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Securing Land Rights under Rapid Population Growth: The Feasibility of Institutional Land Rights Protection in Africa

by

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The paper examines the claim that a virtuous cycle of more secure land rights, more land-saving investments, and denser populations requires the development of institutions that regulate competition over land. We construct a contest model that links the tenure-security–investment relationship to the efforts of land users to enhance land rights themselves and the role of institutional protection. We study the effect of population growth on a close-to-subsistence economy, including the possibility that it weakens institutional protection. We derive sufficient conditions for a positive effect on land investment, but also show that population growth can push the economy into a low-productivity trap.

Keywords: land-rights protection, land investment, institutions, population growth, contest model

JEL classification code: D74, P14, O13, O17, Q15

1 Introduction

Since the 1990s, rapid population growth and increasing agricultural commercialization, including the current boom in biofuel development, have created mounting competition for fertile land in many rural areas of sub-Saharan Africa.¹ As a result,

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¹ There is a large literature showing that the common image of a land-abundant agriculture no longer holds for most of Africa (see, e.g., Atwood, 1990; Migot-Adholla et al., 1991; Platteau, 1996, 2000; Deininger, 2003; Deininger and Castagnini, 2006; Peters, 2004, 2007; Austin, 2008; Toulmin, 2008; Headey and Jayne, 2014). Although, on average, land is more abundant than in other continents, farm households seem to cluster in areas with high population density and therefore have small plots that tend to shrink over

indigenous land tenure systems that in earlier decades assured farmers' land rights have come increasingly under pressure. According to a widely held theory,² these pressures are not alarming, however, because they are just the forebode of a spontaneous process of individualization of land rights. It is claimed that customary land-holding systems are flexible and adaptive enough to allow this endogenous evolution; only in the final stage is a public intervention needed to consolidate and legally enforce the rights that have freely emerged in the field. This process of increasingly secure land rights for individual farmers is expected subsequently to induce, through investments and land transfers, the higher productivity of land that is required for a growing population.

More recently a number of authors have seriously questioned this optimistic view for sub-Saharan Africa (see, e.g., Platteau, 1996, 2000; Deininger, 2003; Peters, 2004; Deininger and Castagnini, 2006; Bromley, 2008). A virtuous cycle of increasingly precise land rights, more productivity-enhancing investments and land exchange, and denser populations necessitates an appropriate pace and scale of institutional innovations. Africa has always been at a disadvantage in this respect compared with Asia (Platteau, Hayami, and Dasgupta, 1998; Platteau, 2000). In Asia, land-holding systems historically have been formed and adapted under conditions of land scarcity, which largely explains why adjustments toward the adoption of modern land-saving technologies have been relatively modest and without much friction. In contrast, in sub-Saharan Africa as well as in other African countries with a recent history of relative land abundance, agricultural modernization has been hampered by a major lag in evolving land institutions.³ This delay in providing adequate tenure security can lead to "a downward spiral of conflict and strife over a rapidly shrinking overall pie" (Deininger and Castagnini, 2006, p. 322).

The literature (discussed in section 2) suggests two broad, interacting factors that explain the lack of responsive land institutions in these African countries: the traditionally egalitarian culture and the weak capacity of the state. In response to growing land scarcity, the historical egalitarian culture in this region has a tendency to distribute the communal land equally, but among fewer people. This has resulted in many local land conflicts, hurting particularly groups of previous users with the least status and power – typically women, migrants, and pastoralists. The struggles

time (Chamberlin, Jayne, and Headey, 2014; Headey and Jayne, 2014; Jayne, Chamberlin, and Headey, 2014).

² This theory is related to the work of, e.g., Boserup (1965, 1981), Cohen (1980), Noronha (1985), and Bruce (1988). It has been endorsed by the World Bank since a report on Africa (The World Bank, 1989, p. 104) and by other major aid agencies, thereby abandoning their earlier conventional view that customary land tenure in Africa impedes agricultural development (Peters, 2004, p. 270). Platteau (1996, 2000) has provided a detailed criticism of this theory, which he called "the evolutionary theory of land rights" (see also Sjaastad and Bromley, 1997).

³ In highly populated parts of Asia, land intensification featured the increased use of fertilizer, mechanization, and irrigation, whereas in Africa smallholder farmers mainly responded to population pressures by more continuous cropping of existing fields, posing problems of sustainability (see Headey and Jayne, 2014).

over land, and the consequences for these groups, often have been aggravated by the inadequacy of formal land institutions devised by the state. While most African governments have started land reform processes, only a little land is yet formally registered and titled. Moreover, these land titles are frequently not seen as legitimate and authoritative by either ordinary people or the elite. This has created a situation where legally backed land claims are overlapping and competing with socially legitimate land claims that are based on customary principles. Many agree that the resulting confusion and insecurity have strongly increased the number and duration of land conflicts and that, through political manipulation, some of these conflicts have escalated in widespread violence.

The objective of this paper is to conceptualize and explore: (1) the interactions between the efforts of competing social groups in trying to secure land rights, their investments in land-saving technologies, and the involvement of traditional and modern land institutions; and (2) how these interactions are affected by population growth. A major finding is that population growth may cause a situation of low tenure security, low land investment, and weak institutions from which it is difficult to escape without access to external resources, because the revenue that can be reaped through land taxation to fund a gradual enforcement of land institutions is essentially limited.

We begin by constructing a simple contest game that examines the dissipation of resources by two groups of land users who attempt to establish property rights to land under conditions of land scarcity. Institutions act as a third party that can reduce conflict and improve tenure security by deterring groups from taking offensive steps. We determine to what extent land tenure security and group payoffs depend on the degree of institutional protection. One conclusion from the basic model is that institutions do not always matter (Propositions 1 and 2). For instance, if the productivity of retained land is sufficiently higher than that of captured land, groups avoid conflict by engaging in defensive activities to the extent that any institutional help is superfluous. However, if the relative productivity of retained land is not that high, then private defensive measures are inadequate, and tenure security is an increasing function of the amount of institutional protection per unit of land.

Next, the model is extended with land investments by letting groups also decide on the level of land productivity. Our purpose is to examine how productivity-enhancing and security-enhancing efforts by farmers respond to lower land tenure security. The answer depends on whether lower tenure security stems from weaker institutions or from more aggression by other land users – a distinction that seems to be overlooked in the literature. Less institutional protection reduces land investment and raises defensive measures by farmers (as in Deininger and Jin, 2006; Fenske, 2011). Thus institutional and self-organized protection are substitutes here. More outward aggression also discourages land investment, but now any defensive measures by farmers will also decrease. This is because the lower productivity resulting from fewer land investments makes it less worthwhile for farmers to arrange more protection of their land or even maintain the same level. It implies

that, in principle, more outward aggression has a stronger negative effect on land investment than weaker institutional protection.

The extended model is subsequently used to investigate the effect of population growth. We start by addressing the above debate on the adaptability of traditional landholding systems to rising population pressure by deriving sufficient conditions for population growth to exert a positive effect on land investment, qualifying also the requirement of having institutional protection of land rights in place (Theorem 1). Then we take the debate one step further by arguing that population growth also tends to undermine institutional protection, whether customarily or legally arranged. We examine to what extent the ability of traditional and state-controlled systems to project power over the land is constrained by the revenues collected through taxes or other contributions that can be extracted from land users. The two may even be interdependent, as the collected revenues for running a land governance program are likely to increase with the degree of tenure security that is accomplished through the program. We provide a tentative analysis of how the budgetary room for improvements in institutional protection of land rights could be affected by population growth. One finding is that, due to the interdependence, population growth may push the economy into a state of low tenure security and land investment where the possibility of funding gradual improvements in land governance through land taxation is limited (Theorem 2).

The paper makes several contributions to the literature. One is a formal, integrated analysis of some key relationships that are put forward in the large, mainly qualitative and empirical literature on the importance of institutional protection under increasing land scarcity (see references above). The paper also connects to a more theoretical literature concerning the relationship between property rights, economic development, and the power of the state (for a survey, see Besley and Ghatak, 2010). For instance, Besley and Persson (2009, 2011) provide a political-economy model where the government chooses the level of institutional enforcement (as well as the tax rate), subject to certain upper bounds that can be loosened in the future by investing in legal and fiscal capacity. Konrad and Skaperdas (2012), McBride, Milante, and Skaperdas (2011), and Garfinkel, McBride, and Skaperdas (2012) study contest games where the contending groups themselves can invest in institutional enforcement as a collective good. Our focus on the budgetary room for improvements in institutional protection treats fiscal and legal capacity as given, yet emphasizes the interdependence between the difficulty of collecting taxes and the inability to define and enforce land rights. Moreover, how effective a given protection of land rights is depends negatively on the size of the population. Hotte (2001) studies a different aspect by assuming that the effectiveness of property rights decreases with distance from the government's administrative centre.

The paper also contributes to research on poverty traps (for a survey, see Barrett, Garg, and McBride, 2016), particularly where poverty becomes self-reinforcing due to investment insecurity arising from appropriation. For instance, Gonzalez (2006) studies a contest game where imperfect appropriation of returns may discourage or prevent investment even when its cost is negligible. Our results suggest

that this may not hold if efforts to strengthen self-protection do not rival with investment, allowing an increase of both. Mehlum, Moene, and Torvik (2003) briefly examine the endogenous relationship between appropriation and budget-constrained law enforcement. They find that improving public protection may require such a high tax that the profitability of productive activities, which have to carry the burden of the tax, will fall below the profitability of predatory activities, thus tying the economy to a low stage of development (see also Dabla-Norris and Freeman, 2004). Similarly, Lloyd-Ellis and Marceau (2003) show that, in the presence of endogenous credit constraints that particularly hamper low-wealth borrowers, reducing the expected payoff to predatory activities through a public policy of harsher punishments may also backfire if the required tax hike is too high. The obstructing mechanism we propose is that a higher tax to fund more institutional protection raises tax revenue per hectare, but lowers the number of taxable hectares as it affects tenure security due to increasing appropriation.

The remainder of the paper is as follows. Section 2 sketches the background of the analysis by discussing the lack of responsive land institutions in many sub-Saharan countries. Section 3 specifies the basic model, a two-stage contest game where land tenure security depends on group effort and an exogenous level of institutional protection. Section 4 extends the model to a three-stage game in which groups undertake investments to increase land productivity. It looks at the interaction between productivity-enhancing and security-enhancing efforts and how this is influenced by institutional protection and outward aggression. Section 5 employs the extended model to discuss the ability of traditional and state-controlled tenure systems to project power over the land under rising population pressure. Section 6 concludes. An appendix contains proofs.

2 Background

At least two broad, interacting factors can be identified that explain the slow adaptation of land institutions in sub-Saharan Africa and in other African countries with a recent history of land abundance: the traditionally egalitarian culture and the weak capacity of the state. As there is an extensive literature on this topic, we only highlight some features that help to put our analysis into perspective.

Land-abundant areas are historically inhabited by egalitarian societies. Communal networks of reciprocal relationships function as a social security system that enables households to recover from setbacks and also guarantee cooperation in the production and maintenance of public goods. The drawbacks of egalitarian norms show up when the underlying stationary conditions change, especially when these changes alter the relative positions of people (Boehm, 1993; Narayan et al., 2000, ch. 4; Platteau, 2000, chapters 3–5; Hoff and Sen, 2006). When population growth turned land into a scarce resource in many regions in sub-Saharan Africa, a first response by social groups was to restrict the access of outsiders to their local territories. For example, herders who historically relied on secondary rights of use

of water and grazing resources found their passage increasingly blocked with crop fields and enclosures. Continued population pressure on the land also tended to affect the cohesion of social groups themselves. Many experts agree that the numerous signs of cultural insecurity and social conflict in sub-Saharan Africa in the recent past are indeed closely connected to the rapid transformation of communal lands (see, e.g., Platteau, 2000; Deininger, 2003; Peters, 2004, 2007; Deininger and Castagnini, 2006; Toulmin, 2008; Jayne, Chamberlin, and Headey, 2014). The privatization of communal lands and the entry into market relations and a money economy undermines traditional social safety nets and causes an erosion of egalitarian norms and values (for a formal analysis, see Haagsma and v. Mouche, 2013). It leads to stricter definitions of those who have legitimate land claims and those who have not. Thus local land conflicts turned up everywhere, hurting particularly those with the least social and political status. In Rwanda, just before the outbreak of civil war in 1994, customary practices with respect to land transfers to family and relatives increasingly refused categories of people that had traditionally received social protection, such as return migrants, divorced women, and orphans. The extreme land scarcity in this country, in combination with inequalities in off-farm employment opportunities, created the tensed atmosphere that contributed to the outbreak of the civil war (André and Platteau, 1998).⁴

The unequal impact of eroding traditional authority systems has often been aggravated by the weak capacity of the state. Herbst (2000, chapters 1 and 4) mentions two broad reasons why land-abundant areas typically provide a major obstacle to state formation. First, low population densities make it costly for the state to exert control over people, especially under the large variety of ecological and geographical conditions that characterize African countries (see also Austin, 2004). Second, because an abundance of open land tends to discourage territorial wars, the necessity to organize an efficient tax administration to cover military expenditures is essentially missing.⁵ Hence, the recent history of land abundance in sub-Saharan Africa gives a clue to why governments in this region are badly equipped to accommodate the rising pressures on the land and to provide tenure security. Not only the ability but also the willingness of central authorities to invest in rural areas has traditionally been lacking, also because African politicians tend to equate their political survival with appeasement of the urban populations (Bates, 1981). As a result, taxation of land or income has always been low, also in colonial times, and today revenue collection still mainly depends on trade taxes and non-tax revenues such as aid and earnings from natural resources (Austin, 2004; Di John, 2009; OECD, 2010). The fiscal weakness has been aggravated by decades of economic stagnation following the oil crises of the 1970s. It halted the gradual progress in the control over the territories. Many basic agents of the state left the rural areas,

⁴ Additional pressure on the land comes from external forces, such as the recent demand for biofuel, the establishment of wildlife conservation areas, and urbanization (Peters, 2004; Toulmin, 2008).

