Materials
With a Twist
Shear-induced instabilities in metamaterials
Aref Ghorbani
Propositions

1. Self-contact mechanism unlocks unique opportunities to design reformable metamaterials. (this thesis)

2. The complex behavior of metamaterials and nature are both governed by simple principles. (this thesis)

3. The complexity of the human brain is responsible for the peculiarity of humankind.

4. The best plot has two axes and one curve.

5. Practicing open science demands strictly adhering to open-source tools.

6. Science only explains how nature works but not why it works that way.

7. Free access to fundamental digital tools is a human right.

8. Fear of competition motivates women suppression.

Propositions belonging to the thesis, entitled

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Shear-induced instabilities in metamaterials

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Shear-induced instabilities in metamaterials

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CONTENTS
Chapter 1

General introduction

1.1 An introduction on metamaterials

Conventional materials obtain their ordinary physical characteristics from their compositions. However, metamaterials exhibit extraordinary properties beyond the reach of conventional materials, originating from the rational design of their building blocks, so-called unit-cells [1, 2]. Metamaterials usually are artificial lattices made of designed unit-cells and are presented in various fields of physics as a novel approach to create odd optical [3, 4] or mechanical [5, 4] properties. Usually, those properties are not observed in nature, even though there is no fundamental limitation to having natural materials with such properties. Metamaterials usually manipulate the propagation of optical or mechanical waves in an abnormal manner, which can be exploited for innumerable applications. For example, an outstanding idea of optical metamaterials with a negative refraction index was theoretically presented in 1968 [6] and recently realized experimentally [7]. Negative refraction index metamaterials has been widely explored for their exceptional implications in building a new generation of optical devices [8, 9].

Mechanical metamaterials are often made of a periodic array of rationally designed unit-cells and exhibit unusual mechanical properties or unique functionalities, not feasible to obtain in bulk materials nor natural systems [5]. It is notable that acoustic metamaterials are a large subclass of mechanical metamaterials, which focus on programming propagation of acoustical waves in designed structures [10, 11]. In this thesis, we predominantly use the term “metamaterials” to refer to mechanical metamaterials, excluding acoustic metamaterials.
Recent studies on flexible mechanical metamaterials have been developing in several directions. Providing a generic classification of mechanical metamaterials is challenging since different metamaterial types share some characteristics. In the following, we specify some types of mechanical metamaterials relevant to our research. Auxetic metamaterials are well-known mechanical metamaterials that display a negative Poisson’s ratio. In 1987, an auxetic system made of re-entrant thin-rod unit-cells was introduced [12, 13]; in a 2D setup, the unit-cell is a bow-tie shaped re-entrant polygon with thin walls, inspired by the honeycomb pattern [14]. A 2D sheet with a square array of closely packed circular voids, known as the holey sheet, also shows auxetic behaviors under compression [15]. Other examples of auxetic metamaterials have been introduced in more recent studies [16, 17]. Another class of metamaterials are topological metamaterials, which display properties that are topologically protected [18]. Topological metamaterials are usually insensitive to local variations or defects in their structures. This characteristic highlights them as robust systems for designing functional mechanical systems [5]. Buckling-driven metamaterials are highly porous structures that obtain their unusual functionalities from the local buckling of their unit-cells [5]. Changing the unit-cells or void geometry results in different classes of mechanical deformations in buckling-driven metamaterials [19].

Various aspects of metamaterials remain unexplored that are not limited to structural designing. Most of the previous studies on mechanical metamaterials have focused on their functionalities under uniaxial deformations. Nonetheless, metamaterials are anisotropic and compressible systems, and their behavior strongly depends on their deformation modes.

1.2 The core objective of this thesis

Isotropic linear materials can be fully characterized only by determining the Young’s modulus and Poisson’s ratio of the system. Mechanical metamaterials are anisotropic and rather complex since they can obtain up to 21 independent coefficients in their stress tensor, which provides a flexible platform to design unusual metamaterials even using linear elastic composition. Therefore, first of all, characterizing metamaterials requires more diverse deformations modes than only uniaxial loading. Secondly, by diversifying the deformation strategies, a broader range of functionalities and exotic properties can be accessible. Additionally, combining various deformations modes can provide a new prospect for the programmability of metamaterials. For example, the compression response of a
1.3. TOPOLOGICAL KINKS AND ELASTIC WAVE POCKETS

A biholar sheet is programmable via lateral confinement, permitting monotonic, nonmonotonic, and hysteretic responses [20, 21]. In our research, we aim to discover exotic shear-induced properties of metamaterials and use such properties to program the behavior of metamaterials.

In this thesis, we investigate four research projects, presented in separate chapters. In the following, we introduce fundamental concepts relevant to the explored topics: topological kinks and elastic wave pockets, shear-induced normal response, cylindrical shell buckling, and Gaussian curvature in plates. Finally, we present an overview of the projects and introduce our studied systems and strategies.

1.3 Topological kinks and elastic wave pockets

The concept of topological properties has been introduced by the quantum Hall effect [22] and electronic topological insulators [23]. Such properties are topologically protected and usually associated with distinct boundary modes. Recent studies revealed the fundamental link between topological electronic systems and topological mechanical metamaterials [24]. Topological metamaterials display a band gap in their vibration spectrum, associated with their edge modes or self-stress states (zero modes) [24, 18]. Zero modes can emerge in an array of elastically coupled spinners [24] and travel in the system via a nonlinear deformation as a localized domain wall [25, 26]. A traveling domain wall represents a nonlinear topological elastic wave pocket, rarely realized in mechanical metamaterials [27, 28, 29, 30, 31, 32]. Localized domain walls in metamaterials are associated with topological kinks. Such kinks usually are described by a system of elastically coupled units with a double-well potential, known as the $\phi^4$ model [33, 34, 35]. The $\phi^4$ model can also realize a negative solution, defined as the antikink in the same system [36]. A topological kink, in principle, separates different domains that exhibit different structural phases in the system. The wave pockets permitted by the $\phi^4$ models are similar to solitons [37, 38] that travel in the system with no dispersion or energy loss and can retain their original shape even after collision with other solitons. Although wave pockets in the $\phi^4$ model behave similarly, they can be altered by interaction with other waves of the same type [36, 39]. Thus, we refer to elastic waves in $\phi^4$ systems as soliton-like waves.

A confined mechanical metamaterial can display structural phase transitions with static kinks and antikinks [40, 41]. In Chapter 2, we study a metamaterial that exhibits topological kinks and antikinks under longitudinal compression.
The kinks and antikinks are initially static but travel in the system as soliton-like wave pockets via shear deformations. In Chapter 3, we study the role of kink in shear-induced mechanical properties of a small system. As we discuss in detail in Chapters 3, one can program the normal and shear response of the system in a broad range, by implementing a kink in the system via a pre-compression. Here, we introduce the shear-induced behaviors of a conventional system.

1.4 Shear-induced normal response

In 1909, Poynting discovered that applying shear deformation on a conventional continuum leads to a quadratic force perpendicular to the shear plane, known as the normal force. Poynting experimentally showed that a loaded piano wire axially dilates due to the shear-induced normal force, known as the Poynting effect [42, 43]. The normal force and Poynting effect are the result of nonlinearity and, therefore, negligible in linear materials. Perpendicular dilation of a sheared isotropic incomprehensible material also is described in well-known nonlinear elasticity theories such as Mooney–Rivlin theory [44, 45]. Positive normal force and the Poynting effect also emerge in viscoelastic materials under shear and significantly contribute to their behavior [46, 47]. Interestingly, some biopolymer gels and soft biomaterials display a negative normal force that tends to induce an axial contraction upon shear deformation, which is known as the reversed Poynting effect [48, 49, 50, 51, 52, 53]. The shear-induced negative normal force has a root in nonlinear strain-stiffening of the polymer network in such systems [48]. Additionally, liquid expulsion upon shearing from the biopolymer gel network is essential, as it causes a transition from positive to negative normal force, where the transition timescale depends on the porosity [53].

We can fully characterize the shearing behavior of isotropic materials and anisotropic systems that are symmetric with respect to shearing direction by two coefficients: (i) the shear modulus, $G_s$, and (ii) the Poynting modulus, $G_n$. $G_s$ is given by the slope of the shear stress-strain curve, which is constant for small shear deformations. $G_n$, is defined as the coefficient of normal stress as a function of the quadratic shear strain. Therefore, the shear stress, $\sigma_s$, and normal stress, $\sigma_n$, are respectively given by $\sigma_s = G_s \gamma$ and $\sigma_n = G_n \gamma^2$, where $\gamma$ is the shear strain. In a cylindrical shell geometry, with an inner radius $R_{\text{min}}$, an outer radius $R_{\text{max}}$, and a height $h$, the normal and shear force responses can be written as $F_n = G_n J (\varphi/h)^2$, and $F_s = \tau / R = G_s J \varphi / (R h)$. Here, $\tau$ is the torque around the axis of the shell, $R = (R_{\text{max}} + R_{\text{min}}) / 2$ is the mean radius, and $J = \pi / 2 (R_{\text{max}}^4 - R_{\text{min}}^4)$ is the second moment of area of the shell. In Figure 1.1,
we show the Poynting and shear moduli of a 3D-printed rubber, rescaled by its Young’s modulus, $Y$, as a function of the pre-compression strain, $\delta$, applied prior to shear. The experimental system is a cylindrical shell with $R_{\text{min}} = 7.5\text{mm}$, $R_{\text{max}} = 12.5\text{mm}$, and $h = 41.4\text{mm}$. We observe that the Poynting and shear moduli are relatively constant as a function of the pre-compression strain, $\delta$, and $G_{n}/G_{s} \simeq 0.5$, as expected for an incompressible isotropic rubber [45].

![Figure 1.1: Shear and Poynting moduli of a 3D-printed rubber as a function of pre-compression strain. The experiment is carried out by a cylindrical shell.](image)

Here, we additionally introduce the inverted Poynting effect as the inverse manifestation of the Poynting effect, where an axial compression induces a nonlinear shear deformation in the system. The inverted Poynting effect should not be confused with the reversed Poynting effect, associated with the shear-induced negative normal force. We elaborate on conventional, reversed and inverted Poynting effects in Chapter 3, where we obtain all these effects in a single metamaterial.

In anisotropic materials, the normal response is non-quadratic. Under low shear strain in anisotropic systems, the normal response usually behaves linearly. Under large shear strains, anisotropic systems can display a nonmonotonic normal response [54, 52, 55, 56]. In Chapter 4, we design auxetic cylindrical shells that are asymmetric (asymmetric meta-shells) with respect to the torsional direction. The asymmetry is implemented by rotating the principal axes of the auxetic design with respect to the cylinder axis. Asymmetric meta-shells demonstrate
a negative radial strain in addition to a nonmonotonic axial strain. We show that cylindrical shells exhibit extreme resistance to torsional buckling, where negative radial strain triggers a unique reconfiguration. In the next section, we discuss buckling behaviors of cylindrical thin and thick conventional shells.

1.5 Buckling of cylindrical shells

Cylindrical shells are ubiquitous in nature (such as hollow-stem plants like bamboo, veins, microtubules) and widely used in engineering (such as silos, space shuttles, bottles, cans). Even though cylindrical structures are suitable for a wide variety of applications for their appropriate geometry and high resistance, they can experience catastrophic buckling under different loading scenarios [57, 58]. Uniaxial compression leads to different buckling types [59, 60, 58]. Torsional loading can also trigger buckling of cylindrical shells, known as the torsional buckling [61, 62], which is common in nature (like in veins [63, 64, 65]) and artificial systems [61, 66, 67, 68]. A common example is observed when an empty beverage can is twisted, which leads to torsional buckling and emergence of creases on the can [62]. In Figure 1.2a, we show a twisted empty can crumpled due to the torsional buckling. A thick shell, shown in Figure 1.2a, flattens due to the torsional buckling, as one can observe in a twisted hose.

![Figure 1.2](image1.png)

**Figure 1.2:** Torsional buckling of a thin metal shell (a) and a thick elastic shell (b). Red lines display the deformation of the vertical lines on the shell after torsion.
In metamaterials, local buckling of the unit-cells usually is used to activate global deformations leading to specific unusual behaviors or novel functionalities, such as negative Poisson’s ratio [15], multi-step pathways transitions [69], encapsulations [70], actuation [71]. On the other hand, in Chapter 4, we exploit local buckling of the unit-cells to suppress a global buckling; we discover that auxetic cylindrical metamaterials, when designed asymmetrically with respect to the torsional direction, induce uniform negative radial strain (radial contraction) upon torsion. The radial contraction coincides with a unique structural reconfiguration that leads to extreme resistance against torsional buckling, keeping the shell stable under large torsional angles.

Longitudinal compression of a 2D plate can initiate bending via out-of-plane buckling. In Chapter 5, we investigate the shape-changing and evolution of auxetic and non-auxetic metamaterial plates under bending.

1.6 Gaussian curvature in plates

Out-of-plane deflection (bending) behavior of plates strongly depends on parameters such as plate geometry, loading scenarios, and boundary conditions. Plates have a significantly smaller thickness compared to the other dimensions. Usually, the thickness-to-length ratio characterizes the plate type and determines the governing plate theory in conventional isotropic plates.

Kirchhoff–Love plate theory explains the deflection of plates with a thickness-to-length ratio smaller than 0.1, so-called thin plates. In thin plates, normal to the mid-surface remains straight and normal after deflection, denoting that transversal shear deformations are negligible [72, 73]. The deflection of thicker plates is explained by Mindlin–Reissner theory, which considers the transversal shear effects across the plate. [72, 73, 74]. Even though the shear deformations are present in thick-plates bending theories, the induced lateral shear effects (transversal shear along the width) still remain negligible in longitudinal bending of rectangular plates (applying curvature along the length), forbidding the formation of lateral curvature along the width (second curvature) [72, 73, 75]. However, a thick beam, with comparable width and thickness but much higher length, has been shown to obtain a second curvature, $\kappa_2$, that linearly developed as a function of the applied curvature (first curvature), $\kappa_1$, given by $\kappa_2 = -\nu\kappa_1$, where $\nu$ is the Poisson’s ratio of the composition [76]. The second curvature effect is less dominant in conventional systems, but for example, can be observed on the surface of a bent pencil eraser (Figure 1.3a). Positive Poisson’s ratio of conventional materials enforces a second curvature with an opposite sign with
respect to the first curvature. Consequently, a thick beam can only induce a saddle-like shape on its surface under bending, representing a negative Gaussian curvature. However, the formation of a dome-like surface with positive Gaussian curvature is expected in metamaterial with a negative Poisson’s ratio as proposed in [17].

![Figure 1.3: The first and second curvatures shown in a bent pencil eraser (a). The unit-cells used for designing the meta-plates (b).](image)

Deflection in structured plates can display different behaviors than conventional plates due to high deformability and large in-plane strains and stress fields. Generally, as in thick beams, Poisson’s ratio has significant effects on plate deflection behavior. In metamaterials, design ingredients can manipulate the Poisson’s ratio in a great span. To design structures with various curvatures, we use honeycomb-inspired patterns. We create unit-cells by gradually varying a hexagon cell to a bow tie-shaped (re-entrant) cell, by varying the control angle $\theta$, shown in Figure 1.3b [14]. A square array of such unit-cells makes rectangular cellular plates, which we refer to as meta-plates. Our designed meta-plates obtain Poisson’s ratios between $-1.35$ and $0.87$, while conventional materials can only exhibit positive Poisson’s ratios between zero and 0.5. In chapter 5, we extensively explore the effect of Poisson’s ratio on the evolution of the second curvature of auxetic and non-auxetic meta-plates under longitudinal bending. We propose a general relation that predicts the lateral curvature of meta-plates, with various geometrical and structural specifications, as a function of applied longitudinal curvature.

1.7 Overview of the thesis

As we generally explained, in this thesis, we aim to investigate specific systems in each chapter Chapters 2–5, with a focus on shear-induced/driven properties. Here, we provide an overview of the studied systems and our approaches.
1.7. OVERVIEW OF THE THESIS

1.7.1 Studied systems and approaches

In Chapter 2, we study metamaterials consisting of a network of non-uniform beams. We investigate the system experimentally and by spring modeling. The model provides a framework to study the system in a discrete limit and by continuum theory. In this research, we focus on the structural phase transitions and shear-driven evolution of the system. In Chapter 3, we investigate a metamaterial sharing the same structural ingredients as the system in Chapter 2. We focus on its mechanical properties, particularly shear and normal responses, and their programmability in a discrete limit. We study the system experimentally and using the model developed in Chapter 2. Structures in Chapter 2 and 3 are designed with unit-cells that provide self-contact transition under confinement. These unit-cells are inspired by a metamaterial made of a square array of closely packed voids with a four-fold symmetric contour carved out of a 2D sheet [19] and remaining forms a network of non-uniform horizontal and vertical beams. Self-contact transitions occur at the unit-cell level and lead to phase transition and significant changes in the behavior and mechanical response of the systems. In Chapter 4, we experimentally explore odd behaviors of asymmetric meta-shells under large torsional angles. Inspired by the holey sheet design [15], we create the meta-shells, where we implement the asymmetry by changing the orientation of the principal axes of the auxetic pattern with respect to the shell axis. In Chapter 5, we experimentally and by the FEM numerical simulations investigate the curvature induced on the surface of 2D meta-plates, with various geometrical and structural properties, upon longitudinal banding. We use a phenomenological model to study the evolution of the curvature in a 2D plate upon bending.

1.7.2 Experimental systems

For experimental studies, we mainly fabricate cylindrical structures suitable to apply shear via torsion, axial compression, and a combination of both deformations. Consequently, we focus on cylindrical shells with minimum and maximum radii of 7.5 and 12.5mm, respectively. The unit-cells are extruded through the shell thickness towards the shell axis. Therefore, the size of the unit-cells varies in depth to keep them compatible with the shell across the thickness. The order of magnitude of length for most shells is 10cm, but their length varies depending on the system. Also, the size of the unit-cells varies depending on the system, but their width and height are typically 1cm. For example, in Chapter 2, we tend to study the system on the verge of continuum limit, and, therefore, we
design some longer shells with smaller unit-cells. Nonetheless, in Chapter 5, we study 2D plate geometries with varying parameters such as thicknesses, widths, unit-cell sizes, wall thickness.

1.7.3 Structure of the thesis

In addition to the General Introduction (current chapter), this thesis consists of four research chapters (Chapters 2–5), a general discussion (Chapter 6), and summary. All research chapters (Chapters 2–5) contain Supplementary Information (SI) in the form of text, figures, or linked videos, included after the main text in each chapter. References are listed at the end of each chapter.
References


CHAPTER 1. GENERAL INTRODUCTION


REFERENCES


CHAPTER 1. GENERAL INTRODUCTION


Chapter 2

Shear-driven topological kinks
CHAPTER 2. SHEAR-DRIVEN KINKS

Abstract

Mechanical metamaterials can exhibit extraordinary properties with a topological origin, such as localized floppy modes at the boundaries that naturally exist in the quantum Hall effect and topological insulators. The boundary modes can steadily propagate as topological kinks by keeping their localized shape intact, representing a soliton-like nonlinear wave pocket described by the well-known $\varphi^4$ model. The propagation of such topological kinks is usually self-induced and beyond the scope of programmability. However, we display a shear-driven transformations of topological kinks in a mechanical lattice consisting of a network of bistable units. We show that the structural characteristic of our system results in the emergence of domains with different structural phases in the structure, separated by topological kinks and antikinks in a longitudinally confined system. Then we show that the kinks/antikinks travel in under a shear deformation (torsion) by altering the domain size and structural phase of the system. Depending on the direction and magnitude of shear deformation, different events such as kink/antikink transformation, creation, or annihilation contribute to the structural phase transition of the system. Our observations suggest that our system is potentially a mechanical counterpart to a ferromagnetic system, where our structural units act as mechanical hysteretic elements and shearing plays as an external magnetic field that similarly alters magnetization in ferromagnetic materials. Controllable kink/antikink transformations provide a flexible platform to program the structural phase and evolution of materials, useful for encoding data, information processing, and energy transportation in a mechanical system.
2.1 Introduction

Mechanical Metamaterials are often periodic mechanical lattices consisting of rationally designed structural units (unit-cells) that show extraordinary behaviors, unlike conventional materials [1, 2]. For example, a chain of elastically coupled rotors is shown to be a mechanical counterpart for the electronic topological insulator, which exhibits zero modes localized at the system boundary within a linear elasticity regime [3]. Moreover, in nonlinear deformation regimes, the system becomes a mechanical conductor, and the zero modes propagate in the system as a localized domain wall (kink) [4, 5], known as soliton that travels in the system with no energy loss and interacts with other solitons without changing its shape [6, 7]. The kink separates two distinct domains of the system before and after its location, and it essentially is a solution of the Klein-Gordon equation, which is the governing equation in a chain of coupled bistable units with a double-well potential, known as the $\varphi^4$ model [8, 9, 10]. The $\varphi^4$ model also allows a negative solution, which is known as antikink [11]. Although traveling wave pockets predicted by the $\varphi^4$ model show similar characteristics as solitons [6, 7], it should be noted that interaction between $\varphi^4$ kinks alters their eventual shape, unlike interactions between true solitons [11, 12]. Therefore, we refer to the solutions of the $\varphi^4$ model as "soliton-like" waves to avoid any possible confusion.

The emergence of topological kinks and propagation of solitary waves has recently been explored in mechanical metamaterials constructed of bistable unit-cells. It has been shown that an impact pulse propagates as a nonlinear elastic wave pocket in a network of bistable unit-cells in 1D [13, 14] or 2D [15]. In a chain of bistable unit-cells, the propagation of soliton-like wave pockets is prohibited for a specific impact amplitude range [16], and two oppositely propagating waves can either cross or reflect after colliding depending on the rotational direction of the impacts [17]. Formation of topological kink and its soliton-like propagation has also been observed in a 1D tristable [18] and 2D bistable [19] metamaterials. A periodic pattern of $\varphi^4$ kink-antikinks could emerge in a sufficiently long metamaterial, made of bistable unit-cells, under a lateral confinement [20]. Such a longitudinal periodic pattern forms a static soliton-like wave along the structure [20]. A static kink line can also emerge in a uniformly frustrated torus-shaped geometry, consisting of bistable unit-cells [21]. The kink line can be altered by an additional set of deformations that lead to a non-Abelian response [21]. The emergence of kinks and soliton-like waves provides a unique tool to locally control the structural phase of topological metamaterials and transfer energy and information within the system. Nonetheless, the discovered
topological kinks have been self-driven or virtually static and nearly impossible to control. However, here we present a metamaterial that exhibits topological kinks under axial confinement, which primarily are static but come to life and travel in the system via quasi-static shear (torsional) deformations. In Figure 2.1, we demonstrate a traveling kink line and phase transition in a cylindrical structure via torsional deformations. The bistable unit-cells exhibit a left-bucked state (blue) below a transitional kink and a right-bucked state (red) above it (Figure 2.1, middle). By applying a clockwise torsion on the cylinder, the kink moves upward by altering the state of the uni-cells, from a right-buckled to a left-buckled (Figure 2.1, right). On the other hand, under a counterclockwise torsion, the unit cells below the kink show a left-bucked to right-bucked state switching, and the kink moves downward (Figure 2.1, left).

Figure 2.1: Shear-driven solitary transitions: Schematic of A cylindrical system with different structural phases, blue (left bucked) and red (right buckled), separated by a transitional kink (middle). The kink travels upward under clockwise torsion (right) and downward under counterclockwise torsion (left) by altering the phase of the system.

We experimentally observe the emergence and transformation of the topological kink and present a spring model to realize the observations numerically in a discrete system and establish the soliton-like nature of the phenomena theoretically using the $\varphi^4$ model in a continuum system. We show that the large system exhibits multiple domains separated by topological kinks and antikinks that can experience creation, transformation, merging, and annihilation, via torsional (shear) deformations, qualitatively analogous to a ferromagnetic system.
Transformation of solitons occurs without energy loss; however, the kinks in our system travel by inserting work on the system, which is the cost of phase transition of the unit-cells and dissipates via snapping. We demonstrate the dissipative features of the system and study the properties of the kink width and its dependency on the system parameters.

Our study provides insight into the formation and evolution of programmable topological kinks in metamaterials and outlines an idea to harness kink propagation and effectively control the structural phase of the mechanical materials.

2.2 Modeling and the continuum solution

We create our metamaterial using a network of bistable beams with a nonuniform cross-section. The unit-cells of this system consist of a vertical beam that is thin at the nodes (hinges) and wide in the middle. The unit-cells can deflect at the nodes, but the deflection is confined due to the self-contact with the horizontal beams on both sides. We have previously shown that a metamaterial with similar properties of the unit-cells and self-contact transitions can exhibit unusual behaviors such as the inverted Poynting effect (compression-induced nonlinear shearing) and can effectively program the sign and magnitude of the Poynting response [22] (shear-induced perpendicular response) [23]. We fabricate our experimental platform by arrays of \( N \) vertical unit-cells and \( n \) radial unit-cells (\( N \)-layer structure) in a cylindrical coordinate, where the length of each unit-cell is \( a_0 \). Thus, the experimental systems are cylindrical shells with inner and outer radii of \( R_{\text{min}} \) and \( R_{\text{max}} \) and the height of \( h_0 = N a_0 \). Further details about the experimental system are provided in section 2.4.

