non-negative dynamic profit inefficiency, $DPI_s(\mathbf{Q}_s, \tilde{\mathbf{P}}_t) \equiv \Pi(\mathbf{Q}_s^*, \tilde{\mathbf{P}}_t) - \Pi(\mathbf{Q}_s, \tilde{\mathbf{P}}_t)$, as the sum of non-negative dynamic technical inefficiency, $D_s(\mathbf{Q}_s; \mathbf{g})$, and non-negative dynamic mix inefficiency, $DMI_s(\mathbf{Q}_s, \tilde{\mathbf{P}}_t; \mathbf{g})$:[4]

$$DPI_s(\mathbf{Q}_s, \tilde{\mathbf{P}}_t) \equiv \Pi(\mathbf{Q}_s^*, \tilde{\mathbf{P}}_t) - \Pi(\mathbf{Q}_s, \tilde{\mathbf{P}}_t) = D_s(\mathbf{Q}_s; \mathbf{g}) + DMI_s(\mathbf{Q}_s, \tilde{\mathbf{P}}_t; \mathbf{g}). \tag{9}$$

Here, $DMI_s(\cdot)$ indicates the deviation from the dynamic profit-maximising point because of an incorrect mix of variable inputs, investments and outputs. According to the *Law of One Price* (LoOP), competitive markets are assumed to clear for the same set of equilibrium prices of homogeneous inputs and outputs (Isard, 1977), as expressed by equation (3a). In practice, the LoOP is usually violated, which results in $DMI_s(\cdot) > 0$. Explanatory factors, which are also relevant in our application, include market distortions (Lau and Yotopoulos, 1971), non-economic objectives (Owusu-Sekyere, Hansson and Telezhenko, 2022) and heterogeneous inputs and outputs (Kuosmanen, Cherchye and Sipiläinen, 2006).

2.2. Dynamic profit change, dynamic price change and dynamic productivity change

This paper introduces the following decomposition of (price-normalised, restricted) dynamic profit *change*, which compares dynamic profit at time

4 Dynamic *mix* inefficiency coincides with dynamic *allocative* inefficiency in the current decomposition framework (Ang, 2019; O'Donnell, 2012).

$t+1$, $\Pi(\mathbf{Q}_{t+1},\tilde{\mathbf{P}}_{t+1})$, to dynamic profit at time t, $\Pi(\mathbf{Q}_t,\tilde{\mathbf{P}}_t)$:

$$\begin{aligned}
&\Pi C(\mathbf{Q}_t,\mathbf{Q}_{t+1},\tilde{\mathbf{P}}_t,\tilde{\mathbf{P}}_{t+1})\\
&\equiv \Pi(\mathbf{Q}_{t+1},\tilde{\mathbf{P}}_{t+1}) - \Pi(\mathbf{Q}_t,\tilde{\mathbf{P}}_t)\\
&= \Bigg[\frac{1}{2}(\mathbf{y}_t+\mathbf{y}_{t+1})(\tilde{\mathbf{p}}_{t+1}-\tilde{\mathbf{p}}_t)\\
&\quad -\frac{1}{2}(\mathbf{x}_t+\mathbf{x}_{t+1})(\tilde{\mathbf{w}}_{t+1}-\tilde{\mathbf{w}}_t)+\frac{1}{2}(\mathbf{NI}_t+\mathbf{NI}_{t+1})(\nabla_K\tilde{\mathcal{J}}_{t+1}-\nabla_K\tilde{\mathcal{J}}_t)\Bigg]\\
&\quad + \Bigg[\frac{1}{2}(\tilde{\mathbf{p}}_t+\tilde{\mathbf{p}}_{t+1})(\mathbf{y}_{t+1}-\mathbf{y}_t)-\frac{1}{2}(\tilde{\mathbf{w}}_t+\tilde{\mathbf{w}}_{t+1})(\mathbf{x}_{t+1}-\mathbf{x}_t)\\
&\quad + \frac{1}{2}(\nabla_K\tilde{\mathcal{J}}_t+\nabla_K\tilde{\mathcal{J}}_{t+1})(\mathbf{NI}_{t+1}-\mathbf{NI}_t)\Bigg]\\
&\equiv [PYC-PXC+PIC]+[QYC-QXC+QIC]\\
&\equiv DBC(\tilde{\mathbf{P}}_t,\tilde{\mathbf{P}}_{t+1};\mathbf{Q}_t,\mathbf{Q}_{t+1})+DBC(\mathbf{Q}_t,\mathbf{Q}_{t+1};\tilde{\mathbf{P}}_t,\tilde{\mathbf{P}}_{t+1})\\
&\equiv DBPC+DBQC.
\end{aligned} \tag{10}$$

Extending Balk (1998)'s and Diewert (2005)'s decomposition of static profit change to the dynamic context, equation (10) makes explicit that a firm can increase its dynamic profit through dynamic Bennet price indicator *DBPC* and dynamic Bennet quantity indicator *DBQC*. Following the terminology of Balk (2018), *DBPC* can be interpreted as a dynamic adaptation of the static total price recovery indicator. It assesses the degree to which variable input price change *PXC* is recovered by output price change *PYC* and change in the shadow price of capital *PIC*. Additionally, *DBQC* is a dynamic total factor productivity indicator, consisting of output quantity change *QYC*, variable input quantity change *QXC* and net investment quantity change *QIC*.

Entailing the component *QIC*, *DBQC* generalises the static Bennet quantity indicator, *QYC* – *QXC*, to the dynamic context. This additional component takes into account changes in dynamic productivity by changes in the capital stock due to net investments. If the level of gross investment is just enough to replace the depreciated capital in both periods (that is, $\mathbf{NI}_l=\mathbf{I}_l-\boldsymbol{\delta}\mathbf{K}_l=0$ with $l=\{t,t+1\}$), then *QIC* equals zero. Thus, *QIC* measures contributions to long-term profitability attributed to changes in the capital stock.[5]

2.3. Decomposing dynamic productivity change

Adapting Ang (2019)'s static framework to the dynamic context, we exploit equations (2), (5a)–(5b) and (9) for a further decomposition of *DBQC* into components of dynamic technical change *DTC*, dynamic technical efficiency change *DTEC* and dynamic mix efficiency change *DMEC*:

$$DBQC\equiv DTC+DTEC+DMEC,\ \text{where} \tag{11a}$$

5 The dynamic Bennet indicator is intransitive. See Appendix B for a discussion.

$$
\begin{aligned}
DTC &\equiv \frac{1}{2}\{[\Pi(\mathbf{Q}^*_{t+1},\tilde{\mathbf{P}}_t)-\Pi(\mathbf{Q}^*_t,\tilde{\mathbf{P}}_t)]+[\Pi(\mathbf{Q}^*_{t+1},\tilde{\mathbf{P}}_{t+1})-\Pi(\mathbf{Q}^*_t,\tilde{\mathbf{P}}_{t+1})]\}\\
&\equiv \frac{1}{2}\{DTC_L+DTC_P\}, \qquad (11b)
\end{aligned}
$$

$$
DTEC \equiv D_t(\mathbf{Q}_t;\mathbf{g})-D_{t+1}(\mathbf{Q}_{t+1};\mathbf{g}), \qquad (11c)
$$

$$
\begin{aligned}
DMEC &\equiv \frac{1}{2}\{[DMI_t(\mathbf{Q}_t,\tilde{\mathbf{P}}_t;\mathbf{g})-DMI_{t+1}(\mathbf{Q}_{t+1},\tilde{\mathbf{P}}_t;\mathbf{g})]\\
&\quad+[DMI_t(\mathbf{Q}_t,\tilde{\mathbf{P}}_{t+1};\mathbf{g})-DMI_{t+1}(\mathbf{Q}_{t+1},\tilde{\mathbf{P}}_{t+1};\mathbf{g})]\}\\
&\equiv \frac{1}{2}\{DMEC_L+DMEC_P\}. \qquad (11d)
\end{aligned}
$$

We dually measure dynamic technical change with the dynamic profit-maximising point on the period-dependent frontier as the relevant reference point. Assessing the shift between t and $t+1$ for $\tilde{\mathbf{P}}_t$ yields the Laspeyres-type measure of dynamic technical change, $DTC_L \equiv \Pi(\mathbf{Q}^*_{t+1},\tilde{\mathbf{P}}_t)-\Pi(\mathbf{Q}^*_t,\tilde{\mathbf{P}}_t)$. Doing so for $\tilde{\mathbf{P}}_{t+1}$ yields the Paasche-type measure of dynamic technical change, $DTC_P \equiv \Pi(\mathbf{Q}^*_{t+1},\tilde{\mathbf{P}}_{t+1})-\Pi(\mathbf{Q}^*_t,\tilde{\mathbf{P}}_{t+1})$. Here, *DTC* is the arithmetic average of DTC_L and DTC_P. If $DTC>0$, there is technological progress.[6]

Additionally, *DTEC* assesses the extent to which the DMU catches up with the frontier along directional vector **g**. If $DTEC>0$, dynamic technical inefficiency decreases, which indicates a catch-up with the best-practice adjustment-cost technology.

Finally, *DMEC* is the change in ability to correctly allocate the mix of variable inputs, outputs and net investments, with regard to the dynamic profit-maximising point on the adjustment-cost frontiers of periods t and $t+1$. Analogously to *DTC*, we assess *DMEC* with respect to $\tilde{\mathbf{P}}_t$ and $\tilde{\mathbf{P}}_{t+1}$, which respectively yields the Laspeyres-type measure of dynamic mix efficiency change, $DMEC_L \equiv DMI_t(\mathbf{Q}_t,\tilde{\mathbf{P}}_t;\mathbf{g})-DMI_{t+1}(\mathbf{Q}_{t+1},\tilde{\mathbf{P}}_t;\mathbf{g})$, and the Paasche-type measure of dynamic mix efficiency change, $DMEC_P \equiv DMI_t(\mathbf{Q}_t,\tilde{\mathbf{P}}_{t+1};\mathbf{g})-DMI_{t+1}(\mathbf{Q}_{t+1},\tilde{\mathbf{P}}_{t+1};\mathbf{g})$. Here, *DMEC* is the arithmetic average of $DMEC_L$ and $DMEC_P$. If $DMEC>0$, the ability to correctly allocate the mix of variable inputs, outputs and net investments improves over time.

The above does not show a component of dynamic scale efficiency change. For completeness, Appendix C shows the decomposition that includes dynamic scale efficiency change.

6 Technological regress *can* be implausible in some settings, as vividly described by the often-used quote by Kumar and Russell (2002: p. 540): 'Does knowledge decay? Were 'blueprints' lost?' One may restrict *DTC* to non-negative values, which can be done straightforwardly in practice by including all observations of preceding periods in the reference technology of the period considered.

2.4. Linking the dynamic Bennet quantity indicator to the dynamic Luenberger indicator

We generalise the equivalence of the static Bennet quantity indicator and static Luenberger indicator shown by Chambers (1996, 2002) and Balk (1998), to the dynamic context. Recently, Oude Lansink, Stefanou and Serra (2015) developed the dynamic Luenberger indicator by generalising the static Luenberger indicator introduced by Chambers, Färe and Grosskopf (1996) to the dynamic context:[7]

$$L_{t,t+1}(\mathbf{Q}_t, \mathbf{Q}_{t+1}; \mathbf{g}) = \frac{1}{2}\left\{[D_t(\mathbf{Q}_t;\mathbf{g}) - D_t(\mathbf{Q}_{t+1};\mathbf{g})] + [D_{t+1}(\mathbf{Q}_t;\mathbf{g}) - D_{t+1}(\mathbf{Q}_{t+1};\mathbf{g})]\right\}. \quad (12)$$

Here, $L_{t,t+1}(.)$ is based on the estimation of four dynamic directional distance functions, as defined in equation (2). Like the dynamic Bennet quantity indicator, it rewards output growth and net investment growth and penalises variable input growth. A positive value indicates dynamic productivity growth.