⁵ A large political-science literature highlights the contribution of war to state building; see, e.g., Herbst (2000, ch. 4) for an overview.

including agricultural extension workers, tax collectors, and census takers (Herbst, 2000, ch. 1). While for some years now most African governments have started land reform programmes, the bulk of land is still not formally registered.⁶ Since demarcating and titling land (including legal machinery to enforce contracts) is a prerequisite for assessing property taxes, it is suggested that states in the sub-Saharan region, to some extent, are caught in a trap: increasing control over rural land requires broadening of the tax base, and vice versa (see section 5.2).

The inadequacy of formal land institutions in much of Africa is demonstrated by not only the low amount of registered land but also its weak social legitimacy (see, e.g., Atwood, 1990; Sjaastad and Bromley, 1997; Platteau, 2000, ch. 4; Deininger, 2003; Toulmin, 2008; Deininger, Hilhorst, and Songwe, 2014). In some cases, land rights are simply ineffective. For example, even when their land is legally backed and formally registered, landholders who intend to sell or rent out land often have to consult their family or community for approval. In other cases, land records are unreliable. Because the records are not timely updated, the registration of land increasingly falls behind the actual distribution of land ownership. All this has produced a situation where traditional and modern authority systems, instead of complementing each other, overlap and compete. It has brought confusion and insecurity regarding whose rights count and will be supported in case of a contest. To secure their land claims, people spend resources to mark their property, such as planting trees and digging irrigation furrows. In case of conflict, they act opportunistically by seeking a favourable judgement through a variety of channels (so-called “institutional shopping”), such as community councils, customary chiefs, local governments, and land agencies (Toulmin, 2008). Not surprisingly, the institutional uncertainty has strongly increased the number and duration of land conflicts, often at the expense of vulnerable categories of local populations (Platteau, 2000, ch. 4; Deininger and Castagnini, 2006). Moreover, it has prevented the high levels of investment in land productivity needed to release the pressure on land and feed a growing population (for an overview, see Place, 2009, and Fenske, 2011; see also section 4).

The interplay of some of these factors is examined in the next sections. We study the impact of population growth on land investment and tenure security for the case where poorly protected land users establish land rights themselves through appropriative activities. Population growth not only fuels conflicts over land but is also considered to undermine institutional protection. We show that this may induce a state of low tenure security, low land investment, and weak institutions from which it is difficult to escape without access to external resources, because the revenue that can be collected through land taxation to fund a gradual enforcement of land institutions is essentially limited.

⁶ In many African countries, formal registration of land covers much less than 10% of the area. For example, in the West African region only 2–3% is held by written title, and in Burundi less than 1% (see, e.g., Deininger and Castagnini, 2006; Toulmin, 2008).

3 A Simple Contest Model with Institutional Protection of Land Rights

3.1 Basic Model

Malthusian pressure of population on limited land typically assumes a production technology in agriculture where land and labour are imperfect substitutes. Below, production is modelled in the most extreme way possible, by assuming that land and labour are combined in fixed proportions:

$$q^i = \min(\alpha_i a^i, \beta l^i),$$

where q^i is agricultural output (food or income), a^i is land area in hectares, l^i is labour time (or effort), and α_i and β are positive parameters indicating land and labour productivity. A fixed-coefficient technology is rather realistic in the case of smallholder farmers, once they have chosen which crop to grow. Superscript i refers to an endogenous variable of a group of agents of size N_i ; subscripts indicate group parameters. Normalizing an individual's time to unity, N_i also indicates the total amount of available time of group i (so $l^i \leq N_i$). Another simplifying assumption is that we ignore within-group distribution and coordination issues and treat groups as unitary actors.

It follows that if land (of uniform quality and location) is abundant, all the available time will be devoted to production, granting that the marginal valuation of the output from an extra hour of labour exceeds the disutility from working. With A_i denoting the available amount of land of group i , the assumption of abundant land can then be formally stated as $A_i \geq \beta N_i / \alpha_i$. If there are two groups in this region, the (notional) demand for land of each group can be satisfied if and only if the total quantity of land (A) satisfies $A \geq \beta N_1 / \alpha_1 + \beta N_2 / \alpha_2$. Thus, in this simple setting, land may become a scarce resource and under pressure of competing claims when population ($N_1 + N_2$) grows, total supply of land falls (e.g., through the establishment of conservation areas), or labour productivities increase (e.g., through mechanization). The restriction on land becomes less binding, however, when land productivities increase (e.g., through the application of fertilizers, more effective cultivation techniques, and other Green Revolution practices).

From now on, we consider an initial phase where land has become the scarce factor ($A < \beta N_1 / \alpha_1 + \beta N_2 / \alpha_2$). In section 1 it was argued that, as competition over land emerges, only a timely development of institutional innovations that establish and enforce precise land rights can prevent an escalation into violent and costly conflict. We construct a simple contest game that is concerned with discovering the relation between the dissipation of resources by groups in trying to establish property rights to land, the involvement of (formal or informal) institutions, and the degree of land tenure security. Although the analysis is in terms of groups and proceeds at a certain aggregate level, sometimes we can also think of local, small-scale settings where, for example, two households dispute the boundary of their neighbouring plots of land. Also, a group may have no land at all, so the

analysis can also address the case of returning migrants who find their previous land occupied.

Therefore, suppose two groups 1 and 2 that are initially endowed with A_1 and A_2 hectares of land. The reallocation of land between the groups depends on how much time each group devotes to appropriative activities, distinguished between protecting its own land and threatening the land of the other, and the extent to which institutions enforce the initial claims to land. The fraction of the initial endowment of land (A_i) that is retained by group i is denoted by p^i and given by

$$(1) \quad p^i := \frac{d^i + s_i}{d^i + s_i + f^j}$$

($i \neq j$). We see p^i as a measure of the security of the land rights of group i . It depends positively on the sum of defensive activities of the group (d^i) and the institutional protection of its land rights (s_i), and negatively on offensive activities (*fighting*) of the other group (f^j). Self-organized and institutional protection are simply considered as perfect substitutes, because we are less interested in to what extent one can be replaced by the other. The underlying assumption here is that authorities agree with the initial claims to land. Further observe that $1 - p^j$ is the fraction of the land of the other group (A_j) that is taken away by group i .

The distinction between defensive and offensive activities in (1) was introduced by Grossman and Kim (1995). It allows the study of a nonaggressive equilibrium with positive defence levels, or an armed peace – a realistic possibility that earlier work on contests cannot address (see, e.g., Tullock, 1980; Skaperdas, 1992; Hirshleifer, 1991, 1995). Examples of defensive activities in our context are building fences and keeping watch over the land and also efforts with the primary purpose of signalling claims to land such as planting trees and cultivating the land instead of including a fallow period (see section 4). Examples of offensive activities range from mild protests at a community meeting or local land agency to brutally chasing people from their land. Sometimes the distinction is not so easily drawn, however, as in the case where attack is considered to be the best form of defence.

The institutional parameter (s_i) reflects the degree of compliance enforcement exerted by the state (at various levels) or the community (or network of communities) to protect land rights. In the case of the state, s_i refers to the efforts of the legal machinery to enforce land contracts and to settle disputes with the help of police, courts, educated judges, and so on. In indigenous tenure systems such third-party enforcement is typically missing. In this case, s_i refers to the efforts of community members themselves to regulate land access through consensus and negotiation, occasionally mediated by local rulers or a council of elders. It may be that the land claims of the two groups are differently supported by the prevailing institutional system. Examples of $s_i \neq s_j$ are suggested by the tendency of indigenous tenure systems to restrict land access along ethnic or gender lines and the inclination of some formal, state-controlled tenure systems to promote land access for groups with close ties to the government. We assume that the level of institutional protection cannot fall to zero, acknowledging that there is always some minimum of

social cooperation through norm compliance in informal systems. Also, if state-governed tenure systems break down, protection will probably fall back to this informal default mode. Section 5 investigates the feasibility of institutional protection and how this may be affected by population growth.

The analysis allows for the possibility that a reallocation of land entails a loss of agricultural production (also here following Grossman and Kim, 1995). Land conflicts in Africa frequently result in a reduction of the total output and income of farmers and herdsman (examples are the destruction of crops, killing of cattle, or damaging of fields (Deininger, 2003; Ofuoku and Isife, 2009). This implies that, at least for a certain period of time, the gain of one group may be less than what the other loses. Therefore, denoting the productivity of retained land by α and normalizing the productivity of captured land to one, it is assumed that the latter is lower: $\alpha_i \geq 1$. So group i loses the capacity for α_i products for every hectare it has to give up, while group j can grow only 1 product for every hectare gained. Section 4 allows groups to decide on the level of land productivity, in this way endogenizing the size of the output losses due to predation.

Given land scarcity and the fixed-coefficient technology, the agricultural output of group i follows as

$$(2) \quad q^i = \alpha_i p^i A_i + (1 - p^j) A_j$$

($i \neq j$). The objective of the group is to maximize this output, taking into account the disutilities or costs related to working and appropriative activities. That is, group i maximizes utility

$$(3) \quad u^i = q^i - \gamma_i l^i - \delta_i d^i - \varepsilon_i f^i,$$

where γ_i , δ_i , and ε_i are positive parameters. Note that the efficient number of working hours is given by $l^i = q^i / \beta$. To make working worthwhile, it is assumed that $\beta > \gamma_i$ (the marginal utility of the output from an extra hour of labour always exceeds the marginal disutility from working, γ_i). The parameters δ_i and ε_i refer to the disutilities or material costs of defending and fighting. A simple interpretation of these cost parameters is the time spent on costly litigation in connection with land disputes, but, as the earlier examples indicated, defensive and offensive activities typically go much further than this. In maximizing its utility, each group faces a simple trade-off between reducing these disutilities or costs and maintaining and expanding its land, and thus agricultural output. Notably, since we want to address situations where land rather than labour has become the scarce factor, it is natural to assume that appropriative activities do not restrict the time available for working on the land. This implies that defending and fighting have no opportunity cost in terms of forgone production.

We are now able to formulate a contest game for the two groups, throughout denoted by $i, j \in \{1, 2\}$ with $i \neq j$. Following Grossman and Kim (1995), we envisage a two-stage game with complete information in which the groups, independently and simultaneously, make their choices in two periods. In the first period, groups

choose their time spent on defence, d^i . In the second period, each group having observed the defence of the other, chooses its time spent on fighting, f^i . Using (1)–(3) and $l^i = q^i/\beta$, each group has payoff

$$(4) \quad v^i((d^i, f^i); (d^j, f^j)) := \left(1 - \frac{\gamma_i}{\beta}\right) \left(\alpha_i \frac{d^i + s_i}{d^i + s_i + f^j} A_i + \frac{f^i}{d^j + s_j + f^i} A_j\right) - \delta_i d^i - \varepsilon_i f^i,$$

where $\alpha_i \geq 1$, $\beta > \gamma_i > 0$, $\delta_i, \varepsilon_i > 0$, $s_i, s_j > 0$, and $A_i, A_j \geq 0$ (but $A_i + A_j = A > 0$). With N_i the total amount of time for group i , we assume defence $d^i \in [0, N_i]$ and fighting $f^i \in [0, N_i - d^i]$.

Solving this game through backward induction yields the set of subgame-perfect Nash equilibria.⁷ Our focus is on equilibria $((d_*^1, f_*^1), (d_*^2, f_*^2))$ where time budgets $[0, N_i]$ and $[0, N_i - d^i]$ are not binding – below we make some additional assumptions for this. Further, if q_*^i is the implied equilibrium production and so $l_*^i = q_*^i/\beta$ is the equilibrium amount of labour, the basic assumption of land as scarce production factor is simply satisfied by assuming N_i so large that $l_*^i + d_*^i + f_*^i \leq N_i$ ($i = 1, 2$).⁸

Observe that the game differs conceptually from Grossman and Kim (1995) in incorporating the role of institutional protection. Moreover, in section 4 we endogenize the choice of land productivity (α_i) through a third stage. In the subsequent analysis we focus on the question: how do a group's tenure security and payoff depend on institutional protection?

