Estimation of feed utilisation matrices and demand for feed using farm data

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In this paper we developed a new method for joint estimation of the feed demand model and feed utilisation matrices on the basis of the farm data supplemented by the macro-data. The theoretical framework for our method forms the non-linear programming model describing the profit-maximising behaviour of the compound feed producers (the compound feed model). The specification of the model ensures continuous and smooth feed allocation responses to price changes and allows to include the linear restrictions to account for both engineering information and other *a priori* restrictions. To estimate the compound feed model, a three-step iterative procedure was developed. For testing, the jack-knife method was proposed. The proposed method was tested using the farm data for the Netherlands provided by the European Farm Accountancy Data Network.

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Preface

The feed-livestock sector plays a key role in European Union (EU) agriculture. Despite its importance, relatively little has been written on empirical modelling of feed demand in the EU. A major modelling problem here is the limited availability of macro-data necessary to complete the so-called feed utilisation matrices (FUM's). In this paper we exploit micro data to enrich the information available from the macro-aggregates and estimate FUM's and demand for feed. The novelty of the approach followed is that the micro- and macro-data are used jointly in the estimation procedure.

The study was conducted under the FEA-project (Future of European Agriculture), a joint venture of the Agricultural Economics Research Institute (LEI) in The Hague, the Netherlands Bureau for Economic Policy Analysis (CPB), also in The Hague, and the Centre for World Food Studies (SOW-VU) in Amsterdam. In the FEA-project, a ready-to-use general equilibrium model of the agricultural sector of the European Union is maintained. It is used to assess the impact of policy in EU agriculture.

We would like to thank Prof. Dr. M.A. Keyzer, leader of the FEA-project, who initiated this study and proposed the estimation procedure. The study benefited greatly from helpful discussions with FEA-team members: Dr. Max Merbis and Paul Veenendaal - FEA project leader at LEI. We would also like to thank Dr. Frank van Tongeren for numerous fruitful comments on an earlier version of this paper.

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Summary

In this paper we develop a new method for the joint estimation of a feed demand model and feed utilisation matrices on the basis of farm data supplemented by macro-data. The theoretical framework for our method is a non-linear programming model describing profit-maximising behaviour of compound feed producers. The specification of the model ensures continuous and smooth feed allocation responses to price changes and allows to include linear restrictions to account for both engineering information and other *a priori* restrictions. To estimate the compound feed model, a three-step iterative procedure was developed. The jack-knife method was used to assess the reliability of the estimates.

The estimation procedure uses individual farm data while additional macro-data are used to ensure the consistency of micro-estimates with macro-aggregates. In this way micro-macro consistency is maintained. To generate unavailable farm data necessary for the estimation, a special data-model that generates the unavailable figures from available farm data was developed.

The method proposed was applied to farm data for the Netherlands provided by the European Farm Accountancy Data Network. Estimation results show that the model performs very well. The model parameters were estimated with high precision but at high computational costs. The feed utilisation matrix obtained is consistent with results presented in other sources. The estimation procedure was, however, slow and should be improved.

The developed method provides a consistent framework, which can be used to estimate feed utilisation matrices as well as other unobserved macro-data from micro (farm) figures. Moreover, the usage of farm data allows for a relatively high disaggregation of the model in terms of numbers of products and production factors.

1 Introduction

The feed-livestock sector plays a key role in European Union (EU) agriculture. Knowledge of the feed-livestock relationship is particularly important in assessing the impact of pricing policies on livestock production, feed use and trade in feed components. The EU's Common Agricultural Policy (CAP) influences prices of agricultural products which in turn affect the growth of livestock production and generate shifts in the composition of European feed demand [1]. Therefore, models describing the feed-livestock sector are of particular interest to policymakers.

Despite the importance of the feed-livestock economy, relatively little has been written on empirical modelling of the feed demand relationship in the European Union (see Peeters and Surry, 1997 for an overview). A major problem concerning feed demand modelling is the limited availability of data. In particular, national feed balances per animal type, so-called feed utilisation matrices (FUM's), are usually not compiled by national statistical institutes. This leads to the need of developing methods and standardised procedures, which consistently estimate FUM's and feed demand relationships.

Three basic approaches can be distinguished for obtaining the feed demand estimates when no FUM's are available. The first two of them employ a dual (cost function) method, while the third approach uses a primal (production function) method.

In the first approach, the feed demand equations are derived from a single-output multiple-input cost function, specified by animal types. The derived equations are estimated using separately constructed FUM's (Folmer et al., 1995 and Tabeau, 1999). FUM's are constructed on the base of information on the total usage of feed, expert knowledge, and feeding norms for animals, such as feeding requirements of particular livestock categories, nutritional contributions of concentrate feeds, and various conversion ratios (see e.g. Wolf (1995)).

The second approach uses multiple-output multiple-input cost functions. This approach makes it possible to estimate the total (i.e., for all livestock categories jointly) demand for feed components without using FUM's. This method assumes that feed input is non-separable among livestock categories and uses information on the total usage of feed components (Surry and Moschini, 1984, Mergos and Yatopulos, 1988, Surry, 1993 and Peeters, 1995). An extension of this method was proposed by Peeters and Surry in 1993. They relaxed the non-separability assumption and used the symmetric McFadden cost function to jointly estimate demand equations and FUM's.

The third approach applies the least-cost linear programming (LP) model with constraints describing technical and nutritional restrictions. This model is used to generate feed inclusion rates and price elasticities of feed demand (Peterson, 1986, Peeters, 1990, McKinzie et al., 1986). The parameters generated are in turn used to calibrate the feed demand equations (Surry, 1993).

[1] In general, cereals have persistently been displaced by so-called cereal substitutes in the last three decades. This was caused by a steady increase in the ratio of the price of cereals to that of the cereal substitutes.

