THE INFLUENCE OF SOIL MANAGEMENT ON THE TEMPERATURE WAVE NEAR THE SOIL SURFACE

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I. THE THERMAL PROPERTIES OF SOIL AND AIR

A. THE THERMAL CONDUCTIVITY OF SOILS

According to De Vries (1952), the thermal conductivity of soils may be calculated from the kind, shape and arrangement of the particles and the moisture content. In this theory the soil is supposed to be a coherent medium, viz. air in dry and water in wet soils, in which several kinds of ellipsoidal particles are present. This leads to the relation:

$$\lambda_s = \frac{k_v X_v \lambda_v + k_w X_w \lambda_w + k_u X_u \lambda_u}{k_v X_v + k_w X_w + k_u X_u} \tag{1}$$

with

$$k_i = \tfrac{1}{3} \sum_{a,\,b,\,c} \left\{ 1 + \left(\frac{\lambda_i}{\lambda_0} - 1 \right) g_a \right\}^{-1} \tag{2a}$$

$$g_a + g_b + g_c = 1 \tag{3}$$

$$X_v + X_w + X_u = 1 \tag{4}$$

in which

λ_s = the thermal conductivity of the soil (cal/cm sec. °C)

$\lambda_v, \lambda_w, \lambda_u$ = the same of the soil components, where v, w and u refer to resp. solid material, water and air. An index o refers to the coherent medium and the index i to the particles; for the coherent medium $\lambda_i = \lambda_0$ and $k = 1$.

X = the volume fraction of the component considered; in dry resp. saturated soils, X_w resp. X_u is zero and therefore the corresponding terms in (1).

g = factor which depends on the shape of the particles, denoted by the relation of its axis: $a = b = nc$; with spherical particles is $n = 1$ and $g_a = g_b = g_c = \frac{1}{3}$.

If $g_b = g_a$ then $g_c = 1 - 2\,g_a$ and (2a) will transform into:

$$k_i = \tfrac{1}{3} \left\{ \frac{2}{1 + \left(\frac{\lambda_i}{\lambda_0} - 1 \right) g_a} + \frac{1}{1 + \left(\frac{\lambda_i}{\lambda_0} - 1 \right) (1 - 2g_a)} \right\} \tag{2b}$$

The value g of the soil particles may be determined from direct measurements of their axes or from measurements of the diffusion rate of vapour through a cylinder with dry soil, since (1) also holds for diffusion. In that case k_v is the only unknown, from which g_a can be derived.

In a wet soil transport of heat by destillation of water also occurs, which results in an increase of the thermal conductivity of the air.

Direct measurements of the thermal conductivity can be executed with the non-stationary method of De Vries (1952). As an example, the calculation of the thermal conductivity of a sandy soil will be given. The results of such measurements have been presented already elsewhere (VAN DUIN, 1956).

The properties of this soil are given in table 1.

TABLE 1. Properties of the soil to which fig. 1 and 2 refer

Composition			Moisture content at field capacity	Specific weight	Grains		Conductivity (10^{-3})		
<2 μ	<16 μ	org. matter			n	g_a	λ_v	λ_w	λ_u
3.5	8.0	3.5	12.5% of weight	2.56	4	0.15	10.5	1.42	0.0615-0.225

Saturated soil: The value of λ_v has been determined from direct measurements of the conductivity of saturated soil. In that case $\lambda_0 = 1.42$, $k_w = 1$, $g_a = 0.15$ and $X_u = 0$; the measured value of λ_s was $4.82 . 10^{-3}$ at $X_v = 0.60$, which gives $k_v = 0.398$ and $\lambda_i = \lambda_v = 10.5 . 10^{-3}$ cal/cm.sec.°C.

Soil at field capacity: $X_v = 0.60$, $X_w = 0.184$, $X_w = 0.216$, $k_w = 1$. With small values of X_u the air-filled pores are approximately spherical and for these pores $g_a = \frac{1}{3}$; if $X_w \to 0$ a value $g = 0.0317$ can be derived. The value of g at field capacity then follows from interpolation between 0.0317 and 0.333, which results in $g_a = 0.170$, $k_u = 1.53$ and $\lambda_s = 4.16 . 10^{-3}$ cal/cm.sec.°C.

Dry soil: $X_v = 0.60$, $X_w = 0$, $X_u = 0.40$, $k_u = 1$. The value of $k_v = 0.028$ follows from (2b) with $\lambda_i = 10.50 . 10^{-3}$, $\lambda_0 = 0.0615 . 10^{-3}$ and $g_a = 0.15$. Since the theory gives too small values for λ_s with small values of X_u, a correction factor must be determined by means of direct measurements. This factor appeared to be 1.31 for this soil, giving $\lambda_s = 1.31 . 0.48 = 0.63$.

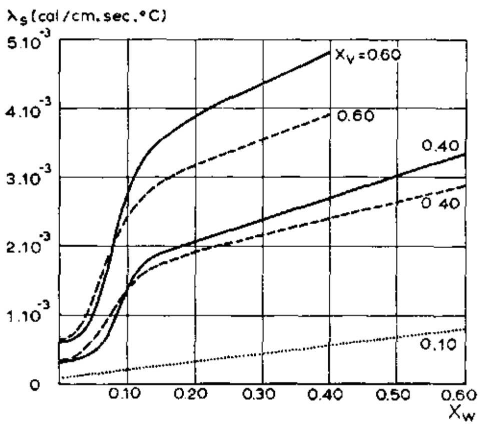

FIG. 1. The thermal conductivity of sand (——), clay (------) and peat (.......) in relation to the volume fractions of solid material (X_v) and water (X_w)

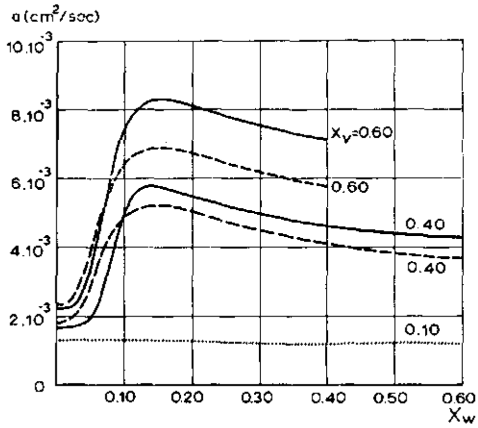

FIG. 2. The thermal diffusivity of sand (——), clay (------) and peat (.......) in relation to the volume fractions of solid material (X_v) and water (X_w)

The influence of the moisture content of this soil on its thermal conductivity has been presented in fig. 1 for several values of X_v, together with this relation for a clay soil and a peat soil.