3.2 Analysis

Let $\lambda_i := 1 - \gamma_i/\beta$ refer to the marginal utility of output corrected for the disutility of the required labour input. Two further parameters will play a key role and also reduce notational clutter:

$$(5) \quad r_j^f := \frac{\lambda_j}{\varepsilon_j} \quad \text{and} \quad r_i^d := \frac{\lambda_i \alpha_i}{\delta_i}.$$

They indicate respectively the *benefit–cost ratio of fighting for a hectare* – the aggressor's marginal valuation of captured land relative to his marginal cost of fighting – and the *benefit–cost ratio of defending a hectare* – the defender's marginal valuation of retained land relative to his marginal cost of defending.⁹

⁷ In general, applying the notion of subgame-perfect Nash equilibrium is somewhat problematic in two-stage games with simultaneous moves (see, e.g., Corchón, 1996). In the current game this is not the case, however, because for each (d^1, d^2) a game in strategic form results where players have a strictly dominant strategy.

⁸ For example, in the case of an equilibrium with $d_*^i = 0 = f_*^i$ ($i = 1, 2$), it holds that $p^i = 1$, and so $q_*^i = \alpha_i A_i$ and $l_*^i = \alpha_i A_i/\beta$. Then N_i must be so large that $\alpha_i A_i/\beta \leq N_i$ ($i = 1, 2$) (these restrictions imply $A < \beta N_1/\alpha_1 + \beta N_2/\alpha_2$, but not the other way around).

⁹ Note that the two parameters ignore the positive effects of defending on p^i and fighting on $1 - p^j$.

To make sure that the time budgets for defence and fighting are not binding, we make two additional assumptions. Let

$$(6) \quad \hat{d}_i := r_j^f A_i - s_i,$$

and define a function $y^j : [0, N_i] \rightarrow \mathbb{R}$ by

$$(7) \quad y^j(d^i) := \sqrt{d^i + s_i} \left(\sqrt{\hat{d}_i + s_i} - \sqrt{d^i + s_i} \right).$$

We assume

$$(8) \quad \hat{d}_i < N_i \quad \text{and} \quad y^j < N_j - \hat{d}_j.$$

As will be seen in a moment, positive levels of private defence and fighting will never exceed \hat{d}_i and y^j , respectively.

We begin by determining the second-stage choice of the time allocated to fighting. Group j takes defensive effort d^i and d^j as given and maximizes payoff v^j in (4) with respect to f^j . Now observe that a group's tenure security is independent of that of the other group: p^i and p^j have no determinants in common. Also, each payoff function is additively separable in p^i and p^j . This implies that group j 's optimal fighting effort does not depend on group i 's fighting effort (it has a strictly dominant strategy) and *only* responds to group i 's defensive effort. The best-response function is given by¹⁰

$$(9) \quad f^j = \underline{f}^j(d^i) := \begin{cases} y^j(d^i) > 0 & \text{if } d^i < \hat{d}_i, \\ 0 & \text{if } d^i \geq \hat{d}_i. \end{cases}$$

It shows that group j is deterred from allocating time to fighting (a corner solution) if group i 's defensive effort exceeds a certain minimum level of private protection, indicated by \hat{d}_i . If this minimum required level for nonviolence is positive and actual protection falls short of it ($0 \leq d^i < \hat{d}_i$), group j engages in fighting (an interior solution). Note that the minimum required level \hat{d}_i is nonpositive if and only if the institutional protection per hectare received by group i exceeds group j 's benefit–cost ratio of fighting: $s_i/A_i \geq r_j^f$.

The first-stage choice of the time allocated to defence takes account of the Nash equilibrium response of fighting to private defence. Group i chooses d_i to maximize payoff v^i in (4) incorporating the effect d^i has on f^j , as implied by $\underline{f}^j(d^i)$ in (9). As before, group i 's optimal defensive effort does not depend on group j 's defence (it has a strictly dominant strategy). We find

$$(10) \quad d^i = \begin{cases} 0 & \text{if } \left(\frac{(r_i^d/2)^2}{r_j^f} \right) A_i - s_i \leq 0 \text{ or } \hat{d}_i \leq 0, \\ \left(\frac{(r_i^d/2)^2}{r_j^f} \right) A_i - s_i < \hat{d}_i & \text{if } 0 < \left(\frac{(r_i^d/2)^2}{r_j^f} \right) A_i - s_i < \hat{d}_i, \\ \hat{d}_i & \text{if } 0 < \hat{d}_i \leq \left(\frac{(r_i^d/2)^2}{r_j^f} \right) A_i - s_i. \end{cases}$$

¹⁰ The results (9) and (10) are derived in the proof of Proposition A1 in the appendix.

Hence, if the minimum required protection $\hat{d}_i \leq 0$, no defensive effort is undertaken. Institutional protection is high enough to prevent any violence by the other group – no additional defence is needed. Now suppose institutional protection is deficient, so that the minimum required protection $\hat{d}_i > 0$. According to the first and second lines of (10), the preferred defence level falls short of this minimum requirement if the group's benefit–cost ratio of defending (r_i^d) is sufficiently low, viz., less than twice the other group's benefit–cost ratio of fighting (r_j^f). In this case, land is lost to the other group through fighting. According to the third line of (10), the preferred defence level is sufficient to deter any aggression if the group's benefit–cost ratio of defending (r_i^d) is sufficiently high.

Combining the results (9) and (10), we immediately find the unique solution of the game, denoted as $((d_*^1, f_*^1), (d_*^2, f_*^2))$ (see Proposition A1 in the appendix). It also allows us to summarize the conditions under which a group will refrain from fighting and claiming land of the other group:

PROPOSITION 1 *Aggression by group j is absent, i.e., $f_*^j = 0$, if and only if*

- (a) $A_i = 0$; or
- (b) $A_i > 0$ and $2r_j^f \leq r_i^d$; or
- (c) $A_i > 0$ and $2r_j^f > r_i^d$ and $s_i/A_i \geq r_j^f$.

The first case is obvious. The second case implies that there is no violence if two times the aggressor's benefit–cost ratio of fighting for a hectare is lower than or equal to the defender's benefit–cost ratio of defending a hectare. The threshold against violence becomes higher if the aggressor's cost of fighting (ε_j) increases or the defender's cost of protecting land (δ_i) decreases (see (5)). Importantly, institutional involvement is superfluous here: the private costs of defending and fighting already allow for protective activities that discourage any level of confrontation. Observe that if preferences and appropriation costs were the same ($\lambda_i = \lambda_j$ and $\delta_i = \varepsilon_j$), a high enough relative productivity of land (α_i) would motivate adequate private defence. The third case requires a relatively high benefit–cost ratio of fighting, where the groups themselves cannot avoid confrontation. Only institutional protection can bring about a nonaggressive equilibrium here (a tuple with $f_*^1 = f_*^2 = 0$). Specifically, conflict is prevented if, for both groups, the received protection per hectare (s_i/A_i) is not less than the aggressor's benefit–cost ratio of fighting for a hectare.

Proposition 1 has some straightforward implications for the relative positions of groups. For instance, group i can do without institutional protection if it is strong in defending its land (δ_i is low) or if group j has difficulties in forming an opposing force or feels itself morally forbidden to do so (ε_j is high). If group j is also vulnerable to opposition (δ_j is high), its claim on land can be easily denied by group i , unless it is sufficiently shielded by institutional protection (s_j/A_j is high enough). We can see here an illustration of the weak position of specific social groups in Africa. For instance, pastoralists and women are especially liable to suffer exclusion from the land and appear to be most in need of institutional protection to secure access to land (see section 2).

It is now a small step to derive the relationship between tenure security and institutional protection. The degree of tenure security is found by calculating the fraction of the initial land that is retained by a group in an equilibrium, p_*^i (using (1)). High tenure security is crucial for the willingness to invest in land productivity (α_i), which would reduce the scarcity of land and take the sting out of land conflicts – a topic we turn to in the next section.

PROPOSITION 2 Suppose $A_i > 0$.

- (a) If $2r_j^f \leq r_i^d$, then $p_*^i = 1$.
 (b) If $2r_j^f > r_i^d$, then

$$p_*^i = \begin{cases} \frac{r_i^d}{2r_j^f} & \text{if } \frac{s_i}{A_i} \leq \frac{(r_i^d/2)^2}{r_j^f}, \\ \sqrt{\frac{s_i}{A_i} / r_j^f} & \text{if } \frac{(r_i^d/2)^2}{r_j^f} \leq \frac{s_i}{A_i} \leq r_j^f, \\ 1 & \text{if } \frac{s_i}{A_i} \geq r_j^f. \end{cases}$$

Figure 1 sketches how land tenure security of group i depends on institutional protection per hectare (s_i/A_i) for the case where groups themselves cannot prevent conflict (Proposition 2(b)). It is seen that a certain minimum of protection is required for institutions to become effective in increasing tenure security. Full tenure security arises when institutional protection per hectare exceeds the other group's benefit–cost ratio of fighting for a hectare (r_j^f). The dashed curve illustrates that, at given protection levels, tenure security of group i generally falls when the other group's benefit–cost ratio of fighting (r_j^f) increases.

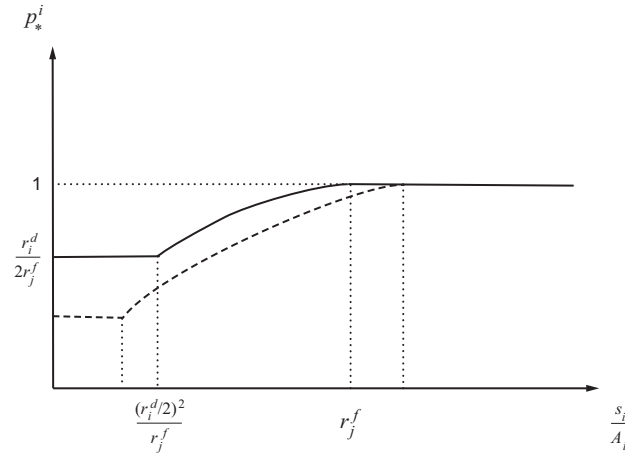
In equilibrium, the payoff of each group (v_*^i) is increasing in the institutional protection of its own land (s_i) and, because capturing land becomes more difficult, decreasing in that of the other group's land (s_j). Hence, it is not immediately clear how a uniform level of institutional protection per hectare covering both groups would influence individual payoffs and the total payoff per hectare. Providing such a uniform level comes closest to a policy of equal land rights for all, particularly supporting (potential) land users with little status and power. For our analysis in section 5 it is sufficient to study here the simple case where there are no group differences in preferences, appropriation costs, or land productivities, and also appropriation costs are the same. So the assumptions are: $\lambda_1 = \lambda_2 =: \lambda$ (so $\gamma_1 = \gamma_2$), $\delta_1 = \delta_2 =: \delta$, $\varepsilon_1 = \varepsilon_2 =: \varepsilon$, $\alpha_1 = \alpha_2 =: \alpha$, and $\delta = \varepsilon$.¹¹

Therefore, suppose $s_1/A_1 = s_2/A_2$ (from now on, $A_1, A_2 > 0$). Then both groups enjoy the same land tenure security, and we can write $p_*^1 = p_*^2 =: p_*$. With a little manipulation (using (4)), the total equilibrium payoff per hectare can be expressed

¹¹ Note that these assumptions do not imply that in equilibrium fighting efforts and defensive efforts are the same, because fighting effort also depends on the size of the other group's land, and defensive effort on the size of own land (see the proof of Proposition A2 in the appendix).

Figure 1

Land Tenure Security as a Function of Institutional Protection per Hectare
(case: $2r_j^f > r_i^d$; the dashed curve arises if r_j^f increases)



in three terms:

$$(11) \quad \frac{v_*^1 + v_*^2}{A} = \lambda\alpha - \lambda(\alpha - 1)(1 - p_*) - \frac{\delta(d_*^1 + d_*^2) + \varepsilon(f_*^1 + f_*^2)}{A}.$$

The first term of the right-hand side refers to the utility of the output per hectare (corrected for the disutility to produce it) that would be attained if institutions prevented any appropriative activity. The second and third terms indicate the potential costs of appropriation. The second term refers to the destructiveness of reallocating land through fighting, which obtains if agricultural production on captured land is lost ($\alpha - 1 > 0$). We have already found that if the benefit–cost ratio of fighting (r_j^f) is sufficiently low, groups themselves can take sufficient defensive measures to secure their land, but otherwise only institutional protection can avoid reallocation losses. The third term refers to the costs per hectare of the resources used for appropriation. These costs are decreasing in institutional protection per hectare (see Proposition A2 in the appendix). In sum, a higher level of institutional protection per hectare unambiguously raises the total payoff (and individual payoffs) per hectare, and it does so by reducing or eliminating reallocation losses and cutting private appropriation costs.

