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Weir aeration - part II: Step weirs or cascades

1. Introduction

In a previous paper the authors have pointed out that very probably step weirs are less efficient in aeration of groundwater as is generally accepted. Intended to have water dispersed frequently, step weirs are unmistakably based on the augmentation of the area of contact between water and air. Indeed, the creation of area of contact is a major factor in aeration and by provision of step weirs oxygen transfer might be promoted, were it not for the fact that the dissolved oxygen content will increase from one step to another. Consequently the oxygen deficit will decrease and as it has been established that oxygen transfer is directly proportional to the oxygen deficit, it is evident that the effect of repeated falls will become successively smaller.

As suggested by the title this paper will concern the performance of step weirs and the height of steps to be practised for economical aeration. The criterion of performance of a given system will be taken to be the raise of the dissolved oxygen content over a flight of cascades, consisting of similar weirs or steps of equal heights. The dissolved oxygen content will be solved analytically on the basis of results obtained on single weirs and the solution will be verified by results obtained after thorough investigation. The latter investigations are described herein and will concern the aeration of a flow being passed more or less plugwise through a circuit at 60 $m^3.hr^{-1}$ capacity. It is noted that the experiments not only serve the above mentioned purpose but may also be regarded as a test case for determining the aeration capacity of an aerator placed in a circulating plug flow.

2. Theoretical considerations

2.1. *Step weirs*

The mathematical analysis is based on the assumption that the action of a weir will not be influenced by its placing in a series of weirs, so that a single weir's share in the performance of such a system only depends on the level of dissolved oxygen and the aeration capacity shown by each step. It has been shown in a previous paper [1] that the aeration at a weir may be expressed by:

$$c_b = c_a + c_o\left(1 - \frac{c_a}{c_s}\right) \qquad (1)$$

where c_a and c_b are the oxygen contents upstream and downstream from the weir resp. and c_s is the oxygen saturation value. It is further recalled that the aeration capacity c_o is expressed in terms of the raise of the dissolved oxygen content, the water arriving at the weir being free of oxygen. So the dissolved oxygen content of the water may be followed on its way down the weirs. Let the dissolved oxygen content of the water arriving at the n - th weir be c_{n-1} and the oxygen content directly downstream from it therefore:

$$c_n = c_{n-1} + c_o\left(1 - \frac{c_{n-1}}{c_s}\right) \qquad (1)$$

This is illustrated in fig. 1 indicating the concentration up- and downstream from the first weir, the second one, etc. It may be seen that each pair of concentrations lies on the straight line starting from point $P_{(c_s,\ c_s)}$ and intersecting the vertical axis in point $P_{(o,\ c_o)}$. From the practical point of view it is interesting that the particular system will show less than 50 per cent of oxygen deficit after the water has been passed over 2 weirs. It is also evident that the water must be passed over many more weirs before reaching saturation. Let the water contain c_i p p m of oxygen when arriving at the system and the concentration after n steps may be solved as follows. The concentration at a point downstream from the first weir obeys:

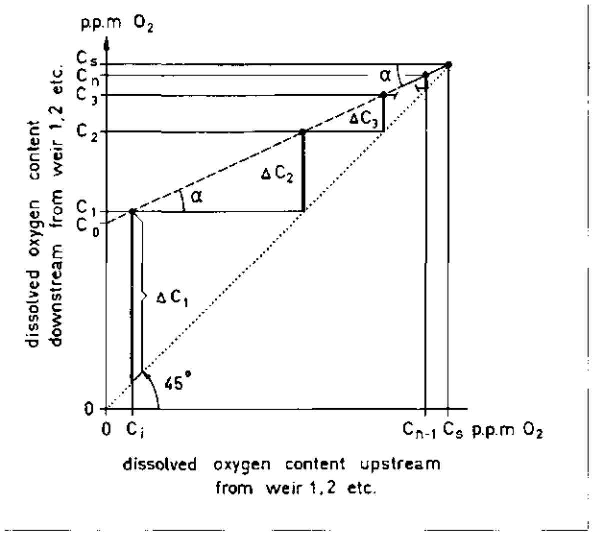
$$c_1 = c_i + c_o\left(1 - \frac{c_i}{c_s}\right) \qquad (1)$$

so that the raise of the concentration becomes:

$$\triangle c_1 = c_o\left(1 - \frac{c_i}{c_s}\right) \qquad (2)$$

Let the raise of the concentration between subsequent weirs be $\triangle c_1$, $\triangle c_2$, ..., $\triangle c_n$, the indices denoting

Fig. 1 - Performance of a step weir.



the numbers of the weirs, then it can be deduced from fig. 1 that:

$$\tan \alpha = (1 - \frac{c_o}{c_s}) = \frac{\Delta c_2}{\Delta c_1} = \frac{\Delta c_3}{\Delta c_2} = \ldots\ldots\ldots = \frac{\Delta c_n}{\Delta c_{n-1}} \quad (3)$$

