LABORATORY OF PHYSICS AND METEOROLOGY
DIRECTOR PROF. DR W. R. VAN WIJK
AGRICULTURAL UNIVERSITY, WAGENINGEN, HOLLAND

A PHYSICAL THEORY ON PLACEMENT OF FERTILIZERS

C. T. DE WIT

STAATSDRUKKERIJ UITGEVERIJBEDRIJF

VERSL. LANDBOUWK. ONDERZ. NO. 59.4 - 'S-GRAVENHAGE - 1953

CONTENTS[1]

[1] De auteur promoveerde op een gelijkluidend proefschrift tot Doctor in de Landbouwkunde aan de Landbouwhogeschool te Wageningen op 10 Juli 1953.

The investigation was performed with the financial aid of the Agricultural Research Council T.N.O., 's-Gravenhage.

I. INTRODUCTION

Artificial fertilizers are of considerable importance in supplementing the supply of plant nutrients. Many investigations have been carried out to determine the best chemical compound, the right amount of fertilizer and the optimum time of application. The method of application, which may be equally important, has received less attention.

Generally, the fertilizers are broadcast over the surface of the soil before or after ordinary cultivation operations. Until some 40 years ago, this procedure was the only one practicable in Western Europe because the seed of common crops was broadcast also. Where the seed is drilled in rows, however, it is feasible to place the fertilizers in bands some distance from the seed. This practice has become common in regions where extensive cropping is practiced and where the seed is drilled in wide-spaced rows. Drilling fertilizer and seed in one operation results in a saving of both fertilizer and labour.

Particularly in the United States, knowledge of the relative value of different methods of fertilizer application has expanded rapidly. The National Joint Committee on Fertilizer Application, founded in 1925, has stimulated investigations on these problems, resulting in a great number of publications (SALTER, 1938). The results of Russian investigations are not so well known in Western Europe and the U.S., although they are of equal importance (AVDONIN, 1949). Under favourable conditions, up to 90 % of the fertilizer has been saved by proper placement in these countries.

Until World War II the American and Russian investigations did not receive much attention in Western Europe, since it was the general opinion, that important savings were possible only in regions where extensive cropping was practiced. When it became necessary to restrict the use of fertilizer during the war, however, more economical use of fertilizers for grain crops was obtained in England by means of localized fertilizer placement. A number of simple field trials were carried out there by the „County Technical Development Sub-Committees". It appeared that on soils seriously deficient in phosphate, more than 50 % of the fertilizer could be saved by drilling seed and fertilizer together (combine-drilling). The farmers adopted this method rapidly („*Agriculture*", 1945). The results were published after the war, and investigations on other crops and placement methods have been carried out (e.g. COOKE, 1949, 1951; STEWART, 1949). Row placement attracted attention in Holland also, and experiments on this subject were started at the Agricultural Experimental Station in Groningen in 1947. Preliminary results of these experiments were published by PRUMMEL (1950).

Despite the extensive work that has been done on fertilizer placement methodology, a theoretical interpretation of the general problem has not heretofore been attempted. A theoretical interpretation would be of considerable practical value in explaining apparently contradictory results, in unifying existing data, and in reducing future work.

A preliminary account of a general theory of fertilizer placement has already been published by VAN WIJK and DE WIT (1951). The present paper gives a detailed account of the development of the theory, and shows the basic data upon which the theory has been constructed.

II. AN OUTLINE OF THE THEORY

1. The basis of comparison

The calculation of the yield for a wide range of fertilizer placement patterns is possible if the relationship between yield, uptake and rate of fertilizer application is known in an arbitrary case, e.g. broadcasting. In this calculation, the different methods are compared in terms of equal amounts of fertilizer per unit of area that actually receives fertilizer, instead of the usual procedure in which the methods are compared in terms of equal amounts of fertilizer per hectare.

The basis of comparison is clarified by figure 1, in which two application patterns are compared on the same field under otherwise equal conditions. In the left hand side of the figure, a common broadcast pattern is given. The plant rows are X_b centimeters apart[1]. In the right hand side of the figure, a placement pattern is given. The fertilizer is applied in bands X_r centimeters wide and parallel to the crop rows. It is assumed that the same amount of fertilizer is applied per unit area of the fertilized parts of the soil in both cases. The amount of broadcast fertilizer is therefore M_b kg per ha, and the amount of placed fertilizer is $(X_r/X_b)\ M_b$ kg per ha. It is further assumed that in both cases the fertilizer is distributed in the same way under the soil surface. For the placement method, the concentration of the fertilizer in the soil is therefore either zero or exactly the same as in the broadcast method.

