GERARD T. M. VAN DER KROON

Flow through filter sand

1. Synopsis

Extension of our knowledge about the flow beyond the region for which Darcy's law is valid is desirable when higher flow rates and coarser materials are to be applied in rapid sand filtration [1]. A modern view [2, 3] is that the flow in the zone of transition remains laminar, which would make it plausible to investigate whether the method of plotting the resistance in natural co-ordinates can be extended to the zone of transition. This is equivalent to treating the transitional flow by Kozeny's formula applied in a modified form. Accurate resistance data must then be available to make this sure. Furthermore, a discussion of the method of dimensionless groups [4] should be presented to justify the abandonment of this method. This was the purpose of a study made by the author.

An important aspect is the particle size relevant to filter flow. The method of permeametry [5, 6] has been applied to samples of Meuse sand of narrow size distribution and the result has been compared with results obtained by other methods like sieving and 'counting and weighing' to decide upon the feasibility of any of these methods.

Finally, some data will be given concerning the influence of particle size and temperature on the expansion of homogeneous sand-beds due to backwashing.

2. Basic considerations

Dimensional analysis shows that the resistance to flow of packed beds consisting of geometrically similar particles may be expressed by:

$$\frac{\frac{\Delta P}{\Delta l} D}{\rho u_p^2} = f\left(\frac{u_p D}{\nu}\right) \qquad (1)$$

The resistance coefficient f is a function of Reynolds number. Accepting Dupuit's assumption ($u_p = \frac{u}{\varepsilon}$) for beds of random packing and adopting a characteristic length dimension of the interstitial space $D = \frac{\varepsilon}{S}$, the modified resistance coefficient is obtained:

$$\frac{\frac{\Delta P}{\Delta l}\,\frac{\varepsilon^3}{S}}{\rho u^2} = f'\left(\frac{u}{\nu S}\right) \qquad (1a)$$

For the region for which Darcy's law is valid and for sand-grains there is experimental evidence [5] that the following equation may be written:

$$f' = \frac{5}{N_{Re}'} \qquad (2)$$

in which $N_{Re}' = \frac{u}{\nu S}$ (2a)

Substitution of (2) in (1a) yields an expression for the bed-resistance as follows:

$$\frac{\Delta P}{\Delta l} = 5\,\frac{S^2}{\varepsilon^3}\,\eta\,u \qquad (3)$$

which after substitution of $S = S_o\,(1-\varepsilon)$ gives:

$$\frac{\Delta P}{\Delta l} = 5\,S_o^2\,\frac{(1-\varepsilon)^2}{\varepsilon^3}\,\eta\,u \qquad (3a)$$

This is recognized as the Kozeny - Carman formula.

This formula, as discussed elsewhere in the paper, will be refashioned for practical purposes as follows:

$$\frac{\Delta P}{\Delta l}\,\frac{\varepsilon^3}{(1-\varepsilon)^2}\,\frac{1}{\eta}\,\frac{1}{S_o^2} = f(u) \qquad (3b)$$

The method of plotting by dimensionless groups enables us to correlate resistance data obtained under widely different circumstances. It is noted that the use of Reynolds number has been criticized [7] in view of the great range of values reported in literature above which Darcy's law would no longer be valid. The vagueness of the size of the pores and the uncertainty about the effect of porosity make the establishment of a general equation troublesome. Relationships for the zone of transition, given in [7, 8, 9], appear in a variety of forms. Quite recently Dudgeon [10] reported straight lines 'bounded by discontinuities of slope', which will be shown to pertain also to the present test data. A straight line is reminiscent of the relationship established by Blasius for turbulent flow through smooth pipes, and in this respect it may be asked whether the new picture is indicative of the onset of turbulence. It is known from experiments [11] that turbulence is not the mechanism causing Darcy's law to be no longer valid. The deviation is attributed to the non-linear acceleration terms in the Navier-Stokes equations, which terms should no longer be dropped when the velocities become larger [12, 11]. It is reasonable to assume, however, that the effect of the acceleration terms is to give rise to a smooth transition from purely laminar to turbulent flow. This picture is

Fig. 1 - Photo of Meuse sand.