While the above illustrates the benefits of institutional protection, it does not deal with the costs of providing this protection, nor how these could be funded. Section 5.2 takes up this issue by exploring the feasibility of institutional protection in traditional and state-controlled systems in relation to the pressure from population growth. To set the stage, we first endogenize a key variable of the model: land productivity α_i . Note that by doing so the size of the reallocation losses will also be endogenized.

4 *An Extension with Investments in Land Productivity*

Secure land rights have long been considered as crucial to bring about the high levels of investment in land productivity required to feed a growing population. When farmers feel more secure in their right to make long-term use of their land, the expected benefits to be reaped from structural land improvements are higher, and thus the farmers are more willing to invest.¹² Empirical research on African tenure systems is, however, far from conclusive. There is indeed evidence suggesting that the causality may run the other way around: investment may be undertaken to increase security rather than in response to more security.¹³ Although sometimes difficult to disentangle in practice, it is critical to distinguish between productivity-enhancing and security-enhancing investments. For example, Deininger and Jin (2006) reported that in Ethiopia farmers may plant eucalyptus trees with the goal of signalling to others a claim to owning the land, while their less visible building of terraces for growing crops is principally undertaken to improve productivity. They found that insecure tenure encourages tree planting but discourages terracing.

In this section we extend the model to a three-stage game in which groups decide on the level of land productivity. While the main purpose of the extended model is to explore the effect of population growth (section 5), the derived Nash equilibrium provides some direct implications for the interaction between productivity-enhancing and security-enhancing activities and how this is influenced by institutional protection and outward aggression.¹⁴

4.1 *Extended Model*

Suppose that, before choosing their time spent on defence (d^i) and fighting (f^i), groups are able to determine the level of land productivity of their initial land (α^i). Raising land productivity above the baseline level 1 is, however, increasingly

¹² Two more effects of secure land rights are mentioned in the literature: they enable land to be used as collateral for loans, and increase the transferability of land so that it may reach more productive farmers. These effects are less relevant for sub-Saharan Africa, where credit and land markets are thin (e.g., Jacoby and Minten, 2007; Fenske, 2011).

¹³ There is a large, mainly empirical literature on the relationship between tenure security and agricultural investment in Africa, including Atwood (1990), Besley (1995), Sjaastad and Bromley (1997), Brasselle, Gaspard, and Platteau (2002), Jacoby and Minten (2007), Deininger and Jin (2006), Bromley (2008), Lunduka (2011), and Melesse and Bulte (2015). Extensive overviews are given by Place (2009) and Fenske (2011).

¹⁴ Only a few attempts have been made to model the relation between tenure security and the two types of activities. Deininger and Jin (2006) and Fenske (2011) provided similar theoretical frameworks where investment is a single decision variable that can jointly produce higher productivity and more security. Their tenure security function is however a black box, and in particular cannot distinguish between insecurity arising from less institutional protection and from more outward aggression. Below it is shown that these two sources of insecurity may have opposite effects on security-enhancing activities.

costly. Investment costs are represented by a function $C_i : [1, \infty[\rightarrow \mathbb{R}$ given by

$$(12) \quad C_i(\alpha^i) := \frac{c_i A_i}{2} (\alpha^i - 1)^2 \quad \text{with} \quad \frac{\lambda_i}{c_i} < \frac{2r_j^f}{\hat{r}_i^d},$$

where $\hat{r}_i^d := \lambda_i / \delta_i$ is the benefit–cost ratio of defending a hectare in the baseline. The specific form is just to ease the exposition – in particular, the assumption that marginal costs are linear in the amount of initial land (A_i). The restriction on c_i ensures that the three-stage game has a unique subgame-perfect Nash equilibrium with finite land investment; the ratio λ_i / c_i is the benefit–cost ratio of investment per hectare. The payoff function of group i is as before, except for the subtraction of $C_i(\alpha^i)$ (a complete definition of this game is after Proposition A2 in the appendix).

4.2 Analysis

The game can be solved through backward induction; thus we can employ the previous results of the basic model and see what in the end determines the optimal choices of α^1 and α^2 .

Therefore, note first that equilibrium defence in the basic model (d_*^i) depends on the group's land productivity, because the benefit–cost ratio of defending (r_i^d) depends on productivity.¹⁵ A higher land productivity may trigger higher expenditure on defence, because output losses loom larger in case of conflict. Let this positive dependence be denoted by $\underline{d}^i(\alpha^i)$. Verify that equilibrium fighting (f_*^i) is not influenced by productivity (once the other group's defence is fixed). Further, it is convenient to make the dependence of equilibrium tenure security (p_*^i) on the group's land productivity explicit by writing $\underline{p}^i(\alpha^i; s_i / A_i)$, in accordance with Proposition 2. Also, \underline{p}^i depends positively on productivity α^i , since tenure security is strictly increasing in private defence and \underline{d}^i is increasing, while the other group's fighting effort (f_*^j) is independent of α^i (this can also be inferred from Figure 1).

The choice of land productivity by group i – in the “new” first stage – then comes down to choosing an $\alpha^i \geq 1$ that maximizes a function $w^i : [1, \infty[\rightarrow \mathbb{R}$ given by

$$(13) \quad w^i(\alpha^i) := \lambda_i \alpha^i \underline{p}^i \left(\alpha^i; \frac{s_i}{A_i} \right) A_i - \delta_i \underline{d}^i(\alpha^i) - C_i(\alpha^i)$$

(see the proof of Proposition A3 in the appendix). It is immediately seen that each group has a dominant strategy. Further observe that not only does the marginal return to land investment depend on tenure security, but tenure security itself also responds to investment. Relatedly, the marginal cost of land investment not only consists of direct costs, but also of expenditure on defence, as noted above. The first-order condition implies the following solution for productivity:

$$(14) \quad \alpha_*^i = 1 + p_*^i \frac{\lambda_i}{c_i},$$

¹⁵ For this paragraph the reader may also wish to consult Proposition A1 in the appendix. There we also provide formal definitions of the below-mentioned functions $\underline{d}^i(\alpha^i)$ and $\underline{p}^i(\alpha^i; s_i / A_i)$ (see (A11) and (A12) with $t = 0$).

where p_*^i is the equilibrium tenure security at a given level of institutional protection:

$$(15a) \quad \text{if } \frac{2r_j^f}{\hat{r}_i^d} - \frac{\lambda_i}{c_i} \leq 1, \quad \text{then } p_*^i = 1,$$

$$(15b) \quad \text{if } \frac{2r_j^f}{\hat{r}_i^d} - \frac{\lambda_i}{c_i} > 1,$$

$$\text{then } p_*^i = \begin{cases} \left(\frac{2r_j^f}{\hat{r}_i^d} - \frac{\lambda_i}{c_i} \right)^{-1} & \text{if } \frac{s_i}{A_i} \leq r_j^f \left(\frac{2r_j^f}{\hat{r}_i^d} - \frac{\lambda_i}{c_i} \right)^{-2}, \\ \sqrt{\frac{s_i}{A_i} / r_j^f} & \text{if } r_j^f \left(\frac{2r_j^f}{\hat{r}_i^d} - \frac{\lambda_i}{c_i} \right)^{-2} \leq \frac{s_i}{A_i} \leq r_j^f, \\ 1 & \text{if } \frac{s_i}{A_i} \geq r_j^f. \end{cases}$$

(The complete solution of the three-stage game is given by Proposition A3 in the appendix (with $t = 0$), including a proof with the derivations of (14), (15a), and (15b).)

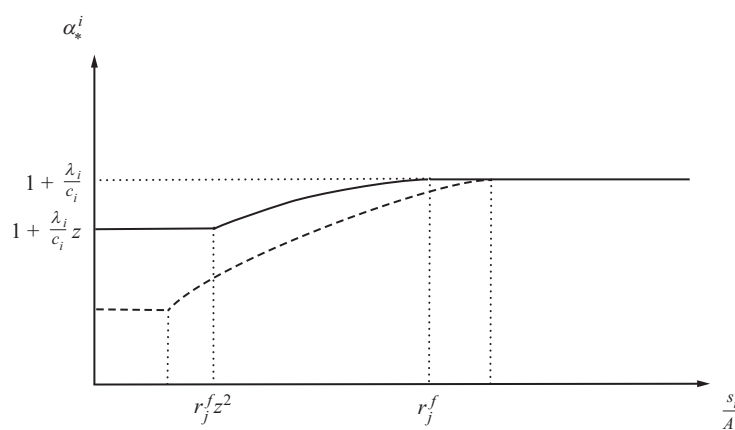
Hence, according to (14), the increase in land productivity relative to the baseline equals the *expected* benefit–cost ratio of investment per hectare, $p_*^i \lambda_i / c_i$. (15a) and (15b) imply that there are two cases to consider: whether $2r_j^f / \hat{r}_i^d - \lambda_i / c_i$ is equal to or less than 1, or is greater than 1. The first case requires a relatively low benefit–cost ratio of fighting or a relatively high benefit–cost ratio of investing. The group will then undertake the highest land investment ($\alpha_*^i = 1 + \lambda_i / c_i$) and (as can be shown) arrange sufficient protection to deter any aggression ($d_*^i = \hat{d}_i$). Just as before, institutional protection is superfluous here for achieving full tenure security. Stronger institutional protection just lowers private protection per hectare by the same amount. When the opposing group becomes more aggressive and tends to threaten tenure security (r_j^f rises, and so does \hat{d}_i), the group simply neutralizes the danger by raising its defence.

Figure 2 illustrates the other, more interesting case of a high benefit–cost ratio of fighting or a low benefit–cost ratio of investing, where institutional protection is crucial for promoting investment. The three sections of the solid productivity curve correspond to low, increasing, and full tenure security. Starting from a state of full security, weaker institutional protection per hectare (a fall of s_i / A_i) will eventually lower tenure security and discourage land investment. Only when institutional protection arrives at the lowest section (the first branch of (15b)) does land investment cease to fall. The reason is that lower protection is now compensated by higher private defence, keeping tenure security constant. Thus institutional and private protection are substitutes here (as in Deininger and Jin, 2006; and Fenske, 2011). Also observe that lower investment costs (c_i) will encourage investment not only because of the higher benefit–cost ratio but also because it may increase tenure security through a rise of private defence.¹⁶

¹⁶ These results are in contrast to Gonzalez (2006), where insecurity of private property may discourage or prevent investment in more productive technology, notably even

Figure 2

Investments in Land Productivity as a Function of Institutional Protection per Hectare
(case: $z = (2r_j^f / \hat{r}_i^d - \lambda_i / c_i)^{-1} < 1$); the dashed curve arises if r_j^f increases)



Do farmers react differently when a fall in tenure security is caused by more outward aggression rather than weaker institutional protection? The dashed curve in Figure 2 depicts the case of a higher benefit–cost ratio of fighting (r_j^f). It is seen that it reduces land investment just as before, but now any undertaken private defence will fall as well. The latter occurs because the lower land productivity makes it less worthwhile for the group to arrange more protection or even maintain its level. Hence, in principle, more outward aggression has a stronger negative effect on a group's land productivity.¹⁷

5 Population Growth and the Feasibility of Institutional Protection

In this section we employ the extended model to investigate the effect of population growth. We start by addressing the debate on the adaptability of traditional land-holding systems to rising population pressure (see section 1) by studying the effect of population growth on land investment and land tenure security. Sufficient conditions are derived for population growth to exert a positive effect on land investment, qualifying also the importance of having institutional protection of land rights in place. We argue, however, that population growth also tends to undermine

when costs are negligible. Our assumptions differ in at least two critical respects: first, the insecurity arising from appropriation is not about the output resulting from investment, but the resource itself – land – is contested; second, defending and fighting do not rival with the scope for investment, allowing an increase of both.

¹⁷ The fall in d_*^i / A_i relates to the first branch of (15b) (see Proposition A3 in the appendix). Here lower s_i / A_i has no effect on α_*^i , whereas higher r_j^f reduces α_*^i .

institutional protection, whether customarily or legally arranged. This sets the stage for a discussion of the ability of traditional systems and state-controlled systems to project power over the land. The critical assumption we make is that this ability depends on the amount of taxes or other contributions that can be extracted from land users, which in turn depends on the arranged tenure security. We show that, due to this interdependence, population growth may push the economy into a state of low tenure security and land investment from which one cannot escape without access to alternative resources.

5.1 Impact of Population Growth

The assumption of land scarcity under a Leontief production technology logically implies that larger group sizes do not affect the equilibrium amounts of defensive and offensive activities (d_*^i and f_*^i) and land investments (α_*^i). In principle, larger group sizes expand time constraints, and since these constraints are not binding, population growth simply increases the abundance of resources used for work and appropriation. Although population growth has no direct effect in the three-stage game, there are two plausible channels through which it may have an *indirect* effect on outcomes. Specifically, a growing population may increase the marginal valuation of output (and land) and worsen the effectiveness of governance institutions.