Let us assume that the dynamic directional distance functions employed in the dynamic Luenberger indicator can be estimated by a quadratic functional form:

$$\begin{aligned} D_h(\mathbf{Q};\mathbf{g}) = {} & \\ & a_0^h + \sum_{i=1}^{u} a_{xi}^h x_i + \sum_{l=1}^{v} a_{yl}^h y_l + \sum_{j=1}^{f} a_{NIj}^h NI_j \\ & + \frac{1}{2}\sum_{i=1}^{u}\sum_{i'=1}^{u} a_{x_i x_{i'}}^h x_i x_{i'} + \frac{1}{2}\sum_{l=1}^{v}\sum_{l'=1}^{v} a_{y_l y_{l'}}^h y_l y_{l'} \\ & + \frac{1}{2}\sum_{j=1}^{f}\sum_{j'=1}^{f} a_{NI_j NI_{j'}}^h NI_j NI_{j'} + \frac{1}{2}\sum_{l=1}^{v}\sum_{j=1}^{f} a_{y_l NI_j}^h y_l NI_j \\ & + \frac{1}{2}\sum_{l=1}^{v}\sum_{i=1}^{u} a_{y_l x_i}^h y_l x_i + \frac{1}{2}\sum_{j=1}^{f}\sum_{i=1}^{u} a_{NI_j x_i}^h NI_j x_i, \end{aligned} \quad (13a)$$

with the restrictions:

$$a_{x_i x_{i'}}^h = a_{x_{i'} x_i}^h, a_{y_l y_{l'}}^h = a_{y_{l'} y_l}^h, a_{NI_j NI_{j'}}^h = a_{NI_{j'} NI_j}^h, \quad (13b)$$

$$\sum_{l=1}^{v} a_{y_l}^h g_l^y + \sum_{j=1}^{f} a_{NI_j}^h g_j^I - \sum_{i=1}^{u} a_{x_i}^h g_i^x = -1; \quad (13c)$$

7 Oude Lansink, Stefanou and Serra (2015) propose a dynamic input-oriented Luenberger indicator in which $\mathbf{g}^y = \mathbf{0}^y$. They also develop a dual dynamic Luenberger indicator based on value functions. Our proposed dynamic Bennet quantity indicator is an empirical indicator not depending on value functions.

$$-\sum_{i'=1}^{u} a^h_{x_i x_{i'}} g^x_{i'} + \sum_{l=1}^{v} a^h_{y_l x_i} g^y_l + \sum_{j=1}^{f} a^h_{NI_j x_i} g^I_j = 0; \qquad i = 1, \dots, u \tag{13d}$$

$$\sum_{l'=1}^{v} a^h_{y_l y_{l'}} g^y_{l'} + \sum_{j=1}^{f} a^h_{y_l NI_j} g^I_j - \sum_{i=1}^{u} a^h_{y_l x_i} g^x_i = 0; \qquad l = 1, \dots, v \tag{13e}$$

$$\sum_{j'=1}^{f} a^h_{NI_j NI_{j'}} g^I_{j'} + \sum_{l=1}^{v} a^h_{y_l NI_j} g^y_l - \sum_{i=1}^{u} a^h_{NI_j x_i} g^x_i = 0; \quad j = 1, \dots, f \tag{13f}$$

The price-normalised dynamic Bennet quantity indicator is a superlative indicator for the dynamic Luenberger indicator in Diewert (1976)'s sense:

Proposition 1. *If the firm is a dynamic profit maximiser and the dynamic directional distance function is quadratic with $a^t_{x_i x_{i'}} = a^{t+1}_{x_i x_{i'}}$ for all i and i', $a^t_{y_l y_{l'}} = a^{t+1}_{y_l y_{l'}}$ for all l and l', and $a^t_{NI_j NI_{j'}} = a^{t+1}_{NI_j NI_{j'}}$ for all j and j', then:*

$$L_{t,t+1}(\mathbf{Q}_t, \mathbf{Q}_{t+1}; \mathbf{g}) = DBC(\mathbf{Q}_t, \mathbf{Q}_{t+1}; \tilde{\mathbf{P}}_t, \tilde{\mathbf{P}}_{t+1}).$$

Proof. See Appendix D. □

3. Data

This paper retrieves data on French meat-processing firms for the period 2012–2019 from the Orbis (2022) database published by Bureau van Dijk. We consider one output, two variable inputs and one quasi-fixed input with the corresponding (gross) investment. The output is annual turnover. The variable inputs are labour and materials. The quasi-fixed input is the opening value of the fixed assets. All variables are expressed in €. We compute implicit quantities by dividing the monetary values by the respective price indices, taken from the public Eurostat (2022) database. The variable on gross investment in fixed assets is constructed using the perpetual inventory method (see for example Kapelko, Oude Lansink and Stefanou (2015)). It is calculated by subtracting the opening value of fixed assets of the current year from the corresponding value of the next year, added with the opening value of depreciation of the next year. The net investment is equal to the gross investment minus the depreciation. To obtain a homogeneous sample and ensure computational feasibility of the employed *m*-out-of-*n* bootstrap, we restrict our analysis to the firms classified by Orbis (2022) as large or very large. These firms match at least one of the following conditions: (i) operating revenue of at least € 10 million, (ii) total assets of at least € 10 million, (iii) at least 150 employees or (iv) listed on the stock exchange. Firms with a labour productivity below € 100 per employee are excluded, as are those with missing values or unrealistic values below € 1,000. We choose the overall averages of variable inputs, output and gross investment as the directional vector. The final data set is an unbalanced panel of 1,683 observations, for which we can compute 1,368 annual growth rates. Table 1 shows the descriptive statistics.

Table 1. Descriptive statistics for French meat-processing firms, 2012–2019

Variable	Mean	St. Dev.	Min	Max
Turnover (implicit quantity in €)	89,251,580	208,562,700	121,484	2,288,931,000
Labour (implicit quantity in €)	10,665,230	26,817,410	38,699	247,161,200
Materials (implicit quantity in €)	57,700,520	141,054,400	2,391	1,864,449,000
Depreciation (implicit quantity in €)	1,635,295	4,560,109	553	72,876,120
Gross investment (implicit quantity in €)	2,103,760	8,148,956	−15,546,580	191,642,400
Price index of turnover (dimensionless)	0.999	0.013	0.978	1.023
Price index of wage (dimensionless)	1.045	0.035	1.000	1.107
Price index of materials (dimensionless)	1.014	0.007	1.000	1.020
Price index of capital (dimensionless)	1.017	0.014	1.000	1.046

Having the most comprehensive coverage of worldwide firm-level financial data, the Orbis database is suitable for firm-level economic analysis, particularly in the productivity context (Gal, 2013). It has recently been employed to this end by, amongst others, Albrizio, Kozluk and Zipperer (2017), Bento and Restuccia (2017), Kapelko (2019) and Duval, Hong and Timmer (2020). However, the Orbis database does not provide detailed information on the quality and composition of inputs and outputs. We implicitly assume that a higher-quality, more expensive good can be represented by a higher implicit quantity (Cox and Wohlgenant, 1986). The fact that there is only one aggregate output and one aggregate measure for materials may be limiting in the context of our application. These aggregations mask the potential heterogeneity in types of products sold and animals purchased.

4. Empirical approach

4.1. Dynamic profit change, dynamic price change and dynamic productivity change

Equation (10) shows that dynamic profit change $\Pi C(\mathbf{Q}_t, \mathbf{Q}_{t+1}, \tilde{\mathbf{P}}_t, \tilde{\mathbf{P}}_{t+1})$ and its decomposition into dynamic price change *DBPC* and dynamic productivity change *DBQC* can be easily computed if quantities and prices are readily available. This is the case for $\mathbf{x}$, $\mathbf{NI}$, $\mathbf{y}$, $\mathbf{w}$ and $\mathbf{p}$.

Being endogenous, $\nabla_K \mathcal{J}_t(\cdot)$ necessitates estimation. Following for example Färe *et al.* (2005) and Ang and Kerstens (2020), we estimate $\nabla_K \mathcal{J}_t(\cdot)$ by fitting a dynamic directional distance function with a quadratic functional form in line with Aigner and Chu (1968)'s linear programming procedure

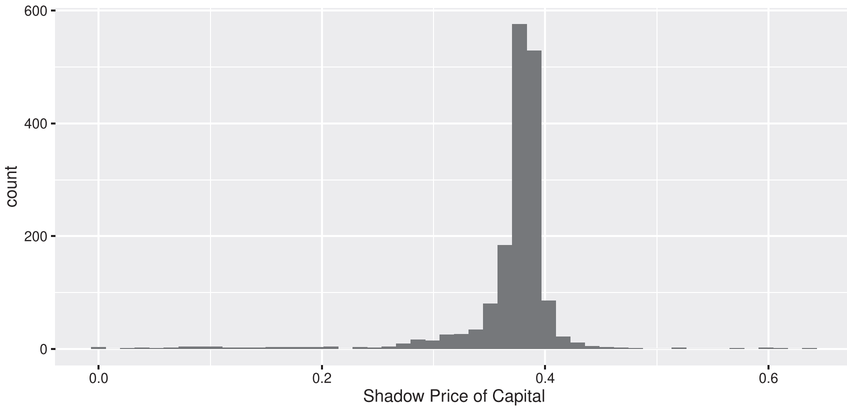

Fig. 1. Histogram with estimated shadow prices of capital.

and exploiting the dual relationship. The quadratic function is smooth and differentiable everywhere and yields linear FOCs retrieved in equations (7). It is consistent with the dynamic Bennet quantity indicator approximating the dynamic Luenberger indicator, as elucidated in sub-section 2.4. We refer to Appendix E for the linear programme yielding $\nabla_K \mathcal{J}_t(\cdot)$. Figure 1 shows a histogram with the estimated shadow prices, which range from 0 to 0.637. A higher $\nabla_K \mathcal{J}_t(\cdot)$ makes investment more valuable for the firm considered. The average is 0.381, which means that increasing the capital stock by € 1 is expected to increase the present value by on average € 0.381 over the infinite time horizon.

The FOCs are satisfied by construction. However, the convexity assumption relying on the second-order conditions can still be violated. The associated Hessian matrix is not negative semi-definite, as evidenced by the eigenvalues $[2.593\times10^4,\ 4.162\times10^{-8},\ -1.468\times10^2,\ -1.208\times10^3]$. This indicates that such a violation may be present for the current sample. As a result, the estimated shadow prices potentially do not yield dynamically profit-maximising allocations of variable inputs, outputs and net investments. As a robustness check, we run our models using alternative shadow prices that comply with the convexity assumption.

4.2. Decomposing dynamic productivity change

The decomposition of dynamic productivity change requires estimation of dynamic profit inefficiency, dynamic technical inefficiency and dynamic mix inefficiency. To this end, we use DEA and a dynamic adaptation of Varian (1984)'s *Weak Axiom of Profit Maximisation* (WAPM) as nonparametric estimators. Neither estimator imposes any functional form, but both are susceptible to outliers and disallow distinguishing inefficiency from noise.

While robust measures of static technical inefficiency are well-established (see Simar and Wilson (1998); Simar and Wilson (2011) and Simar, Vanhems and Wilson (2012)), Simar and Wilson (2020) only recently showed how to apply the *m*-out-of-*n* subsampling bootstrap to obtain robust measures of static profit inefficiency and static mix inefficiency. Henceforth, we obtain robust estimates and confidence intervals for dynamic profit inefficiency. Being computed in an analogous way, the details on dynamic technical inefficiency and dynamic mix inefficiency are relegated to Appendix F for conciseness.

Denote $J_m^b \subseteq \{1, \dots, n\}$ as an index set of size $m \leq n$ for bootstrap iteration $b = 1, \dots, B$. Simar and Wilson (2020) apply a subsampling bootstrap to Varian (1984)'s WAPM, to obtain robust estimates of static profit inefficiency. Adapting the WAPM to the dynamic context, $\forall (\mathbf{Q}_s, \tilde{\mathbf{P}}_t) \in S_{J_m^b} = \left\{(\mathbf{Q}_{is}, \tilde{\mathbf{P}}_{it})\right\}_{i \in J_m^b}$ for quantities at time *s* and prices at time *t*, we compute dynamic (restricted) profit inefficiency (9), where

$$\hat{\Pi}(\mathbf{Q}_s^*, \tilde{\mathbf{P}}_t) \equiv \max_{i \in J_m^b} \Pi(\mathbf{Q}_{is}, \tilde{\mathbf{P}}_t) \tag{14}$$

is the maximum (restricted) dynamic profit computed for subsample size $m \leq n$ and $\Pi(\mathbf{Q}_s, \tilde{\mathbf{P}}_t)$ is the observed dynamic profit (8) for quantities at time *s* and prices at time *t*. Let $\widehat{DPI}_s(\mathbf{Q}_s, \tilde{\mathbf{P}}_t)$ be the full sample estimate with $m = n$. Furthermore, let $\widehat{DPI}_s^b(\mathbf{Q}_s, \tilde{\mathbf{P}}_t)$ be a corresponding bootstrap estimate $b = 1, \dots, B$ for subsample size $m < n$ obtained by drawing *m* independent and uniform samples without replacement from the data. We can then construct $100(1-\alpha)\%$ confidence intervals

$$\left[\widehat{DPI}_s^L(\cdot), \widehat{DPI}_s^U(\cdot)\right] = \left[\widehat{DPI}_s(\cdot) - \frac{\psi_{1-\alpha/2,m}}{n}, \widehat{DPI}_s(\cdot) - \frac{\psi_{\alpha/2,m}}{n}\right], \tag{15}$$

where $\psi_{\alpha/2,m}$ ($\psi_{1-\alpha/2,m}$) is the $\alpha/2$ $(1-\frac{\alpha}{2})$ percentile of the set $\left\{m\left(\widehat{DPI}_s^b(\cdot) - \widehat{DPI}_s(\cdot)\right)\right\}_{b=1}^B$ (Simar, Vanhems and Wilson, 2012). Furthermore, bias-corrected estimates can be computed by (Simar, Vanhems and Wilson, 2012: Eq.(6.1)):

$$\widehat{DPI}_s^{BC}(\cdot) = \widehat{DPI}_s(\cdot) - \frac{m}{n}\frac{1}{B}\sum_{b=1}^{B}\left(\widehat{DPI}_s^b(\cdot) - \widehat{DPI}_s(\cdot)\right). \tag{16}$$

From rearranging equation (9) it follows that bias-corrected estimates of the maximum (restricted) dynamic profit can be computed by:

$$\hat{\Pi}^{BC}(\mathbf{Q}_s^*, \tilde{\mathbf{P}}_t) = \widehat{DPI}_s^{BC}(\mathbf{Q}_s, \tilde{\mathbf{P}}_t) + \Pi(\mathbf{Q}_s, \tilde{\mathbf{P}}_t). \tag{17}$$

Observe that the one-dimensional problem (14) is solved by simple enumeration, while $\hat{D}_t(\mathbf{x}_t, \mathbf{NI}_t, \mathbf{y}_t; \mathbf{g})$ in Appendix F is solved by a linear programme.