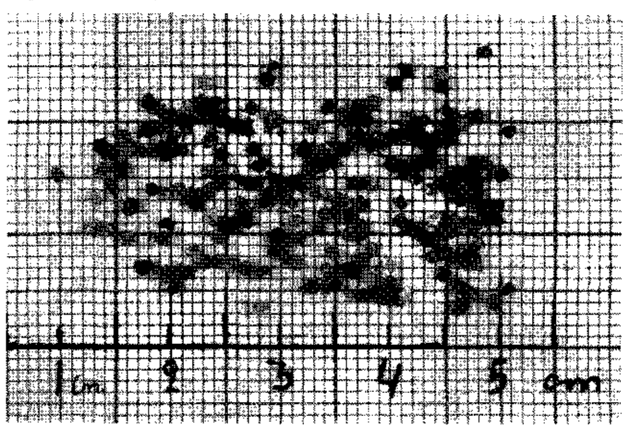


TABLE 1 - *Characteristics of homogeneous Meuse sand samples.*

Sieve numbers	Geometric mean diameter based on square woven sieves d_s in mm	Mean diameter based on 'counting and weighing' d_w in mm	Mean diameter based on permeametry d_f in mm	$\frac{d_f}{d_w}$	$\frac{d_f}{d_s}$
By 0.71 mm On 0.6 mm	0.65	0.75	0.638	0.85	0.985
By 0.85 mm On 0.71 mm	0.78	0.87	0.732	0.84	0.94
By 1.0 mm On 0.9 mm	0.95	1.02	0.84	0.825	0.89
By 1.2 mm On 1.0 mm	1.095	1.18	0.94	0.80	0.86
By 1.4 mm On 1.2 mm	1.295	1.35	1.17	0.87	0.90
By 1.6 mm On 1.4 mm	1.495	1.65	1.245	0.76	0.84
By 1.8 mm On 1.6 mm	1.69	1.86	1.44	0.775	0.85

not in agreement with the actual observations, so that another mechanism must be responsible for the deviation. Though of theoretical interest only, the author will mention an anomaly observed in tube flow, which may provide evidence for this point of view.

3. Experimental

3.1. Permeametric tests

A permeameter of 0.07 m ID was used to measure the resistance of samples of Meuse sand of narrow size distribution (see table and fig. 1) and of stainless steel balls with a diameter of 2.5 mm $\pm$ 0.01. Tap water of various temperatures was passed through beds having thicknesses of 0.5 to 0.8 m. The water temperatures were from 7 to 30 °C and the porosities of the beds were from 38 to 45 %. The beds were supported by a perforated plate covered with a micro-strainer fabric. The permeameter was equipped with a constant head device, a water-jacket to allow the apparatus to be thermostatted with a precision of $\pm$ 0.1 °C and piezometer tappings at a number of levels, being 0.1 m apart, to allow head losses to be measured. The heads were read from precision water manometers. The flow rate was measured by weighing the water discharged during a measured time interval, after the manometer levels had become steady. Investigations on stainless steel balls revealed that the resistance coefficients were probably accurate to within 6 % at a filter rate of 0.4 $cm.s^{-1}$.

