# The principles of surface flux physics: theory, practice and description of the ECPACK library

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# Chapter 1

# Introduction

This text is developed by a special interest group in eddycorrelation. The group and its activities were initiated by Henk de Bruin (Wageningen UR). The group is composed of researchers from:

- the Royal Netherlands Meteorological Institute (KNMI) in De Bilt (Wim Kohsiek, Bart van den Hurk, Cor Jacobs and Fred Bosveld);
- Wageningen University, department of Meteorology and Air Quality (WUR-METAIR) (Arjan van Dijk until April 2000, Henk de Bruin);
- Utrecht University, Institute for Marine and Atmospheric Research Utrecht (IMAU) (Arjan van Dijk: May 2000 – March 2003).

The objective of the special interest group is the development of

- Fundamental understanding of the eddycorrelation method. There are large discrepancies between estimates for terms in the energy balance at the surface obtained via eddycorrelation and estimates found via other methods. With a fundamental basis of the eddycorrelation method we will be better able to eliminate possible causes of differences in observations. Furthermore we will be able to estimate the consequences of certain assumptions for the accuracy of estimated energy fluxes.
- A protocol for practical use of the eddycorrelation method, including corrections for tilt, trends, distortion etcetera. Where possible the protocol will be implemented in a software library. Measurements which have been done and processed according to this protocol will be more versatile in use. Researchers from different groups can compare their results and conclusions, knowing that they have processed their data equally. Consensus about a protocol will facilitate future experiments by providing the conditions and procedures, which have to be taken into account and followed during the measurements.

The present study is meant to provide the basics of the eddy-correlation method. The theory of the eddy-covariance method is deployed in chapter 2. Here the surface fluxes of sensible heat, water vapour, momentum and scalar densities are related to measurable quantities at a finite height above the ground. This is done via continuity and budget equations. In chapter 3 these relations are worked out for practical purposes. A step-by-step recipe is provided for datareduction of eddy-covariance measurements. The software implementation of this recipe in the software library ECPACK is presented in chapter B. A discussion on the definition of the sensible heat flux is given in appendix A

The report and the software were mainly prepared by Arjan van Dijk while at Wageningen University (except for the part on the planar fit method and corrections for flow distortion using 3D ellipsoids). The NetCDF-frontend for ECPACK, the updated documentation and current maintenance is done by Arnold Moene.

The software as well as this report (and updated versions thereof) can be found at the website of the Meteorology and Air Quality Group of Wageningen University: `http://www.met.wau.nl/projects/jep`.

# Chapter 2

# Theory

This chapter provides the theory of the eddy-covariance method for the estimation of surface fluxes. The energy budget equation at the surface is combined with the continuity equation for the layer between the surface and measurement height. Relations are derived between the surface fluxes (sensible heat, evaporation, surface friction and scalar density fluxes) and measurable quantities at finite height.

## 2.1 Definition of the problem

The dynamics of meteorological processes is strongly influenced by the available energy. At night, when the sun is absent, the behaviour of atmospheric boundary layer is totally different than by day. Most of the transfer of incoming energy from the sun to the Earth takes place at the Earth's surface. An incoming energy flux $Q^*$ (the net radiation) is the net effect of incoming and outgoing long and short wave radiation (see figure 2.1):

$$Q^* = K_{\downarrow} + K_{\uparrow} + L_{\downarrow} + L_{\uparrow} \tag{2.1}$$

where $K_{\downarrow}$ is global short wave radiation from the sun (both direct and diffuse), $K_{\uparrow}$ short wave radiation reflected by the surface, $L_{\downarrow}$ incoming long wave radiation from clouds and atmosphere and $L_{\uparrow}$ long wave radiation reflected and emitted by the surface.

Conservation of energy makes that we can formulate a budget equation for the energy flux at the surface, to see where this incoming energy flux is going. The net radiation can do the following (see figure 2.2):

- Vaporize water (vaporization $E$ [kg m$^{-2}$ s$^{-1}$] with evaporation heat $L_v = 2.5\,10^6\, J\, kg^{-1}$ at $0^oC$). Energy flux $L_vE$ is called the latent heat flux.
- Heat the soil (soil heat flux $G$ $[W\, M^{-2}]$)
- Heat the atmosphere (sensible heat flux $H$ $[W\, M^{-2}]$)
- Be absorbed by the crop (strength $\Delta S$ $[W\, M^{-2}]$). We will assume that the crop is not densely covering the surface, and consequently we can neglect $\Delta S$.

In formula:

$$\begin{aligned} Q^* &= L_vE + G + H + \Delta S \\ &\simeq L_vE + G + H \end{aligned} \tag{2.2}$$

This is a balance at the surface, and therefore the quantities involved in this relation should be measured at the surface. In practice one measures soil heat flux $G$ below the surface and evaporation $E$, net radiation $Q^*$ and sensible heat flux $H$ at a certain distance above the surface. The idea is that turbulent transport is the key to defer

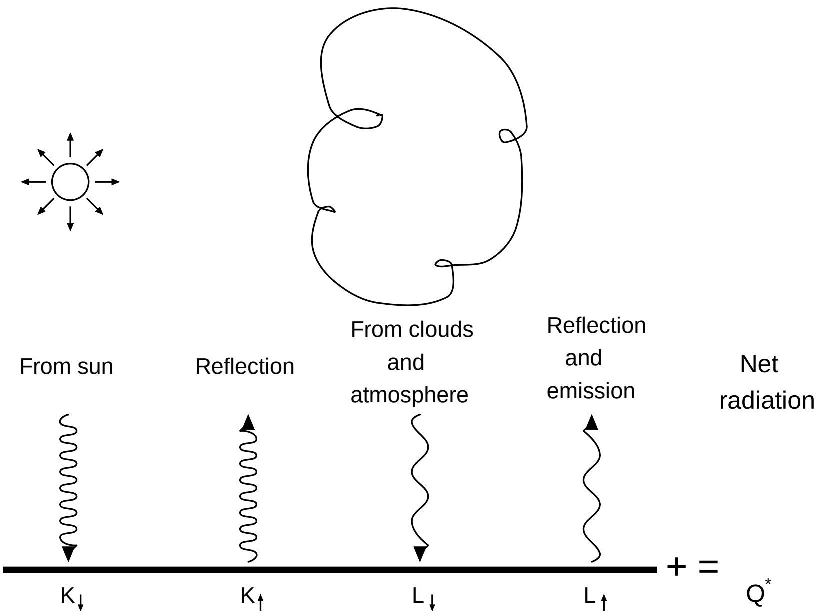


Figure 2.1: Radiation balance at the surface. The net radiation is the heat which is effectively transferred to the surface. It is a composition of incoming and outgoing short and long wave contributions

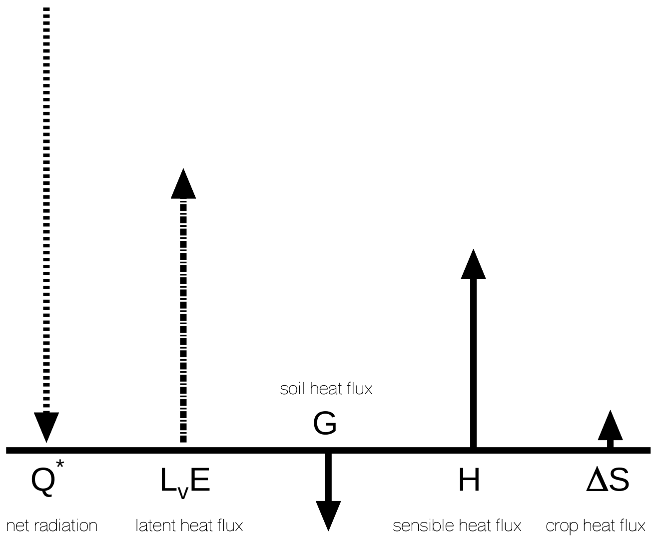


Figure 2.2: Energy balance at the surface

heat and water vapor from the surface. Molecular effects bring heat and vapor in the air, which is just above the surface. The air just above the surface thus will have become warmer and more wet than higher layers of air. There is a vertical gradient of heat and of vapor concentration. When the air of the lowest layer is mixed by turbulence with air from higher layers, the vertical gradients will be levelled and there is vertical transport of heat and of water vapor. This correlation between vertical fluxes and turbulent motions leads to the concept of eddy-correlation. The main concern of eddy-correlation (and therefore of this study) is to translate the signals measured at finite height above the surface into estimates for surface fluxes.

We will consider vertical fluxes $H$, $E$, $\tau$ and $F(\xi)$ of respectively sensible heat, water vapour, horizontal momentum and passive scalar $\xi$ (e.g. $CO_2$). These quantities will be defined in the following section. The definition of the flux $H$ of sensible heat is less straightforward than the definition of the evaporation or of the momentum transfer. Many different definitions of $H$ are made in the literature. Therefore the problem of modelling and estimating the sensible heat flux will be addressed in extra detail.

The continuity equation for an arbitrary quantity will be derived in section 2.3. We will integrate the continuity equation for a horizontally homogeneous situation to construct a budget equation. Budget equations for the thin layer, which is in adjacent to the surface and in which molecular diffusion is transformed into turbulent diffusion, are presented in section 2.4.

The mean vertical velocity plays an important role in the convection of bulk properties. This velocity, which is difficult to measure directly, is related to measureable quantities in section 2.5. Implications of this velocity for evaporation, momentum transfer, sensible heat flux and transfer of $\xi$ will be further elaborated.

In section 2.6 we will discuss some widely spread mis-interpretations of the sensible heat flux and their consequences.

The relation between surface fluxes and quantities measured at a certain height is given in section 2.7 under the assumption of stationary, (statistically) homogeneous conditions.

## 2.2 Basic tools and definitions

In this section some basic tools are explained: continuity relations, which form the core of our physical analysis, and Reynolds-decomposition, with which one separates fluctuations from bulk values.

### 2.2.1 Definition of surface fluxes

Before we can make an analysis of the characteristics of surface fluxes, we first have to make a solid definition of what exactly is a surface flux. In general we can say that **a surface flux is defined as the amount per unit volume of a quantity that passes through a horizontal unit area on the surface**. For the exchange of chemicals (water, $CO_2$, etc.) this definition is sufficiently detailed. For the sensible heat flux and for the shear stress, we have to be more specific.

In appendix A we present a discussion on the definition of the sensible heat flux. It is shown that the following definition is unambiguous and compatible with budget equation 2.2:
**The sensible heat flux $H$ is defined as the flux of heat, which is transferred by the ground to the atmosphere by thermal conduction in the laminar sublayer, during reversibel isobaric processes.**

The definition of the shear stress is similar to the definition of the sensible heat flux: **The shear stress $\tau$ is defined as the friction force exerted by the atmosphere on the Earth's surface.** This definition does not count the force, which is used to accelerate water vapour, which is evaporated with zero velocity at the surface. Once airborne, all forces which act on the vapour are *internal forces* in the atmosphere, and consequently irrelevant for the estimation of the force on the surface.

From the definitions in this section it should be clear that sometimes we are only interested in one contribution to the flux of a certain quantity, and not in the total vertical flux. The strictness of our definitions will be important.

### 2.2.2 Reynolds-decomposition

Instantaneous contributions to the fluxes can vary capriciously from time to time, while the net exchange over a longer time may be relatively constant. To estimate the mean values of the fluxes (over a time $\Delta t$, which we still have to decide about), we introduce the so-called Reynolds decomposition of quantities into their mean and fluctuating parts. Mean values will be marked with an overbar, fluctuating contributions by a prime, e.g.:

$$\xi(t) = \overline{\xi} + \xi'(t) \tag{2.3}$$

$$\overline{\xi} \equiv \frac{1}{\Delta t} \int_{t_0}^{t_0+\Delta t} \xi(t)\,\mathrm{d}t \tag{2.4}$$

$$\xi'(t) \equiv \xi(t) - \overline{\xi} \tag{2.5}$$

Naturally the mean value of a fluctuating term equals zero:

$$\overline{\xi'(t)} = 0 \tag{2.6}$$

Reynolds decomposition will be an important tool in the rest of this study.

## 2.3 Continuity- and budget equations

In this section we will gain insight in quantities by making a relation, in which the time rate of change of the concentration of a quantity is coupled to all known sources for that quantity. Such a relation is called the quantity's **continuity equation**. Integration of the continuity equation over a certain volume gives the **budget equation**. Budget equations will be used to construct expressions for surface fluxes of species, sensible heat and horizontal momentum.

### 2.3.1 The continuity equation

We study the flux of quantity $\xi$, where $\xi$ gives the amount of a physical quantity per volume (its concentration). The flux-vector associated with quantity $\xi$ will be called $\vec{J}(\xi)$, and gives the amount of $\xi$, which passes per unit of time through a unit area. An important contribution to the total flux of quantity $\xi$ is convective flux $\vec{J}_c(\xi)$. The convective contribution gives the amount of $\xi$ that is transported along with a mass flux. The instantaneous value of the convective contribution $\vec{J}_c(\xi)$ to the flux of $\xi$ is given by the product of the physical quantity with velocity vector $\vec{u}$ (The components of velocity $\vec{u}$ will be called $(u, v, w)$):

$$\vec{J}_c(\xi) \equiv \xi\,\vec{u} \tag{2.7}$$

This is seen as follows: The velocity does not only give the pace with which the air is traveling (unit: m/s), it also reflects the amount of volume, which crosses per unit of time through a unit surface (unit: $(\mathrm{m}^3/\mathrm{m}^2)/\mathrm{s}$). In convective flux $\vec{J}_c$ we count both convection and molecular diffusion (which is a mass-bound flux contribution). In the study of fluxes of chemicals, the convective flux vector is the only flux vector involved:

$$\vec{J}(\text{chemicals}) = \vec{J}_c(\text{chemicals}) \tag{2.8}$$

In the study of the transfer of heat or momentum there are more contributions to the total flux than just the convective term. Heat and momentum may also be transferred by conduction or friction. Furthermore heat may be transferred by radiation. We will use symbols $\vec{J}_i$ to denote other contributions to the flux vector of $\xi$ than the convective term (such as radiation and the flow of potential energy in heat exchange processes), where index $i$ counts the contributions, and symbols $S_i$ for sources and sinks of $\xi$.

The sources of quantity $\xi$ will be called $S_i(\xi)$. Sources of chemicals can be found in chemical reactions and in the evaporation of liquid chemicals. Sources of heat can be chemistry, viscous dissipation and phase transitions.

With the above given notation for flux-vector $\vec{J}$ and sources $S_i$ the general form for a continuity equation is:

$$\textbf{Continuity:} \qquad 0 = \frac{\partial \xi}{\partial t} + \mathrm{div}\,\vec{J}(\xi) - \sum_i S_i(\xi) \tag{2.9}$$

$$= \frac{\partial \xi}{\partial t} + \mathrm{div}\left(\xi \vec{u} + \sum_j \vec{J}_j\right) - \sum_i S_i \tag{2.10}$$

When we combine this relation with the following (usual) definition of the convective derivative:

$$\frac{\mathrm{d}}{\mathrm{d}t} \equiv \frac{\partial}{\partial t} + \vec{u}\cdot\nabla \tag{2.11}$$

then we find the following form for the continuity relation:

$$0 = \frac{\mathrm{d}\xi}{\mathrm{d}t} - \sum_i S_i + \xi\,\mathrm{div}\,\vec{u} + \mathrm{div}\left(\sum_j \vec{J}_j\right) \tag{2.12}$$

A quantity is called conserved when an outflux of that quantity implies that there will be accordingly less of it left in the region where it came from. In other words: there are no sources and sinks of $\xi$:

$$\textbf{Conservation:} \qquad 0 = \frac{\partial \xi}{\partial t} + \mathrm{div}\left(\xi \vec{u} + \sum_j \vec{J}_j\right) \tag{2.13}$$

An example of a conserved quantity: Molecules of a certain kind (concentration $\varrho_\xi$) can only be exchanged via either convection or via molecular diffusion (both represented by flux vector $\vec{J}_c(\varrho_\xi)$). Molecular diffusion plays an important role close to boundaries, where e.g. water molecules evaporate into a layer of air, which has very low speed relative to the ground. The budget equation for a single chemical is therefore:

$$0 = \frac{\mathrm{d}\varrho_\xi}{\mathrm{d}t} + \varrho_\xi\,\mathrm{div}\,\vec{u}_\xi \tag{2.14}$$

where $\vec{u}_\xi$ represents the fractional velocity of the $\xi$-molecules. For the mixture of air as a whole, we can use mean velocity $\vec{u}$ in relation 2.14.

For quantity $q_\xi \equiv \xi/\varrho$, which gives the amount per unit mass of the same quantity of which $\xi$ gives the amount per unit volume ($\varrho$ is air density), the following budget equation can be derived:

$$\frac{\mathrm{d}q_\xi}{\mathrm{d}t} = \frac{1}{\varrho}\frac{\mathrm{d}(\varrho q_\xi)}{\mathrm{d}t} + q_\xi\,\mathrm{div}\,\vec{u} = \frac{1}{\varrho}\left(\frac{\partial(\varrho q_\xi)}{\partial t} + \mathrm{div}\left\{\varrho q_\xi \vec{u}\right\}\right) = \sum_i \tilde{S}_i - \frac{1}{\varrho}\mathrm{div}\left(\sum_j \vec{J}_j\right) \quad \text{with} \quad \tilde{S}_i \equiv \frac{S_i}{\varrho} \tag{2.15}$$

In the derivation of relation 2.15 we have used conservation relation 2.14 for $\varrho$. Quantity $q_\xi$ is called a 'specific quantity'. Source strengths $\tilde{S}_i$ give the creation per unit time of $\xi$ per unit mass, while $S$ is per unit volume. Specific quantities can sometimes provide better insight into exchange processes than absolute quantities. When a quantity $\xi$ is conserved, the continuity equation of the associated specific quantity $q_\xi = \xi/\varrho$ is:

$$\frac{\mathrm{d}q_\xi}{\mathrm{d}t} = 0 \tag{2.16}$$

even in situations with non-stationary density. This relation, which expresses that nothing of interest happens to a conserved quantity, is much simpler than relation 2.14, which includes density effects.

### 2.3.2 The budget equation

To relate the surface flux to measurements at height $z_M$ we have to specify all relevant sources and flux contributions in continuity equation 2.10, and integrate it over a virtual box $V$, which is situated directly above the surface patch of our interest. The measurement position is placed somewhere on the upper boundary of the box. The contact area between the virtual box and the Earth's surface will be called $B_0$. The side boundary $B_s$ of the box consists of lines in vertical direction. The top surface of the box is taken parallel to ground surface $B_0$, and is called $B_t$. To achieve a stable estimate, we take the time average of the integrated continuity equation. The integrated continuity equation, which is called the **budget equation** of an arbitrary quantity with concentration $\xi$ is:

$$
\begin{aligned}
0 = & \underbrace{\frac{1}{\Delta t}\int_V \big(\xi(t_2)-\xi(t_1)\big)\mathrm{d}V}_{\text{storage}} - \underbrace{\sum_i \int_V \overline{S}_i \mathrm{d}V}_{\text{creation}} \\
& + \underbrace{\int_{B_t} \left(\overline{\xi\vec{u}}\right)\cdot\vec{n}_{\text{out}}\mathrm{d}B_t}_{\text{convection via top}} + \underbrace{\int_{B_s} \left(\overline{\xi\vec{u}}\right)\cdot\vec{n}_{\text{out}}\mathrm{d}B_s}_{\text{convection via sides}} - \underbrace{\int_{B_0} \left(\overline{\xi\vec{u}_\xi}\right)\cdot\vec{n}_{\text{in}}\mathrm{d}B_0}_{\text{emission from ground}} \\
& + \underbrace{\sum_j \int_{B_t}\left(\overline{\vec{J}}_j\cdot\vec{n}_{\text{out}}\right)\mathrm{d}B_t}_{\text{other fluxes through top}} + \underbrace{\sum_j \int_{B_s}\left(\overline{\vec{J}}_j\cdot\vec{n}_{\text{out}}\right)\mathrm{d}B_s}_{\text{other fluxes through sides}} - \underbrace{\sum_j \int_{B_0}\left(\overline{\vec{J}}_j\cdot\vec{n}_{\text{in}}\right)\mathrm{d}B_0}_{\text{other fluxes from ground}} \qquad (2.17)
\end{aligned}
$$

where $\vec{n}_{\text{out}}$ is a unit vector, perpendicular to the surface of box $V$, and vector $\vec{n}_{\text{in}}$ is a unit vector, which for reasons of convenience points from the Earth's surface perpendicularly into virtual box $V$. Velocity $\vec{u}_\xi$ is the velocity of the gas-component, which carries $\xi$ through the ground/atmosphere interface (most significant candidate is often evaporating water). At all other segments of the boundary of $V$ (i.e. on $B_s$ and $B_t$) we assume that diffusion is insignificant, and hence take $\vec{u}_\xi = \vec{u}$.

### 2.3.3 Fluxes of species

For fluxes of chemicals, e.g. water vapour, we can take $\xi = \varrho_\xi$. Chemical reactions (source strength $S_{\text{chem}}$) and evaporating droplets (source strength $S_{\text{vap}}$) can produce source contributions in the budget equation for a chemical. The only way to gain or lose species through the boundaries of a volume is via convection. For fluxes of species, the term of interest in budget equation 2.17 is the emission from the ground:

$$
\begin{aligned}
F(\varrho_\xi) \equiv & \frac{1}{B_0} \underbrace{\int_{B_0}\left(\overline{\xi\vec{u}_\xi}\right)\cdot\vec{n}_{\text{in}}\mathrm{d}B_0}_{\text{emission from ground}} \\
= & \underbrace{\frac{1}{B_0\Delta t}\int_V\big(\varrho_\xi(t_2)-\varrho_\xi(t_1)\big)\mathrm{d}V}_{\text{storage}} - \underbrace{\frac{1}{B_0}\int_V \overline{S}_{\text{chem}}\mathrm{d}V}_{\text{chemical production}} - \underbrace{\frac{1}{B_0}\int_V \overline{S}_{\text{vap}}\mathrm{d}V}_{\text{vaporization of fog or rain}} \\
& + \underbrace{\frac{1}{B_0}\int_{B_t}\left(\overline{\varrho_\xi\vec{u}_\xi}\right)\cdot\vec{n}_{\text{out}}\mathrm{d}B_t}_{\text{convection via top}} + \underbrace{\frac{1}{B_0}\int_{B_s}\left(\overline{\varrho_\xi\vec{u}_\xi}\right)\cdot\vec{n}_{\text{out}}\mathrm{d}B_s}_{\text{convection via sides}} \qquad (2.18)
\end{aligned}
$$

In this relation density $\varrho_\xi$ measures the density of the gas-fraction of $\xi$. Solid or liquid $\xi$ is not included in $\varrho_\xi$.

### 2.3.4 Sensible heat flux

The first work on the relation between vertical heat flux and eddy-correlation was written by G.I. Taylor (1914, 1915). A study, which started from budget equations in stead of from ad hoc assumptions, was first done by Montgomery (1951, 1954).

We have defined the sensible heat flux as an energy flux. The study of energy involves thermodynamics, and we adopt the definitions and notations developed in a textbook on thermodynamics by Riegel (1992, pages 237-247). Our starting point will be the first law of thermodynamics, which states that energy is a conserved quantity, and that it can only be transferred either via a transfer of heat or by work done on a system. Let $e_t$ be the total specific energy of a system (an amount of mass), and $\text{đ}q$ the heat added to a unit mass and $\text{đ}w$ the work done *by* the system (symbol đ represents an inexact differential). In formula the first law of thermodynamics is:

$$\mathrm{d}e_t = \text{đ}q - \text{đ}w \tag{2.19}$$

where the amount of work done by the system is related to a change in volume via ($\alpha \equiv 1/\varrho$ is the specific volume):

$$\text{đ}w = p\mathrm{d}\alpha \tag{2.20}$$

Total energy $e_t$ includes internal energy $U$ (the kinetic energy of the random molecular motion), the kinetic energy of the center of mass of the system (mean motion energy) $|\vec{u}|^2/2$ and potential energy $\phi$ (e.g. gravitation):

$$\mathrm{d}e_t = \text{đ}q - \text{đ}w = \mathrm{d}U + \mathrm{d}(|\vec{u}|^2/2) + \mathrm{d}\phi \tag{2.21}$$

Velocity $\vec{u}$ is the barycentric velocity field of the gas. In the atmosphere, two constituents are significant for the energy balance: dry air (with fractional velocity $\vec{u}_d$) and water vapour (fractional velocity $\vec{u}_v$). The relation between the atmospheric barycentric velocity of wet air and the velocities of the gas fractions is:

$$\vec{u} \equiv \frac{\varrho_d \vec{u}_d + \varrho_v \vec{u}_v}{\varrho} \tag{2.22}$$

In the literature we find the following definition of the internal energy $U$ (see the book by Riegel):

> Every system contains some quantity which cannot be changed without producing some change in at least one of the state variables. This quantity *takes a unique value* for every state of the system; it is a function of state and is called the *internal energy* of the system. (...). Although the internal energy of a system is uniquely defined by the state of the system, we have no way of knowing the ”value” of the internal energy for any given state. *We can only determine changes in internal energy*. These changes depend only on the beginning and end states of a system, and are independent of the process.

**This definition of the internal energy is equivalent to the definition of a potential energy.** It is important to notice that one **cannot** devise an experiment, which will give the internal energy of a given reference state.

For an ideal gas we may relate changes in internal energy to changes in temperature $T$ via:

$$\mathrm{d}U = c_v \mathrm{d}T \tag{2.23}$$

where $c_v$ is the specific heat at constant volume. Most atmospheric processes do not conserve the volume of the mass, which is involved. In stead they take place with (more or less) constant pressure. Therefore we introduce **specific enthalpy** $h$, which is also a function of state only. It is defined by the following relation:

$$h \equiv U + \alpha p \qquad \Rightarrow \qquad \mathrm{d}U = \mathrm{d}h - \alpha \mathrm{d}p - p\mathrm{d}\alpha = \mathrm{d}h - \alpha\mathrm{d}p - \text{đ}w \tag{2.24}$$

The relation between changes in enthalpy and changes in temperature is:

$$\mathrm{d}h = c_p \mathrm{d}T \tag{2.25}$$

where $c_p$ is the isobaric specific heat.

The total energy balance, presented in relation 2.21, can be formulated via changes in specific enthalpy as:

$$\mathrm{d}e_t = \text{đ}q - \text{đ}w = \mathrm{d}h - \text{đ}w - \alpha\mathrm{d}p + \mathrm{d}(|\vec{u}|^2/2) + \mathrm{d}\phi \tag{2.26}$$

which gives the following conservation equation for specific enthalpy (n.b.: the balance is made per unit-mass):

$$0 = -\frac{\text{đ}q}{\mathrm{d}t} + \frac{\mathrm{d}h}{\mathrm{d}t} - \alpha\frac{\mathrm{d}p}{\mathrm{d}t} + \frac{\mathrm{d}(|\vec{u}|^2/2)}{\mathrm{d}t} + \frac{\mathrm{d}\phi}{\mathrm{d}t} \tag{2.27}$$

The following source strengths (heat generated per unit of mass) are involved in heat transfer term $\text{đ}q/\mathrm{d}t$:

- Chemical or nuclear reactions : source strength $\tilde{Q}$.
- Viscous dissipation of kinetic energy : source strength $\tilde{M}$.
- Condensation heat of water vapour : source strength $\tilde{C}$.

and we can specify the following energy flux vectors, which can contribute to an influx of heat through the boundaries of a system:

- Radiation : $\vec{J_r}$.
- Thermal conduction : $\vec{J_h}$. This term will give us the sensible heat flux.

In formula form the expression for heat transfer is:

$$\frac{\bar{\mathrm{d}}q}{\mathrm{d}t} = \tilde{Q} + \tilde{M} + \tilde{C} - \frac{1}{\varrho}\mathrm{div}\left(\vec{J_r} + \vec{J_h}\right) \tag{2.28}$$

which combined with continuity relation 2.27 for specific enthalpy gives:

$$0 = \frac{\mathrm{d}(h + \phi + |\vec{u}|^2/2)}{\mathrm{d}t} + \tilde{Q} + \tilde{M} + \tilde{C} - \frac{1}{\varrho}\mathrm{div}\left(\vec{J_r} + \vec{J_h}\right) - \alpha\frac{\mathrm{d}p}{\mathrm{d}t} \tag{2.29}$$

This is a balance per unit mass (the balance of a specific quantity). From relation 2.15 for the general shape of the continuity relation for specific quantities, we can see that a balance per unit volume is found by multiplication of the equation by density and inclusion of convective terms through the boundaries:

$$\begin{aligned} 0 =& \frac{\partial\varrho_d(h_d + \phi_d + |\vec{u}_d|^2/2)}{\partial t} + \frac{\partial\varrho_v(h_v + \phi_v + |\vec{u}_v|^2/2)}{\partial t} + Q + M + C \\ &- \mathrm{div}\left(\vec{J_r} + \vec{J_h} + \varrho_d(h_d + \phi_d + |\vec{u}_d|^2/2)\vec{u}_d + \varrho_v(h_v + \phi_v + |\vec{u}_v|^2/2)\vec{u}_v\right) - \frac{\mathrm{d}p}{\mathrm{d}t} \end{aligned} \tag{2.30}$$

Where removal of a tilde from a quantity means multiplication with density, turning a specific quantity into a concentration per unit volume:

$$\tilde{\xi}\varrho \equiv \xi \tag{2.31}$$

and where fractional quantities associated with either the dry air fraction or the water vapour fraction are indicated with indices $d$ for dry air and $v$ for water vapour respectively:

$$\xi = \xi_d + \xi_v \tag{2.32}$$

We have converted most of the total time derivatives into partial derivatives. This is done to facilitate the transformation of the continuity equation into a budget equation. With only partial derivatives we can integrate relation 2.30 over a virtual box, which does not follow the mean motion of the air. Only the pressure term is still given as total derivative. This term needs special attention, when one integrates the continuity equation: only the convective change in pressure is relevant!

The convective terms transport the absolute enthalpy, potential energy and mean motion kinetic energy. This implies that we have to give an expression for the absolute specific enthalpy and the absolute potential energy. As we have pointed out earlier, enthalpy is defined as a potential energy, and hence it can only be known up to a constant value, the reference value. None of the estimates, which we will make in this study, is allowed to depend on the reference values of either enthalpy or of potential energy. The reason is that, just as with the internal energy, there is **no** experiment that can give the enthalpy or the potential energy of a given reference state.

From differential relation 2.25 we can express the absolute specific enthalpy as:

$$h = h(T_{\mathrm{ref}}) + \int_{T_{\mathrm{ref}}}^{T} c_p(T)\mathrm{d}T \tag{2.33}$$

$$\simeq h(T_{\mathrm{ref}}) + c_p(T_{\mathrm{ref}})(T - T_{\mathrm{ref}}) \tag{2.34}$$

$$= c_p(T_{\mathrm{ref}})T + b \qquad \text{with} \qquad b \equiv h(T_{\mathrm{ref}}) - c_p(T_{\mathrm{ref}})T_{\mathrm{ref}} \tag{2.35}$$

Quantity $b$ represents the reference energy. With this expression for absolute enthalpy, we can express the energy flux vectors associated with convection of mass as follows:

- Convection of enthalpy with dry air: $\vec{J}_d$.

$$J_d \equiv (c_{pd}T + b_d + \phi_d)\varrho_d\vec{u}_d \tag{2.36}$$

- Convection of enthalpy with water vapour: $\vec{J}_v$.

$$J_v \equiv (c_{pv}T + b_v + \phi_v)\varrho_v\vec{u}_v \tag{2.37}$$

Constants $b_d$ and $b_v$ are the reference energies of dry air and water vapour respectively. From the inclusion of potential energies $\phi_d$ and $\phi_v$, for respectively dry air and for water vapour, we can already see that these terms will introduce a second reference value problem. In fact there is no such problem: no physical experiment can reveal the reference value of a potential energy field. If gravity is the only potential involved, then specific potentials $\phi_d$ and $\phi_v$ have the same value. Fractional velocities $\vec{u}_d$ of dry air and $\vec{u}_v$ of water vapour can differ when there is molecular diffusion. We assume that conduction $\vec{J}_h$ plays a significant role only close to the ground. Away from the ground turbulent transfer is dominant.

The budget equation for enthalpy is now (symbol $\Delta$ is used to represent the increase of a quantity from $t_1$ to $t_2$; an index $z$ indicates the vertical component of a quantity):

$$\begin{aligned}
H \equiv \overline{J_{hz}}\Big|_{z=0} = & \underbrace{\frac{1}{B_0\Delta t}\int_V \Delta\left(\varrho_d(c_{pd}T + b_d)\right)\mathrm{d}V}_{\text{storage of enthalpy in dry air}} + \underbrace{\frac{1}{B_0\Delta t}\int_V \Delta\left(\varrho_v(c_{pv}T + b_v)\right)\mathrm{d}V}_{\text{storage of enthalpy in water vapour}} \\
& + \underbrace{\frac{1}{B_0\Delta t}\int_V \Delta\left(\varrho_d\phi_d\right)\mathrm{d}V}_{\text{storage of potential energy in dry air}} + \underbrace{\frac{1}{B_0\Delta t}\int_V \Delta\left(\varrho_v\phi_v\right)\mathrm{d}V}_{\text{storage of potential energy in water vapour}} \\
& + \underbrace{\frac{1}{B_0\Delta t}\int_V \Delta\left(\varrho_d|\vec{u}_d|^2/2\right)\mathrm{d}V}_{\text{storage in mean motion of dry air}} + \underbrace{\frac{1}{B_0\Delta t}\int_V \Delta\left(\varrho_v|\vec{u}_v|^2/2\right)\mathrm{d}V}_{\text{storage in mean motion of water vapour}} \\
& - \underbrace{\frac{1}{B_0}\int_V \overline{Q}\mathrm{d}V}_{\text{creation via chemistry}} - \underbrace{\frac{1}{B_0}\int_V \overline{M}\mathrm{d}V}_{\text{creation via viscous dissipation}} - \underbrace{\frac{1}{B_0}\int_V \overline{C}\mathrm{d}V}_{\text{creation via condensation}} \\
& + \underbrace{\frac{1}{B_0}\int_{B_t} \overline{\left(\vec{J}_r + \varrho_d(h_d + \phi_d + |\vec{u}_d|^2/2)\vec{u}_d + \varrho_v(h_v + \phi_v + |\vec{u}_v|^2/2)\vec{u}_v\right)} \cdot \vec{n}_{\text{out}}\mathrm{d}B_t}_{\text{loss via top}} \\
& + \underbrace{\frac{1}{B_0}\int_{B_s} \overline{\left(\vec{J}_r + \varrho_d(h_d + \phi_d + |\vec{u}_d|^2/2)\vec{u}_d + \varrho_v(h_v + \phi_v + |\vec{u}_v|^2/2)\vec{u}_v\right)} \cdot \vec{n}_{\text{out}}\mathrm{d}B_s}_{\text{loss via sides}} \\
& - \underbrace{\frac{1}{B_0}\int_{B_0} \overline{\left(\vec{J}_r + \varrho_d(h_d + \phi_d + |\vec{u}_d|^2/2)\vec{u}_d + \varrho_v(h_v + \phi_v + |\vec{u}_v|^2/2)\vec{u}_v\right)} \cdot \vec{n}_{\text{in}}\mathrm{d}B_0}_{\text{emission from ground}} \\
& - \underbrace{\frac{1}{B_0}\int_V \overline{\frac{\mathrm{d}p}{\mathrm{d}t}}\mathrm{d}V}_{\text{convective change of pressure}}
\end{aligned} \tag{2.38}$$

### 2.3.5 Ground shear stress

To estimate the ground shear stress, we will make a budget equation for the momentum. A term, which is only relevant at the surface, is the momentum flux $\overline{\overline{J}}_f$ by friction with the surface. A volume source of momentum is interaction with rain $\vec{R}$ via the wakes of the drops. The only other significant body-force is buoyancy $\vec{B}$, which is connected with density fluctuations and gravity. For the rest the continuity equation is determined by convection (symbol $\otimes$ gives the diadic product; in coordinates it is defined as $\left(\vec{\xi}\otimes\vec{\eta}\right)_{ij} \equiv \xi_i\eta_j$):

$$0 = \frac{\partial \varrho_d \vec{u}_d}{\partial t} + \frac{\partial \varrho_v \vec{u}_v}{\partial t} - \vec{R} - \vec{B} + \text{div}\left(\overline{\overline{J}}_f + \varrho_d \vec{u}_d \otimes \vec{u}_d + \varrho_v \vec{u}_v \otimes \vec{u}_v\right) \tag{2.39}$$

The associated budget equation is:

$$\begin{aligned}
-\vec{\tau} \equiv \vec{J}_{fz}\Big|_{z=0} = & \underbrace{\frac{1}{B_0 \Delta t}\int_V \left(\varrho_d \vec{u}_d(t_2) - \varrho_d \vec{u}_d(t_1)\right) \mathrm{d}V}_{\text{storage in dry air}} + \underbrace{\frac{1}{B_0 \Delta t}\int_V \left(\varrho_v \vec{u}_v(t_2) - \varrho_v \vec{u}_v(t_1)\right) \mathrm{d}V}_{\text{storage in water vapour}} \\
& - \underbrace{\frac{1}{B_0}\int_V \overline{\vec{R}} \mathrm{d}V}_{\text{pulling by rain}} - \underbrace{\frac{1}{B_0}\int_V \overline{\vec{B}} \mathrm{d}V}_{\text{pulling by buoyancy}} + \underbrace{\frac{1}{B_0}\int_{B_t} \left(\overline{\varrho_d \vec{u}_d \otimes \vec{u}_d} + \overline{\varrho_v \vec{u}_v \otimes \vec{u}_v}\right) \cdot \vec{n}_{\text{out}} \mathrm{d}B_t}_{\text{convection via top}} \\
& + \underbrace{\frac{1}{B_0}\int_{B_s} \left(\overline{\varrho_d \vec{u}_d \otimes \vec{u}_d} + \overline{\varrho_v \vec{u}_v \otimes \vec{u}_v}\right) \cdot \vec{n}_{\text{out}} \mathrm{d}B_s}_{\text{convection via sides}} - \underbrace{\frac{1}{B_0}\int_{B_0} \left(\overline{\varrho_d \vec{u}_d \otimes \vec{u}_d} + \overline{\varrho_v \vec{u}_v \otimes \vec{u}_v}\right) \cdot \vec{n}_{\text{in}} \mathrm{d}B_0}_{\text{emission from ground}}
\end{aligned} \tag{2.40}$$

## 2.4 The homogeneous transition layer

In general relations 2.18, 2.38 and 2.40 for the surface fluxes of respectively species, sensible heat and momentum are not easily related to one-point measurements. In this section we will reduce the amount of work by restricting the calculation to what is called the transition layer. The transition layer is a thin layer just above the ground, which is too thin to contain source terms and in which molecular diffusion is transformed into turbulent diffusion. The transition layer will give a good description of the first half centimeter of the atmospheric boundary layer and in certain cases it is even valid for a layer which includes the sensors (in which case it is called the **constant flux layer**).