B. THE THERMAL DIFFUSIVITY OF SOIL AND AIR

The heat capacity of a soil, per unit volume C_s, is determined by the heat capacity of its components, according to:

$$C_s = C_v X_v + C_w X_w + C_u X_u \tag{5}$$

For water, minerals and organic matter the values of C are respectively 1.0, 0.46 and 0.60 cal/cm^3.°C, so the heat capacity will increase with an increasing moisture content.

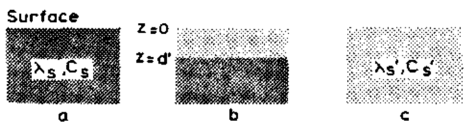

FIG. 3. Model used as basis for the treatment of the temperature wave near the soil surface

An important value to indicate the thermal behaviour of a soil is its thermal diffusivity $a = \lambda_s/C_s$ (cm^2/sec). The greater the value of a, the faster and the deeper will a particular heat wave at the soil surface penetrate into the soil.

The thermal diffusivities of several soils in relation to their moisture content are presented in fig. 2, from which it appears that the diffusivity of mineral soils has a maximum at a moisture content of approx. 15% of volume. In studying heat transport in soils, the assumption of λ_s and C_s being independent of depth, or representing these properties by mean values for certain layers (e.g. according to fig. 3), will give a good approximation in many problems. With heat transport in the air this assumption is a too great simplification since the turbulent diffusion dominates the molecular conduction to a large extent. For the present investigation a linear increase with height of the thermal diffusivity coefficient of the air, K, has been chosen in accordance with LETTAU (1952). This assumption does not hold for larger distances from the soil surface and for that reason the heat flux into the air is somewhat overestimated. The increase of K with height is given by

$$K = \varkappa u_* \; (z + z_0) \tag{6}$$

$\varkappa$ = constant of VON KARMAN, with a value $\varkappa = 0.40$
u_* = friction velocity (cm/sec)
z = distance to the surface (cm)
z_0 = roughness parameter (cm), varying between 0.01 cm for a smooth surface and 10 cm for high grass.

The friction velocity follows from the velocity profile for fully rough flow and neutral stability:

$$\frac{u_z}{u_*} = \frac{1}{\varkappa} \ln \frac{z + z_0}{z_0} \tag{7}$$

where u_z = wind velocity at height z (cm/sec).

Another value which is of importance with reference to the heating and cooling of soils is the product λC, since its values for soil and air determine the distribution of the heat available at the surface among them [compare (23)], while the quotient λ_s/C_s determines the distribution of the heat with depth [compare (27a)]. The equations for a constant flow of heat into soil and air are given by:

$$B = -\lambda_s \frac{\partial T}{\partial z} \tag{8}$$

$$L = -KC_u \frac{\partial \Phi}{\partial z} \tag{9}$$

B = heat flux into the soil (cal/cm^2.sec)
L = heat flux into the air (cal/cm^2.sec)
T = soil temperature (°C)
Φ = air temperature (°C)
C_u = heat capacity of the air per unit volume (cal/cm^3.°C).

The consequences of the available heat at the surface being a periodic function of time, will be dealt with in the next chapter.

II. THE TEMPERATURE WAVE NEAR THE SURFACE OF A HOMOGENEOUS SOIL

A. THEORY

The net amount of radiation reaching the surface of a large homogeneous field, is used for heating the soil and air and will cause evaporation according to:

$$H - E = B + L = U \tag{10}$$

H = radiation that reaches the earth, minus reflection and outgoing long wave radiation (cal/cm^2.sec)
E = energy used in evaporation (cal/cm^2.sec)
U = the available heat for heating soil and air (cal/cm^2.sec)

If the total heat flux at the surface, U, is a periodic function of time, t, according to:

$$U(0, t) = U_{00} + U_0 \cos \omega t \tag{11}$$

and the thermal properties of soil and air are constant with time, the heat fluxes B and L are given by:

$$B(0, t) = B_{00} + B_0 \cos (\omega t + \beta) \tag{12}$$

$$L(0, t) = L_{00} + L_0 \cos (\omega t + \alpha) \tag{13}$$

$$U_{00} = B_{00} + L_{00} \tag{14}$$

$$U_0 = B_0 \cos \beta + L_0 \cos \alpha = L_0 (\cos \alpha + R \cos \beta) \tag{15}$$

$U(0, t)$ = total heat flux into soil and air at depth $z = 0$ and time t.
U_{00}, B_{00}, L_{00} = mean value of the heat waves at the surface
U_0, B_0, L_0 = amplitude of the heat waves at the surface
ω = circle frequency of the periodic fluctuation, where $\omega = 2\pi/\tau$ if τ is its period (sec.)
β, α = shift of phase between the total heat flux and the fluxes resp. into soil and air
R = B_0/L_0

The temperature wave at the surface is given by:

$$T(0, t) = T_{00} + A_0 (\cos \omega t + \gamma) \tag{16}$$

T_{00} = mean value of the temperature wave at the surface
A_0 = amplitude of the temperature wave at the surface
γ = shift of phase between the total heat flux, $U(0, t)$ and the temperature wave at the surface.