3.2. Tube flow tests (see fig. 2)

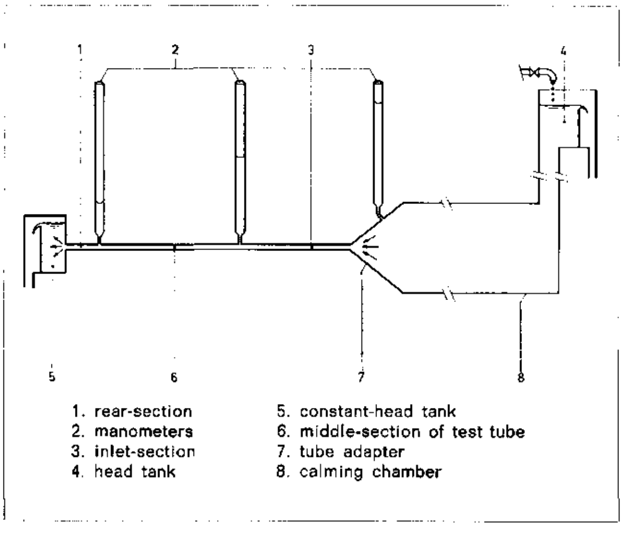
Some systematic efforts have been made to obtain the resistance to flow of tubes with small bore sizes (see table 2). Tap water of room temperature was passed through straight and undulating tubes from a cylindrical calming chamber, 0.1 m ID, 1.2 m long, via a conical tube adapter. The pressure in the chamber was increased by small stages by means of raising a head tank, which was connected to the chamber. Water was supplied to the head tank, which had an overflow device to control the water level. The ends of the tubes were fitted to a constant-head tank. The calming chamber and the test tube were both thermostatted. The normal procedure

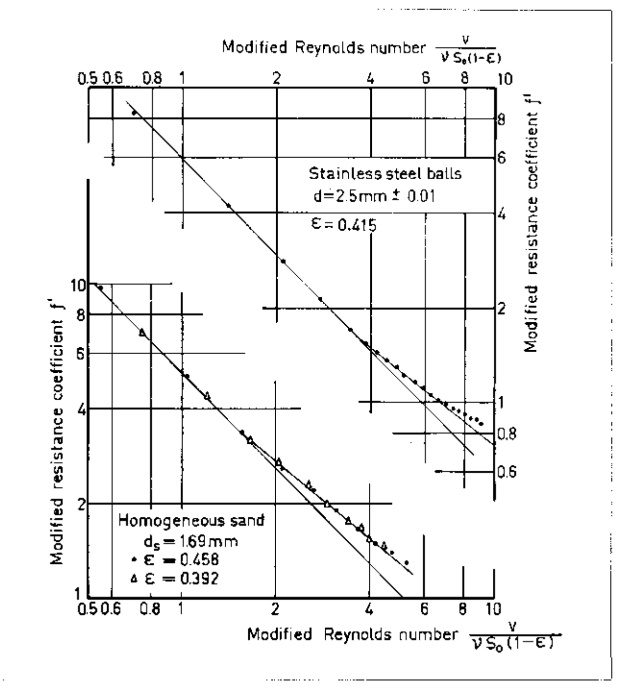
TABLE 2 - *Velocities at which deviations from Poiseuille's law occurred in rear-sections of tubes with small bore sizes.*

Bore size in mm	Material	Velocity $cm.s^{-1}$	Reynolds number
0.80	Tygon	80	640
1.55	Tygon	35	540
2.84	Perspex	25	710
3.09	Tygon	28	860
3.60	Glass	22	790
4.17	Glass	20	830
4.76	Tygon	20	950

Fig. 3 - Modified resistance coefficients for stainless steel balls ▶ and homogeneous sand. Height of the packed bed of stainless steel balls was 49.7 cm. Of homogeneous sand-bed porosity was varied. The height was between 60 and 70 cm.

Fig. 2 - Tube test.