A larger group size (N_i) implies smaller land parcels per user and probably also an increasing incidence of negative externalities among users (e.g., ecological spillover effects). Thus land becomes scarcer and eventually will raise the group's marginal utility of output, particularly when the group starts to live close to subsistence levels and a small drop in consumption can be fatal. Following this lead, note that γ_i (δ_i , ε_i) in (3) implicitly refers to the marginal substitution rate of reducing work effort (defence, fighting) and more output. Making the marginal utility of output explicit for the moment by calling it σ_i , and using γ_i' (δ_i' , ε_i') for the marginal utility of reducing work effort (defence, fighting), we can write $\gamma_i := \gamma_i'/\sigma_i$, $\delta_i := \delta_i'/\sigma_i$, and $\varepsilon_i := \varepsilon_i'/\sigma_i$. Hence, an increase of the marginal utility of output translates into an equiproportionate decrease of γ_i , δ_i , and ε_i . The impact on preferences may be even stronger, because a larger group also increases unemployment. This may reduce the marginal valuation of leisure, and so further decrease the disutilities of appropriative activities. In sum, under subsistence conditions, the consequences of higher N_i can be reasonably inferred from a simultaneous increase in λ_i (recall $\lambda_i := 1 - \gamma_i/\beta$, the marginal utility of output corrected for the disutility of labour) and decrease in δ_i and ε_i .

Let us consider the effects on a group's land productivity and tenure security. A larger group size raises the direct return to land investment (as λ_i rises) and the group's benefit–cost ratio of defending (as \hat{r}_i^d rises). The latter induces additional defence and so higher tenure security at low levels of institutional protection (see the first branch of (15b)). Together the two factors increase the expected benefit–cost ratio of investment per hectare, $p_*^i \lambda_i / c_i$. Thus land productivity improves (see (14)), and also the situation in which institutional protection is not

needed for full tenure security is promoted (as the case $2r_j^f/\hat{r}_i^d - \lambda_i/c_i \leq 1$, becomes more likely). Therefore, we find that, at any institutional protection level, land investment by group i increases when N_i rises. From a broader viewpoint, this result is not far off from the Boserupian thesis that the threat of starvation and the challenge of feeding more mouths motivates people to improve their farming methods and invent new technologies in order to produce more food on their fields.

Yet what happens if the population of the other group also grows? A rise of N_j increases group j 's benefit–cost ratio of fighting (r_j^f), which clearly threatens tenure security. Land investment by group i falls if institutional protection fails to produce full tenure security (i.e., if $s_i/A_i < r_j^f$). More aggression generally lowers tenure security not only directly (as in the basic model), but also because by discouraging land investments it reduces any defensive activities that accompany these investments.

Hence, if both groups become larger, the impact on a group's land productivity and tenure security is unclear. It all depends on the population densities and growth rates of the two groups and to what extent these demographics induce changes in group preferences. Nevertheless, the preceding analysis implies that an unambiguously positive effect on investments in land productivity arises in at least two cases, as summarized by the following result:

THEOREM 1 *Under subsistence conditions,¹⁸ economy-wide population growth increases investments in land productivity by group i if the other group's increased benefit–cost ratio of fighting for a hectare (r_j^f) does not exceed (i) half of its increased benefit–cost ratio of defending a hectare with maximum productivity ($\hat{r}_i^d(1 + \lambda_i/c_i)/2$) or (ii) its existing institutional protection of a hectare (s_i/A_i).*

This result captures elements of the debate between those who see traditional land-holding systems as flexible enough under population pressure to allow a virtuous cycle of increasingly secure individual land rights, more productivity-enhancing investments, and denser populations and those who question this optimistic view for sub-Saharan Africa by arguing that such a virtuous cycle requires an appropriate development of supporting governance institutions (see section 1). Case (i) of the theorem relates to the former view by playing down the role of institutional involvement. Weak institutional protection, whether legally or customarily arranged, is simply supplemented with sufficient private protective activities to prevent disagreements on who owns the land.¹⁹ Case (ii) relates to the latter, more recently held view by emphasizing the necessity of having a third party in place that independently defines and enforces property rights to land.

Another channel through which population growth may affect outcomes is the efficacy of governance institutions. The working of both traditional systems and

¹⁸ That is, in situations where population growth increases the marginal utility of output.

¹⁹ For example, in the absence of any group differences in preferences and costs (and also $\delta = \varepsilon$), both groups would increase investments in land productivity if $\lambda/c \geq 1$ and so if the benefit–cost ratio of investment exceeds the productivity in the baseline.

state-controlled systems of land tenure tends to be threatened by population growth. In traditional systems, where land tenure is governed by customary law within a community or network of communities, third-party enforcement is typically weak. Land access is mainly regulated by the community members themselves through consensus and negotiation, sometimes supported by local rulers or a council of elders. Being based on voluntary information flows and voluntary participation in enforcement, these systems tend to lose their relative efficacy in governance as communities grow, essentially due to an erosion of communitarian values and norms that bind people together. This erosion is likely to result in coordination problems within groups and even more between groups, with the effect of undermining traditional rules on land access and hindering new agreements (see, e.g., North, 1990; Platteau, 2000, ch. 3; Dixit, 2004, ch. 3, for a formal treatment; Tabellini, 2008).

Also state-controlled systems come under pressure as populations grow, although such formal systems are believed to have a greater potential for solving problems in large-scale land access than traditional systems (see section 5.2). State-controlled systems typically consist of legal machinery to enforce land contracts and to resolve disputes with the help of police, courts, judges, and so on. Though operating on a larger scale, a given capacity of legal enforcement also tends to offer less protection to individual citizens as the population grows. In particular, land administration programs with a static enforcement level may become less effective as the number of people with land titles increases. Activities like demarcation of land boundaries, registration and record keeping, adjudication of rights, and resolution of conflicts tend to multiply under a growing population, putting pressure on the existing legal framework (Deininger, 2003; Deininger and Feder, 2009).

Hence, we can safely conclude that, in the absence of formal or informal investments in governance capacity, population growth eventually leads to lower s_i and s_j . It is clear that in situations where institutional protection is necessary for full tenure security, less protection per hectare (s_i/A_i) may reduce a group's land investment and tenure security. One group may be more affected than the other, and indeed different protection levels can create or foster group identities. An example of this is the tendency for traditional systems to discriminate against categories of land users with the least status and power when the communal land becomes scarcer. Group inequalities can even arise for a more subtle reason. It is easy to show that if group i is more vulnerable to opposition (so $\delta_i > \delta_j$), an equal reduction of protection per hectare will particularly lower the tenure security of this group (using (15b) with $\hat{r}_i^d := \lambda_i/\delta_i$).

5.2 Feasibility of Institutional Protection

The crucial role of institutions in establishing a positive link between population growth and land investment can be taken one step further by considering their funding. The ability of traditional systems and state-controlled systems to project power over the land may be constrained by the difficulty in collecting taxes or other contributions from land users. The two may even be interdependent in that the collected

resources for funding governance structures are likely to increase with the degree of tenure security that is accomplished. In Africa, much of the implementation of formal land governance is often delegated to local governments. Insofar as they are eligible to raise property taxes, land taxation could provide incentives to increase land values by developing the local infrastructure to establish and maintain land records. In this way, local land taxes could trigger a virtuous cycle of increasingly precise land rights and higher land productivity. In practice, such a virtuous cycle is often lacking due to a combination of poor infrastructure and low tax revenue (Deininger, Hilhorst, and Songwe, 2014). Below we provide a tentative sketch of how the budgetary room for improvements in the institutional protection of land rights could be affected by population growth. For the sake of clarity, we ignore any group differences in land, size, preferences, and costs, and moreover equate appropriation costs ($\delta = \varepsilon$).²⁰

Suppose each group pays taxes or contributions (T^i) that are proportional to the amount of retained land:

$$(16) \quad T^i := tp^i A_i \quad \text{with } 0 \leq t < \lambda,$$

where t is the tax or contribution rate (it must be lower than λ – the (corrected) marginal utility of output – to make land investment worthwhile). With revenues depending on tenure security, we mimic the general observation that without effective control of the territory it is difficult to collect taxes (Herbst, 2000; Di John, 2009; Deininger, Hilhorst, and Songwe, 2014). Raising revenue through land taxes is common in a system where the state tries to control the land, but traditional authorities in informal tenure systems also raise contributions from villagers to support the communal judgement system. To some extent, the transfer of resources may also reflect the privileges of senior headmen and chiefs gained through their power and reputation (Acemoglu, Reed, and Robinson, 2014; Beekman, 2015).

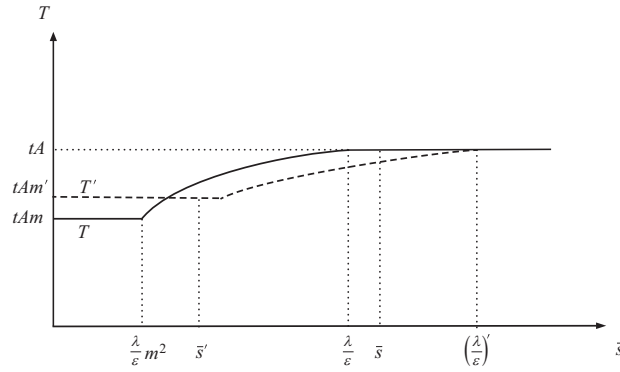
Then, in equilibrium, total tax revenues ($T_*^1 + T_*^2$) depend positively on institutional protection per hectare, $\bar{s} := s/(A/2)$. Defining an auxiliary function $m : [0, \lambda[\rightarrow \mathbb{R}$ given by

$$m(t) := \frac{1 - t/\lambda}{2 - \lambda/c}$$

and noting that $m(t) > 0$ by (12) and (16), total tax revenues equal

²⁰ The extended model with taxation is formulated in the appendix. Proposition A3 specifies the Nash equilibrium and the equilibrium tenure security function – the subsequent analysis uses both results.

Figure 3
Land Tax Revenues under a Growing Population
(case $m(t) < 1$; the dashed curve arises if the population grows)



$$(17) \quad T_*^1 + T_*^2 = T(\bar{s}; t) := tA \cdot \begin{cases} 1 & \text{if } m(t) \geq 1, \\ \left\{ \begin{array}{ll} m(t) & \text{if } \bar{s} \leq \frac{\lambda}{\epsilon} m^2(t) \\ \sqrt{\bar{s} \frac{\epsilon}{\lambda}} & \text{if } \frac{\lambda}{\epsilon} m^2(t) \leq \bar{s} \leq \frac{\lambda}{\epsilon} \\ 1 & \text{if } \bar{s} \geq \frac{\lambda}{\epsilon} \end{array} \right\} & \text{if } m(t) < 1 \end{cases}$$

(using Proposition A3 in the appendix). The solid curve in Figure 3 shows the tax revenue function $T(\cdot; t)$ for the case of $m(t) < 1$, that is, when institutional protection is necessary for achieving full tenure security.

Under subsistence conditions, population growth promotes the case $m(t) < 1$ and eventually causes a shift of the tax revenue function for this case through the rise of λ and fall of ϵ (see section 5.1). The effect is ambiguous. At low protection levels, population growth increases tax revenues, mainly because the induced land investments raise private defence and so tenure security. At high protection levels, however, tax revenues diminish or stay the same, because the absence of private defence allows an increase of fighting and, consequently, a fall in tenure security. The dashed curve in Figure 3 shows the tax revenue function for an increased population.

At the same time, the funds needed to sustain tenure security increase for two reasons. First, as argued in section 5.1, population growth tends to water down the effectiveness of institutional protection at given governance capacity. Second, because population growth raises the benefit–cost ratio of fighting (λ/ϵ), protection levels that initially achieved full tenure security may have to be scaled up to guarantee also full security for the increased population. Figure 3 illustrates both effects

with initial protection \bar{s} and threshold λ/ε versus \bar{s}' (if investment in government capacity stays out) and $(\lambda/\varepsilon)'$ for an increased population.

So far, then, the outcome is that maintaining full tenure security in a growing population is problematic under subsistence conditions. Additional funds will be required to improve governance capacity, but this poses a problem, as the revenues from land taxation tend to fall. The obvious remedy appears to be an increase in (formal) tax rates. However, a maximum limit exists on the revenue that can be collected in a situation of low tenure security. Higher t raises tax revenue per hectare, but lowers the number of taxable hectares, as it affects tenure security. In response to higher taxes the groups reduce their self-defence, since this decreases their benefit–cost ratio of defending $(\lambda(\alpha_* - t/\lambda)/\delta)$, both directly and through its negative impact on land investment. It can be shown that if institutional protection is below a certain low level \bar{s}_* , rates higher than $t = \lambda/2$ cannot increase revenue, implying a relatively low maximum tax revenue $A\lambda m(\lambda/2)/2$ (see Lemma A4 in the appendix).

