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Computing Measures of Identifiability, Observability, and Controllability for a Dynamic System Model with the StrucID App

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Abstract: Identifiability, observability, and controllability are important structural properties of a dynamic system model. Our interest lies in the detection of a lack of identifiability/observability and/or controllability through the computation and subsequent analysis of the exact nullspace of the gramian for non-linear systems. For this analysis we have developed a user-friendly application with the name StrucID which runs in Matlab. The StrucID App requires as input a model definition in (possibly non-linear) state space format. In addition, an output equation that may also be non-linear is required. Through a rank test (SVD) on an associated sensitivity matrix, so-called signature graphs are produced. These represent a model's singular values and nullspace vectors and provide a visual summary. The results can now be used in a substantially reduced symbolic computation (not included yet in the current version of StrucID) that computes a Fliess series expansion of the output signal to arrive at the nullspace of an associated Jacobi matrix. Solving an underlying partial differential equation then completes the structural analysis and generates a re-parametrisation and/or state transformation that allows for model reduction in an exact manner. A few examples will be presented.

Keywords: Identifiability, Observability, Controllability, Sensitivity Analysis

1. INTRODUCTION

The analysis of the structural properties (such as local observability) of large dynamic models may fall victim to insufficient methods available to do so. In this submission, we would like to introduce an application that can be tasked with the preliminary analysis of such models, that is investigating their structural properties. This first examination of a model's structure can be done before any experiments are conducted and can *a priori* highlight some flaws that result in the total correlation between certain model parameters. A structural analysis is needed before any state or parameter inference is performed or before (in case of the dual problem) any control action is taken to steer the state of the system to a desired location in the state space. In practice such an analysis is often omitted because of its complexity. One could even argue that this complexity is an essential weak point in non-linear control theory. Indeed, for a structural analysis a Lie algebra must then be introduced and many Lie derivatives (and Lie brackets) need to be computed symbolically before anything can be said about observability or controllability (Isidori, 1995; Hermann and Krener, 1977). If, for example, a 5-state model is observed through one sensor that reads out one of the states in the model, then this already can become infeasible (Chappell et al., 1990). We think, therefore, that there is a need for software tools that help the user in establishing a quick and accurate answer. For the identifiability question several packages exist that are all based on symbolic computations (see table 1). These are very useful tools establishing local (and in some cases even global) identifiability. In this paper we introduce an alternative application that follows a different route to arrive at the structural properties of a dynamical system. It is based on a combined numerical/algebraic computation of the nullspace of the associated gramian. Its main advantage is computational speed, but it also provides an alternative point of view that has a certain attractiveness to it.

Before we introduce the StrucID version 2.0 package, we first give some background information on the approach we take in analysing structural properties. Essentially, we walk the avenue of parametric sensitivities and this yields a computational attractive point of departure. Since the results obtained in the first stage are numerical, we continue the analysis with a second symbolic computation. The required number of calculations is now reduced using the results obtained from the singular value decomposition (SVD) of the parametric output sensitivity matrix. The second stage of our analysis corroborates the numerical results and subsequently allow for both the precise characterization of the relationship between correlated parameters and the associated re-parametrization of the model at hand.

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Table 1. Some available identifiability software packages and their associated references in the literature.

Name	Reference
COMBOS	Meshkat et al. (2009)
DAISY	Bellu et al. (2007)
EAR	Karlsson et al. (2012)
STRIKE-GOLDD	Villaverde et al. (2016)
GenSSI 2.0	Hong et al. (2019)
Observability Test	Sedoglavic (2002)

2. METHODS

Consider the well-known non-linear state space model and associated observation equation

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = f(x(t), u(t), \theta) \tag{1}$$

$$y(t) = h(x(t), u(t), \theta) \tag{2}$$

with f and h vector functions, $x(t)$ the vector of state variables, $u(t)$ the vector of input variables, and θ the vector that contains all the parameters in the model structure f and/or observation function h. To study the optimal placement of sensors for a fluid model, Krener and Ide review in their CDC paper on observability of large system models the possibility of establishing a *measure* for the degree of observability of the above state space model via the well-known tangent mapping

$$\frac{\mathrm{d}\delta x}{\mathrm{d}t} = \frac{\partial f}{\partial x}\,\delta x \tag{3}$$

$$\delta y(t) = \frac{\partial h}{\partial x}\,\delta x \tag{4}$$

where the Jacobi matrices are evaluated on a reference trajectory that is a solution to (1)-(2), (Krener and Ide, 2009). The local singular values of the non-linear mapping (1)-(2) are defined by the singular values of the tangent mapping. If these singular values are large, then it is relatively easy to reconstruct $x(0)$ from the measurements. The local singular values, in turn, characterize the local observability gramian $P(x^0)$ that can be computed on the basis of the fundamental solution $\Phi(t)$ for the tangent mapping (3)–(4), i.e.

