Dynamic Model of an INAPRO Demonstration Aquaponic System

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Abstract

Although integrated aquaculture and hydroponic (aquaponic) systems are promising food production systems, as yet fish and crop yields are insufficient to compete with uncoupled commercial systems. The availability of nutrients in the water exchanged between aquaculture and hydroponics rarely meets demand for optimal growth of both fish and plants, while at the same time not exceeding toxic levels. In order to investigate minimum water usage and nutrient waste, while maintaining optimal growth in both compartment, a modular and dynamic model was developed. The demo site in Abtshagen is chosen as caste study object.

The water flowrate and nutrient concentrations in the compartments of the aquaponic system were modelled in order to gain insight in limiting water and fertilizer requirement, while still achieving optimal fish and plant growth. Model simulations showed that no water discharge was required under good water exchange management. Sending fish water to the plants once per three days is sufficient to keep the current aquaponic system at Abtshagen supplied with water. Current aquaculture nitrogen was more than sufficient to meet plant nitrogen requirement, with a doubled greenhouse size (384) plants expected to be able to take up all aquaculture nitrogen while not requiring additional nitrogen fertilizer. Significant calcium accumulation was observed, indicating that no Calcium fertilizer is required and no Calcium needs to be added to fish feed, since ground water already contains sufficient levels of Calcium in the Abtshagen area.

Expected fish and fruit yields of 933 kg/y (127 kg/m³/y) and 4145 kg/y (47 kg/m²/y), respectively, corresponding with a fish to fruit ratio of 1:4.0 kg/kg. A 1:8.0 kg/kg ratio is expected under the suggested double greenhouse size.
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1. INTRODUCTION

1.1. Background
The growing world population and changing climate have peaked interest in sustainable food production, resulting in a search for agriculture systems operated in a resource efficient way. Agriculture is currently the largest consumer of water worldwide, accounting for 70% of the global water demand (1). It is also responsible for 30% of the world energy demand and 90% of the mined phosphorus demand (2).

The European Union (EU) participates in the search for innovative and resource efficient food production systems by providing funding for research in these areas (3) (4). One of the projects funded by the EU is the INAPRO project (Innovative model & demonstration based water management for resource efficiency in integrated multitrophic aquaculture and horticulture systems) (5). INAPRO studies aquaponics, a promising food production system based on the principle of water and nutrient exchange between aquaculture and hydroponics (soilless food crop cultivation) in order to reduce water and fertilizer requirement.

The INAPRO project makes use of a model-based optimization approach in order to come up with more insight in aquaponic systems with respect to construction, design and operation. The main focus of the project lies in the recirculating aquaculture systems (RAS) cultivation of Tilapia combined with a nutrient film technique (NFT) based hydroponic cultivation of Tomato. Nutrient rich water from the fish tanks is used to irrigate the tomatoes, which loose water vapour due to evapotranspiration. This evaporated water can be condensed and returned to the fish. Nutrient imbalances are solved by fish water treatment, addition of fertilizer and, to some extent, water exchange.

1.2. Problem
Although aquaponics is a promising food production system, as of yet its yields are significantly lower than individual cultivation of fish or crop and little research is done on the functioning of aquaponic systems on economically viable scale. The availability of nutrients in the water exchanged between aquaculture and hydroponics rarely meets demand for optimal growth of both fish and plants, while at the same time not exceeding toxic levels. This difference is even further increased by the discrepancy in production cycle duration of aquaculture and hydroponics. An completely closed system aquaponic set-up is therefore unlikely. Water exchange and additional fertilizer is still needed in order to guarantee optimal growth of both fish and crop.

1.3. Objectives
Since not only sustainability but also economic viability is required for success, aquaponics should be able to compete on the market with current aquaculture and hydroponic products, meaning the growth of both fish and crop should not be hindered by the coupling of these principles. This study aims to develop a model able to accurately simulate an aquaponic system for feasible control and management strategies.

Objectives of this study are therefore to minimize water and fertilizer requirement, focusing on nitrogen, phosphorus, calcium, magnesium, sodium and potassium, while maintaining optimal growth in both compartments of the INAPRO proposed system as illustrated in Figure 1.
The research questions of the current study are:

1. How to minimize water discharge, fresh water requirement and fertilizer requirement in the Abtshagen system?
2. What is the ideal ratio between fish and plant?

1.4. Approach

In line with the INAPRO objectives, the options for minimizing resource requirement are investigated via an INAPRO modelling step. Focus lies on creating a transparent, easy to edit, INAPRO system model consisting of separate modules for each system component and tank, with applications for easy model simulation, data assimilation, simulation data exportation, parameter sensitivity analysis and optimization. This set-up facilitates the possibility of utilizing and/or expanding the model for future INAPRO simulations.

In order to shed light on how to operate an INAPRO aquaponics system more resource efficient, a system model is constructed using Python 3.4. This model will be based on the aquaponic set-up tested by PAL Anlagenbau GmbH Abtshagen. The computer model consists of a number of process modules, so that simulation of other aquaponic systems should only require slight model changes (5) (6). The dimensions and model parameters of the Abtshagen system are described in appendix A.

Python was chosen because it is an easily accessible, powerful open source programming language with a large build-in library and extensive community created content and tutorials. The aim is to create a transparent computer model using relatively simple equations in order to ensure fast simulation and easy accessibility for future application.

Combining literature, process engineering knowledge, a previous INAPRO model by Reyes Lastiri et al. (2016) and some assumptions about RAS and NFT dynamics, an initial model is created (7). Where possible, parameter estimation will be performed using data. Measurements from the PAL research group will also be used to validate the model and point out equations and parameters that will benefit from more measurements (6). Parameter sensitivity analysis will be performed in order to identify the most important parameters regarding total fresh water usage and discharge. These parameters will then be used to minimize water requirement and discharge. The effect of these
parameters on nitrogen fertilizer requirement will also be investigated, after which the current fish to plant ratio will be compared with different ratios regarding water use and fertilizer requirement.

The completed model will then be used to shed light on possible solutions to limit resource wastage. By minimizing water and fertilizer need under maintained optimal growth conditions of fish and plant, the model can help identify the factors most influential in resource efficiency and thus suggest possible changes in which to limit the wasting of these resources.

1.5. Outline
In chapter 2 the construction of the model will be discussed, together with the mode of implementation in Python, followed by a detailed description of the modelled processes per aquaponic system module. Chapter 3 will tackle parameter sensitivity and key parameter identification regarding total fresh water requirement. In chapter 4 model simulations of the Abtshagen site are described and discussed. Chapter 5 contains a general discussion and conclusions and the references are mentioned in Chapter 6. Two appendices are added detailing model parameters and static model description.
2. Aquaponic system modelling

2.1. Plant Layout

The deterministic, dynamic and mechanistic model simulates fish and crop growth and solute dynamics in an aquaponic system. The considered nutrients are nitrate nitrogen ($N\text{O}_3^− − N$), total ammonia nitrogen ($T\text{AN} − N$), phosphate phosphorus ($P\text{O}_4^{3−} − P$), calcium ($Ca^{2+}$), magnesium ($Mg^{2+}$), potassium ($K^+$), sodium ($Na^+$) and solids as total suspended solids ($TSS$) and total volatile solids ($TVS$). These seven nutrients and two solids are collectively referred to as nutrients. Sulphur and multiple micronutrients are present in feed and required by plants, but are not considered in this study, as these are only present in minute concentrations and/or do not impact the system in a significant way. Sulphur and these micronutrients might need to be added as fertilizer, but since insufficient data is available about their presence in feed, excretion by fish and requirements by plants and fish, no relevant conclusions can be drawn from them when included. Most likely these will not accumulate to dangerous levels, since their presence is very limited.

The model is build assuming zero or first order dynamics and polynomial growth models for plant and fish. It consists of separate modules corresponding to the different operation units in the aquaponic system, as shown in Figure 2 (6). The modules represent the tanks in the system: pump sump (PS), biofilter (BF), reception tank (RE), clean water tank (CW), rearing tanks (RT), sedimentation tank (ST), 3 chamber post purge pit (PP), mixing tank (MT), NFT channel hydroponic greenhouse (HP) and the fresh water tank (TW). The reception tank and clean water tank have a relatively small volume compared to the other tanks. Since no reactions occur in these two tanks, they are considered part of the biofilter. Water circulates in the RAS loop, consisting of pump sump, biofilter, rearing tanks and sedimentation tank in order to keep nutrient concentrations safe for fish cultivation. Occasionally, the sedimentation tank is drained to the post purge. Post purge, mixing tank and hydroponic NFT channels are the greenhouse part of the system, which ensures plants obtain sufficient water and nutrients. Each tank is assume to be ideally mixed, since a simulation time step is chosen significantly smaller than the time constant of most processes involved. However, this is not the case for the solids removal in sedimentation tank and the nitrification processes in the biofilter. However, the rates of these processes increase linearly with influent concentrations and can thus be modelled based on the influent concentration rather than the concentration in various positions in the tank (8) (9). In line with these concentrations, a simulation time step of $1/96$ days (15 minutes) was used when operating the model.

These modules can be coupled using multiple functions to run or optimize the model or to visualise model simulation results. Currently modelled system dimensions and parameters are based on the aquaponic system by PAL, Abtshagen, although computer model modularity enables easy appliance to other systems. A system model representation detailing computer module connectivity is shown in Appendix B.

