Hydraulic resistance in arbitrarily-shaped channel cross-sections

F. Huthoff
Water Engineering & Management, University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands
HKV Consultants, PO Box 2120, 8203 AC Lelystad, the Netherlands

P.C. Roos, D.C.M. Augustijn & S.J.M.H. Hulscher
Water Engineering & Management, University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands

ABSTRACT
A simple model of lateral momentum exchange between adjacent flow sections is used to calculate compartment-averaged flow velocities in a two-stage and in an arbitrarily-shaped channel cross-section. As a result, the calculated flow velocities are based on bed roughness and on turbulent mixing caused by lateral geometry variations. The predicted values are compared to laboratory and field data, referring to flow velocities measured in channels that have varying flow depth across the channel. Good agreement is found between predictions and measurements in the two-stage channel case, showing that with the proposed method the hydraulic resistance in 1D river flow models can be quite easily extended to include the effect of lateral momentum exchange. For arbitrarily-shaped cross-sections, lateral momentum exchange only becomes important if flow variations across the lateral direction are relatively large.

Keywords: open-channel flow, river hydraulics, compound channels, hydraulic resistance

1 INTRODUCTION

The reliability of computational river flow models depends for a large part on the validity and accuracy of the included hydraulic resistance descriptions or parameters. Naturally, the geometrical properties of the river are also important, but with modern measuring techniques topographical inaccuracies are no longer limiting the reliability of model outcomes (e.g. Werner 2004). Consequently, it is common practice to calibrate the roughness parameters in river models such that certain discharge magnitudes match observed water levels (see discussion by Vidal et al. 2007). Such models may give reliable outcomes for situations that are similar to the calibration events, but the reliability of predictions for extrapolated extreme events is unclear. For that reason, it is important that calibration coefficients are well understood, and are either constant or only weakly dependent on flow conditions. This may be achieved by investigating flow dependencies of commonly used calibration coefficients and by replacing the flow-dependencies by actual physical process-descriptions.

In the current work it is shown how a simple modification to the Divided Channel Method may explicitly account for lateral momentum transfer between neighboring flow compartments, thus extracting an implicit flow-dependency usually captured in calibrated roughness parameters. The method is applied to a two-stage and a natural irregularly-shaped channel cross-section.

2 INTERACTING DIVIDED CHANNEL METHOD

2.1 *Two-stage channel*

In Huthoff et al. (2008) a simple method is proposed to calculate the compartment-averaged flow velocity in two-stage channels, while taking into account lateral momentum transfer. The method extends the well-known Divided Channel Method (DCM, e.g. Chow

1959, Yen 2002) with an effective shear stress contribution between neighboring flow compartments, resulting in an *Interacting* Divided Channel Method (IDCM). The underlying equations are a force balance for flow in the main channel (*mc*) and the floodplain (*fp*):

$$\rho g A_{mc} s = \rho f_{mc} U_{mc}^2 P_{mc} + N_{fp} \tau_{\text{int}} h_{\text{int}} \tag{1}$$

$$\rho g A_{fp} s = \rho f_{fp} U_{fp}^2 P_{fp} - \tau_{\text{int}} h_{\text{int}} \tag{2}$$

where ρ is the density of water, g the gravitational acceleration, A and P the cross-sectional area and wetted parameter of the flow compartment, N_{fp} is the number of floodplains, s the streamwise channel slope, f reflects the bed roughness and U is the resulting average velocity in the flow compartments. Main channel and floodplain are divided by artificial vertical division lines which have height h_{int}. Lateral momentum transfer takes place across this interface, represented by the interface shear stress τ_{int}.

Based on scaling assumptions of turbulent velocity fluctuations, the lateral shear stress is parameterized as

$$\tau_{\text{int}} = \frac{1}{2} \gamma \rho \left(U_{mc}^2 - U_{fp}^2 \right), \tag{3}$$

where γ is a dimensionless interface coefficient, which needs to be determined from experimental data. From experiments with 11 different geometrical configurations of two-stage channels, Huthoff et al. (2008) achieve best overall results using $\gamma = 0.020$.

Combining Eqs. (1) - (3) yields analytical solutions for U_{mc} and U_{fp}:

$$U_{mc}^2 = U_{mc,0}^2 - \frac{\frac{1}{2}\gamma N_{fp} \varepsilon_{mc} \left(U_{mc,0}^2 - U_{fp,0}^2 \right)}{1 + \frac{1}{2}\gamma \left(N_{fp} \varepsilon_{mc} + \varepsilon_{fp} \right)}, \tag{4}$$

$$U_{fp}^2 = U_{fp,0}^2 + \frac{\frac{1}{2}\gamma \varepsilon_{fp} \left(U_{mc,0}^2 - U_{fp,0}^2 \right)}{1 + \frac{1}{2}\gamma \left(N_{fp} \varepsilon_{mc} + \varepsilon_{fp} \right)}, \tag{5}$$