$$\frac{\mathrm{d}}{\mathrm{d}t}\Phi(t) = \frac{\partial f}{\partial x}\,\Phi(t) \tag{5}$$

$$\Phi(0) = I \tag{6}$$

The observability gramian is now defined as

$$P(x^0) = \int_0^{t_f} \Phi^T(\tau)\,\frac{\partial h^T(\tau)}{\partial x}\,\frac{\partial h(\tau)}{\partial x}\,\Phi(\tau)\,\mathrm{d}\tau \tag{7}$$

In Krener and Ide (2009) the primary interest is in computing a condition number of the observability gramian as a measure of un-observability. In fact, (7) is *approximated* by the well-known empirical gramian in their paper. This approximation is used frequently in the literature on non-linear model reduction (Lall et al., 2002). Our point of departure, however, is that we use the well-known forward sensitivities equations (that can be shown to be exactly equivalent to (5)–(6)!) and compute these sensitivities as accurate as possible. The reason for focussing on a precise computation of the sensitivity dynamics is that we are primarily interested in the *exact* zero singular values of the observability gramian and so we do not approximate this gramian with an empirical version of it. A few remarks are in place:

- We compute parametric output sensitivities for a given reference trajectory. This trajectory, obviously, is a choice and this *may* influence the outcome of the analysis. This is not very different from a 'normal' non-linear observability analysis (as presented in non-linear control theory on the basis of a Lie-algebra that is constructed from so-called drift and control vector fields (Hermann and Krener, 1977)). Indeed, it is known that there are regular and non-regular points in that setting and the outcome of an observability analysis can also be different in these points. As it turns out, only very specific initial conditions can cause the outcome to be different but, in general, the regular points cover almost the whole state space.
- In our sensitivity analysis the states are treated as special parameters, i.e. the states' *initial conditions* are parametrized and considered as time-invariant parameters whose sensitivity dynamics are calculated. This allows observability of states to be treated in exactly the same way as (time-invariant) parameters.
- We can treat structural controllability in the same framework through computation of the well-known controllability gramian

$$Q(x^f) = \int_0^{t_f} \Phi(\tau)\,\frac{\partial f(\tau)}{\partial u}\,\frac{\partial f^T(\tau)}{\partial u}\,\Phi^T(\tau)\,\mathrm{d}\tau. \tag{8}$$

  The computation of this controllability gramian is facilitated easily by the associated *adjoint equations* of the tangent mapping (3)–(4) on a reference trajectory that starts at final state x^f and is integrated backwards in time. The singular values of $Q(x^f)$ are again, computed as precise as possible via the sensitivity dynamics and we once more emphasize that these are not approximated on the basis of an empirical gramian.
- In the celebrated paper by Moore structural properties of *linear* systems are established in much the same way as we do in this paper, using a principal component analysis, (Moore, 1981). In the case of linear systems the tangent mapping yields the exact same gramian that is analysed in his paper. For non-linear systems the Jacobi matrix in (5) is time-varying but the dynamics are still linear for $\Phi(t)$, allowing a rank test to be performed to check for linear dependencies between sensitivity functions. The possibility of a linear dependence between parametric output sensitivities is well-known in the area of non-linear parameter estimation as an indicator of a possible lack of identifiability (Bard, 1974; Miao et al., 2011).
- In the same paper Moore already pointed out that in order to find an accurate value for the singular values of the gramian, it is better to SVD the matrix $\frac{\partial h}{\partial x}\Phi(t)$ (in case of observability), rather than the observability gramian itself. This comes down to checking linear dependencies between *functions* of the *linear time-varying system* (5)-(6) on a time interval $[0, t_f]$, and this is exactly the same approach as taken in (Moore, 1981).