Hence, without access to external income sources, the economy may be trapped in a state of low tenure security and land investment. This possible effect of population growth is summarized by the following result:

THEOREM 2 *Under subsistence conditions, economy-wide population growth may cause a state of low tenure security, low land investment, and low tax revenues from the land. Raising tax rates to fund gradual improvements in institutional protection can generate only a limited amount of revenue here. Specifically, suppose such a bad state has protection $\bar{s} \leq \bar{s}_*$, where²¹*

$$\bar{s}_* := \frac{\lambda}{\varepsilon} m^2 \left(\frac{3}{4} \lambda \right).$$

Then for all $t \in [0, \lambda]$, it holds that

$$T(\bar{s}; t) = t A m(t) \leq \frac{\lambda}{2} A m \left(\frac{\lambda}{2} \right).$$

The limitations of public policy in a steady state of underdevelopment have been observed before. For instance, Mehlum, Moene, and Torvik (2003) find that improving institutional protection to reduce extortion of productive firms by predatory firms may require such a high tax rate that the profitability of productive firms – the ones that have to pay the tax – becomes insufficient to outperform the profitability of predatory firms, thus keeping the economy trapped in a low stage of development (see also Dabla-Norris and Freeman, 2004). In a similar vein, Lloyd-Ellis and Marceau (2003) show that, in the presence of endogenous credit constraints, which particularly hurt low-wealth borrowers, reducing the expected payoff to predatory

²¹ If $\bar{s} \leq \bar{s}_*$ (and $m(t) < 1$), this state has $p_* < 1$ and $\alpha_* = 1 + p_* \lambda / c < 1 + \lambda / c$. Note that $0 < \bar{s}_* \leq (\lambda/\varepsilon) m^2(t)$ if $t \in [0, 3\lambda/4]$ and that $(\lambda/\varepsilon) m^2(t) \leq \bar{s}_* < \lambda/\varepsilon$ if $t \in [3\lambda/4, \lambda]$ (see (17)). See Lemma A4 in the appendix; Theorem 2 readily follows from this lemma.

behaviour through a public policy of heavier punishments may also backfire if the required increase in taxes is too high. Having access to external sources to fund public policy does not necessarily solve this problem. In our case, income transfers from the central government may enable local governments to enhance the necessary infrastructure, but it also increases their dependence on the politics of the centre, thus weakening the link between taxation and public services and posing problems of legitimacy and accountability (Deininger, Hilhorst, and Songwe, 2014, pp. 81–82).

6 Conclusion

We constructed and analyzed a three-stage contest model to show how greater institutional protection of land rights could improve tenure security under land scarcity conditions, and thus encourage land investment and save resources squandered on land conflicts. The model was subsequently used to examine how increasing population pressure on the land could affect this. Two key mechanisms were discussed: higher population density could raise the marginal utility of output, and it could weaken the protection provided by land institutions. The first mechanism implied higher land values, which promoted land investment, but also lower costs of appropriation, which promoted conflict and thus threatened land investment. This allowed us to address the Boserupian nexus between population and land productivity by deriving sufficient conditions for population growth to exert a positive effect on land investment, qualifying also the need for third-party protection of land rights. The second mechanism posed the question of whether lower protection of land rights because of population growth could be remedied by extracting funds from land users in taxes or other contributions. We found that the two sides of the budget were typically interdependent, as the collected funds from land users increased with the degree of tenure security achieved through higher expenditure on land-rights protection. This implied that population growth could push the economy into a low-productivity trap from which one could not escape by raising taxes to finance increases in land-rights protection.

A basic prediction of our analysis links the growth in land productivity in sub-Saharan countries or within-country regions to the performance of state or local institutions and the growth of population. Countries with low productivity growth would have weak fiscal and legal capacities and relatively high population densities, whereas countries with high productivity growth would show stronger state capacities and face less population pressure. Escaping from the low “development cluster” (Besley and Persson, 2011) is difficult. For countries or regions that were able to jump to a higher stage of development through the build-up of stronger institutions, access to external financial resources might have been a factor.

One weakness of our analysis is that we did not explicitly model the effects of population growth. As a consequence, we could not systematically shut down other channels through which population growth may affect preferences. For instance, in

addition to the two mechanisms discussed, larger groups may also make within-group cooperation more costly, which could raise the marginal costs of defending and fighting. This would oppose the negative effect a higher marginal valuation of output has on appropriation costs. Our model also excluded a long-run Malthusian feedback effect, where land productivity affects population growth (see, e.g., Ashraf and Galor, 2011). A Malthusian steady-state condition would equate the output per group member to some minimum subsistence level. Following this line of reasoning, we would find that if institutional protection provides full tenure security, a unique steady state exists with maximum productivity and high population. At low levels of institutional protection with insecure land rights, group output will be nonlinear in group size, and the economy may well end up in a locally stable low-productivity, low-population equilibrium. For this case the point remains valid that opportunities to improve institutional protection by raising taxes are essentially limited.

Appendix

A.1 Proposition A1

The two-stage game has a unique subgame-perfect Nash equilibrium $((d_*^1, f_*^1), (d_*^2, f_*^2))$ given by

$$d_*^i = \max \left(0, \min \left(\frac{(r_i^d/2)^2}{r_j^f} A_i - s_i, \hat{d}_i \right) \right),$$

$$f_*^i = \begin{cases} \sqrt{d_*^j + s_j} (\sqrt{\hat{d}_j + s_j} - \sqrt{d_*^j + s_j}) & \text{if } d_*^j \leq \hat{d}_j, \\ 0 & \text{if } d_*^j \geq \hat{d}_j. \end{cases}$$

PROOF It will be useful to define

$$(A1) \quad \tilde{d}_i := \frac{(r_i^d/2)^2}{r_j^f} A_i - s_i.$$

Noting (10), \tilde{d}_i is a potential level of defensive effort.

Step 1. Consider the game in strategic form that we obtain by fixing $d^1 \in [0, N_1]$ and $d^2 \in [0, N_2]$. For the payoff functions \tilde{v}^1 and \tilde{v}^2 of this game we have

$$\tilde{v}^i(f^i; f^j) = \alpha_i \lambda_i A_i \frac{d^i + s_i}{d^i + s_i + f^j} + \lambda_i A_j \frac{f^i}{d^j + s_j + f^i} - \delta_i d^i - \varepsilon_i f^i.$$

First we prove that for every strategy d^j player i has a strictly dominant strategy

$$(A2) \quad \underline{f}^i(d^j) := \begin{cases} y^i(d^j) & \text{if } y^i(d^j) > 0, \\ 0 & \text{if } y^i(d^j) \leq 0. \end{cases}$$

Note that (A2) is equivalent to (9) with i and j interchanged. Also, the conditional payoff functions of player i are the functions $\tilde{v}^i(\cdot; f^j)$. If $A_j = 0$, then $f^i = 0$ is the unique maximizer. In this case $y^i(d^j) = -d^j - s_j < 0$, so the desired result holds for $A_j = 0$. Now suppose $A_j > 0$. As $\lambda_i A_j$ and $d^j + s_j$ are positive, all conditional payoff functions of player i are strictly concave. As these functions also are continuous, they have a unique maximizer. As this maximizer does not depend on f^j , player i has a strictly dominant strategy. So this strategy is the unique maximizer of $\tilde{v}^i(\cdot; 0)$ and therefore of

$$\lambda_i A_j \frac{f^i}{d^j + s_j + f^i} - \varepsilon_i f^i.$$

The derivative w.r.t. f^i of this function is

$$\lambda_i A_j \frac{d^j + s_j}{(d^j + s_j + f^i)^2} - \varepsilon_i;$$

it is zero if and only if $f^i = y^i(d^j)$. Using the strict concavity and decreasingness of this function and recalling $\hat{d}_i < N_i$ and $y^i < N_i - \hat{d}_i$ (see (8)), we see that the desired result also holds for the case $A_j > 0$.

Step 2. Consider the game in strategic form that we obtain by inserting in v^1 and v^2 for (f^1, f^2) the Nash equilibrium $(\underline{f}^1(d^2), \underline{f}^2(d^1))$ of the second stage given by (A2). For the payoff functions \bar{v}^1 and \bar{v}^2 of this game we have

$$\bar{v}^i(d^i; d^j) = \alpha_i \lambda_i A_i \frac{d^i + s_i}{d^i + s_i + \underline{f}^j(d^i)} + \lambda_i A_j \frac{\underline{f}^i(d^j)}{d^j + s_j + \underline{f}^i(d^j)} - \delta_i d^i - \varepsilon_i \underline{f}^i(d^j).$$

We now prove that in this game each player i has a strictly dominant strategy d_*^i given by

$$(A3) \quad d_*^i = \begin{cases} 0 & \text{if } \tilde{d}_i \leq 0 \text{ or } \hat{d}_i \leq 0, \\ \tilde{d}_i & \text{if } 0 \leq \tilde{d}_i \leq \hat{d}_i, \\ \hat{d}_i & \text{if } 0 \leq \hat{d}_i \leq \tilde{d}_i \end{cases}$$

(note that this implies (10)). We distinguish between two cases. First suppose $\hat{d}_i \leq 0$: now $f_*^j = 0$ and $d^i + s_i + \underline{f}^j(d^i) = d^i + s_i$. So

$$\bar{v}^i(d^i; d^j) = \alpha_i \lambda_i A_i + \lambda_i A_j \frac{\underline{f}^i(d^j)}{d^j + s_j + \underline{f}^i(d^j)} - \delta_i d^i - \varepsilon_i \underline{f}^i(d^j).$$

We see that $d_*^i = 0$ is a strictly dominant strategy of player i .

Next suppose $\hat{d}_i \geq 0$: now $\underline{f}^j(d^i) = 0$ ($d^i \geq \hat{d}_i$) and $\underline{f}^j(d^i) = y^j(d^i)$ ($d^i \leq \hat{d}_i$). For $d^i \leq \hat{d}_i$ it follows that $d^i + s_i + \underline{f}^j(d^i) = \sqrt{(A_i/r_j^f)}\sqrt{d^i + s_i}$. With this we

obtain

$$\bar{v}^i(d^i; d^j) = \begin{cases} \alpha_i \lambda_i \sqrt{\frac{A_i}{r_j^f}} \sqrt{d^i + s_i} + \lambda_i A_j \frac{\underline{f}^i(d^j)}{d^j + s_j + \underline{f}^i(d^j)} - \delta_i d^i - \varepsilon_i \underline{f}^i(d^j) & \text{if } d^i \leq \hat{d}_i, \\ \alpha_i \lambda_i A_i + \lambda_i A_j \frac{\underline{f}^i(d^j)}{d^j + s_j + \underline{f}^i(d^j)} - \delta_i d^i - \varepsilon_i \underline{f}^i(d^j) & \text{if } d^i \geq \hat{d}_i. \end{cases}$$

We see that the desired results hold if $A_i = 0$. Now suppose $A_i > 0$. Consider the function $\bar{v}^i(\cdot; d^j)$. On the first branch it is strictly concave, and on the second it is affine and strictly decreasing. As the function is continuous, it is strictly quasi-concave and therefore has a unique maximizer. As the maximizer does not depend on d^j , player i has a strictly dominant strategy d_*^i . Noting that the unique maximizer of the function $\mathbb{R}_+ \rightarrow \mathbb{R}$ defined by $x \mapsto \alpha_i \lambda_i \sqrt{A_i/r_j^f} \sqrt{x + s_i} - \delta_i x$ equals $\tilde{d}_i = [(r_i^d/2)^2/r_j^f]A_i - s_i$ if $\tilde{d}_i \geq 0$ and equals 0 otherwise, the desired results follow.

Step 3. Now we can finish the proof of Proposition A1: apply (A2) and (A3).
Q.E.D.

A.2 Proof of Proposition 1

Again define the potential level of defence \tilde{d}_i by (A1).

Step 1. $\hat{d}_i \leq 0 \Rightarrow d_*^i = 0$ and $f_*^j = 0$. And $2r_j^f \leq r_i^d \Rightarrow f_*^j = 0$. Proof of first statement: by Proposition A1, $0 \leq d_*^i = \max(0, \min(\tilde{d}_i, \hat{d}_i)) \leq \max(0, \hat{d}_i) = 0$ and so $d_*^i = 0$. This implies $f_*^j = 0$. Proof of second statement: now $\tilde{d}_i \geq \hat{d}_i$. Proposition A1 implies $d_*^i = \max(0, \hat{d}_i) \geq \hat{d}_i$ and so $f_*^j = 0$.

Step 2. Now we can finish the proof of Proposition 1. If $A_i = 0$, then $\hat{d}_i \leq 0$ and, by step 1, $f_*^j = 0$. If $A_i > 0$ and $2r_j^f \leq r_i^d$, then, by step 1, $f_*^j = 0$. If $A_i > 0$ and $2r_j^f > r_i^d$ and $s_i/A_i \geq r_j^f$, then $\hat{d}_i \leq 0$ and so, by step 1, $f_*^j = 0$.