Next, dynamic mix inefficiency $\widehat{DMI}_s(\mathbf{Q}_s, \tilde{\mathbf{P}}_t; \mathbf{g})$ is computed as the residual from equation (9). Confidence intervals and bias-corrected estimates for dynamic mix inefficiency are constructed analogously as before.

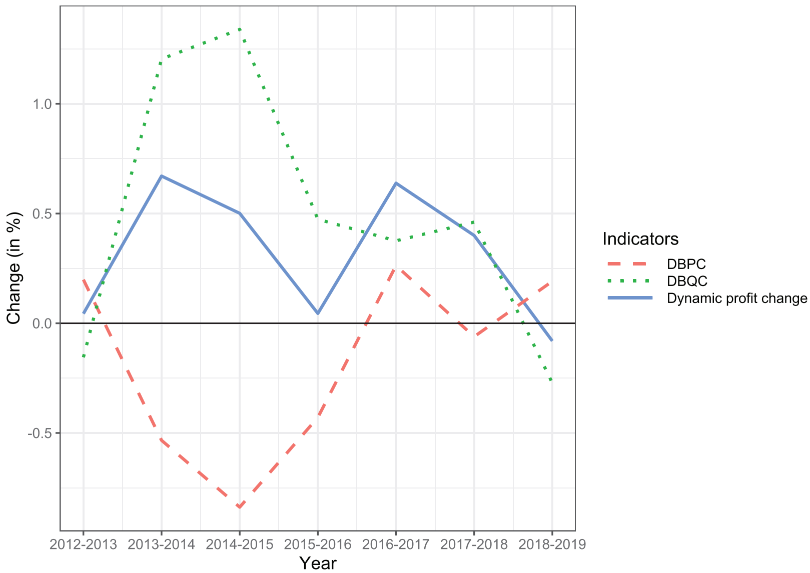

Fig. 2. Average dynamic profit change, dynamic productivity change and dynamic price change over time.

The dynamic profit inefficiency estimator is consistent and converges at rate n^1, while the dynamic technical inefficiency estimator converges at rate $n^{\frac{2}{u+v+f+1}}$. Being the difference between the dynamic profit inefficiency estimator and the dynamic technical inefficiency estimator, the dynamic mix inefficiency estimator inherits the slower rate $n^{\frac{2}{u+v+f+1}}$ of the dynamic technical inefficiency estimator (Simar and Wilson, 2020). Kneip, Simar and Wilson (2008: Theorem 3) show that the m-out-of-n subsampling bootstrap is consistent for any $m<n$. Note that such results on consistency and convergence rate are not yet known for scale inefficiency, which is why we restrict the analysis to assessing the statistical robustness of dynamic technical change, dynamic technical efficiency change and dynamic mix efficiency change. In the main results, we choose $m=0.7n$ for the computation of the bias-corrected estimates and the confidence intervals. A robustness check verifies the results for $m=0.5n$ and $m=0.9n$.

5. Results

5.1. Dynamic profit change, dynamic price change and dynamic productivity change

Following equation (10), we first analyse dynamic profit change as components of dynamic productivity change and dynamic price change. Figure 2 shows the annual average values from 2012–2013 to 2018–2019 in percentage terms. The dynamic profit increases on average by 0.32 per cent *per*

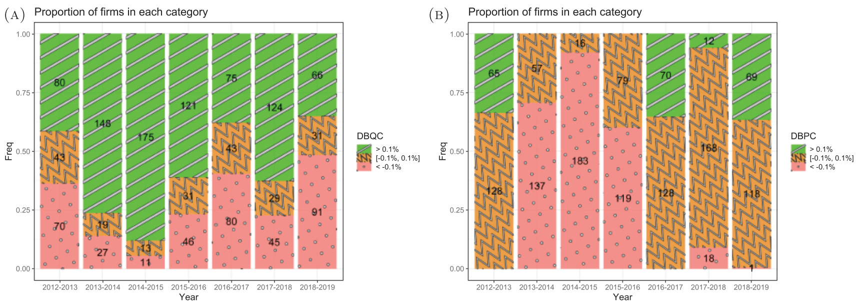

Fig. 3. Proportion of firms by which *DBQC* and *DBPC* between two subsequent years are greater than +0.1 per cent, between –0.1 per cent and +0.1 per cent and smaller than –0.1 per cent.

annum (*p.a.*). There is on average dynamic profit growth in all periods except for 2018–2019, in which there is an average decline of 0.08 per cent. The periods 2013–2014 and 2016–2017 stand out, with a dynamic profit growth averaging, respectively, 0.67 per cent and 0.64 per cent. This dynamic profit growth is mainly driven by dynamic productivity growth and partly offset by dynamic price decline. This is most visible in the periods 2013–2014 and 2014–2015, in which dynamic productivity growth of on average 1.20 per cent and 1.34 per cent is offset by dynamic price decline of on average 0.53 per cent and 0.84 per cent, respectively. The average dynamic price change and dynamic productivity change are, respectively, –0.18 per cent *p.a.* and +0.50 per cent *p.a.* in the studied period.

Figures 3a and b show the proportion of firms for which, respectively, *DBQC* and *DBPC* between two subsequent years are greater than +0.1 per cent, between –0.1 per cent and +0.1 per cent and smaller than –0.1 per cent. In 2013–2014, 2014–2015, 2015–2016 and 2017–2018, the majority of firms change dynamic productivity by more than +0.1 per cent. However, in 2013–2014, 2014–2015 and 2015–2016, this is partly offset by a change in dynamic price below –0.1 per cent in the majority of firms. In every period, only a minority of firms decrease dynamic productivity by more than 0.1 per cent *p.a.* In 2012–2013, 2016–2017, 2017–2018 and 2018–2019, most firms do not show a substantial change in dynamic price, by which *DBPC* ranges between –0.1 per cent and +0.1 per cent.

5.2. Decomposing dynamic productivity change

Table 2 shows summary statistics on *DBQC* and its components, *DTC*, *DTEC* and *DMEC*, following equation (11). Observe that *DBQC* is computed as an empirical indicator without bias correction, while *DTC*, *DTEC* and *DMEC* are bias-corrected estimates using the m-out-of-n bootstrap. Considering annual means, large fluctuations of *DTC* are offset by *DMEC*. The average *DTC* is –0.77 per cent *p.a.* in the studied period and ranges from –24.31 per cent in 2017–2018 to +17.52 per cent in 2014–2015. We observe the reverse trend for *DMEC*, which is averaging +24.12 per cent and –14.56 per cent in the

Table 2. Summary statistics of dynamic productivity change and its components (in %). Bias-corrected estimates are shown for *DTC*, *DTEC* and *DMEC*.

	Year	2012–2013	2013–2014	2014–2015	2015–2016	2016–2017	2017–2018	2018–2019
DBQC	Mean	−0.16	1.20	1.34	0.47	0.38	0.46	−0.27
	Std	2.19	3.18	3.10	3.07	9.39	3.85	2.36
	Min	−11.85	−2.77	−1.91	−13.75	−26.67	−34.40	−19.53
	Max	11.75	23.87	24.02	25.98	123.63	36.46	7.78
DTC	Mean	−9.83	6.76	17.25	10.28	−11.39	−24.32	6.01
	Std	0.53	0.51	0.30	0.39	0.40	0.80	0.18
	Min	−11.83	5.25	15.61	7.77	−13.02	−26.77	5.40
	Max	−6.46	10.11	18.08	11.26	−9.76	−19.03	6.47
DTEC	Mean	0.02	0.41	−1.24	1.90	−0.53	0.56	−3.06
	Std	7.76	4.71	8.76	17.88	47.23	62.51	22.61
	Min	−49.62	−21.30	−63.08	−55.51	−647.54	−608.07	−163.74
	Max	78.57	38.52	94.50	239.49	112.50	629.86	248.10
DMEC	Mean	9.64	−5.38	−14.56	−12.17	12.72	24.12	−3.69
	Std	8.02	5.04	8.86	18.63	56.95	65.37	25.36
	Min	−75.26	−34.89	−95.28	−254.26	−112.53	−646.66	−287.75
	Max	42.85	26.26	49.88	63.64	794.05	646.87	180.18

respective periods. The average *DMEC* is $+1.54$ per cent *p.a.* in the studied period. Furthermore, the average *DTEC* is −0.26 per cent *p.a.* and shows substantially less variation over time. It ranges from −3.06 per cent in 2018–2019 to $+1.90$ per cent in 2015–2016. The standard deviations reveal a high variability of *DTEC* and *DMEC* and a low variability of *DTC* in all periods. The performance of the dynamic profit-maximising peers on the frontier fluctuates substantially. Over time, the distance to the frontier does not change much among laggards in primal terms, while their catch-up to the dynamic profit-maximising peers varies considerably.

Figure 4 shows the proportion of firms for which components of dynamic productivity increase, stagnate or decrease between two subsequent years. Using the *m*-out-of-*n*-bootstrap, we compare the $[2.5 \text{ per cent}, 97.5 \text{ per cent}]$ confidence intervals of maximum dynamic profit, dynamic technical inefficiency and dynamic mix efficiency between two subsequent years. This allows making the assessment on, respectively, *DTC*, *DTEC* and *DMEC*. As shown in equations (11b) and (11d), *DTC* and *DMEC* can be analysed from Laspeyres and Paasche perspectives. We compare the confidence intervals of $\Pi(\mathbf{Q}_t^*, \tilde{\mathbf{P}}_t)$ and $\Pi(\mathbf{Q}_{t+1}^*, \tilde{\mathbf{P}}_t)$ ($\Pi(\mathbf{Q}_t^*, \tilde{\mathbf{P}}_{t+1})$ and $\Pi(\mathbf{Q}_{t+1}^*, \tilde{\mathbf{P}}_{t+1})$) to assess whether DTC_L (DTC_P) is below zero, not significantly different from zero or above zero, at the 5 per cent significance level. We compare the confidence intervals of $D_t(\mathbf{Q}_t; \mathbf{g})$ and $D_{t+1}(\mathbf{Q}_{t+1}; \mathbf{g})$ for assessing *DTEC*. Finally, we compare the confidence intervals of $DMI_t(\mathbf{Q}_t, \tilde{\mathbf{P}}_t; \mathbf{g})$ and $DMI_{t+1}(\mathbf{Q}_{t+1}, \tilde{\mathbf{P}}_t; \mathbf{g})$ ($DMI_t(\mathbf{Q}_t, \tilde{\mathbf{P}}_{t+1}; \mathbf{g})$ and $DMI_{t+1}(\mathbf{Q}_{t+1}, \tilde{\mathbf{P}}_{t+1}; \mathbf{g})$) for assessing $DMEC_L$ ($DMEC_P$).

Figure 4a shows that DTC_L does not differ from zero at the 5 per cent significance level for (almost) all firms in 2012–2013, 2013–2014, 2015–2016, 2016–2017 and 2018–2019. Furthermore, $DTC_L > 0$ for 57 out of 199 firms in 2014–2015 and $DTC_L < 0$ for 197 out of 198 firms in 2017–2018. Figure 4b shows that DTC_P does not differ from zero at the 5 per cent significance level for the large majority of firms in 2012–2013, 2013–2014, 2014–2015, 2015–2016, 2016–2017 and 2018–2019. Additionally, $DTC_P < 0$ for 197 out of 198 firms in 2017–2018.

Figure 4c shows that *DTEC* does not differ from zero at the 5 per cent significance level for the large majority of firms in all periods. Furthermore, $DTEC > 0$ for 21 out of 198 firms in 2015–2016, and $DTEC < 0$ for 57 out of 199 firms in 2014–2015 and for 67 out of 188 firms in 2018–2019. We thus observe that a positive DTC_L is offset by negative *DTEC* in 2014–2015.

Figures 4d–e show that $DMEC_L$ and $DMEC_P$ do not differ from zero at the 5 per cent significance level for (almost) all firms in all years but 2017–2018. In 2017–2018, $DMEC_L > 0$ for 131 out of 198 firms and $DMEC_P > 0$ for 125 out of 198 firms. Both compensate for the negative DTC_L and DTC_P for almost all firms in that period.