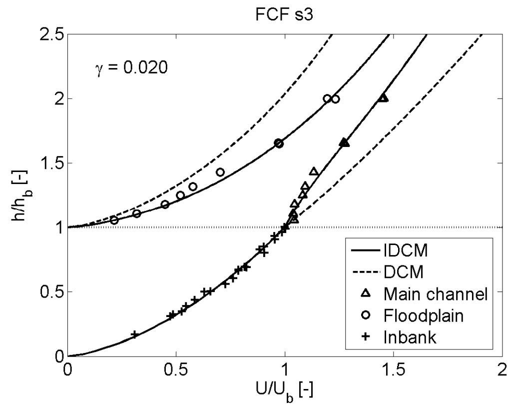

Figure 1. Measured flow velocities in the floodplain and main channel of a symmetrical two-stage channel, compared with predictions using both DCM and IDCM (see text). Flood Channel Facility data (FCF) are taken from Myers & Brennan (1990).

where dimensionless parameters ε_{mc} and ε_{fp} have been defined as

$$\varepsilon_{mc} = \frac{h_{\text{int}}}{f_{mc} P_{mc}}, \tag{6}$$

$$\varepsilon_{fp} = \frac{h_{\text{int}}}{f_{fp} P_{fp}}. \tag{7}$$

If lateral momentum transfer is neglected, then the interface coefficient γ equals 0, which corresponds to results from the standard Divided Channel Method:

$$U_{mc,0}^2 = \frac{g R_{mc} s}{f_{mc}}, \tag{8}$$

$$U_{fp,0}^2 = \frac{g R_{fp} s}{f_{fp}}. \tag{9}$$

From Eqs. (1) and (2) it follows that the hydraulic radii are $R_{mc}=A_{mc}/P_{mc}$ and $R_{fp}=A_{fp}/P_{fp}$. The dimensionless bed roughness f can be described in terms of Manning's roughness coefficient n:

$$f_{mc} = \frac{g n_{mc}^2}{R_{mc}^{1/3}}, \qquad f_{fp} = \frac{g n_{fp}^2}{R_{fp}^{1/3}}. \tag{10}$$

Figure 1 shows the results of the analytical solutions in Eqs. (4) and (5), using $\gamma = 0.020$, for one of the *Flood Channel Facility* experiments (e.g. Myers & Brennan 1990). Agreement of the IDCM with measured velocities is good, while, in contrast, the conventional DCM underpredicts flow velocities in the floodplain and overpredicts velocities in the main channel.

2.2 *Multi-stage channel*

The Interacting Divided Channel Method based on Eqs. (1) - (3) can easily be extended to describe flow in multi-stage, or arbitrarily-shaped, channel cross-sections. The force balance for compartment *j* now becomes

$$\rho g A_j s = \rho f_j P_j U_j^2 + h_{j-1/2}\tau_{j-1/2} + h_{j+1/2}\tau_{j+1/2}, \tag{11}$$

where $h_{j-1/2}$ refers to the interface on the left and $h_{j+1/2}$ to the interface on the right of compartment *j*. The corresponding shear stress is

$$\tau_{j+1/2} = \frac{1}{2}\gamma\rho\left(U_{j+1}^2 - U_j^2\right). \tag{12}$$

For an arbitrarily-shaped channel divided into *N* compartments, Eqs. (11) – (12) yield a set of *N* equations linear in U_j^2. Because individual flow compartments in the channel cross-section interact only with their direct neighbors, this linear system can be written as a matrix equation involving a *tridiagonal* matrix that represents the effect of lateral momentum transfer. Consequently, the linear system can be quite easily solved using the *Thomas algorithm* (Conte & de Boor 1972), which requires $O(N)$ calculation steps and thus does not require much computational effort to solve for the flow velocities.

In Huthoff et al. (2007) IDCM is applied to hypothetical irregular channel cross-sections, showing that in cross-sections with large jumps in depths, or at interfaces with sudden changes in bed roughness, significant flow differences are predicted using IDCM as compared to neglecting the lateral momentum transfer (i.e. by using DCM). IDCM thus provides a promising method to include lateral momentum

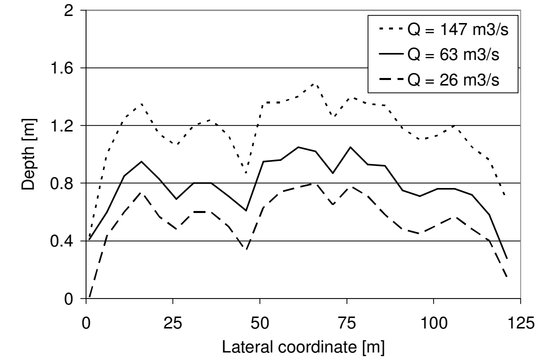

Figure 2. Measured flow depths in a cross-section of the Ebro River for three discharge events (data from Burguete et al. 2007).