We make the following assumptions about the transition layer (see Sun et al., 1995, p.3164):

- **The top of the layer $\delta z$ is above the viscous surface sublayer**. This assumption implies that at the ground fluxes are dominated by molecular diffusion and thermal conduction.
- At the top of the transition layer, the mixing of dry air and water vapour is dominated by turbulent diffusion (see figure 2.3). This means that bits of wet air are torn apart into smaller bits of wet air, a process in which both wet air constituents (dry air and water vapour) are subjected to the same set of velocity fluctuations. From this observation we conclude that at the top of the transition layer velocities $\vec{u}_d$, $\vec{u}_v$ and $\vec{u}_\xi$ of respectively dry air, water vapour and species $\xi$ must be equal:

  $$\vec{u}_d(z=\delta z) = \vec{u}_v(z=\delta z) = \vec{u}_\xi(z=\delta z) = \vec{u}(z=\delta z) \tag{2.41}$$

- At the surface the **mass flux is dominated by the evaporation of water**. This implied that at the surface the only non-zero velocity component is the vertical velocity of water vapour. The assumption is not valid above surfaces with chemical reactions, like burning forests.
- **Homogeneous vertical flow field**. In an inhomogenous vertical flow field localized updraughts or fixed plumes can give a net vertical transport of dry air.
- **Homogeneous terrain**. Consider a wet spot of terrain, surrounded by dry terrain. The wet spot will evaporate water, but the vertical water vapour flux in the column of air straight above the wet spot will not be constant. Advection of dry air into the column and advection of wet air out of the column by horizontal flow gives a net contribution to the balance of dry air. Assuming horizontal homogeneity we can neglect convective terms in the budget equations via the side boundary of the virtual box.

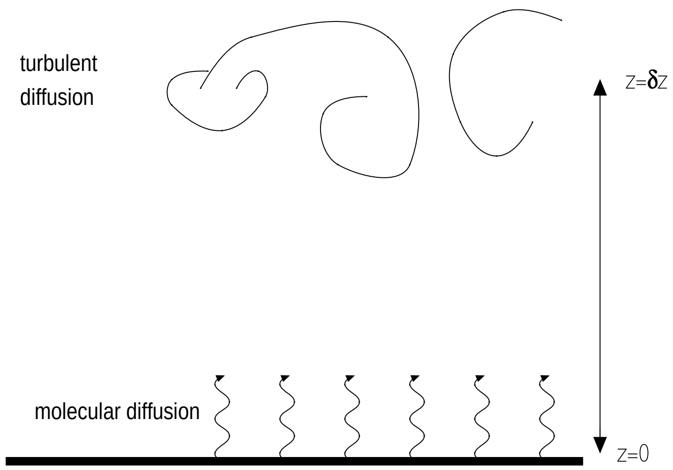


Figure 2.3: Thin layer adjacent to the surface over which budget equations are integrated

- The transition layer is **too thin for storage or creation effects to be significant**.
- **The surface is level**. In non-level situations, the evaporation may interfere with terms in the budget equation for horizontal momentum.
- The potential energy is a function of position and the potential difference between any two points in the virtual box is too small to cause significant contributions to the budget equation.
- The influx of kinetic energy at the surface associated with evaporation is negligible.

With these restrictions, the surface fluxes of species, heat and momentum are given by the following expressions:

$$F(\varrho_\xi) = \overline{\varrho_\xi w}\Big|_{\delta z} \tag{2.42}$$

$$H = \overline{\varrho_d(c_{pd}T + |\vec{u}|^2/2)w}\Big|_{\delta z} + \overline{\varrho_v(c_{pv}T + |\vec{u}|^2/2)w}\Big|_{\delta z} - \overline{\varrho_v c_{pv} T w_v}\Big|_0 \tag{2.43}$$

$$-\tau_\xi = \overline{\varrho_d uw}|_{\delta z} + \overline{\varrho_v uw}|_{\delta z} \tag{2.44}$$

## 2.5 Mean vertical velocity: The Webb-term

In all budget equations, which we have derived in section 2.3, convective fluxes $\vec{J_c}(\xi)$ play an important role. In relations 2.42, 2.43 and 2.44 for the relation between surface fluxes of species, sensible heat and horizontal momentum and measurements at the top of the homogeneous transition layer, these convective fluxes are even dominant. In this section we will show that the vertical convective fluxes cannot be directly estimated from measurements. A method will be presented to circumvent the problems, related to the presence of an inmeasurable but significant mean vertical velocity.

### 2.5.1 Origin of the "eddy-correlation problem"

The mean vertical component of convective fluxes has the general form $\overline{w\xi}$. With Reynolds decompositions for both $w$ and $\xi$ we can write this expression as follows:

$$\overline{J_{cz}(\xi)} = \overline{w\xi} = \overline{w}\,\overline{\xi} + \overline{w'\xi'} \tag{2.45}$$

From relation 2.45 we see that there are two contributions to the mean convective flux of quantity $\xi$: one giving the transport by the mean vertical velocity of the bulk (=mean value) of quantity $\xi$, the other representing a correlated behaviour of the vertical fluctuations and the fluctuations in quantity $\xi$. When we regard the vertical fluctuations as actions of turbulent eddies, then we come to the picture of the second term as turbulent eddies picking up quantity $\xi$ at a lower position, and transporting it to a higher level. This view of the second term in relation 2.45 has led to the expression **eddy-correlation**-method, when this relation 2.45 is used to estimate mean values of vertical fluxes.

One can *in principle* estimate vertical convective fluxes via direct application of relation 2.45 to measured datasets. Nevertheless straightforward integration of instantaneous fluxes will not give a reliable estimate for the mean convective flux. There is a basic problem, which prevents relation 2.45 from being of direct use. This problem, which we will call the **eddy-correlation problem**, is caused by the immeasurability of mean vertical velocity $\overline{w}$. In practical situations the mean vertical velocity is small ($\sim$ 0.1 mm/s), but not exactly zero. We introduce the following notation:

$$
\begin{aligned}
F_m(\xi) &\equiv \overline{w}\,\overline{\xi} \\
F_f(\xi) &\equiv \overline{w'\xi'} \\
\beta(\xi) &\equiv \frac{\overline{w'\xi'}}{\overline{\xi}} = \frac{F_f(\xi)}{\overline{\xi}}
\end{aligned}
\qquad (2.46)
$$

Quantity $\beta(\xi)$, which has dimension m/s, is called the **pushing velocity** of quantity $\xi$. The pushing velocity gives the velocity with which quantity $\xi$ travels relative to the local mean vertical velocity.

In **indifferent situations**, i.e. quantity $\xi$ is not exchanged with the surface, the two terms $F_m(\xi)$ and $F_f(\xi)$ in the flux will cancel (see figure 2.5 for a situation close to indifference with a small flux going off the surface). Consequently we see that the term $F_m(\xi)$ is equally important as the term $F_f(\xi)$. In such situations the mean vertical velocity is cancelled by the pushing velocity of quantity $\xi$ (this was pointed out for $CO_2$-fluxes by Leuning et al. (1982)). We will calculate an indirect method to estimate the mean vertical velocity $\overline{w}$ in an atmospheric boundary layer. Here "indirect" means "via a set of fluctuation terms $F_f(\xi_i)$ of quantities $\xi_i$". The resulting expression will be combined with relation 2.45 for mean vertical fluxes. The best known article on this field has been written by Webb et al. (1980). Inclusion of a mean vertical velocity in estimates for mean vertical fluxes is therefore called the Webb-correction. In our study we will extend Webb's relations to include pressure fluctuations. Flux term $F_m(\xi)$ is just one of the two terms in the expression for the flux and can be of the same order as term $F_f(\xi)$. Therefore we prefer to call flux term $F_m(\xi)$ the **Webb term** in stead of the "Webb correction". The importance of a non-zero mean vertical velocity was pointed out on incorrect grounds by Jones and Smith (1978) (who acknowledge their error later (Smith and Jones, 1979)), and by Brook (1978). Their assumption that there is no net mass flux was shown to be incorrect by Webb et al. (1980), Leuning et al. (1982), Webb (1982), Nicholls and Smith (1982) and Businger (1982). Without any justification Lloyd et al. (1984) subtract the mean velocity contribution from their definition of their surface fluxes, and refer to Webb et al for "some small corrections". The essence of Webb's correction is just to include the term, which Lloyd et al eliminated from their definition. It is peculiar to notice that in recent literature some researchers still omit the Webb term (Kaimal, 1968).

The mean vertical velocity consists of three terms: an evaporation term, a heat-flux term and a friction term. These three terms arise from the following mechanisms (the three mechanisms of mean vertical velocity generation are schematically drawn in figure 2.4):

- **Heat flux induced mean vertical velocity**: When there is vertical transport of heat from the surface into the atmospheric boundary layer, then there is a mechanism of compact blobs of cool air going down, and of expanded blobs (with the same mass) of warm air going up. We see that exchange of heat at the surface is directly coupled with the expansion of air at the surface. This means that, at the surface, there is a net creation of volume, which is the origin of a net vertical velocity.

- **Evaporation induced mean vertical velocity**: Liquid water can evaporate at the surface. Since gaseous water takes three orders of magnitude more space per unit of mass than liquid water, evaporating water is effectively a source of gas at the ground. When 6 millimeters water evaporate in a day, then 6 meters of pure water vapour are "created", and the atmosphere is lifted up 6 m. To satisfy continuity, this water condensates at a higher level in the atmosphere, from where it will be sent back to the surface in its compact liquid form during rain.

- **Surface friction induced mean vertical velocity**: Deceleration of air by friction at the surface induces a negative vertical velocity. When a rough surface forces a blob of air to give up part of its horizontal momentum to that surface, then the velocity decreases. The pressure in that blob will increase, leading to compression. The presence of a horizontal momentum flux indicates that the just described process of blobs transfering horizontal momentum is systematic. We see that a horizontal momentum flux induced by roughness gives a sink of volume at the surface.

The assumption that there is no net transport of dry air will give us sufficient tools to express the mean vertical velocity in terms of fluctuations of the density of dry air. To find an expression for these dry air density fluctuations, we will use standard thermodynamics of ideal gasses. When this will have been done, then we can find an expression for the mean vertical velocity. The resulting expression for the mean vertical velocity will be used to estimate surface fluxes of arbitrary quantity $\xi$. In particular, we will apply the results of our study to the fluxes of heat, water vapour and momentum.

Before Webb's famous article was published, many scientists had assumed that there was no vertical velocity at all (Robinson, 1951, see). Others assumed that there was a small velocity associated with heat transfer, but no mass-flux related velocity component (Kohlsche, 1964, see).

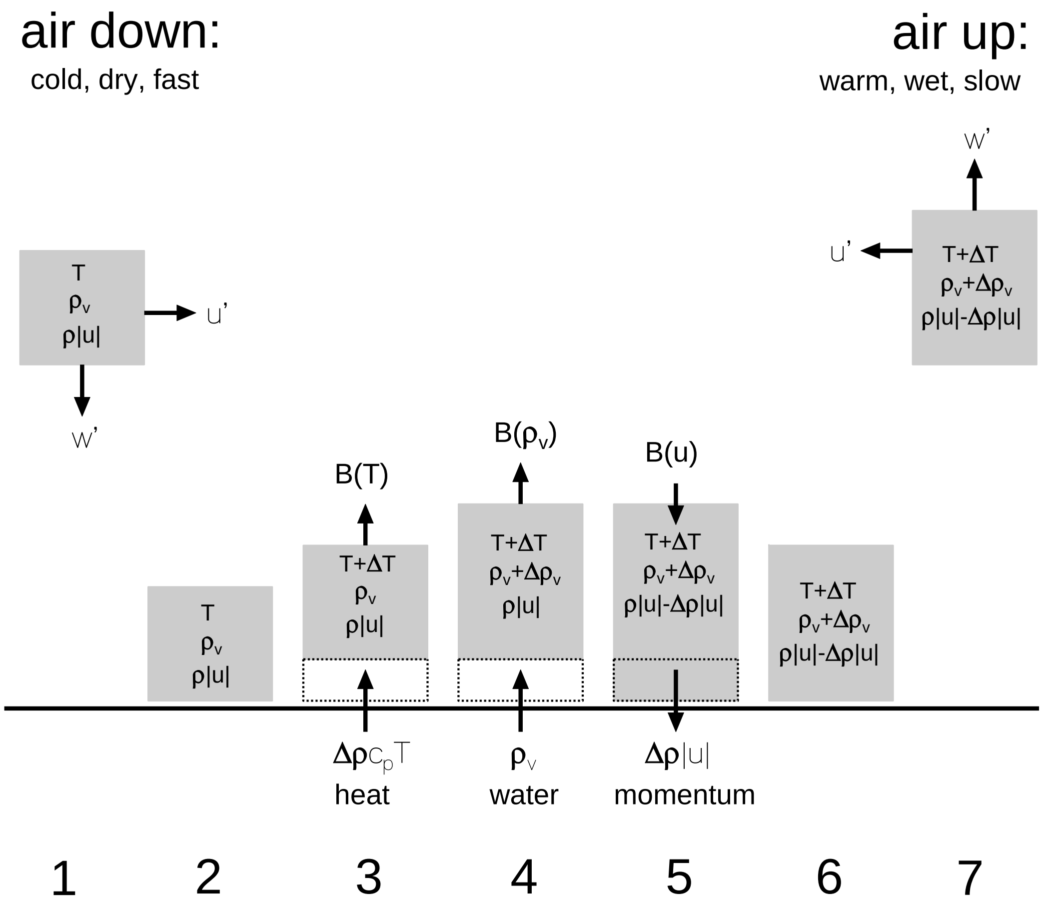


Figure 2.4: Induction of mean vertical velocity via exchange with the surface of heat, water vapour and horizontal momentum. In this example cold dry air (1) with a relatively high velocity approaches the surface (2). When the air absorbs heat, it will expand (3). The evaporation of water into the air gives an increase in volume and hence a vertical velocity (4). When the air looses part of its horizontal momentum (5), the pressure will increase, and consequently the volume will decrease, giving a negative vertical velocity. The air (6) leaves the surface relatively warm, wet and with low velocity (7).

### 2.5.2 Mean vertical velocity in the transition layer

Dry air does neither dissolve in the surface, nor does it evaporate. This implies that there is no surface flux of dry air (density $\varrho_d$) and therefore neither a flux of dry air at the top of the transition layer. This observation gives sufficient grip to estimate the mean vertical velocity at the top of the transition layer, and consequently estimate surface fluxes. With this assumption and a Reynolds-decomposition of the net vertical flux of dry air, we find an expression for the mean vertical velocity at the top of the transition layer:

$$0 = \overline{w\varrho_d} = \overline{w}\,\overline{\varrho}_a + \overline{w'\varrho_d'} \qquad \Rightarrow \overline{w} = -\frac{\overline{w'\varrho_d'}}{\overline{\varrho_d}} = -\beta(\varrho_d) \tag{2.47}$$

The constant-flux character of the transition layer guarantees that this velocity represents the mean vertical velocity throughout the whole transition layer.

We see that the mean vertical velocity equals minus the pushing velocity (defined in relation 2.46) of the exchange of dry air constituent. This means that a mean vertical velocity compensates turbulent transport of dry air, which is conform our assumption of no net dry air flux.

### 2.5.3 Fluctuations in dry air density

In this subsection we will construct an expression for the fluctuations in dry air density, as function of fluctuations in water vapour density, temperature, velocity and pressure. The analysis is still limited to the transition layer. We start with the equation of state of $N$ moles of a mixture of ideal gasses with pressure $p$, volume $V$, temperature $T$. $R$ is the constant of Stefan-Boltzmann.

$$pV = NRT \tag{2.48}$$

The number of moles of gas in the mixture per volume can be found via partial densities $\varrho_i$ and mole-masses $m_i$ for the contributing gasses:

$$\frac{N}{V} = \sum_i \frac{\varrho_i}{m_i} \tag{2.49}$$

According to Dalton's law we can use partial densities $\varrho_i$ to construct the pressure of a gas mixture by superposition of partial pressures from all constituent gasses (substitute $N/V$ from relation 2.49 for the partial densities into equation of state 2.48):

$$\frac{p}{RT} = \sum_i \frac{N_i}{V} = \sum_i \frac{\varrho_i}{m_i} = \frac{\varrho_d}{m_a} + \frac{\varrho_v}{m_v} \tag{2.50}$$

where the last equality follows from restriction of the gas-mixture to a combination of dry air (density $\varrho_d$ and mole-mass $m_a$) and water vapour (density $\varrho_v$ and mole-mass $m_v$). From all gasses, which evaporate at the surface, water evaporates at by far the largest rate. Therefore, in the derivation of the mean vertical velocity we may neglect the evaporation of other gasses like $CO_2$.

We now introduce the Reynolds-decomposition of temperature $T$, pressure $p$ and of partial densities $\varrho_d$ of dry air and $\varrho_v$ of water vapour:

$$\varrho_d = \overline{\varrho}_a + \varrho_d' \tag{2.51}$$

$$\varrho_v = \overline{\varrho}_v + \varrho_v' \tag{2.52}$$

$$p = \overline{p} + p' \tag{2.53}$$

$$T = \overline{T} + T' \qquad \Rightarrow \qquad \frac{1}{T} \sim \frac{1}{\overline{\overline{T}}}\left(1 - \frac{T'}{\overline{\overline{T}}}\right) \tag{2.54}$$

where the last step in decomposition 2.54 for temperature is found via Taylor's expansion up to first order in temperature fluctuations.

We take the mean value of equation of state 2.50 of wet air. The resulting equation is then subtracted from relation 2.50, which gives us a relation between the fluctuating contributions to the equation of state. With use of decompositions 2.51, 2.52, 2.53 and 2.54 we find (up to first order in temperature fluctuations):

$$\frac{\overline{\varrho}_a}{m_a} + \frac{\overline{\varrho}_v}{m_v} = \frac{\overline{p}}{R\overline{T}} \tag{2.55}$$

$$\frac{\varrho_d'}{m_a} + \frac{\varrho_v'}{m_v} = \frac{\overline{p}}{R\overline{T}} \left( \frac{p'}{\overline{p}} - \frac{T'}{\overline{T}} \right) \tag{2.56}$$

In relation 2.55 we have made use of the following assumption:

$$\frac{\overline{p'T'}}{\overline{p}\,\overline{T}} << 1 \tag{2.57}$$

When we substitute the right hand side of relation 2.55 into relation 2.56, then we find the following relation for the fluctuating part of the density of dry air:

$$-\frac{\varrho_d'}{\overline{\varrho}_a} = \mu\sigma\frac{\varrho_v'}{\overline{\varrho}_v} + (1+\mu\sigma)\left( \frac{T'}{\overline{T}} - \frac{p'}{\overline{p}} \right) \tag{2.58}$$

where $\mu$ is the ratio of the mole-masses of dry air and water vapour and $\sigma$ gives the mixing ratio of water vapour and dry air:

$$\mu \equiv \frac{m_a}{m_v} \sim 1.6 \tag{2.59}$$

$$\sigma \equiv \frac{\overline{\varrho}_v}{\overline{\varrho}_a} \sim 0.015 \tag{2.60}$$

### 2.5.4 Pressure fluctuations

To gain better insight into the importance of pressure-fluctuations, we assume that the flow can be considered to be inviscid, and that the velocity fluctuations are laminar disturbances on the mean flow field. With these restrictions on the wind field, we can relate pressure fluctuations to velocity fluctuations via Bernoulli's law:

$$p - p_0 = \frac{1}{2}\varrho|\vec{u}|^2 \tag{2.61}$$

With a Reynolds-decomposition of both pressure, density and velocity we find the following relation for the fluctuating pressure:

$$p' = -\overline{\varrho} \cdot \overline{|\vec{u}|} \cdot |\vec{u}|' - \frac{1}{2}\varrho' \,\overline{|\vec{u}|}^2 \tag{2.62}$$

where $\varrho$ is the density of wet air:

$$\varrho \equiv \varrho_d + \varrho_v \tag{2.63}$$

In atmospheric boundary flow one usually takes a frame of reference such that the direction of the $u$-component of velocity is along the mean flow. In such a frame of reference the length of the velocity vector is therefore given by:

$$|\vec{u}| = \left( (\overline{u} + u')^2 + v'^2 + w'^2 \right)^{0.5} \tag{2.64}$$

$$= \overline{u} \left( 1 + 2\frac{u'}{\overline{u}} + \frac{u'^2 + v'^2 + w'^2}{\overline{u}^2} \right)^{0.5} \tag{2.65}$$

$$\sim \overline{u} \left( 1 + \frac{u'}{\overline{u}} \right) \tag{2.66}$$

where the last equality follows by first order Taylor's expansion. We see from relation 2.66 that only fluctuations in the mean flow direction contribute to the fluctuation in the length of the velocity. This is consistent with the observation that lateral velocity modifications (i.e. perpendicular to the mean flow) merely tilt the velocity without changing its length.

With relation 2.66 for the fluctuating part of the length of the velocity vector we can simplify relation 2.62 for the fluctuating pressure to:

$$\begin{aligned} p' &= -(\overline{\varrho}_a + \overline{\varrho}_v)\,\overline{u}\,u' - \frac{1}{2}(\varrho'_d + \varrho'_v)\,\overline{u}^2 \\ &= -k\overline{p}\left(2\frac{u'}{\overline{u}} + \frac{\varrho'_d + \varrho'_v}{\overline{\varrho}_a + \overline{\varrho}_v}\right) \end{aligned} \tag{2.67}$$

where pressure coefficient $k$ is defined by:

$$k \equiv \frac{\frac{1}{2}\overline{\varrho}\,\overline{u}^2}{\overline{p}} \tag{2.68}$$

When combined with relation 2.67 for the fluctuating pressure, relation 2.58 for the fluctuating density of dry air can be written as:

$$-\frac{\varrho'_d}{\overline{\varrho}_a} = \frac{\sigma\left[\mu(1+\sigma)+k(1+\mu\sigma)\right]}{1+\sigma+k(1+\mu\sigma)}\frac{\varrho'_v}{\overline{\varrho}_v} + \frac{(1+\sigma)(1+\mu\sigma)}{1+\sigma+k(1+\mu\sigma)}\left(\frac{T'}{\overline{T}} + 2k\frac{u'}{\overline{u}}\right) \tag{2.69}$$

Up to first order in $\sigma$ and $k$ relation 2.69 is:

$$-\frac{\varrho'_d}{\overline{\varrho}_a} = (1+\mu\sigma+k)\,\frac{T'}{\overline{T}} + \mu\sigma\,\frac{\varrho'_v}{\overline{\varrho}_v} + 2k\,\frac{u'}{\overline{u}} \tag{2.70}$$

Bernoulli's law is in fact invalid in vortical, or even turbulent, flow, but the weak intensity of most atmospheric turbulence, when compared with the mean velocity, makes that with the analysis in this subsection we have at least an estimate for the order of magnitude of pressure effects. We may assume that pressure coefficient $k$ is fixed by relation 2.68 up to a constant factor, which is or the order 1.

### 2.5.5 Practical estimation of convective fluxes in the transition layer

In expression 2.47 for the mean vertical velocity we can substitute relation 2.70 for the fluctuating part of the density of dry air. This gives the following relation for the mean vertical velocity in the constant flux layer:

$$\overline{w} = (1+\mu\sigma+k)\,\frac{\overline{w'T'}}{\overline{T}} + \mu\sigma\,\frac{\overline{w'\varrho'_v}}{\overline{\varrho}_v} + 2k\,\frac{\overline{w'u'}}{\overline{u}} \tag{2.71}$$

One directly recognizes in relation 2.71 for the mean vertical velocity three contributions: The first term gives the influence of the source of volume at the surface connected with temperature exchange. The second term in relation 2.71 gives the effect of source of volume introduced by evaporation of water. The third term gives the influence of net air compression, when horizontal velocity is transfered to the surface. These three effects are schematically presented in figure 2.4.

The mean vertical velocity is a weighted sum of the pushing velocities connected with the transfers of heat, water vapour and of horizontal momentum. The weighted contributions will be called the **bulk pushing velocities** $B(\xi)$:

$$B(T) \equiv (1+\mu\sigma+k)\,\frac{\overline{w'T'}}{\overline{T}} \tag{2.72}$$

$$B(\varrho_v) \equiv \mu\sigma\,\frac{\overline{w'\varrho'_v}}{\overline{\varrho}_v} \tag{2.73}$$

$$B(u) \equiv 2k\,\frac{\overline{w'u'}}{\overline{u}} \tag{2.74}$$

With neglection of flux dependences on momentum transfer ($k = 0$) our relation 2.71 for the mean vertical velocity reduces to relation 14 in Webb et al. (1980)'s article:

$$\overline{w} = (1+\mu\sigma)\frac{\overline{w'T'}}{\overline{T}} + \mu\sigma\frac{\overline{w'\varrho_v'}}{\overline{\varrho_v}} \tag{2.75}$$

The mean vertical velocity from relation 2.71 can be used in relation 2.45 for the mean vertical convective flux of physical quantity $\xi$ at the top of the transition layer:

$$\begin{aligned}\overline{J_{cz}(\xi)} =& \overline{w\xi} = \overline{w}\,\overline{\xi} + \overline{w'\xi'} \\ =& \overline{\xi}\left(\frac{\overline{w'\xi'}}{\overline{\xi}} + (1+\mu\sigma+k)\frac{\overline{w'T'}}{\overline{T}} + \mu\sigma\frac{\overline{w'\varrho_v'}}{\overline{\varrho_v}} + 2k\frac{\overline{w'u'}}{\overline{u}}\right)\end{aligned} \tag{2.76}$$

## 2.5.6 Fluxes of species, sensible heat and momentum

### 2.5.6.1 Species

According to relation 2.42 the relation between measurements at the top of the transition layer and the actual surface flux of species is fully determined by the convective term. Therefore relation 2.76 gives the correct interpretation of eddycorrelation measurements on the exchange of quantity $\xi$ between the surface and the atmosphere:

$$F(\xi) = \overline{\xi}\left(\frac{\overline{w'\xi'}}{\overline{\xi}} + (1+\mu\sigma+k)\frac{\overline{w'T'}}{\overline{T}} + \mu\sigma\frac{\overline{w'\varrho_v'}}{\overline{\varrho_v}} + 2k\frac{\overline{w'u'}}{\overline{u}}\right) \tag{2.77}$$

The measurement height is still limited to the unpractical distance of about 0.5 cm above the surface, since the analysis was limited to the transition layer. Later in this study the results of this section will be extended to more practical measurement heights.

With neglection of pressure fluctuations ($k = 0$) our relation 2.77 for the flux of quantity $\xi$ is compatible with Webb's relation 24:

$$F(\xi) = \overline{w'\xi'} + (1+\mu\sigma)\frac{\overline{w'T'}}{\overline{T}}\overline{\xi} + \mu\sigma\frac{\overline{w'\varrho_v'}}{\overline{\varrho_v}}\overline{\xi} \tag{2.78}$$

In practical situations the mean vertical velocity is often dominated by the contribution related to the heat flux. When, in such a situation, we neglect the term $\mu\sigma$ with respect to 1, then we find a good estimate for the flux of quantity $\xi$ to be:

$$F(\xi) = \overline{w\xi} = \overline{w'\xi'} + \frac{\overline{w'T'}}{\overline{T}}\overline{\xi} = F_f(\xi) + \beta(T)\overline{\xi} = \big(\beta(\xi)+\beta(T)\big)\,\overline{\xi} \tag{2.79}$$

In other words: the pushing velocity of temperature pushes the mean concentration of quantity $\xi$ in vertical direction away from the surface, giving a contribution to the flux of $\xi$.

If $\xi$ is indifferent, and is consequently not exchanged with the surface, then there is an equilibrium between the pushing velocity of $\xi$ and the pushing velocity of temperature. Via rough estimate 2.79 for the flux of $\xi$ we find:

$$F(\xi) = 0 \qquad \Rightarrow \qquad \beta(\xi) = \frac{\overline{w'\xi'}}{\overline{\xi}} = -\beta(T) = -\frac{\overline{w'T'}}{\overline{T}} \tag{2.80}$$

In words this relation tells us that when the heat flux pushes a quantity, which is indifferent for the surface, to higher levels, then that quantity will push itself back into place. In such cases the fluctuation contribution $F_f(\xi)$ can become negative, even when total flux $F(\xi)$ is positive (see figure 2.5).

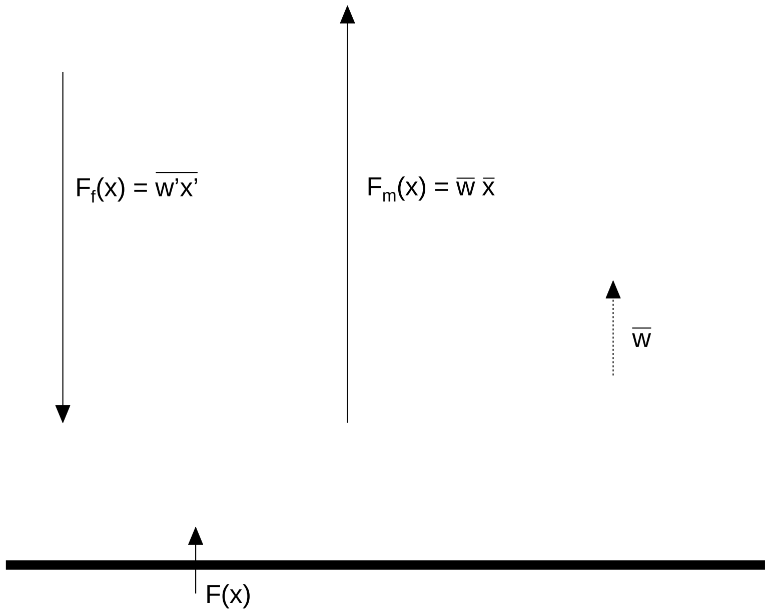


Figure 2.5: Flux contributions for a situation with a small $\xi$-flux away from the surface. The fluctuation term $F_f(\xi)$ is negative, while the total flux $F(\xi)$ is positive.

#### 2.5.6.2 Evaporation

A relation for the evaporation of water is directly found from relation 2.77, when we apply this relation to the flux of water vapour $\varrho_v$:

$$E\left(\simeq F(\varrho_v)\right) = \overline{\varrho_v}\left((1+\mu\sigma+k)\,\frac{\overline{w'T'}}{\overline{T}} + (1+\mu\sigma)\,\frac{\overline{w'\varrho_v'}}{\overline{\varrho_v}} + 2k\,\frac{\overline{w'u'}}{\overline{u}}\right) \tag{2.81}$$

The water involved in the vapour flux is evaporated at the surface. The heat, which is necessary to evaporate the liquid water at the surface, is called the **latent heat flux**. The latent heat flux is given by the following relation:

$$\text{Latent heat flux} \quad = \lambda E \tag{2.82}$$

where $\lambda$ is the evaporation heat of water.

#### 2.5.6.3 Momentum

To apply the currently developed theory to the momentum flux we have to carefully manage all the terms in relation 2.44. We neglect the third order correlation term. We arrive at the following expression:

$$\tau = -\left(\overline{w}\left\{\overline{\varrho}\,\overline{u} + \overline{\varrho'u'}\right\} + \overline{\varrho}\,\overline{w'u'} + \overline{u}\,\overline{\varrho'w'}\right) \tag{2.83}$$

$$= -\left(\overline{w}\,\overline{\varrho}\,\overline{u} + \overline{\varrho}\,\overline{w'u'} + \overline{u}\,\overline{\varrho'w'}\right) \tag{2.84}$$

$$= -\left(\overline{\varrho}\,\overline{w'u'} + \overline{u}\,F(\varrho)\right) \tag{2.85}$$

In relation 2.84 the term $\overline{\varrho'u'}$ is neglected compared with the term $\overline{\varrho}\,\overline{u}$. The mass flux $F(\varrho)$ in relation 2.85 is fully determined by evaporation $E$ (the surface does not evaporate dry air or soil).