It is interesting to compare our findings with those of Kapelko (2019), who applies a dynamic Luenberger indicator to European food-processing firms for the period of 2005–2012, using non-bootstrapped DEA. Kapelko finds

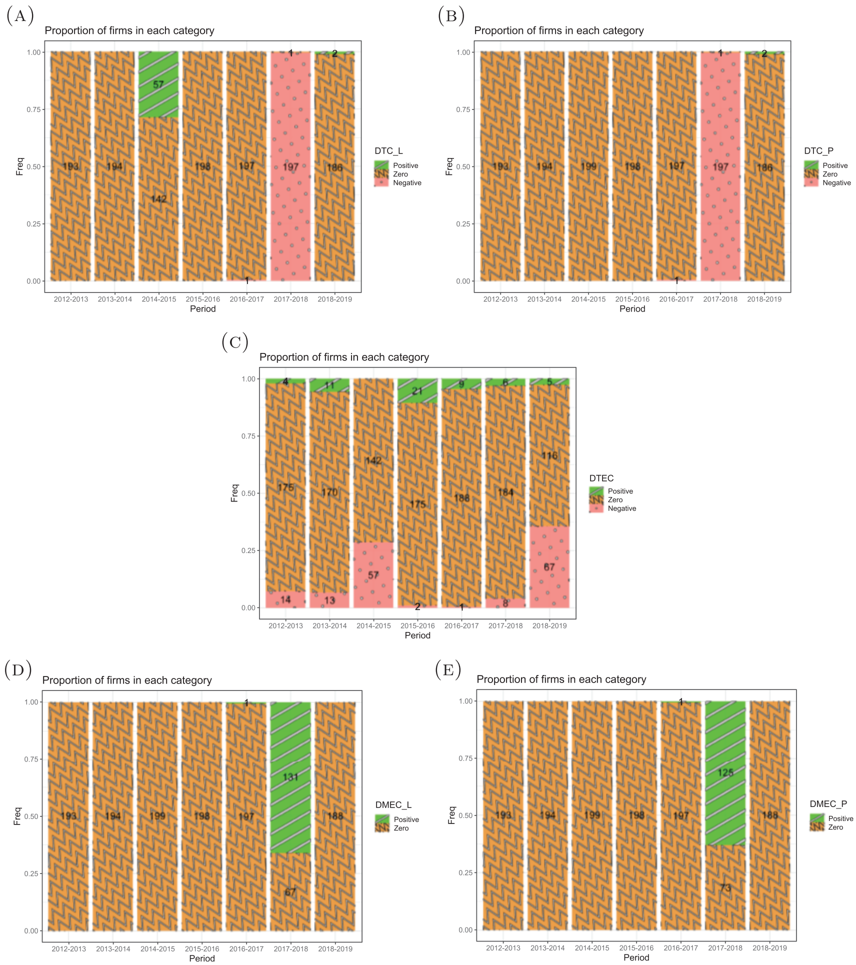

Fig. 4. Proportion of firms for which components of dynamic productivity change are positive, zero and negative at the 5 per cent significance level.

on average a slight dynamic productivity decline for Western European meat-processing firms during that period, with fluctuating underlying components. Our results show that the dynamic productivity of French meat-processing firms has on average modestly increased, with large yet often not significant fluctuating components for the period of 2012–2019. The potential variability of components of productivity growth when using DEA has also been pointed out by Atkinson, Cornwell and Honerkamp (2003). The *m*-out-of-*n* bootstrap employed here thus provides an important qualification of the statistical robustness of this variability over time: most fluctuations are statistically insignificant in the current sample.

5.3. Robustness checks

We conduct three robustness checks, which we relegate to Appendix G for conciseness. We investigate (i) an alternative specification of $\nabla_K \mathcal{J}_t(\cdot)$, (ii) alternative subsample size *m* and (iii) the subsample for which the divergence between *DMEC* and $DBQC - DTC - DTEC$ is small.

Quantitatively, these robustness checks largely yield similar results. We do note that a larger subsample size tends to result in bias-corrected estimates being smaller in absolute terms. This seems especially to be the case for extreme values. Overall, the values of these components are similar, as confirmed by the Pearson correlation, and lead to similar ranks, as confirmed by the Spearman rank correlation. The Li (1996) test shows that the overall distributions are similar for *DBQC*, *DTEC* and *DMEC*, but not for *DTC*, when using an alternative subsample size.

Qualitatively, all robustness checks lead to the same conclusions. The dynamic profit growth is on average low in the studied period. Dynamic productivity growth tends to be accompanied by dynamic price decline and *vice versa*. Overall, there is a modest dynamic productivity growth, which is driven by large fluctuations of its underlying components. However, these fluctuations are largely statistically insignificant.

6. Conclusions

Appropriately considering adjustment costs, this paper develops a statistically robust nonparametric framework for analysing dynamic profits, prices and productivity. We show that change in ‘dynamic profit’ (annual flow version of intertemporal profit in current-value terms) can be expressed as the sum of dynamic price change and dynamic productivity change. These novel components, respectively, represent a dynamic Bennet price indicator and a dynamic Bennet quantity indicator. Adapting Ang (2019)’s static decomposition framework to the dynamic context, the latter is further decomposed into components of dynamic technical change, dynamic technical efficiency change and dynamic mix efficiency change. The dynamic Bennet quantity indicator has theoretical underpinnings. We show that it is a superlative indicator for Oude Lansink, Stefanou and Serra (2015)’s dynamic Luenberger indicator in Diewert (1976)’s sense for appropriately normalised prices: if the dynamic directional distance function can be represented up to the second order by a quadratic functional form with time-invariant second-order coefficients and there is dynamic profit-maximising behaviour, then an appropriately price-normalised dynamic Bennet quantity indicator is equivalent to the dynamic Luenberger indicator.

The potency of our proposed framework is illustrated by an empirical application to 1,638 observations of French meat-processing firms for the years 2012–2019 using DEA. Applying the *m*-out-of-*n* bootstrap recently developed by Simar and Wilson (2020) permits statistical inference in our nonparametric setting.

The results show that dynamic profit has increased on average by 0.32 per cent *p.a.* in the studied period, which is low. It is driven by a modest dynamic productivity growth of on average 0.50 per cent *p.a.*, being partially offset by dynamic price decline of on average 0.18 per cent *p.a.* The decomposition of dynamic productivity growth reveals that the underlying components fluctuate substantially. However, the *m*-out-of-*n* bootstrap casts doubt on the statistical robustness of these fluctuations. Three robustness checks confirm these findings. The results highlight the importance of employing the *m*-out-of-*n* bootstrap beyond the usual deterministic DEA. They also corroborate the broader narrative of the productivity slowdown in Europe in recent decades (Andrews, Criscuolo and Gal, 2016). The statistical insignificance may partly be driven by the aggregate nature of inputs and outputs in the Orbis database. An ideal application of our approach would involve data that include a broader range of inputs, such as animal inputs in the meat-processing case. We offer our advance as a means of guiding future research on dynamic profits, prices and productivity.

We have three recommendations for future research. First, the proposed adjustment-cost framework could also be estimated using stochastic frontier analysis. This requires a suitable specification of a flexible functional form and imposition of additional translation property restrictions, which is in practice challenging (Stefanou, 2020), yet feasible, as shown by Oude Lansink, Stefanou and Serra (2015).

Second, we recommend to further investigate theoretical relationships between dynamic productivity measures. Such relationships have been demonstrated for a vast array of static productivity measures. The extension to the dynamic setting is especially relevant if economic optimising behaviour needs to be modelled as a long-term rather than a short-term endeavour.

Third, our adjustment-cost framework could be adapted to the environmental context. Despite substantial progress in the understanding of modelling environmental bads in a production framework (see for example Coelli, Lauwers and Van Huylenbroeck, 2007; Hoang and Coelli, 2011; Murty, Russell and Levkoff, 2012), the focus has predominantly been on *flows* rather than *stocks*. The field of natural resource economics teaches us that natural resources such as forests and soils should be treated as a capital stock with dynamic properties. As suggested by Ang and Dakpo (2021), modelling this salient feature in a production framework would yield novel insights.

Acknowledgements

The authors thank the Editor Timothy Richards and two anonymous referees for suggestions and feedback, all of which significantly improved this paper. We are grateful to seminar participants at TU Darmstadt, Wageningen University, Paris-Saclay Applied Economics and the Swedish University of Agricultural Sciences for helpful comments on an earlier draft. We thank Benjamin Hampf, Kamel Louhichi, Alfons Oude Lansink and Spiro Stefanou for constructive suggestions. The usual disclaimer applies.

References

Aigner, D. J. and Chu, S. -F. (1968). On estimating the industry production function. *American Economic Review* 58: 826–839.

Albrizio, S., Kozluk, T. and Zipperer, V. (2017). Environmental policies and productivity growth: Evidence across industries and firms. *Journal of Environmental Economics and Management* 81: 209–226. 10.1016/j.jeem.2016.06.002.

Andrews, D., Criscuolo, C. and Gal, P. N. (2016). The best versus the rest: the global productivity slowdown, divergence across firms and the role of public policy, Tech. rep.

Ang, F. (2019). Analyzing components of productivity growth using the Bennet-Lowe indicator: An application to Welsh sheep farms. *American Journal of Agricultural Economics* 101: 1262–1276. 10.1093/ajae/aay111.

Ang, F. and Dakpo, K. H. (2021). Comment: Performance measurement and joint production of intended and unintended outputs. *Journal of Productivity Analysis* 55: 185–188. 10.1007/s11123-021-00606-z.

Ang, F. and Kerstens, P. J. (2017). Decomposing the Luenberger-Hicks-Moorsteen Total Factor Productivity indicator: An application to US agriculture. *European Journal of Operational Research* 260: 359–375. 10.1016/j.ejor.2016.12.015.

Ang, F. and Kerstens, P. J. (2020). A superlative indicator for the Luenberger-Hicks-Moorsteen productivity indicator: Theory and application. *European Journal of Operational Research* 285: 1161–1173. 10.1016/j.ejor.2020.02.030.

Ang, F. and Oude Lansink, A. (2018). Decomposing dynamic profit inefficiency of Belgian dairy farms. *European Review of Agricultural Economics* 45: 81–99. 10.1093/erae/jbx024.

Atkinson, S. E., Cornwell, C. and Honerkamp, O. (2003). Measuring and decomposing productivity change: Stochastic distance function estimation versus data envelopment analysis. *Journal of Business and Economic Statistics* 21: 284–294. 10.1198/073500103288618963.

Atradius (2018). Market monitor: Focus on food performance and outlook, Tech. rep.

Balk, B. M. (1998). *Industrial Price, Quantity, and Productivity Indices: the Micro-Economic Theory and an Application*. Boston: Kluwer Academic Publishers.

Balk, B. M. (2018). Empirical productivity indices and indicators. In: E. Grifell-Tatjé, C. A. K. Lovell and R. C. Sickles (eds.), chap. 2, *The Oxford Handbook of Productivity Analysis*, Oxford: Oxford University Press, 77–120.

Balk, B. M., Barbero, J. and Zofío, J. L. (2020). A toolbox for calculating and decomposing Total Factor Productivity indices. *Computers and Operations Research* 115: 104853. 10.1016/j.cor.2019.104853.

Banker, R. D., Charnes, A. and Cooper, W. W. (1984). Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management Science* 30: 1078–1092. 10.1287/mnsc.30.9.1078.

Bennet, T. L. (1920). The theory of measurement of changes in cost of living. *Journal of the Royal Statistical Society* 83: 455–462. 10.2307/2340960.

Bento, P. and Restuccia, D. (2017). Misallocation, establishment size, and productivity. *American Economic Journal: Macroeconomics* 9: 267–303 10.1257/mac.20150281.

Brea-Solís, H., Casadesus-Masanell, R. and Grifell-Tatjé, E. (2015). Business model evaluation: Quantifying Walmart's sources of advantage. *Strategic Entrepreneurship Journal* 9: 12–33. 10.1002/sej.1190.

Brümmer, B., Glauben, T. and Thijssen, G. (2002). Decomposition of productivity growth using distance functions: The case of dairy farms in three European countries. *American Journal of Agricultural Economics* 84: 628–644. 10.1111/1467-8276.00324.

Caves, D. W., Christensen, L. R. and Diewert, W. E. (1982). Multilateral comparisons of output, input, and productivity using superlative index numbers. *The Economic Journal* 92: 73–86. 10.2307/2232257.

Chambers, R. G. (1996). A new look at exact input, output, productivity, and technical change measurement, Maryland Agricultural Experimental Station.

Chambers, R. G. (2002). Exact nonradial input, output, and productivity measurement. *Economic Theory* 20: 751–765. 10.1007/s001990100231.

Chambers, R. G., Chung, Y. and Färe, R. (1998). Profit, directional distance functions, and Nerlovian efficiency. *Journal of Optimization Theory and Applications* 98: 351–364. 10.1023/A:1022637501082.

Chambers, R. G., Färe, R. and Grosskopf, S. (1996). Productivity growth in APEC countries. *Pacific Economic Review* 1: 181–190. 10.1111/j.1468-0106.1996.tb00184.x.

Coelli, T., Lauwers, L. and Van Huylenbroeck, G. (2007). Environmental efficiency measurement and the materials balance condition. *Journal of Productivity Analysis* 28: 3–12. 10.1007/s11123-007-0052-8.

Cox, T. L. and Wohlgenant, M. K. (1986). Prices and quality effects in cross-sectional demand analysis. *American Journal of Agricultural Economics* 68: 908–919. 10.2307/1242137.

Diewert, W. E. (1976). Exact and superlative index numbers. *Journal of Econometrics* 4: 115–145. 10.1016/0304-4076(76)90009-9.

Diewert, W. E. (2005). Index number theory using differences rather than ratios. *American Journal of Economics and Sociology* 64: 311–360. 10.1111/j.1536-7150.2005.00365.x.

Drechsler, L. (1973). Weighting of index numbers in multilateral international comparisons. *Review of Income and Wealth* 19: 17–34. 10.1111/j.1475-4991.1973.tb00871.x.

Duval, R., Hong, G. H. and Timmer, Y. (2020). Financial frictions and the great productivity slowdown. *The Review of Financial Studies* 33: 475–503. 10.1093/rfs/hhz063.

Eldor, D. and Sudit, E. F. (1981). Productivity-based financial net income analysis. *Omega* 9: 605–611. 10.1016/0305-0483(81)90048-7.