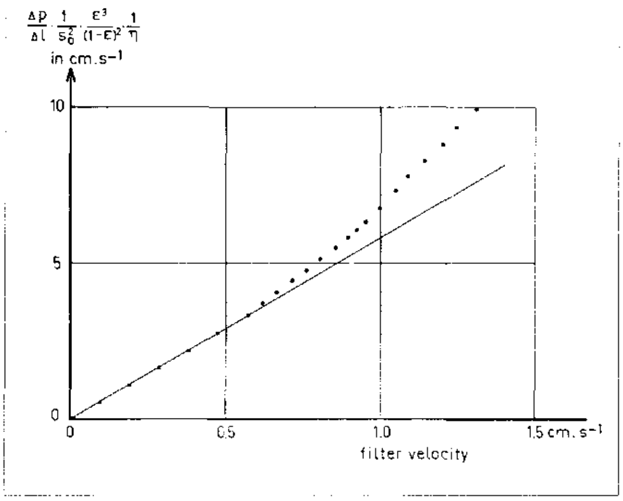


Fig. 4 - Resistance to flow of stainless steel balls (d = 2.5 mm ± 0.01). Height of the bed was 49.7 cm and porosity was 41.5 per cent.

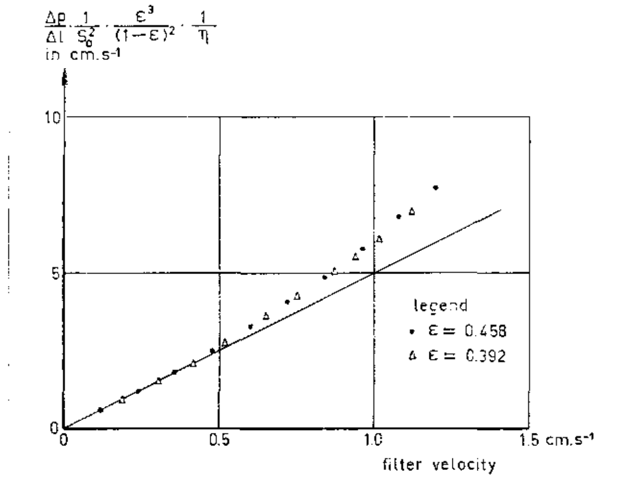


Fig. 5 - Resistance to flow of homogeneous sand (d_g = 1.69 mm) at different porosities.

was followed to obtain the coefficient of resistance for the total tube length [13, 14]. It is recalled that the establishment of the parabolic velocity profile gives rise to an additional resistance, which may be evaluated from $1.12\frac{u^2}{g_c}$ [15, 16]. To obtain resistances across inlet-, middle- and rear-sections of the tubes, piezometer tappings consisting of hypodermic needles were carefully inserted into the tube walls. The tappings were connected to precision water manometers.

3.3. Permeameter results

A typical result is shown in fig. 3. From this it appears that for stainless steel balls and sand the plots of modified resistance coefficients versus Reynolds numbers become straight lines for various ranges of Reynolds number. For the range immediately following the region for which Darcy's law is valid the following exponential relationship may be written:

$$f' = \frac{C}{(N_{Re}')^n} \qquad (4)$$

Values of n for both materials are in the neighbourhood of 0.8, which is in close agreement with values found by Dudgeon [10] for sand and $^3/_{16}''$ crushed dolerite*). Computation shows that the plot becomes almost straight, when resistances are plotted versus filter rates, immediately beyond the region for which Darcy's law is valid. The plotting in natural co-ordinates manifests an abrupt deviation from Darcy's law (see fig. 4). The filter rates at which the deviation occurred with sand samples listed in table 1, showed a trend upwards as the size increased. The range was from 0.3 to 0.5 cm.s⁻¹. From this it may be concluded that Reynolds number is not appropriate to indicate the upper limit of validity of

*) Dudgeon used other parameters, but it can be shown that the representation based on dimensionless groups yields the same values for n as given in [10].

Fig. 6 - Resistance to flow of homogeneous sand (d_g = 1.095 mm) at different temperatures.