From (3) follows:

$$\Delta c_2 = (1 - \frac{c_o}{c_s}) \Delta c_1$$

$$\Delta c_3 = \frac{\Delta c^2_2}{\Delta c_1} = (1 - \frac{c_o}{c_s})^2 \Delta c_1$$

$$\Delta c_n = (1 - \frac{c_o}{c_s})^{n-1} \Delta c_1 \quad (4)$$

The total raise of the concentration after n weirs would thus become:

$$c_n - c_i = \Delta c_1 + (1 - \frac{c_o}{c_s}) \Delta c_1 + \ldots\ldots\ldots + (1 - \frac{c_o}{c_s})^{n-1} \Delta c_1$$

This is a geometrical series the sum of which may be expressed by:

$$\Delta c_1 \sum_{n=1}^{n} (1 - \frac{c_o}{c_s})^{n-1}$$

As $0 < 1 - \frac{c_o}{c_s} < 1$ the following equation for c_n can be written:

$$c_n = c_i + \Delta c_1 \left\{ \frac{1 - (1 - \frac{c_o}{c_s})^n}{1 - (1 - \frac{c_o}{c_s})} \right\} \quad (5a)$$

The latter equation can after substitution of (2) be refashioned and written as follows:

$$c_n = c_s \left\{ 1 - (1 - \frac{c_i}{c_s}) (1 - \frac{c_o}{c_s})^n \right\} \quad (5)$$

With this formula the dissolved oxygen content for any system consisting of n weirs can be solved, provided each weir shows the same aeration capacity.

2.2. *Circuit with single weir*

To verify the performance of step weirs a circulation system was chosen as an experimental model, fitted with a single weir. We shall therefore try to find out if there is an analytical solution of the aeration provided at such a system which render possible the interpretation of test data. It must be admitted that the solution will be complicated while the interpretation might be hampered by the imperfections of the experimental system. However, these considerations have not kept us back from starting up this investigation, attractive as it was to have an installation which required a minimum of spacing. And as it has been mentioned before such a model may also provide information as to how aeration takes place in a circulation system. An example of this is the well-known oxidation ditch. It is noted that quite recently Sweeris has reported about full-scale experiments on brush aeration in the latter type of installation. Though the solution obtained by him has not yet been published [2] it is thought to be in accordance with the present formulation. We can imagine that some readers will take the experimental verification for granted and them we would refer to section 4 in which the evaluation of step weirs will be discussed.

Thus, to solve the aeration capacity consider a well stirred basin receiving the water being dropped from a weir at the rate Q. Let the volume of the receiving basin be V and its nominal residence time therefore:

$$\Theta_m = \frac{V}{Q} \quad (6)$$

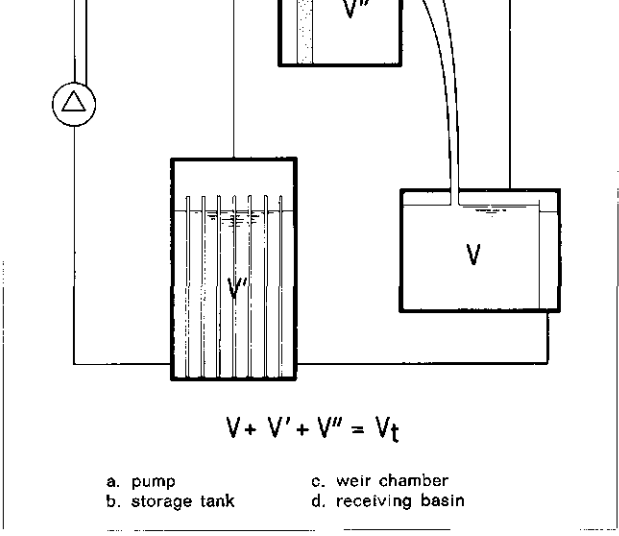
Stirring arises from the agitation by the inflowing water and due to this the concentration c_b in the basin (d) (see fig. 2) will be homogeneously distributed throughout the major part of it. When the concentration of the inflowing water be c_a a material balance then shows that c_b satisfies the differential equation:

$$V \frac{d\, c_b}{dt} = Q\,(c_a - c_b) + W \quad (7)$$

where W is the rate of oxygen supply to the basin owing to the entrainment of air bubbles. For the present we shall assume the aeration capacity to be c_o so that the rate of oxygen supply may be expressed by:

$$W = Q\, c_o\, (1 - \frac{c_a}{c_s}) \quad (8)$$

Fig. 2 - Flow sheet.



Substitution of (6) and (8) in (7) and refashioning of terms will yield the following differential equation:

$$\frac{d\,c_b}{dt} + \frac{1}{\Theta_m} c_b = \frac{1}{\Theta_m} \left\{ c_a + c_o \left(1 - \frac{c_a}{c_s}\right) \right\} \tag{7a}$$

We will further consider the receiving basin to be part of a circuit equipped with a pump. The pump will raise water from a buffertank into the weir chamber as indicated in the flow sheet given in fig. 2.