Eurostat (2022). Eurostat. http://ec.europa.eu/eurostat. Accessed 2 March 2022.

Fallah-Fini, S., Triantis, K. and Johnson, A. L. (2014). Reviewing the literature on non-parametric dynamic efficiency measurement: state-of-the-art. *Journal of Productivity Analysis* 41: 51–67. 10.1007/s11123-013-0349-8.

Färe, R., Grosskopf, S., Noh, D. -W. and Weber, W. (2005). Characteristics of a polluting technology: theory and practice. *Journal of Econometrics* 126: 469–492. 10.1016/j.jeconom.2004.05.010.

Färe, R., Grosskopf, S., Norris, M. and Zhang, Z. (1994). Productivity growth, technical progress, and efficiency change in industrialized countries. *American Economic Review* 84: 66–83.

Gal, P. N. (2013). Measuring total factor productivity at the firm level using OECD-ORBIS.

Grifell-Tatjé, E. and Lovell, C. A. K. (1999). Profits and productivity. *Management Science* 45: 1177–1193. 10.1287/mnsc.45.9.1177.

Grifell-Tatjé, E. and Lovell, C. A. K. (2000). Cost and productivity. *Managerial and Decision Economics* 21: 19–30. 10.1002/1099-1468(200001/02)21:13.0.CO;2-7.

Hoang, V. -N. and Coelli, T. (2011). Measurement of agricultural total factor productivity growth incorporating environmental factors: A nutrients balance approach. *Journal of Environmental Economics and Management* 62: 462–474. 10.1016/j.jeem.2011.05.009.

Isard, P. (1977). How far can we push the 'Law of One Price'? *American Economic Review* 67: 942–948.

Kapelko, M. (2019). Measuring productivity change accounting for adjustment costs: evidence from the food industry in the European Union. *Annals of Operations Research* 278: 215–234. 10.1007/s10479-017-2497-0.

Kapelko, M., Oude Lansink, A. and Stefanou, S. E. (2015). Investment age and dynamic productivity growth in the Spanish food processing industry. *American Journal of Agricultural Economics* 98: 946–961. 10.1093/ajae/aav063.

Kneip, A., Simar, L. and Wilson, P. W. (2008). Asymptotics and consistent bootstraps for DEA estimators in nonparametric frontier models. *Econometric Theory* 24: 1663–1697. 10.1017/S0266466608080651.

Kumar, S. and Russell, R. R. (2002). Technological change, technological catch-up, and capital deepening: relative contributions to growth and convergence. *American Economic Review* 92: 527–548. 10.1257/00028280260136381.

Kuosmanen, T., Cherchye, L. and Sipiläinen, T. (2006). The law of one price in data envelopment analysis: Restricting weight flexibility across firms. *European Journal of Operational Research* 170: 735–757. 10.1016/j.ejor.2004.07.063.

Kurosawa, K. (1975). An aggregate index for the analysis of productivity and profitability. *Omega* 3: 157–168. 10.1016/0305-0483(75)90115-2.

Lau, L. J. and Yotopoulos, P. A. (1971). A test for relative efficiency and application to Indian agriculture. *American Economic Review* 61: 94–109.

Li, Q. (1996). Nonparametric testing of closeness between two unknown distribution functions. *Econometric Reviews* 15: 261–274. 10.1080/07474939608800355.

Lucas, R. E., Jr. (1967). Adjustment costs and the theory of supply. *Journal of Political Economy* 75: 321–334. 10.1086/259289.

Miller, D. M. and Rao, P. M. (1989). Analysis of profit-linked total-factor productivity measurement models at the firm level. *Management Science* 35: 757–767. 10.1287/mnsc.35.6.757.

Murty, S., Russell, R. R. and Levkoff, S. B. (2012). On modeling pollution-generating technologies. *Journal of Environmental Economics and Management* 64: 117–135. 10.1016/j.jeem.2012.02.005.

Nemoto, J. and Goto, M. (1999). Dynamic data envelopment analysis: modeling intertemporal behavior of a firm in the presence of productive inefficiencies. *Economics Letters* 64: 51–56. 10.1016/S0165-1765(99)00070-1.

Nemoto, J. and Goto, M. (2003). Measurement of dynamic efficiency in production: An application of data envelopment analysis to Japanese electric utilities. *Journal of Productivity Analysis* 19: 191–210. 10.1023/A:1022805500570.

Nishimizu, M. and Page, J. M. (1982). Total factor productivity growth, technological progress and technical efficiency change: dimensions of productivity change in Yugoslavia, 1965-78. *The Economic Journal* 92: 920–936. 10.2307/2232675.

O'Donnell, C. J. (2012). Nonparametric estimates of the components of productivity and profitability change in US agriculture. *American Journal of Agricultural Economics* 94: 873–890. 10.1093/ajae/aas023.

Orbis (2022). Orbis. http://orbis.bvdinfo.com/. Accessed 8 April 2022.

Oude Lansink, A. and Stefanou, S. E. (1997). Asymmetric adjustment of dynamic factors at the firm level. *American Journal of Agricultural Economics* 79: 1340–1351. 10.2307/1244290.

Oude Lansink, A., Stefanou, S. E. and Serra, T. (2015). Primal and dual dynamic Luenberger productivity indicators. *European Journal of Operational Research* 241: 555–563. 10.1016/j.ejor.2014.09.027.

Ouellette, P. and Yan, L. (2008). Investment and dynamic DEA. *Journal of Productivity Analysis* 29: 235–247. 10.1007/s11123-007-0079-x.

Owusu-Sekyere, E., Hansson, H. and Telezhenko, E. (2022). Use and non-use values to explain farmers' motivation for the provision of animal welfare. *European Review of Agricultural Economics* 49: 499–525. 10.1093/erae/jbab012.

Plastina, A. and Lence, S. H. (2018). A parametric estimation of total factor productivity and its components in US agriculture. *American Journal of Agricultural Economics* 100: 1091–1119. 10.1093/ajae/aay010.

Rothschild, M. (1971). On the cost of adjustment. *The Quarterly Journal of Economics* 85: 605–622. 10.2307/1882270.

Rungsuriyawiboon, S. and Stefanou, S. E. (2007). Dynamic efficiency estimation: an application to US electric utilities. *Journal of Business & Economic Statistics* 25: 226–238. 10.1198/073500106000000288.

Serra, T., Oude Lansink, A. and Stefanou, S. E. (2011). Measurement of dynamic efficiency: A directional distance function parametric approach. *American Journal of Agricultural Economics* 93: 756–767. 10.1093/ajae/aaq175.

Silva, E., Oude Lansink, A. and Stefanou, S. E. (2015). The adjustment-cost model of the firm: Duality and productive efficiency. *International Journal of Production Economics* 168: 245–256. 10.1016/j.ijpe.2015.06.027.

Silva, E. and Stefanou, S. E. (2007). Dynamic efficiency measurement: Theory and application. *American Journal of Agricultural Economics* 89: 398–419. 10.1111/j.1467-8276.2007.00999.x.

Silva, E., Stefanou, S. E. and Oude Lansink, A. (2020). *Dynamic efficiency and productivity measurement*. New York, NY: Oxford University Press. 10.1093/oso/9780190919474.001.0001.

Simar, L., Vanhems, A. and Wilson, P. W. (2012). Statistical inference for DEA estimators of directional distances. *European Journal of Operational Research* 220: 853–864. 10.1016/j.ejor.2012.02.030.

Simar, L. and Wilson, P. W. (1998). Sensitivity analysis of efficiency scores: How to bootstrap in nonparametric frontier models. *Management Science* 44: 49–61. 10.1287/mnsc.44.1.49.

Simar, L. and Wilson, P. W. (2011). Inference by the *m*-out-of-*n* bootstrap in nonparametric frontier models. *Journal of Productivity Analysis* 36: 33–53. 10.1007/s11123-010-0200-4.

Simar, L. and Wilson, P. W. (2020). Technical, allocative and overall efficiency: Estimation and inference. *European Journal of Operational Research* 282: 1164–1176. 10.1016/j.ejor.2019.10.011.

Stefanou, S. E. (2020). Dynamic Analysis of Production. In: Ray S. C., Chambers R. G. and Kumbhakar S. C. (eds.), *Handbook of Production Economics*. Singapore: Springer Singapore, 1–29. 10.1007/978-981-10-3450-3_17-1.

Treadway, A. B. (1969). On rational entrepreneurial behaviour and the demand for investment. *Review of Economic Studies* 36: 227–239. 10.2307/2296839.

Treadway, A. B. (1970). Adjustment costs and variable inputs in the theory of the competitive firm. *Journal of Economic Theory* 2: 329–347. 10.1016/0022-0531(70)90017-7.

Varian, H. R. (1984). The nonparametric approach to production analysis. *Econometrica* 52: 579–597. 10.2307/1913466.

Appendix A Properties of the adjustment-cost technology set

Following Ang and Oude Lansink (2018) and Silva, Stefanou and Oude Lansink (2020: p. 125-128, Section 4.3), we make the following assumptions on the adjustment-cost technology:

Axiom 1. (Closedness). $\mathcal{T}_t(\mathbf{K}_t)$ *is closed.*

Axiom 2. (Free disposability of inputs and outputs). *if* $(\mathbf{x}'_t, -\mathbf{y}'_t) \geq (\mathbf{x}_t, -\mathbf{y}_t)$ *then* $(\mathbf{x}_t, \mathbf{I}_t, \mathbf{y}_t) \in \mathcal{T}_t(\mathbf{K}_t) \Rightarrow (\mathbf{x}'_t, \mathbf{I}_t, \mathbf{y}'_t) \in \mathcal{T}_t(\mathbf{K}_t)$.

Axiom 3. (Investment Inaction is possible). $(\mathbf{x}_t, \mathbf{0}_t, \mathbf{y}_t) \in \mathcal{T}_t(\mathbf{K}_t)$.

Axiom 4. (Negative monotonicity in investments). *if* $\mathbf{I}_t \geq \mathbf{I}'_t$ *then* $(\mathbf{x}_t, \mathbf{I}_t, \mathbf{y}_t) \in \mathcal{T}_t(\mathbf{K}_t) \Rightarrow (\mathbf{x}_t, \mathbf{I}'_t, \mathbf{y}_t) \in \mathcal{T}_t(\mathbf{K}_t)$.

Axiom 5. (Reverse nestedness in capital stock). *if* $\mathbf{K}'_t \geq \mathbf{K}_t$ *then* $\mathcal{T}_t(\mathbf{K}_t) \subseteq \mathcal{T}_t(\mathbf{K}'_t)$.

Axiom 6. (Convexity). *Technology set* $\mathcal{T}_t(\mathbf{K}_t)$ *is convex.*

Axiom 3 states that production is possible without investment and is consistent with periodic observed investment spikes. Axioms 2, 4 and 6 model the adjustment costs associated with investment. Investment comes at the expense of production (given the level of input use) and requires a higher input use to maintain production. Finally, Axiom 5 models that an addition in the capital stock widens the production possibilities.

Appendix B Transitivity

One can render *DBQC* transitive by choosing (normalised) prices $\bar{\mathbf{P}}$ that are fixed across space and over time. In line with Ang (2019), this would yield the dynamic 'Bennet-Lowe' quantity indicator $DBC(\mathbf{Q}_t, \mathbf{Q}_{t+1}; \bar{\mathbf{P}})$, which can also be decomposed employing equation (11). Transitivity allows straightforward interpretation of multilateral and -temporal comparisons (O'Donnell, 2012), as $DBC(\mathbf{Q}_h, \mathbf{Q}_j; \bar{\mathbf{P}}) = DBC(\mathbf{Q}_h, \mathbf{Q}_i; \bar{\mathbf{P}}) + DBC(\mathbf{Q}_i, \mathbf{Q}_j; \bar{\mathbf{P}})$. The dynamic Bennet quantity indicator is *in*transitive, but allows equal characterisation of observation-specific weights under potentially different market conditions. We refer to Drechsler (1973) and Caves, Christensen and Diewert (1982) for early yet pertinent discussions about the trade-off between transitivity and characteristicity. Following the reasoning of Ang (2019) in the static context, the dynamic Bennet-Lowe quantity indicator can lead to a violation of the identity test in its dual, $\Pi C(\mathbf{Q}_t, \mathbf{Q}_{t+1}, \tilde{\mathbf{P}}_t, \tilde{\mathbf{P}}_{t+1}) - DBC(\mathbf{Q}_t, \mathbf{Q}_{t+1}; \bar{\mathbf{P}})$. This dual thus cannot be regarded as a dynamic *total* price recovery indicator, as an unchanged price can change the dual value. Such a violation does not occur in the dynamic Bennet formulations.

Appendix C Dynamic scale efficiency change

Extending Ang (2019)'s framework to the dynamic context, we can define *benchmark* adjustment-cost technology $\breve{\mathcal{T}}_t(\mathbf{K}_t) \supseteq \mathcal{T}_t(\mathbf{K}_t)$ with the same properties as $\mathcal{T}_t(\mathbf{K}_t)$. Usually, $\breve{\mathcal{T}}_t(\mathbf{K}_t)$ is defined with regard to constant returns to scale. Equations (2)–(9) can then also be established for $\breve{\mathcal{T}}_t(\mathbf{K}_t)$. This permits defining two dynamic scale inefficiency measures. Dynamic primal scale inefficiency $DPSI_t(\mathbf{Q}_t; \mathbf{g})$ is defined as:

$$DPSI_t(\mathbf{Q}_t; \mathbf{g}) = \breve{D}_t(\mathbf{Q}_t; \mathbf{g}) - D_t(\mathbf{Q}_t; \mathbf{g}). \tag{C1}$$

Here, $DPSI_t(\cdot)$ indicates the deviation from the optimal scale in terms of quantities of variable inputs, investments and outputs.