When we substitute relation 2.81 for the evaporation into relation 2.85 for the momentum transfer, then we arrive at the following expression (up to first order in $\sigma$ and $k$):

$$\tau = -\overline{\varrho}\,\overline{u}\left(\frac{\overline{w'u'}}{\overline{u}} + \sigma\,\frac{\overline{w'T'}}{\overline{T}} + \sigma\,\frac{\overline{w'\varrho_v'}}{\overline{\varrho_v}}\right) \tag{2.86}$$

#### 2.5.6.4 Sensible heat

In relation 2.43 for the sensible heat two types of terms can be found: one type associated with vertical convection of enthalpy (this contribution is called $H_{\text{enth}}$), the other with vertical convection of kinetic energy (called $H_{\text{kin}}$):

$$H = \underbrace{\overline{\varrho_d c_{pd} T w}\Big|_{\delta z} + \overline{\varrho_v c_{pv} T w}\Big|_{\delta z} - \overline{\varrho_v c_{pv} T w_v}\Big|_0}_{H_{\text{enth}}} + \underbrace{\frac{1}{2}\,\overline{\varrho |\vec{u}|^2 w}\Big|_{\delta z}}_{H_{\text{kin}}} \tag{2.87}$$

With use of a Reynolds decomposition of the mass-flux and temperature we can write the enthalpy-flux term as:

$$H_{\text{enth}} = \left(c_{pd}\left(\overline{\varrho_d w}\,\overline{T} + \overline{(\varrho_d w)' T'}\right) + c_{pv}\left(\overline{\varrho_v w}\,\overline{T} + \overline{(\varrho_v w)' T'}\right)\right)_{\delta z} - \left(c_{pv}\left(\overline{\varrho_v w_v}\,\overline{T} + \overline{(\varrho_v w_v)' T'}\right)\right)_0 \tag{2.88}$$

We substitute the following observations in relation 2.88 for $H_{\text{enth}}$:

- In the laminar sublayer the fluctuations $\xi'$ of any quantity $\xi$ are dominated by mean values $\overline{\xi}$:

  $$\xi'\Big|_{z=0} = 0 \qquad \Rightarrow \qquad \overline{w'\xi'}\Big|_{z=0} = 0 \tag{2.89}$$

  With this assumption we can eliminate the last term.

- The temperature of the transition layer differs from the surface temperature by an amount $\Delta T_s$. When we combine this assumption with the old assumption that the mean mass flux of water vapour is constant through the transition layer, then we can combine the third and the fifth term:

  $$\left(c_{pv}\,\overline{\varrho_v w}\,T\right)_{\delta z} - \left(c_{pv}\,\overline{\varrho_v w}\,T\right)_0 = c_{pv}\,\overline{\varrho_v w}\,\Delta T_s \tag{2.90}$$

- The first term vanishes by virtue of the assumption that there is no net dry air flux.

With these observations the integrated energy budget equation gives the following relation for the enthalpy convection contribution to the sensible heat flux (notice that we did not assume zero mean vertical velocity!):

$$H_{\text{enth}} = c_{pd}\,\overline{T'\,(w\varrho_d)'}\Big|_{\delta z} + c_{pv}\,\overline{T'\,(w\varrho_v)'}\Big|_{\delta z} + c_{pv}\,\overline{\varrho_v w}\,\Delta T_s \tag{2.91}$$

The specific heat of air is approximately given by the following relation taken from Nicholls and Smith (1982):

$$c_{p,\text{wet air}} = c_{pd}(1 + 0.84q) \tag{2.92}$$

In our study we will assume that this dependency of the specific heat of wet air on the specific humidity is negligible, and assume that $c_p$ (without reference to a gas) is constant. With this assumption, relation 2.91 for $H_{\text{enth}}$ reduces to:

$$H_{\text{enth}} = c_p\,\overline{T'\,(w\varrho)'}\Big|_{\delta z} + c_p\,\overline{\varrho_v w}\,\Delta T_s \tag{2.93}$$

$$= c_p\,\overline{T}\,\overline{\varrho}\left(\left(1 + \sigma\frac{\Delta T_s}{\overline{T}}\right)\frac{\overline{w'T'}}{\overline{T}} + \overline{w}\,\frac{\overline{\varrho' T'}}{\overline{T}} + \sigma\frac{\Delta T_s}{\overline{T}}\,\frac{\overline{w'\varrho_v'}}{\overline{\varrho_v}}\right)_{\delta z} \tag{2.94}$$

In the second equality we have neglected third order correlations. The second term in this relation can be neglected in comparison with the first term. This is easily seen via a consideration of the respective orders of magnitude, indicated with symbol $O$ (estimates are taken from Kohlsche (1964)):

$$O\left(\frac{|w'|}{\overline{w}}\right) \simeq 10^3 \text{ to } 10^4 \tag{2.95}$$

$$O\left(\frac{|T'|}{\overline{T}}\right) \simeq 10^{-3} \text{ to } 10^{-4} \tag{2.96}$$

$$O\left(\frac{|\varrho'|}{\overline{\varrho}}\right) \simeq 10^{-3} \tag{2.97}$$

and for the temperature jump at the surface we take

$$O\left(\frac{|\Delta T_s|}{\overline{T}}\right) \simeq 10^{-1} \text{ to } 10^{-3} \tag{2.98}$$

With these estimates we find (symbol $R(\xi,\eta)$ denotes the normalized correlation function of quantities $\xi$ and $\eta$):

$$\begin{aligned} O\left(\frac{\overline{w}\,\overline{\varrho' T'}}{\overline{\varrho}\,\overline{w' T'}}\right) &= \frac{O\left(\frac{|\varrho'|}{\overline{\varrho}}\right)}{O\left(\frac{|w'|}{\overline{w}}\right)} \frac{R(\varrho,T)}{R(w,T)} \\ &= (10^{-6} \text{ to } 10^{-7}) \frac{R(\varrho,T)}{R(w,T)} \end{aligned} \tag{2.99}$$

We can now derive a criterion for the density/temperature term in relation 2.94 being significant compared with the velocity/temperature term. Let us assume that the density/temperature term has to be taken into account, when it has a value, which is more than one percent of the velocity/temperature term. From the order of magnitude estimates we see that this requires that the correlation coefficient of density with temperature should be at least four orders of magnitude larger than the correlation coefficient of vertical velocity with temperature. In general this condition will not be met. Therefore we have proved our hypothesis and we can neglect the second term in relation 2.94.

The terms involving the temperature jump across the surface can be neglected via a similar reasoning. Only in extreme conditions these terms can become relevant. Consequently relation 2.94 can be written as:

$$H_{\text{enth}} = c_p \overline{\varrho}\, \overline{w' T'}\Big|_{\delta z} \tag{2.100}$$

It is important to recognize from this relation, that there is no Webb-term involved in the contribution to the sensible heat flux from enthalpy convection! This is the most classic relation to have been used for flux estimation (e.g.by Wesely et al. (1970)).

The term involving convection of kinetic energy is related to measurable quantities as follows: Take the mean velocity along the first coordinate (we have done this already in the conversion of pressure fluctuations to velocity fluctuations). The length of the velocity vector is then only modified by fluctuations in mean flow direction. This gives:

$$\overline{|\vec{u}|^2} = \overline{U}^2 + \overline{u'^2} \tag{2.101}$$

$$\overline{(|\vec{u}|^2)'\,\xi'} = 2\overline{U}\,\overline{u'\xi'} \tag{2.102}$$

With these expressions we can expand $H_{\text{kin}}$ as:

$$H_{\text{kin}} = \frac{1}{2} E\left(\overline{U}^2 + \overline{u'^2}\right) + \overline{\varrho}\,\overline{U}\,\overline{w' u'} + \overline{w}\,\overline{U}\,\overline{\varrho' u'} \tag{2.103}$$

The last term can be neglected compared with the middle term via relations 2.95 and 2.97. The first term gives the vertical convection of kinetic energy (of both mean and turbulent flow fields) driven by evaporation. The second term gives the loss of kinetic energy via friction with the surface.

In total the sensible heat flux can be related to measurable quantities at the top of the transition layer via:

$$H = \left\{ c_p \overline{\varrho}\,\overline{w' T'} + \frac{1}{2} E\left(\overline{U}^2 + \overline{u'^2}\right) + \overline{\varrho}\,\overline{U}\,\overline{w' u'} \right\}_{\delta z} \tag{2.104}$$

We will now estimate the relative importance of these three contributions. We use the following characteristic values: $c_p \sim 1000$ J kg$^{-1}$K$^{-1}$, $\varrho \sim 1$ kg m$^{-3}$, $w' \sim u' \sim 1$ m/s, $T' \sim 0.5\ ^o$C, $R(w,T) \sim 0.5$, $E \sim 6$ mm per 12 hours, $\overline{U} \sim 1$ m/s, $R(w,u) \sim -0.3$. With these values we find:

$$H_{\text{enth}} \sim 250 \quad \text{W m}^{-2} \tag{2.105}$$

$$\frac{1}{2} E\left(\overline{U}^2 + \overline{u'^2}\right) \sim 1.4 \cdot 10^{-4} \quad \text{W m}^{-2} \tag{2.106}$$

$$\overline{\varrho}\,\overline{U}\,\overline{w' u'} \sim -0.3 \quad \text{W m}^{-2} \tag{2.107}$$

From these estimates it is clear that, at the top of the transition layer, the sensible heat flux is sufficiently accurately estimated by the enthalpy flux.

## 2.6 Common practice in eddy-correlation

Many researchers use the following definition of the sensible heat flux (Kohlsche, 1964, see):

$$\textbf{Common definition} \qquad \begin{aligned} H \equiv & F(\varrho c_p T) \sim c_p F(\varrho T) \\ = & c_p \left( \overline{\varrho}\,\overline{w}\,\overline{T} + \overline{\varrho}\,\overline{w'T'} + \overline{w}\,\overline{\varrho' T'} + \overline{T}\,\overline{w'\varrho'} + \overline{\varrho' w' T'} \right) \end{aligned} \tag{2.108}$$

At the end of the former subsection we have already seen that we can neglect the third and the fifth terms in this relation:

$$H = c_p \left( \overline{w}\,\overline{\varrho}\,\overline{T} + \overline{\varrho}\,\overline{w'T'} + \overline{T}\,\overline{\varrho' w'} \right) \tag{2.109}$$

$$= c_p \left( \overline{\varrho}\,\overline{w'T'} + \overline{T}\,F(\varrho) \right) \tag{2.110}$$

The second term in this relation measures the flux of enthalpy associated with the net vertical mass flux (which equals evaporation $E = F(\varrho)$), with zero Kelvin as reference temperature. This counts all thermal energy in the water molecules. Such a contribution by the evaporation to the sensible heat is not very realistic. Consider for example a situation in which all radiation, which is received by the surface, is used to evaporate water (no soil heat flux: the surface is a thermally isolated). Assume that there are no temperature fluctuations. In this case there should not be a sensible heat flux, because our energy balance is already closed. Nevertheless, we find a contribution to $H$ via relation 2.110. The necessity to make a specific choice for the reference temperature already tells the incorrectness of this expression. The difference between this expression and our expression 2.100 is relatively small, which is seen as follows: Follow the analysis of the momentum flux in section 2.5.6.3 and substitute relation 2.81 for the evaporation into relation 2.110. We find:

$$H = c_p \overline{\varrho}\,\overline{T} \left( (1+\sigma) \frac{\overline{w'T'}}{\overline{T}} + \sigma \frac{\overline{w'\varrho_v'}}{\overline{\varrho_v}} \right) \tag{2.111}$$

Coefficient $\sigma$ is of the order of 0.015. This implies that the difference between (disputable) relation 2.111 and (correct) relation 2.100 is only small.

One way to eliminate the influence of enthalpy flux contribution measured from zero Kelvin is to assert that there is no vertical net flux of mass. This incorrect reasoning (there is a net mass flux and it equals evaporation) is presented by Robinson (1951, page 65), Bernhardt (1961), Businger and Deardorff (1967), Bakan (1978) and Swinbank (1951, page 142) (he is aware of the possible presence of a mean vertical velocity and couples the elimination of the vertical mass flux to the necessity to make his analysis independent from the value of the reference enthalpy).

Other researchers try to brush absolute enthalpies under the carpet, by invoking a mysterious ”reference temperature” $T_0$. Without any reference to its origin one subtracts this $T_0$, which is usually taken around the mean surface temperature, from all temperatures. With the inclusion of this reference temperature, the unwanted effects are reduced to insignificance, which clears the conscience. This strategy is adopted by e.g. Montgomery (1948, 1951, 1954, page 270), Businger (1982, page 1890), Kraus (1969, page 9) and Webb et al. (1980, page 93). Frank and Emmitt (1981) simply put the reference temperature at 0 Kelvin and adopt a wrong expression for the mass flux. It is no surprise that their findings predict differences in fluxes from other estimation methods of 25 percent. The ”reference temperature escape” from absolute enthalpies is pointed out to be conceptually wrong by e.g. Sun et al. (1995).

Some researchers encounter problems when they neglect terms in relation 2.109 (which itself is incorrect). We will now show that the three terms in relation 2.109 are of comparable absolute value. The reason why we study an incorrect relation is that many discussions are still concerned with these details.

$$H = c_p \left( \overline{w}\,\overline{\varrho}\,\overline{T} + \overline{\varrho}\,\overline{w'T'} + \overline{T}\,\overline{\varrho' w'} \right) \tag{2.112}$$

$$= c_p \left( \overline{\varrho}\,\overline{w'T'} + \overline{T}\,F(\varrho) \right) \tag{2.113}$$

$$= c_p \overline{\varrho}\,\overline{T} \left( (1+\sigma) \frac{\overline{w'T'}}{\overline{T}} + \sigma \frac{\overline{w'\varrho_v'}}{\overline{\varrho_v}} \right) \tag{2.114}$$

From relation 2.113 it is obvious that if the air is dry (and as a consequence there is no net mass flux) then relation 2.114 for the heat flux reduces to:

$$H(\text{dry air}) = \overline{\varrho}\,c_p\,\overline{w'T'} \tag{2.115}$$

We will now show that the three terms in expression 2.112 for the sensible heat flux are equally important:

$$
\begin{aligned}
c_p\,\overline{w}\,\overline{\varrho}\,\overline{T}(\text{dry air}) &= \;\; c_p\,\overline{\varrho}\,\overline{w'T'} \\
c_p\,\overline{\varrho}\,\overline{w'T'}(\text{dry air}) &= \;\; c_p\,\overline{\varrho}\,\overline{w'T'} \\
c_p\,\overline{T}\,\overline{\varrho' w'}(\text{dry air}) &= -\,c_p\,\overline{\varrho}\,\overline{w'T'}
\end{aligned}
$$

The first equality is made via relation 2.75 for the mean vertical velocity for dry air. The third equality is constructed via the observation that, via the equation of state, we can express density fluctuations in terms of temperature fluctuations. Fluctuations in pressure are neglected when compared with fluctuations in temperature and density. Pressure fluctuations are supposed to induce very quick motions in the fluid, which almost immediatelly level out the differences in pressure. Consequently the fluid has no time to build up pressure fluctuations of any importance. This assumption that pressure fluctuations are negligible is called the *Boussinesq approximation*. With this approximation we find:

$$
\frac{\varrho'}{\overline{\varrho}} \simeq -\frac{T'}{\overline{T}} \tag{2.116}
$$

Three (fundamentally disputable) assumptions about the flow, which are often made when one estimates the heat flux from eddy-correlation measurements, lead to serious mis-estimations of the heat flux:

- When the density of air is mistakenly considered to be constant (but the mean vertical velocity is correctly modelled), then relation 2.77 for the flux would have given us:

  $\varrho' = 0$ : assumption by mistake $\quad\Rightarrow\quad$ $H(\text{dry air}) = 2\overline{\varrho}\,c_p\,\overline{w'T'}$

  This relation differs by a factor of 2 from relation 2.115. We conclude that inclusion of density fluctuations in calculations on heat exchange is essential.

- When one neglects the mean vertical velocity in the estimation of the heat flux via eddy correlation (but correctly accounts for density fluctuations), then one will draw the unrealistic conclusion that there is no heat flux *by definition*:

  $\overline{w} = 0$ : assumption by mistake $\quad\Rightarrow\quad$ $H(\text{dry air}) = 0$

- The one who will simulaneously forget to account both for density fluctuations and for the mean vertical velocity will make a correct estimation of the heat flux in dry air. Such a researcher is simultaneously neglecting two contributions to the flux, which both are of the same order as the total flux. His luck and success has its origin in the cancellation of the two terms, which he has neglected:

  $\varrho' = 0$ and $\overline{w} = 0$ : assumptions by mistake $\quad\Rightarrow\quad$ $H(\text{dry air}) = \overline{\varrho}\,c_p\,\overline{w'T'}$

## 2.7 The stationary homogeneous constant flux layer

The results of section 2.4 are impractical since they are derived for measurements at the top of the transition layer, which is about 0.5 cm from the ground. We will now extend the relations from the former section to more realistic measurement height. We will restrict the analysis to situations satisfying the following conditions:

- **Stationary bulk temperature**. If the bulk temperature of the portion of air below the sensor(s) changes during the experiment, then the density will change. The change in mass of the air below the sensor will induce a net flux of dry air through the sensor(s).

- **Stationary bulk specific humidity**. If the boundary layer builds up water content, then water vapour replaces dry air constituent, which consequently has to move in upward direction through the sensor.

- **Stationary bulk pressure**. With an external change in bulk-pressure the density of dry air below the sensor will be modified, resulting in a net vertical flux of dry air through the sensor.

- **Stationary bulk velocity**. A change in bulk-velocity will influence the density of dry air below the sensor. This gives a net vertical flux of dry air through the sensor.

Homogeneity is assumed both for the surface and for the turbulent flow above it (up to measurement height). For the turbulent flow in the atmospheric boundary layer, homogeneity is imposed in a statistical sense: from place to place the distribution of turbulent motions measured over measurement interval $\Delta t$ must be constant. Among others this assumption implies that there are no non-stochastic eddies. Permanent local updraughts are excluded from the analysis in this section. For the fast, small scale structures (with timescale much faster than $\Delta t$) the homogeneity assumption means that the high wavenumber range of the spectrum must be homogeneous. When the surface is homogeneous, this seems a good assumption. Coherent structures with a lengthscale much larger than the distance made by the mean velocity during measurement interval $\Delta t$ will appear in the measurement as a contribution to the mean velocity, and consequently have a homogeneous appearance. Difficulties may be caused by structures with a timescale comparable with $\Delta t$. The effects of such eddies are not statistically averaged during the measurement interval. In practice the velocity spectrum seldomly has a spectral gap at cycle-time $\Delta t$. Therefore in most cases there will be motions with timescale comparable to $\Delta t$. In this section we will assume that there is a spectral gap at $\Delta t$. In practice one has to keep in mind that this may not be a valid assumption! In a situation, which is homogeneous in the way just described, there is no mean contribution to the budget equations from convective or other fluxes via the sides of the virtual box.

In this homogeneous constant flux layer, integration of the continuity equation can, as was done for the transition layer, be restricted to integration over the vertical coordinate.

The assumption of a constant flux implies that storage and creation effects are negligible. The total flux is dominated by convective and other fluxes (like radiation or conduction) and their sum does not vary with height. The ratio of convective and other fluxes may vary with height.

Storage and creation can be neglected when the measurement height is too low to store or create a significant amount of $\xi$ in the atmospheric layer between the sensor and the surface. The height up to which this is a good assumption can be estimated by comparison of the respective terms in the budget equations. A second condition which allows for the neglection of storage is stationarity: at the end of the measurement the average characteristics of the air between the eddycorrelator and the ground are the same as at the start of the experiment.

In contrast with the transition layer, the vertical gradient of physical quantities in the constant flux layer need not be constant with height.

The budget equation for species (and in particular of water vapour) in the constant flux layer equals the budget equation for the transition layer. Therefore relations 2.42 and 2.77 provide the correct relations between measurements at the top of the homogeneous constant flux layer and the surface fluxes. Relation 2.81 gives the corresponding expression for evaporation. Similarly, relation 2.86 gives the relation for the vertical transport of horizontal momentum.

In expression 2.38 for the sensible heat flux the only terms which are significant in the stationary, homogeneous constant flux layer are the ones associated with loss via the top and emission from the ground:

$$\begin{aligned} H = \;& \overline{\left(\vec{J}_r + \varrho_d(h_d + \phi_d + |\vec{u}_d|^2/2)\vec{u}_d + \varrho_v(h_v + \phi_v + |\vec{u}_v|^2/2)\vec{u}_v\right)}_{\delta z} \\ & - \overline{\left(\vec{J}_r + \varrho_d(h_d + \phi_d + |\vec{u}_d|^2/2)\vec{u}_d + \varrho_v(h_v + \phi_v + |\vec{u}_v|^2/2)\vec{u}_v\right)}_0 \end{aligned} \quad (2.117)$$

With only gravitational potential energy involved this relation reduces to:

$$H = c_p\,\overline{\varrho}\,\overline{w'T'}\Big|_{\delta z} + \overline{J}_{rz}\Big|_{\delta z} - \overline{J}_{rz}\Big|_0 + c_p E(T|_{\delta z} - T|_0) + E\left(g\,\delta z + \frac{1}{2}(\overline{U}^2 + \overline{u'^2})\Big|_{\delta z}\right) + \overline{\varrho}\,\overline{U}\,\overline{w'u'}\Big|_{\delta z} \quad (2.118)$$

In practical situations the only significant terms are those associated with enthalpy convection and with radiation divergence:

$$H \simeq c_p\,\overline{\varrho}\,\overline{w'T'}\Big|_{\delta z} + \overline{J}_{rz}\Big|_{\delta z} - \overline{J}_{rz}\Big|_0 \quad (2.119)$$

# Chapter 3

# Practice

## Introduction

In chapter 2 we related surface fluxes of sensible heat, water vapour, momentum and scalar densities to measurable quantities at finite height. In the current chapter we will show which way to follow when one is interpreting eddy-correlation measurements in terms of these surface fluxes. Mean values and covariances have to be estimated from collected time-series. On top of this many corrections are involved. Some of these corrections refer to the concept of eddy-correlation. Examples of this type of correction are tilt-correction for known misalignment of the setup, non-zero mean vertical velocity, storage of relevant quantities in the air below the sensor and trend-correction. Besides conceptual corrections there are many instrument-specific corrections, mostly associated with non-ideal response. Examples of this type of correction are ”tilt-correction” to enforce expected symmetries on the measured flow, humidity- and side-windcorrection for sonic temperature, frequency-response correction for structures smaller or faster than the sensors and oxygen-correction for humidity. In subsequent sections we will address these steps in the analysis of EC-signals. In the final section to this chapter we will make error-estimates for the mean values and for the surface-fluxes.

## 3.1 Recipe for datareduction

The following sequence of steps is made to convert sets of raw measured eddy-covariance data into flux-estimates and associated tolerance levels:

1. Before any record of measured data is touched, correction matrix $\overline{\overline{W}}_{\mathrm{dist}}$ for flow distortion by relatively small ellipsoidal obstacles can be calculated via the procedure given in section 3.2.

2. Raw voltages and bytes are read from file. Known constant delays between the channels are compensated by appropriate shifted reading of the sequences.

3. Synchronized raw data is converted from voltages and bytes into physical quantities with use of known calibration functions. In this calibration step the sonic velocity is corrected for velocity bias following the procedure given in section 3.3.1 and the sonic temperature estimates are corrected for side-wind via the procedure in section 3.3.2. In the second iteration of this recipe, tiltcorrections are applied to the raw data as part of the calibration. In *both* calibration iterations all velocity vectors are corrected for flow-distortion via matrix $\overline{\overline{W}}_{\mathrm{dist}}$.

4. Provisional mean quantities are calculated via the procedure in section 3.4.

5. Slow measurements of a wet bulb system are used to assess whether the optical hygrometer suffers from calibration drift. The procedure in section 3.3.3 is used to correct for this drift.

6. For each run the mean and (co-)variances of the calibrated quantities are estimated via the procedure in section 3.4.

7. Variances and covariances are corrected for linear additive trends following the procedure of section 3.5.

8. The effects of mis-alignment of the set-up on the mean quantities and on the (co-)variances is corrected for via either of the following tilt-corrections:

   (a) Yaw, pitch and roll-corrections according to the procedures outlined in sections 3.6.1, 3.6.2 and 3.6.3. The assumptions are that the mean velocity per run cannot have a vertical component and that lateral velocity correlations must vanish.

   (b) The Planar Fit Method presented in section 3.6.4. This method assumes that the set-up has a stationary misalignment. This misalignment is estimated from the collection of run-mean velocity vectors. The planar fit method can be extended to effectively reproduce the results of the triple-tilt-correction in the former suggestion, with the advantage that now the angles of different runs and of different set-ups are comparable, which was not the case with the yaw, pitch and roll-angles of the triple tilt..

   Both tilt-corrections involve simple matrix-multiplications on the mean quantities and on the (co-)variances.

9. Now that the tilt-angles are known, all previous steps (except for the first two steps) in this data-reduction recipe are repeated, but now the tilt-corrections are carried out on the raw data. In this second iteration tolerance estimates are generated for both mean quantities and for all (co-)variances using the method given in section 3.11. To correctly perform tilt-corrections one would have had to record all possible third- and fourth-order correlations. Application of a second iteration eliminates the necessity for tilting of tolerances of covariances, because the tilting is now performed on the raw data.

10. All mean values and (co-)variances which involve the sonic temperature are corrected for humidity effects via the relations given in section 3.7. This correction is not applied to the raw data in the calibration-process, because in practice the hygrometer may drop out for certain samples or even during (short) periods. Skipping the bad samples of the hygrometer will still permit for the reliable estimation of mean humidity and of covariances with humidity. Therefore humidity corrections which rely on these estimates can still be used, whereas individual samples can no longer be corrected.

11. After correction of the sonic temperature for humidity, the mean sonic temperature $\overline{T}_s$ is compared with the mean thermocouple temperature $\overline{T}_c$. The sonic temperature relies on a single calibration constant: the acoustic pathlength. A small error in the estimation of the acoustic pathlength can easily lead to a systematic error in the sonic temperature of Kelvins. Possible errors in the estimate for the acoustic pathlength are eliminated by mapping $\overline{T}_s$ on $\overline{T}_c$. This is done by multiplication of all factors $\overline{T}_s$ in the (co-)variances with a factor $\overline{T}_c/\overline{T}_s$:

$$\overline{T_s} \to \overline{T_c} \qquad \Rightarrow \qquad \overline{T_s'x'} \to \overline{T_s'x'}\,\frac{\overline{T}_c}{\overline{T}_s} \tag{3.1}$$

   The associated error $\Delta l$ in acoustic pathlength $l$ is found via:

$$|\Delta l| \simeq \frac{1}{2} l \frac{|\Delta \overline{T}|}{\overline{T}} \tag{3.2}$$

   where $|\Delta\overline{T}|$ is the absolute difference between $\overline{T_s}$ and $\overline{T_s}$, and where $\overline{T}$ is either of the two mean temperature estimates.

12. All (co-)variances involving humidity are corrected for oxygen sensitivity of the optical hygrometer via the procedure given in section 3.8. The temperature estimates, which are used in the oxygen correction procedure, were corrected for humidity in a previous step. This indicates that the relations for estimation of temperature and of humidity are coupled and should therefore in principle be solved simultaneously. We assume that our decoupled approach, which is first order in the errors involved, will provide sufficiently accurate estimates when these errors are sufficiently small (corrections on corrections are considered to be second order effects and consequently neglected).

13. The Moore/Horst model presented in section 3.9 fitted with the Kaimal-model spectra are used to correct (co-)variances for all types of frequency-response errors. Half of the absolute corrections which are made to the covariances are quadratically added to the tolerance estimates which were already made for the covariances (see section 3.9.1.9).

14. The Webb-velocity according to the relation presented in section 3.10 is added to the direct estimate for the mean vertical velocity.

15. Surface fluxes are estimated from the mean values and (co-)variances at measurement height $\delta z$ via relations 2.42, 2.119 and 2.81):

$$F(\varrho_\xi) = \overline{w'\varrho'_\xi}\Big|_{\delta z} + \overline{w}\,\overline{\varrho_\xi}\Big|_{\delta z} \qquad \text{for scalars } \xi \text{ with density } \varrho_\xi\text{, e.g. water vapour: } F(\varrho_v) \equiv E$$

$$\tau = -\overline{\varrho}\,\overline{w'u'} - \overline{u}\,E \qquad \text{for surface friction}$$

$$H = c_p\,\overline{\varrho}\,\overline{w'T'}\Big|_{\delta z} + \overline{J}_{rz}\Big|_{\delta z} - \overline{J}_{rz}\Big|_0 \qquad \text{for sensible heat}$$

Radiation divergence terms $\overline{J}_{rz}\big|_{\delta z}$ and $\overline{J}_{rz}\big|_0$ should be available from additional peripheral measurements conducted during each run.

16. Tolerance levels are estimated for the surface fluxes. Since both scalar flux $F(\varrho_\xi)$ and surface friction $\tau$ (and *not* sensible heat flux $H$!!) have a term dependent on mean velocity $\overline{w}$, one should take care in incorporating the tolerance of $\overline{w}$ into the tolerance levels of the surface fluxes. Even when the mean vertical velocity is rotated out with a tilt-correction, then a statistical error in $\overline{w}$ remains. This error espresses how well one can expect to eliminate the mean vertical velocity of other runs with the tilt-angles of this particular run. Here the number of eddies plays a role: the largest turbulent structures will be horizontally oriented. This implies that fluxes which depend on horizontal velocities (e.g. horizontal transport terms) will tend to have larger tolerances than fluxes which depend on vertical velocity (the ground fluxes).

## 3.2 Flow distortion by small obstacles

Though it may seem a bit early in the datareduction process, we will start with the distortion of turbulent flows by obstacles and how to compensate for this. The reasons are that the distortion matrix can be calculated even before the measurement takes place and that the corrections involved influence other corrections in the datareduction process (i.e. the tilt-corrections).

Most turbulence measurements are intrusive. This means that generally the setup (or the platform, the boom or other apparatus) will induce systematic velocity errors. Generally such errors are attributed to tilt-errors and accordingly processed. Wieringa (1980) has shown that the use of tilt-corrections to correct for flow-distortion can cause severe errors in the estimates of fluxes.

For small obstacles Oost (1991) has made the following correction method: model the obstacle (plus its wake!) as a 3D ellipsoid and consider the flow around the obstacle as a time-varying homogeneous potential flow. For relatively large eddies (when compared with the sizes of the obstacle) this assuption is assumed to be reasonable. The relation between distorted and undistorted velocities is now a simple linear tensor-relation. The tensor can be calculated straightforwardly from the classic analytic solution for potential flow around an ellipsoid by (Milne-Thomson, 1938). The possibility to have three unequal axes makes this model more general than the cylindre-correction by Wyngaard (1981), which uses the same idea of time-varying homogeneous potential flow around the obstacle.

Correction matrix $\overline{\overline{W}}_{\text{dist}}$, for an ellipsoid with semi-axes $(b_1, b_2, b_3)$ along the coordinate axes and for a measurement position $(x_1, x_2, x_3)$ (measured relative to the centre of the ellipsoid) is found via the following relations:

$$W_{\mathrm{dist},ij} = \delta_{ij}\left(1+\frac{b_1b_2b_3}{2-\alpha_i}\int_{\lambda}^{\infty}\frac{\mathrm{d}q}{(b_i^2+q)k_q}\right) - x_j\frac{b_1b_2b_3}{(2-\alpha_j)}\frac{1}{(b_j^2+q)k_\lambda}\frac{\partial\lambda}{\partial x_i} \tag{3.3}$$

$$k_q^2 \equiv (b_1^2+q)(b_2^2+q)(b_3^2+q) \tag{3.4}$$

$$\alpha_i \equiv b_1b_2b_3\int_0^{\infty}\frac{\mathrm{d}q}{(b_i^2+q)k_q} \tag{3.5}$$

$$\frac{\partial\lambda}{\partial x_i} = \frac{2x_i(b_j^2+\lambda)(b_k^2+\lambda)}{(\lambda-\mu)(\lambda-\nu)} \tag{3.6}$$

where $i, j, k = 1, 2, 3,\ i \neq j \neq k$ and where $\lambda$, $\mu$ and $\nu$ are the roots of

$$0 = f(\theta) = x_1^2(b_2^2+\theta)(b_3^2+\theta) + x_2^2(b_3^2+\theta)(b_1^2+\theta) + x_3^2(b_1^2+\theta)(b_2^2+\theta) - (b_1^2+\theta)(b_2^2+\theta)(b_3^2+\theta) \tag{3.7}$$

This flow-distortion correction can only be correctly applied when the tilt-errors have been eliminated from the measurements. Otherwise one will use the wrong position $\vec{x}$ for the position of measurement in this flow distortion correction (generally one will provide $\vec{x}$ in a frame of reference which is supposed not to be tilted). For practical problems we assume that both the yaw- and roll-corrections and the flow-distortion correction are small corrections, such that their commutator can be neglected as a second order effect. This means that it is allowed to perform the flow-distortion correction in the first calibration iteration in the recipe of datareduction, even though there has not yet been any tilt-correction.

## 3.3 Calibration of the raw signals

The raw signals collected by the datalogging system has to be calibrated using the calibration relations established in the laboratory prior to the field-experiment. To prevent X-talk between sensors and unnecessary uncertainties it is best to apply these calibration relations on every sample individually. This as opposed to the use of calibration relations linearized around the mean value for the interpretation of covariances.

To exploit the datalogging system to its maximum capacity, one should use analogue gain/offset-amplifiers to map the range of the sensors onto the sampling range of the datalogger (usually ± 10 Volt). The first step in the processing of EC-data will be the compensation for gain and offset.

Calibrated samples should be validated by checking their values. Irrealistic quantities should be tagged as 'suspicious', and not be used in the estimation of mean and (co-)variances.

Three important calibrations should be carried out at calibration time:

- Velocity bias correction of sonic velocities.
- Side-wind correction of the temperature estimated via the sonic anemometer and
- Correction for calibration drift of optical humidity sensors.

Procedures to follow are:

### 3.3.1 Velocity bias correction for sonic

Sonic anemometers can suffer from systematic mis-estimation of the velocity (bias). Without proper correction such persistent velocity contributions will be falsely attributed to a tilt-error in the setup. One should therefore estimate the bias by placing the sonic in a closed box (zero velocity) and record its velocity estimate. This bias should be subtracted from all velocity estimates. Researchers who use their own calibrations instead of factory specifications may have already eliminated velocity bias in their calibration relations.

### 3.3.2 Side-wind correction of sonic temperature

Transmission times $t_1$ (first firing) and $t_2$ (return pulse in reversed direction) of two pulses are used to find the component of the air-velocity in the direction along the axis through the transmitter/receiver couple. From these two transmission times one estimates the so-called *sonic temperature* $T_s$ via the following relation (see Schotanus et al. (1983), relation 4):

$$T_s \equiv \frac{l^2}{4\gamma R}\left(\frac{1}{t_1}+\frac{1}{t_2}\right)^2 \tag{3.8}$$

where $l$ is the separation between transmitter and receiver, and where physical constant $\gamma R$ equals $403\mathrm{m}^2\mathrm{s}^{-2}\mathrm{K}^{-1}$. This relation gives correct temperature estimates when the air flow direction is along the acoustic path. When velocity components normal to the acoustic path are involved, this relation has to be modified.