Three factors can contribute to the energy of such unit-cells under deformations: A unit-cell can stretch/contract in length, deflect at the nodes, and additionally, it can exhibit a self-contact interaction with the horizontal elements. Consequently, each unit-cell under compression can take a right-buckled or left-buckled state, which we code red (0) and blue (1), respectively. To study the system analytically, we model the unit-cell by a straight spring with a pair of arc-shaped transversal springs placed at a distance \( r \) from the nodes and symmetrically spread by \( 2\alpha \), which are responsible for the self-contact interactions. Visit section 2.4 for more details. The potential energy of the unit-cell \( i \) is given
\[ E_i = \frac{1}{2} ka_0^2 e_i^2 + k_b \theta_i^2 + kr^2 \left( \frac{1}{2} (1 + \tanh \frac{\theta_i - \theta_c}{\Delta \theta_c}) \right) (\theta_i - \theta_c)^2 + kr^2 \left( \frac{1}{2} (1 + \tanh \frac{-\theta_i - \theta_c}{\Delta \theta_c}) \right) (-\theta_i - \theta_c)^2 \]

where \( k \) and \( k_b \) respectively are the spring constant and bending coefficient, \( e_i \) is the local strain, \( \theta_i \) is the deflection angle, and \( \theta_c \) is the self-contact angle. The last two terms of the equation introduce the self-contact potential in deflection to the right and left, respectively, where \( \Delta \theta_c \) represents the transition interval from no contact to the self-contact regime since the transition is not abrupt practically. In our experimental system, it can be rationalized and estimated by the difference in the contact angle between the inner and outer radius of the shell (see the Supplementary Information (SI), section 2.6.1).

Here, we obtain the local strain of the beams as a function of the deflection and axial strain, \( \delta_i \), by

\[ e_i = -1 + \frac{1}{\cos \theta_i} (1 - \delta_i) \]

If the structure is under the pre-compression strain of \( \delta = (h_0 - h)/h_0 \), where \( h \) is the height of the system after compression, we can write \( \delta_i \approx \delta + d\delta \), where \( d\delta \ll \delta \) and is negligible in our system. We write the dimensionless potential of the system by \( \xi = E_i/E_0 \), where \( E_0 = (1/2)ka_0^2 \). Using Taylor series, we obtain the potential of a 1D system as \( \xi \approx C_1 \theta^2 + C_2 \theta^4 + C_0 \), where \( C_0, C_1, \) and \( C_2 \) depend on the characteristics of the system and the pre-compression strain, \( \delta \). For the pre-compressed system, \( C_1 < 0 \) and \( C_2 > 0 \) that leads to a double-well potential, where the unit-cells experience two minima at \( \theta_0 = \pm \sqrt{-C_1/(2C_2)} \). We obtain the equilibrium equation in a continuum system using the Euler-Lagrange theorem as

\[ \frac{d}{d\xi} \left( \frac{1}{2} \left( \frac{\partial L}{\partial \dot{\theta}} \right) \theta \right) - \frac{d}{d\xi} \xi + \frac{d^2}{d\xi^2} \theta = 0 \]

where partial differential with respect to the variable \( x \) is given by \( \frac{d}{dx} := \partial_x \), \( k'_b \) is the second gradient coefficient, and \( \ell = z/a_1 \) is the dimensionless length, where \( z \) denotes the height, and \( a_1 = a_0(1 - \delta) \) is the lattice constant. Finally, the equilibrium equation is obtained as a form of the well-known Klein-Gordon equation [24]:

\[ \partial_{\ell\ell} \theta + (E_0 C_1/k'_b) \theta + (2E_0 C_2/k'_b) \theta^3 = 0. \]

The solution to this equation is given by

\[ \theta(\ell) = \pm \theta_0 \tanh \left( \frac{\ell - \ell_0}{\sqrt{2} \eta} \right), \]
were, $\theta_0$ is the equilibrium deflection of the unit-cells, $\eta = \sqrt{-k_b/(E_0C_1)}$, and $\ell_0$ specifies the location of the kink in the system; for a system with $N$ unit-cells $\ell_0 = (N+1)/2$, where the kink is in the middle. In this solution, the deflection of the unit-cells show a sharp transition from $\theta_0$ to $-\theta_0$ via a localized domain wall (kink) with the width of $w = 2\sqrt{2}\eta$. Extended details on theoretical calculations are provided in the SI, 2.6.1. This is a soliton-like solution in anharmonic 1D lattices that, similarly, consist of quasiparticles with double-well potential [9, 10].

Additionally, we can obtain the deflection and local strain of an 1D array of connected unit-cells by numerically minimizing the total energy of the system, given by $E = \sum_{i=1}^{N} E_i + \sum_{i=1}^{N-1} k'_b(\theta_i - \theta_{i+1})^2$. In the following sections, we will present and compare the experimental, continuum, and numerical results.

2.3 Results

2.3.1 Small system with a single kink

First, we study a minimal system consisting of 6 unit-cells in length to understand the behavior of a single kink under deformations. We initially pre-compress the structure, then shear (twist) it in positive (clockwise) and negative (counter-clockwise) directions. In Figure 2.2, we compare the experimental observations with the discrete and continuum solutions and show the shear-driven traveling kink in the system.

Figure 2.2a and b, show the experimental observations and the numerical solutions, respectively. First, we compress the structure without allowing any shear (twist) at the top and bottom boundaries and observe that the three bottom unit-cells buckle to the left, but the three top unit-cells buckle to the right (Figures 2.2a- and b-middle). To study the structural phase transitions and the topological kink in the system, we follow the deflection of the unit-cells along the structure, which shows a sharp transition from $-\theta_0$ to $\theta_0$. In Figure 2.2c, we compare the deflection profile (left) and unit-cells deflection angle (right) predicted by discrete model and continuum theory, which coincide well. Consistent with experimental and numerical results, the continuum solution ($\theta = \theta_0 \tanh(\frac{\ell-\ell_0}{\sqrt{2}\eta})$) shows that the unit-cells obtain a left-buckled profile before and a right-buckled profile after a domain wall, known as the topological kink. Emergence of kink is a result of coupling between bistable unit-cells as predicted in the $\varphi_4$ model [8, 9, 10]. We realize that the local strain of the springs, determined by the color, in the kink area is slightly higher (Figure 2.2b). For the sake of comparison, we highlight the approximate kink area in Figures 2.2a
Here, the kink emerges in the middle height of the system and splits the structure into two domains with different phases. However, we can change the boundary condition in our system via a torsional deformation (shear). Interestingly, we observe that by applying a torsional deformation on the cylinder, the kink travels within the system, upward under a clockwise (cw) torsion but downward under a Counterclockwise (ccw) torsion (Figure 2.2a). The solution of the discrete model coincides with these observations showing the same upward and downward traveling behavior of the kink under the negative or positive shear strains, respectively (Figure 2.2b). The negative and positive shear strains represent the cw and ccw torsions, respectively. When a kink travels, the unit-cells between the initial and final position of the kink undergo a phase switching from right-buckled (red) to left-buckled (blue) or vice versa. In Figure 2.2d, we show the phase of the structure as a function of the torsional angle rescaled by the transition torsion, the torsional angle (shear) that initiates a state switching of a unit-cell from $\pm \theta_0$ to $\mp \theta_0$, $\varphi_t$. The equivalent switching shear strain is given by $\gamma_t = R \varphi_t / \hbar$, where $R$ is the mean radius of the shell, $R = (R_{\text{min}} + R_{\text{max}})/2$. We can obtain the switching shear strain based on the geometrical parameters of the unit-cells for the shear required to drive a unit-cell from $\theta_0$ to $-\theta_0$, as $\gamma_t = 2 \tan \theta_0 / N$.

These observations implicate a soliton-like traveling kink solution for the system. Since the $\phi^4$ model is Lorentz invariant, the time-dependent solution is given by a Lorentz boost factor as $\theta(\ell, t) = \theta(\ell - Ct)$, where $C$ is the kink velocity. This represents that the shape of the solution cannot change in time, but a time-dependent term alters the phase of the equation of motion in a $\phi^4$ model [9, 10]. However, the kink dynamic in our system is shear-driven, unlike the self-driven kink. Therefore, we can obtain the shear dependent solution of the system by shear dependent Lorentz boost as $\theta(\ell, \gamma) = \pm \theta_0 \tanh(\ell - \ell_0 - \frac{ct}{\sqrt{2} \gamma})$, where $c$ is a constant value representing the rate of kink transformation under applied shear strain, $\gamma$. The phase shift by varying the boundary condition via a shear strain is an interesting feature that introduces an external control parameter useful for tuning the phase of the material. The traveling rate, $c$, is a counterpart for the velocity of a self-driven solitary wave. Since the kink travels by one unit-cell under $\gamma_t$, we can obtain the traveling rate of our system as $c = 1 / \gamma_t = N / (2 \tan \theta_0)$. Therefore, the traveling rate of the kink under shear depends on the intrinsic characteristics of the system. In Figure 2.2d-inset, we show the traveling kink under shear deformation in the continuum system.

The phase switching of the unit-cells occurs by a snap, where unit-cells
2.3. RESULTS

Figure 2.2: Soliton-like solution with a single kink: a) Phase switching of the unit-cells in a 6-layer structure, first compressed (middle) and then twisted in cw or ccw directions, demonstrating a traveling kink (yellow shade). b) Shear-driven traveling kink and the local strain of the unit-cells (color bar) in the discrete model. c) The deformation profile (left) and unit-cell deflection (right) along the pre-compressed structure: theory (solid line), discrete model (dashed line and disk), and experiment (cross). d) Phase switching from right-buckled (red) to left-buckled (blue), and vice versa, upon shearing in the experiment and discrete model; the same phase plot for the continuum theory in the same ranges. e) Shear force as a function of torsional angle in the experiment (bottom, $\delta = 0.14$) and discrete model (top, $\delta = 0.2$).
pass the energy barrier at zero deflection \( \theta = 0 \) and switch to the opposite minimum potential. The shear force shows a maximum at the zero deflection and decreases after the unit-cell passes the energy barrier. Consequently, we observe an oscillatory shear force response as a function of torsional (or shear) deformations in discrete systems (Figure 2.2e). It should be noted that we cannot obtain the oscillatory responses in a continuum model, as this is a primary limitation of continuum systems.

Although we obtain a single kink in a small system, a large system can exhibit a set of kink-antikink in the system. An antikink is a negative solution in the \( \varphi^4 \) model and represents a reverse kink, where the structural phase changes from a right-buckled to a left-buckled, before and after antikink, respectively. We study a large system to understand the behaviors of the system with several kinks and antikinks and their interactions.

### 2.3.2 Kink-antikink creation, annihilation, and interaction

To investigate the emergence of multiple kinks/antikinks in the system, we study a large system with \( N = 18 \) unit-cells in length. The experimental structure, the 18-layer cylinder shown in Figure 2.3a, has the same minimum and maximum radii as the 6-layer cylinder, but the unit-cells here are smaller, and each layer contains \( n = 12 \) unit-cells around the perimeter. We draw vertical black lines, connecting the initially vertical elements of the unit-cells, to visualize the local alignment of the structure before and after deformations. By pre-compressing the structure, while torsion (shear) is not allowed at both top and bottom clamps, several regions with right-buckled or left-buckled structural phases appear along the system (Figure 2.3b). Two antikinks (highlighted by a green shade) and one kink (highlighted by a yellow shade) between the antikinks separate different regions of the structure as domain walls. We confirm appearance of multiple kinks-antikinks in the discrete model by numerical minimization (Figure 2.3b-right). These observations represent a static form of soliton-like transitions in the \( \varphi^4 \) model. The static wave in our system is similar to the static waves in a 2D mechanical metamaterial that is sufficiently long and compressed laterally [20]. However, in our system, the confinement is applied in the same direction as the structure is longer. As in the small system (Figure 2.2), we can transfer the kinks along the structure by simply shearing the top boundary.

In order to experimentally study the transitions, we twist the structure between two extreme torsional angles and follow the transitions as demonstrated in Figure 2.3c. Initially, the structure is pre-compressed and twisted to the extreme counterclockwise direction, by \( \varphi = -9 \varphi_t \simeq -4.5 \text{rad} \) Figure 2.3c-left.
2.3. RESULTS

Then we slowly twist the cylinder in the clockwise (cw) direction, under a fixed gap through i–v in Figure 2.3c-top, to its extreme opposite angle as \( \varphi = -9\varphi_t \simeq 4.5\text{rad} \) (Figure 2.3c-right). Finally, we return the structure to the initial state with a counterclockwise (ccw) torsion through vi–x in Figure 2.3c-bottom. The torsions are applied at a low rate (1mrad/s) to ensure a quasi-static process during the deformations. Since the structure is composed of 18 unit-cells in height, a complete cycle of cw-ccw torsions occurs in \( 2 \times 18 \) steps. The system experiences a local minimum potential at each step. In cw torsion, the initially right-buckled unit-cells consecutively switch to the left-buckled phase in a non-intuitive sequence. On the other hand, in ccw torsion, the unit-cells switch their phase from the right-buckled to the left-buckled. In the process of shearing, a series of unusual events occur by the creation and annihilation of kink-antikink pairs and their traveling along the system, which we discuss in the following.

**Shear-driven phase transitions:** In the first step of cw torsion, we observe the creation of a kink-antikink pair at the middle height of the cylinder due to the phase switching of the unit-cells in the 8th layer to the left-buckled (Figure 2.3c-i). In this stage, there is a single layer with an opposite phase almost at the middle of the structure. By increasing the cw torsion, the kink-antikink pair propagate in opposite directions along the cylinder (Figure 2.3c-ii). The size of the newly created region with the left-buckled phase increases due to kink-antikink transformations. The kink (or antikink) disappears when it reaches the boundary of the system (Figure 2.3c-iii). We observe creation of a new pair of kink-antikink in Figure 2.3c-iv. However, by increasing the cw torsion, the bottom antikink approaches the kink above it, and they finally merge and annihilate (Figure 2.3c-v). Similar transitions occur in the ccw torsion, shown in Figure 2.3c-bottom (vi–x). The transitions start with a kink-antikink creation (Figure 3b-vi) followed by a second pair of kink-antikink created below the first one, which we call a double-pair (Figure 3b-vii). Immediately in the next step, the kink from the bellow pair and the antikink from the top pair annihilate (Figure 3b-viii). Then the remaining kink and antikink propagate in opposite directions and finally reach the system boundaries, in Figure 2.3c-ix and x. As a trivial consequence of having two domain types, a kink can only be followed by an antikink along the structure and vice versa. In a full picture, we present all transitions by the color-coded phase of the unit-cells as a function of rescaled torsional angle under cw and ccw torsions, respectively, in Figures 2.3d and e. Additionally, in Video SI 1 (Supplementary Information 2.6.5), we present the dynamics of system in cw-ccw cycle. Here, the border between the red and blue colors shows the existence of kink/antikink, and we can follow its traveling and
interaction with other kinks/antikinks. We can observe the creation of domains in the system and a substantial difference in the phase changes between cw and ccw torsions, where the phase switching events are opposite but similarly initiate with intermediate unit-cells. The number of domains in the system depends on the kink-antikink interaction and the applied shear deformation. To obtain an insight into the number of domain and their evolution, we investigate the energy landscape of a continuum system with a kink-antikink pair, discussed in the next paragraphs.

**Kink-antikink interaction:** To study the interactions between a kink and an antikink, we use the theoretical solution of the system with a kink and an antikink, given by \( \theta'(\ell) = \theta_0(\tanh(\frac{\ell-d/(2a)}{\sqrt{2}\eta}) - \tanh(\frac{\ell-d/(2a)}{\sqrt{2}\eta}) - 1) \), where \( d \) is the kink-antikink separation, which are located at \( \pm d/2 \). To obtain the kink-antikink interaction potential, we numerically calculate the energy of a sufficiently large system \((-15 \leq \ell \leq 15)\) as a function of \( d \) given by \( \xi(d) = \int_{-15}^{15} \xi(\theta'(\ell,d))d\ell \). We subtract the energy of the system with two kinks sufficiently far \( \xi(d=20) \), where their interaction is negligible, from the total energy to obtain the pure kink-antikink interaction energy rescaled by \( E_0 \) as \( \Delta E/E_0 = \xi(d) - \xi(d=20) \), and plot it as a function of \( d/a \) for two pre-compression strains \( (\delta = 0.14 \text{ and } 0.2) \), shown in Figure 2.3f-top. We numerically calculate the kink-antikink interaction force, given by the rescaled force, \( f = a_1F/E_0 = a_1\partial\Delta E/\partial d \), plotted in Figure 2.3f-below. We observe that the kink-antikink repel each other when they are more than two unit-cells apart. However, their interaction is short-range since \( \Delta E \) and \( f \) quickly decline to zero, where they are separated by four unit-cells, \( d/a_1 \approx 4 \). This distance represents the typical domain size of the system and half wavelength of the static soliton-like wave, which is consistent with our observations in Figure 2.3b, although we neglect the effect of boundaries in our calculation. By comparing two curves for \( \delta = 0.14 \) and \( 0.2 \), we note that the domain size is insensitive to the pre-compression strain. On the other hand, the system shows a negative energy and attraction force when the kink and antikink are almost one unit-cell apart \( (d/a_1 \approx 1) \), which can explain the creation of kink-antikink as neighboring pairs in Figure 2.3c.

**Oscillatory response with numerous ground states:** The torque response of the cylinder shows an oscillatory behavior as a function of the torsional angle in both cw and ccw torsions (Figure 2.3f). This oscillatory response occurs due to the snapping of the uni-cells when switching their buckling phase. Each minimum represents a distinct ground state of the system. As we can interpret from the oscillatory torque response, phase switching of the unit-cells occur by inserting energy via applying torque to the system. Although, this energy
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Figure 2.3: A Large system with multiple kinks/antikinks: a) An undeformed 18-layer experimental system. b) The pre-compressed experimental system (left) and discrete model (right, color shows the local strain) showing two antikinks and one kink. c) Evolution of the system upon torsion between its two extreme torsional angles via creation, traveling, and annihilation of kinks and antikinks. d–e) Structural phase evolution upon clockwise (d) and counterclockwise (e) torsions. f) Interaction energy (top) and force (bottom) between a kink and an antikink. g) The experimental torque response as a function of torsional angle.
dissipates in the system after the transition.

*An analogy with ferromagnetic systems:* Our observations suggest that our system is potentially a mechanical counterpart to a ferromagnetic system. Spins tend to align in ferromagnetic materials, similar to the unit-cells in our structure. When our structure is pre-compressed, the unit-cells form different regions with right-buckled or left-buckled states (Figure 2.3b), similar to a ferromagnetic system with local alignments of the spins in several domains. In both systems, domains are separated with localized kinks (domain walls) that can travel in the system and alter the domain size and structural phases. Applying a shear deformation increases the domain size with compatible unit-cell states, comparable with ferromagnetic domain size variation when exposed to an external magnetic field [25]. Similarly, traveling domain walls in a ferromagnetic system are shown to behave as solitary waves [26, 27]. Moreover, oscillatory response due to snapping of the unit-cells in our system is comparable with discontinuous magnetization leading to the Barkhausen effect in ferromagnetic materials [28].

Switching the state of a ferromagnetic element between two opposite magnetization values occurs in an irreversible hysteresis loop. Such a hysteresis also exists in phase switching of a unit-cell in a cyclic shear deformation, from a left-buckled to right buckled and back to the left-buckled. Cyclic shearing coincides with energy dissipation in the loop. In the next section, we investigate the dissipative features of the system, kink properties, and their relationship.

### 2.3.3 Energy dissipation and kink properties

In order to quantify the work needed for phase switching of the unit-cells and the energy dissipation in the system, we conduct a torque-driven cyclic torsion experiment on a 1-layer structure, under constant pre-compression strains, where we can capture snapping of a single layer. In Figure 2.4a, we show the torque response of a 1-layer structure as a function of torsional angle under three pre-compression strains. The area inside each hysteresis cycle represents the amount of energy loss (dissipation), $H$, in the cycle. In Figure 2.4b, we plot the dissipated energy in a cyclic deformation as a function of the pre-compression strain, $\delta$. We observe that the dissipation is almost zero in low pre-compression strains but dramatically increases in the self-contact regime.

In a large system with kink, the dissipation coincides with kink transformation since the applied energy by shear deformation dissipates every time a kink travels by one unit-cell while switching the unit-cell phase. To understand the energy dissipation in the continuum system, we need to quantify the contribution of the kink to the total energy, $E_k$. To this end, we subtract the total energy without a
Figure 2.4: Dissipation and kink properties: a) Hysteresis in the torque response of a 1-layer cylinder in cyclic torque-driven shear deformations under different pre-compression strains and (b) its dissipation energy, $H$, as a function of pre-compression strain, $\delta$. c) Contribution of a single kink to the energy of the system as a function of pre-compression strain, $\delta$. d–f) Kink width as a function of compression strain (d), rescaled spring constant (e), and rescaled bending coefficient (f) of the unit-cells. Here, $k_0$ and $k_{b0}$ are the spring constant and bending coefficients of the 6-layer and 1-layer experimental systems.

Kink width, $w$, is a deterministic characteristic of a $\varphi^4$ system and is associated...
associated with the energy landscape of our systems. We can tune the kink width by varying the pre-compression strain or internal parameters, such as spring constant and bending coefficients. In Figures 2.4d–f, we show nonlinear relationships between kink width rescaled by the lattice constant, \( w/a_l \), as a function of compression strain, spring constant, and bending coefficient, respectively. The spring constant and bending coefficient are rescaled by their values for the 6-layer experimental system, here labeled by \( k_0 \) and \( k_{b0} \), respectively. We observe that the kink width decreases dramatically as a function of the compression strain (Figure 2.4d) and the beams spring constant (Figure 2.4e). However, increasing the bending stiffness of the unit-cells leads to a significant increase in the kink width (Figure 2.4f).

In contrast to a solitary wave that allows the transformation of an impact force with no energy loss, our platform dissipates the impact force, where we can program the dissipative characteristics of the system using a pre-compression strain or intrinsic stiffness of the unit-cells. See SI 2.6.3 for further details on the energy distribution in the system.

2.4 System and methods

**Experimental system:** The unit-cells of our structures consist of a vertical beam with a nonuniform cross-section profile that linearly increases from zero at the nodes (extremes) to a maximum at the middle. Such a profile shape leads to a rhombus-like beam shape with the angle \( 2\alpha \) at the nodes, as shown in Figure 2.5a–unit-cell(2D). Therefore bending is favorable at the nodes, but the beam keeps its profile shape under bending. However, bending is confined on both sides by vertical elements with the same profile shape and angle, \( \alpha \). Additionally, as depicted in the enlarged section in Figure 2.5a-unit-cell(2D), we use small half-circles with a suitable radius at the nodes to keep the nodes connected and obtain a minimum thickness of \( t = 0.7 \text{mm} \) at the hinges (\( t = 0.4 \) for the taller structure). In order to create the unit-cell in a cylindrical coordinate, we extrude the 2D shape in the radial direction. To create the structures, first, we generate a 2D void shape, which is the shape that fills the area between the unit-cells in 2D. Then we extrude the void shape in the radial direction, which leads to the creation of a 3D unit-cell (Figure 2.5a-unit-cell(3D)). Then we generate a circular array of \( n = 8 \) (\( n = 12 \) in the 18-layer) voids around the perimeter, in 6 and 18 layers in length. We carve it out of a bulk shell to obtain our desired structure with the height of \( h_0 = Na_0 \). Additionally, we generate two half-layers at both sides and clamp the structure with two flat rings with the same inner
and outer radii as the cylindrical shell and a height of 3mm (Figure 2.5b).

**Modeling:** To model the system, we use a vertical spring with the length of \( a_0 \) that consists of a pair of arc-shaped parts placed at the distance \( r \) from the nodes and symmetrically spread by \( 2\alpha \). The vertical spring can bend at the edges but stays straight along the length. However, the bending is confined at both sides of the arc-shaped springs (with the angle of \( \alpha \)) connected to the horizontal elements, as shown in Figure 2.5c-unit-cell(model). Under a pre-compression, the unit-cell experiences a double-well potential (equation 2.1), shown in Figure 2.5d for a unit-cell, with the same values for the variables as in the experimental system, under the pre-compression strain of \( \delta = 0.14 \). We theoretically and numerically study a 1D array of unit-cells, where each unit-cell is elastically coupled by neighboring unit-cells through the second gradient potential, given by \( E^{\text{int}}_i = k'_b ((\theta_i - \theta_{i-1})^2, \) where \( i \) labels the unit-cell and \( k'_b \) is the second gradient coefficient. We previously used a similar model without considering a second gradient interaction to study the normal and shear responses in a similar system [23].

**Methods:** We create the 3D CAD model using Blender (open source software) and 3D print the structure using a Formlab Form2 3D printer and elastic resin v1, with a 0.1mm printing resolution. We color the outer surface of the 3D printed structures using a marker for better visualization of design and deformations. Additionally, we draw vertical lines that connect the vertical unit-cells on the 18-layer structure to visualize the deflection and structural phases under deformations. The structures are confined using hard cylindrical cores placed in the middle of the shells to avoid possible side-bucklings under deformations. See SI, section 2.6.4 for complementary details on the experimental setup. The cores are entirely lubricated to minimize friction. We perform deformations on the structure using Anton Paar 302 (or 305) rheometers with minimum accuracies of 1 \( \mu \)m (longitudinal displacements), 0.05 \( \mu \)rad (angular displacements), 0.005 N (normal forces), and 1 nN m (torques).

We investigate the modeled system numerically by minimizing the energy function using the Nelder–Mead method, which allows escaping from local minima to obtain the global minimum. We use Mathematica 11 for numerical calculations, symbolic analysis, and relevant visualizations. We use Python libraries such as Pandas, Plotly, and Scipy in Spyder IDE to analyze and visualize experimental and numerical data.