The ordered differential equations describing the model are solved using the Euler Forward method and can be run for multiple years in order to evaluate multiple production cycles of both plants and fish. Unlike previous models (5) (7), this model can easily be adapted to govern other scenarios, due to its modular approach and easy parameter editing options. Expansion with new modules or other functions is also greatly simplified due to the choice for the open source Python platform and the understandable model structure. Another advantage above existing models is the user defined stringency of model simulation, due to the option to set the simulation time step.
2.2. Pump Sump (PS)

The pump sump is responsible for pumping the water up to the biofilter. In this tank, fresh water is used to keep the total volume of the RAS part of the system constant. The outflow of the pump sump ($F_{out}$) is set to maintain a constant flowrate through the RAS loop using a pump. The pump sump receives inflow from both the sedimentation tank ($F_{in}^{ST}(t)$) and the clean water tank ($F_{in}^{BF}(t)$), which is assumed part of the biofilter. When the two inflows are not sufficient to keep the pump sump tank completely filled, a fresh water input ($u_{H2O}(t)$) is used to compensate this water shortage.
\[
\frac{dV_{PS}}{dt}(t) = F_{in}^{ST}(t) + F_{in}^{BF}(t) + F_{out} + u_{H2O}(t)
\]

Since no reactions occur in the pump sump, the amount of each nutrient \((x)\) in the tank is only influenced by in- and outflow.

\[
\frac{dx}{dt}(t) = C_x^{in,ST}(t) * F_{in}^{ST}(t) + C_x^{in,BF}(t) * F_{in}^{BF}(t) + x(t) * \frac{F_{out}}{V_{PS}(t)}
\]

### 2.3. Biofilter (BF)

![Figure 4 - Map of the Biofilter. PS=Pump sump, BF=Biofilter, CW=clean water tank, RT=Rearing tank, ST=Sedimentation tank.](image)

A biofilter is needed to convert total ammonia nitrogen (TAN as \(\text{NH}_3\) and \(\text{NH}_4^+\)) to the rather harmless nitrate. After the biofilter, the water travels via the reception tank to the clean water tank, from where it is send partly back to the pump sump and partly to the rearing tanks. The water production rate \((r_{H2O}(t))\) is the result of the TAN conversion. The biofilter is assumed to have a constant volume \((\frac{dV_{BF}}{dt}(t) = 0)\), so that outflow \((F_{out}(t))\) can be calculated from the volume balance.

\[
\frac{dV_{BF}}{dt}(t) = F_{in}(t) + F_{out}(t) + r_{H2O}(t) = 0
\]

The TAN conversion is accomplished by nitrifying micro-organisms consuming TAN and \(O_2\) to produce nitrate, water and some microbial biomass (8) (10). The corresponding reaction equation is:

\[
\text{NH}_4^+ + 1.83 \text{O}_2 + 1.97 \text{HCO}_3^- \rightarrow 0.976 \text{NO}_3^- + 2.90 \text{H}_2\text{O} + 1.86 \text{CO}_2 + 0.0244 \text{C}_5\text{H}_7\text{O}_2\text{N}
\]

Looking at the nitrogen compounds in the equation reveals that there is slightly less nitrate production than ammonia consumption. However, the amount of nitrogen lost to biomass is neglected, as this is a significantly small percentage of total nitrate produced. The simplified equation then reads:

\[
\text{NH}_4^+ + 2 \text{O}_2 + 2 \text{HCO}_3^- \rightarrow \text{NO}_3^- + 3 \text{H}_2\text{O} + 2 \text{CO}_2
\]

Besides inflow and outflow of all nutrients \((x)\), there is conversion of TAN \((r_{TAN}(t))\) and production of \(\text{NO}_3^-\) \((r_{NO_3^-}(t))\) and optional addition of minerals to counter loss of alkalinity. All other nutrients are unaffected by any production or removal rate \((r_x(t))\).
\[
\frac{dx}{dt}(t) = c_{i}^{in}(t) * F_{in}(t) + x(t) * \frac{F_{out}(t)}{V_{BF}} + r_{x}(t)
\]  
(2.3.4.)

For TAN, NO₃ and water, the removal rates \(r_{TAN}(t), r_{NO₃}(t)\) and \(r_{H₂O}(t)\), respectively are given by:

\[
\begin{align*}
    r_{TAN}(t) &= -k_{BF}^{TAN} * A_{BF} * c_{TAN}^{in}(t) \\
    r_{NO₃}(t) &= -r_{TAN}(t) \\
    r_{H₂O}(t) &= -\eta_{H₂O}^{*} * r_{TAN}(t)
\end{align*}
\]
(2.3.5.)
(2.3.6.)
(2.3.7.)

Where \(A_{BF}\) is the total active surface area in the biofilter, \(k_{BF}^{TAN}\) the linearity constant of TAN removal and \(\eta_{H₂O}^{*}\) is the mass of water produced per TAN consumed (9).

The biofilter is assumed to be well aerated and thus not oxygen limited. Carbon dioxide is assumed to evaporate from the filter as soon as it is formed. The occurring reaction increases the pH, which is an unwanted side effect of the biofilter. The increased pH does not only affect fish and plant growth, but also changes the TAN ratio of \(NH₄⁺: NH₃\) in favour of unionized ammonia \((NH₃)\), which is very toxic for fish. In order to maintain a constant pH, the loss of alkalinity can be countered by addition of a pH control carbonate to the reactor. For traditional RAS systems, carbonate is usually added as \(NaHCO₃\) or \(CaCO₃\) (11). However, since sodium hinders plant nutrient uptake and growth, this is unsuitable for an aquaponic system. A mixture of \(CaCO₃, MgCO₃\) and \(KHCO₃\) is proposed for aquaponic systems, since these minerals are also taken up by plants and can thus be seen as supplementary fertilizer (12). For minerals \(Ca, Mg\) and \(K\), the rate of these minerals as pH control is described by:

\[
    r_{x}(t) = -\eta_{CO₃}^{x} * \eta_{N}^{x} * r_{TAN}(t)
\]
(2.3.8.)

Where \(y\) is \(Ca, Mg\) or \(K\); \(\eta_{CO₃}^{x}\) the fraction of fertilizer added in the form of \(xC₃\) \((CaCO₃, MgCO₃\) or \(K₂CO₃\)) and \(\eta_{N}^{x}\) the mass of \(x\) needed per mass of TAN converted. For the Abtshagen scenario, no change in pH was observed and therefore no minerals were added to counter loss of alkalinity \((r_{x}(t) = 0)\). The option to model mineral addition is implemented in the model, although \(\eta_{CO₃}^{y}\) is set to zero for all three minerals \((x)\).

### 2.4. Reception Tank (RE)

The reception tank is a small tank between the biofilter and the clean water tank. After the biofilter and reception tank, the water is now sufficiently cleaned to be fed to the rearing tanks. Before being send to the fish, it is collected in the clean water tank, where part of the water is send to the rearing tanks and part is send back to the pump sump to be cycled through the biofilter again. Since no reactions occur in either reception tank or clean water tank and since these tanks has a very small volume, it is assumed to not have a significant contribution concerning concentration changes and they are therefore not taken into account in the model. In order to take their volumes into account, the biofilter volume is considered the sum of biofilter, reception and clean water tank.
2.5. Rearing Tanks (RT)

The rearing tanks are the tanks in which the fish are grown. The currently modelled set up from PAL, Abtshagen contains four 1.84 m$^3$ rearing tanks, which receive a water inflow from the biofilter ($F_{in}(t)$). The volume in the rearing tanks is kept constant, except before fish are introduced and just before harvest. Before fish introduction the tanks are empty and just before harvest, the tanks are drained to the 3 chamber pit post purge ($F_{clean}(t)$), after which the fish are cleaned with two tank volumes of fresh water ($u_{H2O}(t)$) over two days. Before cleaning, the tanks are completely refilled with fresh water ($u_{H2O}(t)$) and further washed with a consecutive rearing tank volume worth of water over the remainder of these two days. The rearing tank outflow to the sedimentation tank ($F_{out}(t)$) is equal to the surplus of water overflowing from the tanks and thus does only occur when more water enters the tank than can be contained. The volume of fish is not taken into account when calculating rearing tank volume. The maximum volume of fish per rearing tank is only 7% of total tank volume. However, when simulating the system for duration longer than a year, this has no noticeable effect on water usage or flowrates. Each rearing tank is modelled separately, where the volume balance per tank is described as:

\[
\frac{dV_{RT}}{dt}(t) = F_{in}(t) + F_{out}(t) + F_{clean}(t) + r_{H2O}(t) + u_{H2O}(t)
\]  

(2.5.1)

The water consumption rate ($r_{H2O}(t)$) is caused by the water the fish take up during growth and based on the fraction of water in fish biomass ($\eta_X^{H2O}$):

\[
r_{H2O}(t) = -\eta_X^{H2O} \times r_X(t)
\]

(2.5.2)

For modelling purposes, fish are modelled as kg fish biomass, instead of individual fish number and weight. Assumed is that the fish in all rearing tanks behave the same and that substrates and environment are sufficient so as to not hinder growth. The only difference is that in each rearing tank, fish are introduced at a different time. The first rearing tank is filled immediately after commencing the aquaponic greenhouse; the second is stocked after $1/N_{RT}$ times the number of days it takes for the fish to grow to harvestable sizes, where $N_{RT}$ is the number of rearing tanks employed. This ensures stacked growth of the fish and levels out the concentrations of nutrients in
solution within the system, preventing concentrations peaks of e.g. TAN, which is toxic to fish in high levels.

The fish growth rate is estimated from growth measurements from the growth data from Coppens International (13). A detailed description of this estimation is described in section 3.1.

\[ r_X(t) = N_X \cdot (p_1^X \cdot t + p_2^X) \]  
(2.5.3.)

In order to ensure maximum growth of fish, sufficient feed (S) is needed. The feed input \((u_S(t))\) needed to support this growth is calculated from the fish feed consumption rate \((r_S(t))\), which in turn is based on the fish growth rate \((r_X(t))\) using a feed conversion ratio \((FCR(t))\).

\[ \frac{ds}{dt}(t) = r_S(t) + u_S(t) + u_{TSS}(t) \]  
(2.5.4.)

\[ r_S(t) = -FCR(t) \cdot r_X(t) \]  
(2.5.5.)

Where \(FCR(t)\) is defined as:

\[ FCR(t) = FCR_0 + \eta_{FCR} \cdot t \]  
(2.5.6.)

It is assumed that all feed is eaten under implementation of the chosen feed regime \(\frac{ds}{dt}(t) = 0, u_{TSS}(t) = 0\). When the provided feed input \((u_S(t))\) exceeds feed requirement \((r_S(t))\), surplus feed will be suspended in the tank and can therefore be treated as an increase in total suspended solids \((u_{TSS}(t))\). The feed consumption rate \((r_S(t))\) is determined by the feed conversion ratio \((FCR(t))\), defined as the amount of feed needed for the production of one unit of biomass, and the fish growth rate \((r_X(t))\). This ratio is assumed to increase linearly over time, from its initial value \((FCR_0)\) up to a maximum value \((FCR_{\text{max}})\) with a specified linearity constant \((\eta_{FCR})\), as described by Coppens International (13).

The nutrients present in feed end up not only in fish biomass, but also in total suspended solids and water, after being digested by the fish. The amount of nutrients \((x)\) present in the rearing tanks thus depends not only on in- and outflow \((C_X^{\text{in}}(t) \cdot F_{\text{in}}(t) \text{ and } C_X(t) \cdot F_{\text{out}}(t))\), but also on nutrient production rate \((r_X)\) caused by feed digestion. Total volatile solids \((TVS)\), total ammonia nitrogen \((TAN - N)\), phosphate \(P (PO_4^{3-} - P)\), \(Ca, Mg, K\) and \(Na\) are all excreted by fish in soluble form. Nutrient production rates depend on the fraction of nutrient in feed \((\eta_S^X)\) and the fraction of nutrient from feed ending up in the tank solution \((\eta_{XS}^{Xaq})\). Appendix A contains a list of nutrient fractions in feed and the distribution of feed nutrients in biomass, solid and soluble waste (14).

\[ \frac{dx}{dt}(t) = C_X^{\text{in}}(t) \cdot F_{\text{in}}(t) + C_X(t) \cdot F_{\text{out}}(t) + C_X(t) \cdot F_{\text{clean}}(t) + r_X(t) \]  
(2.5.7.)

\[ r_X(t) = -\eta_S^X \cdot \eta_{XS}^{Xaq} \cdot r_S(t) \]  
(2.5.8.)