In Stigter and Molenaar (2015); Joubert et al. (2020) it is demonstrated how the zero singular values of the observability gramian can be utilized for an efficient symbolic computation that allows for the re-parametrisation of the non-linear model (1) to be performed. In short: Since unidentifiable parameters come in groups (one group per zero singular value, see e.g. Joubert et al. (2020)), we can focus the symbolic computations on the unidentifiable parameters for each group *separately*. This really is a tremendous (symbolic) computational saving that is harvested from the initial SVD analysis on the output sensitivity matrix, (Stigter et al., 2018).

3. THE STRUCID VERSION 2.0 APP

In figure 1 the opening window of the StrucID_v2.0 App is presented. It consists of 3 panels, namely (from left to right) (i) the user input panel, (ii) the output and parameter selection panel, and (iii) the results panel.

3.1 User Input Panel

In the user input panel we can select the type of analysis (either controllability or observability) to be conducted, specify a length for the integration interval (t_f), and define the desired accuracy of the integration results. For the final integration time t_f it is usually sufficient to specify the length as twice the largest characteristic time constant of the system. These time constants can be easily obtained in Matlab by computation of the eigenvalues of the matrix exponent of the Jacobi matrix associated with model (1). Our experience has shown that the integration length may change the outcome of the structural analysis, but if it is not specified too high then the outcome is very reliable. The accuracy of the parametric output sensitivities solution $y_\theta(t)$ in our App relies heavily on the state-of-the-art Sundials [1] solver that is, in fact, part of the SimBiology toolbox in Matlab. This solver utilizes complex derivatives for a very accurate computation of the Jacobi matrices $\frac{\partial f}{\partial x}$, $\frac{\partial h}{\partial x}$, $\frac{\partial f}{\partial \theta}$, and $\frac{\partial h}{\partial \theta}$ that appear in the forward sensitivity equations:

$$\frac{dx_\theta(t)}{dt} = \frac{\partial f}{\partial x} x_\theta + \frac{\partial f}{\partial \theta} \tag{9}$$

$$y_\theta(t) = \frac{\partial h}{\partial x} x_\theta + \frac{\partial h}{\partial \theta} \tag{10}$$

where $x_\theta(t) = \frac{\mathrm{d}x}{\mathrm{d}\theta}$. The input panel finally includes two buttons with names "Import Model" and "Analyse" that allow the user to import the model input file (in plain ASCII text format) and (after a successful import) start the analysis.

3.2 Selection Panel

Since a structural observability or controllability analysis depends completely on the available input and/or output signals, we have included a so-called selection panel that allows for greater flexibility when the analysis is repeated many times with different sensor combinations. A user can use check marks (✓) to easily select the sensors (or input variables) he/she has available in the given experimental setup. Of course, it is interesting to study how exactly the observability of a system changes if a different selection of sensors is chosen and our App provides a quick tool to find, for example, a *minimal output set*, i.e. the set of sensors that is minimally needed for identification of all the selected states and/or parameters in the model (Joubert et al., 2018). At the bottom of the central panel, a user can specify exactly which state variables and parameters he/she wishes to reconstruct from the available signals. *Initial values* for state variables and/or parameters that are considered unknown can also be specified here. A reasonable random value will be generated by the software if an IC is not specified numerically. Specifying user defined initial values can be important in some cases and can make a difference in the outcome of a structural analysis as clearly demonstrated in Saccomani et al. (2003).

3.3 Result Panel

Once the analysis button has been pressed (after a successful import of the model definition from the text file), the sensitivities are computed and a singular value decomposition of the output sensitivity matrix is found. Results are presented in two figures, namely (i) a graph of the singular values and the null space of the sensitivity matrix (corresponding to the singular values that are considered as zero) and (ii) a directed (adjacency) graph that is based on the Jacobi matrix $\frac{\partial f}{\partial x}$. In this graph an arrow from node j to node i is drawn if the corresponding (i, j) matrix element in the Jacobi matrix is non-zero. The directed graph depicted in figure 2 shows how the 14 state variables in example 4.1 are connected to one-another. In addition, the state variables that are involved in the observation equation (2) are indicated in red so that the user knows which nodes in the directed graph are associated with sensors in the experimental setup. Finally, the result panel also presents the largest gap-distance between consecutive singular values of the sensitivity matrix. This yields a first indication of a possible rank deficiency of the sensitivity matrix since the singular values *after* the gap can be treated as zero and hence, indicate a possible lack of observability. A better underpinning of the rank deficiency is performed in a second stage of our analysis, i.e. via a symbolic computation. Utilising the preceding SVD results for this example allows one to perform an identifiability analysis within a matter of seconds. In the past, reported computation times required hours to complete for large models with only one (or a few) sensor(s) available (Anguelova et al., 2012).