Dynamic dual scale inefficiency $DDSI_t(\mathbf{Q}^*_t, \tilde{\mathbf{P}}_t)$ is defined as:

$$DDSI_t(\mathbf{Q}^*_t, \tilde{\mathbf{P}}_t) = \breve{\Pi}(\mathbf{Q}^*_t, \tilde{\mathbf{P}}_t) - \Pi(\mathbf{Q}^*_t, \tilde{\mathbf{P}}_t). \tag{C2}$$

Here, $DDSI_t(\cdot)$ also indicates the deviation from the optimal scale, but is measured in terms of quantities as well as normalised prices of variable inputs, investments and outputs.

Adapting Ang (2019)'s static framework to the dynamic context, we introduce two further decompositions of $DBQC(\cdot)$. First, $DBQC(\cdot)$ can be decomposed into components of dynamic technical change with regard to the benchmark technology $\widetilde{DTC}$, dynamic technical efficiency change ***DTEC***, dynamic primal scale efficiency change ***DPSEC*** and dynamic mix efficiency change with respect to the benchmark technology $\widetilde{DMEC}$:

$$DBQC(\cdot) \equiv DTC + DTEC + DPSEC + \widetilde{DMEC}, \text{ where} \tag{C3a}$$

$$\begin{aligned}\widetilde{DTC} &\equiv \frac{1}{2}\{[\breve{\Pi}(\mathbf{Q}^*_{t+1}, \tilde{\mathbf{P}}_t) - \breve{\Pi}(\mathbf{Q}^*_t, \tilde{\mathbf{P}}_t)] + [\breve{\Pi}(\mathbf{Q}^*_{t+1}, \tilde{\mathbf{P}}_{t+1}) - \breve{\Pi}(\mathbf{Q}^*_t, \tilde{\mathbf{P}}_{t+1})]\} \\ &\equiv \frac{1}{2}\{\widetilde{DTC}_L + \widetilde{DTC}_P\},\end{aligned} \tag{C3b}$$

$$DTEC \equiv D_t(\mathbf{Q}_t; \mathbf{g}) - D_{t+1}(\mathbf{Q}_{t+1}; \mathbf{g}), \tag{C3c}$$

$$DPSEC \equiv DPSI_t(\mathbf{Q}_t; \mathbf{g}) - DPSI_{t+1}(\mathbf{Q}_{t+1}; \mathbf{g}) \text{ and} \tag{C3d}$$

$$\begin{aligned}\widetilde{DMEC} \equiv & \frac{1}{2}\{[\widetilde{DMI}_t(\mathbf{Q}_t, \tilde{\mathbf{P}}_t; \mathbf{g}) - \widetilde{DMI}_{t+1}(\mathbf{Q}_{t+1}, \tilde{\mathbf{P}}_t; \mathbf{g})] \\ & + [\widetilde{DMI}_t(\mathbf{Q}_t, \tilde{\mathbf{P}}_{t+1}; \mathbf{g}) - \widetilde{DMI}_{t+1}(\mathbf{Q}_{t+1}, \tilde{\mathbf{P}}_{t+1}; \mathbf{g})]\} \\ \equiv & \frac{1}{2}\{\widetilde{DMEC}_L + \widetilde{DMEC}_P\}.\end{aligned} \tag{C3e}$$

We dually measure dynamic technical change with the dynamic profit-maximising point on the period-dependent benchmark frontier as the relevant reference point. Assessing the shift between t and $t+1$ for $\tilde{\mathbf{P}}_t$ yields the Laspeyres-type measure of $\widetilde{DTC}_L \equiv \breve{\Pi}(\mathbf{Q}^*_{t+1}, \tilde{\mathbf{P}}_t) - \breve{\Pi}(\mathbf{Q}^*_t, \tilde{\mathbf{P}}_t)$. Doing so for $\tilde{\mathbf{P}}_{t+1}$ yields the Paasche-type measure of $\widetilde{DTC}_P \equiv \breve{\Pi}(\mathbf{Q}^*_{t+1}, \tilde{\mathbf{P}}_{t+1}) - \breve{\Pi}(\mathbf{Q}^*_t, \tilde{\mathbf{P}}_{t+1})$. Here, $\widetilde{DTC}$ is the arithmetic average of $\widetilde{DTC}_L$ and $\widetilde{DTC}_P$. If $\widetilde{DTC} > 0$, there is technological progress.

In addition, ***DTEC*** assesses the extent to which the DMU catches up with the actual frontier along directional vector **g**. If ***DTEC*** > 0, dynamic technical inefficiency decreases, which thus indicates a catch-up to the best-practice adjustment-cost technology.

Furthermore, ***DPSEC*** gauges the change in scale operation, in terms of quantities. If ***DPSEC*** > 0, the DMU moves closer to the scale operation of the benchmark frontier, in terms of quantities.

Finally, $\widetilde{DMEC}$ is the change in ability to correctly allocate the mix of variable inputs, outputs and net investments, with regard to the dynamic profit-maximising point on the benchmark frontiers of periods t and $t+1$. Analogously to $\widetilde{DTC}$, we assess $\widetilde{DMEC}$ with respect to $\tilde{\mathbf{P}}_t$ and $\tilde{\mathbf{P}}_{t+1}$, which respectively yields the Laspeyres-type measure of mix efficiency change, $\widetilde{DMEC}_L \equiv \widetilde{DMI}_t(\mathbf{Q}_t, \tilde{\mathbf{P}}_t; \mathbf{g}) - \widetilde{DMI}_{t+1}(\mathbf{Q}_{t+1}, \tilde{\mathbf{P}}_t; \mathbf{g})$, and the Paasche-type measure of mix efficiency change, $\widetilde{DMEC}_P \equiv \widetilde{DMI}_t(\mathbf{Q}_t, \tilde{\mathbf{P}}_{t+1}; \mathbf{g}) - \widetilde{DMI}_{t+1}(\mathbf{Q}_{t+1}, \tilde{\mathbf{P}}_{t+1}; \mathbf{g})$. Here, $\widetilde{DMEC}$ is the arithmetic average of $\widetilde{DMEC}_L$ and $\widetilde{DMEC}_P$. If $\widetilde{DMEC} > 0$, the ability to correctly allocate the mix of variable inputs, outputs and net investments improves over time.

Second, $DBQC(\cdot)$ can be decomposed into components of dynamic technical change with regard to the benchmark technology $\widetilde{DTC}$, dynamic technical efficiency change *DTEC*, dynamic dual scale efficiency change *DDSEC* and dynamic mix efficiency change with respect to the actual technology *DMEC*:

$$DBQC(\cdot) = DTC + DTEC + DDSEC + DMEC, \text{ where} \tag{C4a}$$

$$\begin{aligned}\widetilde{DTC} &\equiv \frac{1}{2}\{[\breve{\Pi}(\mathbf{Q}^*_{t+1}, \tilde{\mathbf{P}}_t) - \breve{\Pi}(\mathbf{Q}^*_t, \tilde{\mathbf{P}}_t)] + [\breve{\Pi}(\mathbf{Q}^*_{t+1}, \tilde{\mathbf{P}}_{t+1}) - \breve{\Pi}(\mathbf{Q}^*_t, \tilde{\mathbf{P}}_{t+1})]\} \\ &\equiv \frac{1}{2}\{\widetilde{DTC}_L + \widetilde{DTC}_P\},\end{aligned} \tag{C4b}$$

$$DTEC \equiv D_t(\mathbf{Q}_t; \mathbf{g}) - D_{t+1}(\mathbf{Q}_{t+1}; \mathbf{g}), \tag{C4c}$$

$$\begin{aligned}DDSEC &\equiv \frac{1}{2}\{[DDSI_t(\mathbf{Q}^*_t, \tilde{\mathbf{P}}_t; \mathbf{g}) - DDSI_{t+1}(\mathbf{Q}^*_{t+1}, \tilde{\mathbf{P}}_t; \mathbf{g})] \\ &\quad + [DDSI_t(\mathbf{Q}^*_t, \tilde{\mathbf{P}}_{t+1}; \mathbf{g}) - DDSI_{t+1}(\mathbf{Q}^*_{t+1}, \tilde{\mathbf{P}}_{t+1}; \mathbf{g})]\} \\ &\equiv \frac{1}{2}\{DDSEC_L + DDSEC_P\} \text{ and}\end{aligned} \tag{C4d}$$

$$\begin{aligned}DMEC &\equiv \frac{1}{2}\{[DMI_t(\mathbf{Q}_t, \tilde{\mathbf{P}}_t; \mathbf{g}) - DMI_{t+1}(\mathbf{Q}_{t+1}, \tilde{\mathbf{P}}_t; \mathbf{g})] \\ &\quad + [DMI_t(\mathbf{Q}_t, \tilde{\mathbf{P}}_{t+1}; \mathbf{g}) - DMI_{t+1}(\mathbf{Q}_{t+1}, \tilde{\mathbf{P}}_{t+1}; \mathbf{g})]\} \\ &\equiv \frac{1}{2}\{DMEC_L + DMEC_P\}.\end{aligned} \tag{C4e}$$

Here, $\widetilde{DTC}$ and *DTEC* are components also occurring in the first decomposition, which have the same interpretation.

In addition, analogously to $\widetilde{DTC}$, we assess *DDSEC* with respect to $\tilde{\mathbf{P}}_t$ and $\tilde{\mathbf{P}}_{t+1}$, which respectively yields the Laspeyres-type measure of scale efficiency change, $DDSEC_L \equiv DDSI_t(\mathbf{Q}^*_t, \tilde{\mathbf{P}}_t; \mathbf{g}) - DDSI_{t+1}(\mathbf{Q}^*_{t+1}, \tilde{\mathbf{P}}_t; \mathbf{g})$, and the Paasche-type measure of mix efficiency change, $DDSEC_P \equiv DDSI_t(\mathbf{Q}^*_t, \tilde{\mathbf{P}}_{t+1}; \mathbf{g}) - DDSI_{t+1}(\mathbf{Q}^*_{t+1}, \tilde{\mathbf{P}}_{t+1}; \mathbf{g})$. Here, *DDSEC* is the arithmetic average of $DDSEC_L$ and $DDSEC_P$. If $DDSEC > 0$, the DMU moves closer to the scale operation of the benchmark frontier, in terms of prices and quantities.

Finally, as in the decomposition presented in the main text, *DMEC* is the change in ability to correctly allocate the mix of variable inputs, outputs and net investments, with regard to the dynamic profit-maximising point on the actual frontiers of periods t and $t+1$. We assess *DMEC* with respect to $\tilde{\mathbf{P}}_t$ and $\tilde{\mathbf{P}}_{t+1}$, which respectively yields the Laspeyres-type measure of mix efficiency change, $DMEC_L \equiv DMI_t(\mathbf{Q}_t, \tilde{\mathbf{P}}_t; \mathbf{g}) - DMI_{t+1}(\mathbf{Q}_{t+1}, \tilde{\mathbf{P}}_t; \mathbf{g})$, and the Paasche-type measure of mix efficiency change, $DMEC_P \equiv DMI_t(\mathbf{Q}_t, \tilde{\mathbf{P}}_{t+1}; \mathbf{g}) - DMI_{t+1}(\mathbf{Q}_{t+1}, \tilde{\mathbf{P}}_{t+1}; \mathbf{g})$. Here, *DMEC* is the arithmetic average of $DMEC_L$ and $DMEC_P$. If $DMEC > 0$, the ability to correctly allocate the mix of variable inputs, outputs and net investments improves over time.

