ON THE ROLES OF CHARACTERISTIC LENGTHS AND TIMES IN SOIL PHYSICAL PROCESSES

P. A. C. RAATS

Institute for Soil Fertility, Haren
and
Department of Mathematics, Agricultural University, Wageningen,
The Netherlands

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P.A.C. Raats

Institute for Soil Fertility, Haren, The Netherlands, and
Department of Mathematics, Agricultural University, Wageningen,
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INTRODUCTION

Models can be formulated at different scales, in particular:

- the micro-scale of individual particles of the solid phase and of individual pores;
- the meso-scale of individual roots, aggregates, and "hot spots" of intense biochemical activity due to the presence of organic matter;
- the macro-scale of the entire, potentially rooted zone at a scale of the order of 1 m^2
- the mega-scale of the field;
- the giga-scale of the region.

Soil physicists feel most comfortable facing problems at the macro-scale. This scale offers the best perspective to formulate models in terms of measurable quantities. Most methods for determining physical and physical-chemical properties pertain to the macroscopic scale. Most instruments used to monitor actual conditions in the field deliver information at the macroscopic scale.

The macro-scale is the natural basis for integration to the mega-scale, i.e. the field scale of concern to the farmer. At the mega-scale attention focusses on integral inputs, storages, productions, depletions, and outputs.

The macro-scale itself is based on integration from the micro-scale either directly or indirectly via a meso-scale. The growth and acitivity of organisms is strongly determined by the physical and physical-chemical conditions at the micro- and meso-scales. Therefore there is a real need to study conditions and processes at these smaller scales. Two approaches should be further developed. One is the development of models and experimental techniques appropriate to these smaller scales. The other is to explore the potential of inferring from measured conditions at the macro-scale the conditions at the smaller scales.

This paper reviews some aspects of the origin and roles of length and time scales associated with movement of water in unsaturated soils.

THE PHYSICAL MATHEMATICAL MODEL

The balance of mass for the water may be written as

$$\partial\theta/\partial t = - \underset{\sim}{\nabla}.(\theta\underset{\sim}{v}) - u, \qquad [1]$$

where t is the time, $\underset{\sim}{\nabla}$ is the vector differential operator, θ is the volumetric water content, $\underset{\sim}{v}$ is the velocity of the water, and u is the volumetric rate of uptake. The volumetric flux $\theta\underset{\sim}{v}$ is given by Darcy's law:

$$\theta\underset{\sim}{v} = - k[h]\ \underset{\sim}{\nabla}h + k[h]\ \underset{\sim}{\nabla}z, \qquad [2a]$$

$$\theta\underset{\sim}{v} = - D[\theta]\ \underset{\sim}{\nabla\theta} + k[\theta]\ \underset{\sim}{\nabla z}, \qquad [2b]$$

$$\theta \underset{\sim}{v} = - \qquad \nabla \underset{\sim}{\phi} + k[\phi] \nabla \underset{\sim}{z}, \qquad [2c]$$

where h is the tensiometer pressure head, z is a vertical coordinate with its origin at the soil surface and taken positive downward, and the diffusivity D and the matric flux potential ϕ are defined by:

$$D = k/C, \qquad \phi - \phi_0 = \int_{h_0}^{h} k\,dh = \int_{\theta_0}^{\theta} D\,d\theta, \qquad [3]$$

where $C = d\theta/dh$ is the differential water capacity. Symbols in square brackets denote functional dependence. Unlike the dependence of k upon θ, the dependence of h upon θ is subject to hysteresis. As a consequence, Eqs. [2b] and [2c] are, strictly, only valid for monotonic changes in water content from some initial condition with uniform θ and h.

Two groups of parametric expressions describing the relationships between water content, pressure head, and hydraulic conductivity have received a lot of attention:

I A group yielding flow equations that can be solved analytically, in most cases as a result of linearization following one or more transformations;

II A group that is favored in numerical studies and to a large extent shares flexibility with a rather sound basis in Poiseuillian flow in networks of capillaries.

To group I belong the following classes of soils (Broadbridge and White, 1988; Raats, 1983, 1988; White and Sully, 1987):

1. Linear soils, with h proportional to ln $\{(\theta - \theta_l)/(\theta_u - \theta_l)\}$ and k proportional to $(\theta - \theta_l)$, where the subscripts l and u denote lower and upper limits, respectively.
2. Green-Ampt or delta function soils, with the diffusivity all concentrated at $\theta = \theta_u$;
3. "Mildly" nonlinear soils, with h a linear function of θ and k an exponential function of θ, implying exponential k[h], D[θ], and D[h] relationships and leading to linearization of Darcy's law in terms of the matric flux potential;
4. "Versatile" nonlinear soils, with D[θ] and k[θ] chosen such that the one-dimensional Richards' equation can be linearized by successive application of the transformations of Kirchhoff, Storm, and Hopf/Cole.