4. EXAMPLES

4.1 Identifiability analysis and model re-parametrisation

Consider the well-known unidentifiable JAK/ STAT model, (Raue et al., 2014; Quaiser et al., 2011). The constitutive activation of the JAK (Janus kinase)/STAT signalling pathway forms part of both the primary mediastinal B-cell lymphoma (PMBL) and the classical Hodgkin lymphoma (cHL) Raia et al. (2011). Raue *et al.* investigated the identifiability of this benchmark model using three different approaches and concluded that the model is unidentifiable Raue et al. (2014). We treat the unknown initial value of state x_2 as an additional parameter and so in total, the values of 23 unknown parameters need to be inferred, (Raue et al., 2014; Raman, 2016):

[1] https://computing.llnl.gov/projects/sundials

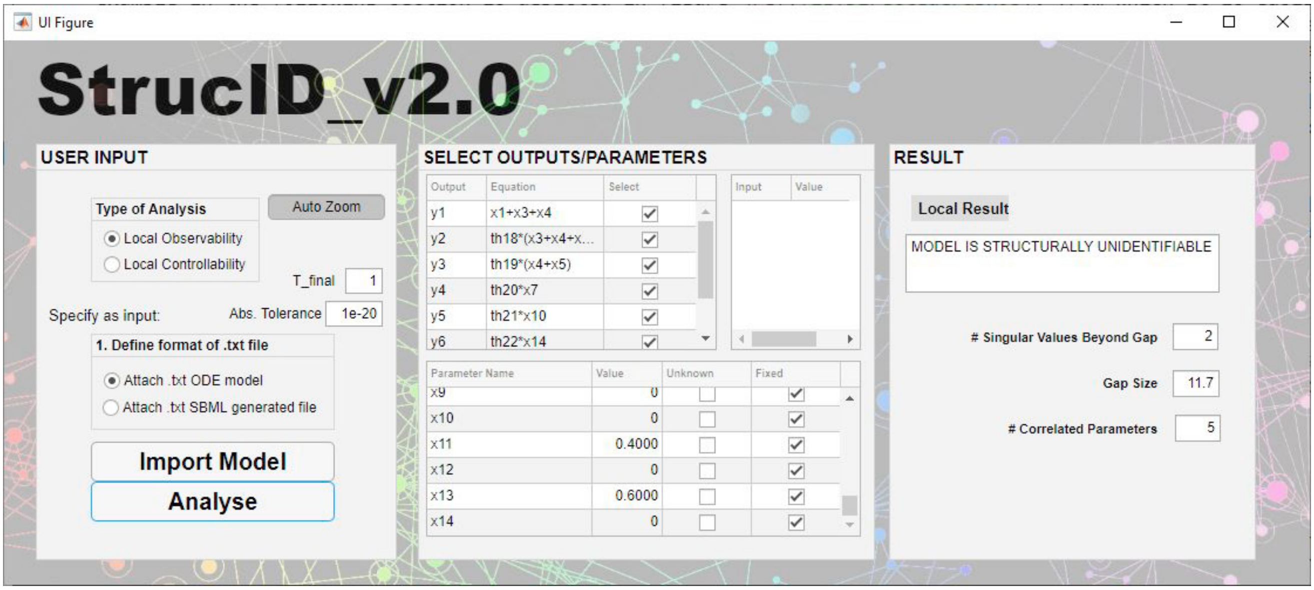

Fig. 1. The main window of the StrucID App version 2.0

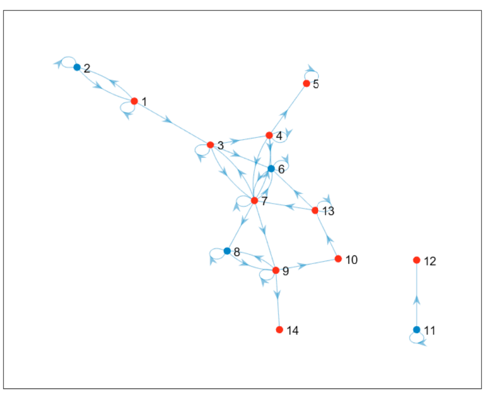

Fig. 2. Directed graph for the JAK/STAT model. The red nodes are associated with an output sensor.