Let the volume of the total system be V_t and assume the flow in the buffertank and the weir chamber to be plug-wise. We may then write for the nominal residence time T of the water in both the buffer tank and the weir chamber:

$$T = \frac{V_t - V}{Q}$$

We shall now try to find out how variations in c_a produce variations in c_b including transients such as arise in starting up an experiment when beginning at a low level of dissolved oxygen c_i prevailing in the whole system. When ignoring the oxygen absorption that might occur in the buffertank and the weir chamber and when substituting $\omega = \frac{t}{T}$ for running time t, the assumptions will enable us to establish the following simple relationship between c_a and c_b:

$$c_{a,\omega} \overset{*)}{=} c_{b,\omega-1}$$

The solution of (7) for $0 < \omega \leqq 1$ will then become:

$$c_a = c_i$$

$$c_{b,\omega} = c_s \left\{ 1 - \left(1 - \frac{c_i}{c_s}\right)\left(1 - \frac{c_o}{c_s}\right) \right\} - c_o \left(1 - \frac{c_i}{c_s}\right) e^{-\frac{T}{\Theta_m}\omega} \tag{9}$$

And for $1 < \omega \leqq 2$ we may write:

$$c_{b,\omega} = c_s \left\{ 1 - \left(1 - \frac{c_i}{c_s}\right)\left(1 - \frac{c_o}{c_s}\right)^2 \right\} - c_o \left(1 - \frac{c_i}{c_s}\right)\left(1 - \frac{c_o}{c_s}\right)\left(1 + \frac{T}{\Theta_m}(\omega-1)\right) e^{-\frac{T}{\Theta_m}(\omega-1)} - c_o \left(1 - \frac{c_i}{c_s}\right) e^{-\frac{T}{\Theta_m}\omega} \tag{10}$$

A further increase of ω causes the values of $c_{b,\omega}$ to become more complicated as is shown for $2 < \omega \leqq 3$:

$$c_{b,\omega} = c_s \left\{ 1 - \left(1 - \frac{c_i}{c_s}\right)\left(1 - \frac{c_o}{c_s}\right)^3 \right\} - c_o \left(1 - \frac{c_i}{c_s}\right)\left(1 - \frac{c_o}{c_s}\right)^2 \left\{ 1 + \frac{T}{\Theta_m}(\omega-2) + \frac{1}{2}\left(\frac{T}{\Theta_m}\right)^2 (\omega-2)^2 \right\} e^{-\frac{T}{\Theta_m}(\omega-2)} -$$

$$- c_o \left(1 - \frac{c_i}{c_s}\right)\left(1 - \frac{c_o}{c_s}\right)\left(1 + \frac{T}{\Theta_m}(\omega-1)\right) e^{-\frac{T}{\Theta_m}(\omega-1)} - c_o \left(1 - \frac{c_i}{c_s}\right) e^{-\frac{T}{\Theta_m}\omega} \tag{11}$$

So that we will confine ourselves to the solution of $c_{b,\omega}$ for $\omega \leqq 3$.

2.3. *Graphical solution of c_o*

It may be seen from (9) that for $0 < \omega \leqq 1$ $c_{b,\omega}$ approaches asymptotically to the first term on the right side of (9), while its maximum value will be obtained at $\omega = 1$. The same applies to (10) and (11). It is clear that the value of $c_{b,\omega}$ depends on the ratio of T over Θ_m and in order to simplify (9), (10) and (11) this ratio must be chosen great enough as to obtain small errors of $c_{b,\omega}$, if the exponential terms are to be dropped. When comparing the dissolved oxygen content in the receiving basin with (5) it is evident that owing to such simplifications of the formulation of a circulated flow, the values of $c_{b,1}$, $c_{b,2}$, etc. to be obtained at $\omega = 1,2$, etc. will be equal to corresponding values at the step weir and which are obtained after the water has been allowed to pass a system being composed of 1,2, etc. steps. It may thus be speculated that:

$$c_{b,n} \approx c_s \left\{ 1 - \left(1 - \frac{c_i}{c_s}\right)\left(1 - \frac{c_o}{c_s}\right)^n \right\} \tag{12}$$

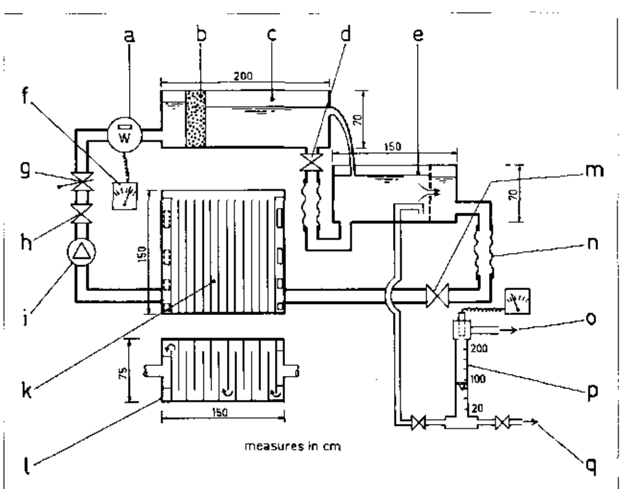
This equation may be written as follows:

$$\frac{c_s - c_i}{c_s - c_{b,n}} \approx \left(\frac{c_s}{c_s - c_o}\right)^n \tag{13}$$