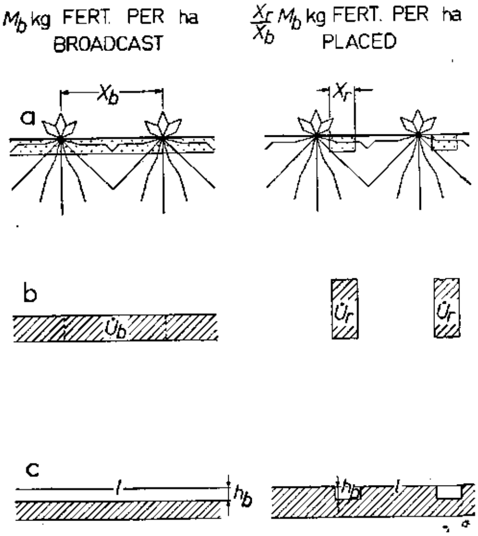


Figure 1.
The basis of comparison between the effect of broadcast (left) and placed (right) fertilizer.
a. X_b = distance between the crop rows. X_r = width of the fertilizer band. M_b kg fertilizer per ha broadcast and $(X_r/X_b)M_b$ kg fertilizer per ha placed.
b. $\dot{U}_b$ = uptake rate from broadcast fertilizer. $\dot{U}_r$ = uptake rate from placed fertilizer. These rates are compared with X_r and X_b in figure 2.
c. Line l = level of availability of a particular nutrient other than the one applied. h_b = the intensity of an unfavourable effect of the broadcast application of M_b kg of the fertilizer per ha on this level of availability. In many cases h_b equals zero.

By varying the width of the bands from 0 to X_b, and the distance from the centre of the bands to the seed row from 0 to $0.5X_b$, a large number of placement methods can be compared with the broadcast method. Since the diffusion of ions in a horizontal direction is negligible, the reactions between soil and fertilizer, such as ion-exchange, fixation and leaching phenomena, are the same for both patterns. The availability to plants of the fertilizer per unit area of fertilized soil is thus likewise the same for both patterns.

[1] List of notation and conversion tables on page 68.

2. Uptake of the nutrient as a function of the fertilized area

There is, however, an important difference in uptake of the given nutrient by the plant from broadcast and placed fertilizer, owing to the fact that a smaller part of the root system is in contact with fertilized soil where the fertilizer is placed than where it is broadcast. This difference is explained by means of figure 2. The abscissa of the

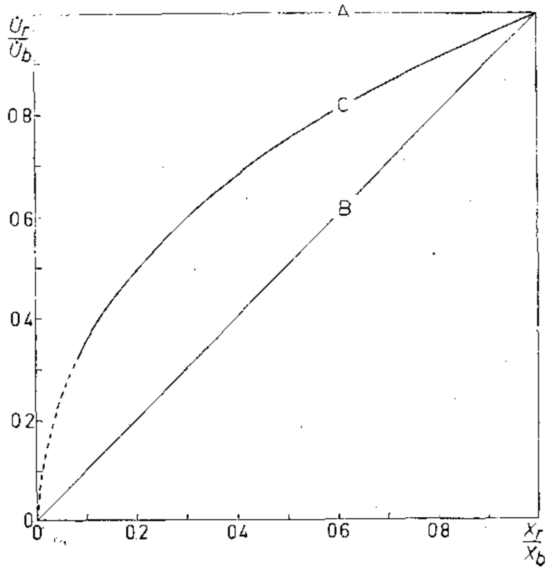


FIGURE 2.
The relationship between the ratio of the uptake rates from placed ($\dot{U}_r$) and broadcast ($\dot{U}_b$) fertilizer and the ratio between the width of the fertilizer band (X_r) and the distance between the crop row (X_b) for the condition described in figure 1a and 1b. If the uptake from fertilizer is small compared with the amount applied, the ratio of the total quantities absorbed from the fertilizer (U_r/U_b) is equal to the ratio $\dot{U}_r/\dot{U}_b$. Line A is to be expected if the rate of uptake from the fertilizer is independent of the fertilized area. Line B is to be expected if the rate of uptake from the fertilizer is directly proportional to the fertilized area. Line C represents the actual relationship.

figure represents the ratio X_r/X_b of the area fertilized by the placed or band application to that fertilized by the broadcast application. Since X_r varies from zero to X_b, this ratio varies from zero to unity. Along the ordinate, the ratio of the rate of nutrient uptake from the placed fertilizer[1] ($\dot{U}_r$) to the rate of nutrient uptake from the broadcast fertilizer ($\dot{U}_b$), represented by $\dot{U}_r/\dot{U}_b$, is given for the condition that the concentration of the fertilizer is the same in the parts of the soil actually fertilized.

It is evident that if X_r is equal to zero, $\dot{U}_r$ and $\dot{U}_r/\dot{U}_b$ likewise equal zero, and also that if X_r is equal to X_b, the placement pattern is identical to the broadcast pattern, so that $\dot{U}_r/\dot{U}_b$ is equal to unity. The points (0, 0) and (1, 1) thus satisfy the functional relationship between these two ratios.