The acoustic pulses emitted by sonic anemometers go with the speed of sound in a frame of reference that moves with the air-flow. Consequently, the wavefront, which in the moving frame of reference has its bearing in the direction of the axis from transmitter to receiver, will never arrive at the receiver (see figure 3.1). The firing of

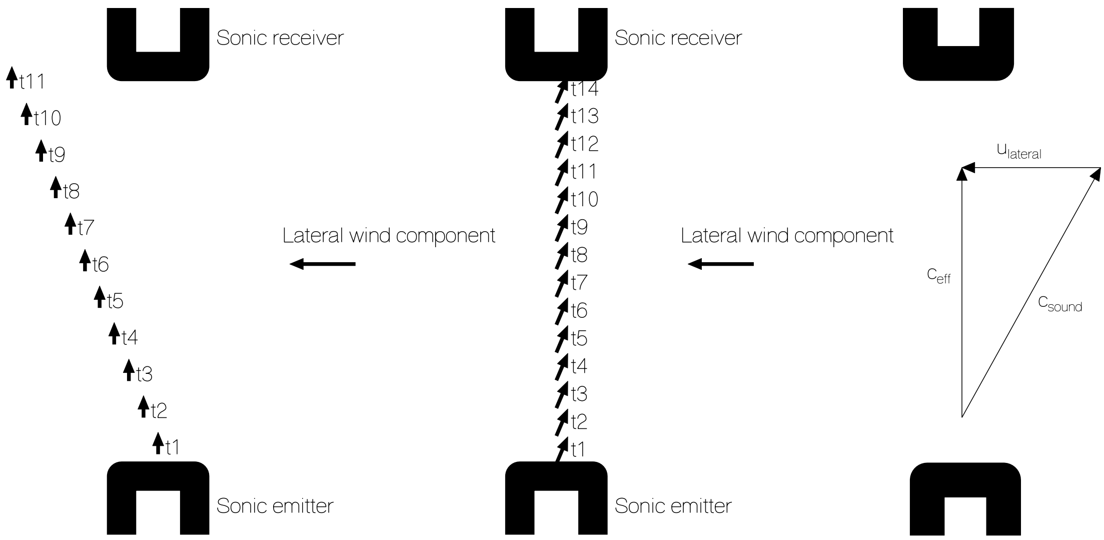


Figure 3.1: Influence of lateral velocity on sonic pulse. On the left: this wavefront will not arrive at the receiver; in the middle: the wavefront which will arrive at the receiver has an inclination and (on the right:) consequently a lower velocity in the direction from transmitter to receiver.

the acoustic transmitter will have to follow an inclined bearing to reach the receiver. This results in an increase of both transmission times $t_1$ and $t_2$, and consequently to underestimation of temperature. Schotanus et al. (1983) have studied the relation between lateral wind $v_n$ and temperature estimation using a sonic anemometer. From their study we can derive a relation between sonic temperature $T_s$ and a side-wind corrected estimate for the temperature of *dry* air, $T_{\mathrm{dry}}$:

$$T_{\mathrm{dry}} = T_s + \frac{v_n^2}{\gamma R} \tag{3.9}$$

For a single-path, vertically oriented sonic anemometer side-wind $v_n$ is related to the velocity components via:

$$v_n^2 = u^2 + v^2 \tag{3.10}$$

where $u$ and $v$ represent the two lateral velocity components normal to the acoustic path. This correction can be performed on all instantaneous data, since the required parameters $u$ and $v$ will be available for any sample with valid $T_s$.

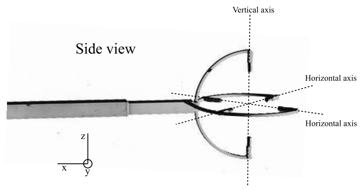


Figure 3.2: Configuration of acoustic anemometer with one axis aligned with the vertical (model studied by Kristensen and Fitzjarrald).

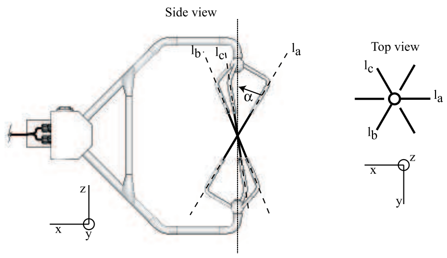


Figure 3.3: Acoustic paths of a 3D sonic anemometer with angle $\alpha$ specifying the deviation from the vertical of *all three* acoustic paths. Note that the CSAT3 depicted here performs the side-wind correction internally.

For a sonic as shown in figure 3.2, with three orthogonal paths of which one is vertically aligned, the relation is:

$$v_n^2 = \frac{1}{3}\left(u^2 + v^2 \quad + w^2 + \frac{1}{2}(u+v)^2 \quad + w^2 + \frac{1}{2}(u-v)^2 = \right) = \frac{2}{3}(u^2 + v^2 + w^2) \tag{3.11}$$

For small vertical velocities when compared with the horizontal velocities, this relation reduces to a relation differing from relation 3.10 by Schotanus et al. (1983) by a factor of 2/3.

For a sonic configured as in figure 3.3, with three acoustic paths $\vec{l}_a$, $\vec{l}_b$ and $\vec{l}_c$, which all have angle $\alpha$ with the vertical axis ($\alpha = 54.7^o$ gives three perpendicular axes):

$$\vec{l}_a = l \begin{pmatrix} -\sin\alpha \\ 0 \\ \cos\alpha \end{pmatrix}; \qquad \vec{l}_b = \frac{l}{2} \begin{pmatrix} \sin\alpha \\ \sqrt{3}\sin\alpha \\ 2\cos\alpha \end{pmatrix}; \qquad \vec{l}_c = \frac{l}{2} \begin{pmatrix} \sin\alpha \\ -\sqrt{3}\sin\alpha \\ 2\cos\alpha \end{pmatrix} \tag{3.12}$$

the squared lateral velocity is the mean value of the squared lateral velocities associated with these three axes:

$$\begin{aligned} v_n^2 &= \frac{1}{3}\left(v_{n,a}^2 + v_{n,b}^2 + v_{n,c}^2\right) \\ &= \frac{1}{2}(1+\cos^2\alpha)(u^2+v^2) + w^2\sin^2\alpha \end{aligned} \tag{3.13}$$

For $\alpha = 0$ this relation reduces to relation 3.10 by Schotanus et al. (1983). For sonics with an angle $\alpha$ of $30^o$ this gives:

$$v_n^2 = \frac{7}{8}(u^2+v^2) + \frac{1}{4}w^2 \tag{3.14}$$

Campbell CSAT sonics *do* have an angle $\alpha$ of $30^o$, however, those sonics do the side-wind correction internally (as mentioned in the manual of the instrument) and a correction in software is not needed.

For the Gill Solent sonics angle $\alpha$ is $45^o$, which gives:

$$v_n^2 = \frac{3}{4}(u^2+v^2) + \frac{1}{2}w^2 \qquad \text{for Gill-Solent} \tag{3.15}$$

This will generally ($u^2 + v^2 >> w^2$) give a smaller side-wind correction than relation 3.10 would have suggested. This difference may even introduce systematic errors in temperature estimates if one erroneously applies relation 3.10 to data collected with Campbell or Gill/Solent sonics, because all correction terms are positive!

Some sonic anemometers include this side-wind correction in the electronics of the sonic (e.g. CSAT3 of Campbell Inc.). Consequently, the correction does not need to be applied to the data. One should consult the manual of the instrument to find out whether the correction is already applied in the instrument.

Schotanus et al. (1983) have included in their study the influence of humidity on temperature estimation via a sonic. It is obvious that, to implement such a correction, one will have to use the signal from a hygrometer. In practice hygrometers may have their black-outs, and as a consequence it may happen that the humidity-correction can only be performed on a small subset of the available time-series. Mixing of humidity-corrected and uncorrected samples would lead to discontinuities in the calibrated time-series. These discontinuities would give unrealistically high variance levels and fluxes. To prevent such artefacts from spoiling our flux-estimates, we postpone humidity-correction of temperature estimates with a sonic until after averaging. Humidity-corrections will be applied to sonic-temperature related variances, covariances and mean values only.

### 3.3.3 Calibration drift of optical hygrometers

All instrumentation should be (re-)calibrated prior to their use in a field-experiment. Factory-provided calibrations may give a guideline for the type of response to be expected in such recalibrations, but one should not rely on their accuracy. Optical hygrometers easily can get dirty or aged, resulting in significant drift of the calibration. Cleaning the lenses may help reduce this unwanted effect, but, to our experience, this does not sufficiently solve the problem. There is however a profitable characteristic in optical hygrometers: in a good approximation humidity $\varrho_v$ is proportional to the logarithm of their response $V$(hygrometer):

$$\log(V(\text{clean hygrometer})) \sim \varrho_v \tag{3.16}$$

When a hygrometer gets dirty, then the amount of light caught by the receiver will be reduced by a constant factor $\alpha_{\rm dirt}$, when compared with the clean situation. When we combine this relation with a Reynolds' decomposition, we find the following relation between the response of a dirty optical hygrometer and humidity:

$$\log(V(t,\text{dirty hygrometer})) = \log(\alpha_{\text{dirt}} V(t,\text{clean hygrometer})) \quad (3.17)$$
$$= \log(\alpha_{\text{dirt}}) + \log(V(t,\text{clean hygrometer})) \quad (3.18)$$
$$\sim c_{\text{dirt}} + \varrho_v(t) \quad (3.19)$$
$$= \left(c_{\text{dirt}} + \overline{\varrho}_v\right) + \varrho_v'(t) \quad (3.20)$$

where $c_{\text{dirt}}$ is proportional to $\log(\alpha_{\text{dirt}})$. From relation 3.20 we see that the signal from a dirty optical hygrometer still correctly gives the fluctuating part of humidity. To find the mean component we have to rely on the signal from a different measuring system, which runs parallel with the optical sensor, e.g. a psychrometer. Attempts to find the mean humidity with the optical sensor are bound to fail: dirt can come onto the sensor during the experiment, making invalid any recalibrations done in the lab.

The mean value of a psychrometer signal, $\overline{\varrho}_v(\text{psychrometer})$, is used to recalibrate optical hygrometers in-situ:

$$\varrho_v(t,\text{from optical hygrometer}) \rightarrow$$
$$\varrho_v(t,\text{from optical hygrometer}) + \left(\overline{\varrho}_v(\text{from psychrometer}) - \overline{\varrho}_v(\text{from optical hygrometer})\right) \quad (3.21)$$

Only the mean value of the parallel system is used. Therefore this system may have slower response than the optical sensor. When records of 30 minutes are analysed, a psychrometer system will be fast enough. Only when (within the record which one is analysing) humidity drifts on a timescale relatively fast when compared with the reaction time of the parallel system, then this in-situ recalibration method will be unreliable.

## 3.4 Estimation of mean values and (co-)variances

The estimation of mean values of huge datasets does involve sums of huge amounts of numbers. It may well be that the dataset is so large that, halfway the establishment of the sum, the fluctuating components in newly added samples vanish when compared to the (huge) running sum due to rounding-off errors. To eliminate this problem some care must be taken. We use sharp brackets to indicate averages over all valid quantities:

$$< y > \equiv \frac{1}{n_{\text{valid}}} \sum_{\text{valid samples}} y(t) \quad (3.22)$$

When in a sum more than one quantity is involved the sum takes only those samples of which *all* relevant quantities are valid.

Mean values $\overline{x}_i$ of the respective quantities $x_i$ are estimated via the following relation:

$$\overline{x}_i \simeq < x_i > + < x_i - < x_i >> \quad (3.23)$$

The first term on the right represents a provisional mean value. In the second term we boost accuracy of the estimate of the mean value by summing the differences of each valid sample from the provisional mean value. In this way the rounding-off problem in the first term is eliminated, making the method apt to cope with large datasets. It should be clear that the mean values are estimated in two scans through the data: one for the estimation of the provisional mean, and a second to complete the estimation.

Once the mean values are known, the covariances are estimated via:

$$\text{Cov}(x_i, x_j) \equiv \overline{(x_i - \overline{x_i})(x_j - \overline{x_j})} \simeq < (x_i - \overline{x_i})(x_j - \overline{x_j}) > \quad (3.24)$$

## 3.5 Trend correction

The signals in eddy-correlation measurements contain variations which play a role in exchange processes and variations which contribute to the RMS, but which are not related to transport phenomena. Diurnal variations e.g.

happen to have major influence on the course of temperature and radiation, but in many cases their net budget per day is relatively small. Consequently one may want to remove the slow fluctuations from the signals and concentrate on the fast fluctuations with a turbulent origin. This action is called *trend correction*. Whether or not trend removal influences the resulting estimates for fluxes may vary from case to case. There will definitely be a reduction in variance and therefore a reduction in estimates which use variances in combination with Monin-Obukhov similarity. Researchers are warned to take caution when trends are removed.

There are many types of trends. The most common types are additive trends and multiplicative trends:

$$x(t) = x_{\text{detrended}}(t) + \text{Trend}(t) \qquad \text{additive trend} \tag{3.25}$$

$$x(t) = x_{\text{detrended}}(t) \cdot \text{Trend}(t) \qquad \text{multiplicative trend} \tag{3.26}$$

Signals can be corrected for such trends by inverting the above relations:

$$x_{\text{detrended}}(t) = x(t) + \overline{x} - \text{Trend}(t) \qquad \text{additive trend} \tag{3.27}$$

$$x_{\text{detrended}}(t) = \frac{x(t)}{\text{Trend}(t)} \qquad \text{multiplicative trend} \tag{3.28}$$

Popular ways for the estimation of trend functions in a set of data are: polynomial regression (first order fits are most popular), a (possibly weighted) moving average applied to the set, (co-)sine functions. Without justification for their preference micrometeorologists generally assume the additive trend model and consequently correct their measurements by subtraction of the trend. Although e.g. in the case of radiation a multiplicative model for diurnal trends gives the best representation of the physics behind the trend.

Linear additive trends are removed from time-series $x(t)$ and $y(t)$ and their covariance as follows (for $x = y$ we find the corresponding relation for variances):

$$x_{\text{detrended}}(t) = x(t) - t' \cdot \frac{\overline{x't'}}{\overline{t'^2}} \tag{3.29}$$

$$\overline{x'y'}_{\text{detrended}} = \overline{x'y'} - \frac{\left(\overline{x't'}\right) \cdot \left(\overline{y't'}\right)}{\overline{t'^2}} \tag{3.30}$$

After trend-correction the fluctuating part of the dataset will have many more changes of sign than before trend-correction. Therefore a major advantage of trend-correction is shown by relations 3.104 and 3.105 for the tolerance estimates for mean values of fluctuating quantities: when trend-correction is allowed then one can get much faster statistical convergence of mean values by removing the trend. In other words: the error of mean values (e.g. covariances) is much smaller after trend-correction.

Other ways to perform trend-correction, which we will not discuss, include digital filtering.

## 3.6 Tilt-correction

When an eddy-correlation setup is erected one will try to orient the vertical axis of the anemometer with the ”true” vertical direction. In practice there will always be a (minute) deviation from the vertical and a corresponding bias in the flux-estimates. To eliminate the bias from the fluxes one has to align the frame of reference with the vertical using a coordinate rotation. The definition of what is ”vertical” may depend on the mean wind direction. Therefore one will have to estimate the misalignment angles of the setup for each wind-sector individually. It is common practice among micrometeorologists to place the first coordinate axis along the mean (horizontal) wind. The subsequent rearrangement of coordinates involves another rotation map.

Let us assume that one individual rotation map is characterised by matrix $\overline{\overline{A}}$:

$$\vec{x} \quad \mapsto \quad \overline{\overline{A}} \cdot \vec{x} \tag{3.31}$$

This same map is used to map velocities, covariances of velocities with scalars $s$ and Reynoldsstresses to the new frame of reference:

$$\vec{u} \mapsto \overline{\overline{A}} \cdot \vec{u} \qquad \Rightarrow \qquad u_i \mapsto \sum_j A_{ij} u_j \tag{3.32}$$

$$\overline{\vec{u}' \otimes s'} \mapsto \overline{\overline{A}} \cdot \overline{\vec{u}' \otimes s'} \qquad \Rightarrow \qquad \overline{\left(u'_i s'\right)} \mapsto \sum_j A_{ij} \overline{\left(u'_j s'\right)} \tag{3.33}$$

$$\overline{\vec{u}' \otimes \vec{u}'} \mapsto \overline{\overline{\overline{A}} \cdot \vec{u}' \otimes \overline{\overline{A}} \cdot \vec{u}'} = \left(\overline{\overline{A}} \otimes \overline{\overline{A}}\right) : \overline{\left(\vec{u}' \otimes \vec{u}'\right)} \qquad \Rightarrow \qquad \overline{\left(u'_i u'_j\right)} \mapsto \sum_k \sum_l A_{ik} A_{jl} \overline{\left(u'_k u'_l\right)} \tag{3.34}$$

The total tilt-correction is performed as a set of subsequent rotations following the above relations. These composing sub-rotations will be presented in this section. In sections 3.6.1, 3.6.2 and 3.6.3 we will address a method to correct for set-up misalignment, which assumes that the tilt of the set-up can be found from the run-mean velocity and from the lateral velocity fluctuation. In section 3.6.4 we will present an alternative method which assumes that the tilt has been stationary throughout the collection of a set of runs. With this latter method one will find nonzero run-mean vertical velocities and lateral velocity covariances.

### 3.6.1 Yaw-correction

The basic laws of turbulent exchange processes are independent of the choice of a frame of reference $u, v, w$. To facilitate discussions on the contributions to such processes we generally place the mean horizontal wind direction along the first coordinate axis. As a consequence the mean wind $\overline{v}$ in lateral direction vanishes identically:

$$\overline{v} \equiv 0 \tag{3.35}$$

This choice is imposed on our data, which had been taken in an arbitrary frame of reference, by application of the following transformation:

$$\overline{\overline{A}}_{\text{yaw}} = \begin{pmatrix} \frac{\overline{u}}{\sqrt{\overline{u}^2+\overline{v}^2}} & \frac{\overline{v}}{\sqrt{\overline{u}^2+\overline{v}^2}} & 0 \\ -\frac{\overline{v}}{\sqrt{\overline{u}^2+\overline{v}^2}} & \frac{\overline{u}}{\sqrt{\overline{u}^2+\overline{v}^2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3.36}$$

### 3.6.2 Pitch-correction

We assume that within measurement accuracy there is no mean vertical velocity over periods of the order of 30 minutes. Any mean vertical velocity arising from eddy-correlation measurements is the consequence of probe-misalignment. When the yaw has already been corrected for using relation 3.36, then the pitch is corrected for using the following transformation:

$$\overline{\overline{A}}_{\text{pitch}} = \begin{pmatrix} \frac{\overline{u}}{\sqrt{\overline{u}^2+\overline{w}^2}} & 0 & \frac{\overline{w}}{\sqrt{\overline{u}^2+\overline{w}^2}} \\ 0 & 1 & 0 \\ -\frac{\overline{w}}{\sqrt{\overline{u}^2+\overline{w}^2}} & 0 & \frac{\overline{u}}{\sqrt{\overline{u}^2+\overline{w}^2}} \end{pmatrix} \tag{3.37}$$

where $\overline{u}$ and $\overline{w}$ are mean velocities after yaw-correction.

An unresolved problem with pitch-correction is connected with the statistical error in the mean vertical velocity. Only a limited number of independent samplings contribute to the mean vertical velocity, which therefore has a non-zero tolerance following relations 3.104 and 3.105. This has an important implication: the pitch-angle will not be accurately estimated and will change from record to record. A time-varying pitch-angle conflicts with the suggestion that this angle represents a systematic misalignment of the setup. Long-term mean values of the velocity components may solve this problem (possibly per wind-sector), but then for each data record there will be a residual mean vertical velocity of the order of the tolerance given by relations 3.104 and 3.105. This residual mean vertical velocity was found to generally exceed the mean vertical velocity according to Webb et al. (1980). It is therefore necessary to find out whether or not the mean vertical velocity per record represents true physics (e.g. updraughts) or time-variations in the alignment of the setup. Experimental evidence is needed to clarify this subject. A suggestion is to place several 3D anemometers close to one another and check if their respective residual tilt-angles (per windsector) correlate in time *after the long term tilt-corrections have been carried out*.

### 3.6.3 Roll-correction

Provided that the mean wind direction does not change with height there is no lateral velocity-correlation. Therefore all correlation of lateral velocity fluctuations $v'$ with vertical velocity fluctuations $w'$ is to be attributed to misalignment of the setup with the vertical. To bring back eddy-correlation measurements to the proper coordinate frame one has to roll the frame of reference around the mean wind direction (now along the first coordinate axis). This involves the following transformation (see Wilczak et al. (2001)):

$$\overline{\overline{A}}_{\mathrm{roll}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & \sin\beta \\ 0 & -\sin\beta & \cos\beta \end{pmatrix} \qquad \text{with} \qquad \beta = \frac{1}{2}\mathrm{atan}\frac{2\overline{v'w'}}{\left(\overline{v'^2} - \overline{w'^2}\right)} \tag{3.38}$$

When the direction of the mean wind changes with height the above roll-correction is incorrect. In the process of datareduction it is therefore important to assess whether or not the assumption $\overline{v'w'} = 0$ is reasonable.

### 3.6.4 The planar fit-method for tilt-correction

Wilczak et al. (2001) have shown that it may be advisable to leave the path of ”classic” tilt-corrections as presented in the former three sections. A collection of runs is assumed to be related to stationary set-up conditions: one unique mis-alignment, involving two angles, describes the tilt-error for all runs. The most commonly used method, the double rotation scheme with just yaw and pitch as presented earlier in this section, is shown to have two disadvantages. The first is that the sampling error of the mean vertical velocity results in a tilt angle estimation error. This adds a random noise component to the longitudinal stress estimate, making individual data run estimates of the stress more uncertain. Second, for measurements over the sea where the cross-stream stress is important, the double rotation method is shown to overestimate the surface stress, due to the uncorrected lateral tilt component.

The triple rotation method of the anemometer axes (yaw, pitch and roll as presented earlier), is shown to result in even greater run-to-run stress errors due to the combined sampling errors of the mean vertical velocity and the cross-wind stress. Also, since it assumes that the true lateral stress is zero, it cannot be used for measurements over the sea where the lateral stress term may be important.

The planar fit method computes a single set of anemometer tilt angles for a set of data runs. Since many data runs are used to determine the tilt angles, it is much less susceptible to sampling errors. The method also allows one to accurately compute the lateral component of the stress. Use of the planar fit method provides greatly improved estimates of the surface stress than the other two commonly used methods.

The planar fit method will lead to non-zero run-mean vertical velocities. Researchers who want to use eddy-coviariance measurements to estimate scalar surface fluxes (e.g. evaporation and $CO_2$) must take these mean vertical velocities into account. The resultant flux contributions reflec the influence of long waves and large eddies on the flux at measurement height.

A disadvantage of the planar fit method may be that, above heterogeneous terrain, the tilt-angles can depend on the mean flow direction. With a higher order fit method (e.g. quadratic) it may be possible to account for such directional dependencies of the tilt-angles. A second disadvantage is the supposition that setup-conditions have remained constant throughout the collection of all runs. When one is measuring above fast growing crop or grass this may not be a good assumption. Furthermore the mechanical structure of the setup (tethering and fixtures) may not be *exactly* stationary up to the millimeter e.g. via temperature effects or via drying of the soil. When in such cases the mean wind-direction changes systematically with time, then the drift in orientation of the set-up will correlate with the run-mean wind direction. The systemacy in the plot of the mean wind directions would then suggest a planar fit, which bears no relation to the actual tiltings of the set-up. A realistic approach may require limitation of the total time over which one wants to use planar fitting to not more than one day.

#### 3.6.4.1 'Classic' planar fit

For each run $i$ the run mean horizontal and vertical velocities ($\overline{u}_{m,i}$, $\overline{v}_{m,i}$ and $\overline{w}_{m,i}$) are recorded (subscript $m$ indicates that the mean values are measured values in the tilted frame of reference). A planar least squares fit is applied to the collection of run mean horizontal and vertical velocities to find constants $b_0$, $b_1$ and $b_2$ in:

$$\overline{w}_m = b_0 + b_1\overline{u}_m + b_2\overline{v}_m \tag{3.39}$$

The solution of the least squares problem is given by the following matrix equation:

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} 1 & \widetilde{\overline{u}} & \widetilde{\overline{v}} \\ \widetilde{\overline{u}} & \widetilde{\overline{u}^2} & \widetilde{\overline{u}\,\overline{v}} \\ \widetilde{\overline{v}} & \widetilde{\overline{u}\,\overline{v}} & \widetilde{\overline{v}^2} \end{pmatrix}^{-1} \cdot \begin{pmatrix} \widetilde{\overline{w}} \\ \widetilde{\overline{u}\,\overline{w}} \\ \widetilde{\overline{v}\,\overline{w}} \end{pmatrix} \tag{3.40}$$

where a tilde is used to denote mean values over the collection of (products of) run-mean values. From coefficients $b_1$ and $b_2$ they extract the following un-tilt tensor $\overline{\overline{A}}_{\text{pf}}$:

$$\overline{\overline{A}}_{\text{pf}} \equiv \begin{pmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & \sin\beta \\ 0 & -\sin\beta & \cos\beta \end{pmatrix} \tag{3.41}$$

where

$$\sin\alpha = \frac{-b_1}{\sqrt{b_1^2+b_2^2+1}} \qquad \sin\beta = \frac{b_2}{\sqrt{b_2^2+1}}$$

$$\cos\alpha = \frac{\sqrt{b_2^2+1}}{\sqrt{b_1^2+b_2^2+1}} \qquad \cos\beta = \frac{1}{\sqrt{b_2^2+1}}$$

One can incorporate into the planar fit matrix the yaw-rotation according to relation 3.36 of the first coordinate axis into the mean velocity over the collection of all runs (see section 3.6.1):

$$\overline{\overline{A}}_{\text{pf + yaw}} = \begin{pmatrix} \frac{u_{\text{day}}}{\sqrt{u_{\text{day}}^2+v_{\text{day}}^2}} & \frac{v_{\text{day}}}{\sqrt{u_{\text{day}}^2+v_{\text{day}}^2}} & 0 \\ -\frac{v_{\text{day}}}{\sqrt{u_{\text{day}}^2+v_{\text{day}}^2}} & \frac{u_{\text{day}}}{\sqrt{u_{\text{day}}^2+v_{\text{day}}^2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \overline{\overline{A}}_{\text{pf}} \tag{3.42}$$

$$\text{where} \quad \begin{pmatrix} u_{\text{day}} \\ v_{\text{day}} \\ w_{\text{day}} \end{pmatrix} \equiv \overline{\overline{A}}_{\text{pf}} \cdot \begin{pmatrix} \widetilde{\overline{u}} \\ \widetilde{\overline{v}} \\ \widetilde{\overline{w}} \end{pmatrix} \tag{3.43}$$

Tensor $\overline{\overline{A}}_{\text{pf + yaw}}$ is applied to all samples or to the run-mean values and (co-)variances to untilt the set-up. In this way measurements from different eddy-covariance setups, which in principle should measure the same turbulent flow, are made directly comparable.

Wilczak et al. (2001) suggest that a possible mean vertical velocity bias in the sonic anemometer can be found via the planar fit method in constant $b_0$ from relation 3.39:

$$w_{\text{bias}} = b_0 \tag{3.44}$$

#### 3.6.4.2 Planar fit with no velocity bias

We have tested the use of relation 3.44 for the estimation of the vertical velocity bias by comparing direct measured values for the bias, which were found by operating an anemometer during half an hour at zero velocity (in its box), with values found for $b_0$ in planar fits to data measured with that same sonic. The mean velocity measured at zero velocity was 4.8 cm/s. The value for $b_0$ varied from 1 mm/s to 5 cm/s, dependent on which subset of runs (all measured on the same day) was used for the planar fit. The capricious dependence of $b_0$ on the specific subset of runs and its strong deviation from the directly measured value of the mean vertical velocity bias to our opinion makes $b_0$ an unreliable estimate for the vertical velocity bias. This is not surprising since the value of $b_0$ is found by extreme extrapolation of the fit plane: from the sector within which all run-mean velocity vectors are found to the point where $\overline{u} = \overline{v} = 0$.

We assume that any bias in the vertical velocity has been measured experimentally and that it is accounted for in the calibration function of the sonic. This implies that the planar fit should be based on a plane through the origin, which leads to the following relations:

$$\overline{w}_m = b_1 \overline{u}_m + b_2 \overline{v}_m \tag{3.45}$$

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} \widetilde{\overline{u}^2} & \widetilde{\overline{u}\ \overline{v}} \\ \widetilde{\overline{u}\ \overline{v}} & \widetilde{\overline{v}^2} \end{pmatrix}^{-1} \cdot \begin{pmatrix} \widetilde{\overline{u}\ \overline{w}} \\ \widetilde{\overline{v}\ \overline{w}} \end{pmatrix} \tag{3.46}$$

with the same relations between $b_1$, $b_2$ and tilt-angles $\alpha$ and $\beta$ as with the original planar fit.

#### 3.6.4.3 Planar fit for triple-rotation correction of one run

The frames of reference, which can be found by performing triple-tilt corrections per run, will generally differ from the frame of reference, which is found by performing a planar fit correction for the collection of runs. The residual tilt-angles, which give the differences between these frames, can best be found in a way similar to the planar fit method. This guarantees that the residual angles of different runs and even of different setups are comparable (i.e. are estimates for the same physical quantities). As with the above planar fit method we define a plane with normal vector $(b_1, b_2, -1)$. Two equations are required to fix constants $b_1$ and $b_2$. The first equation is found by placing the run-mean velocity on the plane:

$$\begin{pmatrix} \overline{u} \\ \overline{v} \\ \overline{w} \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ -1 \end{pmatrix} = 0 \tag{3.47}$$

The second equation is found by imposing zero correlation between lateral velocity fluctuations within the plane and lateral velocity fluctuations perpendicular to the plane:

$$\overline{\left[\left\{\begin{pmatrix} b_1 \\ b_2 \\ -1 \end{pmatrix} \times \begin{pmatrix} \overline{u} \\ \overline{v} \\ \overline{w} \end{pmatrix}\right\} \cdot \begin{pmatrix} u-\overline{u} \\ v-\overline{v} \\ w-\overline{w} \end{pmatrix}\right] \quad \left[\begin{pmatrix} b_1 \\ b_2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} u-\overline{u} \\ v-\overline{v} \\ w-\overline{w} \end{pmatrix}\right]} = 0 \tag{3.48}$$

This coupled set of equations is quadratic in variables $b_1$ and $b_2$ and can be solved using e.g. Maple, which can output the solutions in Fortran-format (expressions are too complicated to include in this text). Like with the planar fit for a collection of runs, coefficients $b_1$ and $b_2$ from the planar fit to a single run are used to estimate angles $\alpha$, $\beta$ and a yaw-angle per run, which fine-tune the planar fit into the classic triple tilt correction. In a practical test on 19 hours of turbulence measurements at 10 meters above the grass in Cabauw angles $\alpha$, $\beta$ were generally smaller than 2 degrees. The residual yaw-angles reflected the expected variation in mean wind direction about the day-mean velocity.

## 3.7 Correction of sonic temperature for humidity

From the signal of the sonic anemometer we can estimate the speed of sound, which is a measure for temperature. To estimate temperature from the speed of sound we have to take into account that this velocity depends both on temperature and on humidity. The relation is (in Kelvin of course):

$$T = \frac{T_{\text{sonic}}}{1 + 0.51\, q} \tag{3.49}$$

where the sonic temperature $T_{\text{sonic}}$ has already been corrected for sidewind via relation 3.9. Relation 3.49 can be used to correct both individual temperature samplings and the mean temperature. For the correction of (co-)variances of temperature with a quantity $s$ one gets correction relations via partial differentiation of relation 3.49:

$$\overline{T's'} = c\, \overline{T'_{\text{sonic}} s'} \tag{3.50}$$

$$\overline{T'^2} = c^2\, \overline{T'_{\text{sonic}}}^2 \tag{3.51}$$

$$c \equiv 1 - 0.51\overline{q} - 0.51\,\overline{T_{\text{sonic}}}\, \frac{\overline{q's'}}{\overline{T'_{\text{sonic}} s'}} \tag{3.52}$$

For the covariance of temperature with vertical velocity, which plays the central role in the estimation of the sensible heat flux, this gives:

$$\overline{T'w'} = \overline{T'_{\text{sonic}} w'} - 0.51\,\overline{q}\,\overline{T'_{\text{sonic}} w'} - 0.51\,\overline{T_{\text{sonic}}}\,\overline{w'q'} \tag{3.53}$$

The second term in this relation is new in comparison with relation 8 in the study by Schotanus et al. (1983).

## 3.8 Oxygen correction for optical hygrometer

An open path optical hygrometer probes air for water vapour as follows: a light-source emits a monochromatic beam of ultraviolet light (for the krypton tube: 123.6 nm with a small secondary band at 116.5 nm and for Lyman-$\alpha$: 121.6 nm). A receiver measures which fraction of the emitted light is received at a distance of typically one centimeter. The frequency of the ultraviolet light is such that water vapour will absorb the light. The fraction of light which is absorbed per unit of length is proportional to the concentration of the vapour. Including non-linear effects the response $V$ of a hygrometer is related to the respective gas concentrations via the following formula:

$$\begin{aligned} -\frac{1}{x} \ln\left(\frac{V}{V_0}\right)(\varrho_w, \varrho_o) = & -\frac{1}{x} \ln\left(\frac{V}{V_0}\right)(\varrho_{w,\text{ref}}, \varrho_{o,\text{ref}}) \\ & + k_w(\varrho_w - \varrho_{w,\text{ref}}) \; + \; k_o(\varrho_o - \varrho_{o,\text{ref}}) \; + \; \text{higher order effects in } \varrho_w \text{ and } \varrho_o \end{aligned} \tag{3.54}$$

where $V_0$ is the response in vacuum, $x$ is separation between transmitter and receiver, $\varrho$ refers to density and $k$ indicates extinction coefficients. Subscripts $w$ and $o$ refer to water vapour and oxygen respectively. The first three terms in the right hand side of relation 3.54 are linearisations of the relation around atmospheric conditions. Tanner et al. (1993) adopts the following reference conditions: pressure 101325 Pa, temperature 305 K and zero humidity. We make a slight modification to these conditions: instead of zero humidity we will refer to 10 gram $H_2O/m^3$. The reason is that estimates for the extinction coefficient for water vapour at zero humidity are highly inaccurate, because they involve the derivative of a fitfunction at its boundary. Our extinction coefficients refer to these modified atmospheric conditions. We estimate reference oxygen concentration $\varrho_{o,\text{ref}}$ (based on 21 percent oxygen volume) to be $0.2685\,\text{kg}\,\text{m}^{-3}$.

To find extinction coefficients $k_i$ we differentiate theoretical response relation 3.54. In formulae:

$$k_w = -\frac{1}{x} \left. \frac{\mathrm{d} \ln\left(\frac{V}{V_0}\right)}{\mathrm{d}\varrho_w} \right|_{\text{atm. cond.}} \qquad \text{for water vapour} \tag{3.55}$$

$$k_o = -\frac{1}{x} \left. \frac{\mathrm{d} \ln\left(\frac{V}{V_0}\right)}{\mathrm{d}\varrho_o} \right|_{\text{atm. cond.}} \qquad \text{for oxygen} \tag{3.56}$$

The latter sensitivity is an unwanted characteristic since via their oxygen sensitivities these hygrometers are sensitive to temperature. A consequence of this double sensitivity of optical hygrometers is that estimates for the latent heat flux will be influenced by the sensible heat flux. This cross-talk will have to be eliminated.

Tanner et al. (1993) suggest that new experiments have shown that the oxygen-sensitivity coefficients published in the past (Tanner, 1989) overestimate the actual oxygen sensitivity by a factor of 2! This sudden change of insight has triggered us to perform calibrations of several Campbell krypton hygrometers and compare the resulting coefficients with those of Tanner et al. (1993) and of Tanner (1989). Moreover two Mierij Lyman-$\alpha$ hygrometers were included in the comparison (see van Dijk et al. (2003) for details). The extinction coefficients found in our experiments indicated that one cannot assume universal values for $k_w$ and $k_o$. The smallest value found for $k_o$ for krypton tubes ($1.3\,10^{-3}\,\mathrm{m}^3\,\mathrm{gram}^{-1}\,\mathrm{m}^{-1}$) differs from the largest value ($3.4\,10^{-3}\,\mathrm{m}^3\,\mathrm{gram}^{-1}\,\mathrm{m}^{-1}$) by a factor of 3, which itself was substantially smaller than the most recent value reported by Tanner ($4.5\,10^{-3}\,\mathrm{m}^3\,\mathrm{gram}^{-1}\,\mathrm{m}^{-1}$). The spread in values of $k_w$ was less dramatic, but still significant. The differences found for the extinction coefficients for Lyman-$\alpha$ hygrometers showed smaller differences than for the krypton-hygrometers, but this may be attributed to the fact that only two Lyman-$\alpha$s were tested. It is clear that for each individual hygrometer $k_w$ and $k_o$ will have to be estimated via calibration.