Besides these soluble wastes, suspended wastes are excreted in the form of total suspended solids. The amount of TSS produced per kg of feed fed \((\eta_{TSS}^S)\) is assumed constant, resulting in a linear relation between feed rate and TSS production rate \((r_{TSS})\). Since excess feed is also considered as TSS, there is an additional TSS input rate \((u_{TSS})\).

\[ \frac{dTSS}{dt}(t) = C_{TSS}^{\text{in}}(t) \cdot F_{\text{in}}(t) + TSS(t) \cdot \frac{F_{\text{out}}(t)}{V_{RT}(t)} + TSS(t) \cdot \frac{F_{\text{clean}}(t)}{V_{RT}(t)} + r_{TSS}(t) + u_{TSS}(t) \]  
(2.5.9.)

\[ r_{TSS}(t) = -\eta_{TSS}^S \cdot r_S(t) \]  
(2.5.10.)

\[ u_{TSS}(t) = u_S(t) + r_S(t) \]  
(2.5.11.)
Both solids, TSS and TVS dissolve over time to release more ammonia nitrogen (TAN-N), phosphate $P$ ($PO_4^{3-}$-$P$), Ca, Mg, K and Na. However, the fraction of these nutrients in solids is limited. Another phenomenon happening is bio-flocculation, the uptake of solutes by bio-flocs, suspended microbial clusters. Besides that, dissolving of solids and bio-flocculation are both slow processes compared to the rate at which solids are removed from the system by the sedimentation tank. Therefore, dissolving and bio-flocculation are assumed negligible in this model.

### 2.6. Sedimentation Tank (ST)

![Figure 6 - Map of the Sedimentation Tank. PS=Pump sump, RT=Rearing tank, PP=3 chamber post purge pit.](image)

The sedimentation tank retains most suspended solids and some volatile solids present in the system. At least once every $t_{ST}^{min}$ days and at most once every $t_{ST}^{max}$ days, depending on the water demand in the greenhouse, the sedimentation tank is decoupled from the RAS loop and drained to the 3 chamber post purge pit ($F_{clean}(t)$). The actual interval of cleaning the tank is based on water level in the post purge. As soon as the post purge volume gets below a certain level ($u_{VP}$), the sedimentation tank is drained to the post purge to prevent water shortages in post purge and consecutive mixing tank and greenhouse NFT channels, within the limits $t_{ST}^{min}$ and $t_{ST}^{max}$. After draining, the tank is reconnected and is filled up again by the rearing tanks outflow ($F_{out}(t)$). When full, constant volume is maintained by an outflow that takes excess water to the pump sump ($F_{out}(t)$).

$$\frac{dV_{ST}}{dt}(t) = F_{in}(t) + F_{out}(t) + F_{clean}(t) \quad (2.6.1.)$$

The suspended and volatile solids ($x$) are retained in the tank by a retention rate ($r_x(t)$), which removes these solids from solution and makes them settle as settled solids (TSS(s)). These settled solids get resuspended in the cleaning outflow to the 3 chamber post purge pit ($F_{clean}(t)$) following a specific resuspension rate ($r_{TSS(s)}(t)$):

$$\frac{dx}{dt}(t) = c_x^{in}(t) * F_{in}(t) + x(t) * \frac{F_{out}(t)}{V_{ST}(t)} + Q_x^{clean}(t) + r_x(t) \quad (2.6.2.)$$
The TSS and TVS settling rates \(r_x(t)\) and TSS\((s)\) resuspension rate \(r_{TSS(s)}(t)\) are defined as:

\[
r_x(t) = -\mu_x \cdot x(t)
\]

Where \(\mu_x\) is the specific removal rate of each individual component.

The non-solid nutrients \((z)\) present in the system do not undergo any conversions and can be described by the following mass balance:

\[
\frac{dz}{dt}(t) = C_z^{in}(t) \cdot F_{in}(t) + z(t) \cdot \frac{F_{out}(t)}{V_{ST}(t)} + z(t) \cdot \frac{F_{clean}(t)}{V_{ST}(t)}
\]

### 2.7. 3 Chamber post purge pit (PP)

![Diagram of 3 Chamber Post Purge Pit](image)

When the sedimentation tank is cleaned, it is drained to the 3 chamber post purge pit, where solids are permanently removed from the system. The post purge is the link between the RAS and greenhouse part of the aquaponic system. Water outflow from the post purge \((F_{out}(t))\) is based on the water demand in the mixing tank. When the water requirement in the greenhouse is low in between planting seasons, water accumulates in the post purge. Excess water that does not fit in the post purge is discharged \((F_{dis}(t))\).

\[
\frac{dV_{PP}}{dt}(t) = F_{in}(t) + F_{out}(t) + F_{dis}(t)
\]

Since no other reactions are assumed to occur in the post purge, the nutrient mass balances depends on in- and outflow \((C_x^{in}(t) \cdot F_{in}(t)\) and \(C_x(t) \cdot F_{out}(t))\), discharge \((C_x(t) \cdot F_{dis}(t))\) and solid removal rates \((r_x(t))\).

\[
\frac{dx}{dt}(t) = C_x^{in}(t) \cdot F_{in}(t) + x(t) \cdot \frac{F_{out}(t)}{V_{PP}(t)} + x(t) \cdot \frac{F_{dis}(t)}{V_{PP}(t)} + r_x(t)
\]

Where the removal rate \(r_x(t)\) only applies to TSS and TVS, following the rate equation:

\[
r_x(t) = -\mu_x \cdot x(t)
\]
2.8. Mixing Tank (MT)

The mixing tank is a large storage tank able to hold water that has passed through the post purge ($F_{in}(t)$). From the mixing tank, water is fed to a mixing station, where small volumes of fertilizer mixture can be added to the water to bring it to the right EC and nutrient concentrations before feeding it to the NFT channels. The volume increase due to fertilizer addition is neglected, as up to two litres of water is added to the NFT channels up to once a week, which is negligible compared to plant water uptake (up to 480 L/d), channel volume (300 L), daily flowrates (360 m$^3$/d) and total water use (over 300 m$^3$/year).

Both storage tank and mixing station are modelled as one large mixing tank, due to the relatively small volume of the mixing station. The outflow to NFT channels ($F_{out}(t)$) is set to automatically refill the channels when their volume drops. When the mixing tank volume is insufficient to maintain the required outflow size, fresh water ($u_{H_2O}(t)$) is added to supplement the outflow.

$$\frac{dV_{MT}(t)}{dt} = F_{in}(t) + F_{out}(t) + u_{H_2O}(t)$$ (2.8.1.)

Nutrient shortages are compensated by adding fertilizer ($r_x(t)$). In reality, various combinations of salt solutions (like $KH_2PO_4$, $Ca(NO_3)_2$) are added to rectify nutrient shortages. However, the model assumes loose ions (like $K^+$, $NO_3^-$) are added. This is done to simplify the model in order to ensure its fast performance. Since normally salts are added in such combinations that all nutrient ions are provided in wanted concentrations, modelling their individual addition results in the same desired mixture, with the single downside of not knowing which combination of salts would be most wise to use in order to obtain the required concentrations. The fertilizer addition in the mixing station, modelled as fertilizer added to the mixing tank outflow, can be described by the following nutrient mass balances:

$$\frac{dx(t)}{dt} = C_x^{in}(t) \cdot F_{in}(t) + Q_x^{out}(t)$$ (2.8.2.)

$$Q_x^{out}(t) = x(t) \cdot \frac{F_{out}(t)}{V_{MT}(t)} + r_x(t)$$ (2.8.3.)

$$r_x(t) = (C_x^{rec} - C_x^{HP}) \cdot V_{HP}(t)$$ (2.8.4.)

Where $r_x(t)$ is only used for the fertilizer components $NO_3^-$, $PO_4^-$, $Ca^{2+}$, Mg$^{2+}$ and $K^+$. Assumed is that nitrogen is only added as $NO_3^-$, since the amount of TAN added as fertilizer is negligible compared to the amount of nitrate added to aquaponic greenhouses and since plants do not show
significant favouring of either nitrogen compound under aquaponics relevant concentrations \((1)\). The amount of fertilizer added is based on the difference between greenhouse nutrient concentration \((C_{x}^{HP}(t))\) and recommended nutrient concentration \((C_{x}^{rec})\). The outflow is supplemented with fertilizer until the outflow contains the recommended amount of nutrient to reach the recommended nutrient concentration \((C_{x}^{rec})\), as proposed by Lattauschke \((15)\).

2.9. **Hydroponic NFT channels (HP)**

![Figure 9 – Map of the Hydroponic NFT channels in the greenhouse. MT=Mixing Tank, HP=hydroponic NFT channels.](image)

The hydroponics module describes the dynamics of the nutrient film technique (NFT) channels in the greenhouse. From these channels water is taken up by the tomato plants \((r_{H_{2}O}(t))\) planted in the channels. All channels are assumed ideally mixed, since a pump cycles water through the channels in less than one minute. The channels are kept at constant volume by an inflow from the mixing tank \((F_{in}(t))\) and the channels are only emptied after the growing season and filled back up when a new stock is planted \((F_{dis}(t))\). Besides these flows, no water is removed from the channels. Condensed water is represented as an outflow from the greenhouse to the fresh water tank in the RAS part of the system \((F_{out}(t))\). Since the condensed water is obtained from the air and not the NFT channels, this flow is not included in the NFT volume balance. Since the amount of condensed water collected is negligible compared to the total water requirement for the aquaponic system, it is set to zero in the model \((1)\).

\[
\frac{dV_{HP}}{dt}(t) = F_{in}(t) + F_{dis}(t) + r_{H_{2}O}(t) \quad (2.9.1.)
\]

Growth of the tomato plants is split between fruit \((r_{X_{f}}(t))\) and non-fruit growth \((r_{X_{nf}}(t))\). Since extensive data was available on fruit growth, a polynomial equation could be used model plant growth \((6)\). No measurements on non-fruit tomato plant were available. However, since the final non-fruit weight \((X_{nf}^{max})\) was present, a logistic model was assumed.

Water uptake in the NFT channels was measured weekly. Together with plant (fruit and non-fruit) growth models, these measurements were used to create a polynomial plant water uptake model. Fruit and non-fruit growth and water uptake models and their parameter estimates are described in sections 3.2 to 3.4, respectively.