It is logical to expect that a decrease of the fertilized part of the root system cannot cause an increase in rate of nutrient uptake from the fertilizer, since the concentration in the fertilized parts remains the same. Therefore, the line $\dot{U}_r/U_b = 1$, represented in the figure by line A, must be an upper limit of the relationship.

If the roots of the plant should act independently of each other, the relationship $\dot{U}_r/\dot{U}_b = X_r/X_b$, represented in figure 2 by line B, should result. This line must be a lower limit. Furthermore, it is logical that the ratio $\dot{U}_r/\dot{U}_b$ increases gradually with increasing X_r/X_b.

Thus, it must be concluded that the relationship between $\dot{U}_r/\dot{U}_b$ and X_r/X_b is represented by a gradually increasing curve, such as line C, which passes through the points (0, 0) and (1, 1) and lies in the area surrounded by the ordinate and the lines A and B.

[1] By rate of uptake is meant the quantity of the nutrient absorbed from the fertilizer per unit time.

Since the total uptake from the fertilizer is in many cases small compared with the total amount applied (for exceptions see III, 4; IX, 3 and 4), the concentration of the fertilizer in the soil is in many cases materially independent of the total uptake. Where these conditions are fulfilled, the above considerations hold for the total uptake from the fertilizer as well as for the rate of uptake from equal concentrations. In the ensuing development, U_b and U_r are designated, respectively, as the total uptake from the broadcast and placed fertilizer[1].

An analysis of data available in the literature (III) shows that as a first approximation the relationship between X_r/X_b and U_r/U_b is independent of the kind of crop, the kind of fertilizer and the environmental conditions. The calculated relationship, called the compensation function, is represented in figure 18 by line C.

If in a certain case the relationship between fertilizer rate and uptake for broadcast fertilizer is known, the compensation function may be used to calculate this relationship for placement patterns with $0<X_r<X_b$ under the same conditions. For purposes of illustration, let it be assumed that the increase in uptake of a particular nutrient resulting from fertilization is equal to U_b kg/ha when M_b kg/ha is broadcast. From the compensation function (figure 2), a placement method with $X_r = 0.5\ X_b$ is found to have a compensation factor of 0.75. The increase in uptake from fertilization is thus 0.75 U_b kg/ha if 0.5 M_b kg/ha is placed under half the soil area with vertical distribution identical with that of the broadcast fertilizer. The whole rate-uptake curve can now be calculated for this placement method by multiplication of the increase in uptake of the nutrient from broadcast fertilization by 0.75, adding the resulting figure to the uptake without fertilizer and plotting the result at a fertilizer rate of one-half the broadcast fertilizer rate (i.e., at 0.5 M_b). Figure 3 shows some curves constructed in this way. For the construction of the curves corresponding to $X_r = 0.25\ X_b$ and $X_r = 0.125\ X_b$ the compensation factors 0.55 and 0.41, respectively, have been used. (The source of the numerical values of the compensation factor will be made evident in III, 3).

It is obvious that with low application rates the increase in uptake from fertilization increases with decreasing width of the fertilizer band. The same is not true with high application rates, however, owing to the fact that the curves for placed fertilizer flatten at progressively lower rates as the band width is decreased. The point of intersection of the curves for broadcast and placed fertilizer thus shifts to lower uptakes and rates as the band width is decreased.

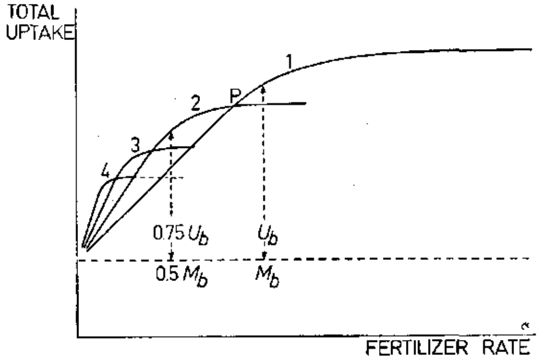


FIGURE 3.
The relationship between fertilizer rate and total uptake for different methods of application.
Curve 1 for broadcast fertilizer ($(X_r/X_b) = 1$), Curve 2 ($(X_r/X_b) = 0.5$), Curve 3 ($(X_r/X_b) = 0.25$) and Curve 4 ($(X_r/X_b) = 0.125$) for placed fertilizer, calculated by means of the compensation function. **P** = point of intersection between the curves 1 and 2.

[1] It is shown in III, 1 that the uptake from the fertilizer can be safely replaced by the increase in uptake resulting from fertilization.