Classes I.3 and I.4 include as extreme subclasses the classes I.1 and I.2. Class I.3 has been useful in obtaining analytical solutions for multidimensional steady flows and for steady flows involving uptake by plant roots. The solutions of the Richards' equation for class I.4 represent a major advance in theoretical soil physics since the last ISSS congress (Broadbridge and White, 1987, 1988; Broadbridge and Rogers, 1990; Sander et al., 1988; Warrick et al, 1990; White and Broadbridge, 1988).

Group II can be best represented by a superclass of soils based on (see Raats 1990a for further details):

a) A four-parameter relationship between the pressure head h and the degree of saturation $S = \theta/\theta_s$:

$$h/h_{ref} = \left(\frac{1 - S^{\alpha}}{S^{\beta}} \right)^{\gamma}, \qquad [4]$$

where h_{ref} is a characteristic pressure head and α, β, and γ are empirical constants;

b) A three parameter dependence of the relative hydraulic conductivity

k/k_s upon h and S:

$$k/k_s = S^r \left(\frac{{}_0\int^S h^{-p}\, dS}{{}_0\int^1 h^{-p}\, dS} \right)^q \qquad [5]$$

where 1, p, and q are constants. Introducing [4] into [5] gives

$$k/k_s = S^r \{ I_{S^\alpha} ((p\beta\gamma + 1)/\alpha,\ (1 - p\gamma))\}^q \qquad [6]$$

where I_{S^α} (. , .) is the incomplete beta function.

The superclass of soils, in sofar it is defined by [4], is related to well-known subclasses (see Raats, 1990a for details). Equation [5] reduces to Burdine's expression if $r = n + m - 1$, $p = 2 + b$, and $q = 1$ and to Mualem's expression if $r = n + m - 2$, $p = 1 + b$, and $q = 2$, where n is the connectivity parameter with $1 < n < 2$, and m and b are tortuosity factors.

INTRINSIC CHARACTERISTIC LENGTHS, VELOCITIES AND TIMES

At any state of an unsaturated soil characterized by particular values of θ, h, and k one can define the <u>differential</u> intrinsic characteristic length, velocity, and time by

$$\lambda \equiv k dh/dk = d\phi/dk, \qquad [7]$$

$$\upsilon \equiv dk/d\theta, \qquad [8]$$

$$\tau \equiv \lambda/\upsilon = k(dh/dk)(d\theta/dk) = (d\phi/dk)(d\theta/dk). \qquad [9]$$

In general these differential parameters will be a function of the water content. However, corresponding robust <u>integral</u> parameters can be defined by averaging λ over k, and υ over θ:

$$\bar{\lambda} = \frac{{}_{h_1}\int^{h_u} k dh}{(k_u - k_1)} = \frac{\phi_u - \phi_1}{k_u - k_1}, \qquad [10]$$

$$\bar{\upsilon} = \frac{k_u - k_1}{\theta_u - \theta_1} \qquad [11]$$

$$\bar{\tau} = \bar{\lambda}/\bar{\upsilon} = \frac{(\theta_u - \theta_1)\ {}_{h_1}\int^{h_u} k dh}{(k_u - k_1)^2} = \frac{(\theta_u - \theta_1)(\phi_u - \phi_1)}{(k_u - k_1)^2}. \qquad [12]$$

The differential parameters λ, υ, and τ were considered earlier in the context of "mildly" nonlinear soils, for which λ = constant, $\upsilon \sim k$, and $\tau \sim k^{-1}$ (Raats, 1976).

For $h_1 = -\infty$, $k_1 = 0$ and $h_u = 0$, $k_u = k_s$, the negative of $\bar{\lambda}$ corresponds to the critical pressure head h_c of Bouwer (1964). Raats and Gardner (1971) derived expressions for $\bar{h}_c$ for six empirical relationships between k and h.

The integral parameters $\bar{\lambda}$, $\bar{\upsilon}$, and $\bar{\tau}$ play a key role in numerous recent papers from the CSIRO Centre of Environmental Mechanics at Canberra (e.g.

White and Sully, 1987, Broadbridge and White, 1988). In those papers $\bar{\lambda}$ is written in the form

$$\bar{\lambda} = b \frac{S^2}{(\theta_u - \theta_1)(k_u - k_1)} \tag{13}$$

where $S[h_1, h_u]$ is the sorptivity and b is obtained by comparing [10] and [13] and using an expression for S^2 based on the quasi-analytical solution of the horizontal adsorption problem of Philip and Knight (1974). For the resulting expressions for $\bar{\lambda}$, $\bar{\upsilon}$, and $\bar{\tau}$, associated with the classes of soils belonging to Group I, the reader should consult the papers of White and Sully, 1987 and Broadbridge and White, 1988.