**Numerical constants:** We obtain the numerical values based on the experimental systems and measurements. Spring constant is calculated based on the effective Young’s modulus (slope of stress-strain curve), \( Y_{eff} \), from the compression test in the pre-buckling regime, where the response is linear. The equivalent
spring coefficient of the cylinder with $n$ unit-cells around the perimeter and $N$ unit-cells in the length is given by $K = Y_{eff}A_s/(Na_0)$, using which we obtain the spring coefficient of the unit-cells as $k = KN/n = Y_{eff}A_s/(na_0)$. For the experimental system with $N = 6$, we obtain the effective Young’s modulus as $Y_{eff} \simeq 0.19Y$, where $Y \simeq 3.6 \times 10^6$Pa is Young’s modulus of the elastomer, calculated by compression test in a buck shell and a bulk cube. Consequently, the value of the spring coefficient is given by $k = 2655$ N m$^{-1}$. However, the experimental system with $N = 18$ has smaller unit-cells with $n = 12$ unit-cells at each layer, and obtain different values of $Y_{eff} \simeq 0.28Y$ and $k = 3912$ N m$^{-1}$. The onset of the transition to the buckling regime, $\delta_b$, depends on the bending and second gradient coefficients, which is obtained as $\delta_b = 0.03$ (0.034) for the structure with $N = 6$ (18) layers (more details in SI 2.6.2). We assume that the bending coefficient and second gradient coefficient to hold the same value since they have the same origin, and we set them accordingly to obtain same values for $\delta_b$ as in the experiments; we obtain $k_b = k'_b = 0.0029$ N m for the structure with $N = 6$, and nearly the same value, $k_b = k'_b = 0.0027$ N m, for the structure with $N = 18$. 

Figure 2.5: The system and its spring model: a) 2D (top) and 3D (top) schematic of the designed unit-cell. b) The CAD image of the 6-layer structure. c) The spring model of the unit-cell undeformed (left) and deformed (right). d) The double-well potential of a unit-cell under $\delta = 0.14$ compression strain.
2.5 Conclusion

We introduce a system that exhibits different structural domains under longitudinal compression due to structural phase transitions, explained by the $\varphi^4$ model. The structural domains are separated by topological kinks/antikinks that can travel along the structure via a shear deformation that coincides with energy dissipation. Shearing can create kink-antikink pairs or result in their annihilation when they meet in the system. By a qualitative comparison, we show that our topological system can act as a mechanical counterpart for ferromagnetic materials since both systems acquire comparable structural ingredients, and the behaviors observed in our system similarly occur in ferromagnetic materials. Our structure obtains numerous ground states under shearing, making a diverse number of global states accessible that can be potentially harnessed for designing pathways and a mechanical memory.

We highlight the shear-driven traveling topological kink as a novel aspect of mechanical metamaterials that can provide a platform for programming structural phase, transpiration, and information coding and processing in mechanical materials. Such characteristics can have implications in building soft structures with diverse functionalities, particularly useful in soft robotics and biomechanics. For example, a multi-stable programmable cylinder can be a functional robot arm, or a mechanical memory unit can provide a substrate to design interactive materials that can interact with the environment, learn, and remember.
2.6 Supplementary Information (SI)

2.6.1 Details of the continuum theory

on-site potential of a unit-cell

Each unit-cell is composed of a vertical beam with an initial length of $a_0$ that can contract or stretch, given by beam length times local strain ($a_0 e$), and bend at the nodes (extremes) by $\theta$, and experience contact with the horizontal elements at $\theta = \theta_c$. Therefore, we can write the on-site potential of the unit-cell $i$ by

$$E_i = \frac{1}{2} k a_0^2 e_i^2 + k_b \theta_i^2 + k r^2 H[|\theta_i| - \theta_c](|\theta_i| - \theta_c)^2,$$

where, $k$ and $k_b$ are the spring constant and bending coefficient of the unit-cells, respectively. The self-contact potential is introduced by the elastic energy due to the contraction of the arc-shaped parts times a Heaviside step function, $H$, which is 0 for $x < 0$ and 1 for $x \geq 0$. We introduced this energy to study the shear and normal force responses in a similar system with a nonuniform beam network and showed that it is capable of reproducing the experimental results in close agreement [23].

In this potential equation, we assumed that the transition to the self-contact regime is discontinuous by using the Heaviside step function in the contact potential. However, in a real system, a transition from a no-contact to the self-contact regime is continuous and can be estimated by a realistic function of the form of $\tanh$ as

$$H[|\theta_i| - \theta_c] \approx \frac{1}{2} (1 + \tanh \frac{|\theta_i| - \theta_c}{\Delta \theta_c}),$$

where, $\Delta \theta_c$ the width of transition from 0 to 1 at the contact angle, $\theta_c$. In our designed system, we can calculate the contact angle by $\theta_c = \pi/2 - (2\alpha)$. Since the experimental systems are cylindrical, the beams are closer to each other on the inner surface of the shell compared to the outer surface. Thus, we can estimate $\Delta \theta_c \approx 0.09 \text{ rad}$ based on the geometry (obtained from the CAD model). However, the continuous transition to the contact regime also has a secondary origin; since all elements in our system are soft, it is trivial that the contact area increases by bringing two elements closer to each other. Therefore, the contribution of contact in the energy of the system will increase continuously from zero to a maximum, even for a 2D geometry. We note that the potential equation is discontinuous at $\theta = 0$ due to having $|\theta_i|$ in the equation. Even
though the terms including $|\theta_i|$ are negligible at $\theta \simeq 0$, this can raise errors when obtaining the derivatives of the potential. To fix this issue, we include the potentials due to contact under deflection to each side separately. Thus, we obtain equation 2.1.

We define the compressive strain of the beam $i$ by $\delta_i = \delta a$, where $\delta a = a_0 - a$ and $a$ is the vertical length of the deformed unit-cell. We obtain the unit-cell strain, $e_i$, as a function of the compressive strain as

$$e_i = -1 + \frac{1 - \delta_i}{\cos \theta_i}. \quad (2.6)$$

Additionally, we scale the energy term with $E_0 = \frac{1}{2} k a_0^2$ to obtain the dimensionless form of the equation as:

$$\xi = \frac{E_i}{E_0} = (-1 + \frac{1 - \delta_i}{\cos \theta_i})^2 + \frac{k b}{E_0} \theta_i^2 + \left(\frac{r}{a_0}\right)^2 \left((1 + \tanh \frac{\theta_i - \theta_c}{\Delta \theta_c}) \right) \left(\theta_i - \theta_c\right)^2 + \left(\frac{r}{a_0}\right)^2 \left((1 + \tanh -\frac{\theta_i - \theta_c}{\Delta \theta_c}) \right) \left(-\theta_i - \theta_c\right)^2 \quad (2.7)$$

We use Taylor series to simplify the equation as $\xi \simeq C_1 \theta^2 + C_2 \theta^4 + C_0$ (Figure 2.6), where:

$$C_1 = (k_b/E_0) + 2(r/a_0)^2 + \delta^2 - \delta - 2(r/a_0)^2 \tanh \left(\frac{\theta_c}{\Delta \theta_c}\right)$$

$$- \frac{2\theta_c^2 (r/a_0)^2 \tanh^3 \left(\frac{\theta_c}{\Delta \theta_c}\right)}{\Delta \theta_c^2} + \frac{2\theta_c^2 (r/a_0)^2 \tanh \left(\frac{\theta_c}{\Delta \theta_c}\right)}{\Delta \theta_c^2}$$

$$- \frac{4\theta_c (r/a_0)^2}{\Delta \theta_c} + \frac{4\theta_c (r/a_0)^2 \tanh^2 \left(\frac{\theta_c}{\Delta \theta_c}\right)}{\Delta \theta_c}, \quad (2.8)$$
\[ C_2 = \frac{1}{4} - \frac{55\delta}{60} + \frac{2\delta^2}{3} \]
\[ + \frac{r^2\theta_c^2 \left( \cosh \left( \frac{2\theta_c}{\Delta\sigma_c} \right) - 5 \right) \tanh \left( \frac{\theta_c}{\Delta\sigma_c} \right) \text{sech}^4 \left( \frac{\theta_c}{\Delta\sigma_c} \right)}{3a_0^2 \Delta\theta_c^4} \]
\[ - \frac{4r^2\theta_c \left( \cosh \left( \frac{2\theta_c}{\Delta\sigma_c} \right) - 2 \right) \text{sech}^4 \left( \frac{\theta_c}{\Delta\sigma_c} \right)}{3a_0^2 \Delta\theta_c^3} \]
\[ + \frac{2r^2 \tanh \left( \frac{\theta_c}{\Delta\sigma_c} \right) \text{sech}^2 \left( \frac{\theta_c}{\Delta\sigma_c} \right)}{a_0^2 \Delta\theta_c^2} \] (2.9)

and

\[ C_0 = \delta^2 - 2\theta_c^2 (r/a_0)^2 \left( -1 + \tanh \left( \frac{\theta_c}{\Delta\sigma_c} \right) \right) \] (2.10)

Figure 2.6: The double-well potential of the modeled system and its estimation (dashed curve) as a function of deflection under the pre-compression of \( \delta = 0.14 \).

The axial confinement applied by a constant compressive strain can transfer the system from a stable state in zero deflection to an unstable state for \( \delta > \delta_b \), where \( \delta_b \) is the onset of buckling under compression. We can theoretically predict the onset of buckling by solving \( \frac{\partial^2 \xi(\theta,\delta)}{\partial \theta^2} \big|_{\theta=0} = 0 \). For a single unit-cell of our system we obtain \( \delta_b = 0.024 \), comparable with the experimental value of \( \delta_b = 0.029 \) for the 6-layer structure.
Interaction potential and the continuum limit

To study the system in a continuum limit we need to include the second gradient effect that reflects the interactions between each unit-cell with the neighboring unit-cells. We assume that such interaction can only happen through deflections and relative shearing are negligible. Thus the rescaled interaction potential is given by

\[ E_{int}^i / E_0 = \left( k_b' / E_0 \right) (\theta_i - \theta_{i+1})^2 \]

where \( k_b' \) is the second order bending coefficient. Thus, the total potential of a unit-cell is given by

\[ L = \left( k_b' / E_0 \right) (\theta_i - \theta_{i+1})^2 + \xi_i \]

In equation 2.7, \( \xi_i \) depends on both \( \theta_i \) and \( \delta_i \). However, we can write it as \( \delta_i = \delta + d\delta_i \), where \( d\delta_i = 0 \) in pre-buckling regime and \( d\delta_i \ll \delta \) in our system. Since all the terms including \( d\delta \) in the Taylor series are smaller than \( \delta d\delta_i \) we neglect them in the equation. We numerically confirm that \( O(d\delta^{-2}) \sim O(\delta) \) in a 6-layer system.

In a continuum limit for our system with a lattice constant of \( a_l \),

\[ L = \left( k_b' / E_0 \right) (a_l \partial_z^2 \theta(z))^2 + \xi(z) \]

Using the Euler–Lagrange theorem, \( \partial L / \partial \theta(z) = 0 \), we can write the equation of equilibrium as

\[
\begin{align*}
(2a_l^3 k_b' / E_0) \partial_{zz} \theta(z) + 2C_1 \theta(z) + 4C_2 \theta^3(z) & = 0, \\
(2.11)
\end{align*}
\]

which can be written in a dimensionless form by \( z \rightarrow \ell := z / a_l \) and \( \theta(\ell) \rightarrow \theta \), and finally simplified as:

\[
\partial_{\ell\ell} \theta + \frac{E_0 C_1}{k_b} \theta + \frac{2E_0 C_2}{k_b} \theta^3 = 0, \tag{2.12}
\]

This equation is a form of Klein-Gordon equation, and can be analyzed in different regimes for our system.

Solutions

**Buckling regime:** We can study the system in different regimes of deformations. The pre-buckling regime is uninteresting since the deflections are zero and the beam configuration does not change. In the buckling regime, \( \theta^3 \ll \theta \), thus we can discard the \( \theta^3 \) term in the governing equation 2.12. In this regime, the beam behaves like a conventional Euler beam, and the deflection along the height has a sinusoidal form,

\[
\theta(\ell) = \alpha \theta_0 \sin\left( j \frac{\ell - \ell_0}{\eta} \right), \tag{2.13}
\]

where \( \eta = \sqrt{-k_b' / (E_0 C_1)} \) and \( \theta_0 = \pm \sqrt{-C_1 / (2C_2)} \). The amplitude is determined by \( |\alpha \theta_0| \) and \( j \) is a positive integer: \( j = 1 \) provides a solution for the first
mode of Euler buckling and we can obtain higher modes for $j \geq 2$. Finally, $\ell_0$ depends on the boundary condition, and if the structure has the length of $l$ and only subjected to compression, $\ell_0 = l/2$.

Self-contact regime: When the structure is compressed to the self-contact regime unit-cells experience a double-well potential. The primary solution is as follows

$$\theta(\ell) = \pm \theta_0 \tanh\left(\frac{\ell - \ell_0}{\sqrt{2}\eta}\right),$$

which extensively is discussed in the main text and compared with the experimental and numerical results. This solution predicts the existence of a single kink in the system, however, a general solution does not limit the system to a single kink and can be extended to a more general form by introducing multiple terms with the form of $\tanh$ to the equation, as is done in the main text to study a large system with multiple kinks, consist with the experimental observations. However, a general solution can be written in terms of elliptic functions that fully captures the properties of the system [9, 10], as also shown to predict soliton-like transitions in mechanical metamaterials [20]. Investigating such general solutions could be useful to provide a more comprehensive explanation in large systems, however, this is beyond the scope of this study.

Deflection profile: We can also obtain the deflection profile of the structure along its length, given by

$$\frac{x}{a_l} = \sqrt{2}\theta_0 \eta \ln(\cosh(\frac{\ell - \ell_0}{\sqrt{2}\eta})) + \text{const.}$$

We compare this solution with the deflection of the structure obtained numerically and observed in the experimental system in Figure 2.2c.

2.6.2 Compression response

We obtain the numerical constants using the slope of the stress-strain curve in the linear pre-buckling regime and the onset of transition to the buckling regime, $\delta_b$, in the compression tests. In Figure 2.7a, we show the stress rescaled by Young’s modulus of the elastomer times the number of unit-cells at each layer, $n$, for both 6-layer and 18-layer structures. We observe that the response in the pre-buckling regime nicely coincides. It clearly shows that the effective Young’s modulus of the structures depends on the number of unit-cells per layer rather than the unit-cell size. The onset of the buckling regime for both structures coincide as well ($\delta_b \approx 0.03$). The onset of the buckling regime depends on the
design parameters, particularly the minimum thickness of the hinges, $t$, shown in the enlarged image in Figure 2.7b. For 6-layer structure $t = 0.75$ mm, and for 18-layer structure $t = 0.4$ mm. To roughly obtain the same value for $\delta_b$, the length the thinnest section is slightly increased in 18-layer structure as $t' = 0.02$ mm. Note that $t' = 0$ in 6-layer structure.

![Figure 2.7: a) The applied stress, $\sigma$, rescaled by the number of unit-cells per layer times Young’s modulus, $nY$, is plotted as a function of the compression strain, $\delta$. b) Unit-cell and hinge design (enlarged) for the 18-layer structure.](image)

### 2.6.3 Energy distribution along the system

We can obtain the energy distribution along the structure using the solution for $\theta$ in the energy function, $E(\theta)$. In Figure 2.8a, we show the energy distribution along the structure as a function of the unit-cell index. We observe that in the vicinity of the kink location, $i = 3$ to 4, the potential shows a sharp peak. The peak raises by increasing the pre-compression strain, as we observe by comparing the peak for the systems with $\delta = 0.14$ and $\delta = 0.2$. The unit-cells in the kink area experience higher energy. In Figure 2.8b, we show the energy within the kink domain wall, $E_w$, as a function pre-compression strain, where $E_w = \int_{\ell_0-w/2}^{\ell_0+w/2} E(\ell)d\ell$ and $\ell_0 = 3.5$.

### 2.6.4 Experimental setup

We perform a combination of compression and torsions tests using a rheometer. In Figure 2.9, we show the designed setup for the experimental procedures. We
use the designed clamps that enclose the structures. Four holders at each side tighten the top and bottom ends of the structure to the clamps to avoid any sliding at the boundaries. Clamps and holders are 3D printed using Formlab Form2 3D printer and tough resin. Metal nuts are placed in the designed voids inside the clamps and holders to be able to tighten the holder to the clamps by screws. The inner core assures that the side deflection of the shells is prohibited. The inner cores are entirely lubricated to minimize friction and allow smooth sliding.
2.6.5 Supplementary Video

The supplementary video is downloadable via the provided link.

**Video 1** Traveling kinks and antikinks in the large system under cw-ccw cycle: Click here or scan the code.

![Supplementary Video Link](QR_CODE)
CHAPTER 2. SHEAR-DRIVEN KINKS

References


Chapter 3

Inverted and programmable Poynting effects
Abstract

The Poynting effect generically manifests itself as the extension of the material in the direction perpendicular to an applied shear deformation (torsion) and is a material parameter hard to design. Unlike isotropic solids, in designed structures, peculiar couplings between shear and normal deformations can be achieved and exploited for practical applications. Here, we engineer a metamaterial that can be programmed to contract or extend under torsion and undergo nonlinear twist under compression. First, we show that our system exhibits a novel type of inverted Poynting effect, where axial compression induces a nonlinear torsion. Then the Poynting modulus of the structure is programmed from initial negative values to zero and positive values via a pre-compression that generates a kink in the system. Our work opens avenues for programming nonlinear elastic moduli of materials and tuning the couplings between shear and normal responses by rational design. Obtaining inverted and programmable Poynting effects in metamaterials inspires diverse applications from designing machine materials, soft robots and actuators to engineering biological tissues, implants and prosthetic devices functioning under compression and torsion.
3.1 Introduction

The Poynting effect is a surprising non-linear elastic effect that makes, in the original experiment of Poynting [1], a hanging piano wire under tension become longer when it is twisted (Figure 3.1a-left). The consequence is also that if the distance between the two ends is fixed, the torsion induces a stress normal to the shear plane (normal stresses) that tends to separate the two ends (Figure 3.1b-left) [2, 3]. Developing normal stresses or axial deformations under torsion are two equivalent manifestations of the Poynting effect. Poynting found that the normal stress as a function of shear strain follows a quadratic relation with a positive coefficient [1, 4, 5], now called the Poynting modulus. While conventional materials such as the piano wires of Poynting show a positive Poynting modulus (Figures 3.1a- and b-left), complex materials such as biopolymer systems [6, 7, 8, 9, 10, 11] and designed structures [12] can exhibit a negative Poynting modulus, causing a shear-induced contraction under a fixed load (Figure 3.1a-right), or negative normal force at a fixed gap (Figure 3.1b-right and Video SI 1, Supplementary Information). Designing metamaterials with exceptional mechanical properties originated from their structure rather than their composition has attracted much research in different disciplines of science [13, 14]. So far, mechanical metamaterials have been studied mostly under compression or tension. The response of metamaterials to direct shear [15] and in particular, the Poynting effect have remained largely unexplored. Understanding the complex coupling between shear and normal responses in metamaterials provides insights for harnessing and programming their shear and Poynting moduli.

Pulling or pushing on isotropic linear elastic objects causes expansion or contraction, but torsion is not allowed [16, 17]. In a limited number of recent studies, induced linear torsion by compression have been uncovered in designed chiral structures [18, 19, 20, 21, 22] mainly due to broken spatial symmetry of the system. Hence we ask whether an object is capable of transforming pure compression into a nonlinear torsion, which we call the inverted Poynting effect (Figure 3.1c and Video SI 1, Supplementary Information). The inverted Poynting effect is not a conventional material property and it has not been observed so far.

In the previous chapter (Chapter 2), we investigated the formation of topological kinks in a confined metamaterial and their shear-driven transformation. In the current chapter, we aim to program the sign and magnitude of the Poynting modulus in a similar metamaterial with few unit-cells (discrete limit). We design a cylindrical metamaterial that exhibits a programmable Poynting modulus and an inverted Poynting effect, where compression induces a torsion. The cylindrical
CHAPTER 3. PROGRAMMABLE POYNTING EFFECT

Figure 3.1: Poynting effect. a) Applying torsion under fixed loads causes dilation in a material with a positive Poynting modulus (left) and contraction in a material with a negative Poynting modulus (right). b) When the material is confined under a fixed gap, dilation and contraction will be manifested as positive (left) or negative (right) normal forces, respectively. c) Schematic of a cylindrical shell showing the inverted Poynting effect, where, in contrast to the Poynting effect’s manifestation in Poynting’s original experiment, an applied compression induces a nonlinear torsion in the cylinder.

The metamaterial is composed of identical units (unit-cells) consisting of beams with a suitably designed cross-section profile. By applying compression, the cylinder induces nonlinear and linear torsions. The sign and magnitude of the normal and shear responses of the designed structure are tuned by the interplay between buckling instability and self-contact interactions of the beams via a pre-compression applied prior to shear deformation. The applied pre-compression induces a topological kink (see chapter 2) that significantly alters the response of system to the shear deformation and switches the sign of the Poynting response. We present a simple spring model that reproduces our experimental results and characterizes the essential design parameters to obtain the inverted Poynting and to program the Poynting modulus. Our spring model follows the same concepts as the model presented in Chapter 2 but here, we exclude the unnecessary details such as the interaction between neighboring unit-cells for the sake of simplicity.

Our findings outline a strategy towards the rational designing of a programmable nonlinear elastic response of metamaterials with potential applications in engineering biomaterials functioning under torsional deformation and designing robot arms, soft rotational actuators and mechanical switches [23, 24, 25]. In soft robotics, for instance, coupling between torsion and compres-
sion can be exploited for the design of twisters, rotational actuators, kinematic controllers and pick and place end-effector [26, 22].

3.2 System and procedure

We design a hollow cylindrical shell composed of an array of unit-cells, which provides a network of nonuniform beams (Figure 3.2a) capable of side-buckling and self-contacting under compression [27]. We apply compression and torsion deformations on the cylindrical metamaterial by clamping the structure between two custom-made plates of an Anton Paar rheometer and measure the mechanical responses. The experimental details are explained in the experimental procedure section.

We first conduct two series of compression experiments with different boundary conditions: in the first series the bottom side of the shell is clamped and the top side is free to rotate (‘torsion-free’), while in the second series rotation is not allowed at both sides (‘clamped’). Then we use the clamped boundary condition and perform two series of torsion experiments under fixed loads and fixed gaps.

The compression strain is defined as \( \delta = |h - h_0|/h_0 \), where \( h_0 = 40.1 \text{ mm} \) is the initial effective height of the cylindrical shell and \( h \) is its height after the pre-compression, under the compression force, \( F \), with \( \pm 0.2\% \) experimental error. Compression stress is defined by \( \sigma = F/A_s \), where \( A_s \) is the cross-section area. Torsional angle, \( \phi \), develops by applying shear force, \( F_s \), and induces the axial deformation of \( \delta_n \) under a fixed load or the normal force of \( F_n \) under a fixed gap. In isotropic elastic materials, the shear stress, \( \sigma_s \), is proportional to the shear strain, \( \gamma \), \( \sigma_s = G_s \gamma \), and the normal stress induced by shear follows a quadratic relation as a function of shear strain, \( \sigma_n = G_n \gamma^2 \) [1, 5]. Here, we characterize the normal and shear responses of the structure with a Poynting modulus, \( G_n \), and a shear modulus, \( G_s \), respectively. Normal and shear force responses of a cylindrical shell under torsion are given by \( F_n = G_n J(\phi/h)^2 \), and \( F_s = \tau/R = G_s J\phi/(Rh) \), respectively, where \( \tau \) is the torque around the axis of the shell, \( R = (R_{max} + R_{min})/2 \) is the mean radius, and \( J = \frac{\pi}{2}(R_{max}^4 - R_{min}^4) \) is the second moment of area of the shell.
3.3 Results

3.3.1 Inverted Poynting effect and three regimes of structural rearrangements

Our designed cylindrical metamaterial is capable of showing the inverted Poynting effect by inducing torsion under compression (Figures 3.2b and c). As shown in Figure 3.2b in the torsion-free compression, we observe the rotational buckling with an affine torsional deformation across the height of the sample. However, in the clamped compression experiment, the torsional deformation accumulated at the middle of the structure (Figure 3.2c). In both compression experiments, three distinct regimes of configurational changes are observable in the structure: (i) pre-buckling (Figure 3.2a), (ii) buckling of the beams (Figures 3.2b- and c-left), and (iii) self-contact (Figures 3.2b- and c-right). Figure 3.2d, shows the compression stress, $\sigma$, rescaled by Young’s modulus of the elastomer, $Y = 3.63$ MPa, as a function of compression strain, $\delta$, for the torsion-free (dashed curve) and clamped (solid curve) experiments, respectively. The three regimes have different effective stiffnesses due to their structural configurations. The effective stiffness of the system in each regime, $Y_{\text{eff}}$, is determined by the slope of the curve times Young’s modulus of the elastomer.

In the pre-buckling regime ($\delta < \delta_b$, where $\delta_b = 0.012$ is the compression strain for the onset of the buckling regime) the vertical beams are stable and resist buckling (Figure 3.2a), resulting in a relatively high stiffness, with $Y_{\text{eff}} = 0.095Y$, in both compression tests. In the buckling regime ($\delta_b \leq \delta < \delta_c$, where $\delta_c = 0.13$ is the compression strain for the onset of the self-contact regime), the stress remains almost constant and the structure softens ($Y_{\text{eff}} = 0.004Y$), due to the buckling instability in the beams (Figures 3.2b- and c-left). Finally, in the third regime ($\delta \geq \delta_c$), the structure becomes stiff again, with $Y_{\text{eff}} = 0.06$, due to the self-contact between the beams (Figures 3.2b- and c-right).