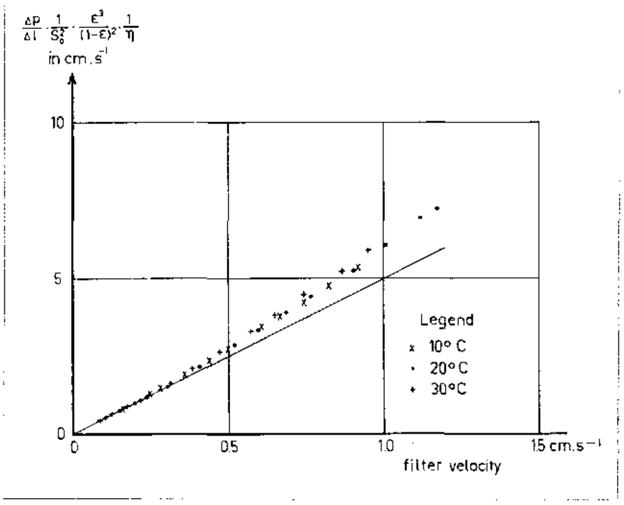
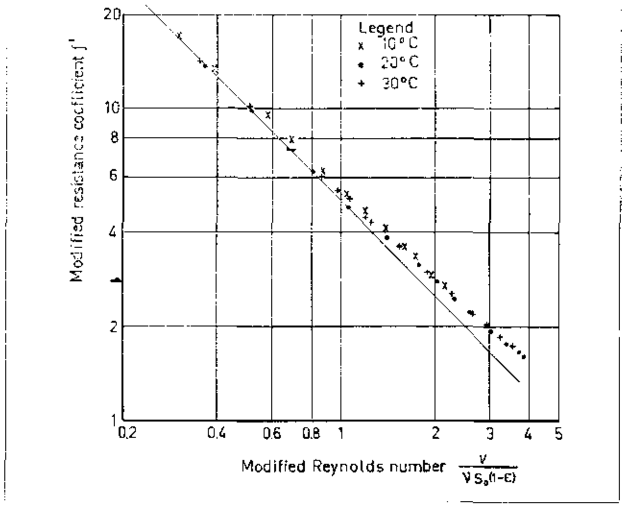


Fig. 7 - Modified resistance coefficients for homogeneous sand (d_g = 1.095 mm) at different temperatures.



Darcy's law. The reason for this is not apparent. It is possible that big sand-grains are not geometrically similar to small sand-grains [17], which might explain the considerable scatter in previous correlations. Henceforth it seems expedient to investigate whether the method of plotting the resistance in natural co-ordinates, according to equation 3b, can be employed over a restricted range of filter rates. An important criterion is in fact, that this method should give as good a correlation for different porosities as the method of dimensionless groups does (see fig. 3).

Computation shows that there is an error of less than 5 % for porosities varying from 38 to 45 % and for filter rates smaller than 1.0 cm.s^{-1} (36 m.hr^{-1}), when the product of $\frac{\triangle P}{\triangle l}$ and $\frac{\varepsilon^3}{(1-\varepsilon)^2}$ is plotted versus the filter rate (see fig. 5).

As regards viscosity [18], data in fig. 6 further reveal that for temperatures within a range of 7 to 30 °C the bed-resistance varies almost inversely proportional to the viscosity, up to a filter rate of 1.0 cm.s^{-1} (36 m.hr^{-1}). At higher filter rates the correlation becomes progressively poorer. For the range of Reynolds number immediately following the region for which Darcy's law is valid the method of dimensionless groups shows an inferior correlation (see fig. 7). Actually, it is believed that the foregoing conditions regarding porosity and viscosity cover ranges, which are in accordance with practical circumstances.

Results obtained so far afford strong presumption that for practical purposes Kozeny's formula can be used beyond the region for which Darcy's law is valid, provided a satisfactory solution is found for the size of the sand-grains.

The oldest and most direct means for determining the size of sand particles is sieving [19, 20]. A more appropriate method being used nowadays is permeametry, which gives the specific surface area S_o [6]. The determination is by means of the viscous resistance offered to flowing water by a column of packed sand-grains, according to equation 3a. In this the factor 5 has been proposed by Carman [5]. To relate the specific surface area of sand to the size of sand based on sieves or following from Hazen's 'count and weigh' method, it is suitable to use a factor accounting for the shape of the particles [21]. For irregular particles we have:

$$S_o = \frac{\phi_s}{d} \qquad (5)$$

$\phi_s = 6.0$ for spheres. Another, probably a more convenient approach that may be employed for homogeneous sand samples is the equivalent filter flow diameter:

$$S_o = \frac{6.0}{d_f} \qquad (6)$$

d_f is defined as the diameter being equivalent to the mean diameter of spherical particles of narrow size distribution, which offer the same resistance to flow as irregular particles do. There are some reasons which might justify this approach, as will be shown later.