The solution of c_o from (13) may be obtained by plotting the values of $\log \frac{c_s - c_i}{c_s - c_{b,n}}$ vs. n T, as is usual in the evaluation of aerators placed in well stirred tanks. The graphical solution would be greatly facilitated might a straight-line relationship be obtained. If this is the case and the angle of the line is denominated by β we may write:

$$\tan\beta = \frac{\log \frac{c_s - c_i}{c_s - c_{b,n}}}{n\,T} \approx \frac{1}{T} \log \frac{c_s}{c_s - c_o} \tag{14}$$

*) This may be understood when following a fluid element along its course down the storage tank and the weir chamber. The assumption of plug flow implies that the fluid element which leaves the weir chamber and thus enters the receiving basin at time ω, will show the same oxygen content as the element which entered the storage tank or left the receiving basin at time ω-1. The above mentioned assumption is adopted not because it is thought to be the most realistic — as is a distribution of residence times rather than a single time T — but because it yields an easily conceivable formulation of the raise of the dissolved oxygen content in the receiving basin.



a. flow meter
b. screens consisting of gravel-packs
c. weir chamber
d. valve fitted in by-pass
e. receiving basin
f. flow indicator
g. check valve
h. valve for flow adjustment
i. pump
k. storage tank
l. plan of storage tank with baffle arrangement
m. valve to be closed when fitting height adapters
n. height adapters
o. flow to storage tank
p. sensor assembly
q. flow to sampler

Fig. 3 - Experimental installation.

The solution of c_0 from (14) will yield the following expression of the aeration capacity:

$$c_0 = c_s\,(1 - 10^{-T \tan \beta}) \qquad (15)$$

This expression may be regarded as the formulation of the aeration capacity of an aerator placed in a circulation system of nominal circulation time $T + \Theta_m$ and under the condition of plugflow in the main part of the system. It is noted that this formula has been first obtained by Sweeris [2].

3. Experimental

Experiments were conducted with the purpose of investigating the correctness of the formulation of the aeration capacity c_0 given above. This purpose may be accomplished in various ways, one of which is the determination of c_0 and thereafter comparing the obtained value with the aeration capacity established previously. The experimental system consisted of basins of various capacities. The contents of the receiving basin e.g. were about 0.6 m^3. The total volume was chosen as to obtain convenient values of the residence time T at flow rates ranging from 20 to about 60 $m^3.hr^{-1}$ and was 2.2 m^3 on the average. The contents were measured volumetrically.

3.1. *Experimental system* (see fig. 3)

The water was allowed to pass from the receiving basin (e) to the buffertank (k) through a plastic pipe by gravity. Height adapters (n) were available for fitting in the pipe in the event of adjusting the position of the receiving basin. Plug flow through the storage tank was produced by the arrangement of baffles (l) and a flow distributor of special design to distribute water evenly over the height of the tank. Discharge of water from the tank was provided by a similar device placed opposite to the inlet. The flow was effectuated by a pump (i), it was adjusted by a valve (h) and measured by a magnetic flow meter (a). The flow rate could be read from a flow indicator (f). The weir chamber was provided with screens (b) to allow water to be distributed evenly over the cross-section of the chamber. Preliminary hydraulic investigations revealed that a stable flow pattern could be obtained by the arrangement of gravel packs at 0.4 m thickness filled with gravel at 10 to 20 mm grain size. Downstream from the weir chamber there was a detachable weir over which the water was allowed to pass to the receiving basin. The widths of both tanks were 0.75 m.

To obtain a sufficient margin of dissolved oxygen in the system to allow of much effective collection of data, water was deoxygenated with sodium sulfite and cobalt chloride as a catalyst at the start of each experiment. Both compounds were added slowly to the circulating flow. It is evident that for the deoxygenation procedure to be facilitated, the water should be prevented from absorbing oxygen and therefore the flow was by-passed from the weir. By opening a valve (d) water was passed through a closed conduct consisting of a flexible hose. The passage of water through the hose was continued until low oxygen concentrations were prevailing in the total system. The investigation was started at the closure of valve (d) so that aeration was resumed and continued until the water had become fully saturated with oxygen. An oxygen sensor was available for continuous measuring of the dissolved oxygen content of the water lead out of the receiving basin. Samples were taken for the chemical analysis of dissolved oxygen and sulfite.

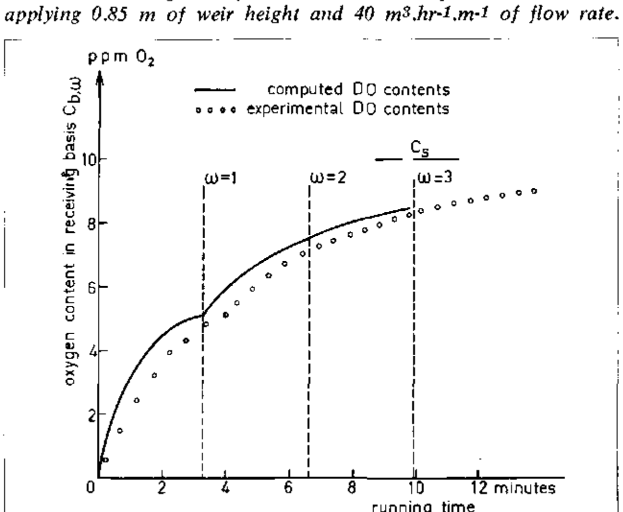
Analysis.