In the particular case that both in soil and air the thermal properties are constant with the distance to the soil surface, the relation between A_0, B_0 and L_0 is given by:

$$B_0 = A_0 \sqrt{\lambda_s C_s \omega} \tag{17a}$$

$$L_0 = A_0 C_u \sqrt{K \omega} \tag{18a}$$

$$R = \sqrt{\lambda_s C_s / C_u \sqrt{K}} \tag{19a}$$

According to SCHMIDT (1918, see LETTAU, 1952)

$$\gamma = -\pi/4 \qquad \alpha = 0 \qquad \beta = 0 \qquad \text{(20a) (21a) (22a)}$$

by which (15) is transformed into $U_0 = L_0 (1 + R)$, and combination with (18a) and (19a) will give:

$$A_0 = \frac{U_0}{\sqrt{\omega}\,(\sqrt{\lambda_s C_s} + C_u \sqrt{K})} \tag{23a}$$

Introducing the turbulent diffusion coefficient, K, increasing with height, gives other values of L_0, R, A_0, α, β and γ (according to LETTAU):

$$L_0 = \frac{A_0 k_0 u_* C_u}{\pi j}; \quad R = \frac{B_0}{L_0} = \frac{\pi j \sqrt{\lambda_s C_s \omega}}{k_0 u_* C_u} \qquad \text{(18b) (19b)}$$

$$\gamma = \operatorname{arctg}\left\{-\frac{R \sin \pi/4 + \sin\,(\operatorname{arctg} 1/2j)}{R \cos \pi/4 + \cos\,(\operatorname{arctg} 1/2j)}\right\} \tag{20b}$$

$$\alpha = \gamma + \operatorname{arctg} 1/2j; \quad \beta = \gamma + \pi/4 \qquad \text{(21b) (22b)}$$

$$A_0 = \frac{U_0}{\sqrt{\lambda_s C_s\,\omega}\cos\beta + C_u k_0 u_* \cos\alpha/\pi j} = \frac{\pi j\, U_0}{k_0 u_* C_u\,(\cos\alpha + R\cos\beta)} \tag{23b}$$

where j depends on the period of a periodic fluctuation and the turbulent parameters according:

$$j = -0{,}367 + \frac{1}{\pi}(\ln\, k_0 + \ln u_* - \ln z_0 - \ln \omega) \tag{24}$$

The temperature at time t and at distance z from the surface now follows from

$$T\,(z, t) = T_{00} + p_z\, A_0 \cos\,(\omega t + \gamma + \psi_z) \tag{25}$$

$$\Phi\,(z, t) = T_{00} + h_z\, A_0 \cos\,(\omega t + \gamma + \eta_z) \tag{26}$$

with

$$p_z = e^{-z/D_s}; \qquad \psi_z = -z/D_s \qquad \text{(27a) (28a)}$$

$$h_z = \frac{(jJ + \frac{1}{4})^2 + \frac{1}{4}(j - J)^2}{j^2}; \; \eta_z = -\operatorname{arctg}\frac{j - J}{2\,jJ + \frac{1}{2}} \qquad \text{(27b) (28b)}$$

where J depends on the wind profile according to:

$$J = j - \frac{1}{\pi} \ln \frac{z + z_0}{z_0} \tag{29}$$

D_s $(= \sqrt{2a/\omega})$ is the damping depth of the temperature wave in the soil (cm). The value of D_s may be calculated from the course of temperature at two values of z according to:

$$D_s = \frac{z_2 - z_1}{\ln A_1/A_2}; \qquad D_s = \frac{z_2 - z_1}{\psi_1 - \psi_2} \qquad \text{(30a) (30b)}$$

where

A_1 and A_2 are the amplitudes at depths z_1 and z_2;
ψ_1 and ψ_2 the corresponding shifts in phase.

B. CALCULATIONS

SOIL STRUCTURE

In fig. 4 the calculated temperature wave during a clear day in the month of May is given for a sandy soil at field capacity with different values of X_v. The corresponding properties are given in table 2 (case 1, 2 and 3), while the properties of the air were as follows:
$z_0 = 0.7$; $u_* = 20.4$; $j = 3.47$; $\pi j/k_0 u_* C_u = 4.46.10^3$; $\omega = 73.10^{-6}$ and $T_{00} = 11.5$ °C.

The amplitude of the daily wave amounts to resp. 9.6 and 13.4 °C when $X_v = 0.60$ resp. 0.40, according to the functions $T(0, t) = 11.5 + 9.6 \cos(\omega t - 0.54)$ and $T(0, t) = 11.5 + 13.4 \cos(\omega t - 0.44)$.

If the heat flux into the soil had been independent from the thermal properties of these soils, the amplitudes would have been A_0 and ρA_0 with $\rho = \sqrt{\lambda_s C_s}/\sqrt{\lambda'_s C'_s}$ and $\lambda_s C_s$ and $\lambda'_s C'_s$ referring to the situation of fig. 3a and 3c respectively. In that case $A_0 = 9.6$ and $\rho A_0 = 9.6.42.1/22.9 = 17.6$ °C; because of the decrease of the amplitude of the heat wave into the soil from $3.44.10^{-3}$ to $2.62.10^{-3}$cal.cm^2.sec, the amplitude of the heat wave only increases from 9.6 to 17.6.2.62/3.44 = 13.4 °C.

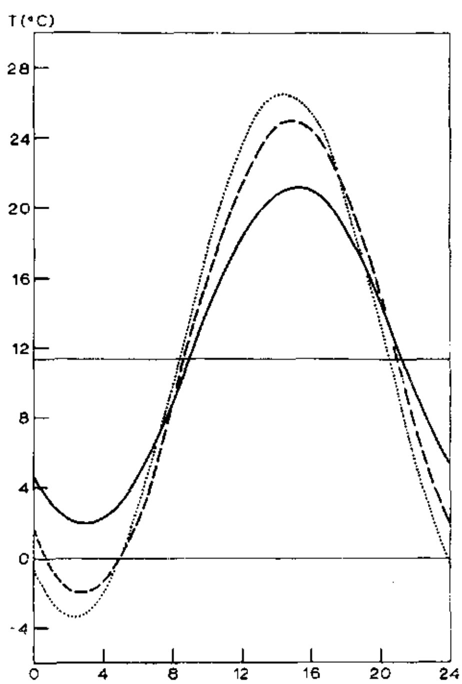

FIG. 4. The daily temperature wave in a sandy soil at field capacity with $X_v = 0.60$ (——); $X_v = 0.40$ (------) and $X_v = 0.40$, crumb structure (.......)

