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Innovative Applications of O.R.

Endogenous dynamic inefficiency and optimal resource allocation: An application to the European Dietetic Food Industry

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The conventional dynamic cost inefficiency model relies on the directional distance function with an exogenous directional vector to measure technical and allocative inefficiency. However, this approach may lead to contradictory recommendations for firms to become technically and allocatively efficient. By definition, the conventional model forces firms to reduce their inputs and increase their investments in order to become technically efficient; for some firms this is followed by the reverse recommendation to become allocatively efficient. This paper proposes a model that endogenizes the directional vector to solve for the cost minimizing combination of inputs and investments. In contrast to the conventional model with an exogenous directional vector, our model provides managers with monotonic prescriptions. We illustrate the superiority of the endogenous directional vector model over its conventional counterpart using a dataset of EU firms in the dietetic food industry. The differences in the managerial prescriptions are striking, with the conventional model wrongly recommending reductions in inputs that are underused with respect to their optimal amounts minimizing cost.

1. Introduction

The measurement and decomposition of economic efficiency into technical and allocative components has a long tradition in economic theory since Debreu (1951), and, particularly, Farrell (1957). The latter author showed that cost efficiency, defined as minimum cost divided by observed cost, can be decomposed into a technical efficiency measure depending only on quantities, and, a residual, which he termed *price efficiency*. While technical efficiency measures the cost excess from the failure to exploit the production frontier, price efficiency measures the additional cost excess in which a (projected) technically efficient firm incurs by failing to use the optimal cost minimizing quantities (mix) of inputs at given market prices. Hence, price efficiency effectively corresponds to the concept of allocative efficiency as defined in the literature on measurement of efficiency of production; at least since Färe et al. (1985). Cost efficiency and its components are bounded by one, and the greater their value, the more efficient is the firm in each of these dimensions.

From an economic perspective, this decomposition is grounded on the duality theory introduced by Shephard (1953), and later extended by Färe and Primont (1995). Duality theory simply states that, assuming cost minimization, it is possible to recover the technology, in this case the associated input production possibility set, from its supporting cost function, and vice versa. The relevance of duality is the possibility of proving the existence of a so-called Mahler inequality by which minimum cost is smaller or equal than observed cost at the technically efficiency projection of any firm under evaluation (Pastor, Aparicio & Zofío, 2022). Hence Farrell's decomposition is theoretically consistent by relating the quantity (primal) and dual (price) spaces.

Beyond the radial model corresponding to the technical efficiency measure introduced by Farrell (1957), several authors have proposed alternative models to decompose cost inefficiency relying on additive measures of technical inefficiency (e.g. Charnes, Cooper, Golany, Seiford & Stutz, 1985). For instance, based on the input oriented Enhanced Russell Graph measure introduced by Pastor et al. (2011) (also known as Slack Based Measure), Aparicio, Ortiz and Pastor (2017) develop the duality that allows decompos-

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ing normalized cost inefficiency into technical and allocative terms. Aparicio, Pastor and Vidal (2016) introduced the weighted additive distance function (WADF), which endows additive-type models with a distance function structure. An appealing feature of the WADF is the flexibility offered when choosing the weights for the input slacks. Hence, the duality theory developed by these authors for the WADF encompasses a wide range of measures, enabling a consistent decomposition of cost inefficiency. For instance, the normalized weighted additive model, the measure of inefficiency proportions (MIP), the range adjusted measure (RAM) of inefficiency, etc.

Recently, Silva, Oude Lansink and Stefanou (2015) have extended duality theory underlying all these models to the cost adjustment model that provides an intertemporal (dynamic) approach to efficiency measurement when some inputs are quasi-fixed. Their analytical proposal departs from the multiplicative approach inherent to the radial approach by adopting the flexible input directional distance function (DDF) introduced by Chambers, Chung and Färe (1996). The DDF is flexible because researchers may choose the directional vector g and, as shown by Chambers et al. (1996), it nests its Farrell radial counterpart. Specifically, if the directional vector is equal to the observed amount of inputs, $g = x$, then it can be proven that the DDF is equal to one minus Farrell's input technical efficiency.[1] Because of the additive nature of the DDF, and based on its duality with the cost function, economic performance defines as the (normalized) difference between observed cost minus minimum cost.

However, the flexibility offered by the DDF comes at the cost of a subjective choice of the directional vector g. Because of its straightforward relationship with the popular radial approach, as shown above, most studies in the empirical literature choose as directional vector the observed amount of input quantities. This allows a straightforward interpretation of the value of the directional distance function as the proportion of inputs that need to be reduced to reach the production frontier. It also makes the DDF units' invariant in the sense that if we multiply inputs and outputs as well as their directional vector by the same vector, then its value remains unchanged. However, choosing different directional vectors across firms was soon identified as a shortcoming in the context of technical and economic efficiency measurement.

Firstly, the value of the DDF—i.e., the technical inefficiency score, depends both on the length and the direction of g, implying that their absolute values are not directly comparable. Secondly, as in the radial approaches, the exogenous choice of the directional vector results, through duality, in an arbitrary decomposition of cost inefficiency into technical inefficiency, and the residual allocative efficiency. To the extent that the value of the DDF depends on the subjective choice of the directional vector g, so does the value of the allocative inefficiency. Consequently, changing the value of g; i.e., projecting the same observation in alternative directions or projecting observations in different directions, changes the value of both the technical and allocative inefficiencies, which, once again, become incomparable across observations. Färe, Grosskopf and Margaritis (2008) discuss several possibilities when choosing a common or 'egalitarian' directional vector g, that makes the distance functions readily comparable in value and imply that the cost inefficiency decomposition is based on the same direction. Two possible choices are the unitary vector, $g = 1$, or the mean of the observed input quantities, $g = \bar{x}$.

[1] Interestingly, although this relationship extends to the cost measure, so additive cost inefficiency is equal to one minus Farrell's cost efficiency, this relationship does extend to the allocative (in)efficiency component, see Aparicio et al. (2017). Hence, a decomposition of cost inefficiency based on the DDF does not generalize that of Farrell

Regardless the trade-off between alternative directional vectors, either different or equal to all observations, they all have in common their subjective nature. For this reason several authors have proposed alternative criteria to endogenize its value by considering specific goals. One option in the primal space, considering quantities only, is the minimization of the distance to the production frontier. This consists in the identification of the closest targets as surveyed by Aparicio & Pastor, 2014a, 2014b). Färe, Grosskopf and Whittaker (2013) also propose to endogenize the values of the directional vector by normalizing the sum of its element to one. Under this restriction, the technical inefficiency measure projecting the observation on the production frontier can be directly related to the slacks-based directional distance function introduced by Färe and Grosskopf (2010). Also, based on the information contained in the sample when identifying peers, several authors have proposed data driven orientations. A recent example is Daraio and Simar (2016), who propose a method that allows choosing context specific (or local) directions for firms, considering as benchmarks those facing similar characteristics. These conditions can be associated with the closeness of those benchmark peers to the input mix of the evaluated firm (again, $g = x$), but also to some other contextual conditions (factors); e.g., benchmarks facing the same non-discretionary inputs and outputs. A characteristic of all these approaches endogenizing the directional vector is that they are technological in nature and therefore unrelated to economic optima involving market prices. As shown below our approach departs from these models by considering as endogenous direction the projection of the observations on the optimal minimizing cost benchmark.

Yet, one additional drawback of the DDF, common to all approaches above, is that it does not satisfy the indication property—e.g., see Färe and Lovell (1978) and Russell and Schworm (2009). This implies that the technically efficient projection does not belong to the strongly efficient production possibility set, and additional individual reductions (slacks) in input quantities may still be feasible, beyond the radial projections. As noted by Fukuyama, Matousek and Tzeremes (2020) the existence of individual slacks in input quantities implies that, when decomposing cost efficiency, technical efficiency is overestimated and, correspondingly, allocative efficiency is underestimated.

The existence of these shortcomings prompts us to propose the endogenization of the directional vector in the dynamic cost model, so the firm is projected on an economically optimal benchmark on the frontier; in this case the optimal input bundle that minimizes dynamic production costs. The endogenous model that we introduce solves these problems, because: i) it ensures that the technical inefficiencies are comparable in monetary values; ii) the decomposition of cost inefficiency is clearly classified into technical or allocative for each observation, so they do not weight differently depending on the length and direction of the directional vector; iii) by searching for minimum cost peers, it prevents that non Pareto-Efficient firms belonging to the weakly efficiency frontier are identified as benchmark; and, therefore, iv) technical efficiency and allocative efficiency cannot be overestimated and underestimated, respectively. The reason is that, under the usual assumptions about the technology and the cost function, firms belonging to the weakly efficient technological frontier cannot define the cost minimizing hyperplanes characterizing the economic frontier because additional input reductions would be feasible.

Finally, in the new approach, by searching for the cost minimizing optima, an additional restriction imposed by the conventional model is overcome. That related to the forceful reduction of inputs, which is unwarranted from a managerial perspective. If inputs are to be reduced, as in the existing radial and DDF approaches, it may be still possible that the best available economic benchmark on the technological frontier, subject to the no-negativity constraint of the directional vector, does not minimize cost. But if, as in reality, inputs can be adjusted at will, so the firm is free to reduce but also increase them if necessary, then the new proposal shows that the cost minimizing benchmark can be reached directly without any intermediate projections because of existing constraints on input changes. As anticipated above, the freedom that managers have to adjust inputs results in a relevant result. Cost inefficiency is either technical or allocative. The reason is rather logical. If a technically inefficient firm laying inside the production possibility set can be directly projected to the cost minimizing benchmark, then all inefficiency is technical and there is no room for allocative inefficiencies. On the other hand, if a technically efficient firm laying on the frontier does not minimize cost, then all its cost inefficiency is allocative. Moreover, as indicated, the subjective—two-stage—decomposition of cost inefficiency into technical and allocative criteria is avoided.

The new method brings simplicity to the managerial decision making process, as it would not make sense that firms are prescribed a reduction in the amount of an input to overcome technical inefficiencies, and yet, in a subsequent stage, they are required increase it so as to demand the optimal amount that minimizes cost. For example, in the case of labor, furloughing employees in a first stage to reach the efficient frontier, and re-hire them in a later moment so as to minimize cost, rises extra costs related to legal and compensation expenses, as well as training activities. These unwarranted costs can be prevented by adopting the monotonic approach proposed here; i.e., setting a final target is dynamically more cost efficient than following the two-stage approach implicit in the conventional model. The monotonic approach makes sense from a managerial perspective when one realizes that firms tend to solve their technical and allocative inefficiencies simultaneously, and therefore do not follow the two-stage process, first technical, then allocative, as directed by the standard model. Indeed, prescribing these conflicting actions results in inconsistencies that raise transaction or adjustment costs, which brings us back and shows the connection to the dynamic cost inefficiency model of Silva et al. (2015).