In the torsion-free experiments, compression induces shear deformation, with distinct behaviors at buckling and self-contact regimes (Figure 3.2e and Video SI 2, Supplementary Information). The coupling between the compression and torsion, in the buckling regime, is nonlinear and the best fit to the experimental torsional angle as a function of compression gives $|\phi| = 4.9/\delta - \delta_b$ (blue curve in Figure 3.2e). The emergence of this square root relation is due to the buckling of the internal beams [28]. However, in the self-contact regime, compression and torsion are linearly proportional (red line in Figure 3.2e). Whereas linear coupling between compression and torsion has been achieved before for chiral structures [18, 19, 20], here for the first time nonlinear couplings between...
3.3. RESULTS

Figure 3.2: Experimental setup and compression tests. a) 3D printed structure is clamped between two plates. Unit-cells and nonuniform beams are magnified. b,c) The compressed cylinder at the buckling (left) and self-contact (right) regimes in torsion-free (b) and clamped (c) compression experiments. Deformed unit-cells are magnified. d) Rescaled nominal normal stress, $\sigma/Y$, as a function of compression strain, $\delta$, for torsion-free (dashed curve) and clamped (solid curve) experiments. e) Torsional angle, $\phi$, as a function of compression with square root (blue) and linear (red) fits, in the buckling and self-contact regimes, respectively. Insets are the predictions of the model and have the same axes and units as the main plots, except for the vertical axis in the inset of (e), which represents the shear strain in the 2D model, $\gamma$. $\gamma$ can be converted to the equivalent torsion in our cylindrical shell via $\phi = (h_0/R)\gamma$. Vertical dashed blue and red lines show the onset of buckling and self-contact regimes, respectively. Three regimes are marked with (i), (ii) and (iii).

Compression and torsion in an originally achiral structure have been studied. Thus, the cylindrical metamaterial translates an axial compression to a nonlinear...
torsion (the inverted Poynting effect) or linear torsion depending on the amount of compression. In the following, we investigate the Poynting response of the clamped structure under different loading conditions.

### 3.3.2 Poynting modulus under fixed loads/fixed gaps

To quantify both manifestations of the Poynting effect for our metamaterial, we apply torsional deformation under fixed loads (Figure 3.1a) or fixed gaps (Figure 3.1b) and follow its normal responses. The first loading scenario is equivalent to Poynting’s original experiment [1]. Initially, the clamped structure is loaded under a force $F$, resulting in an axial compression strain, $\delta$. Then the torsion is applied on the top boundary while $F$ remains constant, causing an axial strain of $\delta_n$. The axial strain and applied shear forces are shown as a function of torsion for a range of loads in Figures 3.3a and b, respectively. To quantify the second manifestation of the Poynting effect (torsion under fixed gap), we apply torsion on the pre-compressed shell while the height of the structure is fixed at $h$. Here, $F$ is the force needed for the pre-compression and $F_n$ is the torsion-induced normal force; $F + F_n$ is the total axial force response of the pre-compressed shell under torsion.

Figures 3.3c and d show the normal forces and applied shear forces as a function of torsion under fixed gaps, respectively (see Video SI 3, Supplementary Information). For both series, the normal responses behave quadratically (Figures 3.3a and c) while the shear responses behave linearly (Figures 3b and d) as a function of torsion in the limit of small torsional deformation, $\phi < 0.2\text{rad.}$ Initially, the non-compressed cylinder shows a contraction/negative normal force under torsion. While not a common material response, the negative Poynting modulus has been observed and investigated in biopolymer networks [6, 7, 8]. The origin of the negative Poynting modulus in biopolymers is rooted in the expulsion of water from their porous networks under deformation allowing them to shrink [10]. Similarly, in our metamaterial, the presence of voids allows to circumvent the volume conservation and to stretch the beams under torsion, which leads to negative normal responses. By applying a pre-compression on our metamaterial the curvatures of the normal response curves in both fixed load and fixed gap experiments (Figures 3.3a and c) change their sign and magnitude similarly. This indicates that the Poynting response of the structure is independent of whether the gap or the load was fixed, however, its sign and magnitude can be tuned by the level of pre-compression.
3.3. RESULTS

3.3.3 Programmable Poynting and shear moduli

To determine how to program the nonlinear moduli of the metamaterial, we quantify shear and Poynting moduli as a function of pre-compression. For torsion under fixed gap experiments, the coefficients of the fits to the normal force, $F_n$, and shear force data, $F_s$, in Figures 3.3c and d, rescaled by $J/h^2$ and $J/(Rh)$, give the Poynting ($G_n$) and shear moduli ($G_s$), respectively. For torsion under fixed load experiments, we define the coefficient of the quadratic fits in Figure 3.3a rescaled by $J/(A_sY_{eff}h^2)$ as the Poynting modulus. Figures 3.3e and f show the Poynting and shear moduli rescaled by Young’s modulus of the elastomer as a function of compressive strain for both loading scenarios.

The Poynting modulus in the pre-buckling regime is negative. For the intermediate pre-compressions (buckling regime), the Poynting modulus becomes zero. However, by approaching the self-contact regime, it rapidly increases and reaches a maximum value at $\delta \simeq \delta_c$, where the transition from the buckling to the self-contact regime occurs. The obtained moduli from both loading scenarios coincide as expected, apart from a deviation occurring at this transient deformation. By further increasing the strain $G_n$ decays sharply and approaches zero. Both shear moduli calculated from the two loading scenarios coincide as well (Figure 3.3f). The shear modulus, $G_s$, first decreases to zero after the transition from the pre-buckling to the buckling regime. Subsequently, $G_s$ increases sharply at the onset of the self-contact regime and keeps increasing at higher pre-compressions but with a lower rate. The absolute value of $G_n/G_s$ varies from 0 to about 13 for our metamaterial, whereas for a bulk cylindrical shell with the same dimensions and made of the same elastomer $G_n/G_s \simeq 0.5$ (see Chapter 1.4), in agreement with the theoretical prediction of $G_n/G_s = 5/8$, for an incompressible isotropic rubber [5].

Misra et al. have found that when a rectangular pantographic plate is being twisted around its long axis its normal and torque responses, can be tuned by modifying the geometry of the ligaments and structural parameters of the unit-cell [12]. In contrast, we program the Poynting effect and the torque response of our designed structure based on an interplay between buckling of the ligaments and self-contact between them (Figures 3.3a–d). Both systems at small torsional angles show linear and quadratic behavior for the shear and normal responses, respectively. The pantographic plates for specific combinations of geometrical parameters can show a transition to negative normal force under torsion. However, in large torsional angles, the responses of the two systems are significantly different. The response of pantographic plates under compression is not studied while for our cylindrical metamaterial under compression the
3.3.4 Oscillatory Poynting modulus in large deformation

In Figures 3.3a–d, we observe deviations from the quadratic response with a nonmonotonic behavior at large torsions. To understand this behavior at

inverted Poynting effect is observed.

Thus, by tuning the level of pre-compression we can program the magnitude and sign of the Poynting modulus and even eliminate it. In addition, our structure shows a strong potential for tuning the shear modulus over a wide range.
large deformations, we follow the structural changes and mechanical responses under large torsional deformation at a fixed pre-compression in the self-contact regime \((\delta_c)\). **Figure 3.4a** shows sequential images of the structural changes in our experiment. In **Figure 3.4b**, we observe a periodic oscillation in both normal (solid line) and shear (dashed line) forces as a function of torsion amplitude. In the course of torsion, one layer slides over another layer by snapping its beams from tilted to vertical and again tilted configurations, causing a local maximum in the normal force. Since the rearrangement occurs layer by layer, the number of the peaks of the normal response is set by the number of layers \((l = 4)\). For large deformation experiments we determine the Poynting and shear moduli as \(G_n = \left(\frac{h^2}{2J}\right)(\partial^2 F_n/\partial \phi^2)\), and \(G_s = \left(Rh/2J\right)(\partial F_s/\partial \phi)\). **Figure 3.4d** shows both rescaled \(G_n\) (solid line, left axis) and \(G_s\) (dashed line, right axis) oscillate between positive and negative values. Negative values of \(G_s\) accompanied by negative slopes in the curves of shear force indicating the occurrence of the snap instability (see **Video SI 4**, Supplementary Information). Thus, snap instability and self-contact under large torsional deformation result in oscillatory nonlinear moduli that are not allowed in conventional materials and rare in metamaterials.

### 3.3.5 Theoretical predictions vs. experimental results

We model the experimental system by using a 2D square network of Hookean springs and energy minimization to predict the configuration and mechanical responses of the system. Details of this model are explained in the modeling procedure section. The results predicted by the model successfully reproduce all the qualitative features of the experiments, as shown in the insets of **Figures 3.2d, 3.2e, 3.3e, 3.3f, 3.4c, and 3.4e**, with minor quantitative differences. For example, the model’s maximum values of \(G_n\) and \(G_s\) are higher than observed experimentally (**Figures 3.3e and f**). This can be attributed to neglect of the thickness of the beams’ middle point in the model, which excludes the bending possibility at the middle of beams and leads to a higher \(G_n\) and \(G_s\). Using this model, the onset of the self-contact regime can be predicted analytically by \(\delta_c \simeq (1 - \cos(\frac{\pi}{2} - 2\alpha)) + \delta_b \simeq 0.13\), in a good agreement with the experimental observations. Moreover, we analytically predict the square root coupling of compression-torsion observed in the inverted Poynting experiment as \(\gamma = \sqrt{2(\delta - \delta_b)}\) (**Figure 3.2e**, inset). Since shear deformation represents itself as a torsion on the cylindrical shell, by converting shear strain to torsion using \(\phi = (h_0/R)\gamma\), we predict a coefficient of 5.7 for the square root relation close to the experimental value of 4.9 (**Figure 3.2e**). The simple linear spring model identifies buckling and self-contact as the minimum ingredients to achieve
Figure 3.4: Periodic Poynting moduli in large strain deformation under the pre-compression of $\delta = 0.13$ in the self-contact regime. a) Sequence of images showing the internal deformations of the cylindrical structure during one cycle of a large amplitude torsion experiment. b,c) Normal response, $F + F_n$, (solid) and shear force, $F_s$, (dashed) as a function of torsion under large torsional deformations, for experiment (b) and model (c). d,e) Rescaled Poynting, $G_n/Y$, (solid) and shear, $G_s/Y$, (dashed) moduli as a function of torsion for experiment (d) and model(e). (c) and (e) are predictions of the model only for positive shear deformations ($\gamma \geq 0$) with a fixed gap boundary. The results of negative shear are mirror images of these curves, with four peaks in one deformation cycle.

programmable/inverted Poynting effects and confirms the structural origin of the nonlinear responses.
3.4 Conclusion

In conclusion, we have engineered a cylindrical metamaterial with programmable Poynting and shear responses. We showed that our designed structure is capable of exhibiting the inverted Poynting effect by translating an axial compression to a nonlinear torsion, in contrast to conventional elastic materials. We also succeeded in programming the Poynting modulus by varying the level of pre-compression/loading prior to torsion. We switched the sign of the Poynting modulus and tuned its value over a wide range, including even eliminating it. Furthermore, we successfully modeled and studied the system using an energy minimization method. The model identifies buckling of the ligaments and self-contact as the essential design elements to achieve programmable and inverted Poynting in a metamaterial. Our analytical approach opens avenues for bottom-up programming of the shear and normal mechanical responses of metamaterials based on self-contact as a mechanical feedback mechanism. Besides the fundamental importance of understanding nonlinear shear and normal moduli, their programmability provides a groundwork to numerous possible applications in solid mechanics. For instance, our system is capable of translating a unidirectional motion into torsion and of switching the mechanical forces. As these are relevant features of machines, we anticipate applications in designing machine materials, robot hands, mechanical force switches, and rotational actuators with simpler and more efficient mechanism compared to the conventional mechanisms. Additionally, the ability to program the Poynting effect inspires applications in biomechanics where couplings of torsional and axial deformations are ubiquitous (e.g. tendons, cartilages, and cardiovascular systems) [29, 30, 31, 21] and may lead to engineer novel biological tissues, implants, and external prosthetic devices functioning under torsion and compression [23, 24, 25].

3.5 Experimental procedure

The cylindrical beam network is created by extracting an evenly distributed circular pattern of eight voids, with a 4-fold symmetry contour, in \( l \) number of layers from a cylindrical shell with an outer radius of \( R_{\text{max}} = 12.5 \text{ mm} \), and an inner radius of \( R_{\text{min}} = 7.5 \text{ mm} \). We use the polar function \( s(u) = c[(1 - (a + b)) + a \cos(4u) + b \cos(8u)] \) to create shape of the pores with a 4-fold symmetry contour, where, \( s(u) \) is the radius at the polar angle \( u \), \( x = s(u) \cos(u) \), and \( y = s(u) \sin(u) \). In this equation, \( a \) and \( b \) are the shape tuning parameters, and \( c \) sets the pore’s size. Considering \( a = b = 0 \), we can create a circle with
a radius of $c$. Overvelde et al. showed that a 2D network with the following parameters for the pore shape of the unit-cell, $a = -0.21$ and $b = 0.28$ (Figure 3.5a), exhibits side-buckling under compression [27]. This shape also provides a nonuniform profile for the cross-section of the beams that form the structure. We use the same parameters to create our unit-cells. First, in CAD software (Blender 2.8), we place the pore contour (Figure 3.5a) at the outer surface of our cylindrical shell so that the plane of the pore is parallel to the axes of the shell. Then, we extrude this pore shape in the radial direction towards the center of the cylinder and create the void volume (Figure 3.5b). Carving a radial array of 8 voids evenly distributed around $2\pi$ rad, in $l$ layers leaves us with a cylindrical network of nonuniform beams composed of $8 \times l$ unit-cells. We set the pore size parameter as $c = 4.7 \text{mm}$, which gives the minimum beam thickness of $0.8 \text{mm}$ at the shell’s outer surface. We create extra half layers on top and bottom and, finally, clamp the structure with two O-rings with the same maximum and minimum radii as the shell and a height of $3 \text{mm}$ (Figure 3.5c). These features enable us to clamp two sides of the cylindrical metamaterial during the measurements.

Figure 3.5: a) The pore contour. b) CAD model of the void, created by extruding the pore contour in radial direction toward z axis. c) An isometric view of the CAD model of the cylindrical metamaterial.

We 3D print the designed cylindrical metamaterial using a Formlab Form2 3D printer and elastic resin v1, with a resolution of $0.1 \text{mm}$. We 3D print a bulk cylindrical shell with the same elastic resin and dimensions as our cylindrical metamaterial, and perform a compression test to determine Young’s modulus of
We conduct the compression and torsion experiments on our cylindrical metamaterial via an Anton Paar MCR302 rheometer with accuracies of 1 µm (longitudinal displacements), 0.05 µrad (angular displacements), 0.005 N (normal forces), and 1 nN m (torques). Using a custom made plate-plate geometry, we clamp the upper and bottom sides of the shell (Figure 3.2a). The deformation angle, \( \phi \), is positive when the torsion is clockwise. In this article, we present the data from single tests. Repeating the experiments with the same structures may lead to negligible changes, smaller than 0.1 N in force responses and 0.2% in strain rates, which can root in the durability of the elastomer. The analysis on data and visualizations has been performed using Mathematica 11.

3.6 Modeling

We model the cylindrical metamaterial as a 2D square network of Hookean elastic beams with the length of \( a_0 \) that could either stretch or contract. The beams can bend at the connecting nodes. Each beam has a pair of arc-shaped elastic arms, which are placed at the distance \( r \) from each end of the beam and symmetrically spread by \( \pm \alpha \) (Figures 3.6a and b). We study this model system in a 2-step deformation procedure: first, compression along the vertical axis, \( z \) (Figure 3.6c), and then shear along the horizontal axis (\( x \)). The connecting arms to the beams are designed to mimic the beam profile's nonuniformity, and they can deliver self-contact under deformation. Self-contacts lead to elastic contractions of the arms that produce the reaction forces.

The total elastic energy due to deformation of each beam is composed of stretching energy, \( E_s = \frac{1}{2} k a_0^2 e^2 \), and bending energy, \( E_b = k_b \theta^2 \). To calculate the contribution of self-contact in the elastic energy we assume that the arc-shaped arms have the same Hookean coefficient as the straight parts of the beam, \( k \), and when subjected to a self-contact, their curvature remains constant but their length, \( s \), changes through change of the arc angle, \( 2\alpha \); thus, \( \delta s = 2r\delta\alpha \). Since the deflection of the beam is divided between to contacting arcs, we can write \( \delta\alpha = \delta\theta/2 \), where \( \delta\theta = |\theta| - \theta_c \) is the deflection of the beam after self-contact at \( \theta_c = \frac{\pi}{2} - 2\alpha \). Due to such deformation, the energy of each arm changes by \( E_c = \frac{1}{2} k r^2 \delta\theta^2 = \frac{1}{2} k_c \delta\theta^2 \), which gives the elastic coefficient of the self-contact as \( k_c = k r^2 \). So the total energy for \( l \) layers of the vertical beams is:

\[
E = k_b \sum_{i=1}^{l} \theta_i^2 + \frac{1}{2} k a_0^2 \sum_{i=1}^{l} e_i^2 + k r^2 \sum_{i=1}^{l} \delta\theta_i^2 H[\delta\theta_i].
\]

(3.1)
Since bending and self-contact occur symmetrically at both nodes of each vertical beam, their energy terms do not have the pre-factor $1/2$. Heaviside step function, $H$, represents the self-contact interactions; $H[x]$ is 0 for $x < 0$ and 1 for $x \geq 0$.

Figure 3.6: a) Schematics of a non-uniform beam with associated parameters in our model. b) Schematic of the network of the beams in 4 layers. c) Under the pre-compression, beams are tilted by $\theta_i$ and when $\theta_i \geq \frac{\pi}{2} - 2\alpha$ self-contact occurs.

By considering a periodic boundary condition in the horizontal direction, we can model a closed structure such as a cylinder. Since each horizontal beam is joined with two beams at its right and left sides, alteration of its orientation is not favorable. Under this condition and since we apply the compression and shear deformations uniformly at the top layer, horizontal beams remain horizontal. Thus we can infer one column structure’s solution to the whole system and define the total energy of a system with $n$ unit-cells in each layer is $E_{tot} = nE$.

We use $n = 8$, $l = 4$, $\alpha = 0.54$ rad, and $r = 4.7$ mm, mimicking our experimental structure. The spring constant of the whole cylinder, $K$, is given by $K = Y_{eff}A_s/h_0$. Since the cylinder consists of $l$ layers with 8 beams in each layer, the spring constant for each beam can be calculated by $k = Kl/8$. We estimate the spring constant from the force response in the compression experiment before buckling as $k = 1331$ N m$^{-1}$. To estimate the bending stiffness of a beam with a rectangular cross-section, we can use $k_b = M/\theta = YI/(r_c\theta) \simeq (2YI/l_b)$, where $I$ is the moment of inertia, $l_b$ is the length of the beam, and $r_c$ is its bending curvature. By assuming that the beam has the same thickness and width as at the thickness and width of the hinge part, we can obtain the moment of inertia. This estimation predicts an order of magnitude of $10^{-4}$ N m for the beams’ bending.
3.7. SUPPLEMENTARY INFORMATION (SI)

stiffness. We finally calibrate the bending stiffness at \( k_b = 8.4 \times 10^{-4} \text{N m} \) to obtain the buckling instability at the same compression strain as in the experiment \( (\delta_0 = 0.012) \). We obtain the modeled system’s configuration by numerically minimizing its energy (Equation 3.1) under different boundary conditions. Numerical minimizations are performed by Mathematica 11 using the Nelder-Mead method and a working precision of 15 digit numbers. We use the presented model to mimic the torsion-free compression, clamped compression, and fixed gap torsion experiments.

In our spring model, we can tune the onset of buckling and self-contact transitions by changing the stiffness coefficients and geometrical properties of beams, such as \( \alpha \). For instance, increasing the bending stiffness of the beams, while keeping the stretching stiffness constant, leads to an increase in the onset of the buckling regime. Additionally, by increasing \( \alpha \) the width of the beam profile increases and therefore, transition to the self-contact regime occurs at a lower pre-compression strain. These parameters would provide a high potential for tuning the mechanical responses of the structure.

We exclude the energy due to the interaction between different layers (second gradient effect) for simplicity. The second gradient interaction term does not significantly change the mechanical response of the system. However, it is necessary to obtain a reliable solution for the structural phase transitions and study the system in the continuum limit, as done in the previous chapter (Chapter 2). A similar approach by including the second gradient effects has been used to describe extensible beams [32] and the pantographic plates under different modes of deformations, such as longitudinal extension, in-plane shearing, and extension combined with the in-plane rotation of an edge [32, 33].

3.7 Supplementary Information (SI)

3.7.1 Supplementary Videos

The supplementary videos are downloadable via the provided links.

Video 1 An introduction to the Poynting and inverted Poynting effects: Click here or scan the code.
CHAPTER 3. PROGRAMMABLE POYNTING EFFECT

**Video 2** Inverted Poynting effect and three regimes of structural rearrangements:
Click here or scan the code.

**Video 3** Programmable Poynting effect:
Click here or scan the code.

**Video 4** Oscillatory Poynting and shear moduli:
Click here or scan the code.
References


Chapter 4

Suppressing torsional buckling using auxetics
CHAPTER 4. SUPPRESSING TORSIONAL BUCKLING

Abstract

The building blocks of metamaterials often consist of unstable thin-walled elements (hinges) that locally buckle when loaded and trigger structural rearrangements to serve a specific functionality or induce an unusual mechanical property. Here, we feature the capability of auxetic metamaterials in preventing disruptive global buckling. By tuning the orientation of principal axes of auxetic patterns, we design auxetic shells (meta-shells) that display an asymmetry with respect to the direction of torsion. The asymmetric meta-shells exhibit a linearly increasing radial contraction (uniform negative strain) and a nonmonotonic axial strain upon torsion. Under small torsional deformation, the asymmetric meta-shells soften, but under large torsion, they stiffen due to structural reconfiguration and ordered compaction of unit-cells. This reconfiguration leads to circumvention of the torsional buckling under a surprisingly large torsional deformation. This study provides insight into tailoring asymmetry and programming shear-induced instabilities in metamaterials, with various potential applications in designing mechanical elements for soft robotics and biomechanics.
4.1 Introduction

Mechanical metamaterials often promote local instabilities to induce unusual functionalities and exotic mechanical properties [1, 2], such as negative Poisson’s ratio [3]. Here we introduce a strategy to utilize local instabilities in metamaterials in order to suppress a global instability, torsional buckling in cylindrical shells [4].

The abundance of cylindrical shells in natural and artificial systems motivates investigation to predict their buckling behaviors [5]. Recent studies reveal the stability landscape of perfect and imperfect cylindrical shells experiencing an axial load and a poker force resembling a single imperfection [6, 7]. Moreover, an effective strategy has been presented for the nondestructive prediction of buckling in imperfect cylindrical shells under axial loads [8]. However, in many circumstances, cylindrical systems are subject to torsional loads that can lead to the failure of a system, known as the torsional buckling [4, 9, 10, 11]. Torsional buckling is abundant in nature and everyday life, at different scales from buckling of an empty silo to buckling of blood vessels and veins [12, 13, 14]. A familiar example of torsional buckling occurs with twisting an empty beverage can, which leads to the emergence of creases on the shell (Figure 4.1a) [4, 15]. Increasing the thickness of the shell does not necessarily prevent buckling under torsion but changes the buckled shell’s shape into a flattened shell, like a buckled twisted hose (Figure 4.1b). In many applications buckling leads to failure of a system and should be prevented. Here, we investigate a strategy for designing cylindrical auxetic metamaterials that suppress torsional buckling by inducing a uniform radial contraction (Figure 4.1c).

We use the well-known auxetic holey sheet [3] to design our cylindrical metamaterials (meta-shells). Holey sheet is designed by applying a 2D array of closely packed circular voids to an elastic sheet that results in creation of a 2D lattice of connected square-like units (unit-cells) with periodicity along the principal axes, which connect the unit-cells. Holey sheets have been studied extensively [16, 17, 18, 19], and inspired designing 2D [20, 21, 22], 3D [2, 23, 24], and cylindrical [25, 26, 27, 28, 29] metamaterials with various unique functionalities that mainly rely on the internal buckling of their building blocks (unit-cells), which we refer as the local buckling. Most previous studies on auxetic structures focus on mechanical properties under uniaxial loading in a relatively small deformation range where an out-of-plane buckling (global buckling) is unfeasible or prevented. However, some studies unraveled novel global buckling behaviors such as discontinuous buckling of auxetic beams [30] and porosity-depended buckling in holey cylindrical metamaterials [26].
In this research, we focus on the large torsional deformation of the meta-shells and discover that the torsional buckling is preventable in auxetic shells under a surprisingly large torsional deformation by introducing an asymmetry with respect to the direction of torsion. We rotate the principal axes of the auxetic lattice, and therefore anisotropy orientation, with respect to the loading direction (or torsion around the main axis of the cylinder) to create asymmetric meta-shells that display different behaviors with respect to the direction of torsional (shear) deformation. The connecting unit-cells along the principal axes are initially vertical and horizontal. But by rotating the principal axes, the connecting unit-cells form helical paths with opposite handedness on the cylindrical coordinate. If the rotation is between 0 and $\pi/2$, the opposite helical paths have different pitch values, and the meta-shell is asymmetric with respect to the torsional direction. If the rotation is 0 (unit-cells alien straight) or $45^\circ$ (unit-cells align diagonally), the cylinder is symmetric. Twisting an asymmetric meta-shell leads to a unique structural reconfiguration with uniform radial contraction (negative radial strain) that effectively prevents torsional buckling.