$$\dot{x}_1(t) = -\theta_1\, u_1\, c_1\, x_1(t) - \theta_5\, x_1(t) + \theta_6\, x_2(t) \tag{11}$$
$$\dot{x}_2(t) = \theta_5\, x_1(t) - \theta_6\, x_2(t) \tag{12}$$
$$\dot{x}_3(t) = \theta_1\, u_1\, c_1\, x_1(t) - \theta_2\, x_3(t)\, x_7(t) \tag{13}$$
$$\dot{x}_4(t) = \theta_2\, x_3(t)\, x_7(t) - \theta_3\, x_4(t) \tag{14}$$
$$\dot{x}_5(t) = \theta_3\, x_4(t) - \theta_4\, x_5(t) \tag{15}$$
$$\dot{x}_6(t) = -\frac{\theta_7\, x_3(t)\, x_6(t)}{(1+\theta_{13}\, x_{13}(t))} - \frac{\theta_7\, x_4(t)\, x_6(t)}{(1+\theta_{13}\, x_{13}(t))} + \theta_8\, c_2\, x_7(t) \tag{16}$$
$$\dot{x}_7(t) = \frac{\theta_7\, x_3(t)\, x_6(t)}{(1+\theta_{13}\, x_{13}(t))} + \frac{\theta_7\, x_4(t)\, x_6(t)}{(1+\theta_{13}\, x_{13}(t))} - \theta_8\, c_2\, x_7(t) \tag{17}$$
$$\dot{x}_8(t) = -\theta_9\, x_8(t)\, x_7(t) + c_2\, \theta_{10}\, x_9(t) \tag{18}$$
$$\dot{x}_9(t) = \theta_9\, x_8(t)\, x_7(t) - c_2\, \theta_{10}\, x_9(t) \tag{19}$$
$$\dot{x}_{10}(t) = \theta_{11}\, x_9(t) \tag{20}$$
$$\dot{x}_{11}(t) = -\theta_{12}\, c_1\, u_1\, x_{11}(t) \tag{21}$$
$$\dot{x}_{12}(t) = \theta_{12}\, c_1\, u_1\, x_{11}(t) \tag{22}$$
$$\dot{x}_{13}(t) = \frac{\theta_{14}\, x_{10}(t)}{(\theta_{15} + x_{10}(t))} - \theta_{16}\, x_{13}(t) \tag{23}$$
$$\dot{x}_{14}(t) = \theta_{17}\, x_9(t) \tag{24}$$

The model output contains 5 additional parameters, $\theta_{18}, \ldots, \theta_{22}$:

$$y_1(t) = x_1(t) + x_3(t) + x_4(t) \tag{25}$$
$$y_2(t) = \theta_{18}\,(x_3(t) + x_4(t) + x_5(t) + x_{12}(t)) \tag{26}$$
$$y_3(t) = \theta_{19}\,(x_4(t) + x_5(t)) \tag{27}$$
$$y_4(t) = \theta_{20}\, x_7(t) \tag{28}$$
$$y_5(t) = \theta_{21}\, x_{10}(t) \tag{29}$$
$$y_6(t) = \theta_{22}\, x_{14}(t) \tag{30}$$
$$y_7(t) = x_{13}(t) \tag{31}$$
$$y_8(t) = x_9(t) \tag{32}$$

Importantly, the predefined initial conditions are,

$$\boldsymbol{x}(0) = [1.3, x_2(0), 0, 0, 0, 2.8, 0, 165, 0, 0, 0.34, 0, 0, 0] \tag{33}$$