However, all the approaches presented above have some shortcomings. The first approach is inconsistent, because it estimates the FUM's and the feed demand equations independently. It ignores the fact that the same technologies may generate quite different feed utilisation patterns due to differences in relative prices. Therefore, FUM's created in this way do not allow us to correctly quantify the impact of prices on feed demand. When feed input is non-separable among livestock categories, the dual approaches make it impossible to estimate feed demand equations by animal types. Moreover, the dual approaches do not take into account the impact of technical-nutritional restrictions on feed substitution. The LP approach considers these restrictions explicitly and can cover a multitude of feed ingredients and feed aggregation levels. This approach suffers, however, from two other limitations. First, the estimated price responsiveness is conditional upon a given level of output since the LP models do not incorporate expansion effects. Second, LP models have a piecewise linear response function which may lead to very large feed composition responses to price changes.

To overcome these problems, we have developed a new method for joint estimation of the feed demand model and the FUM's. The proposed approach has the advantages of both primal and dual approaches without their disadvantages. Our method is in line with the primal approach with production technology being described by a non-linear feed mixing function, technical-nutritional restrictions and feed balances. The non-linear feed mixing function ensures smooth price responses and expansion effects. The total feed balances guarantee the macro consistency of the estimated FUM's.

The method proposed by us is not designed to estimate any specific form of the demand function. Demand for feed will be derived from the non-linear programming model describing optimisation behaviour of the (compound) feed industry. The parameters of this model (parameters of the feed mixing function) will be estimated using data about compound feed cost and animal numbers available from farm data (from the European Farm Accountancy Data Network). The use of primary micro-data improves the quality of the estimates and allows deep disaggregation of the model.

To estimate the model, we developed a three-step iterative procedure, which estimates the unknown parameters of the model and the FUM's jointly. Both micro (farm) and macro (national) data are used to estimate the model. This approach has three advantages. First, it allows us to estimate the model parameters and the FUM's consistently. Second, the estimated FUM's are consistent with macro-data. Finally, the use of the individual data makes it possible to investigate farm specific feed-livestock relationships. This in turn allows the investigation of the impact of various policy measures on the behaviour of different types of farms as well as the impact of farms' decisions on the feed-livestock economy as a whole.

The paper is organised as follows: In Section 2, we formulate a compound feed model. Sections 3 and 4 describe estimation and testing procedures of the model, respectively. Section 5 characterises the data used to estimate the model and deals with data issues. In Section 6, the estimation results are described. Section 7 concludes.

2 The theoretical model

We assumed that allocation of the compound feed components to animal types (i.e. FUM's) results from the profit-maximising behaviour of feed compounders. In this way we ensure a micro-economic interpretation of the results obtained. We have used a non-linear programming model to describe the behaviour of the feed compounders. This has two advantages. First, a properly specified non-linear program ensures continuous and smooth feed allocation responses to price changes and expansion effects. Second, the non-linear programming model can also include linear restrictions to account for both engineering information and other *a priori* restrictions.

To derive the model, we assume that the feed compounders buy the feed components on the market, mix them to produce compound feed, and sell the compound feed to farmers. Since the supply of some feed components is restricted and since different animal types require feed having different nutrient compositions, the feed components are substitutes. This is ensured by assuming a non-linear mixing technology. On the other hand, the produced compound feed has to meet certain nutritional requirements, which is described by linear restrictions on the feed components. To choose the optimal composition of the compound feed components, the feed compounders maximise their profit given the non-linear mixing technology and nutrient restrictions.

To formalise our model, we assume that the compound feed industry produces feed for L animal types using J feed components and that K nutrient ingredients (i.e., metabolised energy, crude proteins, dry matter component, and so on) are distinguished. To describe the model, we use the following symbols:

$\mathbf{A}_l=[a_{kj}]$	- nutrient composition matrix for the feed components;
$\boldsymbol{\beta}_l$	- parameters of the mixing function;
s_l	- share of non-feed component costs in value of the compound feed;
$\mathbf{d}_l=[d_{kl}]$	- required contents of the nutrient ingredients in the compound feed;
$F(\boldsymbol{\beta}_l,\mathbf{v}_l)$	- concave mixing function;
$\mathbf{p}=[p_j]$	- feed components prices;
q_l	- compound feed production;
r_l	- compound feed price;
$\underline{\mathbf{v}}_l=[\underline{v}_{jl}]$	- feed component input;
$\overline{\mathbf{v}}_l=[\overline{v}_{jl}]$	- committed feed component input.

Using the above symbols, the model describing the feed compounders' behaviour can be written as follows:

1. $\max_{q_l, v_l \geq 0} \{\sum_l (r_l\, q_l - s_l\, r_l\, q_l - \mathbf{p}\, \mathbf{v}_l)\}$

subject to:

2. $q_l = F(\beta_l, v_l) \qquad l=1,2,...,L$

3. $\mathbf{A}_l \mathbf{v}_l \geqq \mathbf{d}_l \qquad l=1,2,...,L$

4. $\mathbf{v}_l \geq \bar{\mathbf{v}}_l \qquad l=1,2,...,L$

where symbol $\geqq$ indicates that some costaints are satisfied as equalities and some as inequalities.

We will call the model (1) - (4) 'the compound feed model'. The objective function (1) is the profit function. Profit is equal to the value of production sold ($r_l\, q_l$) minus the cost of input. This cost is equal to the cost of feed components ($\mathbf{p}\, \mathbf{v}_l$) used to produce the compound feed plus other costs (equal to $s_l\, r_l\, q_l$). Equation (2) describes how the feed components v_l are mixed to produce the compound feed q_l. $\boldsymbol{\beta}_l$ is a vector of parameters of the mixing function F. Nutritional constraints (3) ensure that farm demands for nutrient ingredients is fulfilled. It is assumed that there are no nutrient losses in the production process. We also assume that at least one nutrient constraint is satisfied as an equality constraint, so that the mathematical program (1) - (3) is bounded [1]. According to constraint (4), the minimal quantity of the compound feed components used in production is equal to the committed level.

[1] In our application, we assume that the nutrient constraint for metabolised energy is the equality constraint. According to the literature, the metabolised energy provided by compound feed does not meet the total energy requirements necessary to feed animals and the metabolised energy provided by roughage closes the balance. On the other hand, compound feed and roughage usually provide surpluses of the other nutrient components.