With extinction coefficients $k_w$ and $k_o$ we can calculate correction factors $c$, which correct raw latent heat estimates $L_vE(\mathrm{raw})$ for oxygen sensitivity of the hygrometer. We will use relation 18 in the article by Tanner et al. (1993):

$$L_vE(\mathrm{corrected}) = c(\beta)L_vE(\mathrm{raw}) \tag{3.57}$$

$$c(\beta) = 1 + 0.23\frac{k_o}{k_w}\frac{L_v\beta}{c_pT} \tag{3.58}$$

where $\beta$ is the Bowen-ratio and $L_v$ is the evaporation heat of water ($2.45\cdot10^6\mathrm{J\,kg}^{-1}$. Note that a number of errors related to relation 3.58 were present in van Dijk et al. (2003) (Jean-Martial Cohard, pers. comm., 2006): the units of $L_v$ were given incorrectly, $c_p$ was omitted, and the units of the factor 0.23 were given as $\mathrm{gK}^{-2}\mathrm{J}^{-1}$, rather than 0.23 being a dimensionless constant (equal to $\mathrm{Frac}_O\frac{m_O}{m_{air}}$, for symbols see below equation 3.60). Probably, in van Dijk et al. (2003) $c_p$ was absorbed in the constant 0.23, and the units of $L_v$ were mixed up. From relation 3.58 we see that correction factor $c(\beta)$ varies in first order with the extinction coefficients. This implies that small errors in these extinction coefficients lead to second order variations in estimated latent heat. Therefore, with respect to the oxygen correction of estimated latent heat fluxes, the extinction coefficients need not be known with very high accuracy. Extinction coefficient $k_w$ for water vapour is of course not only related to the oxygen correction of the evaporation, but also to the evaporation itself! Therefore for the actual estimation of evaporation one must have better estimates for $k_w$ than necessary for the estimation of correction factors $c(\beta)$. When we select the best hygrometers, which were used in our calibrations (both krypton and Lyman $\alpha$), then the correction factors found with use of relation 3.58 climb to a maximum of about 10 percent for very dry conditions (Bowen-ratio 5). This is much less than found by Tanner. Our upper limit to the correction is small enough to allow for the conclusion that both types of optical hygrometer are equally fit to be used under both wet and dry conditions.

The above method to correct latent heat estimates for oxygen sensitivity of the hygrometer can be generalized to a correction for all (co-)variances involving humidity:

$$\overline{x'\rho_v'} \rightarrow c\,\overline{x'\rho_v'} \tag{3.59}$$

$$c = 1 + \mathrm{Frac}_O\, m_o\,\frac{p}{RT^2}\,\frac{k_o}{k_w}\,\frac{\overline{x'T'}}{\overline{x'\rho_v'}} \tag{3.60}$$

for any quantity $x$, where $\mathrm{Frac}_O$ is the fraction of oxygen molecules in the air (generally 21%; above forests or above burning terrain this constant may have a different value!). To correct the humidity-variance one has to use the square of this factor $c$.

## 3.9 Correction for frequency response and path averaging

The smallest structure size in atmospheric flows is smaller than a millimeter and with high wind speeds (e.g. 10 m/s) this size corresponds with frequencies of the order of 10 kHz. Most instruments perform their probing on larger volumes and their frequency-response is often much worse than 10 kHz. The consequence of such non-ideal measurement is an underestimation of (co-)variances. Still one makes use of these non-ideal instruments assuming that the estimated covariances can be corrected. To allow for such correction it is important that the lost contribution is small. Turbulence spectra generally obey this condition. Using phenomenological spectra measured by Kaimal et al. (1972) Moore (1986) has developed a set of correction relations for measured covariances. We will review these corrections and jointly present the improvements on these corrections proposed by Horst (1999). The results of the model by L.Kristensen and Fitzjarrald (1984) will be presented separately.

### 3.9.1 The Moore-Horst model

The turbulent flux density can be measured using eddy-correlation, provided that fluctuations in the frequency range in which turbulent transport takes place are all sensed. In practice, this condition is hardly met due to a limited frequency response of the sensors and the data acquisition system, averaging over a path rather than taking a point value, separation between sensors for different quantities, and filtering applied. For each of these effects a theoretical co-spectral transfer function can be computed, which is unity for all frequencies for an ideal system. Convolution of this loss factor with the actual turbulent spectrum of the considered quantity gives a fraction of the true covariance that is actually sensed. Application of this method to really measured spectra will not be of much significance, since these spectra show the shortcomings of the sensor configuration we were looking to correct for. Therefore, theoretical spectra are used. The flux loss $\Delta F_{xy}$ is then defined by

$$\frac{\Delta F_{xy}}{F_{xy}} = 1 - \frac{\int_0^\infty T_{xy}(n) S_{xy}(n) \mathrm{d}n}{\int_0^\infty S_{xy}(n) \mathrm{d}n} \tag{3.61}$$

where $n$ is the frequency, $T_{xy}$ the net co-spectral transfer function, and $S_{xy}$ the theoretical co-spectral distribution function.

Moore (1986) worked out most of the frequency response correction for a Hydra flux measurement station (Shuttleworth et al., 1988). The special corrections applicable to closed path sensors as the LICOR6262 have been obtained from Leuning and Moncreiff (1990). An overview of these corrections is also given by Moncreiff et al. (1995). Sections 3.9.1.1 to 3.9.1.8 are a selection of ”Sparse canopy parameterizations for meteorological models”, PhD-thesis Wageningen Agricultural University, ISBN 90-5485-491-X, by B.J.J.M. van den Hurk, 1996. References to specific experiments have been removed.

#### 3.9.1.1 Digital sampling at limited frequency

An analogue-to-digital sampling acquisition method causes aliasing of spectral contributions exceeding the Nyquist frequency. When one is merely interested in the moments of one or more turbulent quantities this is not a problem, since the moments are integrated over the entire spectrum.

Moore (1986) proposed that the effective transfer function for an analog-to-digital sampling system, $T_a(n)$, was given by:

$$T_a(n) = 1 + \left(\frac{n}{n_s - n}\right)^3 \qquad n \leq n_s/2 \tag{3.62}$$

with $n_s$ the sampling frequency. For eq. 3.62 he assumed that aliasing is reduced by prefiltering the raw signal at $n = n_s/2$, causing negligible co-spectral power above the Nyquist frequency (see Figure 3.4 for an example).

However, since the spectral distribution of variance and covariance is irrelevant when determining the total variance or covariance, the digital-sampling correction as given in eq. 3.62 should not be used.

#### 3.9.1.2 Low-pass filtering

Low-pass filtering is applied to prevent aliasing, or folding frequencies higher than the Nyquist frequency $n_s/2$ into lower frequencies (Stull, 1988). The transfer function $T_v(n)$ is given by

$$T_v(n) = \left(1 + \left(\frac{n}{n_0}\right)^4\right)^{-1} \tag{3.63}$$

where $n_0$ is the cut-off frequency (at $n_s/2$). The time constant of the filter is given by $1/2\pi n_0$. Obviously, when no low-pass filtering is applied $T_v = 1$. An example of $T_v$ is shown in figure 3.4.

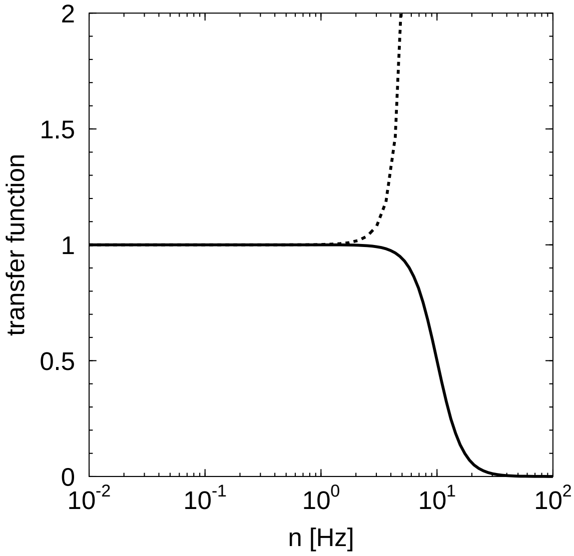


Figure 3.4: Examples of the low-pass filtering transfer function $T_v$(continuous line) and the analog-to-digital transfer function $T_a$ (dashed line) for $n_s = 10$Hz

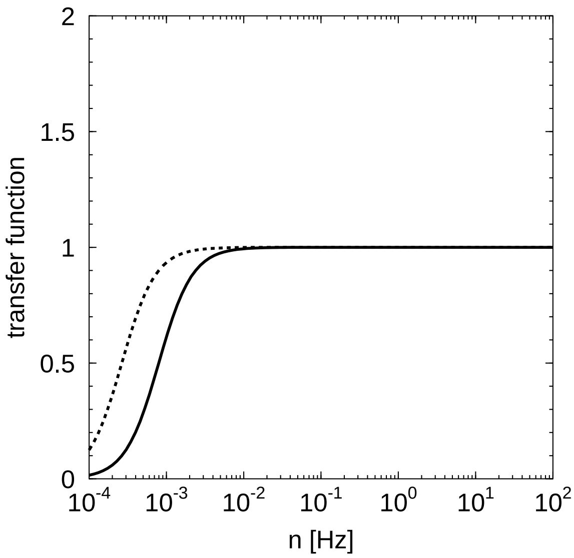


Figure 3.5: Example of the high-pass filtering transfer function $T_d$ for $n_s = 10$Hz; shown are $\tau_d = 200$s (continuous line) and 600 s (dashed line)

#### 3.9.1.3 High-pass filtering (detrending)

The transfer function $T_d(n)$ for a first order digital filter is to a very good approximation given by

$$T_d(n) = \frac{(2\pi n\tau_d)^2}{1+(2\pi n\tau_d)^2/\alpha_d} \qquad n \leq n_s/2 \tag{3.64}$$

An example is shown in Figure 3.5 for $\tau_d$ = 200 and 600 s. For linear detrending the choice of the interval length is very similar to choosing a time constant $\tau_d$ for a running mean interval.

#### 3.9.1.4 Sensor response and tube damping

The dynamic response of many sensors can be described by a simple first-order gain function:

$$T_r(n,\tau_c) = \left(1+(2\pi n)^2\tau_c^2\right)^{-1/2} \tag{3.65}$$

where $\tau_c$ is the time constant of the instrument. An example is depicted in figure 3.6.

A special case of damping of fluctuations is caused by the tube transporting the air from the sonic anemometer volume to a gas analyzer. Leuning and King (1992) present a transfer function $T_t$ given by

$$T_t(n) = \begin{cases} \sqrt{\exp(x/6Du_t)} & \frac{2\pi n r_t^2}{D} < 10 \\ 1 & \text{elsewhere} \end{cases} \tag{3.66}$$

where $x$ is given by $-(\pi n r_t)2l$, $r_t$ the tube radius, $l$ the tube length, $D$ the diffusivity of the gas being analyzed and $u_t$ the air speed in the tube. Eq. 3.66 is strictly valid in cases where the flow within the tube may be considered to be laminar, and density fluctuations at all frequencies travel down the tube with the same velocity, $u_t$. Based on expressions presented by Philip (1963), Leuning and King (1992) state that this applies to frequencies for which $2\pi n r_t 2/D < 10$. For turbulent flow they propose the following transfer function

$$T_t(n) = \sqrt{\exp\left(-160 Re^{-1/8} r_t n^2 l/u_t^2\right)} \qquad Re > Re_c \tag{3.67}$$

where $Re_c$ is a critical Reynolds number, equal to ±2300, and $Re$ is given by $2u_t r_t/\nu$. Figure 3.7 shows an example for both equations.

#### 3.9.1.5 Sensor line averaging

In most cases a scalar quantity is measured over a (finite) path length rather than at a single point. The effect of the spatial averaging involved can be described very well by

$$T_p(p) = \frac{1}{2\pi f}\left(3+\exp(-2\pi f) - 4\frac{1-\exp(-2\pi f)}{2\pi f}\right) \tag{3.68}$$

where $f$ is the normalized frequency $np/u$, $p$ being the averaging distance. Spatial averaging is relevant for all sensors. However, the effect on the temperature measured using a thermocouple is considered small enough to ignore a correction for this. The averaging path for the sonic temperature is equal to that of the vertical wind, and will be discussed hereafter. For the closed- path analyzer the averaging path is determined by the length of the gas chamber. An example is shown in figure 3.8.

The effect of spatial averaging on measurements of vector quantities is different to that for scalar quantities. Moore (1986) gives a simplified transfer function for the vertical wind component, based on findings of Kaimal et al. (1968). The transfer function $T_w$ for averaging the vertical velocity over a path with distance $p$ reads

$$T_w = \frac{2}{\pi f}\left(1+\frac{\exp(-2\pi f)}{2} - \frac{3(1-\exp(-2\pi f))}{4\pi f}\right) \tag{3.69}$$

For the horizontal wind components a general function as eq. 3.69 is not possible to give, since it depends on sensor geometry and wind direction. For a symmetrical orthogonal set of transducers (as for the Kaijo Denki DAT310

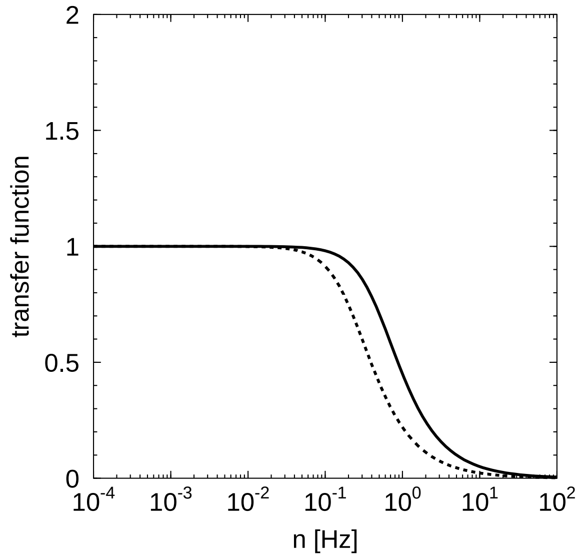


Figure 3.6: Example of the sensor response transfer function $T_r$ for $n_s$ = 10Hz; shown are $\tau_c = 0.1s$ (continuous line) and 0.5 s (dashed line).

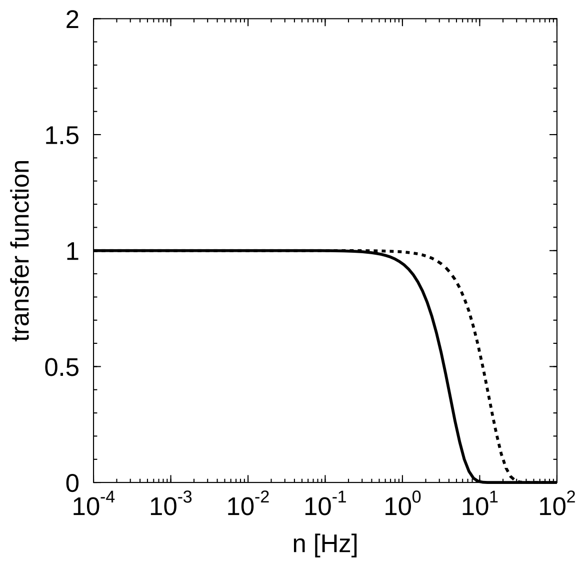


Figure 3.7: Example of the tube damping transfer function $T_t$; shown are eq. 3.66 for laminar flow (continuous line) and eq. 3.67 for turbulent flow (dashed line). In both cases $n_s$ = 10Hz, $l = 4m, r_t = 0.0015m, u_t = 5m/s$ and $D = D_v = 2.5610^{-6}m^2/s$

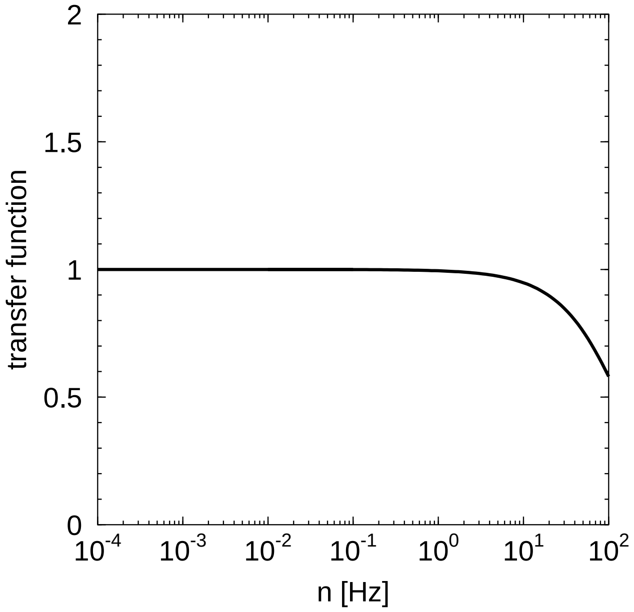


Figure 3.8: Example of the transfer function for sensor line averaging for scalars, $T_p$, for $p = 0.025m$ and $u = 5m/s$

device), the transfer functions can be computed for a horizontal wind from a direction of $45^o$ compared to each component. Then the sensor averaging transfer function can be reduced to a single function $T_u$:

$$T_u = \left(\frac{\sin \pi f}{\pi f}\right)^2 \quad (3.70)$$

No attempt was made to investigate the assumptions leading to this formulation. Figure 3.9 provides an example of $T_w$ and $T_u$.

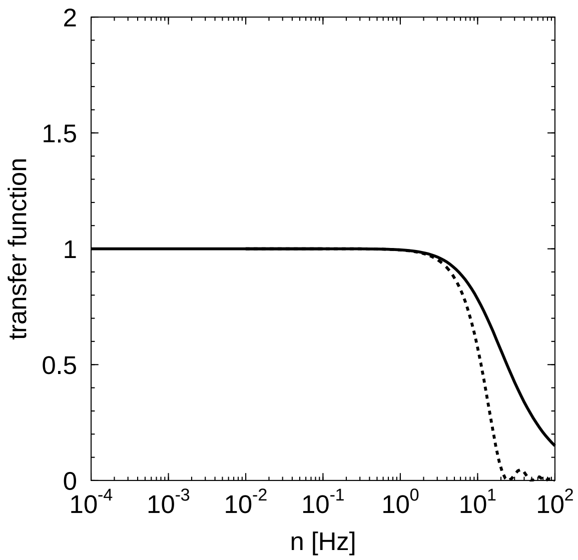


Figure 3.9: Example of the transfer function for sensor line averaging for vectors: $T_w$ (continuous line), $T_u$ (dashed line). In both cases $p = 0.20m$ and $u = 5m/s$

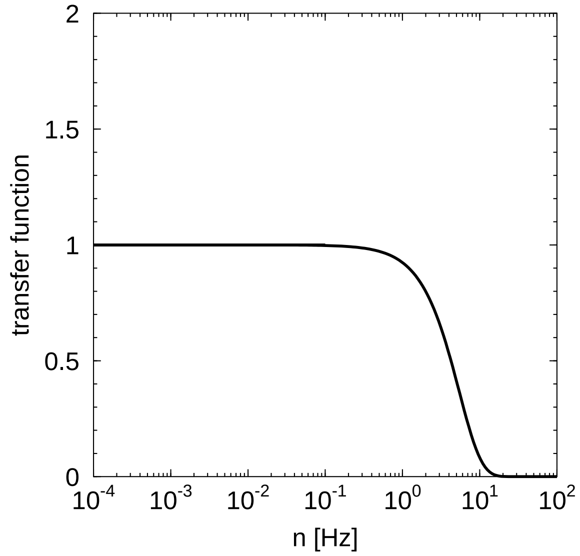


Figure 3.10: Example for the sensor separation transfer function $T_s$ for $s = 0.20m$ and $u = 5m/s$

#### 3.9.1.6 Sensor separation

Ideally, eddy correlation covariances are computed from measurements taken at exactly the same point. In practice, usually a separation between different sensors is necessary. The loss of covariance due to sensor separation is a function of the distance between the sensors and the angle of the wind direction relative to the separation path. For practical purposes Moore (1986) developed a scheme which can be used to correct for both longitudinal and lateral separation, provided that the sensor separation $s$ is small and open to the atmosphere:

$$T_s(f) = \exp\left(-9.9 f^{1.5}\right) \tag{3.71}$$

where $f$ is the normalized frequency, given by $ns/u$ (see Figure 3.10).

#### 3.9.1.7 Net transfer functions

The net transfer functions for the several covariances can be found by multiplying the relevant gain functions given above. A net transfer function for the data acquisition system, $T_n$, can be specified, which applies to all sensors. It is defined by

$$T_n = T_a T_d T_v \tag{3.72}$$

The net transfer functions for the separate variances and covariances depend further on sensor time constant $\tau_x$, averaging path $p_x$, diffusion coefficient $D_x$ and separation from the $w$-sensor $s_{wx}$. The subscript $x$ refers to vertical wind when $x = w$, horizontal wind in both directions for $x = u$, thermocouple temperature for $x = T$, sonic temperature for $x = s$, humidity measured by Lyman-$\alpha$ and Krypton for $x = q$, and humidity and $CO_2$-concentration measured by a closed path device for $x = h$ and $c$, respectively. Then the net transfer functions for the separate variances are given by:

$$\begin{aligned}
T_{uu} &= T_n T_{u,v}(p_u) T_r^2(\tau_u) \\
T_{ww} &= T_n T_w(p_w) \\
T_{TT} &= T_n T_r^2(\tau_T) \\
T_{ss} &= T_n T_w(p_w) \\
T_{qq} &= T_n T_p(p_q) \\
T_{hh} &= T_n T_p(p_h) T_r^2(\tau_h) T_t(D_h) \\
T_{cc} &= T_n T_p(p_c) T_r^2(\tau_c) T_t(D_c)
\end{aligned} \tag{3.73}$$

According to Moore (1986) the covariance transfer functions can be found from the variance transfer functions via

$$T_{xy} = \sqrt{T_{xx}T_{yy}} \tag{3.74}$$

which would consequently yield:

$$\begin{aligned}
T_{wu} &= T_n T_r(\tau_u)\sqrt{T_w(p_w)T_{u,v}(p_u)} \\
T_{wT} &= T_n T_s(s_{wT})T_r(\tau_T)\sqrt{T_w(p_w)} \\
T_{ws} &= T_n T_w(p_w) \\
T_{wq} &= T_n T_s(s_{wq})\sqrt{T_p(p_q)T_w(p_w)} \\
T_{wh} &= T_n T_s(s_{wh})T_r(\tau_h)\sqrt{T_p(p_h)T_w(p_w)T_t(D_h)} \\
T_{wc} &= T_n T_s(s_{wc})T_r(\tau_c)\sqrt{T_p(p_c)T_w(p_w)T_t(D_c)}
\end{aligned} \tag{3.75}$$

In a recent publication Horst (1999) has made clear that this approach to estimate the cospectral transfer function neglects the phase-shift inherent in applying a frequency-dependent filter to time-series data. For two sensors with a simple first-order response, each characterized by a different time constant $\tau$, the spectral variance transfer functions are:

$$T_{xx} = \frac{1}{1+\omega^2\tau_x^2} \qquad T_{yy} = \frac{1}{1+\omega^2\tau_y^2} \tag{3.76}$$

while the cospectral transfer function reads

$$T_{xy} = \frac{(1+\omega^2\tau_x\tau_y) + \omega(\tau_x-\tau_y)Q_{xy}/S_{xy}}{(1+\omega^2\tau_x^2)(1+\omega^2\tau_y^2)} \tag{3.77}$$

where $Q_{xy}$ is the quadrature spectrum and $S_{xy}$ the cospectrum. This relation only reduces to relation 3.76 when the time constants of the two sensors are equal: $\tau_x = \tau_y$.

#### 3.9.1.8 Model spectra

For the description of the atmospheric spectra and cospectra the formulations of Kaimal et al. (1972) have been used. The formulations provide a description of spectral energy $S_{xy}$ as function of (normalized) frequency $f = nz/u$ and stability $z/L_v$, $z$ being the measuring height. The spectra are derived for the variance of the three wind components and temperature, plus their mutual covariances. Moore (1986) concluded that spectra of the other scalars (humidity and CO2) resembled the temperature spectra very well, and thus

$$\begin{aligned}
S_{qq} &= S_{hh} = S_{cc} = S_{TT} \\
S_{wq} &= S_{wh} = S_{wc} = S_{wT}
\end{aligned} \tag{3.78}$$

Furthermore, the spectra for both horizontal wind components are considered equal as well.

The general function of $S_{xx}$ under stable conditions ($z/L_v > 0$) can be represented by

$$nS_{xx}(n) = \frac{f}{A_x + B_x f^{5/3}} \tag{3.79}$$

where $A_x$ and $B_x$ are functions of the atmospheric stability. Also the cospectra are well reproduced under stable conditions using a general equation:

$$nS_{wx}(n) = \frac{f}{A_{wx} + B_{wx} f^{2.1}} \tag{3.80}$$

Table 3.1 gives the formulations of $A_x$, $B_x$, $A_{wx}$ and $B_{wx}$.

| Variance spectra | $A_x$ | $B_x$ |
|---|---|---|
| $x = w$ | $A_w = 0.838 + 1.172(z/L_v)$ | |
| $x = u$ | $A_u = 0.2A_w$ | $B_x = 3.124A_x^{-2/3}$ |
| $x = T$ | $A_T = 0.0961 + 0.644(z/L_v)^{0.6}$ | |
| Covariance spectra | $A_{wx}$ | $B_{wx}$ |
| $x = u$ | $0.124(1 + 7.9z/L_v)^{0.75}$ | $2.34A_{wx}^{-1.1}$ (corrected version - AvD) |
| $x = T$ | $0.284(1 + 6.4z/L_v)^{0.75}$ | |

Table 3.1: Formulations of $A_x$, $B_x$, $A_{wx}$ and $B_{wx}$ for stable (co)variance spectra

Unfortunately, the unstable spectra are not easily defined, due to a dependence on the boundary layer height $z_i$. Hojstrup (1981) developed suitable expressions for the horizontal and vertical wind velocity:

$$nS_{ww}(n) = \left( \frac{f}{1 + 5.3 f^{5/3}} + \frac{16 f \xi}{(1 + 17 f)^{5/3}} \right) C_w^{-1} \tag{3.81}$$

and

$$nS_{uu}(n) = \left( \frac{210 f}{1 + 33 f^{5/3}} + \frac{f \xi}{\zeta + 2.2 f^{5/3}} \right) C_u^{-1} \tag{3.82}$$

where

$$C_w = 0.7285 + 1.4115\xi \qquad C_u = 9.546 + 1.235\xi\zeta^{-2/5}$$

$$\zeta = \left( \frac{z}{z_i} \right)^{5/3} \qquad \xi = \left( \frac{z}{-L_v} \right)^{2/3}$$

Since $z_i$ was not known for most time intervals, a fixed value of 1000 m was chosen, as to represent a typical condition.

No suitable models for atmospheric temperature spectra for unstable conditions are cited in literature. However, Moore (1986) argued that for most conditions the spectra given by Kaimal et al. (1972) could be used. For the temperature variance is given

$$nS_{TT}(n) = \begin{cases} \frac{14.94 f}{(1+24 f)^{5/3}} & f < 0.15 \\ \frac{6.827 f}{(1+12.5 f)^{5/3}} & f \geq 0.15 \end{cases} \tag{3.83}$$

while the temperature cospectra read

$$nS_{wT}(n) = \begin{cases} \frac{12.92 f}{(1+26.7 f)^{1.375}} & f < 0.54 \\ \frac{4.378 f}{(1+3.8 f)^{2.4}} & f \geq 0.54 \end{cases} \tag{3.84}$$

The spectrum of momentum transfer is described by

$$nS_{uw}(n) = \begin{cases} \frac{20.78 f}{(1+31 f)^{1.575}} & f < 0.24 \\ \frac{12.66 f}{(1+9.6 f)^{2.4}} & f \geq 0.24 \end{cases} \tag{3.85}$$

Based on these theoretical spectra and the transfer functions described above, Figure 3.11 gives an example of the net frequency response corrections applied to $\sigma_u^2$ and to $\overline{w'T'}$, for a specified height and wind speed.

#### 3.9.1.9 Consequences of spectral corrections for tolerance estimates

The correction procedure proposed in this section assumes that the spectra found by Kaimal et al. (1972) are representative for all conditions. This is a very idealised point of view. To incorporate the uncertainty of this correction procedure into the tolerance estimates for covariances we propose to (quadratically) add half the absolute difference between corrected and uncorrected estimates to the tolerance estimates for the covariances which were based on other causes (e.g. statistics):

$$\text{Tol} \longrightarrow \sqrt{\text{Tol}^2 + \left(0.5\,\text{Freq.corr.}\right)^2} \tag{3.86}$$

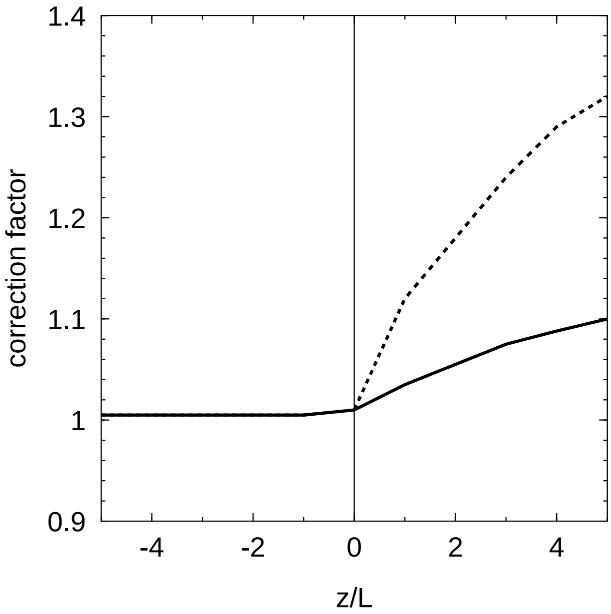


Figure 3.11: Example of frequency response corrections as function of $z/L_v$ : $\Delta F_{uu}$ (continuous line); $F_{wT}$ (dashed line). Configuration parameters are as follows: $z = 10m$, $u = 5m/s$, $p_u = p_w = 0.20m$, $s_{wT} = 0.25m$, $\tau_c = 0.5s$ (thermocouple) and $n_s = 10Hz$

### 3.9.2 The Kristensen-Fitzjarrald model

L.Kristensen and Fitzjarrald (1984) and van Dijk (2002) have developed a procedure to find the transfer function of vertical scalar fluxes, which are estimated from 3D sonic anemometer measurements (with finite paths of length $l$) and scalar point-samples. The sonic is supposed to either have one of its acoustic paths aligned with the vertical direction (L.Kristensen and Fitzjarrald (1984), see figure 3.2) or to have its acoustic paths in a configuration along three axes $\vec{l_a}$, $\vec{l_b}$ and $\vec{l_c}$, which all have angle $\alpha$ with the vertical axis ($\alpha = 54.7^o$ gives three perpendicular axes) (van Dijk (2002), as shown in figure 3.3).

To get a non-zero flux, their model involves anisotropic turbulence and is therefore more complex than studies by e.g. Kaimal et al. (1968) and Oncley (1989). Their statistical model for the true co-spectrum at height $z$ has a $k^{-7/3}$-behaviour for small waves ($\beta$ is a stability dependent parameter):

$$\mathrm{Co}(k) = \beta z\, \overline{wS}\, (|k|z)^{-7/3} \tag{3.87}$$

The measured cospectrum is related to this true cospectrum via

$$\tilde{C}o(k) = \int_0^\infty \int_0^{2\pi} K \cdot A\left(\sqrt{k^2+K^2}\right) \cdot \frac{k^2 + K^2 \sin^2\theta}{k^2+K^2} \cdot \\ \cdot \frac{1}{3}\left(\mathrm{sinc}\left(\frac{\vec{k}\cdot\vec{l_a}}{2}\right) + \mathrm{sinc}\left(\frac{\vec{k}\cdot\vec{l_b}}{2}\right) + \mathrm{sinc}\left(\frac{\vec{k}\cdot\vec{l_c}}{2}\right)\right) \mathrm{d}\theta \mathrm{d}K \tag{3.88}$$

where functions $A$ and $F$ are obtained from relation 3.87 for the underlying true cospectrum via

$$A(\kappa) = \frac{1}{3\pi\kappa^2} \cdot \left(\frac{2}{3\beta}\right)^{0.75} \cdot \overline{wS} \cdot z \cdot F\left(\left(\frac{2}{3\beta}\right)^{0.75} \cdot \kappa \cdot z\right) \tag{3.89}$$

$$F(s) = \frac{91}{30} s^{-7/3} \tag{3.90}$$

The transfer function $T_{\mathrm{flux}}$, which results from these relations:

$$T_{\mathrm{flux}} \equiv \frac{\tilde{C}o(k)}{Co(k)} \tag{3.91}$$

is estimated via numerical integration. The results for popular configurations (Campbell 3D sonic and Gill/Solent with $\alpha = 30^o$) and for a sonic with $\alpha = 45^o$ are plotted in figure 3.12.

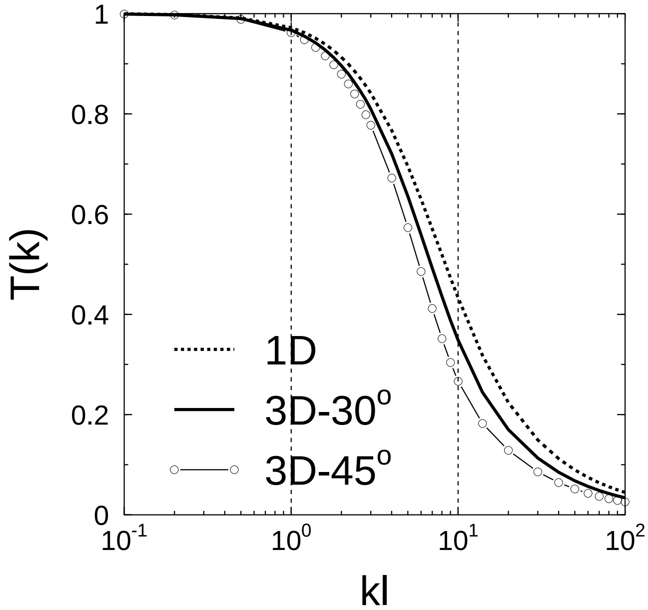


Figure 3.12: Transfer functions for sonic anemometers: the dashed line gives the results by Kristensen and Fitzjarrald, the continuous line is for the 3D Campbell/Solent type of sonic and the circles represent a configuration where the acoustic paths make an angle of 45 degrees with the vertical axis. The quantity on the horizontal axis is wavenumber $k \equiv 2\pi/\lambda$ of a flux-contribution made dimensionles with acoustic pathlength $l$.

## 3.10 Webb term (mean vertical velocity)

In section 2.5 we have seen that, even though the soil may be 'firm' (i.e. non-penetrable), there can be a non-zero run-mean vertical velocity, inducing a flux-contribution $F_m(\xi)$ of transported quantity $\xi$ via (see relation 2.46):

$$F_m(\xi) \equiv \overline{w}\,\overline{\xi} \tag{3.92}$$

Generally the mean vertical velocity is too small to be directly measured. The three-angle tilt-correction method (yaw, pitch and roll, see sections 3.6.1, 3.6.2 and 3.6.3) supposes that any measured mean vertical velocity must be attributed to set-up misalignment. These tilt-corrections will assure that the direct estimate for $\overline{w}$ vanishes. An indirect estimate for $\overline{w}$ is made via relation 2.71:

$$\overline{w} = (1 + \mu\sigma + k)\,\frac{\overline{w'T'}}{\overline{T}} + \mu\sigma\,\frac{\overline{w'\varrho_v'}}{\overline{\varrho_v}} + 2k\,\frac{\overline{w'u'}}{\overline{u}}$$

On the other hand, when the planar fit method (section 3.6.4) is used, there will be non-zero direct estimates for the run-mean vertical velocity. This direct estimate must be added to the above indirect estimate, since the indirect estimate is neglected/eliminated by the planar fit.