The dissolved oxygen content was determined by the Winkler method. Samples were checked for the presence of sulfite with iodine.

3.2. *Results*

About 80 runs were made to determine the aeration capacity of the system under unsteady-state conditions. Like in the former experiments [1] the height of the weir and the flow rate were varied. As the steady-state aeration at single weirs has been discussed at length in a previous paper [1] we will for the present confine ourselves to a comparison of results obtained in both systems and consider more particularly the extent to which the experimental oxygen content in the receiving

Fig. 4 - Relationship between dissolved oxygen content in receiving basin and running time from a circulation experiment at weir F applying 0.85 m of weir height and 40 $m^3.hr^{-1}.m^{-1}$ of flow rate.



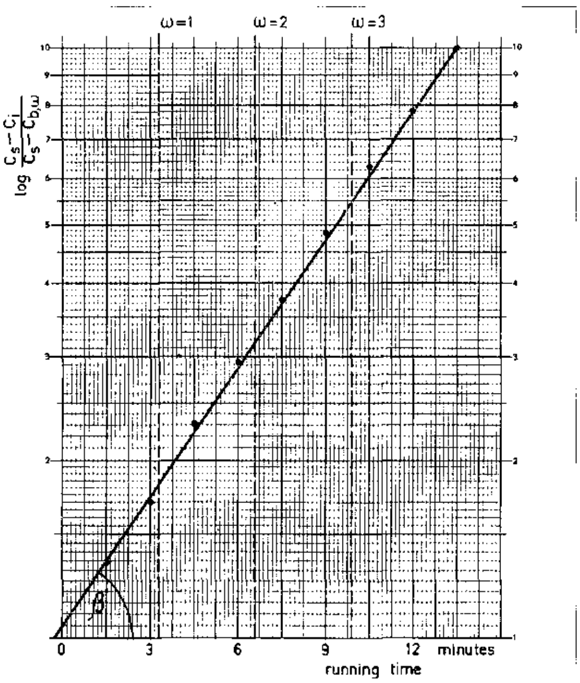


Fig. 5 - Graphical solution of the aeration capacity obtained at weir F applying 0.85 m of weir height and 40 $m^3.hr^{-1}.m^{-1}$ of flow rate.

basin agrees with the analytical solution given by (9), (10) and (11). For instance, results obtained at weir F applying a weir height of 0.85 m and 40 $m^3.hr^{-1}$ of flow rate are shown in fig. 4. The graph indicates that the experimental values are lower than the values predicted by (9), (10) and (11). Very probably, the difference may be due to the imperfections arising from the closure of the valve (d). Yet, the results of the experiments are such as to yield small differences between the fore-mentioned values, so that it seems justified to assess the aeration capacity c_0. It has been assumed elsewhere in this paper that c_0 can be solved by replotting the oxygen data on semi-logarithmic paper as is shown in fig. 5. It has been noted that this solution would be greatly facilitated might a straightline relationship between values of

$\log \frac{c_s - c_i}{c_s - c_{b,\omega}}$ at $\omega = 1, 2$, etc. and $\omega = 1, 2$, etc. be obtained.

Fortunately, such a relationship does exist and it has been found from all our experiments. It is remarkable that intermediate values also obey to it which may obviously be attributed to the smooth curvature of the line that can be drawn as to connect the measuring data (see fig. 4). It would thus seem as if c_0 can be solved from our circulation system, though it may be asked whether its value is still depending on the conditions of flow. This question may be solved from a comparison of measuring data obtained at different flow rates, i.e. at different values of the circulation time. In table 1 there are shown some of the results obtained at a constant weir height. In the table's last column on the right the reader may find computed values of c_0 obtained at weir D and applying an average weir height of 0.42 m. It may be seen that the values are in close agreement with each other and as this finding is consistent with results obtained under steady-state conditions, we may say with reasonable certainty that (15) is valid.

TABLE 1 - *Data obtained at weir D and at an average weir height of 0.42 m.*

Q $m^3.hr^{-1}$	V_t m^3	C_s p p m. O_2	tan β hr^{-1}	C_o p p m. O_2
20	2.85	9.52	0.97	2.13
30	2.85	9.45	1.52	2.17
40	2.85	9.40	2.00	2.12
50	2.85	9.82	2.12	1.92
59	2.85	9.85	2.81	2.12

Many experiments have been carried out both in the steady and the unsteady-state system and the availability of extensive data would therefore enable us to ascertain the accuracy of the latter system.

When tests having been carried out under similar operating circumstances in the steady and unsteady-state systems are chosen as criteria, it would follow that out of a total of about 80 runs 95 per cent showed consistent values of c_0. The accuracy may hence be estimated on the basis of c_0-values which were found to vary from 75 to 85 per cent of the corresponding values in the steady-state system.

When looking at table 1 once more, it appears that c_0 obtained at weir D and 30 $m^3.hr^{-1}$ of flow rate amounts to 2.17 ppm.O_2, while the steady-state value according to fig. 14 (see ref. 1) comes up to about 2.8 ppm. O_2. The former value thus is, like all other unsteady-state values, lower than the value that might have been attained, a result which may at best be explained from fig. 4 in which it is shown the experimental oxygen data also being below expectation.