The induced radial contraction under torsion has only been observed in viscoelastic materials previously [36] and is a counterpart to the perpendicular normal response under shear deformation, which is more typical and a fundamental feature of nonlinear materials [37, 38]. The shear-induced normal force is positive and quadratic for most materials and leads to a perpendicular dilation known as the Poynting effect [39]. The meta-shells, however, show a nonmonotonic normal response with a sign reversal under large torsions, which also is reported in pantographic metamaterial [40]. The normal force is quadratic only for the two symmetric meta-shells, and its sign is negative (pulls by twist) for the shell with straight unit-cells, where principal axes are vertical and horizontal, but positive (pushes by twist) for the shell with unit-cells aligned in the diagonal. Moreover, we can program the sign and magnitude of the Poynting response by applying a pre-compression to the symmetric cylinders, similar to our previous studied system [41].

Preventing torsional buckling and tunable cylinder radius by torsion are functional features exploitable in soft robotics, biomechanics, and material engineering for designing industrial devices such as pumps, valves, and actuators.

4.2 System and procedure

A square arrangement of four circular voids in a sheet forms a square-like unit-cell in the center, with its diagonals forming a plus sign (Figure 4.1d,
Figure 4.1: Buckling vs. radial contraction in cylindrical shells: Under torsion, a very thin shell, like a beverage can (a) is shrunk and randomly buckled, while a thicker shell (b) is buckled by flattening in the middle. However, a designed meta-shell (c) obtains a uniform radial contraction under torsion instead of buckling. Holey shells are designed by varying the orientation of unit-cells (d–f) and their deformations under clockwise (cw) and counterclockwise (ccw) torsions are studied. The asymmetric meta-shell radially contracts under cw torsion (e, right).
We can change the network and its mechanical properties by rotating the unit-cell orientation by angle $\theta$ with respect to the vertical axis. Setting $\theta = 0$ gives a straight arrangement of the unit-cells (Figure 4.1d, left). In this case, the diagonal lines connecting the unit-cells form vertical and horizontal paths, representing the symmetry of the structure. A cylindrical meta-shell designed by such a unit-cell arrangement, behaves similarly under clockwise or counterclockwise torsions since it is symmetric with respect to the torsional deformation around the $z$-axis of the shell (Figure 4.1d). If the rotation angle is in the $0 < \theta < \pi/4$ range, the symmetry is broken, and the meta-shell becomes asymmetric with respect to torsion direction (Figure 4.1e, left). As a result of this modification, the horizontal and vertical lines that connect the unit-cells will change to oppositely rotating helices on the cylindrical shell. However, the shell created by $\theta = \pi/4$ (Figure 4.1f, left) is also symmetric, and the behavior of the shell is insensitive to the direction of torsion. Here, all the shells have the inner and outer radii of $R_{\text{min}} = 7.5\text{mm}$ and $R_{\text{max}} = 12.5\text{mm}$, respectively, and the initial height of $h_0 = 53.3\text{mm}$. See section 4.5 and Supplementary Information (SI), subsection 4.6.2, for more details on designing. Here, we focus on torsion experiments and discuss the results of compression tests in SI, subsection 4.6.1.

To study the properties of the meta-shells, we conduct clockwise ($\text{cw}$) and counterclockwise ($\text{ccw}$) torsions on each cylinder. In the torsion experiments, the cylinders are axially free (axial force is $0 \pm 0.01\text{N}$), which means that the cylinders could freely dilate or contract depending on their normal force response. Similarly, a soda can is free to contract while manually twisting and becomes shorter in length after applying a torsion (Figure 4.1a). We show the applied torsion and torque around the main axis of the shells by $\phi$ and $\tau$, respectively. The shear strain is defined by $\gamma = \phi R/h_0$, and the shear force is $F_s = \tau/R$, where $R = (R_{\text{max}} + R_{\text{min}})/2$ is the average radius. We calculate the compression strain by $\delta = |h - h_0|/h_0$, where $h$ is the height after the applied compression. The compression stress is given by $\sigma = F/A_s$, where $F$ is the compression force, and $A_s = \pi(R_{\text{max}}^2 - R_{\text{min}}^2)$ is the area of a horizontal cross section of the cylindrical shell. We define the axial strains induced via torsion by $\delta_a = (h - h_0)/h_0$ to distinguish it from the applied compression strain, $\delta$. The radial strain of the cylinders is defined by $\delta_r = |r - R_{\text{max}}|/R_{\text{max}}$, where $r$ is the outer radius of the deformed shells at the middle of the structure.
4.3 Results and discussion

4.3.1 Negative radial and nonmonotonic axial strains under torsion

Radial contraction vs. torsional buckling: We start with applying cw and ccw torsions on the designed shells under zero axial load, while the shells can axially dilate or contract. As we see in Figure 4.1b, twisting a rubber shell without the hole patterns leads to buckling of the shell and flattening its sides. For the meta-shells, we observe surprising behaviors depending on the orientation angle of the hole patterns, $\theta$. In the symmetric structures, with $\theta = 0$ and $\pi/4$, we observe torsional buckling, as shown respectively in Figures 4.1d and f for both cw and ccw torsions. As we can see, since the designs are symmetric, the ccw torsion leads to the same buckling behavior with an inverse twist. For the asymmetric meta-shells with $0 < \theta < \pi/4$, the deformation strongly depends on the direction of applied torsion (Figure 4.1e, left, with $\theta = \pi/6$). When the torsion and the direction of $\theta$ are the same, here ccw, we observe torsional buckling (Figure 4.1e, middle shell). However, when the torsion and the direction of $\theta$ are opposite, here cw, the torsional buckling is no longer happening, and the shell uniformly contracts in the radial direction, instead (Figure 4.1e, right shell). The origin of such contraction is rooted in the auxeticity of the structure. The auxetic holey sheet is anisotropic and exhibits a negative Poisson’s ratio under a compressive deformation applied in the diagonal directions of the unit-cells. Moreover, we can translate a shear deformation into a combination of compression and extension perpendicular to each other. Thus, a suitable alignment of the unit-cells can trigger the negative Poisson’s ratio and cause negative radial stress and contraction in a cylindrical shell. To further investigate the shear-induced peculiar behaviors of the meta-shells, we study their radial and axial strains under cw torsions.

A negative radial strain (contraction): A sequence of images in Figures 4.2a–d show the deformation of an asymmetric meta-shell with a unit-cell orientation of $\theta = \pi/6$, under different levels of cw torsion. In Figure 4.2e, we show the radial contraction of the meta-shell (with $\theta = \pi/12$ and $\pi/6$) as a function of the shear strain. The radial contraction in the middle of the shell, $\delta_r$, increases as a function of the applied torsion until the structure reaches a new regime where the radial contraction in the middle is saturated (Figures 4.2c and d). The saturation of the radial strain occurs due to the extreme compaction of the structure, where the deformed unit-cells fill the voids and are in contact with the neighboring unit-cells, sharing a large contact area. The compacted region first
emerges at the middle height of the cylinders (Figure 4.2c), and by increasing the torsion, it symmetrically extends towards the top and bottom boundaries of the cylinders (Figure 4.2d). The structure with $\theta = \pi/12$ shows the same behavior as the structure with $\theta = \pi/6$ but with a lower slope for the radial contraction as a function of shear strain. For the structure with $\theta = \pi/6$, we do not reach the compaction regime, where the radial contraction flattens in this graph. We define the slope of the radial contraction as a function of torsion as the contraction ratio $\mu$, which is constant in the low shear strain regime before
the compaction regime and depends on $\theta$. In the extreme torsions, the cylinders are highly squeezed (Figure 4.2d), and by increasing the torsion further, the shell still resists buckling but some hinges finally break apart before any torsional buckling is observed (see SI Video 1).

A nonmonotonic axial strain: In the course of torsion, in addition to the monotonic negative radial strain, the asymmetric shells exhibit a nonmonotonic axial strain, representing the complex Poynting behavior of the asymmetric structures (Figure 4.2f). The shells initially show a usual positive Poynting response by dilating under torsion (Figure 4.2b). But after the axial strain reaches a maximum value ($\delta_n \approx 0.05$), the Poynting response is reversed, and the shells contract and finally become shorter than their initial height (Figures 4.2c and d). The nonmonotonic axial strain is a result of the sign reversal normal stress due to the complex shear-induced structural interactions in auxetic systems with an asymmetry with respect to the shearing direction. Likewise, intrinsic anisotropy is a crucial factor in determining the normal response of the biological materials such as ligaments and tendons and in heart mechanics [42, 43]. Moreover, nonmonotonic normal response with a sign reversal transition has been observed in anisotropic soft biomaterials with an anisotropy [44, 42].

A comparison with other systems that induce axial and radial stress: The axial stress perpendicular to the shear direction is also present in viscoelastic systems. The axial stress essentially is a result of the Poynting effect and is rooted in the elastic nonlinearity of isotropic solids and viscoelastic materials. Axial stress is usually positive, but negative axial stress is also observed in some biopolymers systems [45, 46, 47, 48, 49, 42] and designed structures [40]. We recently presented a metamaterial that exhibits negative axial stress, programmable to positive [41]. However, our asymmetric meta-shells induce switchable axial stress besides negative radial stress upon torsion. The shear-induced negative radial stress is also observed in viscoelastic systems but is usually negligible compared to the axial normal stress [36]. The radial stress emerges as a result of a coupling between non-linear elasticity and flow in a viscoelastic system. Here, we show the emergence of a significant second normal stress in elastic metamaterials.

### 4.3.2 Mechanical responses and suppressing the torsional buckling

Small deformations and pre-buckling regimes: The unit-cells rotation angle, $\theta$, highly influences the mechanical responses of the shells under torsion. In Figure 4.3a, the torque response is shown as a function of the torsional angle, $\phi$ for different shells with various $\theta$. For small torsional deflections, $\phi < \pm 0.2 \text{rad}$, all
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shells are in the pre-buckling regime, and the responses are almost linear. We use the slope of the linear fit in the pre-buckling regime to obtain the shear modulus (shear stiffness), $G_s$, by considering $\tau = G_s J \phi / h$ where $\tau$ is the torque around the axis of the shell and $J = \frac{\pi}{2} (R_{max}^4 - R_{min}^4)$ is the second moment of area of the shell [41].

In Figure 4.3b, we show the shear modulus of the meta-shells as a function of $\theta$. We observe that the shear stiffness is minimum for the shell with a straight unit-cell orientation $\theta = 0$ and increases to a maximum for the shell with $\theta = \pi/4$. Consequently, setting $\theta = \pi/4$ leads to the highest shear stiffness, where the main axis of unit-cells are aligned in the diagonal direction, which is consistent with the previous studies [17]. Since the unit-cells have a rotational symmetry with respect to the axial direction, variation of the shear stiffness should be insensitive to the direction of the rotation, $G_s(\theta) = G_s(-\theta)$. Consequently, we can estimate the shear stiffness in the first order approximation by a quadratic function, $G_s \approx \theta^2$, which can be verified analytically using a single tilted beam model (see SI, subsection 4.6.3). As it is shown in Figure 4.3b, inset, $G_s$ linearly increases as a function of $\theta^2$.

Large deformation and torsional buckling: In Figure 4.3a, we can observe that the torque response of the asymmetric shells monotonically increases under cw torsions. However, by increasing the amount of ccw torsion, the mechanical behaviors of all shells change dramatically due to the torsional buckling. One can realize that the shells with $\theta = \pi/4 - \delta\theta$ and $\theta = \pi/4 + \delta\theta$ unit-cell orientations are mirror symmetries around the $z-$axis of the cylinder. Therefore, twisting our shell with $\theta$ in the cw direction is equivalent to twisting a shell with a $-\theta$ unit-cell orientation in the ccw direction. Thus, we present a ccw torsion data of the shells (with $\theta$ unit-cell orientation) as a cw torsion but a $-\theta$ unit-cell orientation to obtain a comprehensive understanding of the effects of unit-cell orientation on the buckling of the shells. In Figure 4.3c and d, we show the buckling torsional angle, $\phi_c$, and torque, $\tau_c$, of the shells as a function of their unit-cell orientation, $\theta$. As we showed previously, the asymmetric shells with $\theta = \pi/12$ and $\theta = \pi/6$ do not show buckling under cw torsions, but in the plots (Figure 4.3c and d), the highest value of $\phi$ and $\tau$, obtained in the experiments, are presented with a shaded background. The limitation is set by the maximum torque that the machine can apply (150mN m). Although, within that limit, some hinges are strongly pulled, and two finally break. In Video SI 1 (Supplementary Information 4.6.5), distinct behavior of asymmetric meta-shell is comparable with the symmetric meta-shells.

The results show the potential of the asymmetric shells in delaying/preventing the buckling of the shells even under more than a full turn ($2\pi$) twist (Figure
4.3c). The maximum torque before buckling depends on the unit-cell orientation and is significantly higher for the asymmetric shells. Similarly, the energy consumed for twisting the cylinders before buckling (or until maximum torque is reached) depends on the unit-cell orientations and is considerably higher for the asymmetric shells with $\theta = \pi/12$ and $\pi/6$ (see SI 4.10).

Figure 4.3: Pre-buckling stiffness and the onset of torsional buckling: a) Torque responses of the meta-shells as a function of the torsional angle around the shell axis. b) Pre-buckling shear modulus, rescaled by Young’s modulus of the elastomer, $Y$, as a function of the unit-cell orientation, $\theta$ (or $\theta^2$ at the inset). In the pre-buckling regime, $G_s$ for $-\theta$ is a mirror symmetry with respect to $\theta = 0$ and avoided in this plot. c and d) The buckling torsional angle (c) and buckling torque (d) as a function of the unit-cell orientation, $\theta$. The open circles show the maximum values reached during the test, without showing any buckling.

To understand the mechanism behind the non-trivial buckling properties, we investigate the structural reconfigurations of the shells upon torsion.
Structural reconfigurations and shear softening/stiffening: We present a qualitative explanation of the distinct buckling behavior of the asymmetric shells. In Figure 4.4a and b, we show the rearrangement of the unit-cells under the shear deformations, for a section of the shells with $\theta = \pi/6$ and $\theta = 0$, respectively. Essentially, shear deformation (full black arrow vector) is a combination of compression (dashed green vector) and extension (dot-dashed blue vector) perpendicular to each other. In Figure 4.4a, we observe that the reconfigurations of the asymmetric shell lead to the creation of parallel lines of packed unit-cells, roughly aligned with the compression contribution of shear (green vectors). Meanwhile, after shearing, half of the hinges are stretched along the extension line (blue vector). However, in the shell with $\theta = 0$ (Figure 4.4b), the reconfiguration does not play in favor of shell persistence since the unit-cells packing/pulling alignments are not sufficiently along the compression and extension vectors. In Figure 4.4c, we show the compaction of the unit-cells in a full-size meta-shell (with $\theta = \pi/6$) by drawing guide lines, for the sake of clarity.

These deformations lead to the emergence of a complex shear softening/stiffening, shown in Figure 4.4d. The stiffness modulus is given by $g_s = (h/J)(\partial\tau/\partial\phi)$, where $\partial\tau/\partial\phi$ is the partial differential of the torque with respect to torsion, which is calculated numerically from the experimental data. We observe that the symmetric structure ($\theta = 0$) stiffens by increasing the shear deformation (orange arrows) and buckles at a certain point (red arrows). The asymmetric shell ($\phi = \pi/6$) similarly stiffens and quickly buckles under ccw torsion. However, under cw torsion, it softens initially (green arrow) and slowly stiffens by increasing the torsion. We also show the softening and stiffening regimes upon the structural reconfigurations (Figure 4.4a and b). These observations provide a qualitative description that reveals the distinct structural reconfigurations of the asymmetric meta-shells as the critical factor in preventing the torsional buckling.

4.3.3 Programmable moduli

The nonmonotonic axial deformation of the shell in Figure 4.2f indicates a transition in the sign of the Poynting response of the asymmetric shells under cw torsion. This observation suggests dependency of the Poynting effect on the unit-cell orientation, $\theta$. In small torsional deformations ($\delta < 0.02$), the Poynting response, characterized by the torsion-induced axial strain, is linear for asymmetric shells but quadratic for symmetric shells as for isotropic materials [39]. We further study the Poynting response in symmetric structures. Symmetric
structures can show side buckling under compression (Figure SI 4.7a). To avoid any possible side buckling of the shells under the pre-compression we made symmetric samples with larger unit-cells but the same shell size compared to the previous symmetric shells mentioned in Figures 4.1–4 (see section 4.5 for geometrical details). Figures 4.5a and b show the symmetric shells with $\theta = 0$ and $\theta = \pi/4$, before (left) and after a pre-compression (right) for these samples.

The normal force and the Poynting modulus: To investigate the Poynting effect in symmetric shells in more detail, we apply torsion on the cylinders under a fixed gap in different pre-compression strains and measure the applied shear and normal force responses. Figures 4.5c and d show the total axial force
(summation of compression force, $F$, and induced normal force $F_n$) for the shell with $\theta = 0$ and $\theta = \pi/4$, respectively. In Figure 4.5c, we show the Poynting moduli, $G_n$, of the shells rescaled by Young’s modulus of the composition, $Y$, as a function of the pre-compression strain, $\delta$. The Poynting modulus, $G_n$, is represented by the curvature of the quadratic fit to the plots in Figures 4.5c and d, and is calculated using $F_n = G_n J(\phi/h)^2$ [41]. For non-compressed shells, we observe that the normal force and the Poynting modulus are negative for the shell with $\theta = 0$ but positive for the shell with $\theta = \pi/4$. However, we can significantly change the Poynting modulus by applying a pre-compression prior to torsion in both shells.

For the shell with $\theta = 0$, applying a pre-compression in the intermediate regime (Figure 4.5e, $0.03 < \delta < 0.1$), vanishes the normal force and the Poynting response from initial negative values. In higher compression levels, we observe that the Poynting modulus slightly declines back to negative values. However, the Poynting modulus of the shell with $\theta = \pi/4$ declines linearly from its initial positive value ($G_n \approx 0.07Y$) to a negative value ($G_n \approx -0.07Y$) as a function of the pre-compression strain. This interesting behavior shows the potential of design of the holes to program the Poynting modulus in a wide range.

The shear force and the shear modulus: In Figures 4.5f and g, we show the shear force responses and their linear fits for the shells with $\theta = 0$ and $\theta = \pi/4$, respectively. Figure 4.5h shows the rescaled shear modulus, $G_s$, by Young’s modulus of the elastomer, $Y$, as a function of the compression strain. We observe that the shear modulus of the shell with $\theta = \pi/4$ is significantly higher than the shear modulus of the shell with $\theta = 0$. In other words, the shell with $\theta = \pi/4$ is almost six times stiffer than the shell with $\theta = 0$ under shear deformation. $G_s$ for the shell with $\theta = 0$ declines from $G_s \approx 0.01Y$ to almost zero where the unit-cells of the shells are buckled under compression ($\delta > 0.03$). The shear modulus, for the shell with $\theta = \pi/4$ slowly declines from $G_s \approx 0.06Y$ to $G_s \approx 0.04Y$, by pre-compressing the shell, which indicated that the compressed shell remains considerably stiff against shearing, despite local bucklings of the unit-cells.

A comparison with a previous study: In our previous work [41], we presented a cylindrical metamaterial created by network of beams with a nonuniform profile section, with a programmable Poynting modulus. It is shown that the Poynting modulus can be programmed via a pre-compression from an initially negative value to positive values, where the pre-compression triggers self-contacts between the nonuniform beams. Here, in our meta-shells, we see that the sign of the Poynting modulus can be reversed from negative to positive via changing the unit-cell orientation from $\theta = 0$ to $\theta = \pi/4$. In contrast with the previous system, programming the Poynting modulus in the holy shells does not relay on the
4.3. RESULTS AND DISCUSSION

Figure 4.5: Programmable Poynting and shear moduli: non-compressed (left) and pre-compressed (right) symmetric shells with \( \theta = 0 \) (a) and \( \theta = \pi/4 \) (b). The normal response as a function of torsion of the shells with \( \theta = 0 \) (c) and \( \theta = \pi/4 \) (d), and the resulting rescaled Poynting modulus as a function of compression strain (e). The shear forces as a function of torsions of the shells with \( \theta = 0 \) (f) and \( \theta = \pi/4 \) (g), and the resulting rescaled shear modulus as a function of compression strain (h).

self-contact transition; but rooted in the alignment of the unit-cells axes along diagonals since the shear forces dominantly affect the diagonals [39]. Additionally, here for the shell with \( \theta = \pi/4 \), the Poynting modulus linearly declines by the pre-compression strain, \( \delta \), and changes its sign from positive to negative, in contrast to the previous system [41]. Programming the Poynting modulus in the meta-shell with \( \theta = \pi/4 \) is practical due to its linearity with respect to the pre-compression strain (Figure 4.5e) and its stability against the side buckling (See SI) and remaining stiff against shearing (Figure 4.5h) under compression.
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4.4 Conclusion

We show that introducing asymmetry (with respect to the torsional direction) to the auxetic cylindrical shell triggers uniform radial contractions under torsional deformations and results in suppressing the torsional buckling. We observe that the torsional angle and torque for the onset of torsional buckling are significantly higher for the asymmetric shells. We reveal that the initial shear-softening due to structural reconfiguration followed by compaction with a distinct unit-cell alignment is accountable for the unusual extreme resistance to buckling. The asymmetric meta-shells show a nonmonotonic normal force response and a sign transition in the Poynting response upon torsion. Furthermore, the Poynting responses of the shells strongly depend on the unit-cells orientation. The Poynting modulus is negative for the shell with \( \theta = 0 \). However, it is initially positive for the shell with \( \theta = \pi/4 \), and it can be programmed via a pre-compression to change its sign to negative.

In this article, we introduce novel strategy on harnessing local instabilities along with asymmetry to suppress a global instability and obtain odd shear mechanical responses. As preventing torsional buckling is crucial in many mechanical systems functioning under torsion and compression, from twisters and robot arms to biological systems like blood vessels, this study provides insights on designing robust mechanical components with diverse applications. The radial contraction mechanism introduced here can offer a novel strategy for designing pumps and valves and can be potentially used to mimic pumping blood in the heart, which happens through complex twisting-contraction mechanisms \cite{50, 51}.

4.5 Experimental procedure

*Design principles and parameters:* The 2D array of these unit-cells connected at thin hinges, with the thickness of \( t \), creates the auxetic metamaterial sheet (holey sheet). We show the straight (Figure 4.6a) and rotated (Figure 4.6b) unit-cells and their geometrical parameters. To map the circular void on the cylindrical coordinate, the circle is initially placed at the outer radius of the cylindrical shell and then extruded in the radial direction toward the main axis of the cylinder to create the void volume (Figure 4.6c, left). We map the void network on cylindrical coordinates to create the auxetic cylindrical shells (Figure 4.6c, right).

First, we create a symmetric cylinder with \( n = 12 \) straight unit-cells around
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the circumference and 8 unit-cells in the length of a cylinder with inner and outer radii of \( R_{\text{min}} = 7.5 \text{mm} \) and \( R_{\text{max}} = 12.5 \text{mm} \), respectively (Figure 4.2d). The hinge thickness is set to \( t = 0.6 \text{mm} \) at the outer radius, and the height of the cylinder is \( h_0 = 53.3 \text{mm} \). We clamp the structure using two disks with the same inner and outer radii and a height of 3mm.

Cylindrical boundary condition: To map the tilted unit-cell network on the cylindrical coordinate, we must consider the periodic boundary condition in the circumferential direction. In other words, unit-cells with clockwise and counterclockwise helical tiling must precisely overlap where they meet. See SI, subsection 4.6.1, and Figure 4.8 for further details on designing. If the unit-cell \( n \) in clockwise helix overlaps with the unit-cell \( m \) in counterclockwise helix (counting from 0), the unit-cell’s rotation angle is given by \( \theta = \arctan(n/m) \). Since \( n \) and \( m \) are integers, practically, we can create the rotated designs with particular rotation angles. A pattern of voids with given \((\phi, z_i)\) coordinates for the clockwise and \((\phi, z_j)\) coordinates for the counterclockwise directions creates the helical network of unit-cells on the cylindrical shells (Figure 4.6b). In different cylinders, we keep the hinge thickness at the outer radius constant, \( t = 0.6 \text{mm} \), by slightly varying the radius of the void radius, \( r = 3.1 \pm 0.2 \text{mm} \). The cylinders in Figures 4.1e and f are respectively designed by \( \theta = \arctan(6/10) \approx \pi/6 \) and \( \theta = \arctan(8/8) = \pi/4 \). In total, we design and create meta-shells with various unit-cell rotation angles as \( \theta = 0, \pi/12, \pi/6, \) and \( \pi/4 \). We avoid reproducing the symmetric cylinders with \( \theta = -\theta \), but we perform both \( \text{cw} \) and \( \text{ccw} \) torsions to obtain a comprehensive understanding of the system. An extended description of the designing procedure and calculations of the variables is provided in SI 4.6.2.

Symmetric shells designed for the Poynting response experiments: To avoid the side buckling under axial compression, we design additional symmetric shells (\( \theta = 0 \) and \( \theta = \pi/4 \)) with bigger but less number of uni-cells. We design an additional shell with \( \theta = 0 \) by \( n = 8 \) unit-cells around the circumference in 5 layers along the shell, and another additional shell with \( \theta = \pi/4 \), \( n = m = 6 \). Here, we set the hinge thickness as \( t = 0.8 \text{mm} \) and \( h_0 = 51.1 \text{mm} \) for both cylinders (Figures 4.6 a and b).