Stage one numerical results confirm that the model is indeed structurally unidentifiable if the model is analysed for the defined initial conditions in (33) and where $x_2(0) \neq 0$. This results is evident from the large gap between the singular values in figure 3. The 2 singular values beyond this gap suggest that the null-space contains 2 base vectors and so there are 2 sets of totally correlated parameters. The union of the elements in these 2 sets, $\boldsymbol{\theta}^{\text{unid}} = \{\theta_{11}, \theta_{15}, \theta_{17}, \theta_{21}, \theta_{22}\}$, follows from the nonzero elements in figure 4. The symbolically calculated nontrivial null-space, computed using Mathematica, confirms that there are indeed 2 sets of totally correlated parameters. The 2 base vectors spanning this null-space are computed as $\{0, 0, -\theta_{17}/\theta_{22}, 0, 1\}$ and $\{-\theta_{11}/\theta_{21}, -\theta_{15}/\theta_{21}, 0, 1, 0\}$ respectively.

The coefficients within these base vectors now allow us to formally define the algebraic relationship between the linear dependant columns of the Jacobi matrix $\left(\frac{\mathrm{d}\boldsymbol{G}(\boldsymbol{\theta})}{\mathrm{d}\boldsymbol{\theta}^{\text{unid}}}\right)$. These now become the coefficients of 2 individual partial

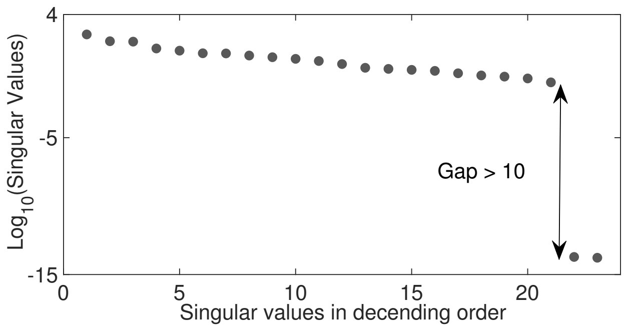

Fig. 3. Identifiability signature of the JAK/STAT model. The 2 singular values beyond the gap suggest that the model is structurally unidentifiable and that there are 2 sets of totally correlated parameters.

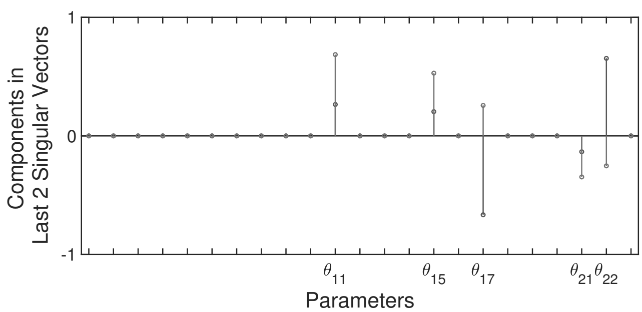

Fig. 4. Identifiability signature of the JAK/STAT model. Entries in the last 2 columns of the right singular matrix, related to the 2 singular values beyond the gap. The non-zero elements indicate the union between 2 potential sets of totally correlated parameters, $\{\theta_{11}, \theta_{15}, \theta_{17}, \theta_{21}, \theta_{22}\}$.

differential equations, described for the functions $\phi_1 = \phi_1(\theta_{17}, \theta_{22})$ and $\phi_2 = \phi_2(\theta_{11}, \theta_{15}, \theta_{21})$:

$$-\frac{\theta_{17}}{\theta_{22}}\frac{\partial \phi_1}{\partial \theta_{17}} + \frac{\partial \phi_1}{\partial \theta_{22}} = 0, \quad (34)$$

$$-\frac{\theta_{11}}{\theta_{21}}\frac{\partial \phi_2}{\partial \theta_{11}} - \frac{\theta_{15}}{\theta_{21}}\frac{\partial \phi_2}{\partial \theta_{15}} + \frac{\partial \phi_2}{\partial \theta_{21}} = 0. \quad (35)$$

One possible solution to (34) is $\phi_{1,1} = \theta_{17}\theta_{22}$, while $\phi_{2,1} = \frac{\theta_{15}}{\theta_{11}}$ and $\phi_{2,2} = \theta_{11}\theta_{21}$ are solutions to (35).