Some results concerning the ratios of d_f to the geometric mean particle size based on sieves, and the size following from the 'count and weigh' method obtained on Meuse sand, have been given in table 1. From this it is clear that the ratios decrease with increasing sand size, which is in agreement with results obtained by other research-workers [22, 23]. It may be concluded that, if the size based on the methods of sieving or 'counting and weighing' were to be used to predict the resistances to filter flow, various correction factors should be applied, which make these methods less attractive.

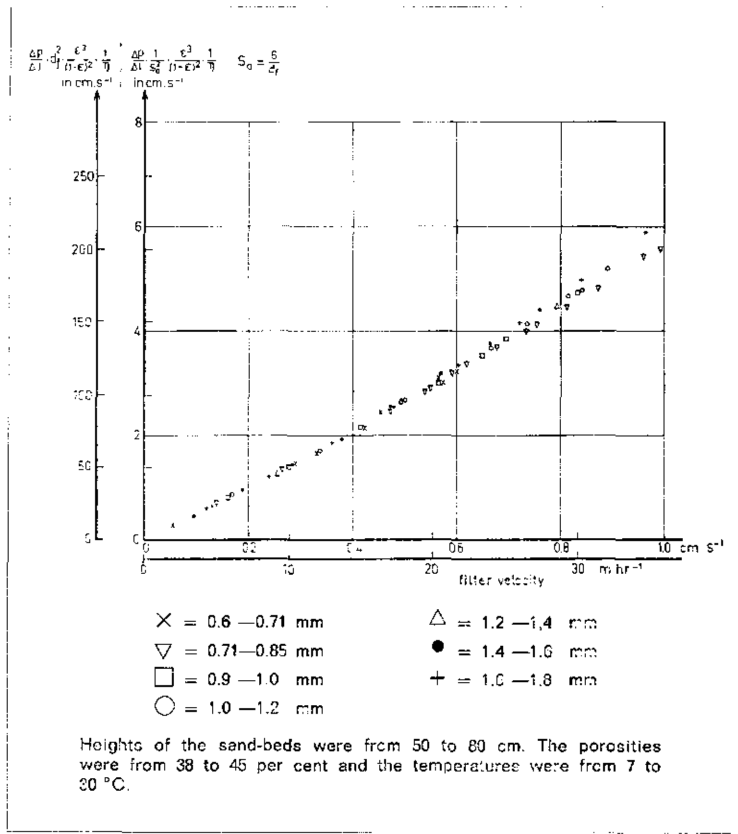


Fig. 8 - Resistance to flow of homogeneous sand samples.

We return to Kozeny's formula and will show that, when the equivalent filter flow diameter d_f is used for the correlation of resistance data obtained on a series of homogeneous Meuse sand samples, the plots in natural co-ordinates coincide almost perfectly for filter rates smaller than 1.0 cm.s^{-1} (36 m.hr^{-1}). This has been illustrated in fig. 8. It is emphasized that coincidence for u $<$0.4 cm.s^{-1} is no matter of argument, since the estimation of d_f is based on Carman's factor 5 (eq. 3a) and $\phi_s = 6.0$ (eq. 6). From the small divergence of plots beyond u = 0.4 cm.s^{-1} it may be concluded that, for the given range of particle size, Kozeny's formula may be applied over a small range of filter rates, beyond the region for which Darcy's law is valid. The formula should then be of a more generalized form:

$$\frac{\triangle P}{\triangle l} \, \frac{\varepsilon^3}{(1-\varepsilon)^2} \, \frac{1}{\eta} \, \frac{1}{S_o^2} = f(u) \qquad (3b)$$

f(u) can be estimated from fig. 8.