Apart from TSS and TVS, the plants take up all nutrients in the water. No distinction between TAN and NO\(_{3}\) absorption is assumed, meaning the tomatoes take up both to satisfy their nitrogen demand. Three nutrient uptake rates \((r_{x}(t))\) were considered. The corresponding equations and the estimated values of related parameters are discussed in section 3.5.

\[
\frac{dx}{dt}(t) = c_{x}^{in}(t) * F_{in}(t) + x(t) * \frac{F_{dis}(t)}{V_{HP}(t)} + r_{x}(t) \quad (2.9.2.)
\]
2.10. Fresh Water Tank (TW)

The fresh water tank is a large fresh water storage tank. The tank is filled with groundwater and condensed water coming from the greenhouse. Since the volume of condensed water is negligible when not mechanically condensing greenhouse air, this input is set to zero. When other tanks require a fresh water input ($u_{H_2O}(t)$), the required water is taken from this large storage tank and send to those modules. Since no other processes take place in this tank, only the total water requirement of the aquaponic system ($u_{H_2O}^{total}(t)$) is implemented in the model.

$$u_{H_2O}^{total}(t) = u_{H_2O}^{PS}(t) + u_{H_2O}^{RT}(t) + u_{H_2O}^{MT}(t)$$

(2.10.1.)
3. Model Parameter Assessment

Chapter 3 details the models used for fish growth, tomato fruit and non-fruit growth and tomato water uptake and details the estimation of parameters in these models. Next, parameter sensitivity analysis regarding water usage is performed in order to identify the parameters which have big impact on total water usage and therefore are the subject of parameter optimization.

3.1. Fish Growth Model

Since no measurements on fish growth were available from PAL, tilapia growth and FCR data obtained from Coppens, was used instead (13). The fish biomass ($X(t)$) model implemented is a polynomial model, a second order polynomial chosen for its low complexity and good fit to data, as seen in Figure 10 (2). Also considered was an exponential model. However, when fitting initial biomass ($X_{stock}$) and final biomass per fish to an exponential model, the growth rate is first underestimated until the very last days of growth, when the growth is significantly overestimated. The inaccurate growth pattern of an exponential equation results in a low fit, as seen when comparing the root-mean-square deviation of the exponential model (0.104) compared to the polynomial model (0.0112). The total fish growth is modelled as a function of time ($t$), number of fish ($N_X$), initial weight per fish ($X_{stock}$) and two fish growth second order polynomial parameters ($p_1^X$ and $p_2^X$).

$$X(t) = N_X * (p_1^X * t^2 + p_2^X * t + X_{stock})$$  \hspace{1cm} (3.1.1.)

$p_1^X$ and $p_2^X$ were obtained by fitting the model to the average weight gain of tilapia. The fitting was performed by minimizing the difference between daily data on average tilapia weight over 173 days and the suggested polynomial model, under the restriction that the average fish weight at the final day should match measurement should be the same ($X^\star(t_f) = X^\star(t_f)$). Since the growth is assumed optimal and is thus not modelled as being dependent on environmental factors, optimizing these parameters can be achieved by investigating only the growth model (without the number of fish term $N_X$), without having to run the entire aquaponic model. The fit of both models, including root-mean-square deviations (RMSE) are shown in Figure 10.
Figure 10 - Average mass per fish, data by Coppens versus exponential (exp) and polynomial (poly) growth models (13).

As can be seen in Figure 10, the polynomial model shows a significantly better fit to the data than the exponential model. This is also reflected by the RMSE of the polynomial model, which is 10 times lower than the RMSE of the exponential model.

3.2. Tomato Fruit Growth Model

Growth of the tomato plants is split between fruit \( r_{X_f}(t) \) and non-fruit growth \( r_{X_{nf}}(t) \). Since extensive data was available on fruit growth, a polynomial model could be used to describe the amount of fruit collected. Other models and simplified versions thereof were considered (16). However, these models are either not suitable for tomato, tackle whole plant growth instead of fruit-only or require environmental factors like temperature, irradiation strength and humidity, which were not measured at the Abtshagen site. However, the model is based on the number of plants \( N_X \) and fruit yield per plant \( Y_{X_f} \):

\[
X_f(t) = N_X \times Y_{X_f} \times \left( p_1^{X_f} \times \left( t - t_0^{X_f} \right)^2 + p_2^{X_f} \times \left( t - t_0^{X_f} \right) \right)
\]

(3.2.1)

\[
r_{X_f}(t) = \frac{dX_f}{dt}(t)
\]

(3.2.2)

The two fruit growth second order polynomial parameters \( p_1^{X_f} \) and \( p_2^{X_f} \) were obtained by fitting the model (without number of plants \( N_X \)) to the average fruit harvest per plant. The measurements were obtained from the aquaponic greenhouse in Abtshagen, from tomato plants fed with fish water supplemented with fertilizer up to an EC value of 3.0 mS/cm (6).
These parameters were then divided by the total fruit yield per plant in order to be able to model different yield per plant, which might be depending on yearly weather, greenhouse size and different planting densities.

### 3.3. Tomato non-fruit Growth Model

Since no measurements of non-fruit growth were performed, logistic growth was assumed (17). Modelling non-fruit does need to be taken into account, since the uptake of water and nutrients (purely based on non-fruit elemental composition and fresh to dry weight ratio) represents a significant part of total water and nutrient uptake. A logistic growth rate \( r_{X_{nf}}(t) \) was chosen due to its simplistic nature, requiring only few parameters, since no measurements of non-fruit growth were available and literature on the subject is scarce.

\[
X_{nf}(t) = N_X \cdot \frac{x_{nf}^{min} \cdot x_{nf}^{max} \cdot e^{\mu_{X_{nf}} \cdot (t-t_0^X)}}{x_{nf}^{max} \cdot x_{nf}^{min} \cdot \left( e^{\mu_{X_{nf}} \cdot (t-t_0^X)} - 1 \right)} \tag{3.3.1}
\]

\[
r_{X_{nf}}(t) = \frac{dX_{nf}(t)}{dt} \tag{3.3.2}
\]

Where \( x_{nf}^{min} \) and \( x_{nf}^{max} \) are minimum and maximum weight of non-fruit from one plant, \( \mu_{X_{nf}} \) the specific logistic growth rate of the non-fruit part of the plant and \( t_0^X \) the planting date of the tomato stock in the NFT channels.

### 3.4. Plant Water Uptake Model

Since fruit and non-fruit growth models together with water uptake measurements are available, a plant growth dependant water uptake model can be derived. The implemented model is also a polynomial model based on measured water uptake per plant, taking into account maximum non-fruit weight \( X_{nf}^{max} \) and fruit yield per plant \( Y_X \). Models from literature were also considered. However, these are often dependant on environmental factors or leaf area index, are based on whole plant growth or too complex and taxing for model performance (18) (19).
\[
\sum_{t_i=0}^{tf} r_{H_2O} = -N_X \cdot (Y_{X_f} + X_{nf}^{max}) \cdot (p_1 r_{H_2O} (t_i - t_0)^3 + p_2 r_{H_2O} (t_i - t_0)^2 + p_3 r_{H_2O} (t_i - t_0))
\]

(3.4.1.)

The cumulative plant water uptake \(\sum_{t_i=0}^{tf} r_{H_2O}\) is described using the planting date \((t_0)\) and three uptake parameters \(p_1 r_{H_2O}\), \(p_2 r_{H_2O}\) and \(p_3 r_{H_2O}\). The volume of the NFT channels in the Abtshagen greenhouse were kept constant, meaning that all water added reflects the water taken up by the plants. The model (without number of plants, \(N_X\)) was fitted to the cumulative amount of water fed to the tomato plants (6). Again, since optimal plant growth is assumed, environmental factors do not affect the modelled water uptake, allowing fitting of data to the plant water uptake model alone, instead of the entire aquaponic model.

Figure 12 - Water uptake per plant data by PAL (Water Uptake) versus polynomial water uptake model (Ploy. (water uptake))

These parameters were then divided by fruit and non-fruit yield per plant \((Y_{X_f} + X_{nf}^{max})\) in order to make the model adaptable to different plant yields and planting densities.

### 3.5. Plant Nutrient Absorption Model

When water shortages arise in the NFT channels at the Abtshagen site, water from the RAS loop is introduced to supplement the channels. The EC of the channels is measured online. When RAS water alone is not sufficiently rich in nutrients to maintain a constant EC, additional fertilizer is introduced to keep the EC constant. The ratio in which nutrients are added relative to each other, is chosen so that not a single nutrient accumulates to surpluses or decreases to shortages. In reality this is difficult, since extensive knowledge on ground water composition and time varying fish water nutrient composition is required in order to find the perfect ratio of fertilizer nutrients relative to each other. Assuming that this ratio is indeed ideal would result in the assumption that all nutrients added to the NFT channels, RAS water and fertilizer both, are taken up by the plants. Assuming the elemental composition of fruit \((\eta_{X_f})\) and non-fruit \((\eta_{X_{nf}})\) is known and constant throughout growth, the nutrient uptake rate can be estimated as having a linear connection with the growth rate of fruit \((r_{X_f}(t))\) and non-fruit \((r_{X_{nf}}(t))\):
\[ r_x(t) = -\eta_x^X f * r_x^f(t) - \eta_x^X n f * r_x^n f(t) \] (3.5.1.)

Some literature data is available on the elemental composition of tomato fruits \( (\eta_x^X f) \), making it possible to estimate growth related nutrient uptake based on fruit composition (20). However, for non-fruit elemental composition \( (\eta_x^X n f) \), literature is scarce. Assuming that the non-fruit composition is equal to that of fruit \( (\eta_x^X n f = \eta_x^X f) \) makes it possible to estimate nutrient uptake rate based on literature:

\[ r_x^{lit}(t) = -\eta_x^X f * \left( r_x^f(t) + r_x^n f(t) \right) \] (3.5.2.)

No measurements on elemental composition of tomato plants were performed at the test system in Abtshagen. However, fertilizer addition, RAS water addition and RAS water concentrations were measured weekly, meaning that the amount of nutrients delivered to the NFT channels can be calculated. Using parameter optimization and the calculated nutrient uptake, elemental composition of tomato \( (\eta_x^X f) \) can be estimated, assuming again equal fruit and non-fruit composition. The equation to estimate growth-based nutrient uptake rate looks similar to the literature uptake rate:

\[ r_x^{growth}(t) = -\eta_x^X f,est * \left( r_x^f(t) + r_x^n f(t) \right) \] (3.5.3.)

This is true for all nutrients, except TSS, TVS and Na. These three are provided in excess to the plants, since the solids TSS and TVS are not taken up and only very slight amounts of Na are absorbed, resulting in ever increasing concentrations of these nutrients. The sodium absorption is modelled as being linearly related to plant growth using a fruit sodium composition from literature, since no significant active sodium uptake is expected (20). This results in a sodium uptake rate significantly smaller than the sodium inflow in the NFT channels. Although this rate is negligible compared to actual sodium concentration in the NFTs, it is implemented taking into account its more relevant contribution in other scenarios.

The disadvantage of assuming a linear connection between growth and nutrient uptake, is that fruit composition can vary depending on environmental factors, like solar radiation, root temperature and water availability and salinity (21) (22). A third option is to model nutrient uptake based solely on passive uptake, meaning that the uptake of nutrients is solely related to the water uptake rate \( (r_{H2O}(t)) \) of the plants:

\[ r_x^{water}(t) = \eta_x^{H2O} * r_{H2O}(t) \] (3.5.4.)