Appendix D Proof of Proposition 1

*Proof.*We can write

$$L_{t,t+1}(\mathbf{Q}_t, \mathbf{Q}_{t+1}; \mathbf{g}) = \frac{1}{2}\left\{L_t(\cdot) + L_{t+1}(\cdot)\right\}$$

where

$$L_z(\cdot) = \left[D_z(\mathbf{Q}_t; \mathbf{g}) - D_z(\mathbf{Q}_{t+1}; \mathbf{g})\right].$$

Analogous to the proof of Chambers (1996); Chambers (2002) and Balk (1998) in the static context, we apply Diewert (1976)'s quadratic lemma to the dynamic context:

$$\begin{aligned} L_z(\cdot) =& \frac{1}{2}\left[\nabla_y D_z(\mathbf{Q}_t; \mathbf{g}) + \nabla_y D_z(\mathbf{Q}_{t+1}; \mathbf{g})\right](\mathbf{y}_t - \mathbf{y}_{t+1}) \\ &+ \frac{1}{2}\left[\nabla_x D_z(\mathbf{Q}_t; \mathbf{g}) + \nabla_x D_z(\mathbf{Q}_{t+1}; \mathbf{g})\right](\mathbf{x}_t - \mathbf{x}_{t+1}) \\ &+ \frac{1}{2}\left[\nabla_I D_z(\mathbf{Q}_t; \mathbf{g}) + \nabla_I D_z(\mathbf{Q}_{t+1}; \mathbf{g})\right](\mathbf{NI}_t - \mathbf{NI}_{t+1}). \end{aligned}$$

The arithmetic average of both $L_t(\cdot)$ and $L_{t+1}(\cdot)$ and assuming $a^t_{x_i x_{i'}} = a^{t+1}_{x_i x_{i'}}$ for all i and i', $a^t_{y_l y_{l'}} = a^{t+1}_{y_l y_{l'}}$ for all l and l', and $a^t_{NI_j NI_{j'}} = a^{t+1}_{NI_j NI_{j'}}$ for all j and j', yields:

$$\begin{aligned} L_{t,t+1}(\cdot) =& \frac{1}{2}\left[\nabla_y D_t(\mathbf{Q}_t; \mathbf{g}) + \nabla_y D_{t+1}(\mathbf{Q}_{t+1}; \mathbf{g})\right](\mathbf{y}_t - \mathbf{y}_{t+1}) \\ &+ \frac{1}{2}\left[\nabla_x D_t(\mathbf{Q}_t; \mathbf{g}) + \nabla_x D_{t+1}(\mathbf{Q}_{t+1}; \mathbf{g})\right](\mathbf{x}_t - \mathbf{x}_{t+1}) \\ &+ \frac{1}{2}\left[\nabla_I D_t(\mathbf{Q}_t; \mathbf{g}) + \nabla_I D_{t+1}(\mathbf{Q}_{t+1}; \mathbf{g})\right](\mathbf{NI}_t - \mathbf{NI}_{t+1}). \end{aligned}$$

The assumption of dynamic profit-maximising behaviour allows exploitation of the dual relationship between the dynamic directional distance function and restricted dynamic profit function and thus also derivation of the FOCs (7). Substitution for the FOCs completes our proof:

$$\begin{aligned} L_{t,t+1}(\cdot) &= \frac{1}{2}\left[-\tilde{\mathbf{p}}_t - \tilde{\mathbf{p}}_{t+1}\right](\mathbf{y}_t - \mathbf{y}_{t+1}) + \frac{1}{2}\left[\tilde{\mathbf{w}}_t + \tilde{\mathbf{w}}_{t+1}\right](\mathbf{x}_t - \mathbf{x}_{t+1}) \\ &\quad + \frac{1}{2}\left[-\nabla_K \tilde{\mathcal{J}}_t - \nabla_K \tilde{\mathcal{J}}_{t+1}\right](\mathbf{NI}_t - \mathbf{NI}_{t+1}) \\ &= DBC(\mathbf{Q}_t, \mathbf{Q}_{t+1}; \tilde{\mathbf{P}}_t, \tilde{\mathbf{P}}_{t+1}). \end{aligned}$$

□

Appendix E Shadow prices of capital

In line with Proposition 1, we use the following quadratic functional form for firm z:

$$D_t(\mathbf{x}_z, NI_z, y_z; (\mathbf{g}^x, \mathbf{g}^I, \mathbf{g}^y)) \tag{E1}$$

$$= a^0 + \sum_{u=1}^{2} a^u x_z^u + by_z + cNI_z + \frac{1}{2}\sum_{u=1}^{2}\sum_{v=1}^{2}\alpha^{uv}x_z^u x_z^v$$

$$+ \frac{1}{2}\beta y_z^2 + \frac{1}{2}\gamma NI_z^2 + \sum_{u=1}^{2}\phi^u x_z^u y_z$$

$$+ \sum_{u=1}^{2}\chi^u x_z^u NI_z + \psi y_z NI_z + a^{time}(t-2012).$$

Equation (E1) satisfies the conditions from the theoretical results, as all coefficients are time-invariant. We use a time trend to account for technical change.

We estimate equation (E1) for firm z by the following linear programme:

$$\min_{e_z \geq 0} \sum_{z=1}^{Z} e_z \tag{E2a}$$

$$\text{s.t. } e_z = (E5) \qquad \forall z = 1, \ldots, Z \tag{E2b}$$

$$(13b) - (13f) \tag{E6c}$$

$$\partial D_t(\mathbf{x}_z, NI_z, y_z; (\mathbf{g}^x, \mathbf{g}^I, \mathbf{g}^y))/\partial x_z^u \geq 0 \qquad \forall z = 1, \ldots, Z; \forall u = 1, \ldots, 2 \tag{E6d}$$

$$\partial D_t(\mathbf{x}_z, NI_z, y_z; (\mathbf{g}^x, \mathbf{g}^I, \mathbf{g}^y))/\partial y_z \leq 0 \qquad \forall z = 1, \ldots, Z \tag{E6e}$$

$$\partial D_t(\mathbf{x}_z, NI_z, y_z; (\mathbf{g}^x, \mathbf{g}^I, \mathbf{g}^y))/\partial I_z \leq 0 \qquad \forall z = 1, \ldots, Z. \tag{E6f}$$

Combining equations (7b), (7c), (E6e) and (E6f) allows us to directly obtain the firm-specific shadow price of capital $\nabla_K \mathcal{J}_z(\cdot)$:

$$\nabla_K \mathcal{J}_z(\cdot) = p\frac{c + \gamma NI_z + \sum_{u=1}^{2}\chi^u x_z^u + \psi y_z}{b + \beta y_z + \sum_{u=1}^{2}\phi^u x_z^u + \psi NI_z}. \tag{E3}$$

Appendix F Linear programmes

Denote $J_m^b \subseteq \{1,\ldots,n\}$ an index set of size $m \leq n$ for bootstrap iteration $b = 1,\ldots,B$. The bootstrap estimate of the dynamic directional distance function $\hat{D}_t^b(\mathbf{x}_{ks},\mathbf{NI}_{ks},\mathbf{y}_{ks};\mathbf{g})$ evaluated for an observation *k* in period *s* against a technology in period *t* in the direction **g** for bootstrap iteration *b* is computed using the following linear programme:

$$\hat{D}_t^b(\mathbf{x}_{ks},\mathbf{NI}_{ks},\mathbf{y}_{ks};\mathbf{g}) = \max_{\beta,\boldsymbol{\lambda}_t} \beta \tag{F1}$$

$$\sum_{i \in J_m^b} \lambda_{it}\mathbf{x}_{it} \leq \mathbf{x}_{ks} - \beta \mathbf{g}^x, \tag{F2}$$

$$\sum_{i \in J_m^b} \lambda_{it}\mathbf{NI}_{it} \geq \mathbf{NI}_{ks} + \beta \mathbf{g}^I, \tag{F3}$$

$$\sum_{i \in J_m^b} \lambda_{it}\mathbf{y}_{it} \geq \mathbf{y}_{ks} + \beta \mathbf{g}^y, \tag{F4}$$

$$\sum_{i \in J_m^b} \lambda_{it} = 1. \tag{F5}$$

If $m = n$, the approximation follows the classical, usual DEA approach of Banker, Charnes and Cooper (1984). Let $\hat{D}_t(\mathbf{x}_t,\mathbf{NI}_t,\mathbf{y}_t;\mathbf{g})$ be the empirical estimate of equation (2) using the full sample DEA technology (that is, $m = n$). Furthermore, let $\hat{D}_t^b(\mathbf{x}_t,\mathbf{NI}_t,\mathbf{y}_t;\mathbf{g})$ be a corresponding bootstrap estimate $b = 1,\ldots,B$ for subsample size $m < n$, obtained by drawing *m* independent and uniform samples without replacement from the data. We can then construct $100(1-\alpha)\%$ confidence intervals:

$$\left[\widehat{D}_t^L(\cdot),\widehat{D}_t^U(\cdot)\right] = \left[\widehat{D}_t(\cdot) - \frac{\psi_{1-\alpha/2,m}}{n^{\frac{2}{u+v+f+1}}}, \widehat{D}_t(\cdot) + \frac{\psi_{\alpha/2,m}}{n^{\frac{2}{u+v+f+1}}}\right], \tag{F6}$$

where $\psi_{\alpha/2,m}$ $(\psi_{1-\alpha/2,m})$ is the $\alpha/2$ $(1-\frac{\alpha}{2})$ percentile of the set $\left\{m^{\frac{2}{u+v+f+1}}(\widehat{D}_t^b(\cdot) - \widehat{D}_t(\cdot))\right\}_{b=1}^B$ (Simar, Vanhems and Wilson, 2012). Furthermore, bias-corrected estimates can be computed (Simar, Vanhems and Wilson, 2012: Eq.(6.1)) by:

$$\widehat{D}_t^{BC}(\cdot) = \widehat{D}_t(\cdot) - (m/n)^{\frac{2}{u+v+f+1}} \frac{1}{B}\sum_{b=1}^{B}(\widehat{D}_t^b(\cdot) - \widehat{D}_t(\cdot)). \tag{F7}$$

Appendix G Robustness checks

G.1. Alternative shadow price specification

We consider another specification for the shadow prices of capital. Specifically, we compute $\nabla_K \boldsymbol{\mathcal{J}}_t(\cdot)$ for the full data set, not only comprising large and very large firms, but also small- and medium-sized firms. The computed shadow prices are generally higher than in

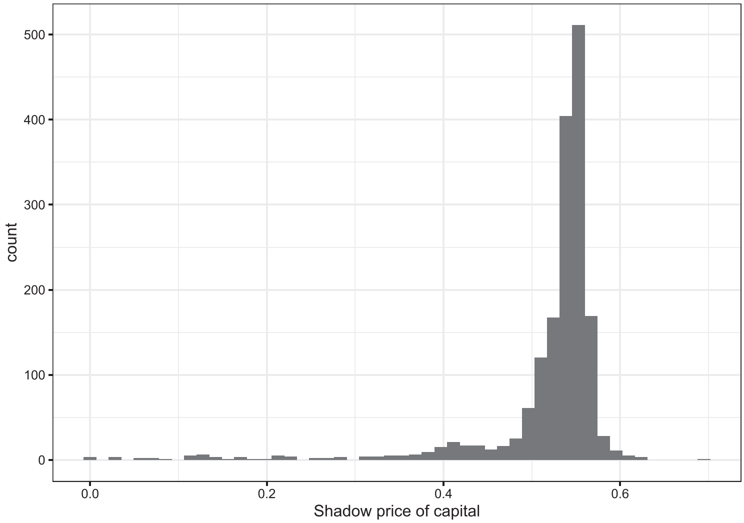

Fig. G1. Histogram with estimated alternative shadow prices of capital.

our main specification. The shadow price of capital is on average 0.521 and ranges from 0 to 0.695. In contrast to our main specification, the Hessian matrix associated with the quadratic dynamic directional distance function is negative semi-definite, as evidenced by the eigenvalues $[6.502\times10^{11}, -5.806\times10^{2}, -5.216\times10^{3}, -3.682\times10^{4}]$. This indicates convexity of the technology set for the full sample, with $\nabla_K \mathcal{J}_t(\cdot)$ consistent with dynamic profit maximisation.

G.2. Alternative subsample sizes

We verify the robustness for the choice of *m*. In the main results, the subsample retains 70 per cent of all observations for the bootstrap. We verify the robustness by also considering subsample sizes retaining 50 per cent and 90 per cent of all observations.[8]

G.3. Decomposition residual check

We investigate the empirical differences between *DMEC*, on the one hand, and $DBQC - DTC - DTEC$, on the other hand. Following equation (11), they should be the same in theory. In practice, however, they can differ as each component is bootstrapped for a different subsample (albeit with the same size *m*). These differences are to a large extent mitigated because of the high number of bootstraps ($B = 2,000$). Indeed, the difference is on average very close to zero (0.00014) and its standard deviation is small (0.01465). Nevertheless,

8 Simar, Vanhems and Wilson (2012) propose a data-driven approach to choose the subsample size of *m*. In their approach, *m* is evaluated from 1 to *n* for *B* bootstraps. Subsequently, *m* is chosen such that the volatility of the confidence intervals evaluated for $[m-i,...,m,...,m+i]$ is minimised given *i*. This yields an observation-specific *m* for the computation of confidence intervals. The same procedure is also conducted for the computation of the bias-corrected estimates, yielding another observation-specific *m*. However, this approach appears to be too computationally heavy for the current application.

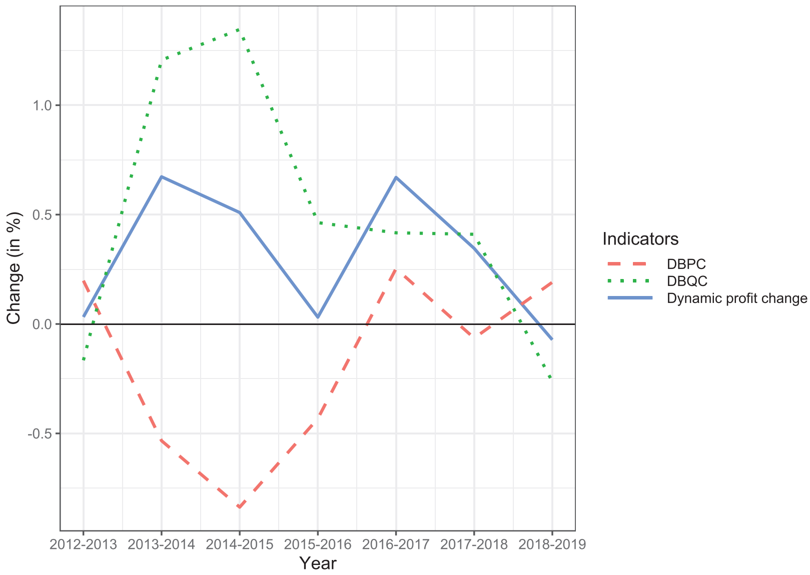

Fig. G2. Average dynamic profit change, dynamic productivity change and dynamic price change over time using alternative shadow prices.