The duality model by Silva et al. (2015) explicitly accounts for the adjustments costs associated with quasi-fixed factors and, in particular, the well-known case of capital stock in tangible assets. The model links optimal decisions related to the optimal flow of variable factors like investments, to the amount of quasi-fixed inputs, which are taken as constant in the short-run. However the model incurs in the same drawback mentioned above, since even if considering adjustment costs in the form of the investment input, reaching the cost minimizing benchmark may require increasing the investment flows in a first stage, and then disinvesting in a second stage so as to reach the optimal amount of capital stock. As before, these conflicting financial decisions are not warranted because of the extra cost that the second stage entails, or even its irreversibility in a 'putty-clay' context; i.e. the impossibility of disinvesting once the firm has committed contractually to a given amount, implying that the elasticity of substitution once the investment has materialized is zero in the short-run, see Baddeley (2003).

The purpose of this article is to enhance the dynamic cost inefficiency model of Silva et al. (2015) by endogenizing the directional function, thereby preventing non-monotonic and unrealistic managerial prescriptions on the intertemporal (dynamic) adjustment of inputs and investment, aiming at achieving the optimal long-run value of gross capital stock of the firm. Nevertheless, the model can be applied to any organizational situation where some inputs are quasi-fixed, and the optimal allocation of resources is based on an intertemporal optimization model that requires the change in some flow variables representing the change in the stock. We also aim at showing how the model can be empirically implemented through Data Envelopment Analysis (DEA) techniques and illustrate its potential to inform managerial decision making by using a real dataset of European firms belonging to the dietetic food industry.

The remainder of this article proceeds as follows: In Section 2 we present the Silva et al. (2015) dynamic cost inefficiency model, with its decomposition into technical inefficiency and allocative inefficiency. Next, in this section, inspired by Zofío, Pastor and Aparicio (2013), we show that it is possible to endogenize the directional vector to prevent unrealistic managerial advice on input changes and save on transaction and adjustment costs. We also show how to operationalize the new model using DEA methods. Section 3 illustrates the numerical differences and alternative managerial advice that emerges when using the two approaches. For this purpose the empirical application focuses on a panel dataset of European firms producing dietetic food. Recent statistics by Eurostat (2019a) show that this is a dynamic industry in the EU exhibiting double-digit growths rates in the last decade. In this context of general expansion, we find indeed that rather than generally reducing inputs and increasing investment so as to reach the production frontier, as the standard model assumes, for the majority of firms inputs should be actually increased, so as to minimize production costs. Section 4 concludes.

2. The dynamic cost inefficiency model: Exogenous and endogenous approaches

2.1. Decomposing dynamic cost inefficiency: the conventional approach with an exogenous orientation

The dynamic cost inefficiency model characterizes the production technology through the input correspondence: $V(y(t)|K(t)) = \{x(t),\ I(t) : (x(t),\ I(t))$ can produce $y(t)$ given $K(t)\}$. It is assumed that at time t, there are $j = 1,...,J$ firms producing a range of M outputs, $y \in \mathbb{R}^M_{++}$, using N variable inputs, $x \in \mathbb{R}^N_{++}$, F investments, $I \in \mathbb{R}^F_{++}$, as well as quasi-fixed factors, $K \in \mathbb{R}^F_{++}$. It also assumed that the N and F prices corresponding to the variable and quasi-fixed factors are observed. These prices are denoted by $w \in \mathbb{R}^N_{++}$ and $c \in \mathbb{R}^F_{++}$, respectively. Following Silva and Stefanou (2003), Silva et al. (2015) and Kapelko, 2017, at any base period $t \in [0, +\infty)$, the firm is assumed to minimize the discounted flow of costs over time subject to an adjustment cost technology characterized by constant returns to scale. Expressing this in terms of the current value, and drooping the subscript t to avoid notational clutter, yields the Hamilton-Jacobi-Bellman equation:

$$rW(y, k, w, c) = \min_{x,I} \left[w'x + c'K + W'_K(I - \delta K) \right] \\ s.t. \\ \vec{D}(y, K, x, I; g_x, g_I) \geq 0 \tag{1}$$

where $W(\bullet)$ represents the discounted flow of costs in all future time periods. $W_K = W_K(y, K, w, c)$ is the vector of shadow values of quasi-fixed factors. The discount rate is $r > 0$ and δ is a diagonal $F \times F$ matrix of depreciation rates, $\delta_f > 0$, $f = 1,\ldots, F$. For simplicity we assume that firms have the same discount rate and depreciation matrix. Finally, $\vec{D}(y, K, x, I; g_x, g_I)$ is the dynamic input directional distance function associated with the dynamic cost inefficiency model and is defined as follows:

$$\vec{D}(y, K, x, I; g_x, g_I) = \max_{\beta} \{\beta : (x - \beta g_x, I + \beta g_I) \in V(y|K)\}. \tag{2}$$

This function measures the distance of firm (x, I) to the frontier in the direction defined by the directional vector $g = (-g_x, g_I)$. In the conventional approach it is assumed that $g = (g_x, g_I) \in \mathbb{R}^N_+ \times \mathbb{R}^F_+ \backslash \{0_{N+F}\}$; i.e., when reaching the production frontier, it is assumed that inputs are reduced—hence the preceding negative sign above, while investments are increased. Silva et al. (2015) and subsequent authors choose the observed quantities of input and investment, $g=(g_x,g_I)=(x,I)$ as directional vector, resulting in the proportional directional distance function. In the endogenous directional vector approach that we propose below in Section 2.3 we relax these assumptions so inputs and investment can be freely adjusted to reach the cost minimizing benchmark. This results in an optimal direction that we will represent by $g^*=(g_x^*,g_I^*)$, where the superscript '*' denotes that the endogenous direction measures cost inefficiency against the economic benchmark.

Given the production possibility set, the minimum cost presented in the objective function of (1) can be calculated resorting to DEA techniques by solving the following model:

$$
\begin{aligned}
rW(y,k,w,c) = \ & \min_{x,I,\gamma}\left[w'x+c'K+W'_K(I-\delta K)\right]\sqrt{a^2+b^2}\\
& s.t.\\
& \sum_{j=1}^{J}\gamma_j y_{jm}\geq y_m,\ \ m=1,..,M,\\
& x_n\geq\sum_{j=1}^{J}\gamma_j x_{jn},\ \ n=1,..,N,\\
& \sum_{j=1}^{J}\gamma_j(I_{jf}-\delta K_{jf})\geq I_f,\ \ f=1,..,F,\\
& \gamma_j\geq 0,\ \ \ j=1,..,J,
\end{aligned}
\tag{3}
$$

where γ is the $(J\times 1)$ intensity vector. Following the standard approach in the literature on cost efficiency decomposition, the technology is characterized by constant returns to scale; see Färe et al. (1985:75) for the traditional model and Silva et al. (2015) for the dynamic cost inefficiency model. Nevertheless, variable returns to scale could be considered by adding the constraint $\sum_{j=1}^{J}\gamma_j=1$.

Based on the representation property of the directional distance function, implying that: $\vec{D}(y,K,x,I;g_x,g_I)\geq 0 \Leftrightarrow (x,\ I)\in V(y\mid K)$, Silva et al. (2015) prove through duality theory the following Mahler inequality:

$$
\begin{aligned}
& CI(y,K,x,I;g_x,g_I;w,W(\cdot))\\
& \quad=\frac{w'x+c'K+W_K(\cdot)'(I-\delta K)-rW(y,K,w,c)}{w'g_x-W_K(\cdot)'g_I}\\
& \quad\geq\vec{D}(y,K,x,I;g_x,g_I),
\end{aligned}
\tag{4}
$$

where the left-hand side represents (overall) dynamic cost inefficiency as the difference between observed cost $w'x+c'K+W_K(\cdot)'(I-\delta K)$ and minimum cost $rW(y,K,w,c)$, normalized by the constraint $w'g_x-W_K(\cdot)'g_I$. This normalization ensures that economic inefficiency is units independent as initially suggested by Nerlove (1965). Calculating the dynamic cost inefficiency $CI(y,K,x,I;g_x,g_I;w,W(\cdot))$ is straightforward once minimum cost is known by solving Eq. (3), and dividing by the normalizing constraint.

The dynamic directional distance function $\vec{D}(y,K,x,I;g_x,g_I)$ can be regarded as a measure of dynamic technical inefficiency, i.e., $\vec{D}(y,K,x,I;g_x,g_I)=TI$, whose empirical value we calculate by resorting to the same DEA approximation of the production technology used to determine minimum cost in Eq. (3). For firm j_o, the DEA model measuring dynamic technical inefficiency (TI) is the solution of the following linear program:

$$
\begin{aligned}
\vec{D}\left(y_{j_o},K_{j_o},x_{j_o},I_{j_o};g_x,g_I\right) = \ & \max_{\beta,\gamma}\beta\\
& s.t.\\
& \sum_{j=1}^{J}\gamma_j y_{jm}\geq y_{j_om},\ m=1,..,M,\\
& x_{j_on}-\beta g_x\geq\sum_{j=1}^{J}\gamma_j x_{jn},\ n=1,..,N,\\
& \sum_{j=1}^{J}\gamma_j(I_{jf}-\delta K_{jf})\geq I_{j_of}+\beta g_I-\delta K_{j_of},\ f=1,..,F\\
& \gamma_j\geq 0,\ \ \ j=1,..,J
\end{aligned}
\tag{5}
$$

As anticipated, in empirical applications of the dynamic cost inefficiency model, it is usual to set the directional vector to be equal to the observed amounts of inputs and investments (or as a fraction of the capital stock): $g=(g_x,g_I)=(x,I)$.This eases the interpretation of the inefficiency score β as the percentage reduction in the amount of inputs and the percentage expansion of investments needed to reach the frontier.