## 3.11 Estimation of tolerance levels

For both the mean values and the covariances we estimate tolerance levels. We define the **tolerance** Tol($x$) of quantity $x$ as **the absolute deviation from that quantity such that the probability of the occurance of a sample further away than the tolerance, after repeated estimation of** $x$**, is reduced to 4 percent**. The tolerance gives us a tool to compare estimates and reject or confirm significant correspondence. For normally distributed samples the tolerance is related to the standard-deviation $\sigma(x)$:

$$\mathrm{Tol}(x) = 2\sigma(x) \qquad \text{for normally distributed samples} \tag{3.93}$$

For skew and nonnormal distributions one has to make separate estimates for upward- and downward tolerances, where the factor 2 in relation 3.93 has to be replaced by factors $c_+$ and $c_-$ taken such that the rejected fractions, which deviate from the mean by more than $c_- \cdot \sigma_-$ in downward sense or by more than $c_+ \cdot \sigma_+$ in upward sense, equal 4 percent.

$$\begin{array}{ll} \mathrm{Tol}_-(x) & = c_- \cdot \sigma_-(x) \\ \mathrm{Tol}_+(x) & = c_+ \cdot \sigma_+(x) \end{array} \qquad \text{for skew and nonnormal distributions} \tag{3.94}$$

Relations 3.93 and 3.94 give the tolerance for individual samples. When $n_{\mathrm{indep}}$ **independent** samples of quantity $x$, each with tolerance $\mathrm{Tol}(x)$, are used to estimate the mean value $\overline{x}$, then the tolerance of that mean value is better than the tolerance of the individual samples by a factor $\sqrt{n_{\mathrm{indep}}}$:

$$\mathrm{Tol}(\overline{x}) = \frac{1}{\sqrt{n_{\mathrm{indep}}}} \mathrm{Tol}(x) \tag{3.95}$$

The problem with this relation is to find the number of independent samples $n_{\mathrm{indep}}$. To make sure that all information is recorded scientists sample their quantities such that the samples have a (slight) overlap. In this way it will be clear that the number of independent samples will often be substantially smaller than the total number of samples in a record. Moreover, the datalogging system will often have one sampling frequency, adjusted to the quantity with the fastest variations. As a consequence, the other quantities will be strongly oversampled.

To estimate the number of independent samples in a time-series we first study the characteristics of a stochastically stationary process. We construct a time-series and suppose that the respective samples $x_k$ are taken independently from a possibly skew distribution around a mean value $\overline{x}$. We also suppose that the time series is long enough to allow for the estimation of probabilities $p_{+,\mathrm{indep}}$ and $p_{-,\mathrm{indep}}$, providing the fraction of time that the signal has positive deviation from the mean value or a negative deviation. With a collection of $N_{\mathrm{indep}}$ samples of which $N_{+,\mathrm{indep}}$ with positive fluctuation around the mean value and $N_{-,\mathrm{indep}}$ with negative fluctuation we estimate $p_{+,\mathrm{indep}}$ and $p_{-,\mathrm{indep}}$ as follows:

$$p_{-,\mathrm{indep}} = \frac{N_{-,\mathrm{indep}}}{N_{\mathrm{indep}}} \qquad \text{and} \qquad p_{+,\mathrm{indep}} = \frac{N_{+,\mathrm{indep}}}{N_{\mathrm{indep}}} \tag{3.96}$$

From sample to sample the sign of the fluctuating part of the signal has a probability $p_{-,\mathrm{indep}} \cdot p_{+,\mathrm{indep}}$ to go from negative to positive and a probability $p_{+,\mathrm{indep}} \cdot p_{-,\mathrm{indep}}$ to go from positive to negative. Together the probability $p_{\mathrm{swap,\ indep}}$ of a change of sign to occur in the fluctuating part of the independent samples is:

$$p_{\mathrm{swap,indep}} = 2\, p_{-,\mathrm{indep}} \cdot p_{+,\mathrm{indep}} = 2\, \frac{N_{-,\mathrm{indep}} \cdot N_{+,\mathrm{indep}}}{N^2_{\mathrm{indep}}} \tag{3.97}$$

This gives an estimate for the number $n_{\mathrm{swap,indep}}$ of sign-changes that can be expected in a new collection of $N_{\mathrm{indep}}$ independent samples:

$$N_{\mathrm{swap,indep}} = N_{\mathrm{indep}} \cdot p_{\mathrm{swap,indep}} \tag{3.98}$$

The ratio of the number of sign-changes $N_{\mathrm{swap,indep}}$ and the probability $p_{\mathrm{swap,indep}}$ of a sign-change brings us back to the number of independent samples $N_{\mathrm{indep}}$.

When the same signal as before is probed with higher and higher sampling frequency, a point will come where the samples will become dependent. We assume that the total record of samples is still large enough to allow for reliable estimation via relation 3.97 of probability $p_{\mathrm{swap,indep}}$ for the change of sign of fluctuations *in case the signal would be independently probed*:

$$p_{\mathrm{swap,indep}} = 2\, \frac{N_- \cdot N_+}{N^2} \tag{3.99}$$

where now $N_-$, $N_+$ and $N$ refer to counting samples in the record of oversampled and hence dependent data. Addition of dependent samples to a dataset consisting of independent samples will form a more or less smooth bridge between the independent samples. As a consequence they will generally not alter the number of sign-changes $N_{\text{swap}}$ of the fluctuating component in the signal, see figure 3.13. This brings us to the conclusion that

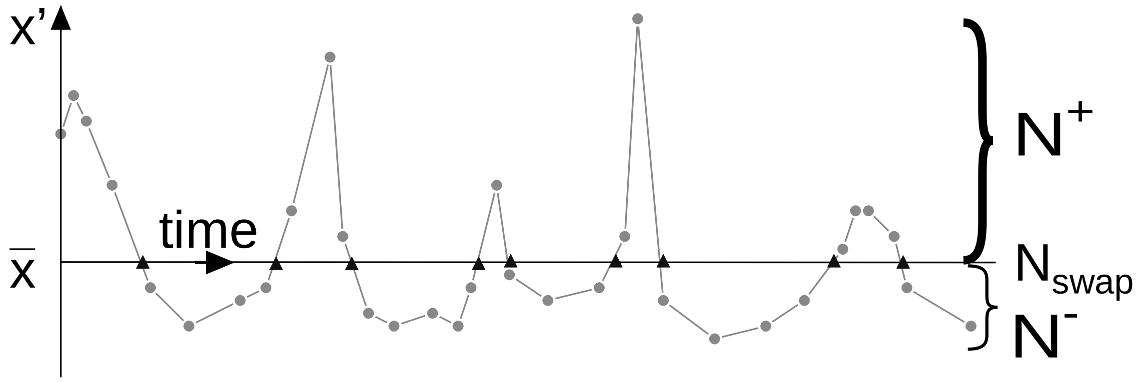


Figure 3.13: Root-counting of fluctuations in oversampled data with a skew distribution

the number of sign-changes in an oversampled signal provides a good measure for the detection of independent samples:

$$N_{\text{swap}} \simeq N_{\text{swap,indep}} = p_{\text{swap,indep}} \cdot N_{\text{indep}} \tag{3.100}$$

Combination of relation 3.100 for the number of sign-changes of the fluctuations in an oversampled record with relation 3.99 for the probability of such sign-changes to occur gives an estimate for the number of independent samples which can be found in the record:

$$N_{\text{indep}} \simeq \frac{N_{\text{swap}}}{p_{\text{swap,indep}}} = \frac{N_{\text{swap}} \cdot N^2}{2N_- \cdot N_+} \tag{3.101}$$

When the mean value, which is subtracted from the samples to find the fluctuating component, is estimated from the record itself, then one has to subtract one from the righthand side of relation 3.101 to compensate for the resulting dependency in the fluctuating component of the data:

$$N_{\text{indep}} \simeq \frac{N_{\text{swap}} \cdot N^2}{2N_- \cdot N_+} - 1 \qquad \text{when mean values are estimated from the same dataset} \tag{3.102}$$

To illustrate the necessity for the extra subtraction, consider the sampling of a linearly increasing signal. In this case all samples are dependent. The mean value estimated from this series will divide the data in two segments: the first half has negative deviation from the mean and the second half has positive deviation. Via relation 3.101 we would consider both segments as representations of two independent samples. It may however be that the whole series represents just a small period in a signal dominated by much larger timescales. This would imply that the record of samples is to be considered as only one independent sample. Relation 3.102 gives the correct number: one independent sample!

In case of a non-skew signal positive and negative fluctuations have equal probabilities, which reduces relation 3.102 for the number of independent samples to:

$$N_{\text{indep}} \simeq 2N_{\text{swap}} - 1 \qquad \text{for symmetric distributions} \tag{3.103}$$

Altogether (relations 3.102, 3.94 and 3.95) we have the following error-estimate for mean quantity $\overline{x}$:

$$\text{Tol}_-(\overline{x}) = \frac{c_-\sigma_-}{\sqrt{\frac{N_{\text{swap}} \cdot N^2}{2N_- \cdot N_+} - 1}} \tag{3.104}$$

$$\text{Tol}_+(\overline{x}) = \frac{c_+\sigma_+}{\sqrt{\frac{N_{\text{swap}} \cdot N^2}{2N_- \cdot N_+} - 1}} \tag{3.105}$$

An important item to bear in mind is the following: fluxes of scalar densities (e.g. evaporation) include a term dependent on the mean vertical velocity. Even when this term is eliminated by a tilt-correction setting this mean vertical velocity to zero, then the tolerance of this term cannot be neglected and should be quadratically added to the tolerance in the covariance term!

# Appendix A

# Discussion on the definition of the sensible heat flux

This appendix provides a discussion in which different points of view on the definition of the sensible heat flux are reviewed. The intention is to facilitate the discussion by explicitly showing the advantages and disadvantages of the different methods. A central role is played by the energy balance at the surface. Therefore we recall relation 2.2:

$$Q^* = L_v E + G + H \tag{A.1}$$

## A.1 Interpretation A

The sensible heat flux is one of the terms in relation A.1 for the energy balance at the surface. In this context, the surface is the thin layer of molecules which receive net radiation. The surface has a thickness comparable with a few times the wavelength of the radiation and consequently can be considered to be two-dimensional. Two-dimensional objects cannot contain much energy and therefore equation A.1 expresses that the energy flux, which is fed to the surface by incoming net radiation, has to be componsated by fluxes which transport this energy away from the surface. Heat is an amount of energy in transfer. Mechanisms for heat transfer are: radiation, convection and conduction.

The radiative energy flux is excluded from the definition of the sensible heat flux since it has already been accounted for in relation A.1 via net radiation term $Q^*$. What remains to be considered is the roles of conduction and of convection for the definition of the sensible heat flux.

The energy, which is transfered in a conductive exchange from the two-dimensional surface to the atmosphere, can be unambiguously counted: just compare the energy content of the thermodynamic system above the surface before and after transfer.

By convection or diffusion molecules can traverse from one side of the two-dimensional surface (the soil) to the other side (the atmosphere). Water is responsible for the lion's share of the exchange of matter between the Earth's surface and the atmosphere. The energy which was already present in those molecules before they entered the "surface" for some milliseconds will be found back in the same molecules when they emerge at the other side of the surface in the atmosphere. The thermal energy of the migrating molecules does (of course) represent energy. Consequently a flux of mass through the surface does represent an energy flux, but this flux cannot be used to close equation A.1. We shall call the convective heat flux $C$. The independence of the energy budget equation at the surface from mass flux through the surface, which is illustrated in figure A.1, is our motivation to exclude $C$ from the definition of the sensible heat flux. We may now define the sensible heat flux as follows:
**The sensible heat flux $H$ is defined as the flux of heat, which is transferred by the ground to the atmosphere by thermal conduction in the laminar sublayer, during reversibel isobaric processes.**

One may nevertheless want to estimate convective heat transfer $C$ to include its effect in large scale atmospheric models. Warm molecules, which enter the atmosphere in warm conditions, can (after a long journey) influence the

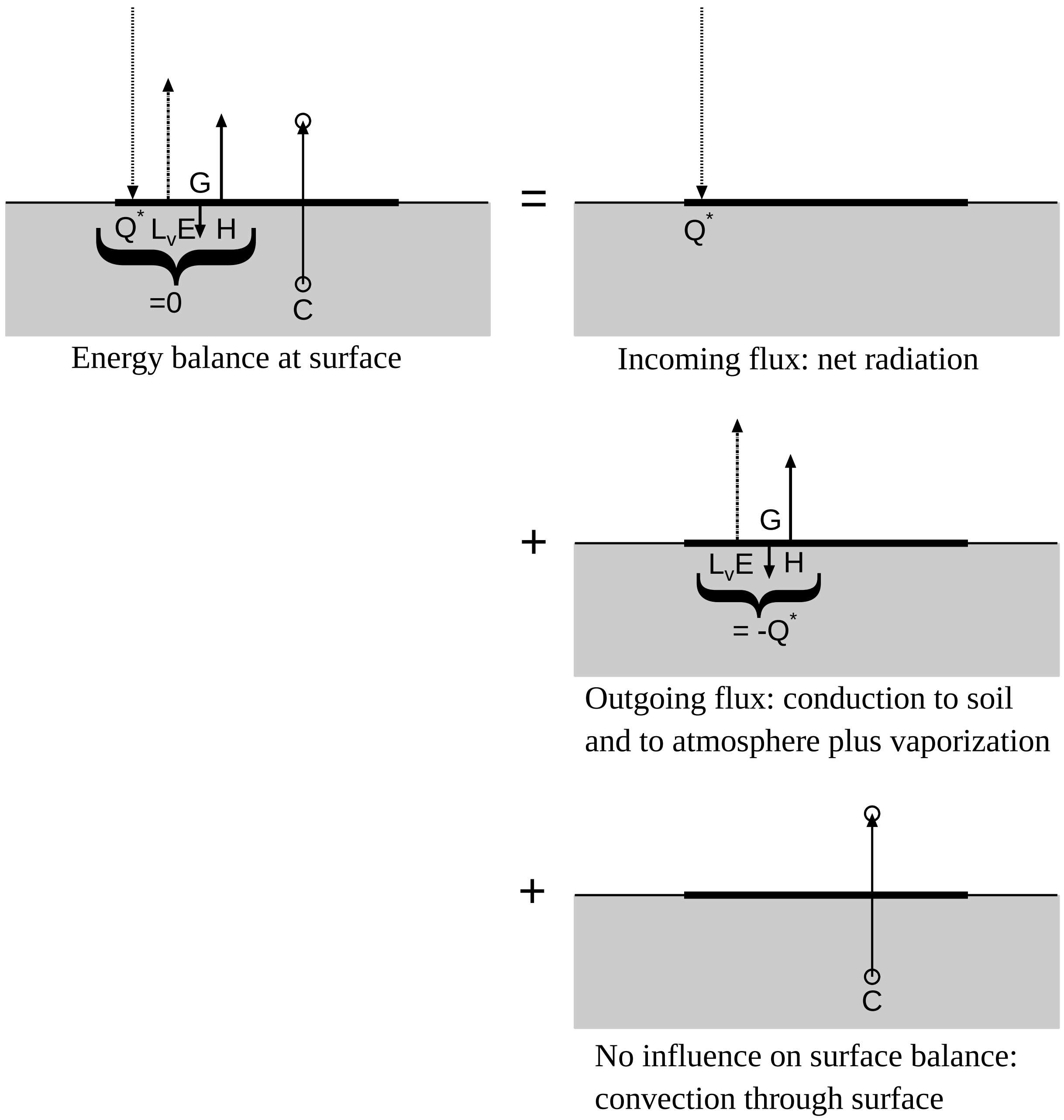


Figure A.1: Interpretation A: convection does not contribute to the energy balance at the surface. Consequently only the conductive heat transfer is of interest for the sensible heat flux.

dynamics of the atmosphere near the poles, where they interact with locally evaporated and consequently much colder molecules. A second example of the influence of the convective heat flux can be observed in plants: plants can extract heat from evaporated molecules, which are warmer than air molecules which were in the atmosphere for a longer while. A large mass-flux through the surface of molecules with a different temperature can even give a drift in temperature above the surface. This literally ”sensible heat” may therefore tempt the researcher to incorporate a fraction $\Delta C$ of the convective energy flux in the definition of sensible heat flux $H$, even though we have already shown that this convective heat flux $C$ is of no importance for the energy budget at the surface. This additional contribution to the sensible heat flux will depend on the temperature difference $\Delta T_{\text{surf/atm}}$ between the surface and some representative parcel of air in the atmosphere. To close energy budget equation A.1 we have to modify the definition of the soil heat flux accordingly:

$$H_{\text{mod}} \equiv H_{\text{conduction}} + \Delta C \quad (A.2)$$

$$G_{\text{mod}} \equiv G_{\text{conduction}} - \Delta C \quad (A.3)$$

$$\Delta C \sim \Delta T_{\text{surf/atm}} \quad (A.4)$$

from which we find

$$Q^* = L_v E + G_{\text{conduction}} + H_{\text{conduction}} \quad (A.5)$$

$$= L_v E + (G_{\text{conduction}} - \Delta C) + (H_{\text{conduction}} + \Delta C)$$

$$= L_v E + G_{\text{mod}} + H_{\text{mod}} \quad (A.6)$$

It seems an undesirable characteristic of these definitions that the soil heat flux depends on a temperature difference between the surface and the atmosphere.

Apart from the absence of necessity and the conceptually peculiar consequences of inclusion of (part of) convective heat flux $C$ in the definition of the sensible heat fux, there is a fundamental problem. Chemical potential $\mu$ expresses the amount of energy per unit mass of the transmigrating molecules involved (indicated with index $v$) and this gives us the following expression for convective heat flux $C$:

$$C = \varrho \mu w_v \quad (A.7)$$

$$\mu = c_{p,v}(T - T_{\text{ref}}) \quad (A.8)$$

$T_{\text{ref}}$ is an arbitrary reference value, which reflects the similarity in definitions of internal energy and of potential energy. For an unambiguous framework it is necessary to formulate definitions and theories about heat flux such that they only depend on energy differences of the masses involved. In this way the reference energies of the masses will not affect our conclusions.

The above definition of the sensible heat flux does not count mass fluxes (convective transport of heat), but only conduction. Therefore it is directly clear that the sensible heat does not depend on reference values of energies in masses. Following interpretation A, one concludes that, with the above motivation, there is a preference to use symbol $H$ for the *conductive* transfer of heat from the surface to the atmosphere.

## A.2 Interpretation B

The name ”sensible heat flux” makes clear that $H$ is that part of the energy flux balance, which can be sensed. As in interpretation A, conduction of heat from the surface to the atmosphere is the most important contribution to the sensible heat flux. Furthermore one cannot neglect the convective transport of heat. Consider a situation where a large, hot mass-flux comes through the surface (for example in volcanic areas or in a kettle which initially contains relatively cold air and which is filled with hot air). In those situations (see figure A.2) a thermometer, which is placed in the system of interest (the atmospheric boundary layer or the kettle), will indicate an increasing temperature and from a physical point of view it would be absurd to negate this measurable effect. Even in the absense of conductive heat transfer, the thermometer ”senses” a heat flux. This makes clear that the convective heat transfer has to be somehow embedded in the definition of the sensible heat flux. Naturally it would be of no physical significance to count all thermal energy in the mass-flux entering the system from zero Kelvin. The reference energy, to which interpretation A refers, can even include relativistic energy $E = mc^2$. This would lead to the assignment of absurdly high values to energy fluxes associated with mass fluxes. It is clear that the *transfer* of energy to the atmosphere from molecules that come from the other side of the surface is proportional to the temperature difference between the molecules initially present in the atmosphere and the surface temperature. The situation is illustrated in figure A.3. One can make estimates for this convective heat transfer by considering the

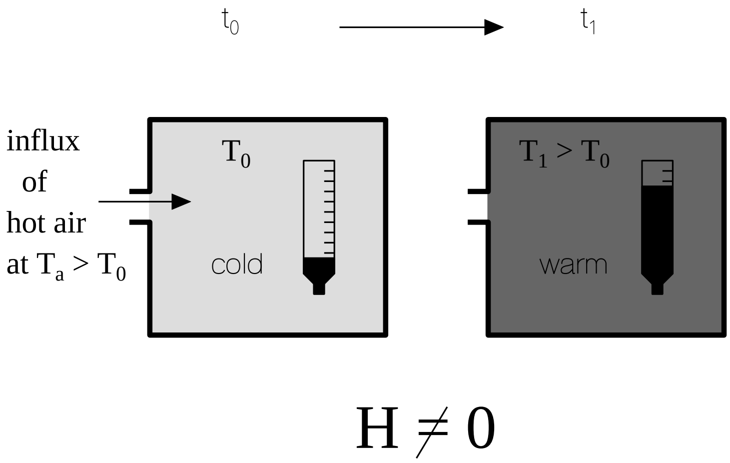


Figure A.2: Interpretation B: convection contributes to the sensible heat flux.

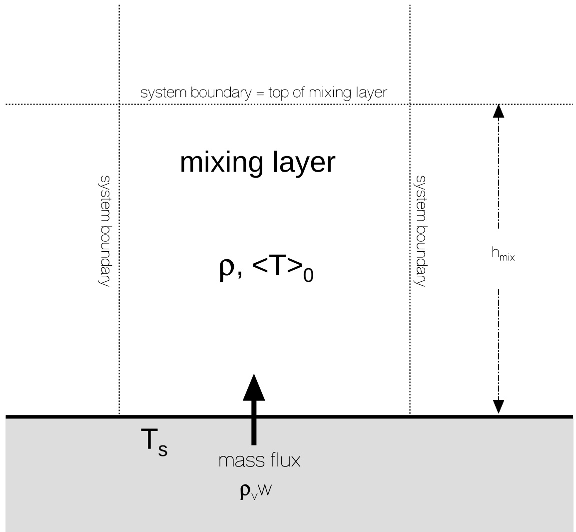


Figure A.3: Interpretation B: integration volume in the atmosphere for sensible heat flux

atmospheric boundary to be a semi-closed system. Collect all evaporating molecules close to the surface and bring them to the average temperature in the mixing layer, by some imaginary heat exchange device. The joules from this pre-heating process are to be counted. The evaporated water has now the average mixing layer temperature and can be admitted to the atmosphere without further necessity to count energy fluxes. When the mass flux is evenly spread over the mixing layer, which has mean temperature $< T >_0$, then we find the following expression for the convective contribution to the sensible heat flux:

$$H_{\mathrm{convect}} = \varrho_v c_{p,v}(T_s - < T >_0)\frac{w_v}{h_{\mathrm{mix}}} \tag{A.9}$$

where $h_{\mathrm{mix}}$ is the height of the mixing layer. If the evaporated mass is not evenly mixed with the atmosphere, then one has to adopt a different value for atmospheric temperature $< T >_0$. To avoid ambiguity we take the average temperature of the mixing layer.

A quantitative estimate for the convective heat flux is found as follows. Let us assume the mixing layer, in which the evaporated water is evenly spread, to have a height of about one kilometer, and assume that 4 millimeters of water evaporate (yielding 4 meters of vapour). The relative mass contribution to the mixing layer, formed by the evaporated water, is four promiles. Let the difference between the temperature of the evaporated water and the average temperature in the mixing layer be some tens of degrees. Then the induced change in temperature of the mixing layer would be some hundredths of degrees. This is too small to represent a significant convective heat flux and therefore we can neglect the (nonzero!) convective contribution to the sensible heat flux.

## A.3 Interpretation C

We define the sensible heat flux as **the rate of increase of the enthalpy of the atmosphere by transformations at the surface.**
We illustrate this definition with the following example: place an imaginary, vertically oriented cylinder of a certain height on the surface. Take the height and conditions such that they can be considered horizontally homogeneous and statistically stationary. Assume that there are no storage or source effects. This implies that there is no flux divergence. Therefore the sensible heat flux which leaves the cylinder via the top equals the sensible flux entering the cylinder via the bottom.

The number of molecules $n_0$, which per unit of time enter the cylinder at the bottom, equals the number of molecules $n_a$, which leave the cylinder at the top:

$$n_0 = n_a \tag{A.10}$$

We assume that the density in the cylinder is constant. Therefore turbulent mixing can change the composition of the air at a certain height, but it does not alter the density.

Dry air and water vapour contribute to the outflux of molecules at the top of the cylinder proportionally to their specific concentrations. The relatively high concentration of dry air constituents at the top of the cylinder will induce an outflux out of the cylinder of mainly dry air molecules (see the first plot in figure A.4). Nevertheless we have assumed stationary conditions, and when more water vapour enters the cylinder than leaves the cylinder via the top, then the cylinder contents will become wetter and wetter (second plot in figure A.4). The assumption of stationarity can therefore only hold when we assume that there is a water vapour concentration gradient. Turbulent mixing will interchange wet air from in the cylinder with drier air from above the cylinder, thus maintaining constant water vapour concentration in the cylinder (last two plots in figure A.4). We see that a mass flux at the surface is transformed into a turbulent flux at the top of the cylinder.

The enthalpy flux $H_E$ at the top consists of two components: the turbulent flux $H_a$, associated with the exchange of parcels of wet and dry air, and a contribution $H_{c,\mathrm{top}}$ associated with the mass flux. The latter can be written as:

$$H_{c,\mathrm{top}} = n_a m_v c_{p,v}(T_a - T_{\mathrm{ref}}) \tag{A.11}$$

where $m_v$ is the mole mass of water. At the bottom of the cylinder we have conduction $H_{\mathrm{conduction}}$ plus a mass flux associated enthalpy flux $H_{c,\mathrm{surf}}$. The latter can be expressed by:

$$H_{c,\mathrm{surface}} = n_a m_v c_{p,v}(T_s - T_{\mathrm{ref}}) \tag{A.12}$$

The situation is illustrated in figure A.5. The budget equation for the energy in the cylinder is:

$$H_a = H_{\mathrm{conduction}} + n_a m_v c_{p,v}(T_s - T_a) \tag{A.13}$$

We see that water vapour, which is warmer than the atmospheric air, contributes to the turbulent heat flux at a certain height.

Let us consider the hydrological cycle. Water comes to the surface in the form of rain. In general the temperature of the rain is lower than of the surface. The soil will have to spend energy to heat the rain while it infiltrates. From this moment on the water is part of the soil and consequently will follow the temperature variations of the soil. Before evaporation the water will first go to the surface and assume surface temperature. This heating of water to surface temperature will be done at the cost of the soil heat flux, which influences the balance in equation A.1.

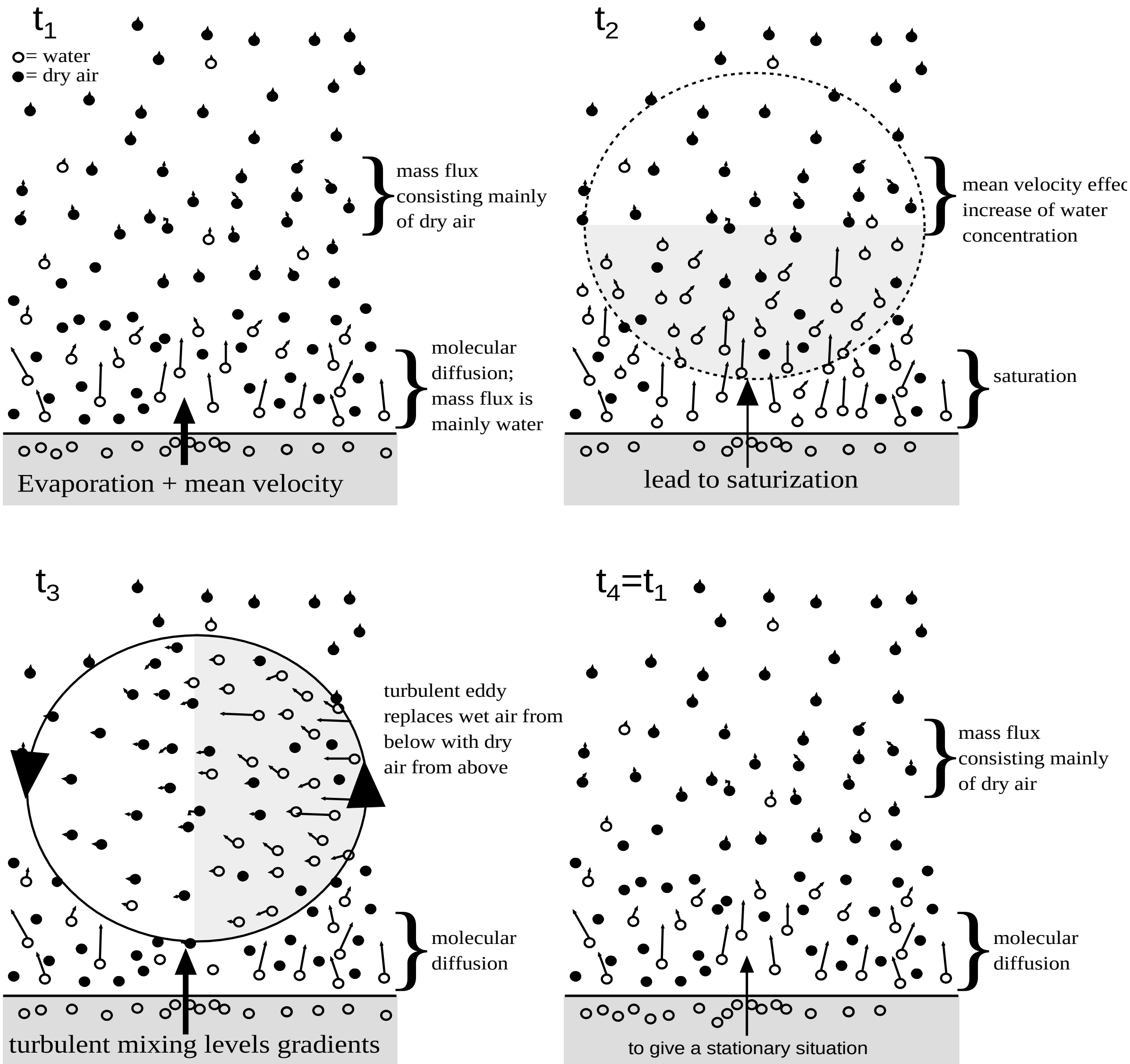


Figure A.4: Interpretation C: An influx of water at the surface leads to an outflux of mostly dry air constituent. Turbulent mixing is required to create a stationary situation.

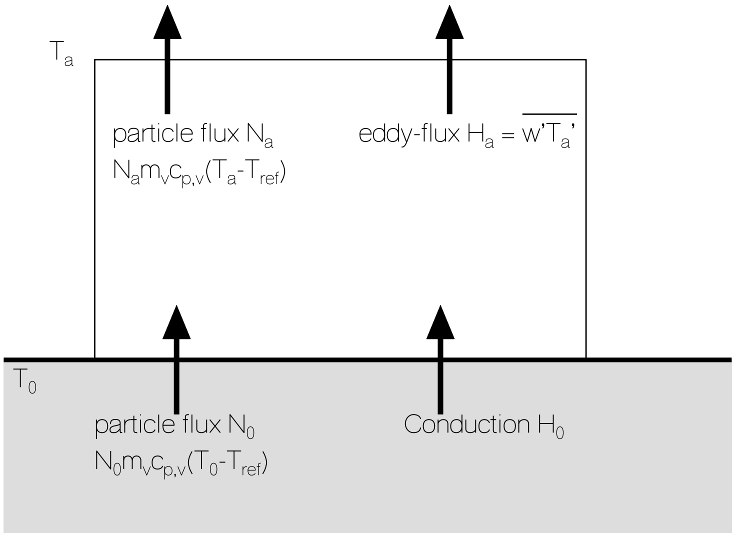


Figure A.5: Interpretation C: integration volume in the atmosphere for sensible heat flux

The situation is comparable with a heater, which takes cold air from outside, heats it and ventilates the heated air into a room. This situation is schematically drawn in figure A.6 and can be compared with figure A.2, corresponding with interpretation B. It seems no longer sustainable to stick to the idea that, when a mass flux is involved, definitions of surface fluxes should be independent of the atmospheric air temperature. To stay with the example of the heater: When the temperature in the room is higher than the temperature of the air coming from the heater, then the heater will have a cooling effect on the room, which is a negative sensible heat flux.

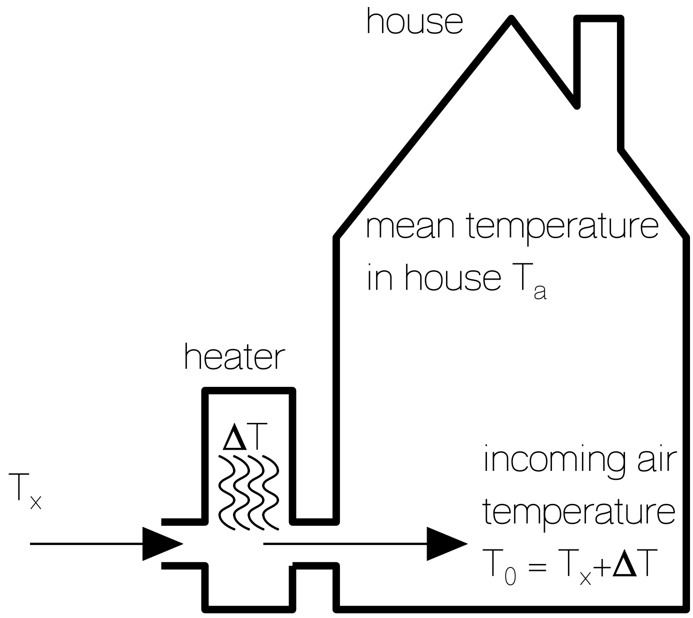


Figure A.6: The change in roomtemperature $T_a$ is related to the convective contribution to the sensible heat flux and depends on the difference in temperature between the air in the room and the air coming from the heater.

This reasoning also applies to the definition of the sensible heat flux in the atmosphere: it depends on the atmospheric temperature. Assume that, at a given moment, the atmospheric temperature rises. Then the sensible heat flux will deminish. As a consequence a new equilibrium will be formed between the terms in relation A.1.

# Appendix B

# Description of software library ECPACK

This appendix describes the core routines that are part of the FORTRAN-77 library ECPACK. The latest version can always be obtained from the website of the Meteorology and Air Quality Group of Wageningen University (under Research → Joint Eddycovariance Project). Apart from the library routines discussed here, a complete data analysis package is available that computes fluxes from eddy-covariance data stored in files using the NetCDF format (http://www.unidata.ucar.edu/packages/netcdf). This package is included in the source code of the library ECPACK as it is available from the website of the Meteorology and Air Quality Group of Wageningen University. Both ECPACK and the complete flux computation package are available for free under the GNU Public License (http://www.gnu.org/licenses).