Now suppose $f_*^j = 0$. We have to prove that one of the above-mentioned three possibilities holds. We may assume $A_i > 0$. Now we have to prove that $2r_j^f \leq r_i^d$, or $s_i/A_i \geq r_j^f$ and $2r_j^f > r_i^d$. This we do by contradiction. So suppose $2r_j^f > r_i^d$, and $s_i/A_i < r_j^f$ or $2r_j^f \leq r_i^d$, i.e., suppose $2r_j^f > r_i^d$ and $s_i/A_i < r_j^f$. This implies $\hat{d}_i > 0$. So $d_*^i = \max(0, \tilde{d}_i) \leq [(r_i^d/2)^2/r_j^f]A_i - s_i < \hat{d}_i$, which in turn implies the contradiction

$$f_*^j = \sqrt{d_*^i + s_i} \left(\sqrt{\hat{d}_i + s_i} - \sqrt{d_*^i + s_i} \right) > 0.$$

Q.E.D.

A.3 Proof of Proposition 2

Again define the potential level of defence \tilde{d}_i by (A1) (see (A3)). In terms of \tilde{d}_i and \hat{d}_i this proposition reads

$$(A4) \quad \text{if } \tilde{d}_i \geq \hat{d}_i, \text{ then } p_*^i = 1,$$

$$(A5) \quad \text{if } \tilde{d}_i < \hat{d}_i, \text{ then } p_*^i = \begin{cases} \frac{r_i^d}{2r_j^f} & \text{if } 0 < \tilde{d}_i, \\ \sqrt{\frac{s_i}{A_i}}/r_j^f & \text{if } \tilde{d}_i \leq 0 < \hat{d}_i, \\ 1 & \text{if } \hat{d}_i \leq 0. \end{cases}$$

The desired result follows from Proposition A1 and the definition of p^i in (1) by noting that

(a) if $\tilde{d}_i \geq \hat{d}_i$ (or $2r_j^f \leq r_i^d$), then $d_*^i = \max(0, \hat{d}_i)$ and $f_*^j = 0$;

(b) if $\tilde{d}_i < \hat{d}_i$ (or $2r_j^f > r_i^d$), then

(i) if $0 \leq \tilde{d}_i$ (or $s_i/A_i \leq [(r_i^d/2)^2]/r_j^f$), then

$$d_*^i = \tilde{d}_i \quad \text{and} \quad f_*^j = y^j(\tilde{d}_i) = \sqrt{\frac{(r_i^d/2)^2}{r_j^f} A_i} \left(\sqrt{r_j^f A_i} - \sqrt{\frac{(r_i^d/2)^2}{r_j^f} A_i} \right);$$

(ii) if $\tilde{d}_i \leq 0 \leq \hat{d}_i$ (or $(r_i^d/2)^2/r_j^f \leq s_i/A_i \leq r_j^f$), then

$$d_*^i = 0 \quad \text{and} \quad f_*^j = y^j(0) = \sqrt{s_i} \left(\sqrt{r_j^f A_i} - \sqrt{s_i} \right);$$

(iii) if $\hat{d}_i \leq 0$ (or $s_i/A_i \geq r_j^f$), then $d_*^i = 0$ and $f_*^j = 0$.

Q.E.D.

A.4 Proposition A2

Suppose $s_1/A_1 = s_2/A_2 =: \bar{s}$. Resource costs in (11) are a decreasing function of \bar{s} , i.e., $(\delta(d_*^1 + d_*^2) + \varepsilon(f_*^1 + f_*^2))/A$ is decreasing in \bar{s} .

PROOF *Step 1.* Note that $\delta = \varepsilon$, $r_i^d/2 = r_j^d =: r^d$ and $r_i^f = r_j^f =: r^f$. Let

$$\tilde{r} := (r^d/2)^2/r^f;$$

\tilde{r} has no direct interpretation, but note that $\tilde{r} - \bar{s}$ is a potential defence level per hectare (see (10)). We have

$$(A6) \quad \frac{d_*^1}{A_1} = \frac{d_*^2}{A_2} = \max(0, \min(\tilde{r} - \bar{s}, r^f - \bar{s})),$$

$$(A7) \quad \frac{f_*^i}{A_i} = \frac{1 - p_*}{p_*} \frac{A_j}{A_i} \left(\frac{d_*^1}{A_1} + \bar{s} \right).$$

Proof of (A6): with Proposition A1. Proof of (A7): use $p_* = (d_*^i + s_j)/(d_*^i + s_j + f_*^i)$ and (A6).

Step 2. (A6) implies, with $A = A_1 + A_2$, that

$$\frac{d_*^1 + d_*^2}{A} = \frac{A_1}{A} \frac{d_*^1}{A_1} + \frac{A_2}{A} \frac{d_*^2}{A_2} = \frac{d_*^1}{A_1}.$$

And with (A7) we obtain

$$\frac{f_*^1 + f_*^2}{A} = \frac{A_1}{A} \frac{f_*^1}{A_1} + \frac{A_2}{A} \frac{f_*^2}{A_2} = \frac{1-p_*}{p_*} \left(\frac{d_*^1}{A_1} + \bar{s} \right).$$

Therefore

$$\frac{d_*^1 + d_*^2 + f_*^1 + f_*^2}{A} = \left(\frac{1-p_*}{p_*} + 1 \right) \frac{d_*^1}{A_1} + \frac{1-p_*}{p_*} \bar{s} = \frac{1}{p_*} \frac{d_*^1}{A_1} + \left(\frac{1}{p_*} - 1 \right) \bar{s}.$$

Now with (A6) we obtain

$$(A8) \quad \frac{d_*^1 + d_*^2 + f_*^1 + f_*^2}{A} = \frac{1}{p_*} \max(0, \min(\tilde{r} - \bar{s}, r^f - \bar{s})) + \left(\frac{1}{p_*} - 1 \right) \bar{s}.$$

Thus resource costs are a function of \bar{s} .

Step 3. First note that $2r^f \leq r^d \Leftrightarrow r^f \leq \tilde{r}$. Now (A8) together with Proposition 2 implies:

(a) if $\tilde{r} \geq r^f$, then

$$\frac{(d_*^1 + d_*^2) + (f_*^1 + f_*^2)}{A} = \begin{cases} r^f - \bar{s} & \text{if } \bar{s} < r^f, \\ 0 & \text{if } \bar{s} \geq r^f; \end{cases}$$

(b) if $\tilde{r} < r^f$, then

$$\frac{(d_*^1 + d_*^2) + (f_*^1 + f_*^2)}{A} = \begin{cases} -\bar{s} + \sqrt{\tilde{r}}\sqrt{r^f} & \text{if } \bar{s} < \tilde{r}, \\ \sqrt{\tilde{s}}(\sqrt{r^f} - \sqrt{\tilde{s}}) & \text{if } \tilde{r} \leq \bar{s} < r^f, \\ 0 & \text{if } \bar{s} \geq r^f. \end{cases}$$

Inspecting the above two formulas shows the decreasingness in \bar{s} . (The decreasingness of the function on the second branch is guaranteed if its derivative is ≤ 0 . This holds if $r^f/4 \leq \tilde{r}$, which is implied by $\alpha \geq 1$.) Q.E.D.

A.5 The Three-Stage Game of the Extended Model

In this game (see sections 4 and 5), $\alpha^1, \alpha^2 \in \Omega := [1, \infty[$ are chosen in the first stage; d^1, d^2 in the second stage; and f^1, f^2 in the third stage. The payoffs may include the subtraction of (taxes) $tp^i A_i$ where $0 \leq t < \lambda_i$ (see (16)). The payoff of player i is given by

$$(A9) \quad \begin{aligned} & v^i((\alpha^i, d^i, f^i); (\alpha^j, d^j, f^j)) \\ & := \lambda_i \left(\left(\alpha^i - \frac{t}{\lambda_i} \right) A_i \frac{d^i + s_i}{d^i + s_i + f^j} + A_j \frac{f^i}{d^j + s_j + f^i} \right) \\ & \quad - \delta_i d^i - \varepsilon_i f^i - C_i(\alpha^i), \end{aligned}$$

where $C_i(\alpha^i)$ is given by (12). Recall $\lambda_i/c_i < 2r_j^f/\hat{r}_i^d$, where $\hat{r}_i^d = \lambda_i/\delta_i$. Observe that the extended model of section 4 is obtained by setting $t = 0$.

A.6 Proposition A3

Suppose $A_i > 0$. Define $\tilde{d}^i(\alpha) := [(\hat{r}_i^d \alpha/2)^2/r_j^f]A_i - s_i$; then $\tilde{d}^i(\alpha)$ is a potential level of defence for given α (see (10)). The three-stage game has a unique subgame-perfect Nash equilibrium $((\alpha_*^1, d_*^1, f_*^1), (\alpha_*^2, d_*^2, f_*^2))$ given by

$$(A10) \quad \begin{aligned} \alpha_*^i &= 1 + p_*^i \frac{\lambda_i}{c_i}, \\ d_*^i &= \max\left(0, \min\left(\tilde{d}^i\left(\alpha_*^i - \frac{t}{\lambda_i}\right), \hat{d}_i\right)\right), \\ f_*^i &= \begin{cases} \sqrt{d_*^j + s_j}(\sqrt{\hat{d}_j + s_j} - \sqrt{d_*^j + s_j}) > 0 & \text{if } d_*^j \leq \hat{d}_j, \\ 0 & \text{if } d_*^j \geq \hat{d}_j, \end{cases} \end{aligned}$$

where

(a) if $2r_j^f/\hat{r}_i^d - \lambda_i/c_i \geq 1 - t/\lambda_i$, then

$$p_*^i = \begin{cases} \frac{1 - t/\lambda_i}{2r_j^f/\hat{r}_i^d - \lambda_i/c_i} & \text{if } \frac{s_i}{A_i} \leq r_j^f \left(\frac{2r_j^f}{\hat{r}_i^d} - \frac{\lambda_i}{c_i}\right)^{-2} \left(1 - \frac{t}{\lambda_i}\right)^2, \\ \sqrt{\frac{s_i}{A_i}}/r_j^f & \text{if } r_j^f \left(\frac{2r_j^f}{\hat{r}_i^d} - \frac{\lambda_i}{c_i}\right)^{-2} \left(1 - \frac{t}{\lambda_i}\right)^2 \leq \frac{s_i}{A_i} \leq r_j^f, \\ 1 & \text{if } \frac{s_i}{A_i} \geq r_j^f; \end{cases}$$

(b) if $2r_j^f/\hat{r}_i^d - \lambda_i/c_i \leq 1 - t/\lambda_i$, then $p_*^i = 1$.

PROOF Observe that the payoff (A9) without the cost C_i is the same as (4) with α_i replaced by $\alpha^i - t/\lambda_i$. This implies that the result of Proposition A1 and its proof concerning the subgame-perfect Nash equilibria of stages 2 and 3 remain valid if we replace r_i^d by $\hat{r}_i^d(\alpha^i - t/\lambda_i)$. Also, Propositions 1 and 2 and their proofs continue to hold for the three-stage game if we make this replacement.

Now note the following: \hat{d}_i and y^i in (6) and (7) depend neither on α^i nor on α^j . Further, d_*^i, f_*^j , and p_*^i may depend on α^i but not on α^j . Then we see that the formulas for d_*^i and f_*^i hold. It remains to prove the formulas for p_*^i and α_*^i .

Define, besides \underline{f}^i given by (A2), also $\underline{d}^i : \Omega \rightarrow [0, N_i]$ by

$$(A11) \quad \underline{d}^i(\alpha^i) := \max\left(0, \min\left[\frac{(\hat{r}_i^d(\alpha^i - t/\lambda_i)/2)^2}{r_j^f}\right]A_i - s_i, \hat{d}_i\right)$$

and in accordance with (1),

$$(A12) \quad \underline{p}^i\left(\alpha^i; \frac{s_i}{A_i}\right) := \frac{\underline{d}^i(\alpha^i) + s_i}{\underline{d}^i(\alpha^i) + s_i + \underline{f}^j(\underline{d}^i(\alpha^i))}.$$

Consider the game in strategic form obtained by substituting in the formulas for v^1 and v^2 , given by (A9), first for (f^1, f^2) the strictly dominant Nash equilibrium $(\underline{f}^1(d^2), \underline{f}^2(d^1))$ of stage 3, and next for (d^1, d^2) the strictly dominant Nash equilibrium $(\underline{d}^1(\alpha^1), \underline{d}^2(\alpha^2))$ of stage 2. Denoting the resulting functions by \hat{v}^1 and \hat{v}^2 , we have

$$\begin{aligned} \hat{v}^i(\alpha^i; \alpha^j) = & \lambda_i \left(\left(\alpha^i - \frac{t}{\lambda_i} \right) \underline{p}^i \left(\alpha^i; \frac{s_i}{A_i} \right) A_i + \left(1 - \underline{p}^j \left(\alpha^j; \frac{s_j}{A_j} \right) \right) A_j \right) \\ & - \delta_i \underline{d}^i(\alpha^i) - \varepsilon_i \underline{f}^i(\underline{d}^j(\alpha^j)) - C_i(\alpha^i). \end{aligned}$$

This implies that the best-reply correspondence of each player is constant and that for player i this constant equals the set of maximizers of the function $w^i : \Omega \rightarrow \mathbb{R}$ given by

$$(A13) \quad w^i(\alpha^i) := \lambda_i \left(\alpha^i - \frac{t}{\lambda_i} \right) \underline{p}^i \left(\alpha^i; \frac{s_i}{A_i} \right) A_i - \delta_i \underline{d}^i(\alpha^i) - \frac{c_i A_i}{2} (\alpha^i - 1)^2$$

(see (13)). Now we are ready to prove the formulas for p^i_* and α^i_* . We do this in three steps.