Afterwards, once program (5) is solved, and based on the Mahler inequality (4), any difference between $CI(y,K,x,I;g_x,g_I;w,W(\cdot))$ and $\vec{D}(y,K,x,I;g_x,g_I)$ can be attributed to allocative inefficiency. Hence, it is possible to decompose dynamic overall cost inefficiency into the contributions of dynamic technical inefficiency (TI), and a residual term defined as dynamic allocative inefficiency (AI). This requires rendering expression (4) an equality:

$$
\begin{aligned}
& CI(y,K,x,I;g_x,g_I;w,W(\cdot))=\vec{D}(y,K,x,I;g_x,g_I)\\
& \quad +AI(y,K,x,I;g_x,g_I;w,W(\cdot))=TI+AI,
\end{aligned}
\tag{6}
$$

and therefore,

$$
\begin{aligned}
AI = & \ \frac{w'x+c'K+W_K(\cdot)'(I-\delta K)-rW(y,K,w,c)}{w'g_x-W_K(\cdot)'g_I}\\
& -\vec{D}(y,K,x,I;g_x,g_I).
\end{aligned}
\tag{7}
$$

2.2. A monetary valued directional distance function

Chambers et al. (1998) did not specify a particular orientation at the time of introducing the decomposition of economic efficiency into the technical component represented by the directional distance function and allocative inefficiency. However, regardless a particular direction $g=(g_x,g_I)$, it can be trivially seen from the denominator in (4) that normalizing its value (length) so the following equality is verified: $w'g_x-W_K(\cdot)'g_I=1$, results in a valuation of cost inefficiency and its components in monetary terms; e.g., dollars. This makes the interpretation of $\vec{D}(y,K,x,I;g_x,g_I)$ straightforward as the cost excess in which the firm incurs due to dynamic technical inefficiency. To prove this assertion we first denote by $g^T=(g_x^T,g_I^T)\in\mathbb{R}_{++}^N\times\mathbb{R}_{++}^F$ any directional vector that projects the firm under evaluation to the frontier point (x^T,I^T), while satisfying $w'g_x^T-W_K(\cdot)'g_x^T=1$. In the dynamic cost inefficiency model, (x^T,I^T) represents technological targets on the production frontier (hence the superscript T). Consequently, given (x^T,I^T), the directional vector can be rewritten as $g^T=(g_x^T,g_I^T)=\varsigma(x-x^T,I^T-I)$, where $\varsigma>0$ is a scalar. From this expression, and recalling the input and investment prices, we note that $\varsigma=[(w'x+W_K(\cdot)'I)-(w'x^T+W_K(\cdot)'I^T)]^{-1}$ since $\varsigma w'(x-x^T)+\varsigma W_K(\cdot)'(I-I^T)=\varsigma[w'(x-x^T)+W_K(\cdot)'(I-I^T)]=1$. Therefore, ς corresponds to the inverse of the cost difference between the observed firm (x,I) and its projection (x^T,I^T) on the frontier. Relevant for our goal is that ς is related to the length of the directional distance function, and therefore the technical inefficiency it represents. Let us define the dynamic input directional distance function subject to the unit valued normalizing constraint as

$$
\begin{aligned}
& \vec{D}^T\left(y,K,x,I;g_x^T,g_I^T\right)\\
& =\max_{\beta}\left\{\beta:\left(x-\beta g_x^T,I+\beta g_I^T\right)\in V(y|K)\middle|w'g_x^T-W_K(\cdot)'g_x^T=1\right\},
\end{aligned}
\tag{8}
$$

Then we can establish the following result:

Proposition 1. *Let* $(w, W_K(\cdot))$ *be the vector of prices and let* $g^T = (g_x^T, g_I^T) \in \mathbb{R}_{++}^N \times \mathbb{R}_{++}^F$ *be a vector such that* $w'g_x - W_K(\cdot)'g_I = 1$. *Let* $(x, I) \in V(y|K)$, *then* $\vec{D}^T(y, K, x, I\,; g_x^T, g_I^T) = 1/\varsigma$, *where* $\varsigma = [(w'x + W_K(\cdot)'I) - (w'x^T + W_K(\cdot)'I^T)]^{-1}$.

Proof. For any input-investment vector $(x, I) \in V(y|K)$, any projected vector in the direction $g^T = (g_x^T, g_I^T) \in \mathbb{R}_{++}^N \times \mathbb{R}_{++}^F$ is feasible; i.e., $(x^T, I^T) = (x - \vec{D}^T(y, K, x, I\,; g_x^T, g_I^T)g_x^T, I + \vec{D}^T(y, K, x, I\,; g_x^T, g_I^T)g_I^T) \in V(y|K)$. Recalling the optimal solution to program (5), i.e., β^*, we see that $(w'x^T + W_K(\cdot)'I^T)$ is equal to $w'x + W_K(\cdot)'I + \beta^*(-w'g_x^T + W_K(\cdot)'g_I^T)$. Now, substituting $g^T = (g_x^T, g_I^T) = \varsigma(x - x^T, I^T - I)$, we have that $(w'x^T + W_K(\cdot)'I^T) = w'x + W_K(\cdot)'I + \beta^*\varsigma(-w'(x - x^T) + W_K(\cdot)'(I^T - I))$. Finally, rearranging terms we obtain that $((w'x^T + W_K(\cdot)'I^T) - (w'x + W_K(\cdot)'I))\ /\ \varsigma = \beta^*(-w'(x - x^T) + W_K(\cdot)'(I^T - I))$. And since $(w'x^T + W_K(\cdot)'I^T) - (w'x + W_K(\cdot)'I) = -w'(x - x^T) + W_K(\cdot)'(I^T - I)$, then $1/\varsigma = \vec{D}^T(y, K, x, I\,; g_x^T, g_I^T)$.■

This result shows that the normalized distance function $\vec{D}^T(y, K, x, I\,; g_x^T, g_I^T) = 1/\varsigma = (w'x + W_K(\cdot)'I) - (w'x^T + W_K(\cdot)'I^T)$ is a natural measure of dynamic technical inefficiency in monetary values. Then, relying on the definition of the directional distance function (8), we can decompose dynamic cost inefficiency in the same vein as (6); i.e.,

$$
\begin{aligned}
CI\left(y, K, x, I; g_x^T, g_x^T; w, W(\cdot)\right) &= \vec{D}^T\left(y, K, x, I; g_x^T, g_x^T\right) \\
&\quad + AI^T\left(y, K, x, I; g_x^T, g_x^T; w, W(\cdot)\right) \\
&= TI^T + AI^T. \qquad (9)
\end{aligned}
$$

And, consequently,

$$
\begin{aligned}
AI^T = {} & \frac{w'x + c'K + W_K(\cdot)'(I - \delta K) - rW(y, K, w, c)}{w'g_x^T - W_K(\cdot)'g_I^T} \\
& -\vec{D}\left(y, K, x, I; g_x^T, g_I^T\right). \qquad (10)
\end{aligned}
$$

But, contrary to (6) and (7), $CI^T(\cdot)$, $TI^T(\cdot)$ and $AI^T(\cdot)$ above are actually measured in monetary units.

From an empirical perspective, calculating the dynamic directional distance function $\vec{D}^T(y, K, x, I\,; g_x^T, g_I^T)$ for firm j_o implies adding the constraint $w'g_x^T - W_K(\cdot)'g_I^T = 1$ when solving linear program (5). Specifically:

$$
\begin{aligned}
&\vec{D}^T\left(y_{j_o}, K_{j_o}, x_{j_o}, I_{j_o}; g_x^T, g_I^T\right) = \max_{\beta,\gamma} \beta \\
&\quad s.t. \\
&\quad \sum_{j=1}^{J} \gamma_j y_j \geq y_{j_o m},\ \ m = 1, .., M \\
&\quad x_{j_o n} - \beta g_x^T \geq \sum_{j=1}^{J} \gamma_j x_{jn}, n = 1, .., N, \\
&\quad \sum_{j=1}^{J} \gamma_j (I_{jf} - \delta K_{jf}) \geq I_{j_o f} + \beta g_I^T - \delta K_{jf}, f = 1, .., F, \\
&\quad w'g_x^T - W_K(\cdot)'g_I^T = 1, \\
&\quad \gamma_j \geq 0, \quad j = 1, .., J \qquad (11)
\end{aligned}
$$

Since prices are given when solving (11), this implies that once an exogenously directional vector is chosen: $g = (g_x, g_I)$, it must be rescaled in advance by the researcher to meet the unit value restriction on the normalizing constraint $g^T = (g_x^T, g_I^T)$. In the empirical application we show one of the many possibilities to implement this approach based on the conventional model that relies on the exogenous orientation.

Fig. 1 illustrates cost inefficiency measurement in the context of the dynamic cost inefficiency model resorting to the dynamic input directional distance functions (2) and (8). Given the vector of prices $(w, W_K(\cdot))$, firm A minimizes cost, whose associated optimal isocost is $w'x^* + W_K(\cdot)'I^*$. Firms B and C are both cost and technically inefficient, while the remaining firm D is technically efficient. The particularity for all these firms is that while their dynamic technical inefficiencies differ, as shown by the values of their respective directional distance functions, they incur in the same production cost (belonging to the same isocost line), and therefore have the same dynamic cost inefficiency, defined as $CI(x, I; g_x, g_I; w, W(\cdot)) = (w'x_j + W_K(\cdot)'I_j) - (w'x_\mathrm{A}^* + W_K(\cdot)'I_\mathrm{A}^*)$, $j=$B, C, and D. We explore first the case of technically inefficient firms to show the relevance of Proposition 1. Taking firm C as reference, and choosing as direction that routinely adopted in empirical studies and corresponding to the observed input and investment amounts $g = (g_x, g_I) = (x_\mathrm{C}, I_\mathrm{C})$, but whose values are rescaled to satisfy the normalizing constraining $w'x_\mathrm{C} - W_K(\cdot)'I_\mathrm{C} = 1$, result in the directional vector $g^T = (g_x^T, g_I^T)$. Relying on this direction, we identify $(x_\mathrm{C}^T, I_\mathrm{C}^T)$ as the projection of firm C on the production frontier, and the value of the corresponding input distance function $\vec{D}^T(y_\mathrm{C}, K_\mathrm{C}, x_\mathrm{C}, I_\mathrm{C}; g_x^T, g_I^T)$ is equal to the cost excess in which firm C incurs as a result of technical inefficiencies: $\vec{D}^T(y_\mathrm{C}, K_\mathrm{C}, x_\mathrm{C}, I_\mathrm{C}; g_x^T, g_I^T) = TI_\mathrm{C}^T = (w'x_\mathrm{C} - W_K(\cdot)'I_\mathrm{C}) - (w'x_\mathrm{C}^T - W_K(\cdot)'I_\mathrm{C}^T)$. Subsequently, since dynamic cost inefficiency is equal to $CI_C^T = (w'x_\mathrm{C} + W_K(\cdot)'I_\mathrm{C}) - (w'x_\mathrm{A}^* + W_K(\cdot)'I_\mathrm{A}^*)$, we can decompose it into TI_C^T and the residual allocative inefficiency AI_C^T given in expression (10), which in this case corresponds to $AI_\mathrm{C}^T = (w'x_\mathrm{C}^T + W_K(\cdot)'I_\mathrm{C}^T) - (w'x_\mathrm{A}^* + W_K(\cdot)'I_\mathrm{A}^*) = CI_C^T - \vec{D}^T(y_\mathrm{C}, K_\mathrm{C}, x_\mathrm{C}, I_\mathrm{C}; g_x^T, g_I^T)$. Consequently, for firm C, dynamic cost inefficiency is due to technical and allocative reasons: $CI_\mathrm{C}^T = TI_\mathrm{C}^T + AI_\mathrm{C}^T$, with $TI_\mathrm{C}^T > 0$ and $AI_\mathrm{C}^T > 0$. Firm C illustrates the shortcoming of the conventional non-monotonic, two-stage approach that motivates this study. It can be seen that to reach the production frontier and eliminate dynamic technical inefficiencies in the usual (rescaled) direction $g^T = (g_x^T, g_I^T)$, firm C should reduce inputs and increase investment simultaneously. However, it can be seen that to match the optimal demand amounts given by the benchmark firm A, firm C would have to *reduce* the level of investment, which implies that committing to investment increases, so as to reach the reference benchmark $(x_\mathrm{C}^T, I_\mathrm{C}^T)$, results in exceeding the optimal investment levels, and leads to conflicting investment strategies. Focusing now on firms B and D, Fig. 1 shows that while the former is technically inefficient, $\vec{D}^T(y_\mathrm{B}, K_\mathrm{B}, x_\mathrm{B}, I_\mathrm{B}; g_x^T, g_I^T) > 0$, it is allocatively efficient, $AI_\mathrm{B}^T = 0$. This result holds because the (rescaled) direction set by its observed amount of inputs and investment, precisely projects it to the cost minimizing firm; i.e., $(x_\mathrm{B}^T, I_\mathrm{B}^T) = (x_\mathrm{A}^*, I_\mathrm{A}^*)$. Hence $CI_\mathrm{B}^T = TI_\mathrm{B}^T > 0$. For firm D, the opposite situation is verified. It is technically efficient $\vec{D}^T(y_\mathrm{D}, K_\mathrm{D}, x_\mathrm{D}, I_\mathrm{D}; g_x^T, g_I^T) = 0$, and therefore all dynamic cost inefficiency is allocative: $CI_\mathrm{D}^T = AI_\mathrm{D}^T > 0$.