The highest level routines plus some general routines are in tt ec_gene.f. The correction routines are in `ec_corr.f`. Routines that refer to physical processes are in `ec_phys.f`. Supporting mathematics routines are located in `ec_math.f`. Furthermore, two include files are used, viz. `physcnst.inc` and `parcnst.inc`. The documentation presented here has been extracted from the source code using the Robodoc package (http://www.xs4all.nl/ rfsber/Robo)

## B.1 parcnst.inc/parcnst.inc

NAME

```
parcnst.inc
```

FUNCTION

```
include file that defines various parameters (indicating
instruments etc.)
```

## B.2 physcnst.inc/Rd

NAME

```
Rd
```

FUNCTION

```
Gas constant for dry air
```

SOURCE

```
      PARAMETER(Rd = 287.04D0)              ! [J/Kg.K]
```

## B.3 physcnst.inc/Rv

NAME

```
Rv
```

FUNCTION

```
Gas constant for water vapour
```

SOURCE

```
      PARAMETER(Rv = 461.5D0)               ! [J/Kg.K]
```

## B.4 physcnst.inc/RGas

NAME

```
RGas
```

FUNCTION

```
Universal gas constant
```

SOURCE

```
      PARAMETER(RGAs = 8314.D0)             ! [J/kmol.K]
```

## B.5 physcnst.inc/Epsilon

NAME

```
Epsilon
```

FUNCTION

```
Infinitisimal number
```

SOURCE

```
      PARAMETER(Epsilon = 1.D-30)           ! [1]
```

## B.6 physcnst.inc/Pi

NAME

```
Pi
```

FUNCTION

```
Pi
```

SOURCE

```
      PARAMETER(Pi=3.1415926535897932385D0) ! [1]
```

## B.7 physcnst.inc/Kelvin

NAME

```
Kelvin
```

FUNCTION

```
Temperature of 0 degree Celsius in Kelvin
```

SOURCE

```
      PARAMETER(Kelvin = 273.15D0)          ! [K]
```

## B.8 physcnst.inc/GammaR

NAME

```
GammaR
```

FUNCTION

```
Constant in correction of sonic temperature in voor corss-wind
(part of the Schotanus correction)
```

SOURCE

```
PARAMETER(GammaR = 403.D0)            ! [m^{2}s^{-2}K^{-1}]
```

## B.9 physcnst.inc/DifCo2

NAME

```
DiffCo2
```

FUNCTION

```
Molecular diffusivity CO2 at 30 degree Celcius (?)
```

SOURCE

```
PARAMETER(DifCO2 = 15.6D-6)  ! [m^{2}s^{-1}]
```

## B.10 physcnst.inc/DifH20

NAME

```
DiffH20
```

FUNCTION

```
Molecular diffusivity H20 at 30 degree Celcius (?)
```

SOURCE

```
PARAMETER(DifH2O = 25.7D-6)  ! [m^{2}s^{-1}]
```

## B.11 physcnst.inc/Karman

NAME

```
Karman
```

FUNCTION

```
Von Karman constant
```

SOURCE

```
PARAMETER(Karman = 0.4D0)    ! [1]
```

## B.12 physcnst.inc/GG

NAME

```
GG
```

FUNCTION

```
Gravitational acceleration
```

SOURCE

```
PARAMETER(GG = 9.81D0)       ! [m s^{-2}]
```

## B.13 physcnst.inc/MO2

NAME

```
MO2
```

FUNCTION

```
molecular weight of oxygen
```

SOURCE

```
PARAMETER(MO2 = 32.D0)       ! [g mol^{-1}]
```

## B.14 physcnst.inc/MAir

NAME

```
MAir
```

FUNCTION

```
molecular weight of dry air
```

SOURCE

```
PARAMETER(MAir = 28.966D0)   ! [g/mol]
```

## B.15 physcnst.inc/MVapour

NAME

```
MVapour
```

FUNCTION

```
molecular weight of water vapour
```

SOURCE

```
PARAMETER(MVapour = 18.016D0)! [g/mol]
```

## B.16 physcnst.inc/Mu

NAME

Mu

FUNCTION

ratio: Mu = Mair/Mv

SOURCE

```
PARAMETER(Mu = 1.6078D0)     ! [1]
```

## B.17 physcnst.inc/Kok

NAME

Kok

FUNCTION

extinction for oxygen for Krypton hygrometer

NOTES

First the value from work of Webb (1980) was used
(KoK = 0.0085 [m^{3} g^{-1} cm^{-1}]). Now
a value due to van Dijk (1999) is used.
This constant may disappear from the library and may become
part of the user-supplied instrument specifications

SOURCE

```
PARAMETER(KoK = 0.0038D0)      ! [m^{3} g^{-1} cm^{-1}]
```

## B.18 physcnst.inc/Kwk

NAME

Kwk

FUNCTION

extinction for water vapour for Krypton hygrometer

SOURCE

```
PARAMETER(KwK = 0.143D0)     ! [m^{3} g^{-1} cm^{-1}]
```

## B.19 physcnst.inc/Kola

NAME

Kola

FUNCTION

Extinction for oxygen for Lymann-alpha

SOURCE

```
PARAMETER(KoLa = 0.001085D0)      ! [m^{3} g^{-1} cm^{-1}]
```

## B.20 physcnst.inc/Kwla

NAME

Kwla

FUNCTION

Extinction for water vapour for Lymann-alpha

SOURCE

```
PARAMETER(KwLa = 0.09125D0)          ! [m^{3} g^{-1} cm^{-1}]
```

## B.21 physcnst.inc/FracO2

NAME

FracO2

FUNCTION

Fraction of O2 molecules in air

SOURCE

```
PARAMETER(FracO2 = 0.21D0)   ! [1]
```

## B.22 physcnst.inc/Cp

NAME

Cp

FUNCTION

Specific heat of air

SOURCE

```
PARAMETER(Cp = 1004.67D0)    ! [J Kg^{-1} K^{-1}]
```

## B.23 physcnst.inc/Lv

NAME

```
Lv
```

FUNCTION

```
Latent heat of vaporization of water
```

SOURCE

```
      PARAMETER(Lv = 2.45D6)       ! [J Kg^{-1}]
```

## B.24 physcnst.inc/MinT

NAME

```
MinT
```

FUNCTION

```
Lower limit for acceptance of temperature sample
```

SOURCE

```
      PARAMETER(MinT    = 243.D0) ! [Kelvin]
```

## B.25 physcnst.inc/MaxT

NAME

```
MaxT
```

FUNCTION

```
Upper limit for acceptance of temperature sample
```

SOURCE

```
      PARAMETER(MaxT    = 333.D0) ! [Kelvin]
```

## B.26 physcnst.inc/MinRhov

NAME

```
MinRhov
```

FUNCTION

```
Lower limit for acceptance of water vapour sample
```

SOURCE

```
      PARAMETER(MinRhoV =   0.D0) ! [kg m^{-3}]
```

## B.27 physcnst.inc/MaxRhov

NAME

```
MaxRhov
```

FUNCTION

```
Upper limit for acceptance of water vapour sample
```

SOURCE

```
      PARAMETER(MaxRhoV =   1.D0) ! [kg m^{-3}]
```

## B.28 physcnst.inc/MinRhoCO2

NAME

```
MinRhoCO2
```

FUNCTION

```
Lower limit for acceptance of CO2 sample
```

SOURCE

```
      PARAMETER(MinRhoCO2 =   0.D0) ! [kg m^{-3}]
```

## B.29 physcnst.inc/MaxRhoCO2

NAME

```
MaxRhoCO2
```

FUNCTION

```
Upper limit for acceptance of CO2 sample
```

SOURCE

```
      PARAMETER(MaxRhoCO2 =   1.D0) ! [kg m^{-3}]
```

## B.30 ec_gene.f/EC_G_Main

NAME

```
EC_G_Main
```

## SYNOPSIS

```
CALL EC_G_Main(OutF,DoPrint,
   RawSampl,MaxChan,Channels,NMax,N,MMax,M, PCal,PIndep,
   Psychro,CalSonic,CalTherm,CalHyg, CalCO2, P,
   Calibr,
   Sample,Flag,Mok,Cok,MIndep,CIndep,Rc,BadTc,
   DoCorr, PCorr, ExpVar,
   DirYaw, DirPitch, DirRoll,
   Apf, SonFactr, O2Factor, FrCor,
   Mean,TolMean,Cov,TolCov,
   QPhys, dQPhys,
   HAVE_UNCAL, Have_cal, DiagFlag, FirstDay)
```

## FUNCTION

```
Integrated routine which:
- Calibrates raw samples
- Estimates mean values and covariances with respective tolerances
- Corrects the resulting mean values and covariances for all
  effects selected by the user
- Estimates, from the final mean values and covariances, the surface-
  fluxes with tolerances.
```

## INPUTS

```
OutF      : [INTEGER]
            unit number of file for intermediate results
DoPrint   : [LOGICAL]
            write intermediate results to file?
RawSampl  : [REAL*8(MaxChan, MMax)
            raw, uncalibrated samples
MaxChan   : [INTEGER]
            maximum number of channels in RawSampl
Channels  : [INTEGER]
            actual number of channels in RawSampl
NMax      : [INTEGER]
            maximum number of calibrated quantities
N         : [INTEGER]
            actual number of calibrated quantities
MMax      : [INTEGER]
            maximum number of samples
M         : [INTEGER]
            actual number of samples
PCal      : [LOGICAL]
            print results of slow sensor correction?
PIndep    : [LOGICAL]
            print number of independent samples?
DoCorr    : [LOGICAL](NMaxCorr)
            which corrections to do?
PCorr     : [LOGICAL](NMaxCorr)
            intermediate results of which corrections?
ExpVar    : [REAL*8](NMaxExp)
            array with experimental settings
Psychro   : [REAL*8]
            water vapour density of slow sensor (kg/m^3)
CalSonic  : [REAL*8(NNQ)]
            array with calibration data of sonic
CalTherm  : [REAL*8(NNQ)]
            array with calibration data of thermocouple
CalHyg    : [REAL*8(NNQ)]
            array with calibration data of hygrometer
CalCO2    : [REAL*8(NNQ)]
            array with calibration data of CO2 sensor
P         : [REAL*8]
            atmospheric pressure (Pa)
Calibr    : [SUBROUTINE]
            calibration subroutine
Apf       : [REAL*8(3,3)]
            planar fit untilt matrix
HAVE_UNCAL:   [LOGICAL(NMax)]
            switch whether data for uncalibrated data
            are available for each channel
FirstDay  : [INTEGER]
            day number of first sample in array (needed for
            detrending data that pass midnight)
```

```
OUTPUTS
Sample    : [REAL*8(NMax, MMax)]
            array with calibrated samples
Flag      : [LOGICAL(NMax, MMax)]
            validity flag (true means invalid) for all
            calibrated samples
Mok       : [INTEGER(NMax)]
            number of valid samples for each quantity
Cok       : [INTEGER(NMax,NMax)]
            number of valid samples for each combination of
            two quantities
MIndep    : [INTEGER(NMax)]
            number of independent samples for each quantity
CIndep    : [INTEGER(NMax, NMax)]
            number of independent samples for each
            combination of two quantities
Rc        : [REAL*8(NMax)]
            slope of linear regression in case liner detrending
            has been done (for each quanity)
BadTc     : [LOGICAL]
            if more than half of the thermocouple samples are
            wrong, flag thermocouple as bad
DirYaw    : [REAL*8]
            yaw angle (degrees)
DirPitch  : [REAL*8]
            pitch angle (degrees)
DirRoll   : [REAL*8]
            roll angle (degrees)
SonFactr  : [REAL*8(NMax)]
            correction factor for the covariances with specific
            humidity
O2Factor  : [REAL*8(NMax)]
            Correction factor due to oxygen correction for
            covariance of humidity with each calibrated
FrCor     : [REAL*8(NMax, NMax)]
            Correction factors for covariances for frequency
            response
Mean      : [REAL*8(NMax)] (in/out)
            Mean values of all calibrated signals
TolMean   : [REAL*8(NMax)] (in/out)
            Tolerances in mean values of all calibrated signals
Cov       : [REAL*8(NMax,NMax)] (in/out)
            Covariances of all calibrated signals
TolCov    : [REAL*8(NMax,NMax)] (in/out)
            Tolerances in covariances of all calibrated signals
QPhys     : [REAL*8](NMaxPhys)] (in/out)
            array with physical quantities
dQPhys    : [REAL*8](NMaxPhys)] (in/out)
            array with tolerances physical quantities
HAVE_CAL  : [LOGICAL(NNMax)]
            switch whether data for calibrated data
            are available for each channel
DiagFlag  : [INTEGER](NMaxDiag)
            count of flags occuring in diagnostic word of CSAT
```

## AUTHOR

```
Arjan van Dijk, Arnold Moene
```

## HISTORY

```
Revision: 03-04-2001: get mean W and its tolerance before tilt
                      correction; export via interface (AM)
Revision: 28-05-2001: added passing of info on whether uncalibrated
                      data are available (AM)
Revision: 18-09-2002: removed calcomm.inc and added FirstDay
                      to interface (to pass it to calibration routine
Revision: 5-12-2002:  added DoPF, PPF, Apf to interface to include planar fit
Revision: 13-01-2003: added vectorwind and dirfrom to interface
```

```
Revision: 27-01-2003: removed physical quantities from interface
                      and replace by QPhys
                      Put all correction info into DoCorr and PCorr, ExpVar
REvision: 29-01-2003: added have_cal to interface to
                      have it available in ec_ncdf
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
EC_C_Main
EC_C_T05
EC_C_T06
EC_C_T08
EC_C_T10
EC_G_Reset
EC_G_ShwInd
EC_G_Show
EC_M_Averag
EC_M_MinMax
EC_M_Detren
EC_Ph_Q
EC_Ph_Flux
parcnst.inc
```

# B.31 ec_gene.f/EC_G_Reset

NAME

```
EC_G_Reset
```

SYNOPSIS

```
CALL EC_G_Reset(Have_cal, Mean, TolMean, Cov, TolCov, MIndep,
CIndep)
```

FUNCTION

```
Routine to reset means and covariances based on availability
of the uncalibrated data
```

INPUTS

```
Have_cal  : [LOGICAL(NMax)]
              switch for each channel whether calibrated
              data are available
Mean        : [REAL*8(NMax)]
              mean of quantities
TolMean     : [REAL*8(NMax)]
              tolerance in mean of quantities
Cov         : [REAL*8(NMax,NMax)]
              covariances of quantities
TolMean     : [REAL*8(NMax,NMax)]
              tolerance in covariances of quantities
MIndep      : [REAL*8(NMax)]
              number of independent samples
CIndep      : [REAL*8(NMax,NMax)]
              number of independent samples in covariances
```

OUTPUT

```
Mean        : [REAL*8(NMax)]
              mean of quantities
TolMean     : [REAL*8(NMax)]
              tolerance in mean of quantities
Cov         : [REAL*8(NMax,NMax)]
              covariances of quantities
TolMean     : [REAL*8(NMax,NMax)]
              tolerance in covariances of quantities
MIndep      : [REAL*8(NMax)]
              number of independent samples
CIndep      : [REAL*8(NMax,NMax)]
              number of independent samples in covariances
```

AUTHOR

```
Arnold Moene
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
parcnst.inc
```

# B.32 ec_gene.f/EC_G_ShwFrq

NAME

```
EC_G_ShwFrq
```

SYNOPSIS

```
CALL EC_G_ShwFrq(OutF,FrCor,NMax,N)
```

FUNCTION

```
Prints correction factors associated with frequency response
```

INPUTS

```
OUTF    : [INTEGER]
          unit number of file
FrCor   : [REAL*8(NMax,NMax)]
          freqency response correction factors for
          each combination of quantities
NMax    : [INTEGER]
          maximum number of quantities
N       : [INTEGER]
          actual number of quantities
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
parcnst.inc
```

# B.33 ec_gene.f/EC_G_ShwInd

NAME

```
EC_G_ShwInd
```

SYNOPSIS

```
CALL EC_G_ShwInd(OutF,MIndep,
                 CIndep,NMax,N,M,Freq)
```

FUNCTION

```
Prints number of independent observations
```

INPUTS

```
OUTF    : [INTEGER]
          unit number of file
MIndep  : [INTEGER(NMax)]
          number of independent samples for
          means
CIndep  : [INTEGER(NMax,NMax)]
          number of independent samples for
          covariances
NMax    : [INTEGER]
          maximum number of quantities
N       : [INTEGER]
          actual number of quantities
M       : [INTEGER]
          total number of samples
Freq    : [REAL*8]
          sampling frequency (Hz)
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
parcnst.inc
```

# B.34 ec_gene.f/EC_G_ShwMinMax

NAME

```
EC_G_ShwMinMax
```

SYNOPSIS

```
CALL EC_G_ShwInd(OutF, N, Mins, Maxs)
```

FUNCTION

```
Prints min/max of series
```

INPUTS

```
OUTF    : [INTEGER]
          unit number of file
N       : [INTEGER]
          number of series
Mins    : [INTEGER(NMax)]
          min value of series
Maxs    : [INTEGER(NMax)]
          max value of series
```

AUTHOR

```
Arnold Moene
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
parcnst.inc
```

# B.35 ec_gene.f/EC_G_ShwHead

NAME

```
EC_G_ShwHead
```

SYNOPSIS

```
CALL EC_G_ShwHead(OutF,String)
```

FUNCTION

```
Prints header to intermediate results file
```

INPUTS

```
OUTF    : [INTEGER]
          unit number of file
N       : [CHARACTER](*)
          String to write
```

AUTHOR

```
Arnold Moene
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

# B.36 ec_math.f/EC_M_ABCForm

NAME

```
EC_M_ABCForm
```

SYNOPSIS

```
CALL EC_M_ABCForm(a,b,c,Root1, Root2, AllReal)
```

FUNCTION

```
Solves ax^2 + bx + c = 0
```

INPUTS

```
a      : [REAL*8]
         first coefficient
b      : [REAL*8]
         second coefficient
c      : [REAL*8]
         third coefficient
OUTPUTS
Root1  : [REAL*8]
         first root
Root2  : [REAL*8]
         second root
AllReal: [LOGICAL]
         all roots real?
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```



## B.37 ec_math.f/EC_M_Averag

NAME

```
EC_M_Averag
```

SYNOPSIS

```
CALL EC_M_Averag(x,NMax,N,MMax,M,Flag,
                 Mean,TolMean,Cov,TolCov,MIndep,CIndep,Mok,Cok)
```

FUNCTION

```
From the given set of calibrated samples, calculate the averages,
variances, covariances and their tolerances.
```

INPUTS

```
x      : [REAL*8(NMax,MMax)]
         Array with calibrated samples.
         First index counts quantities; second counter
         counts samples. Only the first N quantities
         and the first M samples are used.
NMax   : [INTEGER]
         maximum number of quantities
N      : [INTEGER]
         actual number of quantities
MMax   : [INTEGER]
         maximum number of samples
M      : [INTEGER]
         actual number of samples
Flag   : [LOGICAL(NMax, MMax)]
         If flag(j,i) is true, then quantity j in sample
         i is not ok.
OUTPUTS
Mean   : [REAL*8(NMax)]
         The average value array x. Only samples with Flag = 0
         are used.
TolMean: tolerance of Mean, defined as 2 * sigma / sqrt(NIndep)
         where sigma is the standarddeviation of the quantities
         and where the number of independent samples is estimated
         as twice the number of sign-changes of the fluctuations
         of the respective quantities around their means.
Cov    : [REAL*8(NMax,NMax)]
         covariances
TolCov : [REAL*8(NMax,NMax)]
         tolerances of Cov, estimated in same way as tolerances
         of mean.
MIndep : [INTEGER(NMAx)]
         Number of independent samples in time series from
         which means are calculated
CIndep : [INTEGER(NMAx,NMAx)]
         Number of independent samples in time series from
         which covariances are calculated
Mok    : [INTEGER(NMax)]
         number of valid samples for each quantity
Cok    : [INTEGER(NMax,NMax)]
         number of valid samples for each combination of
         two quantities
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
parcnst.inc
```

## B.38 ec_math.f/EC_M_BaseF

NAME

```
EC_M_BaseF
```

SYNOPSIS

```
Value = EC_M_BaseF(x,FType,Order,C)
```

FUNCTION

```
Purpose : Calculate simple functions. Currently implemented:
- Ordinary polynomials
- Polynomials in the natural logarithm of x
```

INPUTS

```
x      : [REAL*8]
         argument of function
```

```
FType  : [INTEGER]
         function type: NormPoly or LogPoly (defined in
         parcnst.inc
Order  : [INTEGER]
         order of the polynomial
C      : [REAL*8(0:Order)]
         array with coefficients
```

RETURN VALUE

```
return value  : [REAL*8]
         value of polynomial
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
parcnst.inc
```

## B.39 ec_math.f/EC_M_Cardano

NAME

```
EC_M_Cardano
```

SYNOPSIS

```
CALL EC_M_Cardano(Poly, Root, AllReal)
```

FUNCTION

```
Uses the Cardano solution to solve exactly:
ax^3 + bx^2 + cx + d = 0
"a" is not allowed to be zero.
```

INPUTS

```
Poly   : [REAL*8(0:3)]
         coefficient of third order polynomial
OUTPUTS
Root   : [REAL*8(2,3)]
         array with roots, first index are
         real and imaginary part, respectively,
         second index for three roots
AllReal: [LOGICAL]
         all roots real?
```

AUTHOR

```
Arjan van Dijk
```

SEE ALSO

```
Abramowitz and Stegun
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

## B.40 ec_math.f/EC_M_Det2

NAME

```
EC_M_Det2
```

SYNOPSIS

```
Value =  EC_M_Det2(x)
```

FUNCTION

```
Give determinant of REAL*8 2*2-matrix
```

INPUTS

```
x      : [REAL*8(2,2)]
         matrix
```

RETURN VALUE

```
return value  :
         [REAL*8]
         determinant
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

## B.41 ec_math.f/EC_M_Determ

NAME

```
EC_M_Determ
```

SYNOPSIS

```
Value =  EC_M_Determ(x)
```

FUNCTION

```
Give determinant of real 3*3-matrix
```

INPUTS

```
x      : [REAL*8(3,3)]
         matrix
```

RETURN VALUE

```
return value  :
         [REAL*8]
         determinant
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

## B.42 ec_math.f/EC_M_Detren

NAME

```
EC_M_Detren
```

SYNOPSIS

```
CALL EC_M_Detren(x,NMax,N,MMAx,M,Mean,Cov,y,RC)
```

FUNCTION

```
Construct a linearly detrended dataset from a given dataset
```

INPUTS

```
x       : [REAL*8(NMax,MMax)] (in/out)
          array with samples: x(i,j) = quantity i in sample j
          only the first N quantities and the first M samples
          are used. On output: detrended series
NMax    : [INTEGER]
          maximum number of quantities in x
N       : [INTEGER]
          actual number of quantities in x
MMax    : [INTEGER]
          maximum number of samples in x
M       : [INTEGER]
          actual number of samples in x
Mean    : [REAL*8(NMax)]
          mean of all quantities
Cov     : [REAL*8 (NMax,NMAx)]
          covariances of quantities used to find trend.
```

OUTPUT

```
RC      : [REAL*8(NMAx)]
          Directional coefficients of linear regression
          trend-lines.
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
Revision 28-01-2003: remove argument y, since it causes aliassing (and is not
                 needed).
```

USES

```
parcnst.inc
```

## B.43 ec_math.f/EC_M_DSwap

NAME

```
EC_M_DSwap
```

SYNOPSIS

```
CALL EC_M_DSwap(x,y)
```

FUNCTION

```
Interchanges x and y
```

INPUTS

```
x,y     : [REAL*8]
          quantities
OUTPUTS
x,y     : [REAL*8]
          quantities
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

## B.44 ec_math.f/EC_M_Ell1Q

NAME

```
EC_M_Ell1Q
```

SYNOPSIS

```
CALL EC_M_Ell1Q(phi,alpha,ff,ee)
```

FUNCTION

```
Calculates the elliptic integrals F(Phi\Alpha) and
E(Phi\Alpha) using the Arithmetic-Geometric Mean process
as described in Abramowitz and Stegun, 17.6 (Numbers in
text refer to equations in A&S). Only ok for first quadrant
```

INPUTS

```
phi     : [REAL*8]
          argument
alpha   : [REAL*8]
          argument
OUTPUTS
ff      : [REAL*8]
```

```
        result
ee     : [REAL*8]
        result
```

AUTHOR

```
Arjan van Dijk
```

SEE ALSO

```
Abramowitz and Stegun, 17.6
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

# B.45 ec_math.f/EC_M_EllCoords

NAME

```
EC_M_EllCoords
```

SYNOPSIS

```
CALL EC_M_EllCoords(x,b,y,DyDx)
```

FUNCTION

```
Calculate the elliptic coordinates (Lambda, Mu, Nu) plus
derivatives Dy[i]/Dx[j] corresponding to the Carthesian
coordinates (x[1], x[2], x[3]) for an ellipsoid with
semiaxes (b[1], b[2], b[3]) with b[1]>b[2]>b[3].
Procedure cannot handle points at coordinateplanes.
Outside the ellipsoid the elliptic coordinates satisfy:
-b[1]^2 < Nu < -b[2]^2 < Mu < -b[3]^2 < 0 < Lambda.
```

INPUTS

```
x    : [REAL*8(3)]
       Carthesian coordinate
b    : [REAL*8(3)]
       semi-axes of ellipsoid
alpha  : [REAL*8]
         argument
OUTPUTS
y    : [REAL*8(3)]
       new coordinate
DyDx : [REAL*8(3,3)]
       derivative
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

AUTHOR

```
Arjan van Dijk
```

USES

```
EC_M_Cardano
EC_M_DSwap
EC_M_SQR
```

# B.46 ec_math.f/EC_M_Ellint

NAME

```
EC_M_Ellint
```

SYNOPSIS

```
CALL EC_M_Ellint(phi,alpha,ff,ee)
```

FUNCTION

```
Calculates the elliptic integrals F(Phi\Alpha) and
E(Phi\Alpha) using the Arithmetic-Geometric Mean process
as described in Abramowitz and Stegun, 17.6 (Numbers in
text refer to equations in A&S). ok for all angles.
```

INPUTS

```
phi    : [REAL*8]
         argument
alpha  : [REAL*8]
         argument
OUTPUTS
ff     : [REAL*8]
         result
ee     : [REAL*8]
         result
```

AUTHOR

```
Arjan van Dijk
```

SEE ALSO

```
Abramowitz and Stegun, 17.6
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
EC_M_Ell1Q
```

# B.47 ec_math.f/EC_M_InvM

NAME

```
EC_M_InvM
```

SYNOPSIS

```
CALL EC_M_InvM(a, aInv)
```

FUNCTION

```
Find the inverse of real 3*3 matrix "a"
```

INPUTS

```
a    : [REAL*8(3,3)]
       matrix
OUTPUTS
inv  : [REAL*8(3,3)]
       inverse matrix
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
EC_M_Determ
```

## B.48 ec_math.f/EC_M_InvM2

NAME

```
EC_M_InvM2
```

SYNOPSIS

```
CALL EC_M_InvM2(a, aInv)
```

FUNCTION

```
Find the inverse of real 2*2 matrix "a"
```

INPUTS

```
a    : [REAL*8(2,2)]
       matrix
OUTPUTS
inv  : [REAL*8(2,2)]
       inverse matrix
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
EC_M_Det2
```

## B.49 ec_math.f/EC_M_ISwap

NAME

```
EC_M_ISwap
```

SYNOPSIS

```
CALL EC_M_ISwap(x,y)
```

FUNCTION

```
Interchanges x and y
```

INPUTS

```
x,y    : [INTEGER]
         quantities
OUTPUTS
x,y    : [INTEGER]
         quantities
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

## B.50 ec_math.f/EC_M_ka

NAME

```
EC_M_ka
```

SYNOPSIS

```
Value =  EC_M_ka(x,b)
```

FUNCTION

```
Returns = DSQRT((x+b(1)**2)*(x+b(2)**2)*(x+b(3)**2))
From equation 8
```

INPUTS

```
x      : [REAL*8]
         argument
b      : [REAL*8(3)]
         coefficient
```

RETURN VALUE

```
return value :
         [REAL*8]
         result
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

# B.51 ec_math.f/EC_M_Map2Vec

NAME

```
EC_M_Map2Vec
```

SYNOPSIS

```
CALL EC_M_Map2Vec(a,x,y)
```

FUNCTION

```
Calculates the image of "x" under the map "a"; y(i) = a(ij)x(j)
```

INPUTS

```
a      : [REAL*8(2,2)]
         the mapping matrix
x      : [REAL*8(2)]
         the vector to be mapped
```

OUTPUT

```
y      : [REAL*8(2)]
         the image of the map
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

# B.52 ec_math.f/EC_M_MapMtx

NAME

```
EC_M_MapMtx
```

SYNOPSIS

```
CALL EC_M_MapMtx(a,x,y)
```

FUNCTION

```
 Calculates the image of "x" under the map "a";
y(ji) = a(ki)a(lj)x(lk)
```

INPUTS

```
a      : [REAL*8(3,3)]
         the mapping matrix
x      : [REAL*8(3,3)]
         the tensor to be mapped
```

OUTPUT

```
y      : [REAL*8(3,3)]
         the image of the map
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
Revision: June 21, 2001:
- indices i and j in the mapping  have
  been interchanged. This has also been done in the
  routines that used EC_M_MapMtx (EC_C_T05)
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

# B.53 ec_math.f/EC_M_MapVec

NAME

```
EC_M_MapVec
```

SYNOPSIS

```
CALL EC_M_MapVec(a,x,y)
```

FUNCTION

```
Calculates the image of "x" under the map "a"; y(i) = a(ij)x(j)
```

INPUTS

```
a      : [REAL*8(3,3)]
         the mapping matrix
x      : [REAL*8(3)]
         the vector to be mapped
```

OUTPUT

```
y      : [REAL*8(3)]
         the image of the map
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

## B.54 ec_math.f/EC_M_MinMax

NAME

```
EC_M_MinMax
```

SYNOPSIS

```
Call EC_M_MinMax(x,NMax,N,MMax, M,Flag,Mins, Maxs)
```

FUNCTION

```
Determines minimum and maximum of quantities
```

INPUTS

```
x      : [REAL*8(Nmax,MMax)]
         the sampled quantities
NMax   : [INTEGER]
         maximum number of quantities
N      : [INTEGER]
         actual number of quantities
MMax   : [INTEGER]
         maximum number of samples
M      : [INTEGER]
         actual number of samples
Flag   : [LOGICAL(NMax, MMax)]
         If flag(j,i) is true, then quantity j in sample
         i is not ok.
```

OUTPUT

```
Mins   : [REAL*8(NMax)]
         the minimum of each quantity, only taking into
         account samples with Flag = 0
Maxs   : [REAL*8(NMax)]
         the maximum of each quantity, only taking into
         account samples with Flag = 0
```

AUTHOR

```
Arnold Moene
```

USES

```
parcnst.inc
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

## B.55 ec_math.f/EC_M_MMul

NAME

```
EC_M_MMul
```

SYNOPSIS

```
CALL EC_M_MMul(a,b,c)
```

FUNCTION

```
Matrix C is product of 3*3-matrices A and B
```

INPUTS

```
a      : [REAL*8(3,3)]
         first matrix
b      : [REAL*8(3)]
         second matrix
```

OUTPUT

```
c      : [REAL*8(3)]
         matrix product
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

## B.56 ec_math.f/EC_M_MulVec

NAME

```
EC_M_MulVec
```

SYNOPSIS

```
CALL EC_M_MulVec(x,y)
```

FUNCTION

```
Multiply vector with a constant
```

INPUTS

```
x      : [REAL*8(3)]
         vector
y      : [REAL*8]
         constant
```

OUTPUT

```
x      : [REAL*8(3)]
         vector multiplied with y
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

# B.57 ec_math.f/EC_M_SortDecr

NAME

```
EC_M_SortDecr
```

SYNOPSIS

```
CALL EC_M_SortDecr(x,permutation)
```

FUNCTION

```
Sorts the elements of vector x in decreasing order;
permutation needed is returned as well
```

INPUTS

```
x      : [REAL*8(3)]
         vector
```

OUTPUT

```
x      : [REAL*8(3)]
         sorted vector
permutation :
         [INTEGER(3)]
         permutation to arrive at sorted vector
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
EC_M_DSwap
EC_M_ISwap
```

# B.58 ec_math.f/EC_M_SortUse

NAME

```
EC_M_SortUse
```

SYNOPSIS

```
CALL EC_M_SortUse(x,permutation)
```

FUNCTION

```
Reorders the elements of x according to permutation
```

INPUTS

```
x      : [REAL*8(3)]
         vector
permutation :
         [INTEGER(3)]
         permutation to arrive at sorted vector
```

OUTPUT

```
x      : [REAL*8(3)]
         sorted vector
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

# B.59 ec_math.f/EC_M_specint

NAME

```
EC_M_specint
```

SYNOPSIS

```
CALL EC_M_specint(lambda,b,integral)
```

FUNCTION

```
Calculates the integrals:
\INT_(\lambda)^(\infty) \frac{dq}{{b_{i}^{2}+q}k_{q}}
References "G&R" in the code are to Gradshteyn and Ryzhik:
Tables of Integrals, Series and Products, 4th ed.,Ac. Press,'65
```

INPUTS

```
lambda   : [REAL*8]
           one lambda
b        : [REAL*8(3)]
           vector of b-values
```

OUTPUT

```
integral : [REAL*8(3)]
           result
```

AUTHOR

```
Arjan van Dijk
```

SEE ALSO

```
References "G&R" in the code are to Gradshteyn and Ryzhik:
Tables of Integrals, Series and Products, 4th ed.,Ac. Press,'65
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

## B.60 ec_math.f/EC_M_SQR

NAME

```
EC_M_SQR
```

SYNOPSIS

```
Value =  EC_M_SQR(x)
```

FUNCTION

```
Give the square of x
```

INPUTS

```
x        : [REAL*8]
```

RETURN VALUE

```
return value :
           [REAL*8]
           result
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

## B.61 ec_math.f/EC_M_UnSort

NAME

```
EC_M_UnSort
```

SYNOPSIS

```
CALL EC_M_UnSort(x, permutation)
```

FUNCTION

```
Unsorts the elements of x originally sorted using permutation
```

INPUTS

```
x       : [REAL*8(3)]
          vector
permutation :
          [INTEGER(3)]
          permutation to arrive at sorted vector
```

OUTPUT

```
x       : [REAL*8(3)]
          unsorted vector
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

## B.62 ec_phys.f/EC_Ph_Flux

NAME

```
EC_Ph_Flux
```

SYNOPSIS

```
CALL EC_Ph_Flux(Mean,NMax,Cov,TolMean,TolCov,p,BadTc,
                WebVel, dirYaw)
```

FUNCTION

```
Construct estimates for surface fluxes from mean values and covariances
```

INPUTS

```
Mean   : [REAL*8(NMax)]
         Means of all variables
NMax   : [INTEGER]
         Maximum number of variables
Cov    : [REAL*8(NMax,NMax)]
         Covariances of all variables
TolMean: [REAL*8(NMax)]
         Tolerances in means of all variables
TolCov : [REAL*8(NMax,NMax)]
         Tolerances in covariances of all variables
p      : [REAL*8]
         atmosperic pressure (Pa)
BadTc  : [LOGICAL]
         indicator whether thermocouple temperature is corrupt
WebVel : [REAL*8]
         Webb velocity (m/s)
DirYaw : [Real*8]
         Yaw rotation angle (degrees)
```

OUTPUT

```
QPhys  : [REAL*8](NMaxPhys)
         array with physical quantities
dQPhys : [REAL*8](NMaxPhys)
         array with tolerances in physical quantities
         tolerance in sensible heat flux with sonic temperature (W/m^2)
```

AUTHOR

```
Arjan van Dijk, Arnold Moene
```

HISTORY

```
07-10-2002: added CO2 fluxes and WebVel to interface. Webb-term
            is now computed with Webvel, rather than Mean(W)
26-01-2003: replaced physical quantities by QPhys
$Name:  $
```

```
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
EC_Ph_RhoWet
parcnst.inc
Cp
Lv
```

# B.63 ec_phys.f/EC_Ph_Q

NAME

```
EC_Ph_Q
```

SYNOPSIS

```
Spec_hum = EC_Ph_Q(RhoV,T,P)
```

FUNCTION

```
Calculate the specific humidity of wet air
```

INPUTS

```
 Rhov : [REAL*8]
        Density of air (kg/m^3)
 T    : [REAL*8]
        Temperature (K)
 P    : [REAL*8]
        Pressure (Pa)
```

RETURN VALUE

```
 return value : [REAL*8]
        Specific humidity (kg/kg)
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
EC_Ph_RhoWet
```

# B.64 ec_phys.f/EC_Ph_QCO2

NAME

```
EC_Ph_QCO2
```

SYNOPSIS

```
Spec_CO2 = EC_Ph_QCO2(RHOCO2,RHOV,T,P)
```

FUNCTION

```
Calculate the specific CO2 concentration of wet air
```

INPUTS

```
 RhoCO2 : [REAL*8]
        Density of CO2 (kg/m^3)
 Rhov : [REAL*8]
        Density of air (kg/m^3)
 T    : [REAL*8]
        Temperature (K)
 P    : [REAL*8]
        Pressure (Pa)
```