3. The influence of the fertilizer pattern on the relationship between uptake and yield

For practical purposes it is desirable to be able to predict for a variety of fertilizer patterns the relationship between fertilizer rate and yield. It is thus necessary to take into account the possible modifications of the relationship between the total nutrient uptake and yield that may result from variations in the pattern of placed fertilizer.

The most common type of relationship between nutrient uptake and yield is reproduced in figure 4. The portion of the curve in which the yield increases markedly with increasing uptake is called the region of increase, and the portion of the curve in which there is little or no increase in yield with increase in uptake is called the region of luxury consumption.

Now the place where a certain quantity of a nutrient is taken up, apparently does not influence the use thereof by the plant. It is presumably for this reason that the

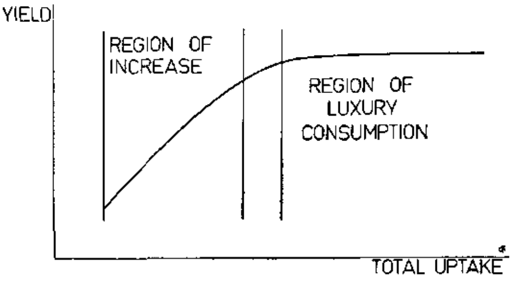


FIGURE 4. A representative relationship between total uptake and yield. In the region of luxury consumption, there is little or no increase in yield with increase in nutrient uptake. This relationship is in many cases independent of the fertilizer pattern.

relationship between uptake and yield is in most cases independent of the fertilizer pattern. One important exception must be made, however. If application of the fertilizer decreases the availability of some other nutrient in the soil, the effect is less serious where the fertilizer is placed than where it is broadcast. As a consequence, the yield corresponding to a given quantity of the fertilizer nutrient absorbed by the plant, may be greater with placed fertilizer than with broadcast fertilizer. The reverse situation, in which fertilization increases the availability of some nutrient not contained in the fertilizer, would result in a greater yield with broadcast than with placed fertilizer for a given quantity of the fertilizer nutrient absorbed by the plant. In practice, this latter situation is probably of minor importance.

4. The influence of the distance between crop row and fertilizer band and the depth of application

At first thought, it would perhaps be expected that the distance between crop row and fertilizer band and the depth of application under the soil surface are of primary importance. Such does not seem to be the case in Western Europe. Aside from the injurious effects that may result from high concentration of fertilizer salts near the row, it appears that within practical limits the distance between crop row and fertilizer band is of importance only where the soil nutrient level is exceedingly low or where environmental conditions are unfavourable during early growth (V).

When the soil remains moist, as is usually the case in Western Europe, the depth of application is without important influence on the yield. In dry seasons, however, any fertilizer localized in the dry upper layer of the soil is inactive and must be considered as wasted in that year (VII, 4).

5. THE EFFECT OF PLACEMENT UNDER DIFFERENT CONDITIONS

The influence of fertilizer placement pattern on crop yield is illustrated in figure 5. This figure contains four diagrams, each corresponding to basically different influences of placed fertilizer. In each diagram the rate-uptake curve for broadcast fertilizer (unbroken lines) is given in the quadrant bounded by the axes marked R (rate) and U (uptake). The uptake-yield curves are represented in the quadrant bounded by the axes marked U (uptake) and Y (yield). These curves may be derived from suitable broadcast experiments. The rate-uptake curves that should have been found with fertilizer placed in bands having the width $X_r = 0.33\ X_b$ have been calculated by means of the compensation function, and are represented by the broken lines. The fertilizer rate-yield curves are constructed from the rate-uptake and uptake-yield curves by eliminating the uptake and are represented in the quadrant bounded by the axes marked Y (yield) and R (rate).

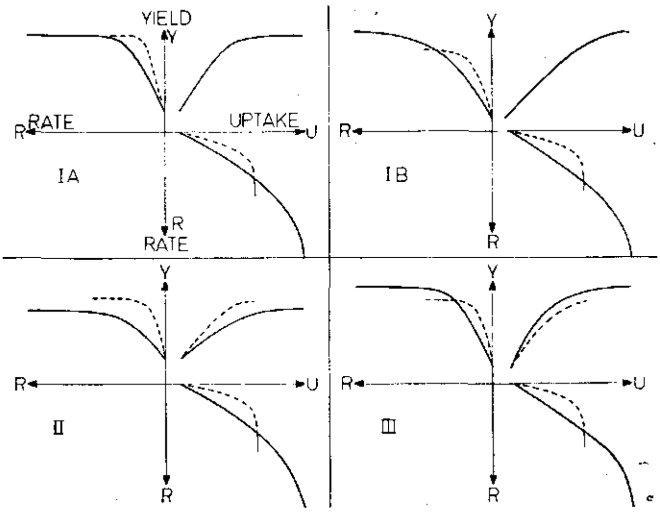


FIGURE 5.
Four diagrams (IA, IB, II, III) representing four basically different effects of placed fertilizer. *Unbroken lines:* broadcast pattern. *Broken lines:* placement pattern with $(X_r/X_b) = 0.33$. The uptake-rate curve and the uptake-yield curve for broadcast fertilizer are taken from experiments. The uptake-rate curve for placed fertilizer is calculated by means of the compensation function. The rate-yield curves are derived from the rate-uptake and the uptake-yield curves by eliminating the uptake.