4. Backwashing

Many empirical relationships have been established

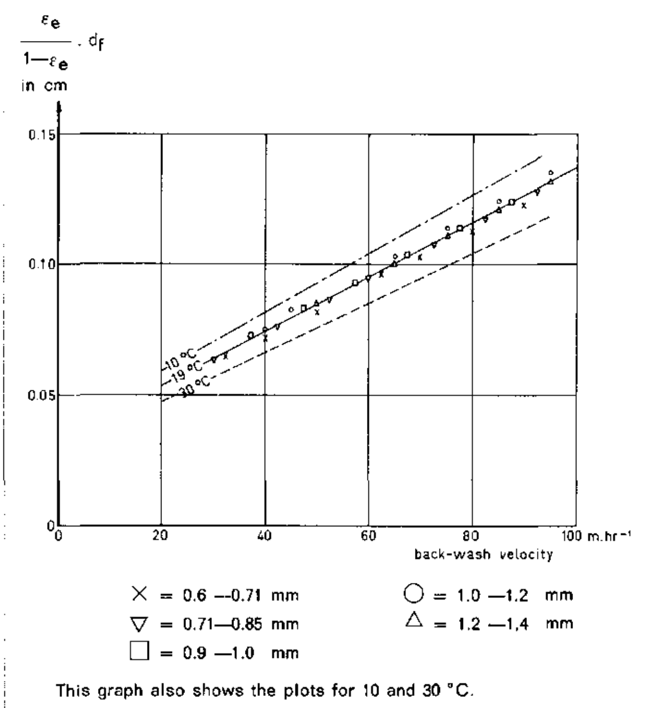


Fig. 9 - Relative sand-bed expansion as a function of backwash velocity for grain sizes varying from 0.6 to 1.4 mm. For these sizes a single relationship has been adopted as is shown for data obtained at sand-beds with heights of 50 to 80 cm at 19 °C.

between the rate of backwashing — also called rate of fluidization — and parameters like porosity [24] and expansion of a fluidized bed [25, 26]. For sanitary engineers the latter relationship is of interest and it is of practical importance to assess the effect of particle size and temperature in this. It is known from experiments that, within certain limits, the height of the expanded bed varies linearly with the rate of backwash [25, 26]. It has also been found that the rate beyond which the relationship is no longer valid increases with increasing particle size. Since normal backwash rates are well within the linear range it seems justified to correlate percentages of sand-bed expansion with backwash rates in natural co-ordinates. Corstjens will explain in a future paper that the correlation may be modified to account for the effect of initial sand-bed height and particle size as follows:

$$u_b = \frac{\varepsilon_e}{1-\varepsilon_e} d_f + C_b \qquad (7)$$

When this expression was used for homogeneous Meuse sand samples with sizes varying from 0.6 to 1.4 mm, which were backwashed at 19 °C, a correlation was obtained as is illustrated in fig. 9. This figure also gives correlations obtained at 10 and 30 °C. It is noted that the amount of sand-bed expansion can be easily computed from $\frac{\varepsilon_e-\varepsilon_i}{1-\varepsilon_i}$ times the height of the fixed bed.

5. Tube flow tests

Previous explanations of the deviation of the filter resistance have depended on whether the flow through filter sand was regarded as analogous to the flow around obstacles or to the flow through coiled or bended tubes [5, 3]. None of the explanations seem to be satisfactory a: it is generally accepted that obstacles should show a smooth transition; b: the analogy to the flow through coiled and bended tubes seems unrealistic. Perhaps the following observation made by the author may throw some light on the problem. When allowing water to flow through straight tubes of small bore sizes to obtain their resistances, a slight deviation from Poiseuille's law was found to occur at low Reynolds numbers in some unaccountable way (see table 2). An additional resistance seemed evident, and it was by the method of inserting piezometer tappings that this resistance could be localized. Fig. 10 shows the resistance coefficients for rear-sections of straight tubes, which were found to differ from those for middle-sections, when measured simultaneously.