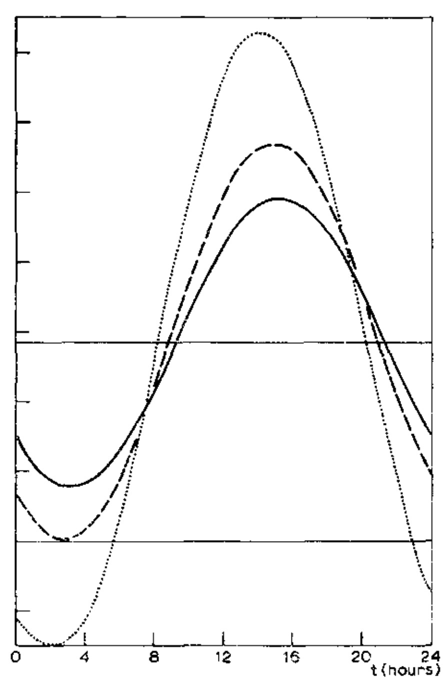

FIG. 5. The daily temperature wave in a sandy soil with $X_v = 0.50$ and moisture contents $X_w = 0.50$ (——), $X_w = 0.16$ (------) and $X_w = 0$ (.......)

If the soil has a crumb structure (soil 4), the crumbs may be considered as particles with their own thermal conductivity (DE VRIES, 1953). In that case a thermal conductivity of the crumbs of $\lambda_c = 3.80 . 10^{-3}$ was found, assuming that all the moisture is concentrated in the crumbs and $X_v = 0.60$ and $X_w = 0.1875$ (fig. 1). A calculation according (1) with these crumbs as particles and $\lambda_w = 0.666$ and $\lambda_w = 0$, gives $k_v = 0.150$ and $\lambda_s = 0.98 . 10^{-3}$.

As a consequence of the lower thermal conductivity of a crumbly structured soil, the amplitude of the temperature wave increases to 15.0 °C and B_0 decreases to 2.22. 10^{-3}cal/cm^2.sec.

MOISTURE CONTENT

The influence of the moisture content of the soil is illustrated in fig. 5 for a sandy soil, which is respectively saturated (soil 4 in table 2), at field capacity (soil 5) or dry (soil 6). It appears from fig. 5 that the difference between soils at field capacity and in a dry state is greater than the difference between saturated and field capacity. The amplitude at the dry soil surface further increases if the available heat, U, increases caused by a decrease of H [compare (10)]. If $E = 0$ the value of U amounts to 8.0. 10^{-3}cal/cm^2.sec and $A_0 = 26$ °C. In that case the importance of horizontal transport of heat increases and (10) does not hold anymore.

DISTANCE TO THE SOIL SURFACE

The decrease of the amplitude with depth according to (27a) is presented in fig. 6 for the heat waves in a peat soil at depths $z = 0$, $z = \frac{1}{4}D_s$, $z = \frac{1}{2}D_s$, and $z = 2D_s$ with $D_s = 5.8$ cm (soil 7 in table 2). The decrease of the amplitude with the distance to the soil surface is presented in an other way in fig. 7, where the vertical line indicates the mean daily temperature.

As follows from this calculation, night-frost will on the peat soil occur in the zone between 1.5 cm below and 6 cm above the soil surface, while the temperature near the moist sandy soil does not fall below this critical value. With a dry soil the situation with reference to night-frost is very unfavourable since the amplitude of the surface temperature is even larger than for a wet peat soil and the more so, the more

TABLE 2. Properties of the soils used in the calculations

Nr.	Soil	X_v	X_w	X_u	λ_s	C_s	$\sqrt{\lambda_s C_s}$	$\sqrt{\omega}$
1	sand, f.c.	0.60	0.19	0.21	$3.80 . 10^{-3}$	0.466	$42.1 . 10^{-3}$	$8.52 . 10^{-3}$
2	sand, loose	0.40	0.125	0.475	$1.70 . 10^{-3}$	0.309	$22.9 . 10^{-3}$	$8.52 . 10^{-3}$
3	sand, crumbly	0.40	0.125	0.475	$0.98 . 10^{-3}$	0.309	$17.4 . 10^{-3}$	$8.52 . 10^{-3}$
4	sand, sat.	0.50	0.50	0	$4.00 . 10^{-3}$	0.73	$54.0 . 10^{-3}$	$8.52 . 10^{-3}$
5	sand, f.c.	0.50	0.16	0.34	$2.70 . 10^{-3}$	0.39	$32.4 . 10^{-3}$	$8.52 . 10^{-3}$
6	sand, dry	0.50	0	0.50	$0.48 . 10^{-3}$	0.23	$10.5 . 10^{-3}$	$8.52 . 10^{-3}$
7	peat, f.c.	0.10	0.50	0.40	$0.68 . 10^{-3}$	0.56	$19.5 . 10^{-3}$	$8.52 . 10^{-3}$
8	peat, dry	0.10	0	0.90	$0.08 . 10^{-3}$	0.06	$2.19 . 10^{-3}$	$8.52 . 10^{-3}$
9	sand, sat.	0.60	0.40	0	$4.80 . 10^{-3}$	0.68	$57.1 . 10^{-3}$	$0.446 . 10^{-3}$
10	sand, f.c.	0.60	0.19	0.21	$3.80 . 10^{-3}$	0.47	$42.1 . 10^{-3}$	$0.446 . 10^{-3}$