Case IA: Uptake-yield curve independent of the fertilizer pattern. Point of intersection of the rate-uptake curves within the region of luxury consumption. The same maximum yields with broadcast and placed fertilizer. At lower rates higher yields with placed fertilizer.
Case IB: Uptake-yield curve independent of the fertilizer pattern. Point of intersection of the rate-uptake curves within the region of increase. At higher rates, higher yields with broadcast fertilizer. At lower rates higher yields with placed fertilizer.
Case II: More favourable uptake-yield curve with placed fertilizer. Maximum yield with placed fertilizer higher than with broadcast fertilizer.
Case III: More favourable uptake-yield curve with broadcast fertilizer. Maximum yield with placed fertilizer lower than with broadcast fertilizer.

Diagrams IA and IB of figure 5 apply to conditions where the uptake-yield curve is independent of the application pattern. In diagram IA, the uptake-yield curve has a region of luxury consumption, and the rate-uptake curve for the placed fertilizer intersects the rate-uptake curve for the broadcast fertilizer within this region of luxury consumption. Thus, the maximum yields obtainable from placed and broadcast fertilizer are the same. The amount of fertilizer necessary to get a certain yield below the maximum is lower with placed fertilizer than with broadcast fertilizer. As a result, a certain amount of fertilizer may be saved by placement. For the part of the curve in which uptake and yield increase about linearly with rate, the yields in case of broadcasting and placement are about the same if M_b kg/ha is broadcast and (X_r/X_b) (U_b/U_r) M_b kg/ha is placed The value of (X_r/X_b) (U_b/U_r), called the coefficient of equivalence, is smaller than unity and decreases with decreasing width of the fertilizer band, as shown in figure 6.

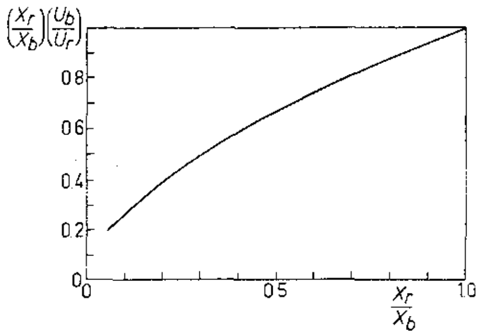


FIGURE 6.
The relationship between the coefficient of equivalence and X_r/X_b. This coefficient, (X_r/X_b) (U_b/U_r) $= (X_r/X_b)^{0.56}$ (see III, 3), gives the value by which the fertilizer rate in case of broadcasting must be multiplied to obtain the fertilizer rate that would have given the same yield in case of placement. The curve has been calculated from the compensation function and holds in Cases IA and IB (figure 5) before the point of intersection and within the region of increase.

In diagram IB of figure 5, the point of intersection between the uptake curves for broadcast and placed fertilizer lies within the region of increase and below the region of luxury consumption. The maximum yield obtainable is thus greater where the fertilizer is broadcast than where it is placed. For fertilization rates in excess of that corresponding to the point of intersection, therefore, placement is less efficient than broadcasting. For lower rates, placement is more efficient than broadcasting, as in diagram IA.

Thus, although for lower rates more fertilizer is saved by placing the fertilizer in narrower bands (compare figure 3), the maximum yield at higher rates can be diminished. This effect limits markedly the conditions under which placement of fertilizer in narrow bands is advantageous.

Diagram II of figure 5 represents the situation in which the uptake-yield curve is improved by placing the fertilizer. In this case it is possible to obtain a higher maximum yield by placing the fertilizer than by broadcasting it.

In diagram III the uptake-yield curve is unfavourably affected by placing the fertilizer. The beneficial effects of placement are reduced by this phenomenon.

Examples of the occurrence of these different cases in practice (figures 7 to 17) are discussed in subsequent chapters. Since the data in these figures have been used for the calculation of the compensation function, the rate-uptake curves for the placement methods have not been calculated, but have been fitted to the observations. The inset-graphs concern the calculation of the ratio U_r/U_b.