We are now in a position to define the 3 new systems parameters $\boldsymbol{\theta}^{\text{id}} = \{\phi_{1,1}, \phi_{2,1}, \phi_{2,2}\}$, that will replace the 5 unidentifiable parameters, $\boldsymbol{\theta}^{\text{unid}} = \{\theta_{11}, \theta_{15}, \theta_{17}, \theta_{21}, \theta_{22}\}$ (an overall reduction of 2 parameters since there are 2 sets of totally correlated parameters).

This example is unique in the sense that the required state transformations are not related to unidentifiable initial conditions. Instead, they are required since the measured output vector contains additional unidentifiable parameters. The interested reader is referred to the work of Chappell and Evans for additional background on the topic of state transformations, (Chappell and Gunn, 1998; Evans and Chappell, 2000). We further refer to Joubert et al. (2020) for a detailed description of the re-parametrisation process.

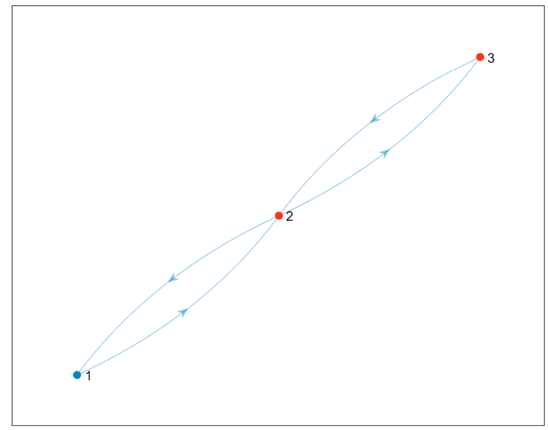

Fig. 5. Directed graph of 3-state model. The two red nodes are directly accessible with a control signal.

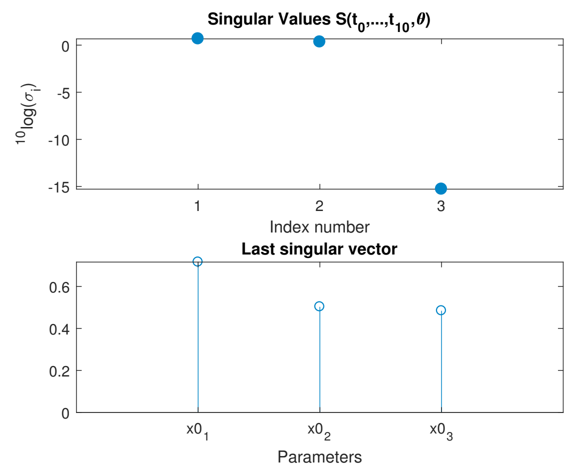

Fig. 6. Controllability signature of 3-state model defined in example 4.2.

4.2 Controllability – example of a confined movement on a sphere

Consider the following control system, whose directed graph is depicted in figure 5.

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix} = \begin{pmatrix} x_2(t) \\ -x_1(t) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ x_3(t) \\ -x_2(t) \end{pmatrix} u(t) \quad (36)$$

If we calculate the product $x^T(t)\,\frac{\mathrm{d}}{\mathrm{d}t}x(t)$, we quickly see that it equals zero for any input $u(t)$ and, therefore, the integral $\frac{1}{2}\,x^T\,x$ is a constant of motion. In other words, once started at a fixed point x_0 in state space, the dynamics of this system continue to evolve on the sphere $x_0^T\,x_0 = R^2$, with R the radius of the sphere. This lack of controllability is confirmed by the controllability signature as presented in figure 6.

5. CONCLUDING REMARKS

We have introduced the user-friendly StrucID v2.0 App which allows for the structural analysis of dynamic system models. The analysis includes both observability and controllability features. It is of particular value when analysing large models, offering quick results and the ability to explore numerous experimental setups within seconds. Combined with symbolic algebra software for further analysis and verification of the numerical (SVD) results, one is able to analyse large systems within computationally trackable times. This makes StrucID v2.0 an attractive tool. We foresee many applications where StrucID has potential, e.g. in aerospace engineering where observability is a major issue for navigation of space vehicles (Woodbury et al., 2018), or in microbiology where large dynamic models need to be calibrated, (Kim et al., 2018). The StrucID v2.0 App runs in Matlab 2019b and is available upon request from the authors.

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