3 The estimation procedure of the compound feed model

The parameters $\mathbf{A}_l$, s_l, $\mathbf{d}_l$, $\mathbf{p}$ and r_l of the compound feed model (1) - (4) have been calculated using different data sources (see section 5). To apply the compound feed model (1) - (4), the unknown parameters $\boldsymbol{\beta}_l$ of the mixing function F have to be estimated. Then the optimal values of the model variables can be calculated. They can be used in turn to compile the FUM.

To estimate the compound feed model we assumed that the farm gate cost $r_l q_l$ of the compound feed should fit as much as possible farmers' expenditures on the compound feed available in the FADN database. Moreover, we assumed that the estimated feed components input $\mathbf{v}_l$ should be as much as possible consistent with the available macro-figures on feed components supply V_j. The estimated feed component input is the optimal solution of the compound feed model and as such it solves the first order conditions of the optimisation problem (1) - (4). Therefore, we solve the following mathematical programming problem to estimate the parameters $\boldsymbol{\beta}_l$ and feed component input ($\mathbf{v}_l$):

5. $$\min_{\beta_{nl}\geq 0} \{L(\boldsymbol{\beta}_l);\ L(\boldsymbol{\beta}_l) = \Sigma_{nl}(c_{nl}^{-1}(c_{nl}- r_l F(\boldsymbol{\beta}_l,\mathbf{v}_{nl})))^2+\Sigma_j(V_j^{-1}(\Sigma_l M_l(\Sigma_{nj}/\Sigma_n\mu_{nl})-V_j)))^2+ (\Sigma_{nl}\, c_{nl}^{-1}(c_{nl}- r_l\, F(\boldsymbol{\beta}_l,\mathbf{v}_{nl})))^2\}$$

subject to:

6. $$(-\mathbf{p} - (1-s_l)\, r_l\, F'_{v_{nl}}(\boldsymbol{\beta}_l,\mathbf{v}_{nl}) + \lambda^d_{nl}\, \mathbf{A}_l + \lambda^v_{nl}) = 0 \qquad \text{for all l, n}$$

7. $$\lambda^d_{nl}\, (\mathbf{A}_l\mathbf{v}_{nl} - \mathbf{d}_{nl}) = 0 \qquad \text{for all l, n}$$

8. $$\lambda^v_{nl}\, (\mathbf{v}_{nl} - \overline{\mathbf{v}}_{nl}) = \mathbf{0} \qquad \text{for all l, n}$$

9. $$\lambda^d_{nl} \geq \mathbf{0}, \qquad \lambda^v_{nl} \geq \mathbf{0}, \qquad \mathbf{A}_l\mathbf{v}_{nl} \geq \mathbf{d}_{nl} \qquad \mathbf{v}_{nl}\geq \mathbf{v}_{nl} \qquad \text{for all l, n}$$

where the following represent:

n	- farm index (n=1,...,N);
$L(\boldsymbol{\beta}_l)$	- loss function
c_{nl}	- cost of the compound feed provided by the FADN data-base;
μ_{nl}	- number of animals in the FADN data-base;
M_l	- number of animals in the macro data;
V_j	- quantity of the feed components provided by the macro-data;
$F'_{v_{nl}}(\boldsymbol{\beta}_l,\mathbf{v}_{nl})$	- first derivatives of $F(\boldsymbol{\beta}_l,\mathbf{v}_{nl})$ with respect to $\mathbf{v}_{nl}$;
λ^d_{nl}, λ^v_{nl}	- Lagrangean multipliers associated with the constraints (3) and (4).

Constraints (6) - (9) are the first order conditions of the optimisation problem (1) - (4) [1]. The loss function (5) consists of three terms. The first term is a weighted (by c_{nl}^{-1}) sum of squared differences between the observed farmers' expenditures on the compound feed and farm gate compound feed cost calculated from the compound feed model (1) - (4). The second term represents the micro-macro consistency conditions weighted by V_j^{-1}, which compare the estimated amount of the compound feed components used by compound feed industry with the available macro-figures. The third term is the usual condition that the sum of residuals (weighted by c_{nl}^{-1}) of the estimated model should be equal to zero.

Program (5) - (9) is hard to solve because it is highly nonconvex. Therefore, we have developed an iterative procedure to solve this problem [2]. The procedure developed solves three programs in each iteration 't': the inner program, the outer program and the step-length determination program.

Given the starting values of the model parameters $\boldsymbol{\beta}^{*t}_l$, for the iteration 't', the inner program solves the model (1) - (4) for every farm 'n'. It generates the optimal values of the feed component input ($\mathbf{v}^*_l$) and the Lagrangean multipliers λ^d_{nl} and λ^v_{nl} associated with the constraints (3) and (4).

The outer program calculates a gradient λ^β_l of the loss function $L(\boldsymbol{\beta}_l)$ with respect to $\boldsymbol{\beta}_l$ and for $\mathbf{v}_l = \mathbf{v}^*_l$, which provides direction for adjusting $\boldsymbol{\beta}_l$. The following method of steepest descent is applied to calculate the new betas:

$$10. \quad \boldsymbol{\beta}^{*t+1}_l = \boldsymbol{\beta}^{*t}_l - \omega\, \lambda^\beta_l \qquad \text{for all } l$$

where $\omega>0$ is the step-length and λ^β_l serves as the search direction.

Since it is not possible to derive the loss function analytically in our case, we calculate the gradient λ^β_l as the Lagrange multiplier associated with restriction $\boldsymbol{\beta}_l = \boldsymbol{\beta}^{*t}_l$ imposed on the parameters $\boldsymbol{\beta}_l$. To calculate this multiplier the optimisation program (5) - (9) is solved with the additional restriction:

$$11. \quad \boldsymbol{\beta}_l = \boldsymbol{\beta}^{*t}_l \qquad \text{for all } l$$

The outer program (5) - (9), (11) is highly nonconvex similarly to the program (5) - (9), but if the optimal solution $\mathbf{v}^*_l$, λ^d_{nl} and λ^v_{nl} of the inner program (1) - (4) is locally unique, it will be the single feasible solution of the outer program. Therefore, the optimisation procedure is only needed to compute the Lagrange multiplier λ^β_l. It is normally calculated very rapidly since the outer program is initialised at optimal $\mathbf{v}^*_l$, λ^d_{nl} and λ^v_{nl}.