III. THE COMPENSATION FUNCTION

1. The uptake from soil and fertilizer

In figure 1, the fertilized volumes are represented schematically for placed and broadcast fertilizer. The respective volumes are in the ratio X_r/X_b. Since diffusion of elements in a horizontal direction in soil is very small (Sayre and Clark, 1935), the only significant displacement of the fertilizer with time will be in a vertical direction. Despite a change in vertical distribution of the fertilizer with time, however, the broadcast and placed fertilizer will be equally affected. As a result, the ratio X_r/X_b will remain unchanged. From the instant the roots are developed under the entire soil surface, the relative availabilities of the placed and broadcast fertilizer will be in the ratio X_r/X_b. As indicated in II, 2 a simple relationship exists between the volume ratio X_r/X_b and the nutrient uptake ratio U_r/U_b if the same amounts of fertilizer are applied per unit of surface area. To determine this relationship experimentally, the uptake from the fertilizer must be calculated from a number of application experiments.

The uptake of the nutrient from the fertilizer[1] can be estimated by subtracting the uptake of the nutrient by the crop grown without fertilizer from the uptake of the nutrient by the crop grown with fertilizer (that is, by calculating the increase in uptake from fertilization), as has been done by several investigators (Russell and Watson, 1940). This procedure involves the assumption that the uptake of the nutrient from the soil is the same in the presence as in the absence of the fertilizer.

Recent experiments with radioactive phosphate (Spinks and Barber, 1947, 1948; Spinks and Dion, 1949; *Soil Science*, 1949) indicate that this is not always the case.[2] If the fertility level of the soil is low, the uptake of the nutrient from the soil usually increases somewhat with increasing fertilizer rates, as can be seen from table 1.

Table 1. Uptake of phosphorus by tobacco from soil and fertilizer, as determined by means of radioactive P (compiled from Woltz et al., (1949); exp.: Oxford + P; –Ca)

Placement pattern and amount of fertilizer per acre	Yield per acre	Total uptake per acre	Uptake from soil per acre
	lb	lb	lb
No phosphate	988	5.0	5.0
40 lb mixed in rows	1618	7.4	4.0
80 lb mixed in rows	2085	10.6	5.4
40 lb placed in bands	1618	8.0	5.6
40 lb side dressed 10 days after planting	1616	8.2	5.6

[1] The distribution of the nutrient between the different parts of the plant varies with the total amount absorbed (van Itallie, 1938). Therefore, in this paper consideration is given only to the total quantity of the nutrient absorbed and not to the quantity of the nutrient in any particular part of the plant. Except for root crops, the amounts of elements in the roots are small compared with the total amounts absorbed (Pfützer, 1933). Except for root crops, therefore, the quantity of the nutrient contained in the roots can be neglected.

[2] If kinetic exchange between soil and fertilizer phosphorus takes place, the net uptake from the soil is overestimated by means of experiments with radioactive fertilizers. It has been shown by Mc Auliffe et al. (1947) and by Wiklander (1950) that such exchange reactions occur in the soil. Even by means of radioactive isotopes, therefore, it is not possible to determine exactly the net amount of phosphorus taken up from the soil.

On the other hand, with soils having a high nutrient level, the uptake from the soil may decrease somewhat with increasing fertilizer rates (table 2).

TABLE 2. Uptake of phosphorus by potatoes from a high-phosphate soil fertilized with different amounts of radioactive superphosphate (compiled from JACOB (1949); exp.: Long Island; P-level 868 lbs)

Amount of phosphate applied in bands per acre	Yield per acre	Total uptake per acre	Uptake from soil per acre
lb	cwt	lb	lb
0	261	51	51
50	265	51	47
100	266	49	45
200	272	51	43

In the present usage, the error resulting from the assumption that the quantity of a nutrient absorbed from the soil is independent of the quantity of the same nutrient absorbed from the fertilizer is largely compensated by the fact that the error usually affects both the numerator and denominator of the ratio U_r/U_b in the same direction.

This behaviour may be illustrated by a numerical example derived from some data of table 1:

Treatment per acre	Uptake from fertilizer per acre	Increase in uptake per acre
	lb	lb
40 lb in bands	2,4	3.0
40 lb side dressed	2.6	3.2

The ratio of the uptake from the two placements is 2.4/2.6 = 0.924, when estimated from the uptake from the fertilizer, and 3.0/3.2 = 0.948, when estimated from the increase in uptake from fertilization. These two values differ less than 3 %, although the uptake from the fertilizer differs by more than 20 % from the increase in uptake.

The results of one experiment with phosphate have been noted, however, where the uptake from the soil increased with increasing application rate of broadcast fertilizer, and decreased with increasing application rate of placed fertilizer (figure 15). In this case, the ratio U_r/U_b estimated from the increase in uptake is quite different from the ratio U_r/U_b estimated from the uptake from the fertilizer. One of the investigators (pers. comm. WIKLANDER, 1952) communicated that he supposed that in this experiment the development of active roots in the unfertilized part of the soil was smaller where the fertilizer was placed, although this is not indicated by the root weights. In Chapter VI, it is shown that such a response of root development to placement is not often encountered.

2. EXPERIMENTS

It is possible to calculate the ratios U_r/U_b and X_r/X_b obtained under different conditions from various data available in the literature.