In order to estimate the nutrient uptake, all three options were investigated. The measured nutrient uptake was compared to the growth-related nutrient uptake model based on elemental composition from literature \( (r_x^{lit}(t), \text{literature model}) \), growth-related nutrient uptake model based on estimated elemental composition, \( (r_x^{growth}(t), \text{growth model}) \) and the water uptake related nutrient uptake model \( (r_x^{water}(t), \text{water model}) \). Figure 13 show the calculated uptake based on measurements and the three models described above.
The estimated tomato composition and the composition found in literature are described in Table 1. When comparing the two growth-based nutrient uptake models (literature and estimated), it becomes clear that the estimated uptake is significantly higher for all nutrients. Especially the estimated calcium composition of tomato is significantly higher than previously reported values, taking into question the assumption of constant calcium concentration in the NFT channels. Most likely not all calcium is taken up and the calcium fertilizer rate is overestimated in the Abtshagen system. Looking at the groundwater composition in Abtshagen reveals a very high calcium concentration of around 390 mg/L, which is already significantly higher than the recommended concentration of 128.3 mg/L (15) (23). Knowing this, additional calcium fertilizer will be redundant, since the RAS water already contains sufficient calcium. Most likely this is also the case for magnesium, since hard water often contains high levels of both calcium and magnesium.
Table 1 – Elemental composition of tomato fruit ($\eta_{x}^{y}$), literature values compared to model estimates regarding nutrient uptake coupled to elemental fruit composition (20). Relative difference is calculated by first calculating the difference between model estimates and literature values and then dividing by the literature values.

<table>
<thead>
<tr>
<th>$\eta_{x}^{y}$ in kg/m$^3$</th>
<th>Element</th>
<th>Literature</th>
<th>Model Estimates</th>
<th>Relative Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>0.001398</td>
<td>0.00285</td>
<td></td>
<td>104%</td>
</tr>
<tr>
<td>P</td>
<td>0.000264</td>
<td>0.000632</td>
<td></td>
<td>139%</td>
</tr>
<tr>
<td>Ca</td>
<td>0.0000981</td>
<td>0.00271</td>
<td></td>
<td>2662%</td>
</tr>
<tr>
<td>Mg</td>
<td>0.0000971</td>
<td>0.000876</td>
<td></td>
<td>802%</td>
</tr>
<tr>
<td>K</td>
<td>0.00253</td>
<td>0.00522</td>
<td></td>
<td>106%</td>
</tr>
<tr>
<td>Na</td>
<td>0.0000521</td>
<td>-</td>
<td></td>
<td>-</td>
</tr>
</tbody>
</table>

The estimated tomato composition regarding the other nutrients (N, P and K) reveals values about twice as high as literature compositions. The differences, although significant, might be explained by one or more of the following factors:

1. Occasional water and thus nutrient leaking, settling or discharge. When water is taken from the greenhouse, the concentration in the NFT channels is altered drastically and significant nutrients are lost, making it difficult to estimate uptake based on the amount of nutrients fed to the NFT channels.

2. Another interesting factor that might explain the higher compositions found, is the fact that low nutrient fresh water was added to the NFT channels when planting the tomatoes, while NFT concentrations at harvest were significantly higher due to the RAS water and fertilizer added during the growing of the crops. This difference between initial and final concentration has not been taken into account when calculating the total amount of nutrients provided to the NFT channels, since no measurements exist of initial and final concentrations.

3. The assumption that non-fruit composition equals that of fruit also contributes to this problem, since there most likely is less water and more nutrients in non-fruit (24) (25).

4. Yet another factor influencing the difference might be the result of stressed, sub-optimal conditions for growth, since only around 35 kg/m$^2$ fruits were obtained compared to the reported 50+ kg/m$^2$ from other purely hydroponic systems (26). Assuming a more suitable fertilizer composition is applied in the future, in combination with better greenhouse regulation should result in higher yields and fruit compositions closer to other reported values.

The three suggested nutrient uptake models are all implemented, so that different options can be selected, depending on the simulated scenario. However, in order to try and model the true nutrient uptake in plants, the literature based, growth coupled tomato nutrient uptake is used, since the estimation data deviates too drastically from previous reported findings. In reality adding small excesses of fertilizer might be recommended to ensure prevention of nutrient shortages. However, this does not mean that the actual uptake rate of nutrients is also larger.
3.6. Water management parameter sensitivity analysis

By knowing the key parameters in water usage, optimal values for said parameters can be identified, in order to obtain knowledge on better system management and future system design. The parameter sensitivity regarding water usage was investigated for multiple parameters. Parameter sensitivity was defined as:

\[ S_p = \frac{Y(1.1+\bar{p}) - Y(0.9+\bar{p})}{Y(\bar{p})} \]  

(3.6.1.)

Where \( p \) is the parameter being investigated, \( \bar{p} \) the reference parameter value and \( S_p \) this parameters sensitivity regarding water use \( Y \) when deviating its initially assumed value with 10% in both directions. Water usage \( Y \) is either yearly total fresh water requirement, yearly mixing tank fresh water requirement or yearly pump sump fresh water requirement. Parameters representing physical properties, such as water density and molar mass of water, were not taken into account, due to being assumed accurate already. The parameters determining planting density, greenhouse area and number of NFT channels in the greenhouse were not investigated, since these parameters are all used to calculate the total number of plants, which is a parameter that is included in the sensitivity analysis. Water discharge and rearing tank fresh water demand were also investigated. However, a small change in parameters had no significant impact on the resulting discharge or fresh water demand. Water discharge only occurs when the post purge overflows and at the end of the tomato cultivation season the NFT channels are discharged. Since good system management The only parameter directly influencing discharge is the switch determining the lower limit of post purge volume before water is requested from the sedimentation tank. The default value is 6%. By increasing this value at least tenfold, the tank would always be filled more. During the period that no crops are present in the greenhouse and thus no water is needed there, the post purge can overflow causing more discharge. However, a big change in this switch parameter is needed before more discharge is realised.

The rearing tank fish are cleaned with fresh water before harvest. The only variable significantly impacting the rearing tank fresh water demand is \( V_{clean}^{RT} \), defined as the total volume of water used to clean the fish. Normally, the fish are cleaned with a volume of water equal to two times the rearing tank volume. However, since two tank volumes is assumed ideal and since the impact on total water use is insignificant for small changes in \( V_{clean}^{RT} \), this parameter was not considered a key parameter in total water usage. The parameters that may significantly impact water usage are shown in Table 2.

Table 2 - Parameter sensitivity \( S_p \) regarding yearly fresh water requirement of the entire aquaponic system (\( u_{H2O}^{total} \), default value 361 m³), mixing tank yearly fresh water requirement (\( u_{H2O}^{MT} \), default value 0.18 m³) and pump sump yearly water requirement (\( u_{H2O}^{PS} \), default value 335 m³).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
<th>( S_p(u_{H2O}^{total}) )</th>
<th>( S_p(u_{H2O}^{MT}) )</th>
<th>( S_p(u_{H2O}^{PS}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_X )</td>
<td>192</td>
<td>-</td>
<td>19%</td>
<td>1186%</td>
<td>19%</td>
</tr>
<tr>
<td>( Y_{Xf} )</td>
<td>22.72</td>
<td>kg</td>
<td>16%</td>
<td>901%</td>
<td>16%</td>
</tr>
<tr>
<td>( t_{ff} )</td>
<td>339</td>
<td>d</td>
<td>1%</td>
<td>0%</td>
<td>1%</td>
</tr>
<tr>
<td>( p_1^{H2O} )</td>
<td>-1.46E-09</td>
<td>m³/kg/d³</td>
<td>-35%</td>
<td>-1357%</td>
<td>-37%</td>
</tr>
<tr>
<td>( p_2^{H2O} )</td>
<td>6.75E-07</td>
<td>m³/kg/d²</td>
<td>51%</td>
<td>4927%</td>
<td>53%</td>
</tr>
<tr>
<td>( p_3^{H2O} )</td>
<td>-7.25E-06</td>
<td>m³/kg/d</td>
<td>-3%</td>
<td>0%</td>
<td>-3%</td>
</tr>
<tr>
<td>( t_{max}^{ST} )</td>
<td>3</td>
<td>d</td>
<td>-1%</td>
<td>793%</td>
<td>-1%</td>
</tr>
</tbody>
</table>
Assuming that no control can be exerted over the plant harvest rate \( t_f^{HP} \) (which depends on weather and outside temperature), plant yield \( Y_{X_f} \) and plant water uptake \( (p_1^{H_2O}, p_2^{H_2O} \text{ and } p_3^{H_2O}) \), leaves total number of plants \( N_X \) as a system design parameters to investigate. Another interesting parameter impacting total water use is the sedimentation tank cleaning regime. The sedimentation tank is emptied at least once a week \( (t_{ST}^{min}) \), to prevent nutrient accumulation in the RAS loop, and up to once every \( t_{ST}^{max} \) days, depending on water demand in post purge, which in turn is based on water demand in mixing tank and thus plant water demand in the NFT channels. Higher values of \( t_{ST}^{max} \) might be insufficient to meet this water demand in the NFT channels, resulting in the need of additional fresh water in the mixing tank to prevent water shortage in the greenhouse. Lower values might mean intensive management of sedimentation tank cleaning.

### 3.7. Parameter Calibration

The dominant parameters concerning water usage, \( N_X, t_0^{HP}, t_f^{HP}, Y_{X_f}, p_1^{H_2O}, p_2^{H_2O} \text{ and } p_3^{H_2O} \), are all related to plant growth or water uptake, since the vast majority of water uptake can be contributed to the plants. Assuming a yield of 50 kg/m\(^2\) as is easily achieved in traditional, commercial hydroponic tomato culture, leaves total plant number and sedimentation tank cleaning regime as variables to be investigated. Knowing the ideal number of plants per number of fish is an important design feature to be taken into account for future aquaponic system design, while the ideal sedimentation cleaning regime might help improve current Abtshagen system management.

An ideal fish to plant ratio does not exist, since this depends on which of multiple criteria are seen as most important in determining the ideal ratio. Focus of this study are water use and fertilizer (especially nitrogen) needed. However, both of these goals are not only dependent on the ratio between fish and plants, but also on the regime of sedimentation tank cleaning.