3.3. *Discussion*

From this study it thus appears that aeration experiments carried out under unsteady-state conditions may yield satisfactory results. A first estimation of the above mentioned data would suggest that our success might be attributed to the arrangements made for the production of plug flow.

From the exponential terms of (9), (10) and (11) it would seem as if the accuracy of measuring results might be promoted by the choice of great values of the ratio of T to Θ_m. An increase of this ratio very probably would have caused the dissolved oxygen content in the receiving basin to reach quasi-stationary levels and so have given rise to plateaus in the experimental graph of fig. 4. So long as stationary levels are absent the concentration of $c_{b,\omega}$ for $\omega = 1, 2$, etc. may be expected to be lower than the values following from the first terms of (9), (10) and (11). As this was always the case in our experiments it is clear that merely dropping the exponential terms must give erroneous results, though it would seem that by appropriate measures as may be the production of plug-flow, the magnitude of the error may be greatly reduced. Meanwhile, it may be concluded that our speculations about the correspondence of the circulation system with the step weir is very near the truth and that the performance of a step weir may be expressed by (5).

4. **Evaluation of a step weir or cascade**

We now come to the ultimate purpose of our study, i.e. the evaluation of a step weir. As (5) has been verified, the criterion of performance of a given system will be taken

TABLE 2 - *Analytical solution of the aeration capacity at step weirs at 10°C and 40 $m^3.hr^{-1}.m^{-1}$ flow rate. (based on weirs D, E or F).*

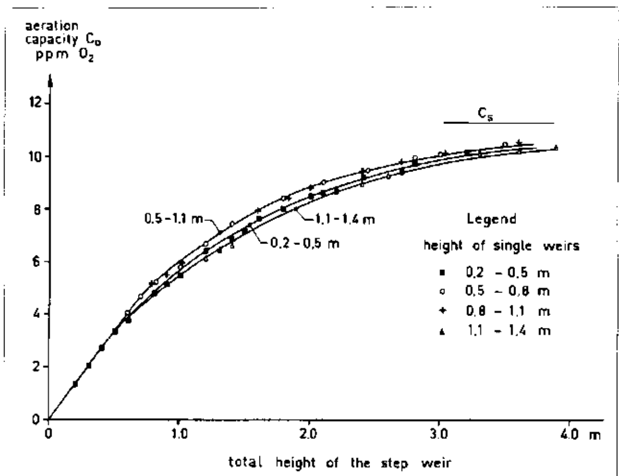
head loss in cm	height of steps 10 cm n	C_n	20 cm n	C_n	30 cm n	C_n	40 cm n	C_n	50 cm n	C_n	60 cm n	C_n	70 cm n	C_n
10	1	0.7												
20	2	1.4	1	1.4										
30	3	2.0			1	2.1								
40	4	2.5	2	2.7			1	2.8						
50	5	3.1							1	3.4				
60	6	3.6	3	3.7	2	3.8					1	4.1		
70	7	4.0											1	4.7
80	8	4.5	4	4.7			2	4.9						
90	9	4.9			3	5.2								
100	10	5.3	5	5.5					2	5.8				
110	11	5.7												
120	12	6.0	6	6.2	4	6.3	3	6.5			2	6.7		
130	13	6.3												
140	14	6.6	7	6.9									2	7.5
150	15	6.9			5	7.2			3	7.4				
160	16	7.2	8	7.4			4	7.7						
170	17	7.5												
180	18	7.7	9	7.8	6	8.0					3	8.4		
190	19	7.9												
200	20	8.1	10	8.3			5	8.6	4	8.6				
210	21	8.3			7	8.6							3	9.1
220			11	8.7										
230														
240			12	9.0	8	9.1	6	9.3			4	9.5		
250									5	9.4				
260			13	9.3										
270					9	9.5								
280			14	9.5			7	9.8					4	10.0
290														
300			15	9.8	10	9.8			6	10.0	5	10.1		
310														
320			16	10.0			8	10.2						
330					11	10.1								
340			17	10.2										
350									7	10.4			5	10.5
360			18	10.3	12	10.3	9	10.4			6	10.5		
370														
380			19	10.4										
390					13	10.5								
400			20	10.5			10	10.7	8	10.7				
410														
420			21	10.6	14	10.7					7	10.8	6	10.9
430														
440							11	10.8						
450					15	10.8			9	10.9				
460														
470														
480					16	10.9	12	10.9			8	11.0		
490													7	11.1
500									10	11.0				

head loss in cm	80 cm n	C_n	90 cm n	C_n	100 cm n	C_n	110 cm n	C_n	120 cm n	C_n	130 cm n	C_n	140 cm n	C_n
10														
20														
30														
40														
50														
60														
70														
80	1	5.2												
90			1	5.5										
100					1	5.8								
110							1	6.0						
120									1	6.2				
130											1	6.5		
140													1	6.7
150														
160	2	8.0												
170														
180			2	8.4										
190														
200					2	8.8								
210														
220							2	8.9						
230														
240	3	9.5							2	9.0				
250														
260											2	9.3		
270			3	9.8										
280													2	9.5
290														
300					3	10.0								
310														
320	4	10.3												
330							3	10.2						
340														
350														
360			4	10.5					3	10.3				
370														
380											3	10.4		
390														
400			5	10.8	4	10.7								
410														
420													3	10.5
430														
440							4	10.8						
450			5	10.9										
460														
470														
480	6	11.0							4	10.9				
490														
500					5	11.0								

to be the raise of the dissolved oxygen content over a flight of cascades consisting of similar weirs at equal heights.