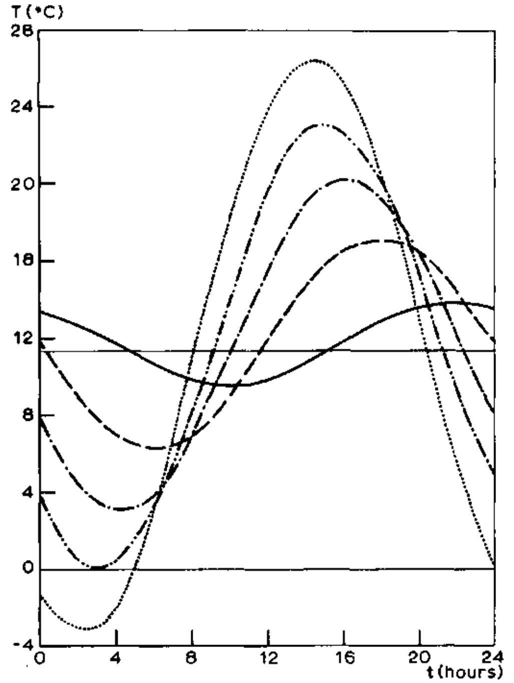

FIG. 6. The daily temperature wave in a peat soil at field capacity and at depths $z = 0$(.......), $z = \frac{1}{4} D_S$ (..-..-..-), $z = \frac{1}{2} D_S$ (.-.-.-), $z = D_S$ (------) and $z = 2\ D_S$ cm(——)

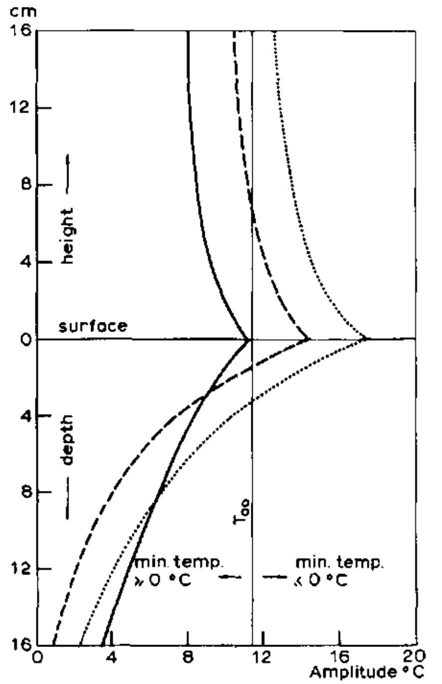

FIG. 7. The decrease of the amplitude of the daily wave of temperature with increasing distance from the surface in a dry sandy soil (.......), a sandy soil at field capacity (——) and a peat soil at field capacity (------)

the wave of the available heat increases. Drying the peat soil makes the situation worse.

Nr.	D	R	α	β	γ	U_0	A_0	B_0	L_0
1	15.0	1.600	–0.40	0.24	–0.54	$5.30 \cdot 10^{-3}$	9.6	$3.44 \cdot 10^{-3}$	$2.15 \cdot 10^{-3}$
2	12.3	0.874	–0.30	0.34	–0.44	$5.30 \cdot 10^{-3}$	13.4	$2.62 \cdot 10^{-3}$	$3.00 \cdot 10^{-3}$
3	9.35	0.660	–0.25	0.38	–0.40	$5.30 \cdot 10^{-3}$	15.0	$2.20 \cdot 10^{-3}$	$3.36 \cdot 10^{-3}$
4	12.3	2.02	–0.43	0.21	–0.58	$5.30 \cdot 10^{-3}$	8.2	$3.71 \cdot 10^{-3}$	$1.84 \cdot 10^{-3}$
5	13.8	1.23	–0.35	0.28	–0.50	$5.30 \cdot 10^{-3}$	11.2	$3.08 \cdot 10^{-3}$	$2.51 \cdot 10^{-3}$
6	7.57	0.40	–0.18	0.46	–0.33	$5.30 \cdot 10^{-3}$	17.6	$1.58 \cdot 10^{-3}$	$3.94 \cdot 10^{-3}$
7	5.80	0.71	–0.26	0.37	–0.51	$5.30 \cdot 10^{-3}$	14.6	$2.44 \cdot 10^{-3}$	$3.27 \cdot 10^{-3}$
8	6.04	0.085	–0.05	0.60	–0.19	$5.30 \cdot 10^{-3}$	22.1	$0.41 \cdot 10^{-3}$	$4.95 \cdot 10^{-3}$
9	266	0.21	–0.12	0.58	–0.20	$1.16 \cdot 10^{-3}$	8.2	$0.21 \cdot 10^{-3}$	$0.99 \cdot 10^{-3}$
10	284	0.16	–0.09	0.82	–0.17	$1.47 \cdot 10^{-3}$	10.8	$0.20 \cdot 10^{-3}$	$1.30 \cdot 10^{-3}$

YEARLY TEMPERATURE WAVES

If the period of the function is a year instead of a day, the damping depth must be multiplied with a factor $\sqrt{365}$. The influence of the moisture content of a sandy soil (9 and 10 in table 2) on the yearly wave has been calculated with $z_s = 0.10$, $u_* = 18.9$, $j = 5.91$ and $\pi j/k_0 u_* C_u = 8.25 . 10^3$. It has been assumed that with the available heat the evaporation from the saturated soil varies during the year from 0.25 and 2.5 mm/day and from the soil at field capacity from 0.15 and 1.5 mm/day.

The calculated surface amplitudes with $U_0 = 1.16 . 10^{-3}$ and $1.47 . 10^{-3}$ cal/cm^2.sec are 8.2 and 10.8 °C respectively, while these values would have been 10.4 and 10.8 °C if in both cases the total heat flux was $1.47 . 10^{-3}$ cal/cm^2.sec. This is an indication of the importance of evaporation in relation with the yearly course of temperature in wet soils.