RETURN VALUE

```
 return value : [REAL*8]
        Specific CO2 (kg/kg)
```

AUTHOR

```
Arnold Moene
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
EC_Ph_RhoWet
```

# B.65 ec_phys.f/EC_Ph_RhoDry

NAME

```
EC_Ph_RhoDry
```

SYNOPSIS

```
Rho_dry = EC_Ph_RhoDry(RhoV,T,P)
```

FUNCTION

```
Calculate the density of dry air component in wet air
Via Dalton's law : Pressure is sum of partial pressures :
P = RhoV*Rv*T + RhoD*Rd*T
```

INPUTS

```
 Rhov : [REAL*8]
        Density of air (kg/m^3)
 T    : [REAL*8]
        Temperature (K)
 P    : [REAL*8]
        Pressure (Pa)
```

RETURN VALUE

```
  return value : [REAL*8]
         Density of dry part of air (kg/m^3)
```

AUTHOR

```
 Arjan van Dijk
```

HISTORY

```
 $Name:  $
 $Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
 Rd
 Rv
```

## B.66 ec_phys.f/EC_Ph_RhoWet

NAME

```
 EC_Ph_RhoWet
```

SYNOPSIS

```
 Rho_dry = EC_Ph_RhoWet(RhoV,T,P)
```

FUNCTION

```
 Calculate the density of wet air
```

INPUTS

```
  Rhov : [REAL*8]
         Density of air (kg/m^3)
  T    : [REAL*8]
         Temperature (K)
  P    : [REAL*8]
         Pressure (Pa)
```

RETURN VALUE

```
  return value : [REAL*8]
         Density of wet air (kg/m^3)
```

AUTHOR

```
 Arjan van Dijk
```

HISTORY

```
 $Name:  $
 $Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
 EC_Ph_RhoDry
```



## B.67 ec_phys.f/EC_Ph_Struct

NAME

```
 EC_Ph_Struct
```

SYNOPSIS

```
 CALL EC_Ph_Struct(Sample,NMax,MMax,M,Flag,
                  XIndex,YIndex,
                  R,dR,Freq,CIndep,Cxy,dCxy)
```

FUNCTION

```
 Calculate structure parameters <(x(r)-x(r+R))*(y(r)-y(r+R))>/R^2/3
```

INPUTS

```
 Sample   : [REAL*8(NMax,MMax)]
            Samples (quantities in first dimension, samples in
            second dimension)
 NMax     : [INTEGER]
            physical first dimension of array Sample.
 MMax     : [INTEGER]
            physical second dimension of array Sample.
 M        : [INTEGER]
            actual number of meaningful samples in array Sample.
 Flag     : [LOGICAL(NMax,MMax)]
            if Flag(i,j) is true, then something is wrong with
            quantity i in sample j.
 XIndex   : [INTEGER]
            indicator of first quantity involved in
            structure function.
 YIndex   : [INTEGER]
            indicator of second quantity involved in
            structure function.
 R        : [REAL*8] :
            separation in meters at which one wants to estimate
            the structure function.
 Freq     : [REAL*8] :
            Sampling frequency in s^-1.
 CIndep   : [INTEGER[NMax,NMax])
            Number of independent contributions by array Sample
            to covariance between quantities selected with
            XIndex and YIndex.
 Rhov     : [REAL*8]
            Density of air (kg/m^3)
 T        : [REAL*8]
            Temperature (K)
 P        : [REAL*8]
            Pressure (Pa)
```

OUTPUT

```
 dR       : [REAL*8]
            separation in meters corresponding with a delay
            of one sample (i.e. tolerance in R)
 cxy      : [REAL*8]
            Structure parameter.
 dcxy     : [REAL*8]
            Tolerance of cxy.
```

AUTHOR

```
 Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
Revision:  20-9-2002: added implicit none and inclusion of
                      parcnst.inc (needed for constants U,V, and W)
Revision:  27-01-2003: removed N from interface
```

USES

```
parcnst.inc
```

## B.68 ec_phys.f/EC_Ph_Obukhov

NAME

```
EC_Ph_Obukhov
```

SYNOPSIS

```
L =  EC_Ph_Obukhov(Ustar,Tstar,Qstar ,MeanT)
```

FUNCTION

```
Calculate Obukhov length (taking into account buoyancy effect of
water vapour)
```

INPUTS

```
Ustar   : [REAL*8]
           u* (m/s)
Tstar   : [REAL*8]
           T* (K)
Qstar   : [REAL*8]
           q* (kg/kg)
MeanT   : [REAL*8]
           mean temperature (K)
           actual number of meaningful samples in array Sample.
```

OUTPUT

```
return value  : [REAL*8]
           obukhov length (m)
```

AUTHOR

```
Arnold Moene
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
physcnst.inc
GG
Karman
```

## B.69 ec_corr.f/EC_C_D01

NAME

```
EC_C_D01
```

SYNOPSIS

```
CALL EC_C_D01(x, b, aInv)
```

FUNCTION

```
This routines computes and applies the matrix for the
correction of turbulent air flow measurements for the
presence of small disturbing objects, like a box with
electronic apparatus. The approach followed is described in:
(see SEE ALSO). The actual matrix is computed
in EC_C_D02.
```

INPUTS

```
 x : [REAL*8(3)]
     Position vector of the point where measurements have been taken.
     The ellipsoid is placed in the origin.
     A right-handed frame of coordinates is chosen.
     The flow is supposed to be expressed in this coordinate frame.
     Therefore, when the flow velocity has positive components,
     upstream measurement points are selected when by giving
     vector x negative components!
 b : [REAL*8(3)]
     Three ellipsoid semi-axes in meters. The ellipsoid is
     supposed to be oriented along the coordinate axes.
     Somehow the algorithm does not seem to like it when two or more
     semi-axes are equal, or when your point x is in one of the
     coordinate planes (one component of x equal to zero).
     To circumvent problems one can take values slightly off
     the problematic values.
```

OUTPUT

```
aInv : [REAL*8(3,3)]
     The matrix which can be used to correct samples and
     covariances for flow distortion.
     Sum_j aInv(i,j)*u(j)     gives the distortion-corrected
     image of measured velocity u.
```

AUTHOR

```
Arjan van Dijk
```

SEE ALSO

```
Oost, W. (1991).  Flow distortion by an ellipsoid and its
application to the analysis of atmospheric measurements.
J.Atm.Oc.Tech., 8 No 3:331-340.
References in the code are to this article.
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
EC_C_D02
EC_M_InvM
```

# B.70 ec_corr.f/EC_C_D02

NAME

```
EC_C_D02
```

SYNOPSIS

```
CALL EC_C_D02(x, b, a)
```

FUNCTION

```
This routines computes the distortion matrix for the
correction of turbulent air flow measurements for the
presence of small disturbing objects, like a box with
electronic apparatus. The approach followed is described in:
(see SEE ALSO).
Calculates the distortion matrix according to equation 12.
```

INPUTS

```
b : [REAL*8(3)]
    a vector containing the three semiaxes of the
    ellipsoid
x : [REAL*8(3)]
    position where the distortion-matrix will be
    calculated; coordinates are relative to the
    center of the ellipsoid
```

OUTPUT

```
a : [REAL*8(3,3)]
    the distortion matrix, when applied to an
    undisturbed wind, "a" gives the disturbed wind
```

AUTHOR

```
Arjan van Dijk
```

SEE ALSO

```
Oost, W. (1991).  Flow distortion by an ellipsoid and its
application to the analysis of atmospheric measurements.
J.Atm.Oc.Tech., 8 No 3:331-340.
References in the code are to this article.
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
EC_M_SortDecr
EC_M_SortUse
EC_M_EllCoords
EC_M_specint
EC_M_MulVec
EC_M_UnSort
```



# B.71 ec_corr.f/EC_C_F01

NAME

```
EC_C_F01
```

SYNOPSIS

```
CALL EC_C_F01(Mean, Cov, NMax, NSize, WhichTemp,
              NSta, NEnd, NumInt, NS, TauD, TauV, CalSonic,
              CalTherm, CalHyg, CalCO2,WXT)
```

FUNCTION

```
Calculate frequency response corrections for sensor response, path
length averaging, sensor separation, signal processing and dampening
of fluctuations in a tube. Based on publications in SEE ALSO.
```

INPUTS

```
Mean    : [REAL*8(NMax)]
          Array of mean values of the quantities in
          this experiment (only the first N quantities are used).
Cov     : [REAL*8(NMax,NMax)]
           covariances of the fluctuations.
NMax    : [INTEGER]
          Physical dimension of array Mean
NSize   : [INTEGER]
          Number of quantities actually involved in this
          experiment.
WhichTemp: [INTEGER]
          Which temperature to use: thermocouple (Tcouple)
          or sonic (TSOnic): see parcnst.inc
NSta    : [REAL*8]
          Start frequency numerical integration.
          Popular value: -5.D0 [unit?].
NEND    : [REAL*8]
          End frequency numerical integration.
          Popular value: LOG(5) = 0.69897D0 [unit?].
NumINT  : [INTEGER]
          Number of intervals in integration.
          Popular value: 19.
NS      : [INTEGER]
TauD    : [REAL*8]
          Interval length for running mean.
          Popular value: 0.D0 [unit?].
TAUV    : [REAL*8]
          Low pass filter time constant.
          Popular value: 0.D0 [unit?]
CalSonic : [REAL*8(NQQ)]
          Calibration specification array of
          sonic anemometer.
CalTherm : [REAL*8(NQQ)]
          Calibration specification array of
          thermometer.
CalHyg  : [REAL*8(NQQ)]
          Calibration specification array of
          hygrometer.
CalCO2  : [REAL*8(NQQ)]
          Calibration specification array of
          CO2 sensor.
```

OUTPUT

```
WXT     : [REAL*8(NMax,NMax)]
          Correction factors for covariances.
```

BUGS

```
In the present configuration, only correction factors are computed
for the variances and for the covariances involving the vertical
velocity. Other covariances get a factor of 1.
```

AUTHOR

```
Arjan van Dijk
```

SEE ALSO

```
  Moore, C.J. (1986): 'Frequency Response Corrections for Eddy
  Correlation Systems'. Boundary Layer Met. 37: 17-35.

  Philip, J.R. (1963): 'The Damping of Fluctuating Concentration
  by Continuous Sampling Through a tube' Aust. J. Phys. 16: 454-463.

  Leuning, R. and K.M. King (1991): 'Comparison of Eddy-Covariance
  Measurements of CO2 Fluxes by open- and closed-path CO2 analysers'
  (unpublished)
```

HISTORY

```
  28-05-2001: added info on which temperature should be used
              in corrections (Sonic or thermocouple)
  19-09-2002: added CalCO2 in interface
  27-01-2003: replaced Nint by NumInt
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
Pi
EC_Ph_Obukhov
```



# B.72 ec_corr.f/EC_C_F02

NAME

```
EC_C_F02
```

SYNOPSIS

```
CALL EC_C_F02(WXT, NMax, NSize, Lower, Upper, Cov, TolCov)
```

FUNCTION

```
Apply frequency response corrections for sensor response, path
length averaging, sensor separation, signal processing and dampening
of fluctuations in a tube. Based on publications given in SEE ALSO
```

INPUTS

```
WXT    : [REAL*8(NMax,NMax)]
         Correction factors for covariances.
NMax   : [INTEGER]
         Physical dimension of array Mean
NSize  : [INTEGER]
         Number of quantities actually involved in this
         experiment.
Lower  : [REAL*8]
         Lower acceptance limit for frequency-response
         factors. Correction factors smaller than Lower are
         set to 1.
Upper  : [REAL*8]
         Upper acceptance limit for frequency-response
         factors. Correction factors larger than Upper are
         Which temperature to use: thermocouple (Tcouple)
         or sonic (TSOnic): see parcnst.inc
```

OUTPUT

```
Cov    : [REAL*8(NMax,NMax)]
         covariances of the fluctuations.
TolCov : [REAL*8(NMax,NMax)]
         tolerances in covariances
```

SEE ALSO

```
Moore, C.J. (1986): 'Frequency Response Corrections for Eddy
Correlation Systems'. Boundary Layer Met. 37: 17-35.

Philip, J.R. (1963): 'The Damping of Fluctuating Concentration
by Continuous Sampling Through a tube' Aust. J. Phys. 16: 454-463.

Leuning, R. and K.M. King (1991): 'Comparison of Eddy-Covariance
Measurements of CO2 Fluxes by open- and closed-path CO2 analysers'
(unpublished)
```

HISTORY

```
28-05-2001: added info on which temperature should be used
            in corrections (Sonic or thermocouple)
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

# B.73 ec_corr.f/EC_C_Main

NAME

```
EC_C_MAIN
```

SYNOPSIS

```
CALL EC_C_Main(OutF,
  DoPrint, Mean,NMax,N,TolMean, Cov,TolCov,
  DoCorr, PCorr, ExpVar,
  DirYaw, DirPitch, DirRoll,
  SonFactr, O2Factor,
  CalSonic,CalTherm,CalHyg,
  CalCo2,FrCor,
  P, Have_Cal)
```

FUNCTION

```
Integrated correction routine applying ALL (user-selected)
corrections in this library on mean values and covariances.
All intermediate results can be output to a file.
Moreover they are returned to the calling routine in
respective variables.
```

INPUTS

```
(outputs and combined inputs/outputs are given as well,
but also under OUTPUT)
OutF    : [INTEGER]
          Unit number of file for intermediate results
DoPrint : [LOGICAL]
          Print intermediate results ?
Mean    : [REAL*8(NMax)] (in/out)
```

```
          Mean values of all calibrated signals
NMax    : [INTEGER]
          Size of arrays with calibrated signals
N       : [INTEGER]
          Actual number of calibrated signals
TolMean : [REAL*8(NMax)] (in/out)
          Tolerances in mean values of all calibrated signals
Cov     : [REAL*8(NMax,NMax)] (in/out)
          Covariances of all calibrated signals
TolCov  : [REAL*8(NMax,NMax)] (in/out)
          Tolerances in covariances of all calibrated signals
DoCorr  : [LOGICAL](NMaxCorr)
          which corrections to do?
PCorr   : [LOGICAL](NMaxCorr)
          intermediate results of which corrections?
ExpVar  : [REAL*8](NMaxExp)
          array with experimental settings
DirYaw  : [REAL*8] (out)
          Yaw angle (degrees)
DirPitch: [REAL*8] (out)
          Pitch angle (degrees)
DirRoll : [REAL*8] (out)
          Roll angle (degrees)
SonFactr: [REAL*8(NMax)] (out)
          Correction factor due to Schotanus correction for
          covariance of sonic temperature with each calibrated
          signal.
O2Factor: [REAL*8(NMax)] (out)
          Correction factor due to oxygen correction for
          covariance of humidity with each calibrated
CalSonic: [REAL*8(NQQ)]
          Calibration info for sonic anemometer
CalTherm: [REAL*8(NQQ)]
          Calibration info for thermocouple
CalHyg  : [REAL*8(NQQ)]
          Calibration info for hygrometer
CalCO2  : [REAL*8(NQQ)]
          Calibration info for CO2 sensor
FrCor   : [REAL*8(NMax,NMax)] (out)
          Correction factors for covariances for frequency
          response
P       : [REAL*8]
          Atmospheric pressure (Pa)
Have_Cal : [LOGICAL(NMax)]
          Calibrated signal available for given quantity ?
```

OUTPUT

```
Mean    : [REAL*8(NMax)] (in/out)
          Mean values of all calibrated signals
TolMean : [REAL*8(NMax)] (in/out)
          Tolerances in mean values of all calibrated signals
Cov     : [REAL*8(NMax,NMax)] (in/out)
          Covariances of all calibrated signals
TolCov  : [REAL*8(NMax,NMax)] (in/out)
          Tolerances in covariances of all calibrated signals
DirYaw  : [REAL*8] (out)
          Yaw angle (degrees)
DirPitch: [REAL*8] (out)
          Pitch angle (degrees)
DirRoll : [REAL*8] (out)
          Roll angle (degrees)
SonFactr: [REAL*8(NMax)] (out)
          Correction factor due to Schotanus correction for
          covariance of sonic temperature with each calibrated
          signal.
O2Factor: [REAL*8(NMax)] (out)
          Correction factor due to oxygen correction for
          covariance of humidity with each calibrated
FrCor   : [REAL*8(NMax,NMax)] (out)
          Correction factors for covariances for frequency
          response
WebVel  : [REAL*8] (out)
          Webb velocity
```

HISTORY

```
28-05-2001: added info on whether uncalibrated data are
            available for a given variable (mainly important
            for sonic and/or Couple temperature since that
            is used for various corrections)
07-10-2002: added WebVel to interface to pass
            Webb velocity independent from Mean(W)
27-01-2003: removed BadTC from interface
            replaced list of correction switches by DoCorr and PCorr, ExpVar
            replaced Nint by NumInt
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
parcnst.inc
EC_C_T05
EC_C_T06
EC_C_T07
EC_C_T08
EC_C_T09
EC_C_T10
EC_C_T11
EC_C_Schot1
EC_C_Schot2
EC_C_Oxygen1
EC_C_Oxygen2
EC_C_F01
EC_C_F02
EC_C_Webb
EC_G_Show
EC_G_ShwFrq
EC_G_Reset
```

# B.74 ec_corr.f/EC_C_Oxygen1

NAME

```
EC_C_Oxygen1
```

SYNOPSIS

```
CALL EC_C_Oxygen1(MeanT, NMax, N, Cov, P,
     HygType, WhichTemp, Factor)
```

FUNCTION

```
Contribution of other gases especially oxygen absorb
some of the radiation at the wavelengths at which the
hygrometer works.
This routine computes the correction factors. The
factors are applied in EC_C_Oxygen2
```

INPUTS

```
MeanT     : [REAL*8]
            Mean temperature (Kelvin)
NMax      : [INTEGER]
            Size of array dimensions
N         : [INTEGER]
            Actual number of calibrated signals
Cov       : [REAL*8(Nmax,Nmax)]
            Covariances
P         : [REAL*8]
            Atmospheric pressure (Pa)
```

```
HygType   : [INTEGER]
            Type of hygrometer (see parcnst.inc for codes)
WhichTemp : [INTEGER]
            Use thermocouple temperature or sonic temperature ?
            (Tcouple or TSonic, codes in parcnst.inc)
```

OUTPUT

```
Factor    : [REAL*8(NMax)]
            Correction factor for the covariance with each of
            calibrated signals
```

NOTES

```
Currently this routine knows about three types of hygrometer:
   ApCampKrypton : the KH20 krypton hygrometer from Campbell Sci.
   ApMierijLyma  : the Lymann-alpha hygrometer from Mierij Meteo
   ApLiCor7500   : the LiCor IR  hygrometer (not sensitive to O2)
```

SEE ALSO

```
EC_C_Oxygen2
```

HISTORY

```
28-05-2001: added info on which temperature should be used
            in corrections (Sonic or thermocouple)
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
parcnst.inc
Kok
KwK
Kola
Kwla
FracO2
MO2
RGas
```



# B.75 ec_corr.f/EC_C_Oxygen2

NAME

```
EC_C_Oxygen2
```

SYNOPSIS

```
CALL EC_C_Oxygen2(Factor, Nmax, N, Cov)
```

FUNCTION

```
Contribution of other gases especially oxygen absorb
some of the radiation at the wavelengths at which the
hygrometer works.
This routine applies the correction factors that were
computed in EC_C_Oxygen1
```

INPUTS

```
Factor    : [REAL*8(NMax)]
            Correction factor for the covariance with each of
            calibrated signals
NMax      : [INTEGER]
            Size of array dimensions
N         : [INTEGER]
            Actual number of calibrated signals
Cov       : [REAL*8(Nmax,Nmax)] (in/out)
            Covariances
```

OUTPUT

```
Cov       : [REAL*8(Nmax,Nmax)] (in/out)
            Covariances
```

SEE ALSO

```
EC_C_Oxygen1
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
parcnst.inc
```

# B.76 ec_corr.f/EC_C_Scal

NAME

```
EC_C_Scal
```

SYNOPSIS

```
CALL EC_C_Scal(Cal, UDum, VDum, WDum,
               UError, VError, WError)
```

FUNCTION

```
To calibrate a sonic signal according to wind tunnel
calibration (for the moment this works for apparatus 6,
i.e. a wind tunnel calibrated sonic)
```

INPUTS

```
Cal   : [REAL*8(NQQ)]
        Array of length NQQ with calibration info
UDum  : [REAL*8 ] (in/out)
        One horizontal component (on exit: calibrated)
VDum  : [REAL*8 ] (in/out)
        Another horizontal component (on exit: calibrated)
WDum  : [REAL*8 ] (in/out)
        Vertical component (on exit: calibrated)
```

OUTPUT

```
UDum  : [REAL*8 ] (in/out)
        One horizontal component (on exit: calibrated)
VDum  : [REAL*8 ] (in/out)
        Another horizontal component (on exit: calibrated)
WDum  : [REAL*8 ] (in/out)
        Vertical component (on exit: calibrated)
UError: [LOGICAL]
```

```
        error flag for U (.TRUE. if wrong data)
VError: [LOGICAL]
        error flag for V (.TRUE. if wrong data)
WError: [LOGICAL]
        error flag for W (.TRUE. if wrong data)
```

AUTHOR

```
Arnold Moene
```

CREATION DATE

```
September 26, 2000
```

NOTES

```
The method is based on a wind tunnel calibration of the sonic
The real velocity components can be derived from the
measured components and the real azimuth and elevation angle.
But the latter are not known and have to be determined
iteratively from the measured components. The relationship
between the real components and the measured components is:

Ureal =  Umeas/(UC1*(1 - 0.5*
              ((Azi + (Elev/0.5236)*UC2)*
               (1 - UC3*Abs(Elev/0.5236)))**2 ))
Vreal =  Vmeas*(1 - VC1*Abs(Elev/0.5236))
Wreal =  Wmeas/(WC1*(1 - 0.5*(Azi*WC2)**2))
and
Azi = arctan(V/U)
Elev = arctan(W/sqrt(U**2 + V**2))

where UC1, UC2, UC3, VC1, WC1, WC2 are fitting coefficients.
An azimuth angle of zero is supposed to refer to a wind
direction from the most optimal direction (i.e. the 'open'
side of a sonic). Samples with an absolute azimuth angle of
more than 40 degrees are rejected.
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
parcnst.inc
```

# B.77 ec_corr.f/EC_C_Schot1

NAME

```
EC_C_Schot1
```

SYNOPSIS

```
CALL EC_C_Schot1(MeanQ,MeanTSon,NMax,N,Cov,Factor,TSonFact)
```

FUNCTION

```
Compute correction factor for partial Schotanus et al. correction.
for humidity of sonic temperature, and of all covariances with
sonic temperature.
Sidewind-correction has already been applied in the
routine where the sonic signal is calibrated.
```

INPUTS

```
MeanQ    : [REAL*8]
           mean specific humidity  (kg/kg)
MeanTSon : [REAL*8]
           mean sonic temperature (Kelvin)
NMax     : [INTEGER]
           size of arrays
N        : [INTEGER]
           actual number of calibrated signals
Cov      : [REAL*8(NMax,NMax)]
           covariance matrix of calibrated signals
```

OUTPUT

```
Factor   : [REAL*8(NMax)]
           correction factor for the covariances with specific
           humidity
TSonFact : [REAL*8]
           correction factor for sonic temperature
```

SEE ALSO

```
EC_C_Schot1
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
parcnst.inc
```

# B.78 ec_corr.f/EC_C_Schot2

NAME

```
EC_C_Schot2
```

SYNOPSIS

```
CALL EC_C_Schot2(Factor,TSonFact,MeanTSoc, NMax,N,Cov)
```

FUNCTION

```
Apply correction factor for partial Schotanus et al. correction
as computed in EC_C_Schot1.
for humidity of sonic temperature, and of all covariances with
sonic temperature.
Sidewind-correction has already been applied in the
routine where the sonic signal is calibrated.
```

INPUTS

```
Factor   : [REAL*8(NMax)]
           correction factor for the covariances with specific
           humidity
TSonFact : [REAL*8]
           correction factor for sonic temperature
MeanTSon : [REAL*8]
           mean sonic temperature (Kelvin)
NMax     : [INTEGER]
           size of arrays
```

```
N        : [INTEGER]
           actual number of calibrated signals
Cov      : [REAL*8(NMax,NMax)] (in/out)
           covariance matrix of calibrated signals
```

OUTPUT

```
Cov      : [REAL*8(NMax,NMax)] (in/out)
           covariance matrix of calibrated signals
```

SEE ALSO

```
EC_C_Schot1
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
physcnst.inc
```

## B.79 ec_corr.f/EC_C_Schot3

NAME

```
EC_C_Schot3
```

SYNOPSIS

```
TCORR = EC_C_Schot3(Temp,Rhov, Press)
```

FUNCTION

```
To do humidity part of Schotanus et al. correction on a
raw sample of sonic temperature, while specific humidity is not
yet known: to get specific humidity from the measured
Rhov one needs a temperature: if no thermocouple available
the only temperature is the sonic temperature.
(TSonic depends on specific humidity, to compute specific
humidity, one needs a temperature, Tsonics depends on
specific humidity ....etc.)
Sidewind-correction has already been applied in the
routine where the sonic signal is calibrated.
```

INPUTS

```
TEMP    : [REAL*8]
          Sonic temperature (without hyumidity correction) (Kelvin)
Rhov    : [REAL*8]
          absolute humidity (kg/m^3)
Press   : [REAL*8]
          atmospheric pressure (Pa)
```

RETURN VALUE

```
return value : [REAL*8]
          corrected sonic temperature (Kelvin)
```

AUTHOR

```
Arnold Moene
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
EC_Ph_Q
```

## B.80 ec_corr.f/EC_C_T01

NAME

```
EC_C_T01
```

SYNOPSIS

```
CALL EC_C_T01(DoWBias,SingleRun,uMean,NRuns,Apf,
              Alpha,Beta,Gamma,WBias)
```

FUNCTION

```
Subroutine performs some preparations for the actual
Planar Fit Method. The actual work is done in routine EC_C_T02.
```

INPUTS

```
DoWBias:   [LOGICAL]
           compute bias in mean vertical wind (FALSE
           implies that the mean vertical wind over all
           runs is assumed to be zero
SingleRun: [LOGICAL]
           Determine rotation for a single run
uMean    : [REAL*8(3,Nmax)]
           matrix of run mean velocity vectors
NRuns    : [INTEGER]
           the number of runs
```

OUTPUT

```
Apf      : [REAL*8(3,3)]
           the planar fit 3*3 untilt-matrix
Alpha    : [REAL*8]
           tiltangle alpha in degrees
Beta     : [REAL*8]
           tiltangle beta in degrees
Gamma    : [REAL*8]
           Fixed yaw-angle in degrees associated with mean over all runs
WBias    : [REAL*8]
           The bias in the vertical velocity
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
EC_C_T02
```

## B.81 ec_corr.f/EC_C_T02

NAME

```
EC_C_T02
```

SYNOPSIS

```
CALL EC_C_T02(DoWBias,SingleRun,uMean,NRuns,Apf,
              Alpha,Beta,Gamma,WBias)
```

FUNCTION

```
Subroutine computes angles and untilt-matrix needed for
tilt correction of Sonic data, using Planar Fit
Method, as described in James M. Wilczak et al (2001), 'Sonic
Anemometer tilt correction algorithms', Boundary Meteorology 99: 127:150
References to formulae are to this article.
The planar fit matrix is extended with an additional yaw-correction
to turn the first coordinate into the direction of the mean wind
over all runs. This extra rotation makes results from different
eddy-covariance systems comparable.
Furthermore, there is the option to determine a planar fit for
a single run (using all individual samples within a run,
rather than the mean velocities from a collection of runs as
in the classic planar fit method).
```

INPUTS

```
DoWBias:   [LOGICAL]
           compute bias in mean vertical wind (FALSE
           implies that the mean vertical wind over all
           runs is assumed to be zero
SingleRun: [LOGICAL]
           Determine rotation for a single run
uMean    : [REAL*8(3,Nmax)]
           matrix of run mean velocity vectors
NRuns    : [INTEGER]
           the number of runs
```

OUTPUT

```
Apf      : [REAL*8(3,3)]
           the planar fit 3*3 untilt-matrix
Alpha    : [REAL*8]
           tiltangle alpha in degrees
Beta     : [REAL*8]
           tiltangle beta in degrees
Gamma    : [REAL*8]
           Fixed yaw-angle in degrees associated with mean over all runs
WBias    : [REAL*8]
           The bias in the vertical velocity
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```



USES

```
EC_M_InvM
EC_M_InvM2
EC_M_MapVec
EC_M_Map2Vec
EC_M_MMul
```

## B.82 ec_corr.f/EC_C_T03

NAME

```
EC_C_T03
```

SYNOPSIS

```
CALL EC_C_T03(Mean,NMax,N,Cov,Speed,Stress,DumVecs,NN)
```

INPUTS

```
Mean    : [REAL*8(NMax)]
          means of all variables
NMax    : [INTEGER]
          maximum number of variables (i.e. size of
          various matrices)
N       : [INTEGER]
          number of variables in used
Cov     : [REAL*8(NMmax, NMax)]
          covariances of all variables
NN      : [INTEGER]
          maximum size of second axis of DumVecs
```

OUTPUT

```
Speed   : [REAL*8(3)]
          copy of means of all variables
Stress  : [REAL*8(3,3)]
          copy of covariances of all variables
DumVecs : [REAL*8(3,4:NN)]
          copy of covariances of all variables with velocity components
```

FUNCTION

```
Help routine for routines for correction of coordinate system
Temporaly stores means and covariances elsewhere
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

## B.83 ec_corr.f/EC_C_T04

NAME

```
EC_C_T04
```

SYNOPSIS

```
CALL EC_C_T04(Speed,Stress,DumVecs,NN, Mean,NMax,N,Cov)
```

INPUTS

```
Speed   : [REAL*8(3)]
          copy of means of all variables
Stress  : [REAL*8(3,3)]
          copy of covariances of all variables
DumVecs : [REAL*8(3,4:NN)]
          copy of covariances of all variables with velocity components
NMax    : [INTEGER]
          maximum number of variables (i.e. size of
          various matrices)
N       : [INTEGER]
          number of variables in used
NN      : [INTEGER]
          maximum size of second axis of DumVecs
```

OUTPUT

```
Mean    : [REAL*8(NMax)]
          means of all variables
Cov     : [REAL*8(NMmax, NMax)]
          covariances of all variables
```

FUNCTION

```
Help routine for routines for correction of coordinate system
Copies back temporarily stored copies of means and covariances
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
parcnst.inc
```

## B.84 ec_corr.f/EC_C_T05

NAME

```
EC_C_T05
```

SYNOPSIS

```
CALL EC_C_T05(Mean,NMax,N,Cov,Map)
```

INPUTS

```
Mean    : [REAL*8(NMax)]
          means of all variables
NMax    : [INTEGER]
          maximum number of variables (i.e. size of
          various matrices)
N       : [INTEGER]
          number of variables in used
Cov     : [REAL*8(NMmax, NMax)]
          covariances of all variables
Map     : [REAL*8(3,3)]
          rotation tensor
```

OUTPUT

```
Mean    : [REAL*8(NMax)]
          means of all variables
Cov     : [REAL*8(NMmax, NMax)]
          covariances of all variables
```

FUNCTION

```
Routine to change coordinate system according to tensor "Map".
This routine is called by all tilt-correction procedures.
Both the mean velocity and the Reynoldsstresses and the
covariances of all velocity components with other quantities
are rotated.
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
EC_C_T03
EC_C_T04
EC_M_MapVec
EC_M_MapMtx
parcnst.inc
```

## B.85 ec_corr.f/EC_C_T06

NAME

```
EC_C_T06
```

SYNOPSIS

```
CALL EC_C_T06(Direction,Yaw)
```

INPUTS

```
Direction : [REAL*8]
            yaw angle
```

OUTPUT

```
Yaw       : [REAL*8(3,3)]
            rotation tensor
```

FUNCTION

```
Construct rotation matrix for coordinate system
about a KNOWN yaw-angle around the vertical
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
Pi
```

## B.86 ec_corr.f/EC_C_T07

NAME

```
EC_C_T07
```

SYNOPSIS

```
CALL EC_C_T07(MeanU,MeanV,Direction)
```

INPUTS

```
MeanU     : [REAL*8]
            mean u-velocity
MeanV     : [REAL*8]
            mean v-velocity
```

OUTPUT

```
Direction : [REAL*8]
            yaw angle (degree)
```

FUNCTION

```
Give yaw-angle to transform coordinate system such that
v_mean = 0 (no mean lateral horizontal velocity component)
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
Pi
```

## B.87 ec_corr.f/EC_C_T08

NAME

```
EC_C_T08
```

SYNOPSIS

```
CALL EC_C_T08(Direction,Yaw)
```

INPUTS

```
Direction : [REAL*8]
            pitch angle
```

OUTPUT

```
Pitch     : [REAL*8(3,3)]
            rotation tensor
```

FUNCTION

```
Give matrix for rotation about a KNOWN pitch-angle
around vector (0,1,0).
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
Pi
```

## B.88 ec_corr.f/EC_C_T09

NAME

```
EC_C_T09
```

SYNOPSIS

```
CALL EC_C_T09(MeanU,MeanW,Direction)
```

INPUTS

```
MeanU     : [REAL*8]
            mean u-velocity
MeanV     : [REAL*8]
            mean W-velocity
```

OUTPUT

```
Direction : [REAL*8]
            pitch angle (degree)
```

FUNCTION

```
Give pitch angle to transform coordinate system such
that w_mean = 0 (no mean vertical velocity component)
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
Pi
```

## B.89 ec_corr.f/EC_C_T10

NAME

```
EC_C_T10
```

SYNOPSIS

```
CALL EC_C_T10(Direction,Roll)
```

INPUTS

```
Direction : [REAL*8]
            yaw angle
```

OUTPUT

```
Roll      : [REAL*8(3,3)]
            rotation tensor
```

FUNCTION

```
Give matrix to rotate coordinate system about a KNOWN
roll-angle around vector (1,0,0).
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
Pi
```

## B.90 ec_corr.f/EC_C_T11

NAME

```
EC_C_T11
```

SYNOPSIS

```
CALL EC_C_T11(MeanU,MeanW,Direction)
```

INPUTS

```
CovVV     : [REAL*8]
            vv-covariance
CovVW     : [REAL*8]
            vw-covariance
CovWW     : [REAL*8]
            ww-covariance
```

OUTPUT

```
Direction : [REAL*8]
            roll angle (degree)
```

FUNCTION

```
Give roll angle to transform coordinate system such
that Cov(V,W) = 0  (vertical velocity fluctuations are
independent from horizontal fluctuations)
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
Pi
```

## B.91 ec_corr.f/EC_C_Webb

NAME

```
EC_C_Webb
```

SYNOPSIS

```
CALL EC_C_Webb(Mean,NMax,Cov,P,WhichTemp, WebVel)
```

INPUTS

```
Mean     : [REAL*8(NMax)]
           mean of all variables
NMax     : [INTEGER]
           maximum number of variables
Cov      : [REAL*8(NMax,NMax)]
           covariance of all variables
P        : [REAL*8]
           atmospheric pressure
WhichTemp: [INTEGER]
           Use thermocouple temperature or sonic temperature ?
           (Tcouple or TSonic, codes in parcnst.inc)
Mean     : [REAL*8(NMax)]
           mean of all variables
```

OUTPUT

```
WebVel    : [REAL*8]
           Webb velocity (m/s)
```

FUNCTION

```
Mean vertical velocity according to Webb, Pearman and Leuning
```

AUTHOR

```
Arjan van Dijk
```

HISTORY

```
28-05-2001: added info on which temperature should be used
            in corrections (Sonic or thermocouple)
07-10-2002: Webb velocity no longer passed through Mean(W). Now
            separate variable, WebVel is used for this.
$Name:  $
$Id: ecpack.tex,v 1.2 2004/02/26 17:05:24 arnold Exp $
```

USES

```
Mu
parcnst.inc
EC_Ph_RhoDry
```

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