To calculate the optimal step-length ω for the steepest descent method (10), we solve a step-length determination program in each iteration 't'. This program follows the outer program and has the same specification as the outer program except for the last equation (11), which is now replaced by an equation analogous to (10), which has the following form:

[1] Using (2), we replaced q_l by $F(\boldsymbol{\beta}^t_l, \mathbf{v}_{nl})$ in the problem (5) - (9).

[2] The estimation procedure uses an approach proposed by Keyzer, 2000.

12. $\boldsymbol{\beta}_l = \boldsymbol{\beta}^t_l - \omega\lambda^\beta_l$ for all l

where the step-lengths ω are choice variables of the step-length determination program.

A loop over all three programs with an adjustment of parameters based on a gradient of the loss function yields the best possible fit with the observations. The iterative procedure is repeated until convergence is reached. The proposed procedure is a steepest descent approach that generally converges to a local optimum.

4 Jack-knife testing procedure for the compound feed model

The mathematical programming (MP) estimation technique has one important shortcoming. It produces estimates without any statistical properties because in general the underlying sampling distributions of the error terms and parameters are either unknown or have no analytical representation. Hence, in this case, it is impossible to evaluate the estimated model statistically. This limitation of the MP method can be overcome by using the jack-knife method to assess the statistical characteristics of the compound feed model.

The jack-knife technique is a non-parametric approach based on a resampling estimation procedure. This procedure is used to generate pseudodata for the parameters of the model by resampling the original observations and calculating pseudovalues for the parameters of interest for each sub-sample. By resampling from the original sample (randomly or based on a certain rule), each new sub-sample will be different from the original one. Hence, each new sub-sample will likely generate different pseudovalues for the parameters. By generating many sets of pseudodata, and hence estimating many pseudovalues, the relevancy of the parameters of interest can be statistically tested by examining the stability of their associated pseudovalues. The generated pseudovalues can therefore be used to calculate model statistics of interest, e.g., measures of variability and confidence intervals for parameters.

The core of the jack-knife technique is to partition out the effect of a particular subset of the data on an estimate of parameters derived from the total sample (see Tukey, 1958). The effect of a particular subset of the data on the target parameter is determined by deleting that subset and re-estimating the parameters. In the most frequently used version of the jack-knife procedure only one data point is deleted each time from the original data set and the estimator is calculated based on the rest of data. For large databases, however, 'z' observations are deleted. This procedure is called deleted-z ($z>1$) jack-knife procedure. The deleted observations can be chosen in different ways (see Shao and Tu, 1995).

The deleted-z jack-knife estimator θ^* of the parameter θ and variance estimator S^2_θ of θ are given by the following formulas:

13. $$\theta^* = \frac{1}{T}\sum_t^T \theta^*_t$$

14. $$S^2_\theta = \frac{N-z}{zT}\sum_t^T (\theta^*_t - \theta^*)^2$$

where θ^*_t is the estimator of θ after deleting the subset t of size z from the complete sample, T is the total number of subsets and N is a size of the complete sample.

The jack-knife variance estimator S^2_θ is consistent under some smoothness conditions for many statistics including functions of sample mean (see Shao and Tu, 1995). The interesting feature of the jack-knife procedure is that the pseudovalues θ^*_t can be treated as independent and identically distributed random variables and, hence, can be used to infer statistical significance test using t-Student statistic with T-1 degree of freedom (Mosteller and Tukey, 1968).

To test the compound feed model for the Netherlands, we used the deleted-z jack-knife procedure with z=5. Observations were deleted sequentially (starting with first five observations, then the second five observations, and so on). In our case N=50 and therefore T=10.

5 Data used to estimate the compound feed model for the Netherlands

There are two types of data necessary to estimate or derive the parameters of the compound feed model: the micro-data from the FADN database and the macro-data from SPEL, CRONOS and other sources. In our research we used data for the Netherlands for 1994.

5.1 The micro-data

The necessary micro-data were extracted from the FADN database for 1994. They provide information about the number of animals and feed costs per farm for 1528 farms. A preliminary analysis of the micro-data was necessary to compute their characteristics, compare them with the macro data, and to develop a procedure to create a database for the estimation of the compound feed model. This preliminary analysis was done using micro-data for all farms having animals. The main characteristic features of the micro-data are as follows:

- the price of a unit of metabolised energy required to feed animals differs substantially between farms. For instance, for poultry, it varies from 86 to 403 ECU per unit and its variation coefficient is equal to 25%;
- the number of hectares which can be used to produce roughage for grazing animals differs substantially by farm (the variation coefficient is 62%) and the metabolised energy which can be produced using roughage ranges from 0 to 1500% metabolised energy requirements. Therefore, these data do not give a reliable indication of the production and use of roughage and we estimate the amount of roughage used to feed animals using other information;
- farms having contract production (farms which have animals but do not own them) do not have any feed cost. The feed cost has to be estimated for these farms.

The micro-data provide the number of animals μ_{nl} per farm. The cost of the compound feed purchased by the farm is available in FADN only for grazing animals, pigs and poultry and it should be further disaggregated to match the disaggregation level assumed for the model. The required content $\mathbf{d}_{nl}$ of the nutrient ingredients in the compound feed is not available in the database. A data-model was built to generate these figures.

5.2 The macro-data

The macro-data contain information about the animal population (SPEL and CRONOS), available feed (SPEL), feed prices (SPEL) and metabolised energy, crude proteins and dry matter contents of feed (SPEL). They show that about 63% of metabolised energy for grazing animals is provided by non-roughage feed. There are substantial differences between the animal populations provided by SPEL and CRONOS. For example, CRONOS reports that there were 680,000 (4.7%) more pigs than SPEL registers for the Netherlands in 1994.