The more often the sedimentation tank is cleaned, the more water is send to the greenhouse. However, when the tank is cleaned more often, more discharge will occur when the post purge tank receives more water from the sedimentation tank than it can contain. In the following, it is assumed that the sedimentation tank is emptied to the post purge at least once a week, in order to ensure that solids and nutrients do not accumulate to dangerous levels for the fish in the RAS loop and to ensure that the sedimentation tank does not clog with solids. If this frequency does not cover the water demand of the greenhouse, the tank can be emptied more often, up to a set interval \( t_{ST}^{max} \) (t_clean_ST_max in Figure 14), allowing tank emptying at least once a week and up to once per \( t_{ST}^{max} \) days, depending on water demand in the greenhouse. In order to investigate the ideal ratio between fish and plant, this sedimentation tank emptying limit and the number of plants have been optimized regarding minimum water usage and minimum nitrogen fertilizer addition, for four rearing tanks of 1.84 m\(^3\) with 175 fish per tank. Comparing the resulting mixing tank fresh water requirement and amount of RAS nitrogen send to the greenhouse under varying both number of plants and interval \( t_{ST}^{max} \) enables estimating the ratio between fish and plant and the ideal sedimentation tank cleaning regime.
Figure 14 - Mixing Tank fresh water requirement over three years based on total number of plants in the greenhouse and sedimentation tank cleaning regime. Top: 14a, Contour plot. Bottom: 14b, Surface plot.

Allowing sedimentation tank cleaning on a daily basis will always meet water demand for up to 550 plants. When allowing only cleaning up to once per three days, only up to 200 plants can be maintained. When more plants are present, water is not fed to the post purge fast enough to meet plant water demand, resulting in a fresh water requirement in the mixing tank to keep the NFT channels filled. In contrast to the other curves in Figure 14a, the graph representing the lowest water requirement (the ‘2 m$^3$’-line) does not seem smooth. When planting the tomatoes, all then empty NFT channels need to be filled completely. A small volume of fresh water is needed to complement the available RAS water at this planting date. Therefore, the fresh water requirement in the mixing
tank is never completely zero. Its irregular character is the result of fluctuations in the mixing tank volume, which the water demand is very sensitive to at the planting date for these low water requirements. This evens out to smooth graphs at higher mixing tank water requirements.

As expected, limiting the number of times the sedimentation tank is emptied, results in a higher water demand in the mixing station. When the sedimentation tank is not emptied as a result of water shortages in post purge and mixing tank, additional fresh water is needed to meet plant water demand. When composing an ideal sedimentation tank cleaning regime optimized to plant water demand, the lowest water demand graph (‘2’-line) seen in Figure 14 reveals at which interval the tank should be cleaned in order to meet plant water demand at all times. For the Abtshagen system with a sedimentation tank of 1.32 m³, for 192 plants the tank should be cleaned once per 3 days in order to meet plant water demand. Cleaning less often is definitely possible, but does result in an additional fresh water requirement in the mixing tank in order to ensure the NFT channels are kept at constant volume.
This cleaning regime is independent of fish number, since nutrients are not taken into account in these calculations. Apparently the effect of the sedimentation tank cleaning regime on fertilizer requirement is very minor, as seen in Figure 15. When the sedimentation tank is cleaned very often, only low concentrations of nitrogen will accumulate in the RAS part of the system, even though this low concentration RAS water is send to the greenhouse very often. At higher values for $t_{ST}^{max}$, the concentration in the RAS loop is significantly higher, but this water is send to the greenhouse less
often, where it is diluted with added fresh water, as seen in Figure 14. For a constant number of plants, the sedimentation tank cleaning regime is therefore of limited impact on the total nitrogen fertilizer need.

At lower plant numbers, a surplus of RAS nitrogen is fed to the greenhouse, as will be described in section 4. This surplus of RAS nitrogen means that less nitrogen fertilizer is required to meet plant nitrogen demand. This can be seen in Figure 15b, as indicated with the red dots. The low amount of fertilizer needed for the current greenhouse size of 192 plants (and plant numbers smaller than indicated by the red dots in Figure 15b), is used at the planting date, to ensure initial concentrations in the NFT channels are optimal for plant growth. At later stages of plant growth, no additional fertilizer is needed.

The current Abtshagen greenhouse can contain up to 192 plants, which, if fed with 3.0 EC RAS water solution, assuming literature nutrient uptake rates of tomato, literature fish excretion rates and fish growth equal to the growth observed at Coppens, would provide a surplus of nitrogen from the RAS alone, meaning further nitrogen fertilizer would be redundant (13). Weekly cleaning of the sedimentation tank would suffice to meet water demand and nitrogen demand. However, monitoring of TAN and solid levels in the RAS loop will be important in order to prevent TAN and solid levels toxic to fish and to prevent clogging of the sedimentation tank. Purely looking at plant growth, weekly cleaning of the sedimentation tank suffices. However, taking into account nutrient accumulation in the RAS loop, more regular cleaning might be required.

When looking at ideal ratio of fish to plant, assuming the RAS system is identical to that of Abtshagen, would mean having double the number of plants in order to ensure all RAS nutrients are taken up by the plants. When minimizing fertilizer use, under twice as many plants (384) per four 1.84 m³ rearing tanks containing 175 fish each (23.7 m2 greenhouse per 1 m3 rearing tank), is more ideal, as will be illustrated in section 4. A double greenhouse size means that there no longer is a surplus of RAS nitrogen send to the plants, while still only very limited amounts of nitrogen fertilizer are required. However, taking into account accumulation of toxic components in the RAS loop might require more stringent sedimentation tank cleaning, resulting in more available water and thus the capacity to water more plants. Maximizing number of plants that can be watered with this four tank RAS system, would depend on the highest acceptable sedimentation tank cleaning interval $t_{ST\text{max}}$. For example, when cleaning the tank up to once per day would provide enough water for about 545 plants (33.7 m2 greenhouse per m3 rearing tank), as can be deduced from the near zero water requirement for around 530 plants with a smallest allowable sedimentation tank cleaning interval $t_{ST\text{max}}$ of one day in Figure 14b (’2‘-line). However, this scenario does require more intensive fertilizer addition.
4. Simulation Results and Discussion
As can be deduced from Figure 16, for the current Abtshagen greenhouse size of 192 plants, cleaning the sedimentation tank at most once per three days is sufficient to ensure sufficient water is sent to the plants. To check this, cumulative water input in pump sump, rearing tank and mixing tank are depicted in Figure 16. Since no water input in the mixing tank is needed, the chosen stringency of sedimentation tank cleaning is sufficient to meet water demand.

4.1. Water Discharge
The current system also does not require water discharge, with the exception of emptying the NFT channels after tomato plant harvest. This emphasises the lower environmental impact of aquaponics compared to uncoupled recirculating aquaculture, which requires discharges of 1-10% of total RAS volume per day. If applied to the uncoupled RAS at Abtshagen, based on weekly sedimentation tank cleaning, this would mean a discharge of at least 70 m$^3$ per year (27).

The mixing tank does also not require additional fresh water in this scenario. Meaning that the plants are fed with RAS water only.

![Figure 16 – Model simulated fresh water requirement in m$^3$ of Mixing tank (blue), Rearing tanks (green) and Pump Sump (red). Total water requirement (cyan) is the sum of the three fresh water requirements.](image)

4.2. Nutrient Concentrations
Under this cleaning regime, the TAN concentration in the rearing tanks reaches up to 0.20 g/L, which at pH 7.0 and 32 °C corresponds to 0.20 mg/L of NH$_3$. This is significantly lower than the concentration of 2.0 mg/L deemed safe for tilapia (28) (29). Thus, under the assumption that the fish excretion ratios from literature are applicable to the Abtshagen system, the chosen sedimentation tank cleaning regime is sufficiently stringent for both water supply to the plants and keeping TAN concentrations low in the RAS loop. The other nutrients are also present in concentrations deemed safe for tilapia cultivation and the chosen sedimentation tank cleaning regime is sufficient to prevent dangerous levels of solids accumulation in the RAS loop (8).
Under optimal fish growth, the RAS nitrogen was sufficient to meet the plant nitrogen demand and additional fertilizer would be redundant, with the exception of the first year, where a small quantity of nitrogen fertilizer was needed due to insufficient nitrogen from the RAS loop caused by not yet stocking of all rearing tanks. Since the model automatically calculates fertilizer requirement based on the nutrient content of the RAS water from the post purge, no nitrogen fertilizer is expected to be needed for the greenhouse the second planting year and onwards. Figure 18 shows the cumulative fertilizer provided to the greenhouse over three years. A clear requirement for potassium ($K^+$), phosphate ($PO_4^{3-}$) and magnesium ($Mg^{2+}$) fertilizer is evident, while the stairlike cumulative nitrogen and calcium fertilizer addition reveals that these two nutrients are only required at the planting dates of each season to ensure the NFT channels contain recommended nutrient concentrations when the tomato plants are introduced. Additional nitrogen is needed in the first year, during the start up phase of the stacked fish growth, resulting in insufficient RAS nitrogen to fully supply all plants. However, for consecutive years, no nitrogen fertilizer is required on any day during the growth of plants, except the initial planting day.
Figure 18 - Model simulated total fertilizer requirement in the aquaponic system over three years. Note that the required fertilizer for elements N and Mg are multiplied by 5 and 10, respectively, in order to be better visible in the figure.

To confirm if nitrogen and calcium do indeed accumulate in the NFT channels, the nutrient concentrations in the greenhouse channels is depicted in Figure 19. The accumulation of nitrogen and calcium can clearly be seen in their rising concentrations, which in the current study reaches significantly higher concentrations than is recommended for hydroponic tomato cultivation. Nitrogen (TAN and nitrate together) and calcium concentrations of 3.9 and 1.67 are 27 and 13 times higher than the recommended concentrations, respectively (15). These substantial concentrations are most likely caused by uncertainties in the fish excretion parameters calculated from literature, an overestimated fish stocking density and/or growth pattern or an underestimated nitrogen and calcium requirement of the plants. Another reason that might play a role in the high RAS nitrogen delivery to the greenhouse might be denitrification, the microbial conversion of nitrates to gaseous nitrogen. Denitrification was assumed to play only a minor role in the nitrogen balances and was therefore neglected. However, especially for longer residence times in the post purge, denitrification might have a significant impact on soluble nitrogen removal.
A slow increase in the NFT channel sodium concentration can also be observed over time. The maximum value of 0.058 g/L of sodium is five times higher than the recommended concentration (15). Giuffrida et al. reported an optimal concentration for irrigation water in Mediterranean conditions of 7 mM NaCl, corresponding to 0.16 g/L sodium. Assuming the maintained greenhouse climate resembles a Mediterranean climate would mean that the simulated concentration is not toxic to the plants, but safe enough to assume plant growth unhindered by sodium (30).