2.3. Optimal resource allocation and endogenous orientation

We are now able to introduce endogenous directions. Assuming that the managers' final goal is to minimize production costs accounting for capital adjustment costs, it seems sensible to choose a direction that projects the firm to that *locus*. From a modeling perspective, this requires a flexible approach that endogenizes the directional vector, thereby removing the existing constraints on inputs and investments of the conventional model. In particular, the possibility of increasing the amount of inputs to be employed if necessary or, as shown above, reducing the investment flows of the firm. Following Zofío et al. (2013), in this section we show that by endogenizing the choice of direction we can define a measure of dynamic cost inefficiency that gets rid of the allocative residual as-

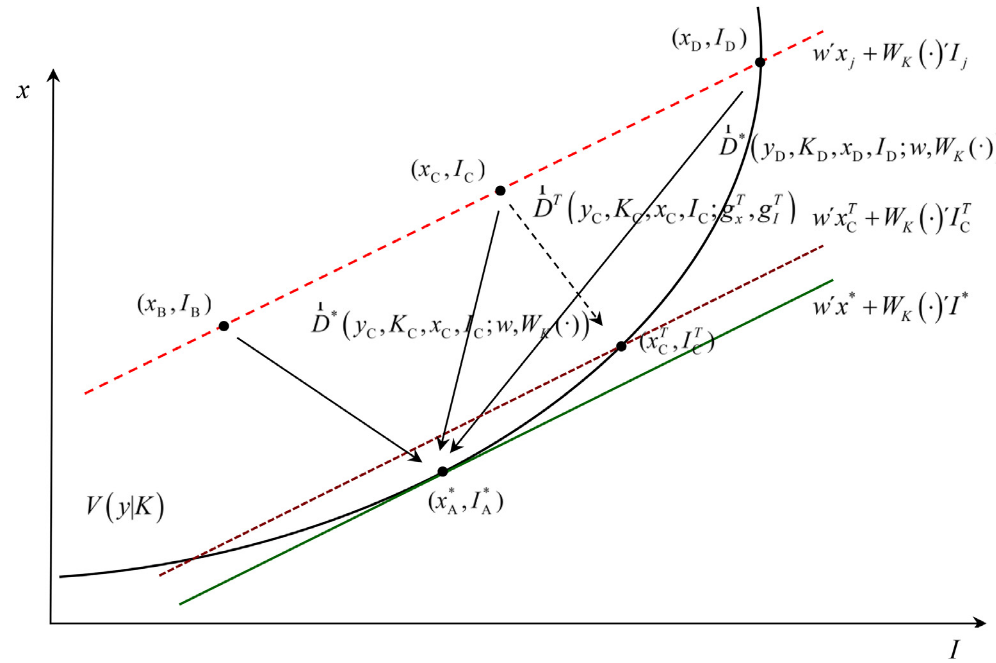

Fig. 1. Cost inefficiency measurement in the dynamic cost inefficiency model.

sociated to the Mahler's inequality in (6) and (10), and that along with the value of the standard distance function (2)—or its monetary valued counterpart (8), allows to determine whether cost inefficiency is due to technical or allocative reasons. The new framework prevents the *ad-hoc* decomposition of dynamic cost inefficiency in these two components because of the subjective choice of directional vector.

To this end we set the value of the directional vector free: $g^* = (g_x^*, g_I^*) \in \mathbb{R}^N \times \mathbb{R}^F \backslash \{0_{N+F}\}$, where the superscript '*' denotes that the endogenous direction will now measure cost inefficiency against the optimal cost minimizing benchmark (x^*, I^*), while satisfying $w'g_x^* - W_K(\cdot)'g_I^* = 1$. For market prices $(w, W_K(\cdot))$ we identify (x^*, I^*) as $(x^*, I^*) = \underset{(x,I),\in V(y|K)}{\arg\min} \; (w'x + W_K(\cdot)'I)$ and define the directional vector we are searching for as:

$$g^* = (g_x^*, g_I^*) = \tau(x - x^*, I^* - I), \tag{12}$$

where the scalar τ corresponds to the following expression:

$$\tau = \left[w'x + c'K + W_K(\cdot)'(I - \delta K) - rW(y, K, w, c)\right]^{-1}, \tag{13}$$

Thanks to this expression, (g_x^*, g_I^*) satisfies $w'g_x^* - W_K(\cdot)'g_I^* = 1$. Therefore, $(g_x^*, g_I^*) = [w'x + c'K + W_K(\cdot)'(I - \delta K) - rW(y, K, w, c)]^{-1}(x - x^*, I^* - I)$.

We can now define the dynamic directional input cost inefficiency measure in the dynamic cost inefficiency model for any observation $(x, I) \in V(y|K)$ by way of (12), and determined from $(w, W_K(\cdot))$, $V(y|K)$ and (x, I). For this purpose, we assume that the minimum cost is not achieved at (x, I), and therefore $w'x + c'K + W_K(\cdot)'(I - \delta K) \geq rW(y, K, w, c)$. Then, given (g_x^*, g_I^*) as in (12), the directional input cost inefficiency measure $\vec{D}(y, K, x, I; g_x^*, g_I^*)$ is defined as

$$\begin{aligned}
&\vec{D}^*(y, K, x, I; w, W_K(\cdot)) := \vec{D}^*(y, K, x, I; g_x^*, g_I^*) \\
&= \max_{\beta} \left\{\beta : (x - \beta g_x^*, I + \beta g_I^*) \in V(y|K) \;\middle|\; w'g_x^* - W_K(\cdot)'g_x^* = 1\right\},
\end{aligned} \tag{14}$$

which corresponds to the definition of the dynamic cost directional distance function (8) with a relevant qualification. In (14) the directional vector (g_x^*, g_I^*) may present negative elements; i.e., it is possible to increase inputs and decrease investment when reaching the cost minimizing benchmark. Now, mirroring Proposition 1, $\vec{D}^*(y, K, x, I; w, W_K(\cdot)) = 1/\tau$ with τ defined as in (13).

Lemma 1. *Let $(w, W_K(\cdot))$ be the vector of market prices. Let $(x, I) \in V(y|K)$ such that $w'x + c'K + W_K(\cdot)'(I - \delta K) > rW(y, K, w, c)$. Then $\vec{D}^*(y, K, x, I; w, W_K(\cdot)) = 1/\tau$ as in (13).*

As with the conventional and monetary valued dynamic directional distance functions, (4) and (11), $\vec{D}^*(y, K, x, I; w, W_K(\cdot))$ can also be derived from the cost function resorting to duality, even if some of its elements are negative.

Proposition 2. . *Let $(w, W_K(\cdot))$ be the vector of market prices. Let $(x, I) \in V(y|K)$ such that $w'x + c'K + W_K(\cdot)'(I - \delta K) > rW(y, K, w, c)$. Then $\vec{D}^*(y, K, x, I; w, W_K(\cdot)) = \min_{w, W_K(\cdot)} \{w'x + c'K + W_K(\cdot)'(I - \delta K) - rW(y, K, w, c) : w'g_x^* - W_K(\cdot)'g_I^* = 1\}$.*

Proof. Given the vector of market prices $(w, W_K(\cdot))$, (g_x^*, g_I^*) defined as in (12) and ensuring that $w'g_x - W_K(\cdot)'g_I = 1$, the value of the objective function at this feasible solution $(w, W_K(\cdot))$ is $(w'x + c'K + W_K(\cdot)'(I - \delta K)) - rW(y, K, w, c)$. Hence, we have that $\min_{w, W_K(\cdot)} \{w'x + c'K + W_K(\cdot)'(I - \delta K) \geq rW(y, K, w, c) : w'g_x^* - W_K(\cdot)'g_I^* = 1\} \leq w'x + c'K + W_K(\cdot)'(I - \delta K) - rW(y, K, w, c) = \vec{D}^*(y, K, x, I; w, W_K(\cdot))$, where the last equality holds thanks to Lemma 1. To prove the reverse equality, we note that $(x - \vec{D}^*(y, K, x, I; g_x^*, g_I^*)g_x^*, I + \vec{D}^*(y, K, x, I; g_x^*, g_I^*)g_I^*) \in V(y|K)$. Then by the definition of the dynamic cost function, we have that for all $(w, W_K(\cdot))$ such that $w'g_x^* - W_K(\cdot)'g_I^* = 1$, $w'(x - \vec{D}^*(y, K, x, I; w, W_K(\cdot)))g_x^* + c'K + W_K(\cdot)'((I + \vec{D}^*(y, K, x, I; w, W_K(\cdot))g_I^*) - \delta K) = w'x + c'K + W_K(\cdot)'(I - \delta K) + \vec{D}^*(y, K, x, I; w, W_K(\cdot)) \; (w'g_x^* - W_K(\cdot)'g_I^*) = w'x + c'K + W_K(\cdot)'(I - \delta K) + \vec{D}^*(y, K, x, I; w, W_K(\cdot)) \geq rW(y, K, w, c)$. Rearranging terms, we obtain that $\vec{D}^*(y, K, x, I; w, W_K(\cdot)) \leq (w'x + c'K + W_K(\cdot)'(I - \delta K)) - rW(y, K, w, c)$. And by the definition of minimum, we finally have that $\vec{D}^*(y, K, x, I; w, W_K(\cdot)) \leq \min_{w, W_K(\cdot)} \{w'x + c'K + W_K(\cdot)'(I - \delta K) - rW(y, K, w, c) : w'g_x - W_K(\cdot)'g_I = 1\}$. ■