III. THE TEMPERATURE WAVE NEAR THE SURFACE OF A LAYERED SOIL

A. THEORY

The influence of depth dependent thermal properties on the course of the heat and temperature waves in the soil, has been treated by PEERLKAMP (1944) and applied by DE VRIES and DE WIT (1954) to calculate the risk of night-frost as influenced by a sandy layer covering a peat soil. VAN DUIN (1954) has also considered the influence of such a layered soil on the distribution of heat between soil and air and applied this on the influence of tillage on the microclimate.

If a soil consists of a bottom layer (to infinity) with thermal properties λ_s and C_s, a top layer with thickness d' and thermal properties λ'_s and C'_s (fig. 3b) the following equations are valid:

$$B_0 = \frac{A_0\sqrt{\lambda_s C_s \omega}}{f}; \qquad R' = \frac{\pi j\sqrt{\lambda_s C_s \omega}}{f\, k_0 u_* C_u} \tag{17b) (19c}$$

$$\gamma = \operatorname{arctg}\left\{\frac{R' \sin(\pi/4 - \varphi) + \sin(\operatorname{arctg} 1/2j)}{R' \cos(\pi/4 - \varphi) + \cos(\operatorname{arctg} 1/2j)}\right\} \tag{20c}$$

$$\beta = \gamma + \frac{\pi}{4} - \varphi \tag{22c}$$

$$A_0 = \frac{\pi j\, U_0}{k_0 u_* C_u (\cos\alpha + R' \cos\beta)} \tag{23c}$$

with

$$f = \rho\left\{\frac{r^2 e^{-4\delta} + 2\, r e^{-2\delta} \cos 2\delta + 1}{r^2 e^{-4\delta} - 2\, r e^{-2\delta} \cos 2\delta + 1}\right\}^{\frac{1}{2}} \tag{31}$$

$$\varphi = \operatorname{arctg} \frac{2\, r e^{-2\delta} \sin 2\delta}{r^2 e^{-4\delta} - 1} \tag{32}$$

and

$$\delta' = \frac{d'}{D'_s}; \; \rho = \frac{\sqrt{\lambda_s C_s}}{\sqrt{\lambda'_s C'_s}}; \; r = \frac{1-\rho}{1+\rho}$$

The value of f varies between 1 and ∞ when d' varies from 0 to ∞, with a maximum of $\operatorname{tg} 2\delta = (r^2 e^{-4\delta} - 1)/(r^2 e^{-4\delta} + 1)$ or $\delta = 1.2$, while $f = \rho$ for more than one value of δ. The minimum value of δ for which $f = \rho$ is found if $\delta = 0.8$ (see fig. 8).

The values of p_z and ψ_z in the upper layer ($0 \leq z \leq d'$) are now given by:

$$p_z = \left\{\frac{r^2 e^{-4\delta} e^{2\zeta} + 2\, r e^{-2\delta} \cos(2\delta - 2\zeta) + e^{-2\zeta}}{r^2 e^{-4\delta} + 2\, r e^{-2\delta} \cos 2\delta + 1}\right\}^{\frac{1}{2}} \tag{27c}$$

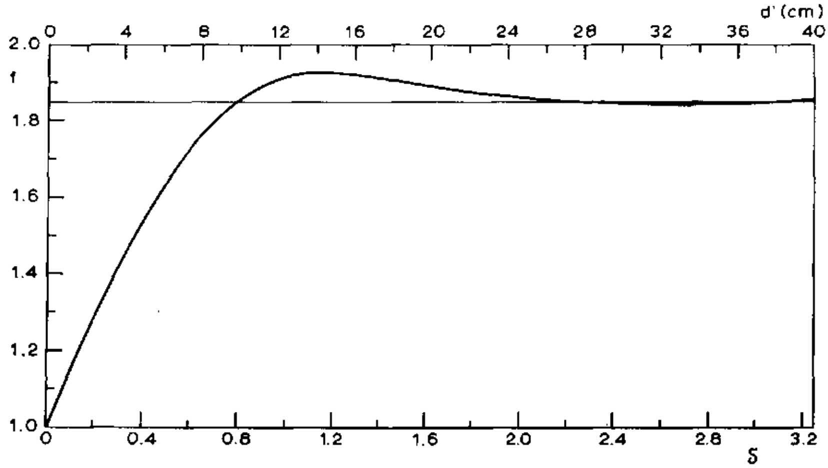

FIG. 8. Influence of δ on the value of f according (31)

$$\psi_z = \operatorname{arctg}\frac{(r^2e^{-4\delta}\,e^{2\zeta}+1)\,\sin\zeta + re^{-2\delta}(e^{2\zeta}+1)\,\sin(2\delta-\zeta)}{(r^2e^{-4\delta}\,e^{2\zeta}-1)\cos\zeta - re^{-2\delta}(e^{-2\zeta}-1)\cos(2\delta-\zeta)} - \varphi \qquad (28c)$$

with $\zeta = z/D'_s$

The values of p_z and ψ_z in the bottom layer ($z \geqslant d'$) are the same as in a homogeneous medium, thus:

$$p_z = p_{d'}e^{-(z-d')/D_s}\ ;\ \psi_z = \psi_{d'} - \frac{z-d'}{D_s} \qquad (27d)\ (28d)$$

In the boundary layer ($z = d'$) the values of p_z and ψ_z are simplified to:

$$p_{d'} = \left\{\frac{e^{-2\delta}\,(r+1)^2}{r^2e^{-4\delta} + 2re^{-2\delta}\cos 2\delta + 1}\right\}^{\frac{1}{2}} \qquad (27e)$$

$$\psi_{d'} = \operatorname{arctg}\frac{(re^{-2\delta}+1)\sin\delta}{(re^{-2\delta}-1)\cos\delta} - \varphi \qquad (28e)$$

The model of fig. 3b and the accessory formulae can also be extended to three or more layers. Although this may give a better resemblence of reality, this refinement will for many problems not be necessary also because other limitations may be much more important, as for example the changes of the soil properties with time.