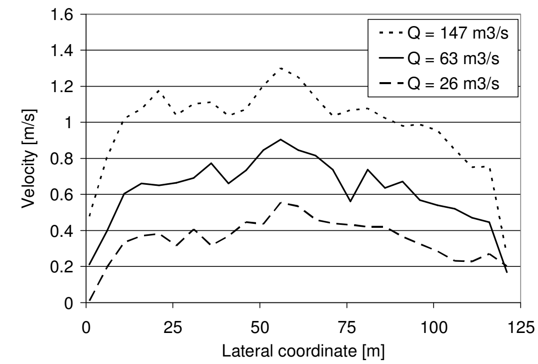

Figure 3. Measured flow velocities (see Fig 2. for corresponding flow depths).

transfer in 1D flow models, without leading to large increase of computation time.

3 HYDRAULIC ROUGHNESS IN A NATURAL RIVER CROSS-SECTION

Next, flow velocities in a natural channel cross-section are considered, to evaluate whether lateral momentum transfer is important (according to IDCM).

3.1 *Measurements in the Ebro River*

In the summer of 2003 the *Statistics and Gauging Service* of the *Ebro River Basin Water Authority* measured flow velocities in a channel cross-section of the Ebro River. The measurement location was chosen at a relatively straight channel section of the river, near the City of Zaragoza (Spain). The three discharge events included magnitudes of Q = 26, 63 and 147 m^3/s. Figures 2 and 3 show the measured flow depths and depth-averaged flow

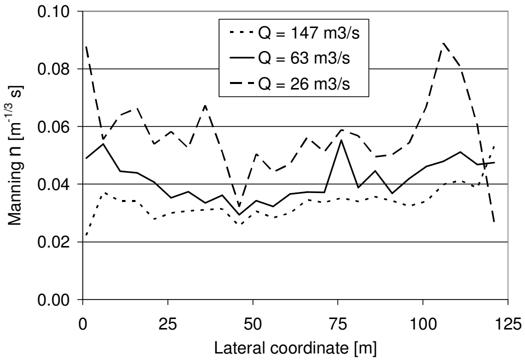

Figure 4. Lateral bed roughness profiles based on depth and velocity measurements in Figs. 2 & 3.

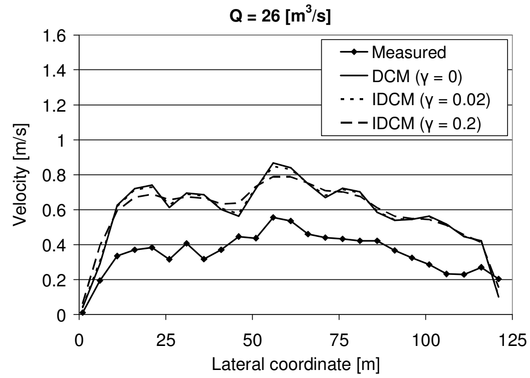

Figure 5. Measured and predicted flow velocities in the Ebro River cross-section, for discharge event Q = 26 m^3/s. For the predicted flow velocties, three different values for the interface coefficient γ have been used.

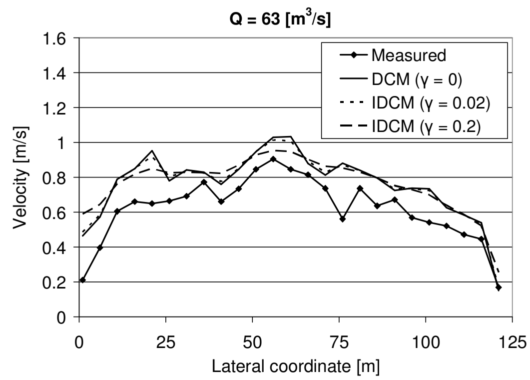

Figure 6. Measured and predicted flow velocities for discharge event Q = 63 m^3/s.

velocities for these respective events (see also Burguete et al. 2007).

The channel section was approximately 121 m wide, and measurements were collected at locations 5 m apart (laterally across the channel). The data collected at each of these locations we treat as representative of the local flow compartment with width 5 m. Next, for each flow compartment the effective local Manning roughness coefficient is calculated based on the hydraulic radius and depth-averaged velocity in each separate compartment (Figure 4). In doing so, a constant value for the streamwise channel bed slope of $9*10^{-4}$ has been adopted (Ebro mean channel slope, e.g. Vericat 2005).