Also, using Proposition 2, and since $\frac{w}{w'g_x^* - W_K(\cdot)'g_I^*}\prime g_x^* - \frac{W_K(\cdot)}{w'g_x^* - W_K(\cdot)'g_I^*}\prime g_I^* = 1$, we obtain the following Mahler inequality counterpart to (4):

$$\begin{aligned} CI(y, K, x, I; g_x^*, g_I^*; w, W(\cdot)) &= \frac{w'x + c'K + W_K(\cdot)'(I - \delta K) - rW(y, K, w, c)}{w'g_x^* - W_K(\cdot)'g_I^*} \\ &\geq \vec{D}^*(y, K, x, I; w, W_K(\cdot)). \end{aligned} \tag{15}$$

To finish our exposition, we need to show that the equality always holds, implying that $\vec{D}^*(y, K, x, I; w, W_K(\cdot))$ can be interpreted as a measure of *overall dynamic cost inefficiency* instead of the money valued dynamic technical inefficiency given by its counterpart $\vec{D}^T(y, K, x, I; g_x^T, g_I^T)$. Here, from Lemma 1 we have that $\vec{D}^*(y, K, x, I; w, W_K(\cdot)) = (w'x + c'K + W_K(\cdot)'(I - \delta K)) - rW(y, K, w, c)$. Additionally, (g_x^*, g_I^*) must satisfy $w'g_x^* - W_K(\cdot)'g_I^* = 1$. Therefore, trivially, the equality holds in (15), implying that $\vec{D}^*(y, K, x, I; w, W_K(\cdot)) = CI(y, K, x, I; g_x^*, g_I^*; w, W(\cdot))$. Petersen (2018: 1074) provides a geometric interpretation of Proposition 2 by showing that the approach introduced by Zofío et al. (2013), endogenizing the directional vector (g_x^*, g_I^*) and including the normalizing constraint $w'g_x^* - W_K(\cdot)'g_I^* = 1$, is equivalent to the requirement that the scalar projection of (g_x^*, g_I^*) onto $(w, W_K(\cdot))$ with $\|(w, W_K(\cdot))\|_2 = 1$ must equal 1. This result holds since $(w, W_K(\cdot))\prime(g_x^*, g_I^*) = \|(w, W_K(\cdot))\|_2 \|(g_x^*, g_I^*)\|_2 \cos\varphi$ by the definition of the cosine, where φ is the angle between $(w, W_K(\cdot))$ and (g_x^*, g_I^*). Therefore, the endogenous dynamic distance function $\vec{D}^*(y, K, x, I; w, W_K(\cdot))$ can be interpreted as the Euclidean distance between (x, I) and the supporting hyperplane characterized by the cost minimizing firm, given market prices.

Resorting to definitions (8) and (14) we can determine whether dynamic cost inefficiency is either technical or allocative. To achieve this categorization we note first that $\vec{D}^*(y, K, x, I; w, W_K(\cdot)) \geq \vec{D}^T(y, K, x, I; g_x^T, g_I^T)$ and, consequently, we have that $AI^* = \vec{D}^*(y, K, x, I; w, W_K(\cdot)) - \vec{D}^T(y, K, x, I; g_x^T, g_I^T)$. Second, if $\vec{D}^T(y, K, x, I; g_x^T, g_I^T) = 0$ the firm is technically efficient, lying on the production frontier of the input set, while if $\vec{D}^T(y, K, x, I; g_x^T, g_I^T) > 0$, the firm is technically inefficient; i.e. an interior point. Consequently, if $\vec{D}^T(y, K, x, I; g_x^T, g_I^T) = 0$, all dynamic cost inefficiency is allocative, and equal to $\vec{D}^*(y, K, x, I; w, W_K(\cdot))$ in monetary terms. But in the latter case $\vec{D}^T(y, K, x, I; g_x^T, g_I^T) > 0$, we have shown that the directional cost inefficiency measure projects the firm under evaluation to the cost minimizing benchmark, where it is allocatively efficient: $AI^* = 0$. Consequently, all cost inefficiency is technical and equal to $\vec{D}^*(y, K, x, I; w, W_K(\cdot))$, again in monetary terms. Finally, it follows that if $\vec{D}^*(y, K, x, I; w, W_K(\cdot)) = 0$, the firm is cost efficient by minimizing production costs. We conclude that the assumption of a consistent dynamic behavior on the part of managers, aiming at monotonic adjustments of inputs and investment to prevent conflicting strategies that entail additional transaction costs, directly results in the categorization of dynamic cost inefficiency as either technical or allocative. We can summarize these results as follows:

$$\begin{aligned} CI(y, K, x, I; w, W(\cdot)) &= \vec{D}^*(y, K, x, I; w, W_K(\cdot)) \\ &= TI^* \ iff \ \vec{D}^T\left(y, K, x, I; g_x^T, g_I^T\right) > 0, \end{aligned} \tag{16}$$

and

$$\begin{aligned} CI(y, K, x, I; w, W(\cdot)) &= \vec{D}^*(y, K, x, I; w, W_K(\cdot)) \\ &= AI^* \ iff \ \vec{D}^T\left(y, K, x, I; g_x^T, g_I^T\right) = 0. \end{aligned} \tag{17}$$

Resorting to DEA methods, it is possible to calculate the dynamic directional cost inefficiency measure solving the following program:

$$\vec{D}^*\left(y_{j_o}, K_{j_o}, x_{j_o}, I_{j_o}; g_x^*, g_I^*\right) = \max_{\beta, \gamma, g_x^*, g_I^*} \beta \tag{18}$$

s.t.

$$\sum_{j=1}^{J} \gamma_j y_{jm} \geq y_{j_o}, \ m = 1, .., M, \tag{18a}$$

$$x_{j_o n} - \beta g_n^* \geq \sum_{j=1}^{J} \gamma_j x_{jn}, \quad n = 1, .., N, \tag{18b}$$

$$\sum_{j=1}^{J} \gamma_j (I_{jf} - \delta K_{jf}) \geq I_{j_o f} + \beta g_f^* - \delta K_{j_o f}, \ f = 1, .., F, \tag{18c}$$

$$w'g_x^* - W_K(\cdot)'g_I^* = 1, \tag{18d}$$

$$\gamma_j \geq 0, \quad j = 1, .., J. \tag{18e}$$

Although this program is nonlinear, it can be linearized by replacing the constraints (18b) and (18c) by writing: $\sigma_{xn} = \beta g_n^*$ and $\sigma_{If} = \beta g_f^*$. In addition, the constraint $w'g_x^* - W_K(\cdot)'g_I^* = 1$ is rewritten as: $w'\sigma_x - W'\sigma_I = \beta$. Program (18) differs from (11) in that the directional vector is not preassigned but endogenous, and therefore (18) searches for the direction (g_x^*, g_I^*) that projects the firm under evaluation to the cost minimizing benchmark (x^*, I^*). From a managerial perspective, rendering the directional vector endogenous allows a direct evaluation of firm's performance in terms of technical or allocative inefficiency, and, more importantly, prescribes input and output adjustments that avoid conflicting non-monotonic changes. Hence, additional transaction and adjustment costs resulting from a subjective decomposition of cost inefficiency are bypassed. This is critical for investment decisions since it is not unusual that the dynamic adjustments of capital stocks actually require *reducing* investment levels, while the conventional model of Silva et al. (2015) forces an increase in its magnitude. This result, which is observed in our empirical application, is illustrated in Fig. 1 by way of firm C, (x_C, I_C). As already discussed, choosing the exogenous vector (g_x^T, g_I^T) projects it on a first stage to its benchmark on the frontier represented by (x_C^T, I_C^T); i.e. $TI^T = \vec{D}^T(y_C, K_C, x_C, I_C; g_x^T, g_I^T)$. But it turns out that to minimize cost, the actual investment effort in new capital should be lower than *both* the observed and the projected ones, I_C and I_C^T, thereby matching the amount of the cost minimizing firm (x_A^*, I_A^*). Moreover, choosing the exogenous vector (g_x^T, g_I^T), results in unwarranted dynamic allocative inefficiencies (i.e., signaling the wrong bundle of input demands), $AI_D^T = CI(y_D, K_D, x_D, I_D; w, W(\cdot)) - \vec{D}^T(y_C, K_C, x_C, I_C; g_x^T, g_I^T) > 0$, because (x_C^T, I_C^T) represents an unnecessary intermediate step, whose interpretation in terms of dynamic technical inefficiency (i.e., wrong engineering practices associated to investment levels lower than those that would be technically optimal) cannot be justified, unless there are convincing reasons to choose (g_x^T, g_I^T), thereby forcing investment increases. Consequently, by solving the endogenous model, $\vec{D}^*(y_C, K_C, x_C, I_C; g_x^T, g_I^T) = TI_D^*$, we rightly learn that all dynamic cost inefficiency is due to technical reasons. While firm C illustrates the arbitrary decomposition of cost inefficiency into technical an allocative components when using the conventional model, firm D illustrates the case of a technically efficient firm, with an exogenous directional distance function $TI_D^T = \vec{D}^T(y_D, K_D, x_D, I_D; g_x^T, g_I^T) = 0$, and therefore, solving the endogenous model tells us that all dynamic inefficiency is allocative $CI(y, K, x, I; w, W(\cdot)) = AI_D^* = \vec{D}^*(y_C, K_C, x_C, I_C; w, W(\cdot)) > 0$. Note that it is necessary to solve both programs because having information on $\vec{D}^*(y_C, K_C, x_C, I_C; w, W(\cdot))$ only, does not allow to identify whether cost inefficiency is due to technical or allocative reasons. Finally, as previously commented, firm B represents the specific case for which the directional vector (g_x^T, g_I^T), corresponding to the observed amounts

(x_B, I_B), once rescaled so as to satisfy the unit valued normalizing constrain, projects the firm exactly onto the cost minimizing firm (x^*_A, I^*_A). Therefore $(g^T_x, g^T_I) = (g^*_x, g^*_I)$, resulting in $CI(y, K, x, I; w, W(\cdot)) = \vec{D}^T(y_B, K_B, x_B, I_B; g^T_x, g^T_I) = \vec{D}^*(y_B, K_B, x_B, I_B; w, W(\cdot)) > 0$. Hence all dynamic inefficiency is technical.