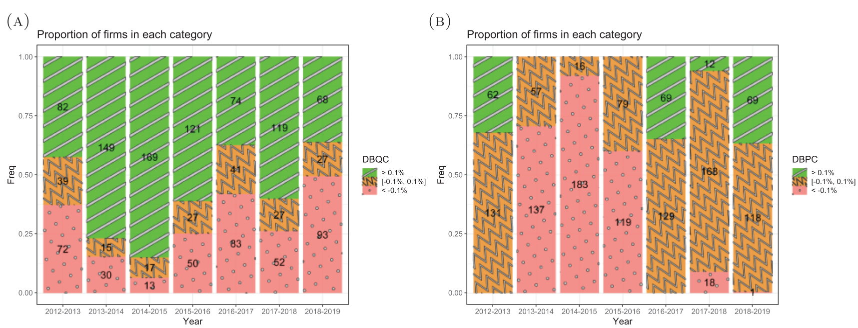

Fig. G3. Proportion of firms for which *DBQC* and *DBPC* increase, stagnate or decrease between two periods using alternative shadow prices.

divergences exist: the differences range from −0.31658 to 0.14550. Our robustness check entails a verification of the main results for the subsample for which these divergences are small ($< 5\%$), in line with our theoretical expectation.

Table G1. Summary statistics on dynamic productivity change and its components (in %) using alternative shadow prices.

	Year	2012–2013	2013–2014	2014–2015	2015–2016	2016–2017	2017–2018	2018–2019
DBQC	Mean	−0.17	1.21	1.35	0.46	0.42	0.41	−0.26
	Std	2.23	3.21	3.14	3.03	9.82	4.05	2.46
	Min	−11.88	−2.79	−2.62	−13.74	−26.71	−37.80	−19.54
	Max	11.71	23.87	24.01	25.11	129.67	36.72	11.18
DTC	Mean	−11.26	5.26	18.08	11.36	−12.80	−26.57	6.15
	Std	0.76	0.75	0.41	0.56	0.69	1.14	0.18
	Min	−12.16	4.43	15.73	7.80	−13.93	−28.21	5.52
	Max	−6.45	10.07	18.73	12.13	−9.82	−19.76	6.60
DTEC	Mean	0.02	0.41	−1.24	1.90	−0.53	0.56	−3.06
	Std	7.76	4.71	8.76	17.88	47.23	62.51	22.61
	Min	−49.62	−21.30	−63.08	−55.51	−647.54	−608.07	−163.74
	Max	78.57	38.52	94.50	239.49	112.50	629.86	248.10
DMEC	Mean	11.14	−3.81	−15.39	−13.27	14.52	26.15	−3.85
	Std	8.19	4.95	8.89	18.64	57.31	65.48	25.49
	Min	−75.24	−33.56	−95.33	−255.41	−112.53	−648.24	−287.82
	Max	42.87	26.30	48.64	63.60	800.00	647.23	183.42

Table G2. Results of nonparametric kernel density test: the samples have the same underlying distribution under the null hypothesis.

		Kernel density test		Pearson correlation coefficient		Spearman's rank correlation	
	Indicator	Tn	p-value	r	p-value	ρ	p-value
$m = 70\%$ vs. $m = 90\%$	*DBQC*	−0.612	1	1	0	1	0
	DTC	198.573	0	0.998	0	0.987	0
	DTEC	1.667	0.882	0.924	0	0.937	0
	DMEC	146.578	0.296	0.891	0	0.980	0
$m = 70\%$ vs. $m = 50\%$	*DBQC*	−0.612	1	1	0	1	0
	DTC	139.969	0	1.000	0	0.981	0
	DTEC	43.197	0.789	0.988	0	0.914	0
	DMEC	−128.044	0.053	0.984	0	0.985	0

Table G3. Summary statistics on dynamic productivity change and its components (in %) for observations where decomposition residual is smaller or equal than 5%.

	Year	2012–2013	2013–2014	2014–2015	2015–2016	2016–2017	2017–2018	2018–2019
$\Pi C(\mathbf{Q}_t, \mathbf{Q}_{t+1}, \tilde{\mathbf{P}}_t, \tilde{\mathbf{P}}_{t+1})$	Mean	0.04	1.61	0.39	0.06	0.72	0.40	−0.16
	Std	1.88	3.39	1.05	2.55	9.93	4.38	2.49
	Min	−11.00	−4.19	−3.48	−16.80	−25.02	−44.87	−18.45
	Max	13.76	11.17	6.00	20.39	131.26	36.81	9.06
DBPC	Mean	0.15	−1.42	−0.65	−0.41	0.22	−0.06	0.16
	Std	0.32	2.21	1.94	1.08	0.71	0.75	0.34
	Min	0.01	−9.43	−18.89	−8.54	−0.01	−10.47	−0.10
	Max	2.32	−0.01	−0.02	−0.02	7.62	0.91	2.18
DBQC	Mean	−0.11	3.03	1.04	0.47	0.49	0.46	−0.32
	Std	1.96	5.11	2.45	3.03	9.47	3.85	2.50
	Min	−11.85	−2.77	−1.91	−13.75	−26.67	−34.40	−19.53
	Max	11.75	20.60	24.02	25.98	123.63	36.46	7.78
DTC	Mean	−9.85	6.82	17.25	10.29	−11.41	−24.32	6.03
	Std	0.42	0.79	0.28	0.37	0.36	0.80	0.18
	Min	−11.83	5.25	15.61	7.77	−13.02	−26.77	5.45
	Max	−7.37	10.11	17.83	11.26	−9.87	−19.03	6.47
DTEC	Mean	0.19	1.72	−1.00	2.02	−1.22	0.56	−1.90
	Std	6.91	10.26	8.85	18.06	48.06	62.51	24.98
	Min	−7.23	−21.30	−63.08	−55.51	−647.54	−608.07	−163.74
	Max	78.57	38.52	94.50	239.49	112.50	629.86	248.10
DMEC	Mean	9.63	−5.05	−15.10	−12.34	13.55	24.12	−4.93
	Std	7.65	11.13	8.65	18.79	57.97	65.37	28.10
	Min	−75.26	−34.89	−95.28	−254.26	−112.53	−646.66	−287.75
	Max	19.90	26.26	49.88	63.64	794.05	646.87	180.18

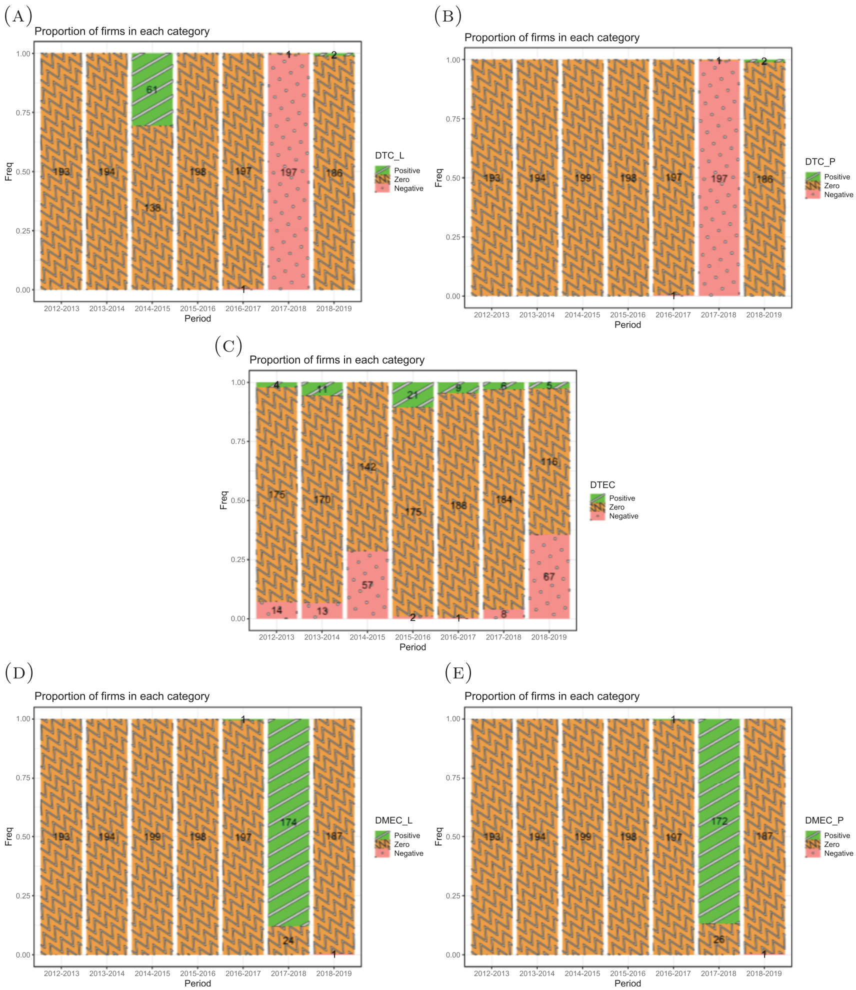

Fig. G4. Proportion of firms for which components of dynamic productivity change increases, stagnates or decreases in consecutive periods using alternative shadow prices.

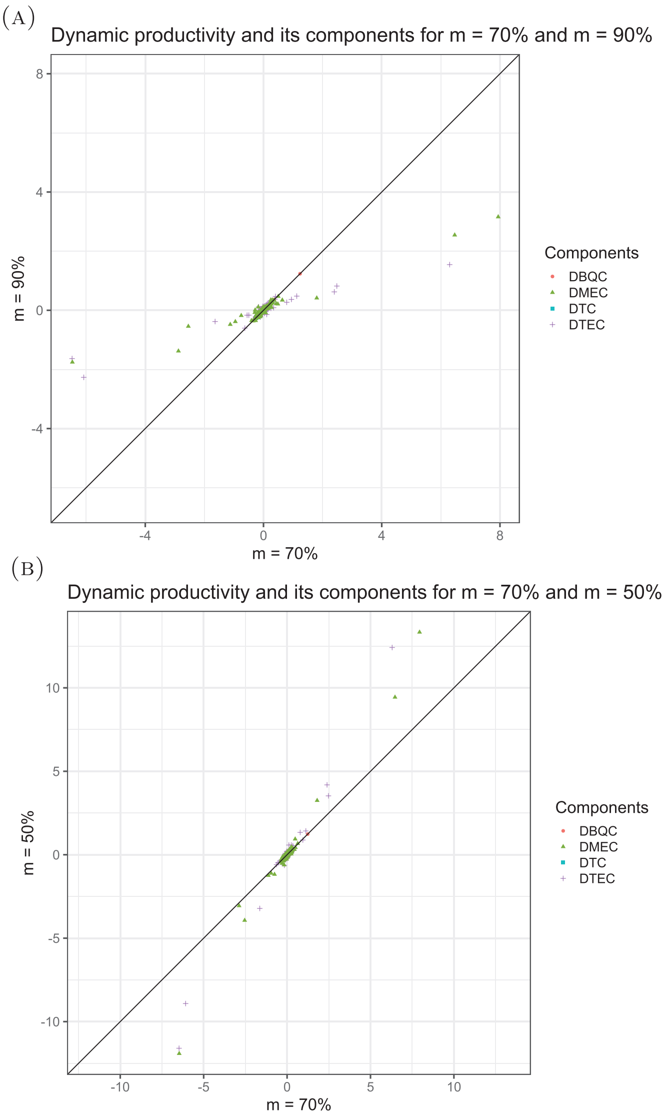

Fig. G5. Comparison of dynamic productivity change and its components for different subsample sizes ($m = 70\%$ vs. $m = 90\%$ and $m = 70\%$ vs. $m = 50\%$) with the 45 degree line as a reference.

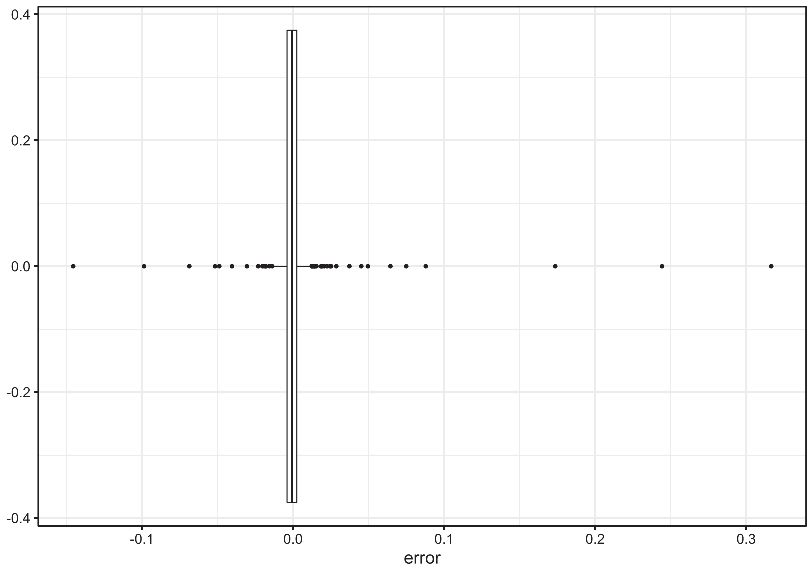

Fig. G6. Boxplot of decomposition residual.