The macro-data provide figures for the feed component prices **p**, compound feed prices r_l, cost ratios c_l, nutrient composition matrix $\mathbf{A}_l$, number of animals M_l and quantity of the feed components V_j.

We used the following additional data sources to specify nutrient requirements and nutrient constraints by type of animal: Bolhuis, et al. (1995), CVB (1997) and OECD (1986).

5.3 Comparison of data from different sources and disagregation level of the model

Comparison of data coming from different sources is hampered by three obstacles:
- not fully representative micro-data;
- differences in animals' classification;
- differences in nutrient requirements for particular types of animal.

A preliminary investigation of the data set for the Netherlands for 1994 shows that the FADN data are not fully representative for all herds. For example, the number of dairy cows in the FADN equals 1.08 times the number of dairy cows in the Netherlands according to CRONOS data. For different types of pigs, this proportion varies between 0.63 and 1.2. This was taken in to account when the micro-macro consistency conditions were specified in the model.

The classification of animals differs across statistical sources. The classification used in the model was obtained by grouping animals belonging to the same animal category (i.e., grazing animals, pigs and poultry) and having similar metabolised energy requirements. In this way, we lowered the impact of the internal structure of aggregates on the nutrient requirements for animal groups distinguished in the model. The metabolised energy requirements for the animal groups present in the model were calculated using data for the Netherlands (Bolhuis, et al., 1995).

For most animals, the nutrient requirements used in SPEL are lower than those published in Bolhuis, et al. (1995). This implies that the total metabolised energy requirement calculated using SPEL data is lower by 24% than the results obtained from data from Bolhuis, et al., 1995. These latter data are considered to be more reliable and therefore they are used in the model.

After the analysis of data sources, we chose the most suitable for our research classification of animals, feed components and nutrient ingredients. We distinguish ten animal types (index l): horses and pony's (HOPO), calves (CACA), dairy cows (CADC), other cattle (CAOT), sheep and goats (SHGO), pigs for fattening (PIFA), sows and stock boars (PISB), piglets (PIPI), laying hens (POLH) and poultry for fattening (POFA); five compound feed components (index j): cereals (FCER), rich protein fodder (FPRO), energy rich fodder (FENE), milk and dairy products (FMIL), and other fodder (FOTH); and three nutrient ingredients (index k): metabolised energy (ENE), crude proteins (PRO) and dry matter component (DRM). The dry matter component applies only to grazing animals. The other feed components are internally (on farm) produced (compound) feed (INTF), roughage (ROUG) and suckled milk (SUMI).

5.4 Data-model

The data-model generates the farm and animal specific data about cost c_{nl} and desired content of the compound feed d_{nkl} that are unavailable in the FADN database. The FADN database provides information about cost of the compound feed purchased by farms [1] and cost of the internal feed produced on farm [2] for three animal groups: grazing animals, pigs and poultry. The data model disaggregates these cost over the animal types in the model disaggregation. Figures about nutrient content of the compound feed are not provided by the FADN data. We derive these figures using the FADN data about number of animals and some supplementary technical coefficients.

We disaggregate the compound and internal feed cost by animal type proportionally to the metabolised energy provided by compound and internal feed. In the disaggregation procedure, differences between the metabolised energy prices for different animal types are taken into account. The relative prices are calculated using the FADN data. As result, the compound and internal feed cost c_{nl} and c'_{nl} by farm and animal type are computed. The following equations are applied:

15. $c_{nl(i)} = \gamma_{ni}\ \rho_{nl(i)}\ f_{nENE\ l(i)} / (\Sigma_{l(i)}\ \rho_{nENEl(i)}\ f_{nENEl(i)})$

16. $c'_{nl(i)} = \gamma'_{ni}\ \rho_{nl(i)}\ f_{nENE\ l(i)} / (\Sigma_{l(i)}\ \rho_{nENEl(i)}\ f_{nENEl(i)})$

where the following represent:

i	- animal group index: pigs (i = PI), poultry (i=PO) and grazing animals (i=GA);
$l(PI)$	- different types of pigs (l(PI)= PIFA, PISB, PIPI);
$l(PO)$	- different poultry types (l(PO)= POLH, POFA);
$l(GA)$	- different types of grazing animals (l(GA)=HOPO, CACA, CADC, CAOT, SHGO);
γ_{ni}, γ'_{ni}	- cost of the compound and internal feed respectively for different animal groups;
$\rho_{nENEl(i)}$	- metabolised energy price index;
$f_{nENEl(i)}$	- total metabolised energy provided by the compound and internal feed.

[1] In the FADN, this cost is called the purchased feedingstuffs for pigs and poultry and purchased concentrated feedingstuffs for grazing animals. It includes not only feed cost but also some other costs.

[2] In the FADN, this cost is called the feedingstuffs produced and used on the farm and includes only the marketable products used as feedingstuffs. Therefore, data on these feedingstuffs do not provide full information about the cost of internally produced and used feed.

The minimal nutrient content f_{nkl} of the compound and internal feed for pigs and poultry is given by the formula:

17. $f_{nkl} = \omega_{kl}\, \mu_{nl}$ for l = PIFA, PISB, PIPI, POLH, POFA; k= ENE, PRO

where ω_{kl} denotes nutrient requirement per animal. We assume that the compound and internal feed provide just the minimal amount of metabolised energy required for pigs and poultry [1].

The formula (17) is not applicable for grazing animals because they eat roughage and the amount of roughage used to feed these animals is unknown. According to feed norms, roughage has to provide some minimal amount of the metabolised energy for grazing animals [2]. Therefore, for grazing animals we assume that $f_{nENEl(GA)}$ is equal to the maximal amount of the metabolised energy which can be provided by the compound and internal feed. The following formula is applied:

18. $f_{nENEl} = (1-\alpha_{ENEl})\, \omega_{ENEl}\, \mu_{nl} - \sigma_{ENEl}\, \mu_{nl}$ for l = HOPO, CACA, CADC, CAOT, SHGO

where the following represent:

α_{kl} - minimal roughage share in total nutrient supply;

σ_{kl} - suckled nutrient ingredients per animal type.