We 3D print the designed structures using a Formlab Form2 3D printer and elastic resin v1 with a 0.1mm printing resolution. A bulk cylinder, 3D printed using the same machine and under the same conditions as the cylindrical metamaterials, has Young’s modulus of \( Y = 2.7 \text{MPa} \). We apply the deformations using an Anton Paar 300 rheometer and measure the torque, normal force, torsion, and axial deformations. We apply the deformations in low strain rates of \( \approx 0.25 \text{ mm/min} \) compressions and \( \approx 0.1 \text{ rad/min} \) torsions to obtain a quasi-
Figure 4.6: Design and parameters: a) the unit-cell, shown by its contour, is created by extracting circular voids of the sheet that forms a square-like shape with a diagonal along the \( z \)-axis. b) The same unit-cell rotated by \( \theta = \pi/6 \) and the position of the voids in the 2D coordinate. c) The void shape in cylindrical coordinate is created by extruding circular voids towards the shell axis and the holy shell, designed by \( \theta = \pi/6 \).

static deformation process.
4.6 Supplementary Information

4.6.1 Buckling under pure compression

It has been shown that buckling mode under compression depends on the porosity of the meta-shell with \( \theta = 0 \) [26]. However, here we observe a transition in the buckling mode depending on the unit-cell orientation. In Figure SI 4.7a, we compare the buckling modes of the shells with different unit-cell orientations. We observe that the shells with zero or small unit-cell rotation (e.g. \( \theta = 0 \) and \( \pi/12 \)) show a side-buckling, like Euler buckling of a beam. On the contrary, the shells with \( \theta = \pi/6 \) and \( \pi/4 \) only buckle internally, keeping the shell intact within a considerable compression strain (see SI Video 2).

In Figure SI 4.7b, we show the rescaled compression stress as a function of the applied compression strain. We observe that the effective Young's modulus of the meta-shells in the pre-buckling regime, \( \delta < 0.02 \), decreases as a function of their unit-cell orientation, \( \theta \) (Figure 4.7c), representing a softening by increasing \( \theta \). Although the buckling mode depends on the unit-cell orientation, the critical compression stress for buckling transition is almost constant \( (\sigma_c \approx 0.006Y) \) for all shells, (Figure SI 4.7d) but the onset of compression strain, \( \delta_c \), is rather higher for the shells with \( \theta = \pi/6 \) and \( \pi/4 \), where the buckling occurs internally and the structure is softer under compression (Figure SI 4.7e). However, the maximum stress and strains that the internally buckled shells experience is an order of magnitude higher than their value at the buckling onset (shaded area in Figure SI 4.7d and e), indicating the high compaction capability and compressibility (\( \sim 50\% \)).

4.6.2 Designing asymmetric meta-shells

To have a consistent helical tiling in the cylindrical structure, counting from zero, the \( n^{th} \) unit-cells of the clockwise helix must fully overlap with the \( m^{th} \) unit-cell in the counterclockwise helix. Likewise, the corresponding void volumes overlap at the same values (Figure SI 4.8). This is satisfied by \( m\phi_m + n\phi_n = 2\pi \), where

\[
\phi_n = \arccos \left( 1 - \frac{(2r' + t)^2 \sin^2 \theta}{2R_{max}^2} \right) \tag{4.1}
\]

and

\[
\phi_m = \arccos \left( 1 - \frac{(2r' + t)^2 \cos^2 \theta}{2R_{max}^2} \right). \tag{4.2}
\]
Figure 4.7: The meta-shells under a compression: a) The shells with $\theta = 0$ and $\theta = \pi/12$ show global buckling (side buckling) under compression, while the shells with $\theta = \pi/6$ and $\theta = \pi/4$ buckle internally, keeping the shell intact. b) Rescaled compression stress as a function of compression strain, and (c) rescaled effective Young’s modulus of the meta-shells as a function of the unit-cells orientation, $\theta$. The onsets of buckling under compression, determined by rescaled stress (d) and compression strain (e), are shown as a function of $\theta$. The empty circles in d and e show the maximum values experimentally obtained, while shell buckling does not occur yet.
In these equations: $\theta$ is the grid rotation angle, $t$ is the hinge thickness, and $R_{max}$ is the maximum radius of the shell. The parameter $r'$ is the radius of the void circle, vertically placed at the outer shell surface, touching it at the sides but fully inside the shell. However, for precise design we need to place a circle touching the outer surface at the top and bottom, which is slightly bigger, given by $r = r'/\sqrt{1 - (r'/R_{max})^2}$. The values of $z_i$ and $z_j$ (Figure 4.6), showing the height difference of the center of the next void along $n$ and $m$ helices, are given by $z_i = (2r + t) \cos \theta$ and $z_j = (2r + t) \sin \theta$, respectively.

This boundary condition imposes a limitation on the void size given by the radius of the outer circle, $r$ (outlined in Figure SI 4.8) or thickness of the hinges, $t$ (the minimum gap between two neighboring voids). We keep the hinge thickness constant for all cylinders at $t = 0.6$ to assure consistency among different cylinders.

Figure 4.8: The boundary condition in a cylindrical design: The 6$^{th}$ void in clockwise and the 10$^{th}$ void in counterclockwise helical tiling fully overlap, shown from the $x$–view (a) and $y$–view (b).
4.6.3 Shear modulus of a tilted beam

We can predict the shear modulus of the asymmetric design using a simple Hookean spring with the length of $a_0$, initially tilted by $\theta$ with respect to the vertical direction (Figure SI 4.9). The shear deformation is applied in horizontal direction and the force is given by $F_s = k\delta l \sin(\theta - \delta\theta)$, where $\delta l$ is the spring strain, $k$ is the spring constant, and $\delta\theta$ is the change in the angle. Using the geometrical constraints and the Taylor series, we can obtain the equilibrium equation,

$$F_s = k a_0 \cos \theta \sin^2 \theta \gamma,$$

where $\gamma = \delta x / h_0$. If the effective shearing area is $A$, the shear modulus is given by $G_s = (ka_0 / A) \cos \theta \sin^2 \theta \approx (ka_0 / A) \theta^2$, which is consistent with our observation of the quadratic relationship between the shear modulus and unit-cell orientation (Figure 4.3b-inset).

![Figure 4.9: A tilted Hookean spring sheared by $\delta x$.](image)

4.6.4 Energy perspective

We observe considerable differences in the energy cost to reach the buckling threshold in both torsion and compression experiments (Figure SI 4.10). The maximum energy cost for the asymmetric structures under torsion is significantly higher.
4.6. **SUPPLEMENTARY INFORMATION**

![Graphs showing critical buckling energy under compression and torsion](image)

Figure 4.10: Critical buckling energy for the shells under compression (a) and torsion (b).

### 4.6.5 Supplementary Video

The supplementary video is downloadable via the provided link.

**Video 1** Behavior of symmetric and asymmetric meta-shells upon torsional deformation under a zero axial force:

[Click here](link) or scan the code.
References


CHAPTER 4. SUPPRESSING TORSIONAL BUCKLING


CHAPTER 4. SUPPRESSING TORSIONAL BUCKLING
Chapter 5

Curvature induced by deflection in thick meta-plates
CHAPTER 5. CURVATURE IN META-PLATES

Abstract

The design of advanced functional devices often requires the use of intrinsically curved geometries that belong to the realm of non-Euclidean geometry and remain a challenge for traditional engineering approaches. Here, it is shown how the simple deflection of thick meta-plates based on hexagonal cellular mesostructures can be used to achieve a wide range of intrinsic (i.e., Gaussian) curvatures, including dome-like and saddle-like shapes. Depending on the unit cell structure, non-auxetic (i.e., positive Poisson ratio) or auxetic (i.e., negative Poisson ratio) plates can be obtained, leading to a negative or positive value of the Gaussian curvature upon bending, respectively. It is found that bending such meta-plates along their longitudinal direction induces a curvature along their transverse direction. Experimentally and numerically, it is shown how the amplitude of this induced curvature is related to the longitudinal bending and the geometry of the meta-plate. The approach proposed here constitutes a general route for the rational design of advanced functional devices with intrinsically curved geometries. To demonstrate the merits of this approach, a scaling relationship is presented, and its validity is demonstrated by applying it to 3D-printed microscale meta-plates. Several applications for adaptive optical devices with adjustable focal length and soft wearable robotics are presented.
5.1 Introduction

Curved surfaces are ubiquitous in nature. Biology, in particular, thrives on curved objects, as a growing body of recent evidence suggests [1, 2, 3]. Fundamental processes, such as cell migration [4, 5], morphogenesis [6, 7], and tissue regeneration [8, 9] are often dependent on the curvature of their surrounding environments [10]. The design of dynamic and programmable curved surfaces, however, remains challenging for engineers. These challenges can, for example, be observed in the case of thin-walled structural elements, which are very popular due to their combination of lightness, load transfer efficiency [11, 12, 13], and low cost. Although an initially flat panel can be reasonably bent in a single direction to adapt the shape of an arch, transforming the panel into a shell dome or a saddle and, thus, changing its Gaussian curvature remains challenging [14, 15, 16, 17]. Several studies have used computer-aided design using conformal geometry [18, 19], origami [20, 21, 22, 23, 24, 25, 26, 27, 28], kirigami [29, 30], or crumpling [31, 32] approaches to create curved objects from flat sheets. However, these approaches rely on very thin sheets and only lead to an approximation of the desired curvature. Other techniques based on non-uniform swelling [33, 34] or inflation [35] or liquid crystal phase transition [36] have also been proposed, but these structures are made of soft materials and may not be suitable for large-scale and/or load-bearing structures. Using “Bending-active systems” is another approach to create form-finding structures [37, 38]. These form-finding structures rely on the elastic deformation of a combination of several structural elements (e.g., vector-active, surface-active, form-active, etc.) that are initially planar or straight [38, 39, 37, 40, 41]. Usually, individual curved beam, shell, or membrane elements of bending-active systems remain elastically constrained and can carry residual bending stresses [38]. Therefore, patterning individual elements and their pre-stress condition can control the shape of the bent elements. This is why bending-active design strategy is mostly considered as a suitable approach for building arbitrarily curved objects rather than continuous structures. This approach can also be used for the design of compliant mechanisms [38].

Following Carl Friedrich Gauss’s “Egregium” theorem, the only theoretically admissible way of obtaining intrinsically curved geometries from flat sheets is to allow for in-plane deformations [20, 42]. The physics of continuum plates has been extensively studied using classical elasticity theories. As an example, Kirchhoff-Love’s classical theory of plates [43] explains the mechanics of thin plates, where through-the-plane shear effects are eliminated, but the Reissner-Mindlin’s theory describes the mechanics of thick plates by considering through-the-plane shear deformations [44, 45, 46]. Some other examples of the approaches that have
CHAPTER 5. CURVATURE IN META-PLATES

been applied to improve our current understanding of the mechanical behavior of curved structures are higher-order shear deformation theory [47, 48] and bending gradient theory [49] that have been used to analyze the curvature of composites plates and differential geometry [50, 51] that can be utilized to study curved objects in the 3D Euclidean space. It is often challenging to derive exact solutions for these analytical problems. Therefore, finite element (FE)-based (homogenization) approaches can be used to analyze the bending behavior of cellular structures [52, 53, 54, 55].

Even though the mechanics of continuum plates have been well studied, “thick” non-isotropic plates that exhibit auxeticity (i.e., a negative Poisson’s ratio) have been less explored [56, 57, 58], particularly under bending deformations. We propose a combination of computational modeling and experiments to study how curvature develops in thick plates exhibiting auxetic or nonauxetic behaviors as a result of mechanical deformation. One way to manipulate the Poisson’s ratio of a plate is to use architected designs such as those found in cellular metamaterials. Rationally designing the small-scale geometry of metamaterials allows for creating unusual macroscale properties, such as negative values of the Poisson’s ratio [57, 58, 59, 60, 61, 62] and stiffness [63, 64], as well as for shape adjustments, such as shape-matching [65], shape morphing [66, 67, 68], shape-shifting [18], or shape integrity [69]. The prefix “meta” is sometimes also applied to structural elements with similar unusual behaviors (e.g., metabeams [70]). It is within this context that we refer to our thick plates with different distributions of the Poisson’s ratio as “meta-plates.”

Here, we focus on the Gaussian curvature that describes the intrinsic curvature of a surface. This is in contrast with the extrinsic curvature, which requires the embedding into a space of higher dimension to be observed. The Gaussian curvature of a surface, $\kappa$, is obtained as the product of its two principal curvatures, $\kappa_1$ and $\kappa_2$: $\kappa = \kappa_1 \kappa_2$ (Figures 5.1a and b). Depending on the values of these principal curvatures, three types of surfaces, namely synclastic (i.e., dome-like), monoclastic (i.e., zero-curvature), or anticlastic (i.e., saddle-like) can be defined.

5.2 Results and discussion

Bending a rubber eraser along its length results in a curvature of opposite sign along the transverse direction (Figure 5.1a). The origin of such induced curvature is the Poisson’s ratio, $\nu$, which relates the strain observed along a transverse direction to the strain applied along the longitudinal direction. Usual materials have a positive Poisson’s ratio and tend to get compressed in the
transverse direction under uniaxial stretching. As the eraser is bent with a curvature $\kappa_1$, its upper part is stretched while the lower part is compressed, leading to opposite strains in the transverse direction. As a result, a curvature $\kappa_2 = \nu \kappa_1$, appears in the transverse direction giving rise to a familiar saddle shape (Figures 5.1a and b). However, this anticlastic effect is only observed in the case of small-width plates that approach the slender shape of a beam. Bending a wide plate generally results in a single curvature (i.e., zero Gaussian curvature), except for a fraction of the width of the plate, which is of the order $\sqrt{h/\kappa_1}$, where $h$ is the thickness of the plate [71]. The absence of the Poisson-induced curvature along the whole width is due to the additional cost in the stretching energy corresponding to the change of the Gaussian curvature. This paradigm is, however, challenged when considering cellular panels in which transverse curvatures are observed even for relatively large specimen widths [71, 72]. This effect can be interpreted as a consequence of a relatively low in-plane stretching stiffness of such meta-plates, while the flexural rigidity remains high [73]. Such combinations of in-plane and flexural mechanical properties can be used to accommodate Gaussian curvatures in thick meta-plates.

We designed thick cellular plates using hexagonal lattices characterized by an angle $\theta$. Depending on the value of this angle, such meta-plates display either a positive ($\theta > \pi/2$) or a negative ($\theta < \pi/2$) value of the Poisson’s ratio, respectively leading to negative (Figures 5.1c and d) or positive (Figures 5.1e and f) Gaussian curvatures upon bending. Numerical simulations based on the FE method display the same features and will be used to study the impact of the different geometrical parameters on the induced transverse curvature.

Consider the out-of-plane buckling in the $Z$-direction of a meta-plate under compression in the $Y$-direction, while hinges provide free rotation at the ends (Figures 5.1c–f). From the classical theory of buckling, we expect the vertical displacement of the plate to follow a sinusoidal function of the longitudinal coordinate with a half period equal to the length of the plate [74, 75]. From symmetry, the displacement of the transverse coordinate should follow an even function. As a simplifying approximation, a quadratic function multiplied by the sinusoidal function can be used to predict the out-of-plane deformation of a thick meta-plate:

$$w(x, y) = A + Bx^2 \sin(Cy), \quad (5.1)$$

where $C = \pi/L$ and $A$ and $B$ are constants determined by fitting Equation 5.1 to our experimental data points. We deduce the main curvatures at the middle point of the surface $\kappa_1 = -AC^2$ and $\kappa_2 = 2B$. We expect $\kappa_2$ to vanish.
Figure 5.1: Curvature in a thick beam and meta-plates: a) An example of an intrinsically curved shape with two non-zero principal curvatures resulting from the bending of a rubber eraser. b) A sketch of the positive second curvature induced on a 2D surface. Experimental setups and computational models of non-auxetic (respectively c and d) and auxetic (respectively e and f) meta-plates. The actual geometrical parameters used in the experiments and computations are presented in Table 5.1, Supplementary Information. The numerical models depict (in contour lines) the vertical displacement, $U_z$.

for small values of the meta-plate thickness. To estimate the Poisson’s ratio of our structures, we used the existing theoretical relations based on rigid frames...
5.2. RESULTS AND DISCUSSION

connected by hinges [76]:

\[ \nu = -\frac{\cos \theta}{a/b - \cos \theta}, \]  

(5.2)

where \(a\) and \(b\) are the lengths of the struts of the hexagonal lattices (Figures 5.1c and e). The values of \(\nu\) vary between 48\(^\circ\) and 120\(^\circ\). Although the actual values of \(\nu\) should also depend on the applied strain, we assumed that \(\nu\) does not change significantly during bending. The reported values of the Poisson’s ratios, thus, pertain to the initial stage of the deformations. Beyond the Poisson’s ratio, several other parameters characterize the metastructure, including the cell size, \(w = 2b \sin \theta\), the thickness of the cell wall, \(t_w\), the thickness of the plate, \(h\), and the width of the plate, \(W\) (Figure 5.1c). The applied load controls the longitudinal curvature, \(\kappa_1\), through uniaxial compression. We used experiments and FE simulations to study how the transverse curvature, \(\kappa_2\), varies as a function of these different parameters.

In the experiments and numerical simulations, our reference meta-plate is based on an array of 15 \(\times\) 18 hexagonal cells with the following dimensions, \(w_0 = 5.1\) mm, \(c = 7.65\) mm, \(t_w = 0.51\) mm, and \(W = 92\) mm (Figure 5.1). Other specimens were designed based on multiples or fractions of these nominal parameters.

5.2.1 Evolution of the second curvature

The specimens were uniaxially compressed in a stage. Beyond a buckling load, they adopt a longitudinal curvature \(\kappa_1\) and a transverse curvature \(\kappa_2\). As the load is increased, the out-of-plane buckling is amplified. Initially, \(\kappa_2\) increases proportionally to \(\kappa_1\), as in the case of a bent eraser, but then saturates to the value of \(\kappa_{max}\) (Figure 5.2). This behavior can be approximately captured by a saturating exponential of the form \(\kappa_2 = \kappa_{max}(1 - e^{-\kappa_1/\kappa_{sat}})\), where \(\kappa_{sat}\) represents the onset of transition to the saturation regime. For \(\kappa_1 \ll \kappa_{sat}\), we expect a linear evolution of the form \(\kappa_2 = (\kappa_{max}/\kappa_{sat})\kappa_1\). We find that the maximum curvature (\(\kappa_{max}\)) reached by the meta-plates is linearly correlated to its Poisson’s ratio (Figure 5.2a, bottom-left), thickness (Figure 5.2b, bottom-left), as well as to the thickness (Figure 5.2c, bottom-left) and width (Figure 5.2d, bottom-left) of its unit cells. While the initial slope, \(\kappa_{max}/\kappa_{sat}\), varies linearly with the Poisson’s ratio (Figure 5.2a, bottom-right), it is approximately independent of the other geometrical parameters (Figures 5.2b–d, bottom-right).

We also observe a decrease of \(\kappa_{max}\) as the thickness of the walls increases (Figure 5.2c, bottom-left). Increasing the thickness of the meta-plates for the
Figure 5.2: Evolution of the second curvature: a–d) The evolution of the second principal curvature with respect to the first principal curvature for meta-plates with different values of the Poisson’s ratio (a), as well as different values of the thickness, \( h \) (b), unit cell thickness, \( t_w \) (c), and unit cell width, \( w \) (d), in two extreme configurations (for each variation of a geometrical parameter, the other parameters were kept constant). The dependence is linear only for the lowest value of \( \kappa_1 \kappa_2 \), as in the classical case of a rubber eraser but tends to saturate for the higher values of \( \kappa_1 \). The variation of \( \kappa_2 \) with \( \kappa_1 \) can be described using an exponential function (i.e., \( \kappa_2 = \kappa_{\text{max}} (1 - e^{-\kappa_1/\kappa_{\text{sat}}}) \)). Insets show the evolution of the fitting parameters with the geometrical characteristics of the meta-plates.
extreme values of the Poisson’s ratio increases the rate of the evolution of the second principal curvature (Figure 5.2b). Increasing the thickness of the unit cell struts, \( t_w \), affects the evolution of the second principal curvature as well (Figure 5.2c). We also evaluated the evolution of the curvature when the unit cell size was scaled. Toward that end, we changed the number of the unit cells while keeping their relative size constant (i.e., \( c/w = 3/2 \)). Meta-plates with a higher number of overall unit cells (i.e., \( w/w_0 = 3 \)) reach higher values of the second principal curvature (Figure 5.2d), and the rate of the curvature evolution for those upscaled structures is lower.

### 5.2.2 The effect of the plate width on the second curvature

To study the effects of the boundary conditions on the second principal curvature, we created computational models of meta-plates with smaller and larger widths (i.e., \( W/4, W/2, 2W, \) and \( 4W \)) than that of our reference models (i.e., \( W \)) for both the maximum and minimum values of the Poisson’s ratio. For those cases, we kept the out-of-plane thickness of the meta-plates (i.e., \( h = 5 \text{ mm} \)) constant. We find the second principal curvature to be highly dependent on the width of the specimens for both auxetic and non-auxetic meta-plates (Figures 5.3k and l). As the width of the meta-plates increases, the absolute value of \( \kappa_2 \) at the center of the meta-plates decreases (Figures 5.3a–j). For sufficiently large widths (i.e., plate’s width \( \geq 2W \)), the middle part of the meta-plates exhibits a near-zero value of \( \kappa_2 \) (Figures 5.3g–j). Closer to the boundaries, however, curvatures similar to what was observed for the reference models are observed (Figures 5.3g–j). An important observation is that when the width of the plate is much larger than the length, curvatures are always localized in the free boundaries with a specific penetration depth.

Global bending in the transverse direction is only observed for meta-plates with relatively small widths (i.e., plate’s width \( \leq W \)) (Figure 5.3). As in the classical case of isotropic plates [77], the bending of wide meta-plates is limited to their edges. Nevertheless, in contrast with classical plates, this maximum width is much larger than the typical limit \( \sqrt{h/\kappa_1} \). For instance, in the specimens shown in Figure 5.3, we observe uniform transverse bending up to a width of 92 mm (Figures 5.3e and f), for an applied curvature equal to \( \kappa_1 = 36 \text{ m}^{-1} \). In the case of a plain material, the limit in width for a uniform transverse bending under the same applied curvature is on the order of \( \sqrt{h/\kappa_1} \approx 12 \text{ mm} \).

A comparison of the deformation of plain (i.e., non-architected) plates with equivalent isotropic elastic properties with those of meta-plates showed similar non-linear behaviors for the negative and positive values of the Poisson’s ratio.
Figure 5.3: The effect of the plate width \((W_0 = 92 \text{ mm})\) of the meta-plates on their shape under compression for two extreme cases with the a,c,e,g,i) negative and b,d,f,h,j) positive values of the Poisson’s ratios. The values of \(\kappa_1\) in \(\text{m}^{-1}\) are shown on different profiles in the left insets of (a–f) (see Videos 1–4, Supplementary Information 5.5.4). The other parameters were maintained constant \((h = 5 \text{ mm})\). The insets in these sub-figures show the displacement distribution in the Z-direction under equal induced curvature (i.e., \(\kappa_1 \simeq 20 \text{ m}^{-1}\)). k–m). The insets show the parameters of the exponential fit for \(\kappa_2 = \kappa_{\text{max}}(1 - e^{-\kappa_1/\kappa_{\text{sat}}})\).
Both types of plates (i.e., plain and meta-plates) were linearly deformed and the transverse induced curvature saturated by imposing the longitudinal curvature. The plain plates, however, cannot predict the maximum curvature achieved by the meta-plates. For an equivalent set of geometrical parameters, the results obtained with plain plates cannot be directly extrapolated to meta-plates and the induced curvature is significantly larger in the case of meta-plates (Figure 5.7, Supplementary Information).

### 5.2.3 A generic relationship describing the second curvature

The linear dependence of the transverse curvature on the thickness of the meta-plate and the Poisson’s ratio is confirmed experimentally. In Figures 5.4a–c, experimental measurements are compared with FE simulations for an imposed curvature (i.e., \( \kappa_1 \approx 20 m^{-1} \)). There is a linear relationship between the thickness of the meta-plate, \( h \), and its second principal curvature, \( \kappa_2 \) , (i.e., \( \kappa_2 \propto h \)) obtained using our computational models for extremely negative (Figure 5.4a) and extremely positive (Figure 5.4b) values of the Poisson’s ratio. This is similar to the linear dependency observed in beams [77]. Our experimental observations show a clear linear relationship between the second principal curvature and the Poisson’s ratio (i.e., \( \kappa_2 \propto \nu \)) as expected (Figure 5.4c). As the thickness of the meta-plate increases, the proportionality constant between the second principal curvature and the Poisson’s ratio increases (Figure 5.6 and Table 5.2, Supplementary Information). Furthermore, regardless of the value of the Poisson’s ratio, the second principal curvature linearly decreases as the in-plane thickness of the struts of the meta-plates increases (Figure 5.6, Supplementary Information).