3. Empirical application to the European dietetic food industry

3.1. Data set

Our empirical application focuses on EU firms in the dietetic food manufacturing industry which represents an interesting case study given its dynamism in the past decade. As consumers are increasingly equating food with health and wellness, the growth of dietetic food industry is inevitable and evident. Dietetic food is associated with sustainability and corporate social responsibility because healthy food choices are often sustainable choices (Esteve-Llorens et al., 2020). According to Eurostat (2019a), this industry has grown rapidly in the EU between 2011 and 2016, with the number of firms and employees increasing by 51% and 24%, respectively, while its value added increased by 11% in the same period. Examples of dietetic foods include: infant and young children food, slimming foods (that is foods for people undertaking energy-restricted diets to lose weight), food for special medical uses (such as food for diabetics), sports foods and food for people with gluten intolerance (Bragazzi et al., 2017). Our dataset on firms' inputs and outputs were taken from the AMADEUS database provided by Bureau van Dijk and corresponds to NACE Rev. 2 code 1086, identified as "Manufacture of homogenized food preparations and dietetic food". This database comprises financial information on public and private companies in Europe. We focused on a balanced panel of firms that were observed in the database in three years, 2011, 2014 and 2017. These three years were chosen because this is the period when the industry under study was increasing in terms of the number of companies, employment, and value added (Eurostat, 2019a).

The application distinguishes two variable inputs, i.e., materials and labor; their costs were taken from the firms' profit and loss account. Quasi-fixed input (capital) was measured as the starting value of fixed assets from the firms' balance sheet (i.e., the end of year value of the previous year). Gross investments in fixed assets in year t were computed as the starting value of fixed assets in year $t+1$ minus the beginning value of fixed assets in year t plus the value of depreciation in year t. The firm-specific values of depreciation were directly taken from the firms' profit and loss accounts. A single output is distinguished in the model as the aggregation of all products in monetary terms, i.e., this corresponds to the revenue reported by the firms in their profit and loss accounts. Such configuration of inputs and outputs is based on prior research (e.g., Kapelko & Oude Lansink, 2017; Kapelko, Oude Lansink & Stefanou, 2014).

The variables downloaded from AMADEUS were measured in local currencies and in current prices. To obtain a common currency, these variables were adjusted by the Purchasing Power Parity (PPP) of the local currency to the US dollar (World Bank, 2019). To obtain input and output values at constant prices, these variables were deflated using country-specific price indices obtained from Eurostat (2019b): material costs were deflated using the producer price index for intermediate goods, labor costs by the labor cost index in food manufacturing, fixed assets by the producer price index for capital goods, and revenues by the producer price index for food manufacturing.

The price indices for materials and labor were used as an approximation of the prices of variable inputs w. The cost price of quasi-fixed factors was calculated as: $c_i = (r + \delta_i)z_i$, where r is the interest rate, δ_i is depreciation rate, and z_i is the price index of the quasi-fixed input. The interest rate r is approximated by the long-term interest rates, collected from the Eurostat (2019c) database. Following Silva et al. (2015), the shadow values of the quasi-fixed factors were determined separately, using a quadratic specification of the optimal value function and rewriting it as: $w'x = rW(y, K, w, c) - c'K - W'_K(I - \delta K)$. When this specification is fitted, the shadow values are obtained using the parameter estimates.

The final sample of dietetic food producers in the EU was obtained after eliminating observations with missing data as well as outliers following the method of Simar (2003). The final sample consisted of 143 firms, divided into Eastern European firms (27 firms), Southern European firms (91 firms) and Western European firms (25 firms). Table 1 presents averages and standard deviations of the input-output variables valued in monetary terms (i.e., multiplied by their corresponding prices), separately for 2011, 2014 and 2017 and the regional composition of the sample. The table indicates that Western firms have, on average, the greatest values for the variable inputs (materials and labor) and output for both years (resulting from higher prices and larger quantities), while the opposite is observed for Eastern European firms, exhibiting the smallest values.

3.2. Results

The computation of dynamic inefficiency measures was undertaken for all three years and including the whole set of firms belonging to all European regions to be able to appropriately compare inefficiencies between regions. We compare the dynamic cost inefficiency results and interpretation of the conventional, monetary valued, model (9) with an exogenous directional vector and the endogenous model (16)–(17). For the exogenous model we choose a directional vector $g^T = (g^T_x, g^T_I) \in \mathbb{R}^N_{++} \times \mathbb{R}^F_{++}$ that is neutral with respect to the orientation, common to all firms, and assigns equal weight to all inputs and investment. This implies that $g^T_x = g^T_I = 1/(w'1_N + W_K(\cdot)'1_F)$, where 1_N and 1_F are unit vectors with dimensions $(N \times 1)$ and $(F \times 1)$.

3.2.1. Dynamic cost inefficiency and its decomposition

Table 2 presents the firm average values for dynamic cost inefficiency (*CI*), dynamic technical inefficiency (*TI*), dynamic allocative inefficiency (*AI*), and the directional vectors for materials (x_1), labor (x_2), and investments (I), in case of the conventional (g^T_x, g^T_I) and endogenized (g^*_x, g^*_I) models. On average, dynamic cost inefficiency in Europe amounts to 7.054 million dollars in 2011, 8.623 million dollars in 2014 and 8.461 million dollars in 2017. These values imply that the potential dynamic average cost saving in Europe is equal to 15.7% (=7.054/44.896 × 100) of total cost in 2011, 15.0% (=8.623/57.405 × 100) in 2014, and 13.5% (=8.461/62,829 × 100) in 2017. In absolute terms, the potential cost saving is largest in Western Europe, and smallest in Southern Europe. These are relevant figures deserving further analysis of the sources of cost inefficiency. Before discussing these sources we remark that, throughout the results section, to test the differences in dynamic inefficiencies between regions we perform the adapted Li test proposed by Simar and Zelenyuk (2006), which is an extension of the nonparametric test for the equality of two densities of Li (1996). To test the differences in directional vectors between regions we use the standard Li (1996) test. Regarding dynamic inefficiency (cost, technical and allocative), the differences between regions reported in Table 2 show that at the critical 5% level the results for firms in Southern Europe are generally different from those of its Western and Eastern counterparts. As for the directional vectors, both for the conventional $g^T = (g^T_x, g^T_I)$ and endogenous $g^* = (g^*_x, g^*_I)$ models, the differences are mostly significant in the last two years, 2014 and 2017.

Table 1
Averages and standard deviations of the firm data, 2011, 2014 and 2017 (million US dollar, constant prices of 2010).

Variable	Eastern[a]	Southern[b]	Western[c]	Whole Europe
2011				
Capital (K)	11.084 (34.342)	23.410 (180.769)	11.603 (29.284)	19.019 (145.278)
Materials (x_1)	11.428 (23.133)	16.518 (113.276)	34.299 (64.054)	18.665 (94.763)
Labor (x_2)	1.858 (4.035)	5.010 (29.981)	10.380 (22.675)	5.354 (25.815)
Investments (I)	0.985 (1.562)	1.862 (11.283)	2.789 (5.411)	1.858 (9.294)
Revenue (y)	19.539 (41.403)	31.138 (200.662)	78.715 (177.596)	37.266 (177.626)
2014				
Capital (K)	9.353 (26.365)	27.360 (209.047)	15.965 (32.389)	21.968 (167.504)
Materials (x_1)	13.790 (25.499)	23.071 (157.181)	50.979 (107.107)	26.198 (133.642)
Labor (x_2)	2.223 (4.246)	6.777 (43.515)	11.640 (23.452)	6.767 (36.118)
Investments (I)	0.611 (0.701)	1.985 (10.850)	6.252 (17.853)	2.472 (11.485)
Revenue (y)	22.623 (46.536)	42.994 (287.930)	92.899 (191.732)	47.873 (244.228)
2017				
Capital (K)	11.566 (33.666)	29.514 (203.548)	25.469 (53.413)	25.418 (164.306)
Materials (x_1)	15.648 (30.614)	24.388 (158.620)	49.324 (87.277)	27.097 (132.372)
Labor (x_2)	2.345 (4.335)	6.739 (39.249)	11.833 (23.436)	6.800 (32.876)
Investments (I)	2.149 (7.930)	3.644 (20.094)	4.511 (8.811)	3.514 (16.765)
Revenue (y)	26.529 (56.236)	44.855 (279.329)	95.743 (178.885)	50.291 (236.494)
No. of firms per year	27	91	25	143

Note: Standard deviations are in parentheses.
[a] Bulgaria, Croatia, Czech Republic, Hungary, Poland, Romania, Slovakia, Slovenia.
[b] Italy, Portugal, Spain.
[c] Belgium, France, Germany, Sweden.

Table 2
Dynamic cost inefficiency and components, including directional vectors. Average values.

Region	*CI*	Normalized directional vector, model (9)			Endogenous directional vector, model (16)-(17)				
		$g_x^T = g_I^T$	TI^T	AI^T	g_{x1}^*	g_{x2}^*	g_I^*	TI^*	AI^*
2011									
Europe	7.054	0.370	1.571	5.482	0.640	0.305	−0.070	5.810	1.244
Eastern	7.952	0.362	1.293	6.659	0.799	0.137	−0.079	7.882	0.070
Southern	3.932	0.370	1.618	2.314	0.577	0.365	−0.063	3.349	0.583
Western	17.446	0.379	1.703	15.743	0.697	0.266	−0.090	12.531	4.915
Significance	c	a	–	c	–	a	–	–	–
2014									
Europe	8.623	0.375	2.415	6.208	0.440	0.136	0.596	6.277	2.346
Eastern	8.352	0.364	2.185	6.166	0.582	0.027	0.466	8.326	0.025
Southern	4.624	0.376	1.990	2.633	0.378	0.162	0.691	3.968	0.656
Western	23.474	0.382	4.211	19.263	0.512	0.163	0.389	12.472	11.001
Significance	c	a b		c	a c	a c	a c	c	
2017									
Europe	8.461	0.363	2.961	5.500	0.425	0.209	0.156	6.716	1.745
Eastern	7.652	0.336	2.248	5.403	0.730	0.074	−0.016	6.620	1.031
Southern	5.302	0.368	2.861	2.442	0.519	0.401	−0.162	5.035	0.268
Western	20.833	0.372	4.096	16.737	−0.244	−0.342	1.499	12.938	7.896
Significance	–	a b c	–	c	a b c	a c	a b	–	a, b

a Denotes a significant difference between Eastern and Southern Europe at the critical 5 percent level.
b Denotes a significant difference between Eastern and Western Europe at the critical 5 percent level.
c Denotes a significant difference between Southern and Western Europe at the critical 5 percent level.
Notes: *CI*, *TI* and *AI* are expressed in million US dollars.

For Europe as a whole in 2011, the conventional approach (under the normalized directional vectors, g^T) attributes 1.571million dollars to technical inefficiency (22.3% of total cost inefficiency, *CI*) and 5.482 million dollars to allocative inefficiency (77.7%). These figures remain stable in the following years. In 2014, technical inefficiency accounts for 29.3% (2.415 million dollars out of 8.263 million dollars) and allocative inefficiency accounts for the remaining 70.7%. In 2017 these numbers are 2.961 million dollars (35.0%) and 5.500 million dollars (65.0%), respectively. The results of the conventional method show then that, on average, allocative inefficiency is the largest component of dynamic cost inefficiency in all regions. Therefore, by following the conventional cost efficiency approach, all we know is that rather than focusing on engineering planning errors that result in technical inefficiency, firms' managers should focus on changing the amount of inputs demanded to meet the optimal quantities (input-mix) that minimize cost. However, how inputs should be changed to reach the cost minimizing benchmark cannot be discerned, because the directional vector is exogenous. This relevant information is obtained by resorting to our new approach.