THE MILLER SCALING THEORY

The scaling theory of Miller and Miller (1956) is concerned with geometrically similar media characterized by length scales $\lambda_{m*} = 1$ and λ_m. For geometrically similar media with geometrically similar distributions of water and of air, the STVF (= surface tension, viscous flow)-theory implies that simple relationships exist between water contents, pressure heads, and hydraulic conductivities:
1. Geometric similarity implies $\theta = \theta_*$;
2. The inverse relationship between the pressure head and the mean radius of curvature implies $h = \lambda_m^{-1} h_*$;
3. The linearized Navier-Stokes equation at the microscopic scale implies that in Darcy's law at the macroscopic the hydraulic conductivity satisfies $k = \lambda_m^{-2} k_*$.

The three primary scaling rules just given are of the form (Raats, 1983):

$$\omega = \lambda_m^{\,n} \omega_*. \tag{14}$$

with integer n. Using the three primary scaling rules, secondary scaling rules can be inferred from Darcy's law and from the balance of mass for the water. Darcy's law implies simple scaling rules for the spatial coordinates x, y, and z, the velocity $\underset{\sim}{v}$, the volumetric flux $\theta\underset{\sim}{v}$, the total head $H = h + z$, the diffusivity D, and the matric flux potential ϕ. The volume balance for the water implies scaling rules for the time t, and the volumetric rate of uptake u. Scaling rules for the differential water capacity C, characteristic length λ, and characteristic speed υ, all three potentially a function of the water content, can also be inferred easily. Table 1 of Raats (1983) gives the values of n in Eq. [14] associated with various parameters. Most noteworthy are the scaling rules for the spatial coordinates and the time
- the spatial coordinates, and hence all macroscopic length scales in processes, should be inversely proportional to the microscopic length scale;
- the time coordinate, and hence all time scales in processes, should be inversely proportional to the cube of the microscopic length scale.

The various parametric expressions for the relationships between θ, h, and k (Groups I and II above) can be interpreted so as to be consistent with the scaling theory of Miller and Miller. For example, if equations [4] and [5] are so interpreted, then h_{ref} is inversely proportional to λ_m

and k_s proportional to λ_m^2.

An important implication of the power function dependence λ_m^n of all the variables ω in Table 1 of Raats (1983) is that if the length scale λ_m is lognormally distributed, then all the variables ω/ω_* will also be lognormally distributed. This is a consequence of the reproductive rule for lognormal distributions: if the variable χ is lognormally distributed with mean μ and variance σ^2 then $e^a x^b$ is lognormally distributed with mean $a + b\mu$ and variance $(b\sigma)^2$.

DISCUSSION AND CONCLUSIONS

The characteristic lengths and times discussed so far are intrinsic soil properties. In actual flow problems two other categories of lengths and times arize

- lengths and times associated with geometry and boundary conditions;
- lengths and times emerging as consequences of a flow process.

Examples of length scales associated with the geometry of the region in which the flow takes place are: the length of a soil column, the depth of a water table, the location of a drip irrigation source within a flow region and/or relative to other sources. Length scales may also arize from boundary conditions: the size of a disc source imposed at the soil surface, the characteristic length associated with a pattern of wetting. Boundary conditions may also imply characteristic times: the duration of an input at a certain rate, the period of a cyclic input.

Examples of lengths emerging as consequences of a flow process are the maximum height of capillary rize of a given upward flux from a water table, the size of a pond at the soil surface resulting from an imposed flux, the effective thickness of the capillary fringe with regard to lateral flow, and the maximum thickness of the zone depleted ahead of a moving root front.

The characteristic times emerging from flow processes are of two types. One type is the time elapsed when a process settles down to some asymptotic behaviour such as a steady state or a time invariant travelling wave, or a regime in which the growth or decay of the profile is a simple, explicit function of time. Another type marks moments at which imposed flux boundary conditions become untenable. Three important examples of this in soil physics are:

1. The rate of evaporation imposed by the evaporative demand of the atmosphere cannot be sustained and switches to a falling rate. The falling rate will be inversely proportional to the time lapsed since some properly chosen instant.
2. The rate of infiltration imposed by patterns of rainfall or irrigation cannot be sustained and switches to a falling rate. The time at which the switch occurs is called the ponding time. This ponding time turns out to be a robust parameter, e.g. being nearly independent of soil type within the wide class of "Versatile" nonlinear soils.
3. The rate of uptake of water imposed by plant demand cannot be sustained and switches to a falling rate, thus leading to deficiencies.

In many cases there emerge in the formulations and solutions of flow problems ratios of the intrinsic characteristic lengths λ and $\bar{\lambda}$ and times τ and $\bar{\tau}$ on the one hand and the characteristic lengths and times associated with geometry and boundary conditions or emerging as consequences of a flow process.

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SUMMARY

This paper reviews some aspects of the origin and role of length and time scales associated with the movement of water in unsaturated soils. A distinction is made between differential and integral scales. The links with parametric expressions describing the relationship between water content, pressure head, and hydraulic conductivity and with the Miller scaling theory are indicated. Also discussed is the interplay between, on the one hand, the intrinsic length and time scales of a soil and, on the other hand, the length and time scales associated with geometry and boundary conditions or emerging as consequences of a flow process.