From the above-mentioned analyses, we propose the following dimensionless scaling relationship between the second principal curvature and the other geometrical parameters of the meta-plate (Figures 5.4d and e).

\[
\kappa_2 = \frac{\kappa_{max} \nu h w}{(t_w W^2)} \cdot \left(1 - e^{-\kappa_1/\kappa_{sat}}\right),
\]

where, \( \kappa_{max} \approx h w/(t_w W^2) \) and \( \kappa_{max}/\kappa_{sat} = \alpha \nu \). Here, we found \( \alpha = 0.02 \). This low value may be interpreted as a consequence of the anisotropy of the meta-plate. Using this expression, we can readily tailor the intrinsic curvature of the meta-plates simply by adjusting the corresponding geometrical parameters of the plates. We can then use this prediction to design the micro-architecture of the meta-plates.
Figure 5.4: A generic relationship between first and second curvature: a,b) Change in the absolute value of the second principal curvature with respect to the thickness of the plate for the most positive (a) and most negative (b) values of the Poisson’s ratios, respectively corresponding to $\theta = 120^\circ$ ($\nu = 0.87$) and $\theta = 48^\circ$ ($\nu = -1.35$). The other parameters were maintained constant ($\kappa_1 \approx 20 \, \text{m}^{-1}, W_0 = 92 \, \text{mm}$). The filled and unfilled markers respectively denote the numerical and experimental data points. The lines in (a) and (b) are fits to the numerical results. c) The evolution of $\kappa_2$ as a function of the Poisson’s ratio for the plates with $h = 5 \, \text{mm}$ ($\kappa_1$ and $W$ were kept constant). d) The relationship between the maximum curvature and the geometrical parameters of the meta-plate in Equation 5.3. e) The dimensionless relationship between both principal curvatures in the form $\kappa_2 = \kappa_{\text{max}} \left(1 - e^{-\kappa_1/\kappa_{\text{sat}}} \right)$. f,g) The SEM images of the 3D printed microscale meta-plates. The bottom images show the deformation of the microscale meta-plates due to the capillary forces (see Videos 5 and 6, Supplementary Information 5.5.4).
To show the length scale independence of the proposed empirical equation between two principal curvatures, we scaled down our reference meta-plates and fabricated auxetic and non-auxetic meta-plates at the microscale using a submicron 3D printing technique (i.e., two-photon polymerization). The microscale meta-plates were made of a resin (i.e., IP-Q, see Experimental Section). The addition and subsequent evaporation of ethanol induced deformations in the micro-plates as a result of a capillary force exerted during the solvent evaporation (Figures 5.4f and g). This leads to dynamic changes in the curvature of the top surface of the micro-plates as shown in Figures 5.4f- and g-bottom and Videos 5 and 6, Supplementary Information 5.5.4. We quantified the level of the induced curvatures through optical microscopy. In Figures 5.4h and i, we show the CAD images of the structures with minimum and maximum Poisson’s ratio, used for fabricating the microscale meta-plates.

After the application of the scaling law (Figure 5.4e), the $\kappa_2$ values of the micro-plates were in the range of those determined for the macro-plates, confirming that the proposed relationship in Equation 5.3 captures the essential physics of the problem across multiple length scales.

5.2.4 Designing curvature

We have shown that, in contrast to plain plates (i.e., a plate with negligible thickness) whose Gaussian curvature under compression is invariably zero, meta-plates could be used to fabricate surfaces with a wide range of positive or negative Gaussian curvature. This approach provides a route for the rational design of thin-walled engineering structures with a wide range of applications and complex curvature requirements. As an illustration, a meta-plate divided into regions with positive and negative values of the Poisson’s ratio could be used to create spatial variations in the Gaussian curvature from positive to negative values along the width (Figure 5.5a). A meta-plate with a gradient of the Poisson’s ratio (gradual variations from positive to negative values) allows for adjusting the location of the maximum curvature as well as for modulating the shape of the curved surface (Figure 5.5b). Many other design approaches where regions with different thicknesses and/or Poisson’s ratios are combined to meet complex curvature requirements can be envisioned as well. This could, for example, be used in soft wearable robotics (e.g., exoskeleton [78, 79]) that need to morph the curved contours that define the shape of the human body (Figures 5.5c and d). With a simple tuning of the geometrical parameters of the elementary cells, a wide range of curvatures could also be exploited to create adaptive optical devices (e.g., mirrors) whose focal length is dependent on the level of the applied
compressive load (Figures 5.5e–h).

Figure 5.5: Designing adaptable curvature and its applications: a) Complex curvatures profile on a bent meta-plate consists of two equal regions with positive and negative Poisson’s ratios. b) A bent meta-plate with a gradient of Poisson’s ratio in length, varying linearly from negative to positive values. The color code in (a-right) and (b-right) show the displacements in the Z-direction). c,d) A demonstration of potential soft wearable devices, especially at the joints, where double curvatures (c,d) are required to create a shape-fitting surface. e–h) Adaptive mirrors with tunable focal points (e) made by covering an auxetic meta-plate (f) with a highly reflective aluminum foil (g). e,h) The adaptive mirrors were then exposed to laser beams. e) The focused laser beams reflected off the surface show flat, concave, or convex mirrors.
5.3 Conclusion

The meta-plates proposed here combine the advantages of thin-walled structures, such as efficiency in load transfer and low weight with those of intrinsically curved designs, such as adaptive shape morphing features and ease of interaction with the human body. Ultimately, the possibility to start from an initially flat shape translates into additional benefits, including the opportunity to incorporate complex surface-related functionalities, including nanopatterns and flexible electronics into the final 3D device [80]. The relation we found between the different physical length scales remains so far empirical. We hope our study will motivate additional research that provides a deeper physical ground to the scaling law we obtained and enables its extension to nonlinear regimes. Nevertheless, this simple law offers a simple basis for programming cellular meta-plates and achieving complex 3D structures that hold great promise for practical applications.

5.4 Experimental section

The macroscale specimens and their grippers from poly(lactic acid) (PLA) filaments (MakerPoint PLA 750 gr Natural) were additively manufactured using a fused deposition modeling 3D printer (Ultimaker 2+). The experimental setup was designed such that a fixed level of longitudinal deformation (i.e., $\delta L = 30 \text{ mm}$) was applied to the upper side of the specimens while fixing the lower sides. The hinge joint at the boundaries of the fixture allowed both ends of the meta-plates to rotate freely and to create an out-of-plane deformation under compression (Figures 5.1c and e). The size of the unit cells (i.e., $2c/w = 3$) and the overall size of the meta-plates (i.e., $W_0 = 92 \text{ mm}$, $L_0 = 130 \text{ mm}$) were kept constant for all the reference designs. The wall thickness, $t_w$, of the struts making up the unit cells was 0.51 mm. The only variable, therefore, was the angle of the unit cells, $\theta$, which determined the Poisson’s ratio of the meta-plate. All dimensions of the meta-plates are presented in Figure 5.1 and Table 5.1, Supporting Information. 10 specimens with $\theta$ angles varying between $48^\circ$ and $120^\circ$ were fabricated to cover a wide range of Poisson’s ratios (Table 5.1, Supporting Information). Although the thickness, $h$, of the meta-plates was set to 5 mm, two additional specimens with lower out-of-plane thicknesses (i.e., $h = 2.5$ and 1 mm) were also fabricated using the maximum and minimum values of the $\theta$ angle (i.e., the most negative and most positive values of the Poisson’s ratio).

The outer contour of the deformed structures was captured by a 3D scanner
(Scan-In-A-Box, FX, ASUS mini beamer, resolution of both cameras: 1280 × 800 pixels). The specimens were photographed from at least eight different angles. The images were then rigidly registered using the software accompanying the 3D scanner (IDEA). After noise removal, the point clouds were imported into CloudCompare software (V.2.9.1) for further analysis.

FE calculations were conducted with Abaqus (Dassault Simulia, V6.14). After importing the geometry of the specimens, a linear brick element (C3D8R, Abaqus) was used for the simulations. A linear elastic material model was used for PLA (\( E = 3.5 \text{ GPa}, \nu = 0.3 \)). Two reference points were placed on the top and bottom of the meta-plates. These points were kinematically coupled with the corresponding upper and lower nodes lying on the surface of the specimens. To perform the buckling analysis, a unit concentrated force was applied to the upper reference node. The upper and lower sides of the structures were set free to rotate perpendicularly to the applied loading direction while their other degrees of freedom were constrained. Linear buckling analysis was then performed using the eigenvalue solver available in Abaqus. The displacements of the nodes corresponding to the first buckling mode were then introduced as geometrical imperfections to perform the nonlinear post-buckling analyses.

A compressive displacement equal to 16 mm was set in the computational models so as to achieve a similar out-of-plane deformation as observed in the experiments, after registering the deformations obtained from the post-buckling analyses to those of the experimental data (Figures 5.1c–f). The displacements in the out-of-plane direction (i.e., \( Z \)-direction, \( U_z \)) of the top surfaces of the computational models were extracted as point clouds. The numerical and experimental point clouds were then registered in CloudCompare (V.2.9.1). The first, \( \kappa_1 \), and second, \( \kappa_2 \), principal curvatures were defined based on Equation 5.1 to the data points at the center point of the meta-plates (Figures 5.1d and f). A surface-fitting algorithm available in Mathematica 11.3 was used for that purpose.

Four additional plate thicknesses (i.e., \( h = 1 \text{ mm}, 2.5 \text{ mm}, 5 \text{ mm and 7.5 mm} \)) were also considered in the computational simulations to evaluate the effects of the thickness of the meta-plates on their curvatures. For the meta-plates with a thickness of 5 mm, 22 additional simulations with varying in-plane thicknesses (i.e., \( t_w/4, 2t_w/4, 3t_w/4, \text{ and } 5t_w/4 \)), widths (i.e., \( W/4, W/2, 2W, \text{ and } 4W \)) and unit cell sizes (i.e., 75\%, 150\% and 300\% of \( w \) while maintaining \( 2c/w = 3 \)) were performed for the cases with the most negative and positive values of the Poisson’s ratio.

Microscale meta-plates were fabricated with a two-photon polymerization 3D printer (Photonic Professional GT machine, Nanoscribe, Germany). A laser power of 100\% and a scanning speed of 0.1 m s\(^{-1} \) were applied to print the
structures in the DiLL configuration using a 10x objective. A droplet of the IP-Q resin (Nanoscribe) was placed on a silicon substrate and was exposed to a femtosecond infrared laser beam (wavelength = 780 nm) to fabricate the designed structures. The samples were then developed in propylene glycol monomethyl ether acetate (from Sigma Aldrich) for 25 min and were dried at room temperature. Furthermore, a scanning electron microscope (SEM, JSM IT100, JEOL) was used to acquire high-resolution images after gold-sputtering (JFC-1300, JEOL, Japan) of the dried specimens (Figures 5.4f and g). The dynamic curvature of the micro-structures was evaluated in air and ethanol (Sigma Aldrich) through an analysis of optical microscopy images (Keyence Digital Microscope VHX-6000) (Figures 5.4h and i).

Soft meta-plates (Figures 5.5c and d) were additively manufactured using a polyjet 3D printer technique (Objet350 Connex3 3D printer, Stratasys, USA) that works on the basis of inkjet-deposited droplets of a photopolymer followed by curing under ultraviolet light. A commercially available hyperelastic polymer (i.e., Agilus30 Black, FLX985, Stratasys, USA) was used for the fabrication of these specimens.

To create the adaptive mirrors (Figures 5.5e–h), the top surface of a meta-plate with the most negative value of the Poisson’s ratio was covered by aluminum foils with a high degree of reflectivity (Figures 5.5f and g). The specimen was then placed in a holder and was subjected to the laser beams passed from expanders (Figure 5.5h). The laser beams were used to demonstrate the change in the focal point and curvature of the mirror as a function of the deformation.
5.5 Supplementary Information (SI)

5.5.1 Designing parameters

In Table 5.1, we provide the parameters used for designing the meta-plates with different Poisson’s ratios.

Table 5.1: The geometrical parameters and Poisson’s ratio of the specimens tested in this study. The parameters are defined in Figures 5.1c and 1e.

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<th>b[mm]</th>
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</tr>
</tbody>
</table>

5.5.2 The second curvature as a function of Poisson’s ratio

Here, we evaluate the second curvature for meta-plates with various thicknesses and Poisson’s ratios, where we keep the applied longitudinal curvature almost constant in all cases. In Figure 5.6, we show the second curvature, \( \kappa_2 \), as a function of the Poisson’s ratio, \( \nu \), for several plate thicknesses. Here, we can compare the FE simulation and experimental results for the meta-plate with \( h = 5 \text{ mm} \). We observe a linear relationship between \( \kappa_2 \) and \( \nu \) with the slope of \( m \), which depends on the meta-plate thickness. The values of \( m \), presented in Table 5.2, are calculated by the slope of linear fits to the data points.

5.5.3 A comparison with plain plates

We use FE simulation to investigate hypothetical non-architected bulk plates (plain plates) with equivalent elastic properties as the meta-plates, such as negative Poisson’s ratio and density. In Figure 5.7, we compare the evolution of
5.5. SUPPLEMENTARY INFORMATION (SI)

Figure 5.6: The second curvature, $\kappa_2$, as a function of the Poisson’s ratio, for plates with the same width but different thicknesses, subjected to a constant applied longitudinal curvature, $\kappa_1$. $\kappa_2$ exhibits a linear relationship as a function of $\nu$ with the coefficients of $m$, presented in Table 5.2 for each plate thickness.

Table 5.2: The parameters of the linear regression fit ($\kappa_2 = m \times \nu$) for meta-plates with different out-of-plane thickness, $h$, values (Figure 5.6).

<table>
<thead>
<tr>
<th>$h$ [mm]</th>
<th>$m$ [m$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2</td>
</tr>
<tr>
<td>2.5</td>
<td>1.4</td>
</tr>
<tr>
<td>5</td>
<td>4.2</td>
</tr>
<tr>
<td>7.5</td>
<td>6.7</td>
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<tr>
<td>5 (Exp)</td>
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</table>

the second curvature in meta-plates with equivalent plain plates. We observe that the rate of the second curvature ($\kappa_{max}/\kappa_{sat}$) and maximum second curvature ($\kappa_{max}$) are significantly higher and increase with a higher slope versus $\nu$ in meta-plates rather than the plain plates.
CHAPTER 5. CURVATURE IN META-PLATES

Figure 5.7: Curvature evolution in conventional plates: a) Induced curvature, $\kappa_2$, vs. imposed curvature, $\kappa_1$, calculated for meta-plates (circular markers) and plain (non-architected) plates with equivalent elastic properties (crossed markers). Maximum second curvature, $\kappa_{\text{max}}$, reached (b), and the initial slope of $\kappa_2$-$\kappa_1$ curves, given by $\kappa_{\text{max}}/\kappa_{\text{sat}}$ (c), as a function of Poisson’s ratio.

5.5.4 Supplementary Videos

The supplementary videos are downloadable via the provided links.

Video 1 Evolution of the meta-plate with $\theta = 48$ and $W = W_0 = 92$ upon bending: Click here or scan the code.
5.5. SUPPLEMENTARY INFORMATION (SI)

**Video 2** Evolution of the meta-plate with $\theta = 120$ and $W = W_0 = 92$ upon bending: Click here or scan the code.

**Video 3** Evolution of the meta-plate with $\theta = 48$ and various width upon bending: Click here or scan the code.

**Video 4** Evolution of the meta-plate with $\theta = 120$ and various width upon bending: Click here or scan the code.

**Video 5** Evolution of the micro-plate with $\theta = 48$ under capillary forces: Click here or scan the code.

**Video 6** Evolution of the micro-plate with $\theta = 120$ under capillary forces: Click here or scan the code.
References


REFERENCES


CHAPTER 5. CURVATURE IN META-PLATES


[34] Jungwook Kim, James A. Hanna, Myunghwan Byun, Christian D. Santangelo, and Ryan C. Hayward. Designing responsive buckled surfaces by


CHAPTER 5.  CURVATURE IN META-PLATES


Chapter 6

General discussion

In this final chapter, we present the main results of this thesis and discuss them from an integrated perspective. We feature the main structural ingredients involved in our presented systems relevant in the context of metamaterials. We address the implications of our findings and their possible applications in a broader perspective. We also articulate particular ideas for future directions based on the results of our research. First, we discuss the general objective of this thesis.

6.1 Objectives

Surprising properties of metamaterials, unraveled in recent studies, indicate their crucial role in designing the next generation of advanced materials [1, 2, 3, 4, 5]. However, the knowledge gap is evident and extensive research is expected to follow up the current discoveries. Particularly in the context of mechanical metamaterials, most studies focus on their exotic behaviors under uniaxial deformations [6, 7], and minor attention has been devoted to shear-induced behaviors of metamaterials [8, 9, 10]. Shear deformations on mechanical systems reveal fundamentally different aspects of materials and deserve independent consideration. Additionally, shear forces are ubiquitous and responsible for various phenomena in mechanical systems. In this thesis, we aim to investigate the shear-induced properties of mechanical metamaterials and design metamaterials exhibiting extraordinary behaviors upon shearing. Furthermore, we aim to exploit such shear-induced unusual characteristics to program behaviors of materials...
and provide platforms for novel functionalities.

In this thesis, we generally investigate the systems under direct shearing or induced shear deformation under various boundary conditions and deformations. We use combination of experimental, modeling, theoretical, and simulation techniques to investigate the properties of designed structures upon different loading scenarios, with a particular focus on shear-induced properties. For experimental studies, we mainly focus on cylindrical structures suitable to apply shear via torsion, axial compression, and a combination of both deformations. Additionally, in a specific case, we study 2D plates under bending, where transversal shear forces are significant, unlike conventional plates [11]. To explain the behaviors of the systems, we theoretically treat different problems either by a known theory in elasticity or developing a new analytical or phenomenological framework based on the modeled system. Spring modeling is a crucial strategy used in this thesis to model the structures solved by numerical and analytical approaches. In the followings, we shortly introduce the projects and results in this thesis and describe their significance and interrelationships.

6.2 Overview of the main findings

First, in this section, we highlight the main findings of this research. Then, we explain the relevance and interconnections between chapters of this thesis and provide a future perspective concerning our studies. In Figure 6.1, we illustrate the overview of the projects and main results. In this figure, we present the experimental systems and their unusual behaviors studied in each chapter and capture the specification of the studied systems and their connections via a wheel diagram.

In Chapter 2, we present metamaterials composed of a network of nonuniform beams that exhibit unique structural phase transitions under compression by creating different structural domains separated by topological kinks and antikinks (Figure 6.1, top-left). We demonstrate that the initially static kinks/antikinks exhibit a soliton-like behavior and propagate along the system via applying a shear deformation. Shear deformations contribute to phase transitions in the system by driving the kinks/antikinks and triggering the creation and annihilation of kink-antikink pairs. We qualitatively show that the system is a mechanical analogous of a ferromagnetic system. We experimentally study the system in a cylindrical platform and fully capture the experimental observations by a spring model. We describe the system as a chain of elastically coupled unit-cells, which undergo a self-contact interaction upon deflection and experience a double-well
6.2. OVERVIEW OF THE MAIN FINDINGS

potential, theoretically investigated by the $\varphi^4$ model [12, 13]. The results confirm the soliton-like origin of the phase transitions. In the following research (Chapter 3), we investigate the mechanical aspects of the system in a discrete limit.

In Chapter 3, we investigate the role of kink and self-contact interaction in programming the shear-induced normal and shear responses, characterized by the Poynting and shear moduli, respectively (Figure 6.1, top-right). We unravel that the Poynting response is highly programmable in our metamaterials by a pre-compression. The normal force and Poynting modulus are initially negative (showing reversed Poynting effect). However, we can program the structure to exhibit a positive normal force and Poynting modulus in the self-contact regime via applying a pre-compression. In this system, the shear and Poynting moduli can increase by one order of magnitude in this structure. Additionally, an axial force induces a nonlinear shear deformation in our metamaterials; since this behavior is an inverse manifestation of the conventional Poynting effect [14], we call it the inverted Poynting effect.

Furthermore, in Chapter 4, we design auxetic cylindrical shells inspired by the well-known auxetic holey sheet [15] but asymmetric with respect to the torsional direction. Similar to the negative axial force (the reversed Poynting effect) we discuss in Chapter 3, we discover a negative radial force response upon torsion, acting radially toward the central axis of the cylindrical shell. The negative radial force represents itself as a uniform radial contraction (Figure 6.1, bottom-right), which initiates a unique structural reconfiguration that unexpectedly suppresses the torsional buckling in surprisingly large torsional angles. Most design strategies in mechanical metamaterials involve the exploitation of local bucklings of unit-cells to trigger exotic behaviors; In contrast, we exploit local bucklings to eliminate the global torsional buckling in auxetic asymmetric shells. Here, we introduce the combination of asymmetry and auxeticity as the complementary factors to obtain a radial contraction upon torsion, resulting in a shape transformation from Euclidean to non-Euclidean hyperboloid surface with negative Gaussian curvature. In the next chapter (Chapter 5), we highlight the effect of auxeticity in the non-Euclidean evolution of thin-walled plates.

In Chapter 5, we explore non-Euclidean shape morphing on the surface of a range of non-auxetic and auxetic cellular plates made of hexagon-based unit-cells (meta-plates) under bending. We discover that auxetic meta-plates obtain an unconventional positive Gaussian curvature upon bending. We extensively study the effect of geometry and structural ingredients, particularly Poisson’s ratio, in shape morphing of meta-plates (Figure 6.1, bottom-left). We investigate experimental and simulation data and propose a general scaling theory to
Figure 6.1: Overview of the research chapters in this thesis, our findings and their interrelationships: The inner layers of the wheel diagram show the unit-cells and their characteristics, and the outer layer specifies a significant aspect of the studied system in each chapter. The experimental systems and their unconventional responses are demonstrated in four corners of the figure. The green and blue arrows, respectively, display the applied deformations and responses of our studied metamaterials.
predict the second curvature as a function of the applied longitudinal curvature, applicable for meta-plates with various geometrical and structural properties. We verify the obtained relationship to be scale-free by experimentally testing it on a micron-scale sample.

In this thesis, we uncover novel aspects of metamaterials suggesting various shear-induced or shear-driven functionalities. Our research sheds light on the relationship between shear-induced properties and structural characteristics of elastic materials, as it provides insight into the behavior and evolution of systems experiencing shear deformation, which are ubiquitous. We proceed to elaborate on the importance of shear deformation in natural and artificial systems. We discuss the origins of our interest in the shear-induced properties of metamaterials and hope to motivate future studies regarding the relationship between shear deformation and structural characteristics.

6.3 Why shear deformations

Shear deformation is present in natural systems, from cell-scale to Earth-scale, and understanding its relation to the structural ingredients provides insight into predicting and controlling mechanical behaviors of such systems. For example, shear strains are abundant in cell growth and biomechanical systems such as tissues, veins, tendons, joint cartilages, and cardiovascular mechanics, to name a few [16, 17]. On a large scale, shear deformation is significant in the evolution of Earth’s crust and is associated with geological structures and shear zones [18, 19]. Investigating shear waves can reveal critical information about multilayered mediums [20] and is used to explore Earth’s crust and underground reservoirs [21, 22]. By better understanding shear-induced properties and their relation to the structural ingredients we can better predict the mechanics of materials and gain an insight into their formation.

Shear-induced normal force can reveal critical information about the composition and structure of a material. In a continuum solid, the normal force response is positive and follows a quadratic relationship as a function of shear strain, leading to perpendicular dilation [14, 23, 24, 25]. However, the quadratic normal response is reversed in some viscoelastic bio-polymers due to the non-linear response of the bio-polymer network [26, 27, 28, 29, 30, 31]. Nonetheless, anisotropic materials show non-quadratic normal force response [32, 31, 31, 33]. Understanding the relationship between normal force and structural ingredients provides an insight to connect the macroscopic properties of materials to their microscopic structure. In this thesis, we show that the behavior of the normal
force depends on the orientation of the ligaments forming the unit-cells. Generally, if the unit-cells have ligaments along the diagonal direction with respect to the shear strain, the normal force observed to be positive (in Chapter 3, in the self-contact regime, and in Chapter 4, the structure with $\varphi = \pi/4$) otherwise the normal force is negative (in Chapter 3, in the pre-buckling regime and in Chapter 4, the structure with $\varphi = 0$). If buckling is dominant and stretching/contraction negligible, the normal force is observed to be zero (in Chapter 3 and 3, in the buckling regime) [34]. However, the normal force is not quadratic in structures with asymmetry with respect to the shear direction (Chapter 4). These observations give insight into the structure of a system regarding the normal force response. Such understanding is valuable in studying elastic and viscoelastic materials since it helps to analyze and interpret rheological measurements, which is a widespread technique.

Shear itself can exhibit extraordinary mechanics in metamaterials that can potentially promote novel mechanical functionalities (Chapters 2–4). Such characteristics allow designing a new class of mechanical devices such as shear-driven pumps, mechanical switches, twisters, and actuators. Moreover, shearing can provide a platform for programming material response in combination with axial strain/confainment, as programming mechanical properties is one of the main aspects of designing metamaterials (Chapters 2 and 3). The combination of shearing and compression is a flexible mechanism to control the mechanical properties of metamaterials and their structural phases on a local scale. Internal shear forces are essential in studying the deflection of 2D elastic metamaterials under deformations [35, 36, 37]. Shear forces are present under various loading modes, but in conventional plate theories, lateral shear deformations are neglected under pure bending [38, 36]. However, they can significantly contribute to the formation of Gaussian curvatures under simple bending in metamaterials (Chapter 5) [39].

The noted reasons motivate studying shear deformation as an essential factor in metamaterials. We hope our results contribute to obtaining a more comprehensive insight into the shear-induced properties of conventional materials and metamaterials, in particular, concerning their structural characteristics. However, our understanding of the shear-induced behavior of numerous metamaterials remains limited, suggesting a significant potential for future research focusing on this problem. Additionally, combining uniaxial and shear deformations as an efficient strategy for programming metamaterials (Chapters 2 and 3) motivates designing systems by exploiting a similar approach.

To outline a general interpretation, we highlight the structural ingredients and their relationship to shear-induced behaviors based on the outcomes of this
6.4 Structural ingredients

6.4.1 Self-contact mechanism

In the structures studied in Chapters 2 and 3, unit-cells are confined due to self-contact interaction between their vertical and horizontal elements. We observe that self-contact is an essential mechanism to obtain novel properties such as structural phase transitions and shear-driven topological kinks in the system (Chapter 2). The origin of the kink is rooted in local confinements of the unit-cells and the double-well potential they experience due to self-contact. The unit-cells in system is inspired by the structure created using a 2D array of voids made of a four-fold symmetric contour [40], but made with simplified unit-cell. The unit-cells of the system with four-fold symmetric voids and our simplified unit-cells share essential ingredients, including deflection of the beams and self-contact transitions under a compression. In Chapter 3, we use the structure with four-fold symmetric voids [40] to design cylindrical metamaterials and observe identical behavior as for our simplified unit cell (Chapter 2). In Chapter 3, self-contact is accountable for the positive and significantly elevated Poynting and shear moduli obtained by pre-compression.