4.3. Biomass Yields

When a rearing tank is harvested, 129 kg fish per rearing tank is obtained. Since fish can be harvested after about 180 days, all rearing tanks can be harvested twice yearly. This way, 1034 kg tilapia per year can be obtained, corresponding to 141 kg/m3/year. The exception to this is the first year of tilapia cultivation, when only 5 harvests are available due to the start up phase of the stacked fish cultivation. Assuming a 5% caging loss and a mortality rate of 5% as was observed at ASTAF-PRO, leaves 933 kg/year (127 kg/m3/year) (31). The amount of fish and plant biomass present in the system is illustrated in Figure 20.
Since the fruit yield of the Abtshagen situation was quite low, the model assumes a yield of 50 kg/m², since commercial tomato greenhouses obtain yields between 50 and 55 kg/m², even as high as 90 kg/m² for professional and experimental systems (32) (33). For the 192 plants in the Abtshagen greenhouse this corresponds to 4363 kg/year. The Abtshagen greenhouse deemed less than 5% of fruits as of insufficient quality for market. Assuming similar loss for the modelled scenario results in 4145 kg/year (47.5 kg/m²/year). The ratio of fish to fruit yield equals 1 : 4.0 kg/kg. Comparing with the ASTAF-PRO systems reported ratio of 1 : 5 kg/kg fish : fruit, further strengthens the hypothesis that a larger hydroponic area can be sustained under the current aquaculture set-up at Abtshagen (5).

The found fish yield of 127 kg/m³/year is close to commercial yields of 150 kg/m³ for tanks up to 4 m³, with more advanced set-ups reaching up to 300 kg/m³ with up to three harvests per tank per year (34). So, under a stocking density of 175 fish per 1.84 m³ tank, the Abtshagen system might be of too small a scale to compete with commercial aquaculture-only systems. However, costs of fish cultivation might be lowered due to lower waste disposal costs and tomato profit. The assumed tomato yield of 47.5 kg/m² is close to 50-60 kg/m² yield of European and American hydroponic greenhouses (33). Even though individual yields of the RAS and hydroponic part of Abtshagen’s aquaponic system are expected to be lower than commercial systems, as a integrated aquaponic system it shows promising yields to compete on the market.

### 4.4. Optimal Greenhouse size

An increased greenhouse size, as suggested in section 3.7, would help to take up more of the high nutrient concentrations. However, increasing greenhouse size would also increase water demand and thus more often RAS water will be used to water the plants, meaning more RAS nitrogen and calcium end up in the NFT channels. Net result will be less accumulation due to higher demand for both nutrients, although the effect of increasing greenhouse size is limited. Assuming the fish nitrogen excretion and plant nitrogen demand are representative would suggest a greenhouse of about double the size to take up all provided nitrogen. Figure 21 depict the concentrations of...
nutrients in a greenhouse with 384 plants over three years and the cumulative amount of fertilizer needed to maintain recommended concentrations. Again, the first year of growing tomato required fertilizer due to the start up phase of stacked fish growth. Consecutive years require slight amounts of nitrogen fertilizer at the end of the growing season. The RAS nitrogen peaks observed in the greenhouse indicate that significant amounts of RAS nitrogen are sent to the greenhouse, but that all this nitrogen is taken up before the end of the growing season.

A large peak in provided nitrogen will still be observable in this scenario. However, that nitrogen is all taken up during the tomato growing season. The doubled calcium uptake in this scenario still results in tenfold calcium excesses, suggesting not only quitting calcium fertilizer, but also calcium free fish feed, since ground water seems to tackle both fish and plant calcium demand already.
Figure 21 - Top: Nutrient concentrations in the NFT channels for a greenhouse of double the size of the Abtshagen greenhouse (384 plants). Bottom: Cumulative fertilizer requirement of different nutrients, again assuming a double greenhouse size (384 plants).

Under this double greenhouse size, fruit yield will double to 8290 kg/year. Since fish yield is unaffected by introducing more tomato plants (933 kg/year), the fish to fruit yield ratio doubles to 1:8.0 kg tilapia per kg tomatoes, which is even higher than the ASTAF-PRO ratio of 1:5 (5), indicating that a future Abtshagen aquaponic system can greatly benefit from a bigger number of plants per fish than currently used.
5. General Discussion and Conclusions

An integrated RAS and hydroponics system model was constructed successfully. The aquaponic model was constructed using measurements, literature and some assumptions with respect to fish and plant growth and nutrient excretion and uptake. The modular computer model is capable of simulating various aquaponic set ups and is easily further adapted using newly collected data or different system dimensions. Fish growth and plant growth were modelled polynomially based on available data and measurements. The growth of either is considered equal to optimal growth under uncoupled cultivation, ensuring the model reflects optimal growth of both fish and plant. The equations used to model growth of either are quite general, since no environmental factors have been taken into account due to the assumption of optimal growth. When modelling a system under sub-optimal conditions or e.g. another fish species, known growth data of this new scenario is needed in order to adjust the polynomial parameters. However, for growth of fish and fruit, a polynomial model is capable of describing growth, in contrast to e.g. an exponential model, since these higher organisms do not grow exponentially, but a rather more gradual growth best described by a polynomial model.

Fish nutrient excretion and plant nutrient uptake were modelled based on literature feed rate and elemental composition, respectively. Simulating the Abtshagen system resulted in suggestions for better system management and improved system design. More system measurements are required to increase model accuracy under different conditions, regarding fish nutrient excretion and plant nutrient uptake.

5.1. Minimizing Water Requirement and Discharge

Using the model, the ideal water exchange flowrate between RAS loop and hydroponics area has been determined. At the Abtshagen system, the sedimentation tank was emptied once a week to supply water to the hydroponic area. This interval was sufficient to keep nutrient concentrations in the RAS loop safe for optimal fish growth and was enough to supply all eight NFT channels with enough water to cultivate tomato with optimal fruit yield.

Using the model, it was determined that up to once per three days emptying the sedimentation tank would suffice to meet water demand in this scenario. When the water exchange is managed such that emptying can occur up to once per day, more than two times the greenhouse size can be provided with water, showing the even larger capacity of the system when fully stocked with fish and RAS water for all tomato plants. Model simulation of the Abtshagen site also points out that no discharge is required under a greenhouse water demand based sedimentation tank cleaning regime.

5.2. Minimizing Fertilizer And Nutrient Waste

Since no discharge is required, no nutrients from the RAS are wasted. Besides that, the current model calculated fertilizer requirement based on shortage in nutrient from RAS send to the greenhouse, allowing prediction of ideal fertilizer regime for optimal plant growth. This can potentially limit addition of excess fertilizer and thus limit nutrient waste. Predictions also indicate that calcium fertilizer is not needed for small greenhouse areas and can even be limited in the feed composition, due to the high calcium concentrations observed in North East Germany ground water.

The current greenhouse size also receives a surplus of nitrogen from the RAS loop under optimal fish stocking densities, contrary to studies on other aquaponic systems (7). This nitrogen surplus suggests
a larger greenhouse area can be maintained under optimal fish stocking, although further model calibration of fish stocking, growth and nutrient excretion is required to confirm this.

On the contrary, sodium does not accumulate to dangerous levels. Since even low sodium concentrations already hinder plant nutrient uptake, sodium should be limited. However, current system does not expect sodium to accumulate to dangerous levels within the NFT channels during the plant growing period.

5.3. Recommendations (Assumptions and Literature Dependencies)
The optimal water exchange regime is based mostly on data from Abtshagen, making it largely independent of the assumptions used to construct the model. The optimal fertilizer addition regime makes use of assumptions and literature data to predict RAS nutrient concentrations and plant nutrient uptake rates. Knowing this, model validation under multiple conditions will allow further parameter optimization and higher model accuracy in these predictions. More measurements from the Abtshagen system concerning fish growth, stocking densities, feed rate and feed composition are required to increase the model accuracy, as well as additional measurements on plant nutrient uptake. Measuring rearing tank in and outflow concentrations and NFT nutrient inflow and NFT nutrient concentrations on regular intervals, will also shed more light on tilapia nutrient excretion rates.

One of the goals of this study was to make a computer model that can easily be adapted to simulate other systems than the Abtshagen site. However, preferably, additional data concerning nutrient dynamics from either the Abtshagen system or other systems is collected, as this would increase model certainty.
6. References


## A. Appendix 1 – List of Parameters

Appendix A details the parameters present in the model, accompanied by relevant symbols, model abbreviations, value or calculation, units, description and references.

### Model Parameters

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1: This parameter is a vector with a value every time step