To calculate the desired nutrient contents of the compound and internal feed (d_{nkl} and d'_{nkl} respectively) for pigs and poultry, we disaggregate the minimal nutrient contents f_{nkl} (see 17) of the compound and internal feed proportionally to the compound and internal feed cost obtained from formulas (15) - (16). In the disaggregation procedure, differences between the nutrient prices for the compound and internal feed are taken into account. The price correction coefficients are calculated using the FADN data. This results in the following formulas:

19. $d_{nkl} = (\phi_{kl}\, c_{nl}\, f_{nkl})/(\phi_{kl}\, c_{nl} + c'_{nl})$ for l = PIFA, PISB, PIPI, POLH, POFA; k= ENE, PRO

20. $d'_{nkl} = (c'_{nl}\, f_{nkl})/(\phi_{kl}\, c_{nl} + c'_{nl})$ for l = PIFA, PISB, PIPI, POLH, POFA; k= ENE, PRO

where ϕ_{kl} represents nutrient price of the internal feed compared with the nutrient price of the compound feed.

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[1] See footnote 1 page 14.

[2] The feed norms provide the coefficient α_{kl} for the minimal dry matter contents of feed provided by roughage. We applied the same coefficient for the metabolised energy.

The calculation procedure described above cannot be applied to grazing animals, because for these animals the minimal amount of nutrient ingredients provided by the compound and internal feed cannot be calculated. To calculate the desired nutrient contents of the compound and internal feed for grazing animals we assume that the unit prices of the nutrient components for these animals are proportional to the average unit price of the nutrient components for pigs and poultry. The proportionality coefficient takes into account differences between the nutrient prices for grazing animals and pigs and poultry and is calculated using the FADN data. The following formulas are used:

21. $d_{nkl} = c_{nl} / \pi_k$ for l = HOPO, CACA, CADC, CAOT, SHGO; k= ENE, PRO

22. $d'_{nkl} = c'_{nl} / \pi'_k$ for l = HOPO, CACA, CADC, CAOT, SHGO; k= ENE, PRO

where:

23. $\pi_k = \varepsilon(\sum_{i=PI,PO}\sum_{nl(i)} c_{nl})/(\sum_{i=PI,PO}\sum_{nl(i)} d_{nkl})$ for k= ENE, PRO

24. $\pi'_k = \varepsilon(\sum_{i=PI,PO}\sum_{nl(i)} c'_{nl})/(\sum_{i=PI,PO}\sum_{nl(i)} d'_{nkl})$ for k= ENE, PRO

and where π_{nk} and π'_{nk} represent nutrient prices for compound and internal feed respectively and ε is a relative nutrient price for grazing animals compared with the nutrient price for pigs and poultry.

Finally, we calculate the roughage intake for grazing animals as a closing variable for the metabolised energy balance:

25. $y_{nl} = ((\omega_{ENEl} - \sigma_{ENEl})\ \mu_{nl} - d_{nENEl} - d'_{nENEl})/\kappa_{ENEl}$ for l = HOPO, CACA, CADC, CAOT, SHGO

where y_{nl} denotes roughage used to feed grazing animals and κ_{kl} is the nutrient contents of a unit of roughage. This in turn allows us to apply the following formula to calculate the desired dry matter contents of the compound feed for grazing animals:

26. $d_{nDRMl} = \alpha_{DRMl}\ y_{nl}\ (1+ c'_{nl}/(\phi_{DRMl}\ c_{nl})$ for l = HOPO, CACA, CADC, CAOT, SHGO

This formula results from the feed norms for grazing animals according to which some minimal dry matter contents of feed has to be provided by roughage.

6 Estimation results of the compound feed model for the Netherlands

In this section we report results obtained from estimating the compound feed model for the Netherlands. The model was estimated using data for 50 aggregated farms obtained by aggregation of farms represented in FADA database. To program, estimate and test the compound feed model the General Algebraic Modelling System (GAMS) was used. To solve a model with the required precision 46 iterations and almost 4 hours were necessary [1]. Testing a model takes 10.5 hours. The biggest optimisation problem is the outer problem, which contains almost 3,400 equations and 3,600 variables.

To apply the model (1) - (4), we assumed that the mix function has the constant return to scale Cobb-Douglas form [2], i.e.:

27. $q_{nl} = \beta_{0l} \prod_j v_{njl}^{\beta jl}, \qquad \sum_l \beta_{jl} = 1$

As a starting point for our estimation, we used betas which were equal to shares of the compound feed components in the total metabolised energy provided by the compound feed for different animal types. These shares were calculated using data published in Helming et al., 1995 [3].

[1] We assume that the sum of relative differences between parameters' values obtained in two following iterations should be lower than 0.1%.

[2] To ensure that $\sum_l \beta jl = 1$, we calculate one parameter residually in the program.

[3] Helming et al., 1995 provide data only for cattle, pigs, poultry for fattening, laying hens and other animals. Therefore we applied betas calculated for cattle for all types of cattle distinguished in our model. The same procedure was applied for pigs.

The measures of fit for the compound feed model for the Netherlands:

- the percentage estimation error [a)]: $5*10^{-8}$
- the correlation coefficient [b)]: 0.991

The accuracy of the micro-macro consistency conditions [c)]:

- cereals (FCER) 92.8%
- rich protein fodder (FPRO) 101.6%
- energy rich fodder (FENE) 74.5%
- milk and dairy products (FMIL) 100.0%
- other fodder (FOTH) 90.6%

Note:

a) Sum of absolute differences between the observed farmers' expenditures on the compound feed and the estimated feed costs related to the total observed farmers' expenditures on the compound feed;

b) The correlation coefficient between the observed farmers' expenditures on the compound feed and the estimated feed costs;

c) Accuracy of matching of the observed macro data V_j on available quantities of the feed components by the estimated micro data v^*_{nl} on the compound feed components calculated as a percentage of macro-data covered by micro-data.

Source: Owncalculations.