6.4.2 Double-well potential

Bistable unit-cells are popular among metamaterial researchers for their ability to exhibit kinks and trigger unusual effects [41, 42, 43, 44]. Most known bistable unit-cells rely on the combination of bending, relative shearing, and stretching/contraction of their hinges to obtain a double-well potential. In our system, however, self-contact is responsible for the double-well potential, tunable by varying the nonuniformity. Additionally, these unit-cells maintain enough freedom to deflect with respect to each other; this leads to extreme shear deformation possibility, useful for programming and tuning the system.

6.4.3 Auxeticity and asymmetry

Auxetics are a class of mechanical metamaterials that exhibit a negative Poisson’s ratio. A 2D plate consist of re-entrant honeycomb (bow-tie shaped) unit-cells (meta-plates) is the first auxetic system, which was introduced in 1987 [45, 46]. Additionally, a 2D elastic sheet with a square array of circular voids exhibit
a negative Poisson’s ratio upon compression, which is known as the holey sheet [15]. In Chapter 4, we design meta-shells inspired by the holey sheet pattern; additionally, we introduce asymmetry with respect to the shearing (torsional) direction of the shells by varying the orientation of the unit-cells array. In Chapter 4, combination of auxeticity and asymmetric designing triggers the shear-induced radial contraction leading to shape-changing from a plain cylinder to a hyperboloid. In Chapter 5, we determine auxeticity as an essential ingredient in obtaining unusual surface curvature with a positive Gaussian curvature in meta-plates. Therefore, auxeticity is fundamentally linked to intrinsic deformations and exploitable to tune shape-changing properties in mechanical systems.

6.5 Outlook and future directions

We hope that the results of this thesis provide insight into understanding and designing metamaterials from a broader perspective. Here, we describe different aspects of our results, discuss their implications in designing metamaterials, and outline particular ideas for future directions.

6.5.1 Zero modes and mechanical insulators

Topological insulators are well-known examples of topological materials in electronic systems, which are conductive in the bulk, but insulator at the surface (boundary) due to boundary zero modes [47]. Recent studies unraveled the fundamental connection between the topological zero modes in mechanical metamaterials and quantum systems [48].

Zero modes result in the emergence of a gap in the vibrational spectrum of a solid. Zero modes are associated with the domain walls in the bulk or floppy modes at boundaries of a topological system, which are insensitive to perturbations. The emergence of kinks in our metamaterials in Chapters 2 and 3 is a footprint of a topological zero mode that permits designing mechanical insulators, which can be floppy at boundaries and functional for mechanical noise canceling. Boundary modes with interesting features can exist in a 2D system in the lateral direction relative to the compression. Since our previous experiments are on cylindrical systems, lateral boundary modes cannot exist. However, the structure in a 2D topology could allow the emergence of zero modes in the lateral direction. A preliminary test on a 2D structure made of the same unit-cell as the cylindrical structures in Chapter 2 (Figure 6.2a) shows that
the compressed system exhibits substantial differences between the left and the right boundaries (Figure 6.2b). Especially at the location of the kink, the left horizontal beam is confined, like the bulk beams, but the right horizontal beam is floppy. This boundary effect reflects the topological difference between a 2D plate and a cylindrical shell. The horizontal beams at the bulk seem to keep their horizontal fixed position, similar to horizontal beams in our cylindrical systems.

Verifying the topological origin of the boundary modes requires studying the dynamical modes and obtaining the vibrational spectrum of the system. To this end, we need to investigate the Hamiltonian of the system in the framework of topological band theory. The Hamiltonian is given by introducing the kinetic energy to the potential of the system as $\mathcal{H} = (k'_b/E_0)(a_1 \partial_z \theta(z))^2 + \xi(z) + 1/2(I/E_0)\dot{\theta}^2$ (see Chapter 2-SI: 2.6); the last term gives the kinetic energy, where $I$ is the moment of inertia and $\dot{\theta}$ is the angular velocity of the beams. Such an investigation leads to understanding a different aspect of our system and is relevant for designing mechanical insulators.

Figure 6.2: Boundary modes in 2D topology: a) Nonuniform beam network in a 2D plate topology. b) The structure deformed under longitudinal pre-compression.
6.5.2 State evolution and memory

We showed that the units of our structure (Chapter 2) switch between right-buckled (0) and left-buckled (1) states in a hysteresis loop. Therefore, the sequence of unit-cell states determines the global state of the system. Such bistable units are known as hysterons, and they can exhibit a surprisingly complex sequence of transitions, known as pathways, in mechanical metamaterials [49]. At each step, the specific sequence of unit-cell state gives the global state of the system. Such mechanical systems are recently introduced as potential mechanical memory units [50, 51, 52]. Systems of hysterons have also been used to study transitions in ferromagnetic materials [53, 54, 55, 56].

In our experiments, we drive the system in the complete cycle from one extreme (all unit-cells left-buckled), by a clockwise (cw) torsion to another extreme (all unit-cells right buckled), and by a counterclockwise (ccw) torsion back to the first extreme. Driving the system in the complete cycle is a primary driving strategy, which is known as the main loop in these systems [49, 50]. As we showed in Chapter 2, our system exhibits one local minimum at each step, in total $2 \times 18$ minima, evolving in the main loop. By observing the global states of the large system in the main loop (Chapter 2), we can confirm that the system experiences $2 \times 18$ distinct global states, which reflects the potential of this system in exhibiting various states in the simplest driving strategy. However, we can implement the pre-compression strain as a second degree of freedom to examine if we can obtain different sets of system states in the main loop. To this end, we drive the system in the main loop under different pre-compression strains.

In Figure 6.3, we show the phase of the system and its changes over the main loop for different pre-compression strains, as $\delta = 0.17, 0.18$ and $0.21$, and driving directions (a, cw and b, ccw). We observe that our system acquires a sequence of global states when driving forward (cw, Figure 6.3a) that is entirely different compared to when returning it (ccw, Figure 6.3b), as the colored regions are distinct. One can realize that our system potentially has $2^N = 2^{18}$ stable state. However, the accessible number of states in the main loop is limited to $2 \times 18$. Although, we can tune the interaction between layers by changing the pre-compression strain and triggering a different set of switching events in the system. To understand these variations, we demonstrate the transition graphs (t-graphs) of the system, Figure 6.3c, to visualize different pathways in the main loops and compare its variations by pre-compression strain. In t-graphs, we code the regions of the system with the same state by a sequence of $0_i$ and $1_j$ (from bottom to top), where 0 and 1 represents the right-buckled and left-buckled
states, respectively, and $i$ and $j$ show the number of unit-cells with the same state in the domain. In Figure 6.3c, the arrows with light green ($\delta = 0.17$), green ($\delta = 0.18$), and dark green ($\delta = 0.21$) show the transition events, where we only show the states at 12 specific steps out of 36, with $\varphi/\varphi_t = -9, -6, -3, 0, 3, 6, 9$). Arrows pointing to the right and left represent the cw and ccw driving directions, respectively. We observe that the switching events occur under similar pathways in $\delta = 0.17$ and $\delta = 0.18$ with minor variations. However, under $\delta = 0.21$, a new switching event, $\varphi/\varphi_t = 6 \rightarrow 3$ (dark green arrow) in Figure 6.3c, triggers a set of new states, which are not present in previous cases. The new transition considerably increases the number of newly accessed states. Figure 6.3c also shows that driving in the main loop in different directions leads to obtaining global states with an anti-symmetry, where the same unit-cells have opposite phases (shown by a rectangular contour with similar dashing).

The transition pathways in previously studied metamaterials are surprisingly complex and difficult to control for practical applications, even in a small system of three unit-cells [50, 51, 52, 49]. However, our system shows practical potential as a memory unit due to combining compression and shearing to control the system. Although these observations imply the existence of interesting effects in the system, the presented t-graphs are incomplete since they only show the main loop transitions. To testify phenomena such as the return point memory effect [57], diverse driving strategies are needed, which is beyond the scope of this thesis but an interesting direction for further research.

### 6.5.3 Stable vs. unstable kink

In Chapter 2, we discuss the emergence of topological kinks under a pre-compression localized within a small domain wall, including two unit-cells in our discrete systems. Two unit-cells in the kink region obtain opposite deflections in the vicinity of the minima ($\pm \theta_0$) of the double-well potential. As delineated in Chapter 2, structural phase transitions occur by unit-cells switching between two stable states under shear deformations. However, an intermediate unstable state exists between two minima of the double-well potential, where the unit-cell is not deflected but maintains a straight orientation. This intermediate state represents an unstable kink and can be realized by shearing the system with a stable kink by a certain amount. Moreover, we can capture this unstable intermediate state solely under a pre-compression (at zero-shear) if the system has an odd number of unit-cells along its length. In Figure 6.4a, we compare the odd-layer (3-, 5-, and 7-layer) and even-layer (4-, 6-, and 8-layer) pre-compressed structures from the experiment and discrete model. The experimental structures
Figure 6.3: Tunable transitions pathways by a pre-compression strain: a–b) Phase evolution of the domains under clockwise (a) and counterclockwise (b) torsions under different pre-compression strains as $\delta = 0.17$ (left), $\delta = 0.18$ (middle), and $\delta = 0.2$ (right). c) Transition-graphs showing the state transition of the system under different pre-compression strains with light green ($\delta = 0.17$), green ($\delta = 0.18$), and dark green ($\delta = 0.2$) arrows. States are given by the sequence of indexed zeros (for right-buckled) and ones (for left-buckled) coding the domains from the bottom layer to the top layer, where their index shows the number of unit-cells with the same state in the same domain. Right-arrows show the clockwise, and the left-arrows show the counterclockwise torsions.
are made of the simplified nonuniform unit-cell (Chapter 2), and the discrete model is based on the model presented in Chapter 3. Plots presented in Figure 6.4b, show the deflection as a function of the unit-cell label for 3- to 8-layer structures. We observe that the unit-cells in unstable kinks experience a significantly higher contraction strain compared to the other unit-cells.

We further investigate the 4-layer and 5-layer structures under shear deformations to compare their mechanical properties. Shearing both odd-layer or even-layer systems lead to oscillatory shear force \( F_s \) (dash) and normal response \( F + F_n \), solid) but with a phase shift. Here, \( F \) is the compression force, and \( F_n \) is the shear-induced normal force. At zero shear, normal force is minimum for the 4-layer structure but maximum for the 5-layer structure (Figure 6.5a, experiment and 6.5b, discrete model). The slope of the shear response at zero-shear is positive for the 4-layer but negative for the 5-layer structure. Therefore, shear and Poynting moduli both are negative, where the system exhibits an unstable kink. In Figure 6.5c, we show the normal force as a function of the equivalent small-range torsion for the 5-layer modeled system \( \delta x/R \) under different pre-compression strains, \( \delta \). As a reminder, \( a_0 \) is the initial length of the unit-cell, and \( R \) is the average radius of the cylindrical shells. The width of the normal force peak becomes narrower by increasing the pre-compression strain, representing a smaller transitional shear strain and higher absolute Poynting modulus. In Figure 6.5d and e, we respectively visualize the unstable-kinks (highlighted area) in 4- and 5-layer modeled structures at various shear strains. We observe a soliton-like traveling behavior as we expected based on the results in Chapter 2.

Consequently, the unstable kink is associated with entirely different mechanical properties of the system, where both the shear and Poynting moduli are negative. Therefore, switching between stable and unstable kinks is the origin of the oscillatory Poynting and shear moduli, presented in Chapter 3. Negative shear modulus potentially can function as a mechanical actuator in soft systems. The unstable kinks, however, can only be realized in discrete systems, and the smaller the number of unit-cells easier capturing the unstable kink is. This explains the relative deflection of the middle unit-cell in the 7-layer experimental structure in Figure 6.4a, which contrarily exhibits an unstable straight position in the modeled system. The relationship between the system size and the shear and Poynting moduli remains a question for systems with stable or unstable topological kinks.
Figure 6.4: Stable vs. unstable kink: a) The pre-compressed experimental and modeled systems with 3–8 unit-cells in length. b) Unit-cell deflection as a function of its label for the modeled systems with 3–8 unit-cells in length, under $\delta = 0.17$ pre-compression strain.

6.5.4 An optimized buckling-resistance system

In Chapter 4, we show that combining auxeticity with asymmetric designing is essential in obtaining unusual shear-induced radial contraction, leading to suppression of the torsional buckling in the system. Nonetheless, an analytical approach quantifying the effect of both ingredients is missing. One can develop a spring model as the framework for numerical analysis and theoretical investigations. Moreover, finite element simulations can complement this study to understand the radial contraction and buckling-resistance properties in an exclusive picture. For example, the dependency of the contraction ratio (slope in contraction strain-shear strain curve) is a crucial parameter, and its relation to the structural ingredients and geometrical properties is unclear. For example, properties such as asymmetry orientation, unit-cell type/scale, and shell dimensions can be studied, in more detail. A complete understanding is essential for designing relevant mechanical components.
6.5. OUTLOOK AND FUTURE DIRECTIONS

Figure 6.5: Unstable kinks under shear deformations: Oscillatory responses of 3-layer and 4-layer experimental (a) and discrete (b) systems as a function of torsion in experimental or its equivalent, $\delta x/R$, in the modeled system. c) Normal response of the 5-layer discrete system in a small-range shear deformation under various pre-compression strains. The traveling unstable kink under shear deformation in 4-layer (d) and 5-layer (e) systems.

6.5.5 Shape programming

In Chapter 4, we realize the role of auxeticity combined with an asymmetric parameter, implemented by rotation of the principal axes of auxetic system, in the radial contraction of a twisted cylindrical shell. The radial contraction leads to negative surface curvature and shape-changing from a Euclidean to a non-Euclidean hyperboloid surface. In Chapter 5, we also observe a Euclidean to a non-Euclidean geometry transformation as a 2D auxetic meta-plate shows an unusual positive Gaussian curvature upon bending. A new direction can involve introducing asymmetry in auxetic plates; as evident in Chapter 4, it can potentially trigger novel shape morphing and surface curvatures under various
deformation scenarios.

As we introduce some of the relevant implications of our presented metamaterials, these systems provide flexible platforms to obtain various unusual functionalities that are inaccessible in the realm of conventional materials. Such functionalities can potentially be utilized for designing advanced structures for different applications. We introduce some particular applications of the presented systems in this thesis.

6.6 Applications

Designing functional mechanical parts has been challenging in different areas such as Soft Robotics, Biomedical Engineering, and Biomechanics, to name but a few. Inspired by the results of this thesis, we introduce some ideas for engineering mechanical compartments.

In Chapter 2, we introduce a system with traveling topological kinks that allows local phase programming of the structure. The system exhibits numerous stable states upon shear, tunable with the number of unit-cells, exploitable as a robot arm functional under a combination of compression and torsion. Traveling kink coincides with a programmable dissipation that enables the system as a medium for energy dissipation and damping impact force. The dissipative feature is usually emphasized in mechanical metamaterials [7]. Additionally, in the section 6.5.2, we discussed how such behavior is relevant for designing mechanical memory units. Designing a functional mechanical memory is a step toward building smart mechanical materials that can interact with their environment, remember, and process information. Intrinsic plasticity can complement the system for learning and mechanical sensing applications. In Chapters 3 and 4, we observe the practicality of the presented systems for tuning the coupling between shear and normal forces and programming the magnitude of the shear rigidity and normal response of the system. These are relevant features in engineering mechanical components such as twisters, mechanical switches, actuators, implants, and prosthetic devices.

Suppressing torsional buckling, obtainable in the auxetic asymmetric meta-shells (Chapter 4), is considerable for designing cylindrical structures that experience torsional loads. Additionally, a squeezable cylindrical geometry suggests applications as a shear-driven pumping mechanism. Due to similar blood pumping mechanisms in the heart and considering the soft composition and potential biocompatibility of the meta-shells, it can provide a strategy for developing an artificial heart.
Functional shape-morphing platform has been interesting for engineering complex 3D objects by shape transformation of 2D panels [58, 59, 60]. A system with Euclidean to non-Euclidean shape transformation, as presented in Chapters 4 and 5, is relevant in designing a wide variety of facilities with desirable shape-changing. Accordingly, designing soft wearables and a curved mirror with adaptive focal length are suggested in Chapter 5.

Relevance in food systems: Although structuring viscoelastic materials is a challenge at the microscopic level, recent advancements and substantial interest in 3D food printing promise micro-structuring in future food engineering. It should be noted that a growing interest in 3D food printing has been observed due to its potential to revolutionize the sustainable food industry. Therefore, design ingredients and structural strategies developed in metamaterials can inspire efficient food micro-structuring, for example, to obtain desired mouthfeel and sensory properties in meat analogous as we can program shear and normal force responses. Moreover, shape-changing is interesting in food systems, especially for post-processed foods, where structural properties such as auxeticity can be implemented for shape programming.

6.7 General conclusion

In this thesis, we explore shear-driven structural phase transitions, mechanical response, instabilities, and shape-changing of metamaterials. We discover the shear-driven evolution of the structural phase via traveling and interacting kinks in metamaterials. Such a system exhibits distinct shear-induced mechanical properties, programmable via implementing kink under confinement. We also highlight the essential role of combining auxeticity and asymmetry in shape-changing and preventing torsional buckling in cylindrical shells. Finally, we propose a generic relation that rationalizes the unconventional shape-changing behavior of auxetic and non-auxetic meta-plates upon bending.

The studies in this thesis unravel new possibilities in controlling and programming the shear-induced behaviors in metamaterials, which has numerous applications in engineering novel soft systems in biomechanics, soft robotics, etc. Our studies can motivate designing and exploiting coupling between shear and axial deformations, particularly in cylindrical systems. Our research provides insight into the formation and evolution of soft materials by elaborating on problems such as structural phase transitions, buckling instabilities, and shape-changing. Our focus on shear deformations extends our understanding of the shear-induced responses (like programmable Poynting effect and shape-changing)
in mechanical systems and their relation to structural characteristics.

A systematic self-contact transition practically can be equivalent to switching to a different structure obtainable only by confinement. Therefore, designing novel self-contacting unit-cells can be a strategy to obtain reformable metamaterials.

In the end, we hope this thesis is a small step to broaden our understanding of the shear-induced behavior of materials with respect to their structure and motivates future research in metamaterials.
References


Summary

In this thesis, we explore shear-induced unusual behaviors of metamaterials. We show that topological kinks emerge in a confined system with self-contacting unit-cells and travel by shear deformations as a soliton-like wave pocket (Chapter 2). The emergence of kink in the confined system allows programming shear-induced responses to a great extent, even reversing the perpendicular shear-induced strain (Chapter 3). We investigate the significant role of the orientation of the principal axes of auxetics with respect to the shearing direction in suppressing shear-induced buckling (Chapter 4). A generic relation is developed that describes the evolution of Gaussian curvature in cellular auxetic and non-auxetic meta-plates under longitudinal bending (Chapter 5). Here, we highlight the main findings of our investigations.

In Chapter 2, a system of interacting unit-cells that exhibit self-contact transition under confinement is investigated with experimental methods, continuum theory, and a discrete model. We discover the formation of topological kinks and antikinks in the system under longitudinal pre-compression that travel by shear deformation as a soliton-like localized $\varphi^4$ wave pocket. The quasi-static evolution of the system and structural phase transitions are further investigated upon shear deformations. Hereto, shear-driven traveling kinks and antikinks and the creation and annihilation of kink-antikink pairs are explored. These events contribute to the structural phase transition of the material, which qualitatively is comparable with a ferromagnetic system. We hereby show that the transformation of kinks/antikinks coincides with a considerable dissipation, which is programmable by the level of pre-compression strain.

In Chapter 3, we show that the mechanical properties of the system are programmable via transition to the self-contact regime and emergence of kink in a discrete limit. The shear-induced response of the system is studied by the means of experimental methods and a discrete model. It is demonstrated that the shear-induced normal force in a non-confined system is negative, where
the system contracts perpendicularly upon shear (the reversed Poynting effect). However, it is programmable via longitudinal confinement (axial pre-compression in the cylinder) and switches to positive, where shear induces a perpendicular dilation (the Poynting effect). Additionally, compression induces nonlinear shear deformations in the system (the inverted Poynting effect). To describe the system quantitatively, the normal response is characterized by defining the Poynting modulus as the coefficient of the quadratic relationship between the normal stress and the shear strain. We show that the Poynting modulus initially is negative, displaying reversed Poynting effects unlike a continuum solid. However, via a pre-compression, it can be eliminated in the buckling regime (zero Poynting modulus) and reversed in the self-contact regime, where the unit-cells undergo self-contact interactions (positive Poynting modulus). We show that the Poynting and shear moduli are highly programmable, and the ratio of Poynting to shear modulus can increase by one order of magnitude in our metamaterial compared to the conventional solids. Oscillatory shear and Poynting moduli (with normal and shear forces) are obtainable upon a large shear deformation. The origin of the oscillatory response is the phase switching of the unit-cells and traveling kink upon shearing (Chapter 2). All the experimental results are also reproduced in a discrete modeled system.

In Chapter 4, we investigate the shear-induced properties of meta-shells inspired by auxetic sheets (holey sheets) designed by implementing a square array of closely packed circular voids. The meta-shells are designed using the same pattern but in a cylindrical coordinate and by implementing asymmetry with respect to the torsional direction around the cylinder axis. The asymmetry is implemented and tuned by rotating the principal axes of the holey sheet (two perpendicular axes of unit-cells periodicity) relative to the cylinder axis. We show that similar to the negative axial strain under torsion (the reversed Poynting effect obtained in Chapter 3), a negative torsion-induced radial strain is achievable in asymmetrically designed auxetic meta-shells, leading to their uniform contraction. Twisting an asymmetric meta-shell firsts leads to its softening due to the unit-cells deflection. By twisting it further, a unique structural reconfiguration results in ordered compaction of the unit-cells that coincides with the stiffening of the asymmetric meta-shells. As a result of this reconfiguration, the asymmetric meta-shells circumvent the torsional buckling, even under extreme applied torque and torsional angle (more than a complete turn, $> 2\pi$). In addition to the radial contraction, the asymmetric meta-shells display a nonmonotonic axial strain, representing the complex Poynting response of the system. Radial contraction in auxetic asymmetric shells governs a shape-changing from a cylinder into a hyperboloid with negative surface curvature.
In Chapter 5, shape-changing of thin-walled auxetic and non-auxetic cellular plates (meta-plates) with various Poisson’s ratios and geometrical properties are investigated upon bending. Applying a longitudinal curvature (bending) along the length leads to the emergence of significant curvature along the width (the second curvature) in meta-plates. In conventional bulk plates, the second curvature is absent since the lateral shear effects are negligible, unlike meta-plates. By tuning the Poisson’s ratio in meta-plates, we can obtain a saddle-shaped surface with negative Gaussian curvature (in non-auxetics) or a dome-shaped surface with positive Gaussian curvature (in auxetics). The second curvature strongly depends on the applied longitudinal curvature. The evolution of the second curvature versus the longitudinal curvature is investigated in meta-plates with a broad range of Poison’s ratio. The meta-plates are designed by gradually varying their unit-cell from honeycomb (non-auxetic) to bow tie-shaped (re-entrant, auxetic). Here, we present a generic relationship to describe the evolution of the second curvature on meta-plates upon bending, considering the structural and geometrical properties of the meta-plates: including the unit-cell’s scale, proportion and wall thickness, and plate’s width, height and Poisson’s ratio. We confirm our scaling theory with extensive simulation and experimental data and verify the reliability of the proposed relation across different length scales, including microscale meta-plates.
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It is not a secret that doing a PhD is not a one-person job. My PhD was not an exception either. I received substantial professional and personal support from my advisers, collaborators, colleagues, friends, and family, which enabled and encouraged me to complete this stage of my life.

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About the author

Aref Ghorbani is an Iranian physicist born in 1987 in Zanjan, Iran. He obtained his BSc in Physics from the University of Zanjan (2007–2011), where he developed a deep enthusiasm for Physics. He continued his MSc studies at the Institute for Advanced Studies in Basic Sciences (IASBS), a top Physics institute in Iran (2011–2014). For his MSc thesis, he theoretically investigated the hydrodynamic synchronization of cilia and the propagation of metachronal waves on ciliated microorganisms. His results were published in the journal of Physical Review E (PRE) in 2017. After his MSc study, he worked as an application specialist in a mobile app market company (Cafebazar) in Tehran, Iran (2015–2016). After less than two years of working, Aref quit his job in order to start an academic career, and in 2018, he was hired as a PhD candidate by Wageningen University. During his PhD, he designed and investigated metamaterials with unusual shear-induced properties. The results of his PhD research are presented in this thesis. Email: arefpj@gmail.com
List of publications

This thesis

• A. Ghorbani, C. Coulais, A. Najafi, D. Bonn, E. van der Linden, and M. Habibi, Shear-driven Topological Kinks. *To be submitted*


• A. Ghorbani, T. Roebroek, C. Coulais, D. Bonn, E. van der Linden, and M. Habibi, Suppressing Torsional Buckling Using Auxetics. *To be submitted*


*These authors have contributed equally to this work.

Other research

• G. Gimenez-Ribes, S. Y. Teng, A. Ghorbani, E. van der Linden, M. Habibi, Quantification of the structural asymmetry using the normal force response. *In preparation*
Overview of completed training activities

* Poster/pitch presentation
† Oral presentation

**Discipline specific Courses**

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## Conferences and symposiums

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### Other activities

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