Step 1.

- (a) If $s_i/A_i \geq r_j^f$, then $\underline{p}^i(\alpha^i; s_i/A_i) = 1$ and $\underline{d}^i(\alpha^i) = 0$.
 (b) If $s_i/A_i < r_j^f$, then

$$\begin{aligned} \underline{p}^i \left(\alpha^i; \frac{s_i}{A_i} \right) &= \sqrt{\frac{s_i}{A_i} / r_j^f} \quad \text{and} \quad \underline{d}^i(\alpha^i) = 0 \quad \text{if} \quad \alpha^i - \frac{t}{\lambda_i} \leq \frac{2r_j^f}{\hat{r}_i^d} \sqrt{\frac{s_i}{A_i} / r_j^f}, \\ \underline{p}^i \left(\alpha^i; \frac{s_i}{A_i} \right) &= \frac{\hat{r}_i^d}{2r_j^f} (\alpha^i - t/\lambda_i) \quad \text{and} \quad \underline{d}^i(\alpha^i) = \frac{(\hat{r}_i^d (\alpha^i - t/\lambda_i))^2 / 2}{r_j^f} A_i - s_i \\ &\quad \text{if} \quad \frac{2r_j^f}{\hat{r}_i^d} \sqrt{\frac{s_i}{A_i} / r_j^f} < \alpha^i - \frac{t}{\lambda_i} < \frac{2r_j^f}{\hat{r}_i^d}, \\ \underline{p}^i \left(\alpha^i; \frac{s_i}{A_i} \right) &= 1 \quad \text{and} \quad \underline{d}^i(\alpha^i) = \frac{r_j^f}{A_i} - s_i \quad \text{if} \quad \alpha^i - \frac{t}{\lambda_i} \geq \frac{2r_j^f}{\hat{r}_i^d}. \end{aligned}$$

- (c) If $s_i/A_i \geq r_j^f$, then $w^i(\alpha^i) = g_0^i(\alpha^i) - C_i(\alpha^i)$, where $g_0^i(x) := \lambda_i A_i (x - t/\lambda_i)$.
 (d) If $s_i/A_i < r_j^f$, then

$$w^i(\alpha^i) = \begin{cases} g_1^i(\alpha^i) - C_i(\alpha^i) & \text{if } \alpha^i - \frac{t}{\lambda_i} \leq \frac{2r_j^f}{\hat{r}_i^d} \sqrt{\frac{s_i}{A_i} / r_j^f}, \\ g_2^i(\alpha^i) - C_i(\alpha^i) & \text{if } \frac{2r_j^f}{\hat{r}_i^d} \sqrt{\frac{s_i}{A_i} / r_j^f} < \alpha^i - \frac{t}{\lambda_i} < \frac{2r_j^f}{\hat{r}_i^d}, \\ g_3^i(\alpha^i) - C_i(\alpha^i) & \text{if } \alpha^i - \frac{t}{\lambda_i} \geq \frac{2r_j^f}{\hat{r}_i^d}, \end{cases}$$

where

$$\begin{aligned} g_1^i(x) &:= \lambda_i A_i \sqrt{\frac{s_i}{A_i} / r_j^f} \left(x - \frac{t}{\lambda_i} \right), \quad g_2^i(x) := A_i \frac{(\hat{r}_i^d / 2)^2}{r_j^f} \delta_i \left(x - \frac{t}{\lambda_i} \right)^2 + \delta_i s_i, \\ g_3^i(x) &:= \lambda_i A_i \left(x - \frac{t}{\lambda_i} \right) - \delta_i \left(\frac{\lambda_j}{\varepsilon_j} A_i - s_i \right). \end{aligned}$$

PROOF The statements for $\underline{p}^i(\alpha^i; s_i / A_i)$ follow from (A4) and (A5) with \tilde{d}_i as in (A1), i.e., here

$$\tilde{d}_i = \frac{(\hat{r}_i^d (\alpha^i - t / \lambda_i))^2 / 2}{r_j^f} A_i - s_i.$$

Indeed, just note that in (a) $\hat{d}_i \leq 0$ and in (b) $\hat{d}_i > 0$, and that the three branches relate to the cases $\tilde{d}_i \leq 0$, $0 < \tilde{d}_i < \hat{d}_i$, $\tilde{d}_i \geq \hat{d}_i$. Having this and the expressions for $\underline{p}^i(\alpha^i; s_i / A_i)$, the statements for $w^i(\alpha^i)$ follow with (A11) and (A13). Concerning the results in (d), we note that the following holds. Case $\tilde{d}_i \leq 0$: $\underline{d}^i(\alpha^i) = 0$. Case $0 < \tilde{d}_i < \hat{d}_i$: $\underline{d}^i = \tilde{d}_i$. Case $\tilde{d}_i > \hat{d}_i$: $\underline{d}^i = \hat{d}_i$.

Step 2. With $g_0^i, g_1^i, g_2^i, g_3^i : \mathbb{R}_+ \rightarrow \mathbb{R}$ as in step 1, the functions $g_0^i - C_i$, $g_1^i - C_i$, $g_2^i - C_i$, and $g_3^i - C_i$ are continuously differentiable and strictly concave. They all have a unique stationary point and therefore a unique maximizer: $\hat{\alpha}_0$, $\hat{\alpha}_1$, $\hat{\alpha}_2$, $\hat{\alpha}_3$, respectively, where

$$\hat{\alpha}_0 = \hat{\alpha}_3 := 1 + \frac{\lambda_i}{c_i}, \quad \hat{\alpha}_1 := 1 + \frac{\lambda_i}{c_i} \sqrt{\frac{s_i}{A_i} / r_j^f}, \quad \hat{\alpha}_2 := 1 + \frac{\lambda_i}{c_i} \left(\frac{1 - t / \lambda_i}{2r_j^f / \hat{r}_i^d - \lambda_i / c_i} \right).$$

Also the w^i in (c) and (d) in step 1 are continuously differentiable and strictly concave.

PROOF It is straightforward to check the statements about $g_0^i - C_i$, $g_1^i - C_i$, $g_2^i - C_i$, $g_3^i - C_i$. As for w^i , the case in (c) is evident. Regarding (d), it is straightforward to check that w^i is continuously differentiable. The derivative of w^i is strictly decreasing on the first branch, the second branch, and the third branch (remembering that $\lambda_i / c_i < 2r_j^f / \hat{r}_i^d$). It follows that w^i is strictly concave.

Step 3. Now we are ready to finish the proof. Step 2 implies that each stationary point of w^i is a unique maximizer and so equals α_*^i . Below we determine α_*^i . Having α_*^i , it is straightforward with step 1 to verify that the formula (A10) holds. We distinguish between four cases.

By step 2, $\hat{\alpha}_0$ is the unique maximizer of $g_0^i - C_i$. Step 1 implies that $\hat{\alpha}_0$ also is the unique maximizer of w^i if $s_i / A_i \geq r_j^f$. Thus in this case $\alpha_*^i = \hat{\alpha}_0$.

By step 2, $\hat{\alpha}_1$ is the unique maximizer of $g_1^i - C_i$. Steps 1 and 2 imply that $\hat{\alpha}_1$ also is the unique maximizer of w^i if

$$\frac{s_i}{A_i} < r_j^f \quad \text{and} \quad \hat{\alpha}_1 - \frac{t}{\lambda_i} \leq \frac{2r_j^f}{\hat{r}_i^d} \sqrt{\frac{s_i}{A_i} / r_j^f}.$$

A calculation shows that these two inequalities are equivalent with

$$r_j^f \left(\frac{2r_j^f}{\hat{r}_i^d} - \frac{\lambda_i}{c_i} \right)^{-2} \left(1 - \frac{t}{\lambda_i} \right)^2 \leq \frac{s_i}{A_i} < r_j^f.$$

Thus in this case $\alpha_*^i = \hat{\alpha}_1$.

By step 2, $\hat{\alpha}_2$ is the unique maximizer of $g_2^i - C_i$. Steps 1 and 2 imply that $\hat{\alpha}_2$ also is the unique maximizer of w^i if

$$\frac{s_i}{A_i} < r_j^f \quad \text{and} \quad \frac{2r_j^f}{\hat{r}_i^d} \sqrt{\frac{s_i}{A_i} / r_j^f} < \hat{\alpha}_2 - \frac{t}{\lambda_i} < \frac{2r_j^f}{\hat{r}_i^d}.$$

A calculation shows that these two inequalities are equivalent with $s_i/A_i < r_j^f$ and $2r_j^f/\hat{r}_i^d - \lambda_i/c_i \geq 1 - t/\lambda_i$. Thus in this case $\alpha_*^i = \hat{\alpha}_2$.

By step 2, $\hat{\alpha}_3$ is the unique maximizer of $g_3^i - C_i$. Steps 1 and 2 imply that $\hat{\alpha}_3$ also is the unique maximizer of w^i if $s_i/A_i < r_j^f$ and $\hat{\alpha}_3 - t/\lambda_i \geq 2r_j^f/\hat{r}_i^d$. A calculation shows that these two inequalities are equivalent with $s_i/A_i < r_j^f$ and $2r_j^f/\hat{r}_i^d - \lambda_i/c_i \leq 1 - t/\lambda_i$. Thus in this case $\alpha_*^i = \hat{\alpha}_3$. *Q.E.D.*

A.7 Lemma A4

Suppose $m < 1$. Define

$$\bar{s}_* := \frac{\lambda}{\varepsilon} m^2 \left(\frac{3}{4} \lambda \right).$$

Let $\bar{s} \in]0, \bar{s}_*]$ and $t \in [0, \lambda[$. (a) $T(\bar{s}; t) \leq A(\lambda/2)m(\lambda/2)$; (b) $p_* < 1$.

PROOF It is immediate that $\lambda/2$ is the unique maximizer of the function $[0, \lambda[\rightarrow \mathbb{R}$ given by $t \mapsto tm(t)$. We further distinguish between two cases.

Case $t \in [0, 3\lambda/4]$: As m is decreasing, we have

$$\frac{\lambda}{\varepsilon} m^2(t) \geq \frac{\lambda}{\varepsilon} m^2 \left(\frac{3}{4} \lambda \right) = \bar{s}_*.$$

As $\bar{s} \leq \bar{s}_*$, it follows that $p_* = m(t) < 1$ and thus

$$T(\bar{s}; t) = Atm(t) \leq A \frac{\lambda}{2} m \left(\frac{\lambda}{2} \right).$$

Case $t \in [3\lambda/4, \lambda[$: As m is decreasing, we have

$$\frac{\lambda}{\varepsilon} m^2(t) \leq \frac{\lambda}{\varepsilon} m^2\left(\frac{3}{4}\lambda\right) = \bar{s}_* < \frac{\lambda}{\varepsilon}.$$

This implies for $\bar{s} \in]0, (\lambda/\varepsilon)m^2(t)]$ that $p_* = m(t) \leq m(3\lambda/4) < 1$ and thus

$$T(\bar{s}; t) = Atm(t) \leq A\frac{\lambda}{2}m\left(\frac{\lambda}{2}\right),$$

and for $\bar{s} \in](\lambda/\varepsilon)m^2(t), \bar{s}_*]$ that

$$p_* = \sqrt{\bar{s}\frac{\varepsilon}{\lambda}} \leq \sqrt{\bar{s}_*\frac{\varepsilon}{\lambda}} \leq m\left(\frac{3}{4}\lambda\right) < 1$$

and thus

$$T(\bar{s}; t) = At\sqrt{\bar{s}\frac{\varepsilon}{\lambda}} \leq A\lambda\sqrt{\bar{s}\frac{\varepsilon}{\lambda}} \leq A\lambda\sqrt{\bar{s}_*\frac{\varepsilon}{\lambda}} = A\frac{\lambda}{2}m\left(\frac{\lambda}{2}\right).$$

Q.E.D.

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