Figure 1 Goodness of fit indicators

To calculate a measure of fit for the compound feed model (1) - (4) for the Netherlands, we compared the observed compound feed cost c_{nl} obtained from the micro-data and the theoretical cost r_l $F(\boldsymbol{\beta}_l^*, v^*_{nl})$ calculated from the model. Figure 1 shows some indicators of goodness of fit. They indicate that the model fits the data very well. The micro-macro consistency conditions perform quite well, however, the use of cereals, energy rich fodder and other fodder are underestimated by 7.2%, 25.5% and 9.4% respectively.

Table 1 Estimation results:

- mathematical programming method

l/j	β_{jl} FCER	FPRO	FENE	FMIL	FOTH	β_{0l}
HOPO	0.229	0.252	0.361	0.011	0.147	3.919
CACA	0.011	0.395	0.424	0.028	0.141	3.266
CADC	0.026	0.375	0.411	0.047	0.140	3.265
CAOT	0.012	0.394	0.429	0.030	0.141	3.266
SHGO	0.224	0.251	0.361	0.015	0.148	3.919
PIFA	0.056	0.259	0.319	0.110	0.157	3.594
PISB	0.132	0.380	0.315	0.046	0.128	3.596
PIPI	0.136	0.378	0.316	0.041	0.128	3.596
POLH	0.338	0.236	0.132	0.056	0.238	5.596
POFA	0.249	0.424	0.065	0.100	0.162	4.333

- Jack-knife method

l/j	β_{jl} FCER	FPRO	FENE	FMIL	FOTH	β_{0l}
HOPO	0.229	0.252	0.361	0.011	0.147	3.919
	(+∞)	(5300)	(+∞)	(500)	(4600)	(+∞)
CACA	0.012	0.395	0.424	0.028	0.141	3.266
	(6.729)	(290)	(680)	(500)	(700)	(+∞)
CADC	0.027	0.375	0.411	0.047	0.141	3.265
	(4.945)	(80.493)	(210)	(220)	(560)	(+∞)
CAOT	0.012	0.393	0.423	0.030	0.141	3.266
	(5.696)	(240)	(550)	(450)	(560)	(+∞)
SHGO	0.224	0.251	0.361	0.015	0.146	3.919
	(+∞)	(880)	(6.600)	(160)	(1.100)	(+∞)
PIFA	0.057	0.357	0.319	0.110	0.157	3.595
	(5.806)	(32.788)	(99.914)	(170)	(41.956)	(+∞)
PISB	0.132	0.379	0.315	0.046	0.128	3.596
	(61.027)	(860)	(750)	(28.118)	(92.465)	(+∞)
PIPI	0.136	0.378	0.316	0.041	0.129	3.596
	(120)	(170)	(490)	(68.052)	(120)	(+∞)
POLH	0.338	0.235	0.132	0.056	0.238	5.184
	(110)	(110)	(400)	(22.664)	(100)	(+∞)
POFA	0.249	0.424	0.065	0.100	0.163	4.333
	(160)	(64.799)	(97.364)	(34.321)	(35.950)	(+∞)

Note: T-student statistic in brackets. +∞ means the T-student statistic higher than 10,000. All parameters are significant at 0.0005% significance level.
Source: Own calculations.

In Table 1, the estimated coefficients of the Cobb-Douglas mix functions and results of the Jack-knife testing are presented. All estimated coefficients are significantly different from zero at the 0.0005% significance level. We conclude that estimates are statistically reliable.

Table 2

Compound feed structure (metabolised energy units):

	HOPO	CACA	CADC	CAOT	SHGO
FCER	0.301	0.015	0.036	0.016	0.279
FPRO	0.332	0.424	0.410	0.423	0.253
FENE	0.362	0.448	0.441	0.448	0.358
FMIL	0.001	0.003	0.005	0.003	0.001
FOTH	0.104	0.110	0.108	0.110	0.109
	PIFA	**PISB**	**PIPI**	**POLH**	**POFA**
FCER	0.098	0.094	0.200	0.471	0.386
FPRO	0.413	0.674	0.387	0.214	0.407
FENE	0.368	0.174	0.324	0.122	0.063
FMIL	0.010	0.004	0.004	0.006	0.011
FOTH	0.111	0.054	0.085	0.187	0.133

Total feed structure (metabolised energy units):

	HOPO	CACA	CADC	CAOT	SHGO
FCER	0.054	0.004	0.008	0.004	0.080
FPRO	0.042	0.106	0.087	0.103	0.072
FENE	0.065	0.112	0.094	0.109	0.102
FMIL	$2.3*10^{-4}$	$7.6*10^{-4}$	0.001	$7.9*10^{-4}$	$4.2*10^{-4}$
FOTH	0.019	0.028	0.023	0.027	0.031
INTF	0.095	0.110	0.099	0.111	0.141
ROUG	0.724	0.634	0.688	0.644	0.508
SUMI		0.005			0.066
	PIFA	**PISB**	**PIPI**	**POLH**	**POFA**
FCER	0.098	0.093	0.199	0.471	0.385
FPRO	0.410	0.671	0.385	0.214	0.407
FENE	0.367	0.173	0.322	0.122	0.063
FMIL	0.010	0.004	0.004	0.006	0.011
FOTH	0.111	0.054	0.085	0.187	0.133
INTF	0.004	0.005	0.005	$5.0*10^{-4}$	0.001

Source: Own calculations.

We used the estimated v^*_{nl} to calculate the compound feed composition for animal types and the feed utilisation matrix (FUM). Table 2 provides data on the compound feed structure and total feed structure for the animal types. The estimation results show that the main components of compound feed for cattle and pigs are protein rich fodder and energy rich fodder. Their total share amounts to more than 71%. There are three main compound feed components for horses, pony's, sheep and goats: cereals, protein rich fodder and energy rich fodder. Their total share in compound feed is about 89%. Cereals and protein rich fodder provide about 69% of the metabolised energy for poultry. These outcomes are consistent with estimations results obtained for the mix function. The most important compound feed components have the largest parameter values. The feed composition obtained is in general consistent with results, which can be calculated from data published in Helming et al., 1995 (see Table 3).