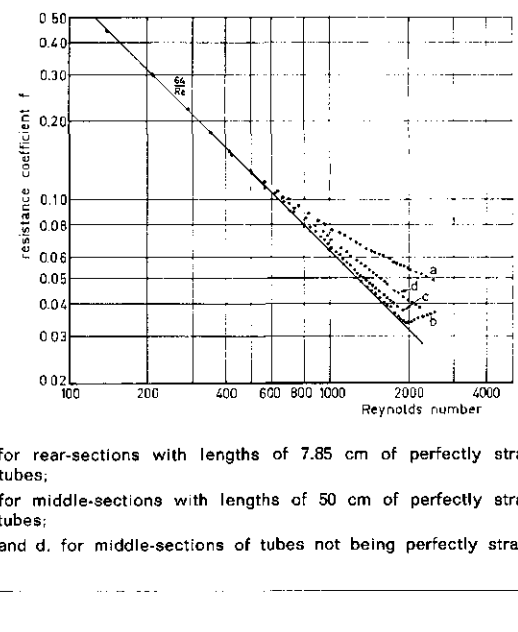
Historically, the first evidence for this are some tests carried out in july 1968. A similar anomaly was found for middle-sections of undulating tubes, though the deviation from Poiseuille's law was smaller (see fig. 10 c and d). It seems unlikely that turbulence could begin at such low Reynolds number [27]. It is possible that the anomaly is linked with changes in the flow pattern. The same suggestion has been put forward to explain the abrupt changes in regime occurring in filter flow [10].

6. Conclusions

6.1. Judging from the straight lines on the log plots it may be concluded that different flow regimes are apparent. It is in the zone of transition between one regime and another that the plots show a considerable scatter for materials of non-uniform size. For materials of uniform size the regimes are most clearly perceptible.

6.2. In the transition zone immediately following the region for which Darcy's law is valid a satisfactory

Fig. 10 - Resistance coefficients for Tygon tubes of 1.5 mm bore size. Tube lengths from 80 to 150 cm.



a. for rear-sections with lengths of 7.85 cm of perfectly straight tubes;
b. for middle-sections with lengths of 50 cm of perfectly straight tubes;
c. and d. for middle-sections of tubes not being perfectly straight.

correlation is obtained for materials of uniform size, when Kozeny's formula is applied. The correlation has been established for the practice of rapid sand filtration. It is emphasized that the correlation is restricted within narrow limits. These are:

temperatures from 7 to 30 °C;

porosities from 38 to 45 %;

filter rates smaller than 36 $m.hr^{-1}$;

size of the sand-grains based on permeametry and from 0.6 to 1.8 mm.

6.3. The size based on permeametry also permits a simple correlation between relative sand-bed expansions and backwash rates, provided the size is from 0.6 to 1.4 mm.

List of symbols:

$\frac{\Delta P}{\Delta l}$ = pressure drop
u_p = characteristic pore velocity dimension
u = filter rate, mean velocity of tube flow
u_b = backwash rate
C_b = backwash constant
N_{Re} = Reynolds number
N_{Re}' = modified Reynolds number
f = resistance coefficient
f' = modified resistance coefficient
g_c = gravity
S = surface of grains per unit volume packed space
S_o = specific surface of grains
D = characteristic length dimension of interstitial space
d_f = equivalent filter flow diameter based on permeametry
d_s = geometric mean diameter based on square woven sieves
d_w = mean diameter based on 'counting and weighing'
ϕ_s = shape factor
ρ = density of water
ν = kinematic viscosity
η = dynamic viscosity
ε = porosity of fixed bed
ε_e = porosity of fluidized bed
ε_i = initial porosity of fixed bed

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