GILE *and* CARRERO (1917) *carried out a fundamental experiment* with the purpose of determining the uptake when different fractions of the roots are fertilized. Plants were grown with their roots divided in different proportions between two Erlenmeyer flasks, one containing a complete nutrient solution and the other containing a nutrient solution lacking one element. Control plants were grown with their roots divided in the same way between two Erlenmeyer flasks, but with a complete solution in both flasks. The elements tested were N, P, K and Fe. For present purposes, it may be considered that the flasks containing a nutrient solution lacking N, P, K or Fe correspond to the portion of soil not fertilized with the lacking element and the flasks with a complete nutrient solution correspond to the portion that receives placed fertilizer. Plants with their roots divided in the same way between two flasks containing a complete solution correspond to plants fertilized by a broadcast application. Plants with 50 % of their roots in a complete solution thus correspond to plants for which X_r/X_b is 0.5, and plants with 25 % of their roots in a complete solution correspond to plants for which X_r/X_b is 0.25. The nutrient solutions were renewed frequently, so that the initial concentrations were not appreciably altered as a result of the growth of the plants. In all experiments, rice was grown for 40 days and corn was grown for 21 days. The yield and chemical composition of the plants were determined at the end of the experiment. The essential results of the experiments are given in table 3.

TABLE 3. Summary of results of ten experiments in which plants were grown with their roots divided in different proportions between various nutrient solutions (compiled from GILE and CARRERO, 1917)

1	2	3	4	5	6	7	
Experiment No	Plant	Element lacking in incomplete nutrient solution	Composition of complete nutrient solution	Percentage of roots in complete solution	U_r/U_b	Average	
						X_r/X_b	U_r/U_b
1	Rice	N	Neutral	50 (50)	0.77		
2	Corn	N	Neutral	50 (56)	0.76	0.50	0.76
3	Rice	N	Neutral	50 (54)	0.74		
3	Rice	N	Neutral, double N	50 (54)	0.77		
4	Rice	N	Neutral	25 (27)	0.55	0.25	0.55
5	Rice	P	Neutral	50 (47)	0.76	0.50	0.77
5	Rice	P	Neutral, double P	50 (50)	0.78		
6	Rice	P	Neutral	33 (33)	0.65	0.333	0.65
7	Rice	K	Acid	50 (51)	0.66	0.50	0.66
8	Rice	K	Acid	25 (29)	0.61	0.25	0.61
9	Rice	Fe	Neutral, Fe as ferrous sulphate	50 (54)	0.66		
9	Rice	Fe	Acid, Fe as ferrous sulphate	50 (57)	0.72	0.50	0.68
10	Rice	Fe	Acid, Fe as ferric tartrate	50 (56)	0.66		

Explanation of table 3:

Column 4. The experiments were conducted using a neutral or an acid nutrient solution (the composition is given in the original paper). In experiment 3 and 5 double amounts of N and P, respectively, (those are the elements lacking in the incomplete solution) were supplied. In experiment 9 and 10 the iron was supplied in the form of ferrous sulphate and ferric tartrate, respectively.

Column 5. The value without brackets is the estimated proportion of the roots in the complete solution expressed as a percentage of the total amount at the beginning of the experiments. The value in brackets is the dry weight percentage of the roots in the complete solution at the end of the experiments.

Column 6. The ratio U_r/U_b has been found by dividing the uptake of the nutrient by the incompletely-fertilized plants by the uptake of the nutrient by the completely-fertilized plants grown under the same environmental conditions.

Column 7. The ratio X_r/X_b has been calculated from the estimated proportion of the roots in the complete nutrient solution at the beginning of the experiment, and not from the weight of the roots at the end. Since there are no significant differences between nutrient solutions and between crops, the results are averaged.

It is evident that under all conditions a compensation in uptake took place, since the ratio U_r/U_b is much greater than the ratio X_r/X_b. Moreover, the ratio U_r/U_b is practically the same for different crops, for different nutrient solutions and for the elements N, P, K and Fe.

The detailed results of experiment 3 with nitrogen are reproduced in figure 7. The relationship between total uptake and fertilizer rate for „broadcast" and „placed" fertilizer is given in the fourth quadrant. The ratio U_r/U_b has been calculated in the inset-graph. Here, the uptake (U_r) at the rates of z and $2z$ g of „placed" nitrogen are plotted against the uptake (U_b) at the rates of $2z$ and $4z$ g of „broadcast" nitrogen,