Table 3 Differences between the compound feed structure calculated using the compound feed model and the compound feed structure obtained form data provided by Helming et al., 1995.

	Cattle	Pigs for	Poultry for fattening	Laying hens	Other animals
FCER	2	-5	0	5	5
FENE	1	0	1	-1	-1
FMIL	-1	0	1	1	0
FOTH	-3	-2	1	-3	-4
FPRO	1	7	-3	-2	0

Source: Own calculations.

According to results presented in Table 2, compound feed satisfies almost 100% of demand for metabolised energy for pigs and poultry. The remaining demand, which is less than 0.5% of the total demand, is satisfied by internal feed. For grazing animals, compound feed satisfies between 18% and 29% demand for metabolised energy. The rest of metabolised energy is provided by internal feed (between 9.5% and 14%), roughage (from 51% to 73%) and, for calves and sheep and goats, by suckled milk (0.5% and 6.6% respectively).

According to our results, there is a surplus of crude proteins in compound feed. All animal types except sows and stock boars get more crude proteins than required. The surplus is equal to 76% on average.

Table 4 provides the structure of the feed utilisation matrix. The distribution of the compound feed components can be characterised as follows:

- cereals are mainly consumed by poultry (65.5% of the total use);
- protein rich fodder is mainly consumed by pigs for fattening (30%) and poultry for fattening (23.7%);
- energy rich fodder is mainly used by dairy cows and pigs for fattening (19.4% and 41.2% respectively);
- milk and dairy products and other fodder are mainly consumed by poultry for fattening (42% and 31.4% respectively) and pigs for fattening (30.7% and 23.3% of the total use respectively).

Using the estimated data on individual farms, we can also calculate farm specific feed characteristics including the feed utilisation matrices (FUM's). Analysis of these data shows that the compound feed composition for given animal type is the same for all farms [1]. This is because only one constraint of the compound feed model (1) - (4) is binding for the optimal solution and because we used the constant returns to scale mix function. This result is, however, consistent with the theoretical specification of the compound feed model that describes behaviour of the compound feed industry. In this context it is reasonable to assume that compound feed production technology does not depend on characteristics of individual farms.

Table 4 Structure of the feed utilisation matrix FUM (quantities)

	FCER	FPRO	FENE	FMIL	FOTH
HOPO	0.003	0.001	0.003	$3.2*10^{-4}$	0.002
CACA	0.003	0.032	0.053	0.011	0.032
CADC	0.027	0.113	*0.194*	0.076	0.116
CAOT	0.004	0.039	0.066	0.018	0.040
SHGO	0.018	0.006	0.014	0.002	0.010
PIFA	0.166	*0.300*	*0.419*	*0.420*	*0.314*
PISB	0.048	0.150	0.061	0.047	0.047
PIPI	0.076	0.064	0.083	0.036	0.054
POLH	*0.237*	0.058	0.051	0.082	0.152
POFA	*0.418*	*0.237*	0.056	*0.307*	*0.233*

Note: The most significant shares are in italic.
Source: Own calculations.

[1] It means that FUM's for farms are linearly dependent.

This last result has however two negative consequences. First, it hampers the possibility to apply the standard regression method to estimate and test statistically the following relationship between the observed and theoretical compound feed cost:

28. $c_{nl} = r_l F(\boldsymbol{\beta}_l, v^*_{nl})$

where the estimated values v^*_{nl} are treated as the given data.

Second, independence of the compound feed composition and farm characteristics means degeneration of the model what can be a source of numerical problems when the model is solved. There are three possibilities to overcome this problem in the future. First, we can use the decreasing return to scale mixing function F instead of constant return to scale function. Second, we can assume the mix function is farm specific, which can by done by introducing some farm dependent variables in its specification. Thirdly, we can assume that it is a *trade-off* between feed components purchased and produced by farms. This means that the nutritional constraints and feasibility constraints of the compound feed model (1) - (4) should include the compound feed produced by farms (called 'the internal feed' in our paper) [1].

[1] We applied the first solution proposed above in our investigation, but it did not improve the estimation results significantly. The two other solutions lead to a model explaining livestock producers' behaviour. It will be a subject of future research.

7 Conclusions

In this paper we developed a new method for the joint estimation of a feed demand model and feed utilisation matrices based on farm data supplemented the macro-data. The theoretical framework for our method is a non-linear programming model describing profit-maximising behaviour of compound feed producers. The specification of the model ensures continuous and smooth feed allocation responses to price changes and allows to include linear restrictions to account for both engineering information and other a priori restrictions. To estimate the compound feed model, a three-step iterative procedure was developed. The jack-knife method was use to assess the reliability of the estimates.

The estimation procedure uses individual farm data while additional macro-data are used to ensure the consistency of micro-estimates with macro-aggregates. In this way micro-macro consistency is maintained. To generate unavailable farm data necessary for the estimation, a special data-model that generates unavailable figures from available farm data was developed.

The method proposed was applied to farm data for the Netherlands provided by the European Farm Accountancy Data Network. Estimation results show that the model performs very well. The model parameters were estimated with high precision but at high computational costs. The feed utilisation matrix obtained is consistent with results presented in other sources. The estimation procedure was, however, slow and should be improved.

The developed method provides a consistent framework, which can be used to estimate feed utilisation matrices as well as other unobserved macro-data from micro (farm) figures. Moreover, the usage of farm data allows for a relatively high disaggregation of the model in terms of numbers of products and production factors.

The modelling framework proposed in this paper can be extended to model compound feed and roughage production on farms consistently with compound feed production by the feed industry. In this case the compound feed model should be reformulated to describe farmers behaviour with respect to animal production. It should take also into account the vertical integration of the compound feed industry and farms having livestock production. The advantage of such an approach would be the consistent modelling of the whole feed-livestock sector. Such a model could be easily extended to represent the farmer's decision process concerning all agricultural production. Placed in a partial or general equilibrium framework, such a model would be a powerful tool to answer policy questions related to particular farms.

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