Solving the endogenous model we learn that for the whole Europe the elements of the optimal directional vector in 2011 corresponding to materials, labor and investments are, on average: $g_{x1}^* = 0.640$, $g_{x2}^* = 0.305$ and $g_I^* = -0.070$. A negative direction states that, to match the optimal input demands, firms would have to expand the input or, alternatively, contract investment. Since the sign of the average input directions is positive, our results suggest the contraction of materials and labor. On the contrary, the sign for the average investment direction is negative, prescribing a mild reduction in investments, most likely because of the financial crisis that started in 2008 and was at its peak in 2011. In 2014, the average directional vectors in Europe still suggest the same reduction in both inputs, and the increase in investments. This last result, which also holds for Europe as a whole in 2017, suggests that by 2014 the financial crisis was overcome and therefore achieving the optimal amount of capital stock K required constant investments in European regions—except Southern Europe that presents a relevant negative value for investment in 2017.

As expected, the need for all these technological changes shows up in the decomposition of dynamic cost inefficiency as technical inefficiency, with allocative inefficiency playing a minor role. This simply reflects that most of the firms lay inside the input set, and therefore monotonic adjustment are feasible in terms of technical efficiency improvements that reduce cost inefficiency. The values of dynamic allocative inefficiency only concern the inefficiency of firms that are technically efficient; i.e. firms that are on the production frontier, which also must change the inputs according to the optimal directions.

On-line appendices A1, A2 and A3 present the kernel density plots of all dynamic inefficiency components in 2011, 2014 and 2017 per region. These plots have been constructed using the procedure outlined in Simar and Zelenyuk (2006). In short, a Gaussian kernel is used, and the reflection method is employed to overcome the issue of a zero-bounded support of the inefficiency scores (Silverman, 1986). Bandwidths are based on Sheather and Jones's (1991) method—for better visualization of the differences in distributions, we cut off the top 5% of observations with the highest inefficiency values. The kernel density plots of the inefficiency components show that the distributions of all inefficiency components in Eastern and Southern Europe tend to have a higher kurtosis than the distributions of Western Europe. This suggests that the inefficiency values are more dispersed for Western Europe than for the other two regions.

Attempting now to compare our inefficiency results with those of previous literature, we should note that research into the economic efficiency of dietetic food industry is very scarce. Most of the studies focus only on the technical efficiency dimension, expressed in terms of conventional technical efficiency scores (% reductions *a la* Farrell, $0 < TE \leq 1$), and not in monetary terms as in the current paper (exact dollar reductions). That makes a direct comparison impossible. However, we can express our results as efficiency scores bounded between 0 and 1. The Farrell equivalent technical efficiency scores may be calculated for the exogenous model as follows: $TE^T = (C_o - TI^T)/C_o$, where TI^T is the technical inefficiency under $g^T = (g_x^T, g_I^T)$, while for the endogenous model $TE^* = (C_o - TI^*)/C_o$, where TI^* is the technical inefficiency under $g^* = (g_x^*, g_I^*)$. We find that, on average, technical efficiency for Europe as a whole in 2011 is 0.965 for the exogenous model and 0.870 for the endogenous model, in 2014 they are 0.957 and 0.891, respectively, while in 2017 they are 0.953 and 0.893, respectively. This makes our findings similar to the studies for other food manufacturing industries (without distinguishing the dietetic food sector) that find relatively high efficiency scores of 0.987 for Greek industry (Rezitis & Kalantzi, 2016), and moderate values of 0.787 for Czech Republic (Rudinskaya, 2017). Smaller values of efficiency are reported in the study by Kapelko, Harasym, Orkusz and Piwowar (2022), which shows rather high average inefficiency scores of 0.508 for 2009–2017 in the same dietetic food industry.

3.2.2. Optimal resource allocation in the dietetic food industry

We now discuss the differences in the managerial prescriptions resulting from the conventional and endogenous models, including the contradictions associated to the former, considering the sign of the optimal directional vectors. On-line appendices A4, A5 and A6 present the standard Gaussian kernel plots for each input direction in 2011, 2014 and 2017 per region. The kernel density plots of the endogenous directions show that the adjustments in the two inputs and investment may be positive or negative. In 2011, 2014 and 2017 the distributions of the endogenous directional vector for materials $g_{x_1}^*$ and labor $g_{x_2}^*$ are mostly positive, while in the distribution of investment $g_{x_1}^*$ we observe many negative values, mostly centered around zero. It is noteworthy that the dispersion in the values of the endogenous directional vectors increases over the period.

In relation to these distributions of the optimal endogenous directions, we have classified in Table 3 the recommended changes in the use of inputs in 2011, 2014 and 2017. We shade in gray those changes that are consistent with the assumptions of the conventional model forcing input reductions and investment increases, $g^T = (g_x^T, g_I^T) \in \mathbb{R}_{++}^N \times \mathbb{R}_{++}^F$; e.g., the case of (x_C^T, I_C^T) as the projection of firm C on the production frontier in Fig. 1. The table shows that for the whole Europe about 76% of the firms should decrease their amount of materials in 2011, 2014 and 2017 to minimize cost (presented in the second to fourth columns, ≤ 0), while the remaining 24% were supposed to increase its use in at least one of the three years (columns identified with >0 under either 2011, 2014 or 2017), which goes against the prescriptions of the standard model. For example, 8% of the firms were supposed to increase materials in 2017, 10% in 2014 and 2% in 2011 (while reducing it in the other two remaining years, respectively). This simply shows that it is quite possible to underuse inputs with respect to the cost minimizing benchmark and, therefore, it makes no sense to force their reduction through g_x^T. Indeed, this situation is aggravated for labor, where less than half the firms, 48%, should reduce employment in all three years, while as many as 7% of the firms should hire more people in the three years (last three columns before *Total*). Where there is almost total disagreement between the endogenous and standard models is in investment, because just 1% of the firms should increase their investing efforts in the three years to minimize cost given the optimal dynamic adjustment of the quasi-fixed capital input (again reported in the last three columns before *Total*). Moreover, as reported in the second to fourth columns, almost one third of the firms, 31%, are prescribed to reduce investment in all three years. In sum, from the new model we conclude that, opposite to the standard assumption, 24% of the firms were underusing materials in at least one year, 52% underused labor, while 99% were overinvesting.

The relatively high underuse of labor is found in all regions, but particularly in Eastern Europe and Southern Europe. Only 41% of the firms in Eastern Europe and 36% in Southern Europe would find advantageous to reduce the labor input in all three years. The underuse suggests frictions in the labor market which withhold firms from achieving a cost minimizing size of their labor force (Wijnands & Verhoog, 2016). Such frictions could take the form of, for example, insufficient labor supply, both in absolute numbers and required abilities (quality), inflexible labor contracts preventing firms from hiring or terminating employees in the short term, or other transaction costs (Kapelko & Oude Lansink, 2017, European Commission, 2016). As for the capital input, as stated above almost all firms face overinvestment, with as many as 31% aiming at disinvestments in all three years, which implies that capital amortization should not be balanced with new investments. This also suggest the existence of frictions in the capital market resulting from credit constraints, uncertainty about future market conditions or rapid technological progress, all giving rise to a high value of the option to wait (European Commission, 2016). Credit constraints can be particularly relevant in the agribusiness, where cooperatives are often the dominant organizational form, with slow reactions when adjusting to changing market condition, which prevents them to reduce capital in a timely manner, as happened in 2011 after the 2008 financial crash. Regarding the materials input, the reduction in the amount used in all three years is dominant. This implies that firms used materials in excess over the whole period. Moreover, the relatively high overuse of materials occurs in

Table 3
Optimal allocation of inputs and investments in 2011, 2014 and 2017 (% of firms).

Year	2011	2014	2017	2011	2014	2017	2011	2014	2017	2011	2014	2017	2011	2014	2017	2011	2014	2017	2011	2014	2017	2011	2014	2017	*Total*
Recommended Change	≤ 0	≤ 0	≤ 0	≤ 0	≤ 0	> 0	≤ 0	> 0	> 0	≤ 0	> 0	≤ 0	> 0	≤ 0	≤ 0	> 0	> 0	≤ 0	> 0	≤ 0	> 0	> 0	> 0	> 0	
Europe																									
Materials	76%			8%			0%			10%			4%			0%			2%			0%			**100%**
Labor	48%			17%			6%			14%			6%			1%			1%			7%			**100%**
Investment	31%			20%			2%			5%			17%			5%			19%			1%			**100%**
Eastern																									
Materials	92%			4%			0%			4%			0%			0%			0%			0%			**100%**
Labor	41%			3%			7%			15%			4%			4%			0%			26%			**100%**
Investment	37%			26%			0%			0%			18%			4%			15%			0%			**100%**
Southern																									
Materials	70%			10%			0%			12%			5%			0%			3%			0%			**100%**
Labor	54%			19%			2%			15%			8%			0%			1%			1%			**100%**
Investment	26%			21%			3%			7%			17%			3%			23%			0%			**100%**
Western																									
Materials	80%			4%			0%			12%			4%			0%			0%			0%			**100%**
Labor	36%			28%			16%			8%			0%			0%			4%			8%			**100%**
Investment	40%			12%			0%			4%			20%			12%			8%			4%			**100%**

Table 4
Change in dynamic cost inefficiency decomposition and in directional vectors between 2011 and 2017. Average values.

Region	ΔCI	Normalized directional vector, model (9)			Endogenous directional vector, model (16)-(17)				
		$\Delta g_x^T = \Delta g_I^T$	ΔTI^T	ΔAI^T	Δg_{x1}^*	Δg_{x2}^*	Δg_I^*	ΔTI^*	ΔAI^*
Europe	1.407	−0.007	1.390	0.018	−0.214	−0.095	0.226	0.906	0.502
Eastern	−0.301	−0.026	0.955	−1.256	−0.068	−0.063	0.062	−1.262	0.961
Southern	1.370	−0.002	1.243	0.127	−0.058	0.036	−0.099	1.686	−0.316
Western	3.387	−0.007	2.393	0.994	−0.942	−0.608	1.589	0.406	2.981
Significance	–	a b c	–	–	a c	a c	–	–	b c

a Denotes significant difference between Eastern and Southern Europe at the critical 5 percent level.
b Denotes significant difference between Eastern and Western Europe at the critical 5 percent level.
c Denotes significant difference between Southern and Western Europe at the critical 5 percent level.
Notes: ΔCI, ΔTI and ΔAI are expressed in million dollars.

all regions, particularly in Eastern Europe, where 92% of the firms should reduce their use in all three years. Markets for commodities were characterized by high volatility in the past decade which adds to the business risk of agribusiness firms (Kapelko & Oude Lansink, 2017, European Commission, 2016).