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<td>$2 \times M_K/M_N$</td>
<td>kg/kg</td>
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<td>$3 \times M_{H_2O}/M_N/\rho_{H_2O}$</td>
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<td>K added per TAN consumed</td>
<td>(8)</td>
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<td>Water produced per TAN</td>
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<td>Initial fish weight</td>
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<td>Initial Feed Conversion Ratio (FCR)</td>
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<td>Maximum FCR</td>
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</tr>
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<td>Fraction Mg in feed</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>Fraction K in feed</td>
<td>(35)</td>
<td></td>
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<td>Fraction Na in feed</td>
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<tr>
<td>Fraction N from feed ending up in tilapia biomass</td>
<td>(36)</td>
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<tr>
<td>Fraction P from feed ending up in tilapia biomass</td>
<td>(36)</td>
<td></td>
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<td>Fraction Ca from feed ending up in tilapia biomass</td>
<td>(37)</td>
<td></td>
<td></td>
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<td>Fraction Mg from feed ending up in tilapia biomass</td>
<td>(38)</td>
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<td>Fraction K from feed ending up in tilapia biomass</td>
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<td>Fraction Na from feed ending up in tilapia biomass</td>
<td>(38)</td>
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<td>Fraction N from feed ending up in TSS</td>
<td>(36)</td>
<td></td>
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<td>Fraction P from feed ending up in TSS</td>
<td>(36)</td>
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<td>Fraction Ca in Trout TSS DW</td>
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<tr>
<td>Fraction Mg in Trout TSS DW</td>
<td>(39)</td>
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<td>Fraction Na in TSS</td>
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<tr>
<td>Fraction N from feed ending up in soluble waste</td>
<td>$\eta_{Naq}$</td>
<td>Naq_NS</td>
<td>Fraction N</td>
<td>(36)</td>
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<td>Fraction P from feed ending up in soluble waste</td>
<td>$\eta_{Paq}$</td>
<td>Paq_PS</td>
<td>Fraction P</td>
<td>(36)</td>
<td></td>
</tr>
<tr>
<td>Fraction N in feed</td>
<td>$\eta_{FS}$</td>
<td>N_S</td>
<td>Fraction N</td>
<td>–</td>
<td></td>
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<tr>
<td>Fraction Ca from feed ending up in TSS</td>
<td>$\eta_{CaSS}$</td>
<td>CaTSS_CaS</td>
<td>Fraction Ca</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>Fraction Mg from feed ending up in TSS</td>
<td>$\eta_{MgSS}$</td>
<td>MgTSS_MgS</td>
<td>Fraction Mg</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>Fraction K from feed ending up in TSS</td>
<td>$\eta_{KS}$</td>
<td>KTSS_KS</td>
<td>Fraction K</td>
<td>–</td>
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<td>Ca concentration</td>
<td>$\eta_{Caaq}$</td>
<td>Caa_CaS</td>
<td>Ca concentration</td>
<td>1 – $\eta_{CaSS}$ – $\eta_{Caaq}$ kg/kg</td>
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<tr>
<td>Mg concentration</td>
<td>$\eta_{Mgaq}$</td>
<td>Mgaq_MgS</td>
<td>Mg concentration</td>
<td>1 – $\eta_{MgSS}$ – $\eta_{Mgaq}$ kg/kg</td>
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<td>$\eta_{Ka}$</td>
<td>Kaq_KS</td>
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<td>$\eta_{Naq}$</td>
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<td>Na concentration</td>
<td>1 – $\eta_{NaSS}$ – $\eta_{Naq}$ kg/kg</td>
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<td>Fraction water in fish biomass</td>
<td>$\eta_{X}$</td>
<td>H2O_X</td>
<td>Fraction water</td>
<td>$0.8/\rho_{H2O}$ m³/kg</td>
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<td>Volume needed to wash the fish in one tank</td>
<td>$V_{stock}$</td>
<td>X_stock</td>
<td>Volume</td>
<td>$N_X \times X_0$ kg</td>
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<tr>
<td>Total volume sedimentation tank</td>
<td>$V_{ST}$</td>
<td>V0_ST</td>
<td>Total volume</td>
<td>1.32 m³</td>
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<tr>
<td>Clean ST at most once per this many days</td>
<td>$t_{clean}$</td>
<td>t_clean_ST</td>
<td>Cleaning duration</td>
<td>0.5/24 d</td>
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<tr>
<td>Clean ST at least once per this many days</td>
<td>$t_{min}$</td>
<td>t_clean_ST_min</td>
<td>Cleaning duration</td>
<td>7 d</td>
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<td>Specific TVS removal rate</td>
<td>$\mu_{TSS}$</td>
<td>mu_TSS</td>
<td>Specific TVS removal rate</td>
<td>0.97 kg/kg/d</td>
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<td>Specific TSS removal rate</td>
<td>$\mu_{TSS}$</td>
<td>mu_TVS</td>
<td>Specific TSS removal rate</td>
<td>0.10 kg/kg/d</td>
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<td>Specific TSS(s) dissolving rate</td>
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<td>mu_clean</td>
<td>Specific TSS(s) dissolving rate</td>
<td>0.97 kg/kg/d</td>
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<td>Total volume post purge</td>
<td>$V_{PP}$</td>
<td>V0_PP</td>
<td>Total volume</td>
<td>9 m³</td>
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<td>Lower limit PP volume before ST-&gt;PP flow</td>
<td>$u_{VPP}$</td>
<td>u_VPP</td>
<td>Lower limit</td>
<td>0.5/9 m³</td>
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<tr>
<td>Specific TSS removal rate</td>
<td>$\mu_{TSS}$</td>
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<tr>
<td>Specific TVS removal rate</td>
<td>$\mu_{TSS}$</td>
<td>mu_TVS</td>
<td>Specific TVS removal rate</td>
<td>0.80 kg/kg/d</td>
<td></td>
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<td>Total volume mixing tank</td>
<td>$V_{MT}$</td>
<td>V0_MT</td>
<td>Total volume</td>
<td>1.1 m³</td>
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<tr>
<td>Lower limit MT volume before PP-&gt;MT flow</td>
<td>$u_{VMT}$</td>
<td>u_VMT</td>
<td>Lower limit</td>
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<td>Optimal N fertilizer concentration</td>
<td>$C_{N}^{rec}$</td>
<td>C_N</td>
<td>Optimal N</td>
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<tr>
<td>Optimal P fertilizer concentration</td>
<td>$C_{P}^{rec}$</td>
<td>C_P</td>
<td>Optimal P</td>
<td>0.0372 kg/m³</td>
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<tr>
<td>Optimal Ca fertilizer concentration</td>
<td>$C_{Ca}^{rec}$</td>
<td>C_Ca</td>
<td>Optimal Ca</td>
<td>0.1283 kg/m³</td>
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<td>Description</td>
<td>Unit</td>
<td>Notes</td>
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<td>--------------------------------------------------</td>
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<tr>
<td>( C_{\text{rec}}^{\text{Mg}} )</td>
<td>C_Mg</td>
<td>kg/m³</td>
<td>Optimal Mg fertilizer concentration</td>
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<td>( C_{\text{rec}}^{\text{K}} )</td>
<td>C_K</td>
<td>kg/m³</td>
<td>Optimal K fertilizer concentration</td>
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<td>( N_{\text{NFT}} )</td>
<td>N_NFT</td>
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<td>Number of NFT channels</td>
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<td>( X_{\text{NFT}} )</td>
<td>X_NFT</td>
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<td>Number of plants per NFT channel</td>
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<tr>
<td>( V^0_{\text{NFT}} )</td>
<td>V0_NFT</td>
<td>m³</td>
<td>Total volume one NFT channel</td>
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<td>( t_0^X )</td>
<td>t0_HP</td>
<td>d</td>
<td>Days after 01-01 tomatoes are planted</td>
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<tr>
<td>( t_f^X )</td>
<td>tf_HP</td>
<td>d</td>
<td>Days after 01-01 plants are removed</td>
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<tr>
<td>( t_0^X_f )</td>
<td>t0_Xf</td>
<td>d</td>
<td>Days after 01-01 start first harvest</td>
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<tr>
<td>( p_1^X_f )</td>
<td>p1_Xf</td>
<td>d⁻²</td>
<td>Fruit growth parameter 1</td>
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<td></td>
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<tr>
<td>( p_2^X_f )</td>
<td>p2_Xf</td>
<td>d⁻¹</td>
<td>Fruit growth parameter 2</td>
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<td></td>
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<tr>
<td>( Y_{\text{Xf}} )</td>
<td>Y_Xf</td>
<td>kg</td>
<td>Fruit yield per plant, environment dep.</td>
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<td>( a_{\text{HP}} )</td>
<td>a_HP</td>
<td>m⁻²</td>
<td>Plants per area, planting density</td>
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<tr>
<td>( p_1^{r\text{H}2\text{O}} )</td>
<td>p1_rH2O</td>
<td>m³/kg/d³</td>
<td>Water uptake parameter 1</td>
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<tr>
<td>( p_2^{r\text{H}2\text{O}} )</td>
<td>p2_rH2O</td>
<td>m³/kg/d²</td>
<td>Water uptake parameter 2</td>
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<tr>
<td>( p_3^{r\text{H}2\text{O}} )</td>
<td>p3_rH2O</td>
<td>m³/kg/d</td>
<td>Water uptake parameter 3</td>
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<td>( r_{\text{Xnf}} )</td>
<td>Xnf_r</td>
<td>d⁻¹</td>
<td>Non-fruit specific growth rate</td>
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<td>( X_{\text{nf}}^{\text{max}} )</td>
<td>Xnf_max</td>
<td>kg</td>
<td>Non-fruit maximum mass per plant</td>
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<tr>
<td>( X_{\text{nf}}^{\text{min}} )</td>
<td>Xnf_min</td>
<td>kg</td>
<td>Initial non-fruit mass per plant</td>
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<tr>
<td>( H_{2\text{O}}^X_f )</td>
<td>H2O_Xf</td>
<td>kg/kg</td>
<td>Fraction dry weight in fruit</td>
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<tr>
<td>( \eta_{\text{Na}}^X_f )</td>
<td>Na_Xf</td>
<td>(0.000569 + 0.0011667)/2 * ((1 - \eta_{\text{H}_{2}\text{O}}^{X_f})) kg/kg</td>
<td>Ratio Na in fruit</td>
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<tr>
<td>( \eta_{\text{Prot}}^X_f )</td>
<td>N_prot</td>
<td>0.16</td>
<td>kg/kg</td>
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<tr>
<td>( \eta_{\text{Na}}^{X_f} )</td>
<td>Na_Xf</td>
<td>(0.000569 + 0.0011667)/2 * ((1 - \eta_{\text{H}_{2}\text{O}}^{X_f})) kg/kg</td>
<td>Ratio Na in protein</td>
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<tr>
<td>( \eta_{\text{N}}^{X_f} )</td>
<td>N_Xf</td>
<td>((0.1167 + 0.1638)/2 * \eta_{\text{prot}}^N + 0.0008621) * ((1 - \eta_{\text{H}_{2}\text{O}}^{X_f})) kg/kg</td>
<td>Ratio N in fruit</td>
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<tr>
<td>( \eta_{\text{P}}^{X_f} )</td>
<td>P_Xf</td>
<td>(0.0037931 + 0.005)/2 * ((1 - \eta_{\text{H}_{2}\text{O}}^{X_f})) kg/kg</td>
<td>Ratio P in fruit</td>
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<tr>
<td>( \eta_{\text{Ca}}^{X_f} )</td>
<td>Ca_Xf</td>
<td>(0.0014516 + 0.0018182)/2 * ((1 - \eta_{\text{H}_{2}\text{O}}^{X_f})) kg/kg</td>
<td>Ratio Ca in fruit</td>
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<tr>
<td>( \eta_{\text{K}}^{X_f} )</td>
<td>K_Xf</td>
<td>(0.036 + 0.0483333)/2 * ((1 - \eta_{\text{H}_{2}\text{O}}^{X_f})) kg/kg</td>
<td>Ratio K in fruit</td>
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<td></td>
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<td>( N_X )</td>
<td>N_Hp</td>
<td>X_{NFT} * N_{NFT}</td>
<td>Total number of plants in the greenhouse</td>
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<tr>
<td>( V^0_{\text{HP}} )</td>
<td>V0_HP</td>
<td>m³</td>
<td>Total volume mixing tank</td>
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<tr>
<td>( A_{\text{HP}} )</td>
<td>A_HP</td>
<td>m²</td>
<td>Total greenhouse area</td>
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</table>

**Hydroponics (HP)**

Total greenhouse area

Total number of plants in the greenhouse

Total volume mixing tank

Total greenhouse area

Plants per area, planting density

Ratio K in fruit growth parameter

Ratio Ca in fruit growth parameter

Ratio P in fruit growth parameter

Ratio Na in fruit growth parameter

Ratio N in fruit growth parameter

Optimal K fertilizer concentration

Optimal Mg fertilizer concentration

Ratio Na in fruit

Ratio K in fruit

Ratio Ca in fruit

Estimate non-fruit specific growth rate

Non-fruit maximum mass per plant

Initial non-fruit mass per plant

Water uptake parameter 1

Water uptake parameter 2

Water uptake parameter 3

Fruit growth parameter 1

Fruit growth parameter 2

Fruit yield per plant, environment dep.
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<th>Component</th>
<th>Assumption</th>
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<tr>
<td>$\eta_{Xnf}$</td>
<td>Assumed equal to fruit</td>
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B. Appendix 2 – System Model Representation

Figure B1 - System Model Representation detailing connectivity computer modules. The red lines represent the input data needed to run the model; The black lines represent module dependencies: Output from one module is used as input for another; Green lines represent the biomass model outputs (fish and tomato); Blue lines represent the fresh water input.