B. CALCULATIONS

CULTIVATION

The influence of a loose top soil covering a dense subsoil with the properties of soils 2 and 1 in table 2, on the amplitude of the temperature wave at the surface varies somewhere between 1 and 13.4/9.6 = 1.4, where 9.6 and 13.4 are the surface amplitudes in the case these soils are homogeneous.

An important question is now, at what depth of the upper layer this soil approxi-

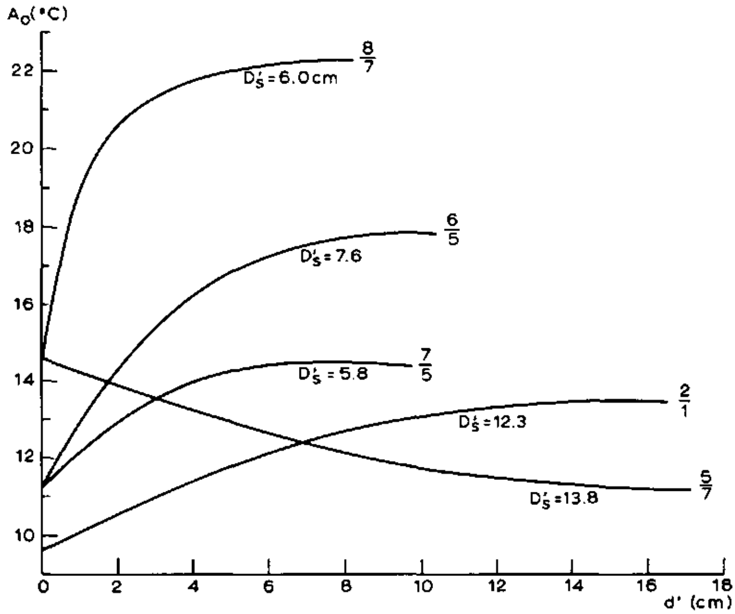

FIG. 9. *Influence of the thickness of the upper layer (d') on the amplitude of the daily temperature* wave (A_0) at the surface. Near the curves numbers of the toplayer and subsoil are given corresponding with table 2

mately behaves itself as a homogeneous soil with the properties of the upper layer. This depth depends of course in a large degree on the damping depth, since this depth determines the depth of penetration of the heat- and temperature waves. From this it is clear that the influence of cultivation that is limited to a small depth, is of minor importance for the yearly course of soil- and air temperature.

Fig. 9 gives the relation between the thickness of the upper layer, d', and the amplitude of the daily temperature wave at the surface for several improvements. First, the influence of a loose by packed upper layer (curve 2/1). The amplitude reaches a maximum of 13.6 °C at $d' = 14.5$ cm and the value of 13.4 °C if $d' = 12$ cm, which value is about equal to the damping depth of the upper layer, D'_s. The influence of such a layer on the yearly wave is next to nothing. With a depth of 30 cm the increase of the amplitude will be 0.1 °C and if the upper layer has a crumb structure 0.15 °C.

TABLE 3. The temperature waves at depths $z = 0$, $z = 10$ and $z = 20$ cm in not cultivated and cultivated clay at Wageningen during the period from January 8 to February 7, 1954 (VAN DUIN, 1956)

	$z = 0$	$z = 10$	$z = 20$ cm
not cultivated ($d' = 0$)	$-0.4+5.9 \cos \omega t$	$-0.5+5.5 \cos (\omega t - 0.09)$	$0.4+4.2 \cos (\omega t - 0.25)$
cultivated ($d' = 20$)	$-0.9+6.5 \cos (\omega t+0.05)$	$0.0+5.5 \cos (\omega t - 0.10)$	$0.9+4.0 \cos (\omega t - 0.25)$
minima: not cultivated	-6.3	-6.0	-3.8
cultivated	-7.4	-5.5	-3.1

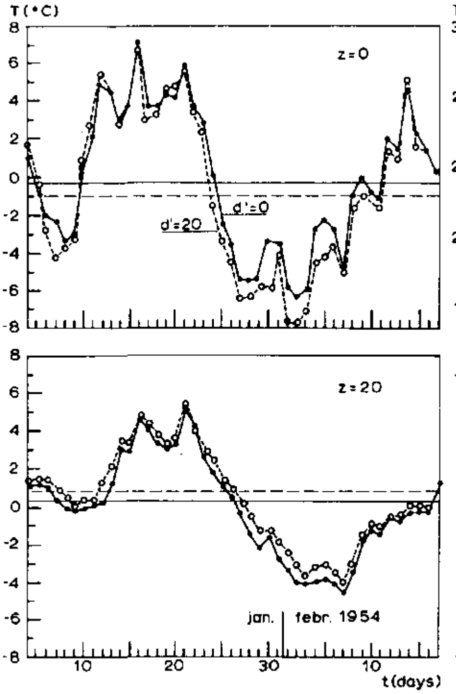

FIG. 10. Course of mean daily temperature at the surface ($z = 0$) and at a depth of 20 cm ($z = 20$) in not cultivated ($d' = 0$) and cultivated ($d' = 20$) unplanted river clay at Wageningen

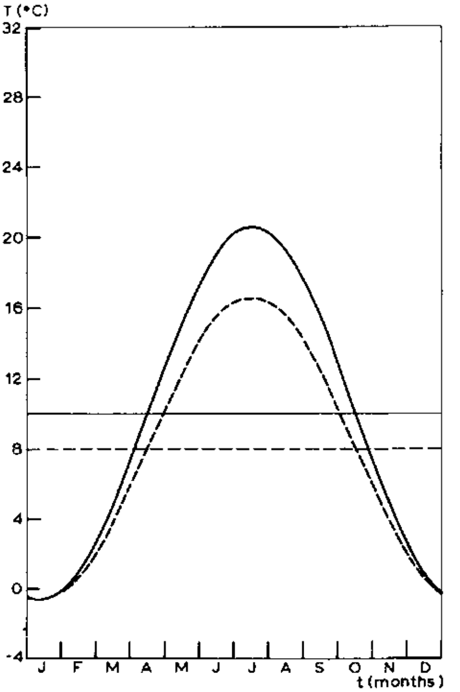

FIG. 11. The calculated yearly course of the mean daily temperature at the surface of a soil with a deep (——) resp. shallow (------) groundwater level, according the data of table 4

Apart from daily and yearly fluctuations irregular fluctuations also occur. This is represented in fig. 10 for the course of temperature during a period of one month for two depths. The corresponding functions are given in table 3.