We stress the finding that the optimal recommendations implied by the endogenized directional vectors may differ from those of the conventional model (shaded in gray). For Europe as a whole, the differences are particularly noticeable for labor and capital investments (for materials 76% of the firms were supposed to reduce its use in accordance with the conventional model). Indeed, less than half of the firms, 48%, see a reduction of their labor force as the recommended change in all three years, while for investments the proportion of firms which should increase their capital stock is non-existent, i.e., 99% of the firms should disinvest in at least one year, and 31% in all three years. By years, 2011 and 2014 are the periods when most disagreement is observed regarding investment, most probably because of the effects of the 2008 financial crisis that was at full swing in the first two years, signaling a reduction in investments to reach minimum cost. On the contrary, 2017 is the year where the increase in labor was mostly prescribed, probably because firms were recovering from the same crisis and needed additional labor to produce larger output at minimum cost. The results for the different regions are generally similar.

3.2.3. Change in dynamic cost inefficiency and decomposition

The cross-section differences between the endogenous and conventional model also emerge when looking at the evolution of the inefficiencies. Table 4 presents the cumulative change in dynamic cost inefficiency, and its decomposition into technical and allocative inefficiencies together with the changes in directional vectors between 2011 and 2017, computed as the value in the final year minus the value in the initial year. The results show that dynamic cost inefficiency in the European dietetic food industry increased by 1.407 million dollars in 2017 compared to 2011. The average differences are positive for all regions, except Eastern Europe, showing a slight reduction in cost inefficiency. However, these changes in cost inefficiency and its components are not statistically different across regions, suggesting that the overall trend is the increase of cost inefficiency. Although Table 4 only reports cumulative changes between 2011 and 2017, looking at Table 2 we observe that most of the increase in cost inefficiency took place in the first period between 2011 and 2014, while cost inefficiency reduced substantially in the second period from 2014 to 2017. The net effect, however, is that of cost inefficiency increments.

The decomposition shows that worsening dynamic technical inefficiency was the main contributor to declining cost inefficiency for Europe as a whole, both under the conventional and endogenous models. The decline in technical efficiency amounts 1.390 and 0.906 million dollars on average for each European firm, representing 98.8% (=1.390/1.407 × 100) and 64.34% (=0.906/1.407 × 100) of the increase of cost inefficiency, respectively. In Eastern Europe, where cost inefficiency improved, technical inefficiency change was also the main driver, whereas in Western Europe the opposite took place. In this regard, it is worth remarking that for Eastern Europe technical and allocative inefficiencies contributed in opposite ways depending on the approach. That is, in the exogenous model, forcing input reductions and investment increases, firms experienced declining technical inefficiency that is counterbalanced by allocative inefficiency improvements. On the contrary, the opposite is observed in the endogenous approach taking firms directly to the cost minimizing benchmark.

Appendix A7 presents the kernel density plots for the changes in dynamic cost inefficiency, dynamic technical inefficiency, and dynamic allocative inefficiency between 2011 and 2017 for each region—on this occasion, for better visualization we cut off the 5% of the observations with the lowest and the largest values of the changes. Again, the kernel density plots suggest a higher kurtosis of the distributions of all inefficiencies for Eastern and Southern Europe rather than for Western Europe. Hence, the distributions for Western Europe are more spread out. The values at which the distributions of the changes in dynamic technical inefficiency and dynamic cost inefficiency peak are around zero for the endogenized directional vector, which is generally in line with the average values presented in Table 4. Furthermore, the kernel density plots suggest that the distributions of the changes in technical inefficiency and cost inefficiency are right skewed for all regions. Appendix A8 presents the kernel density plots for the changes in the optimal directions between 2011 and 2017 for each region. Again, as for the plots for kernel densities for directional vectors in each year, both positive and negative adjustments for the two inputs and investment are possible.

Finally, regarding the evolution of each individual firm within the distributions of technical and allocative inefficiencies corresponding to the endogenous model (TI^* and AI^*), Table 5 reports the transition matrices of the firms' dynamic technical and allocative inefficiencies from 2011 to 2017. By rows, in 2011 there were 22 firms that were technically efficient and 127 that were allocatively efficient. Out of these totals, 6 firms were both technically and allocatively efficient, thereby minimizing cost. Recall that in the endogenous model, the 121 technically inefficient firms are projected to the cost minimizing benchmarks, becoming allocatively efficient. Therefore, these 121 firms show up in the group of allocatively efficient firms in Table 5, while adding the 6 firms that are both technically and allocatively efficient, yields the total 127 firms that are reported as allocatively efficient. In the same vein, the 22 firms that are technically efficient are deemed allocative inefficient unless they minimize cost, and therefore 16 firms are reported as allocatively inefficient in 2011. Focusing now on the transitions, out of the 22 firms that were technically efficient in 2011, about one third (7) continued to be technically efficient in 2017, whereas 15 became inefficient, showing up in the group of

Table 5
Transition matrix of dynamic technical and allocative inefficiencies (number of transitions).

		*TI** in 2017			AI* in 2017		
		Efficient	Inefficient	*Total*	Efficient	Inefficient	*Total*
*TI** in 2011	Efficient	7	15	*22*	18	4	*22*
	Inefficient	14	107	*121*	110	11	*121*
	Total	*21*	*122*	*143*	*128*	*15*	*143*
*AI** in 2011	Efficient	17	110	*127*	115	12	*127*
	Inefficient	4	12	*16*	13	3	*16*
	Total	*21*	*122*	*143*	*128*	*15*	*143*

18 allocatively efficient firms in 2017. Also, only about one tenth (14 firms) of the 121 firms that were technically inefficient in 2011 succeeded in becoming technically efficient in 2017. The bottom part of the table shows that out of the 127 firms that were allocatively efficient in 2011 (and therefore technically inefficient except for the 6 firms minimizing profit), 17 became technically efficient, while 115 continued being allocatively inefficient. Clearly, we could read Table 5 by columns to establish the complementary transitions, i.e., how the different groups of technically and allocatively efficient and inefficient firms observed in 2017 were performing in 2011. The results in Table 5 suggest that in the period between 2011 and 2017 there have not been significant changes in the distributions of efficient and inefficient firms from the technical and allocative perspectives. This implies that the performance of the firms in the industry remains stable. Indeed, in 2017 there were also just 6 firms that minimized cost, 2 of which were also efficient in 2011.

Due to the space limitation, we omit here the analysis of the changes including the intermediate year. Nevertheless, all results for the changes between 2011 and 2014, and 2014 and 2017 (average values, kernels and transition matrices) are presented in online Appendices A9 and A10.

4. Conclusions

The objective of this paper was to develop the endogenous approach to economic efficiency measurement within the context of the dynamic cost inefficiency model introduced by Silva et al. (2015), and demonstrate how it can be used to determine optimal resource allocation and inform managerial decision making. The consideration of an endogenous directional vector in the dynamic cost inefficiency model is critical because it solves known problems of the exogenous approach, resulting from the subjectivity of the choice of different directions, while ensuring that firms' adjustments are monotonic. This approach rules out contradictions in the prescribed changes in input quantities which may occur in the conventional model which decomposes cost inefficiency into technical and allocative inefficiency. This inconsistency of the standard model is solved by endogenizing the directional vector, implying that firms simultaneously address their technical and allocative inefficiencies, resulting in monotonic changes of inputs and investment. In this study we develop the theory behind the endogenous directional vector approach for the dynamic cost inefficiency model and apply it to a dataset of European dietetic food firms.

The results suggest an average potential for cost saving of 7.054 million dollars in 2011, 8.263 million dollars in 2014 and 8.461 million dollars in 2017, representing 15.7%, 15.0% and 13.5% of total cost in these years, respectively. From 2011 to 2017 dynamic cost inefficiency increased on average by 1.024 million dollars showing that firms in the sample endured a worsening in their economic performance. The largest average inefficiency growth is observed in Western Europe (3.387 million dollars on average) while firms in Eastern Europe fared better with a slight reduction in cost inefficiency to the tune of 0.301 million dollars. Interestingly, this negative trend took place between 2014 and 2017, since cost efficiency improved in the first period from 2011 to 2014.

The main take away from our study is that the solutions to the model with endogenized directional vectors may yield very different recommendations from those of the conventional model. The conventional model always recommends contraction of inputs and expansion of investments for firms to become technically efficient. Yet our endogenous results show that, for Europe as a whole, 24% of the firms should increase their used amounts of materials in at least one of the three years. This percentage increases to 52% for labor. In particular, the relative underuse of labor in the last year is completely missed by the conventional model. For investments, the disparity between the conventional and the endogenous directional vector model is even greater. Only 1% of the inefficient firms should increase investments throughout the whole period, while the model indicates that 31% of the firms should reduce their long-term capital stock in all three years. The underuse of labor and overinvestment is observed across all regions. The difference between the conventional and the exogenous models can also be observed in the sources of dynamic cost inefficiency. For Europe as a whole, both technical inefficiency and allocative inefficiency contribute to the growth in cost inefficiency under the two approaches, yet for some regions like Eastern Europe, the conventional model signals that technical inefficiency increments are counterbalanced by allocative inefficiency decreases, while the opposite is observed in the endogenous model. Results also show that there is a clear path dependency in the performance of firms regarding technical and allocative inefficiencies. That is, firms that were efficient in one of the two dimensions in 2011 also more likely remained efficient in the same dimension in 2017.

Our proposal, however, also presents some particularities that may be seen as limitations. For example, by endogenizing the directional vector, cost inefficiency is categorized as either technical or allocative. This is because technical inefficient firms are considered allocatively efficient, since their projection on the production frontier does not need to be followed by a further projection toward the cost minimizing firm, i.e., this benchmark is feasible and allocatively efficient by definition. By contrast, a technically efficient firm is, except for the cost minimizing firm, allocative inefficient, because the associated projection is from an already technically efficient firm onto the cost minimizing frontier. Some authors do not accept the view that the conventional decomposition of economic efficiency into technical and allocative efficiency is actually an artificial construct; artificial because the decomposition is based on a subjectively chosen exogenous directional vector. These authors criticize the endogenous directional vector approach for attributing all inefficiency to either technical or allocative inefficiency, e.g., Petersen (2018; 1074). From a statistical perspective, future research could explore the potential for using a first stage bootstrap approach to correct for sample biases in the measurement of cost inefficiency. Empirically, the consideration of labor as a variable input rather than a quasi-fixed input can also be questioned in those cases where the labor market presents rigidities, e.g., large dismissal costs when terminating contracts. Also, it would be relevant to distinguish multiple categories of quasi-fixed inputs, i.e., factors with different economic lifetime such as buildings and machinery. A further avenue for future research could be the exploration of the role of market structure in assessing economic and endogenous inefficiency. In markets where firms have market power, the assumption of exogenous output and input prices is unlikely to hold.

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Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2022.05.017.

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