3.2 *Application of Divided Channel Methods*

In Figure 4 it can be seen that in the considered channel cross-section lower discharge levels are associated with relatively larger roughness *n* values. Can this trend partly be explained by increased lateral mixing? At lower flow depths the lateral variability becomes more pronounced, hence potentially increasing the flow resistance component that is due to transverse mixing. By comparing how successful both IDCM and DCM (with and without lateral momentum transfer, respectively) are in representing the sampled cases, we get an indication of the importance of lateral momentum exchange.

Assuming that at the largest discharge rate the hydraulic resistance is least affected by transverse mixing, we use the set of these Manning *n* values to asses the impact of transverse mixing at lower flow rates (i.e. we adopt Manning bed roughness values that correspond to Q = 147 m^3/s in Figure 4). In Figures 5 and 6 it can be seen that the predicted flow velocities using these Manning values in the standard DCM for discharge events Q = 26 and 63 m^3/s gives quite poor agreement with measurements. Alternatively, when adopting IDCM and including lateral momentum exchange corresponding to γ = 0.020 or even γ = 0.20 results hardly get any better.

In both Figures 5 and 6 the difference between DCM and IDCM is evident, but instead of significantly decreasing the overall flow velocities, lateral momentum exchange only levels out lateral velocity variations. In effect, for the considered channel cross-section the increase in channel roughness with decreasing discharge is practically unaffected by transverse mixing. Therefore, other processes must be responsible for this observed trend.

4 DISCUSSION

Application of the IDCM to straight two-stage channels have shown that a constant coefficient of $\gamma = 0.020$ can account for the compartment-averaged flow velocities, which are significantly affected by transverse mixing (Huthoff et al. 2008). Extending this finding to a natural channel cross-section has not shown a comparable importance of lateral momentum transfer. Apparently, in the investigated natural cross-section the lateral variability of channel properties (depth, bed roughness) is not large enough to generate important transverse mixing. In the natural channel, lateral jumps in flow depth were mostly smaller than 20%, while in the laboratory two-stage cases the flow depth could suddenly change by 50% or more.

The variation of hydraulic roughness with changing discharge is thus most likely due to actual changes of the bed roughness. A probable cause for this trend is that at lower flow velocities the form drag of the bed becomes more dominant. This could be due to the presence of bed forms (ripples, dunes), larger stones or vegetation on the river bed.

Figure 7 illustrates how the Manning coefficient may change with changing flow depth, when presence of vegetation causes a layer above the bed to be dominated by form drag. Above the vegetation (above height *k*) the flow behaves similar to flow over a rough surface and approaches a constant Manning coefficient if the flow depth is relatively large (i.e. if h >> k).

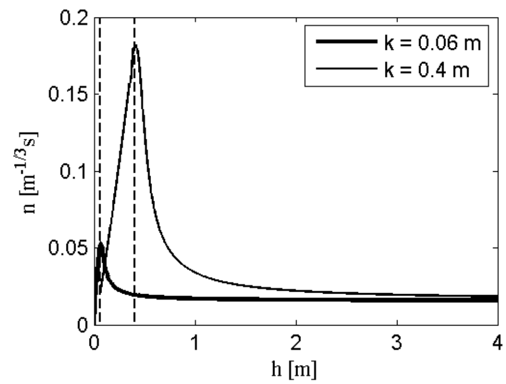

Figure 7. Schematical representation of the dependence of bed roughness on flow depth, whenever flow near the bed is dominated by form drag (k = height of drag-dominated layer). Figure from Huthoff (2007).

5 CONCLUSIONS

The Interacting Divided Channel Method (IDCM) that successfully accounts for the compartment-averaged flow velocities in two-stage compound channels was shown to have only marginal impact on flow velocities in an investigated channel cross-section in the river Ebro. Although some significant depth variations were present in this channel cross-section, the lateral exchange calculated by IDCM could not account for the increasing flow resistance with decreasing discharge.

Instead, the increased flow resistance with smaller flow depths in the investigated Ebro data is more likely due the relative contribution of a drag-dominated flow layer near the bed. It seems that, in general, larger lateral flow differences are needed to make the impact of lateral mixing on the overall flow field important.

To describe the overall roughness of arbitrarily-shaped channel cross-sections based on physical processes, it is of foremost importance to understand the depth-dependence of bed roughness (as affected, for example, by bed forms or vegetation). Lateral momentum exchange may easily be included using IDCM, but its effect becomes only relevant when neighboring flow compartments have large differences in average flow velocities (due to either sudden jumps in bed level or bed roughness). For channels with homogeneous bed roughness properties, changes in flow depth larger than 20% are required to make lateral momentum transfer an important contributor to overall roughness.

ACKNOWLEDGMENTS

We thank Pilar García-Navarro and Javier Burguete for providing the Ebro flow data, and the *Ebro River Basin Water Authority* (*Confederación Hidrográfica del Ebro*) for their permission.

This work is supported by the Netherlands Technology Foundation STW, the applied science division of NWO and the Netherlands Ministry of Economic Affairs.

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