As follows from the temperature functions for one month, the minimum value of the mean daily temperature is approximately 1 °C lower at the surface of the cultivated soil, but at depths of 10 and 20 cm the not cultivated soil cools down less. This difference in temperature may be more accentuated when shorter periods with a rapid change of temperature occur.

The answer to the question to what extent a cultivated soil is colder or warmer than a soil that has not been tilled depends of course on the time of the year and the depth that is considered. In periods of heat increase (spring), the upper layer of a cultivated soil warms up somewhat faster, while the lower layer stays cooler; in periods of heat decrease (autumn) the reverse is true. From this point of view loosening the soil may be preferable for summer crops, compacting the soil for winter crops. On the other hand if night-frost threaten in spring compacting may also be desirable, even as sprinkling.

SOIL MULCH

The influence of desiccation of a soil on its daily amplitude near the soil surface was already illustrated in fig. 5 and 7 for the extreme situation of a homogeneous dry soil in comparison with a homogeneous wet soil. The intermediate situations are given in fig. 9 for a sandy soil (curve 6/5) and a peat soil (curve 8/7), assuming that the available heat is independent of the moisture content. The influence of a decrease in evaporation has been already discussed in a preceding paragraph. The amplitude in soils with a dry upper layer and in homogeneous dry soils is about the same if $d' = D'_S$.

The decrease in risk of night-frost by improving a peat soil by applying a sand cover can be seen from curve 5/7 in fig. 9. If the thickness of this layer amounts to approximately 12 cm, this soil behaves as a sandy soil regarding the daily temperature wave. The properties of these soils are the same as those mentioned in table 2 under 5 (sandy soil at field capacity) and 7 (peat soil at field capacity), with amplitudes A_0 of 11.2 and 14.6°C respectively. If the soil dries out the influence of a sandy top layer when it has a thickness of at least D'_S cm, can be seen when comparing the curves 6/5 and 8/7.

The reverse situation i.e. a top soil of peat on a mineral soil in the form of for example peat litter to protect the subsoil from extreme temperatures, is given in curve 7/5. The amplitude at the original surface decreases from 11.2°C ($d' = 0$ cm) to resp. 8.2 ($d' = 1.5$), 6.3 ($d' = 3$) or 3.8°C ($d' = 6$ cm). Besides this, the mean value of the daily temperature will be higher in winter, although this effect is small except when shorter fluctuations occur.

DRAINAGE

The influence of the moisture content of a soil on its thermal properties, the heat waves into soil and air and the surface temperature is illustrated already by the data of soils 9 and 10 in table 2. Two intermediate conditions are given in fig. 11. In the calculations it has been assumed that the upper layer is at field capacity, while the subsoil is saturated. Some of the properties are presented in table 4.

The mean value of the yearly wave of temperature amounts in the Netherlands to approximately 10°C and it has been assumed that the minimum value for shallow- and deeply drained soils is the same, i.e. 1.6°C. Assuming a yearly mean surface

TABLE 4. Properties of the soils fig. 11 refers to

d'	δ	f	φ	R/f	α	β	γ
50	0.176	1.11	-0.073	0.192	-0.12	0.66	-0.20
150	0.106	1.29	-0.091	0.164	-0.10	0.70	-0.18
U_0	A_0	B_0	L_0	T_{00}	T_{min}	T_{max}	
$1.16 \cdot 10^{-3}$	8.4	$0.22 \cdot 10^{-3}$	$1.02 \cdot 10^{-3}$	10.0	1.6	18.4	
$1.47 \cdot 60^{-3}$	10.8	$0.28 \cdot 10^{-3}$	$1.31 \cdot 10^{-3}$	12.4	1.6	23.2	

temperature of 10°C for the shallow drained soil, this will result in a mean value of 12.4°C for the deeply drained soil. The maximum of the mean daily temperature will amount to resp. 18.4 and 23.2°C. The difference between these maxima will of course depend to a large degree on the postulations that have been made concerning the influence of drainage on the available heat at the soil surface and on the mean value of the yearly temperature waves. If, for instance, the amplitude of the available heat, U_0, amounts in both cases to $1.16 \cdot 10^{-3}$ cal/cm^2.sec, the amplitudes of the surface temperature wave are 8.4 and 8.55°C for shallow respectively deeply drained soils. When in that case the yearly mean temperatures are the same, the difference between the yearly temperature waves at the surface of shallow- and deeply drained soils will be very small.

SUMMARY

The temperature wave near the soil surface depends on the available heat at the soil surface and the thermal properties of soil and air, viz. the thermal conductivity and the thermal diffusivity. The thermal conductivity of a soil may be determined from measurements and with calculations according the theory of DE VRIES (1952).

In first approximation, the thermal properties of the soil are considered to be independent of depth, and those of the air to be increasing with height, according to LETTAU (1952). The influence on the temperature wave of a top layer with different properties has been calculated according VAN DUIN (1954). From this it appears that a soil with a top layer approximately as deep as the damping depth, behaves itself as a homogeneous soil having the thermal properties of the top layer.

The influence of cultivation on the amplitude of the daily temperature wave at the soil surface, during a clear day in the month of May, is given in fig. 9, as well as the effect of other methods of soil management. The desiccation of the topsoil in particular has a large influence on this amplitude, which is important with reference to the risk of night-frost. This indicates the effect of sprinkling. In other cases compacting of the soil may be an effective measure. With peat soils the risk of night-frost is much higher, which explains the use of a sand cover. The influence of drainage on the temperature wave depends to a large degree on its influence on evaporation.

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