The aeration capacities obtained at single weirs, approving to be most successful in aeration (i.e. weirs D, E and F) have been taken for the calculation of the height of the steps to be practised for economical aeration. It is evident that for the utilization of e.g. 2 m of nett loss of head, 4 steps at 0.5 m each or 2 steps at 1.0 m each may be chosen. By way of example dissolved oxygen contents have been computed for one and more steps utilizing a great variety of total head losses at 0.1 m interval, in the case of oxygen-free water arriving at the cascade. We have further taken a flow rate of about 40 $m^3.hr^{-1}.m^{-1}$ and a water temperature of 10°C. The results are collected in table 2. The computed values are plotted in fig. 6. From this it appears that cascades consisting of weirs at 0.2 to 0.5 m of weir heights will yield similar results. This is also true of cascades with

Fig. 6 - The aeration capacity of step weirs consisting of equal steps (weir types D, E or F) as a function of total height at 10°C and a capacity of 40 $m^3.hr^{-1}.m^{-1}$.



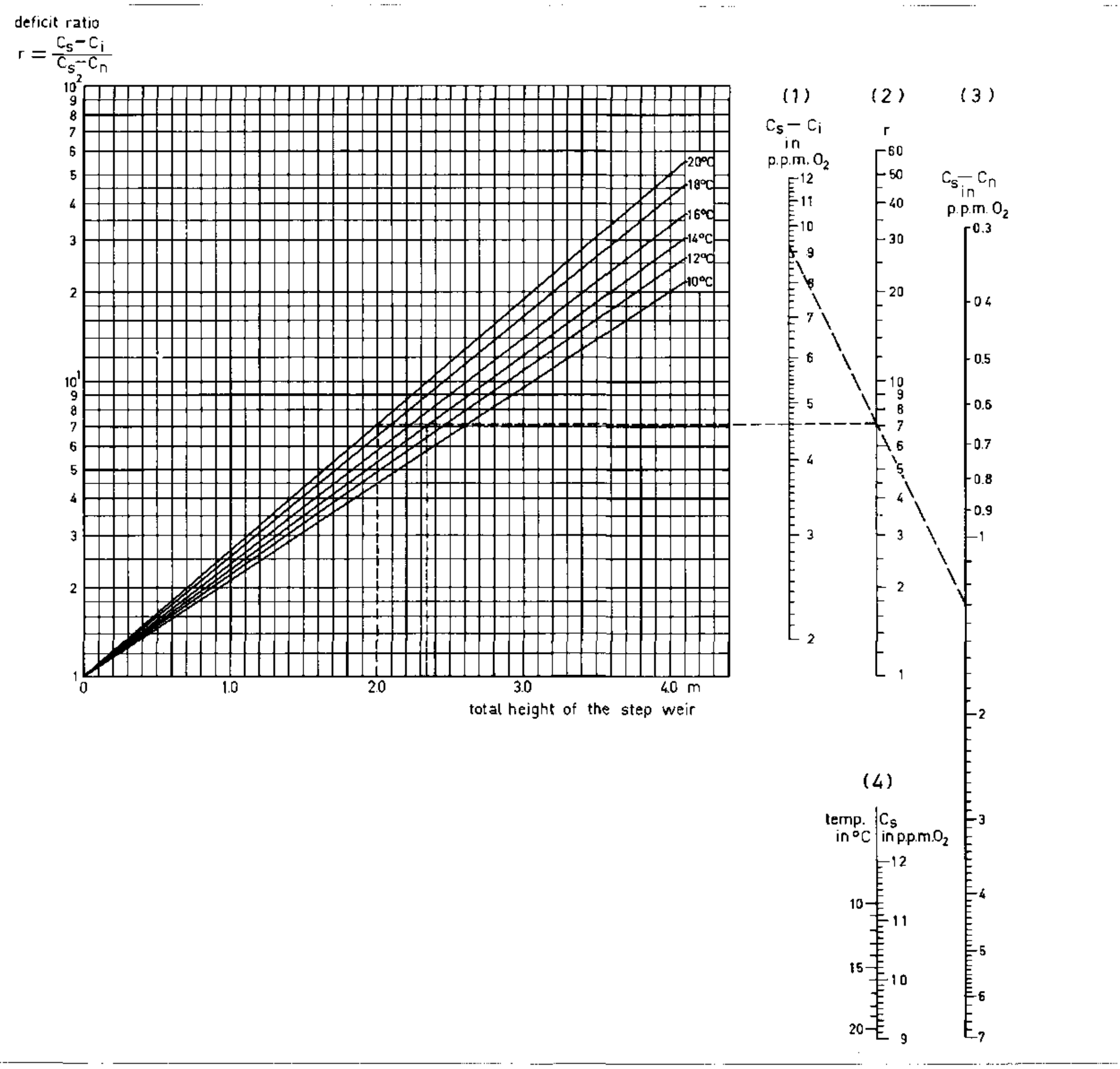


Fig. 7 - Nomograph for evaluation of the performance of step weirs at different temperatures and a capacity of 40 $m^3.hr^{-1}.m^{-1}$.

steps of 1.1 to 1.4 m. The best results are to be obtained at cascades being composed of steps of 0.5 to 1.1 m. It further appears that there is no difference between steps of 0.5 and 1.1 m. This result has been found to be valid irrespective of the initial dissolved oxygen content and temperature, within a range of 10 to 20°C. The choise of this range is connected with the temperature range involved in the former study [1], though it has at present been slightly expanded on both sides. It is recalled that within this range aeration capacities of single weirs may be assumed to be almost independent of temperature.

The above mentioned result throws new light on the performance of step weirs and it is concluded that in the practice of groundwater treatment, water being deficient in dissolved oxygen should not necessarily be aerated at cascades consisting of small steps. On the contrary, it seems justified that cascades, from the economical point of view, might consist of high steps as well, the maximum height of the steps coming up to about 0.8 to 1.0 m.

5. **Practical formula**

To establish a formula expressing the relationship between oxygen deficits and heights of a step weir, a system being composed of weirs D, E or F will be chosen as a basis having a step height of 0.6 m.

The dissolved oxygen content to be obtained after n steps may be expressed by:

$$c_n = c_s \left\{ 1 - \left(1 - \frac{c_i}{c_s}\right) \left(1 - \frac{c_o}{c_s}\right)^n \right\} \qquad (5)$$

When the height of the system amounts to h meter it would follow from the above mentioned reference:

$$n = \frac{h}{0.6}$$

$c_o = 4.1$ p p m.O_2 (see ref. 1 fig. 14).

After substitution of the latter specifications and refashioning of terms the latter equation may be written as follows:

$$\frac{c_s - c_n}{c_s - c_i} = \left(\frac{c_s - 4.1}{c_s}\right)^{h/0.6} \qquad (16)$$

The term on the left side of (16) will be recognized as the ratio $\frac{1}{r}$, r being the deficit ratio introduced by Gameson.

To solve c_n at a given height of the system, at a given concentration of the water arriving at the weir and at a given temperature, or otherwise to solve h for a certain concentration to be acquired when oxygen arriving at the weir and temperature are given, use will be made of a nomograph (see fig. 7). This nomograph may be read as follows:

Example 1: determine c_n at a total height of $2 \times 1 = 2$ m of the step weir at a temperature of 20°C and zero initial dissolved oxygen content. First read the deficit ratio r from the semi-logarithmic graph indicated on the left-hand side of fig. 7. For h = 2.0 m and 20°C we obtain r = 7.1. Read from the auxiliary scale 4 for 20°C c_s = 9.2 p p m. O_2. Calculate $c_s—c_i$ = 9.2 p p m. O_2 Align $c_s—c_i$ (scale 1) and r = 7.1 (scale 2) and read from scale 3: $c_s—c_n$ = 1.3 p p m. O_2. From this it may be calculated a dissolved oxygen content of the water which has been allowed to pass the step weir (c_n) being equal to 9.2—1.3 = 7.9 p p m. O_2.

Example 2: determine the height of a step weir and the size of the steps at a temperature of 14°C and 0.8 p p m. O_2 arriving at the weir for a dissolved oxygen content of 8.7 p p m. to be procured after the water has been allowed to pass the step weir (c_n). Read from the auxiliary scale 4 for 14°C c_s = 10.0 p p m. O_2. Calculate $c_s—c_n$ = 1.3 p p m. O_2 and $c_s—c_i$ = 9.2 p p m. O_2.

Align $c_s—c_i$ (scale 1) and $c_s—c_n$ (scale 3) and read a deficit ratio of 7.1. Determine the height h from the semi-logarithmic graph for r = 7.1 and 14°C. We obtain h = 2.34 m. As the nomograph is based on heights of the steps varying from 0.5 to 1.1 m, it would follow a step weir consisting of 3 steps at 0.78 m each.

Similarly, the nomograph will enable us to predict dissolved oxygen contents for any given system, provided the step weir consists of equal steps being efficient in aeration (weirs D, E and F) and provided temperatures are within the range of 10 to 20°C. Strictly speaking the evaluation is based on the assumption that the aeration capacity within this range is independent of temperature. Further research is required to elucidate the precise influence of temperature and to expand the nomograph for water of other temperatures (viz. from 4 to 10°C). It must also be noted that the nomograph is valid for flow rates up to 60 $m^3.hr^{-1}$ per m weir crest.

6. Conclusion

From this study it may be concluded that in the practice of groundwater treatment, water being deficient in dissolved oxygen should not necessarily be aerated at step weirs or cascades consisting of small steps.

Our data suggest that these aeration facilities, from the economical point of view, may consist of high steps as well. The maximum height of the steps comes up to about 0.8 to 1.0 m.

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