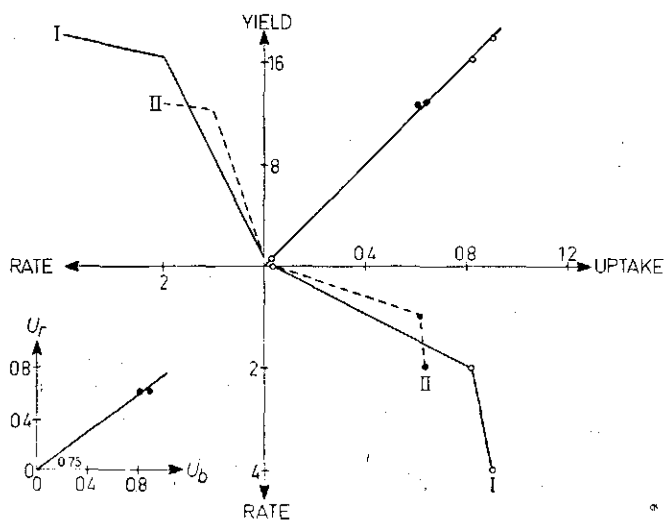


FIGURE 7*a*.
Rate of fertilization, nitrogen uptake and crop yield in a nitrogen-fertilization experiment.
Compiled from GILE and CARRERO (1917).
Crop: rice. *Fertilizer:* nitrogen. *Experiment:* water culture in greenhouse conditions. *Scale units:* Rate in z g N per plant. Uptake in g N per plant. Yield in g dry matter per plant. *Fertilizer patterns:* I: whole root system supplied with nitrogen ($(X_r/X_b) = 1$). II: one-half the root system supplied with nitrogen ($(X_r/X_b) = 0.5$). The ratio U_r/U_b (U in g N) is calculated in the inset-graph. ***Results:*** $(X_r/X_b) = 0.5$; $(U_r/U_b) = 0.75$. Example of Case IA (figure 5).

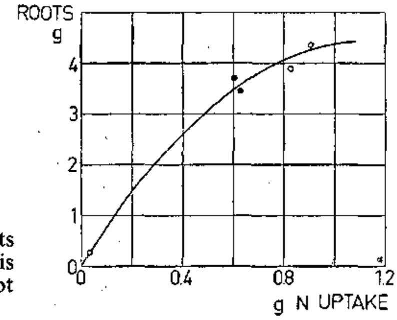


FIGURE 7*b*.
The relationship between total uptake and weight of roots in the experiment of figure 7*a*. The curve holds if this relationship is independent of the fertilizer pattern. It cannot be determined whether this is the case.

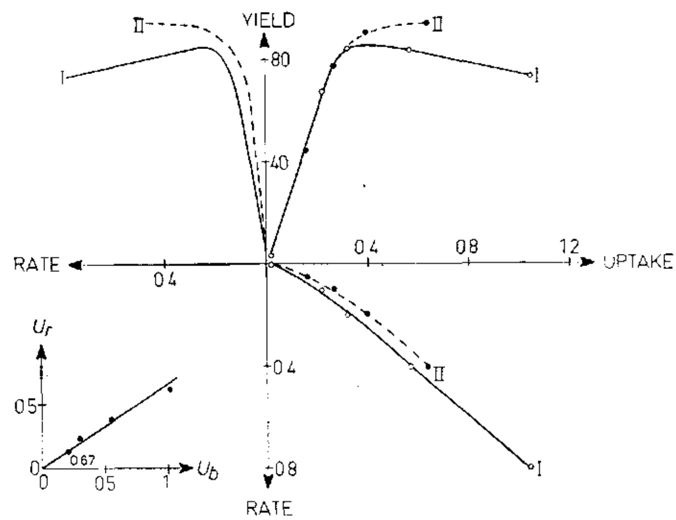


FIGURE 8*a*.
Rate of fertilization, phosphorus uptake and crop yield in a phosphorus-fertilization experiment.
Compiled from GOEDEWAAGEN (1942).
Crop: oats. *Fertilizer:* dibasic calcium phosphate.
Experiment: in containers filled with sand. *Scale units:* Rate in z g P_2O_5 per container. Uptake in g P_2O_5 per container. Yield in g dry matter per container. *Fertilizer patterns:* I: fertilizer mixed with the whole soil volume ($(X_r/X_b) = 1$). II: fertilizer mixed with the right half of the soil volume ($(X_r/X_b) = 0.5$). The ratio U_r/U_b (U in g P_2O_5) is calculated in the inset-graph.
Results: $(X_r/X_b) = 0.5$; $(U_r/U_b) = 0.67$. Example of Case II (figure 5).

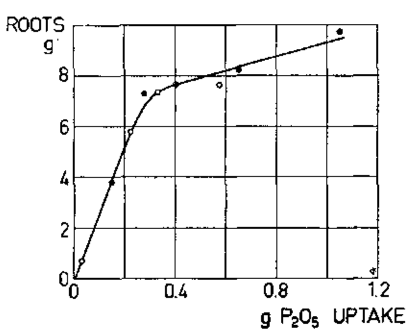
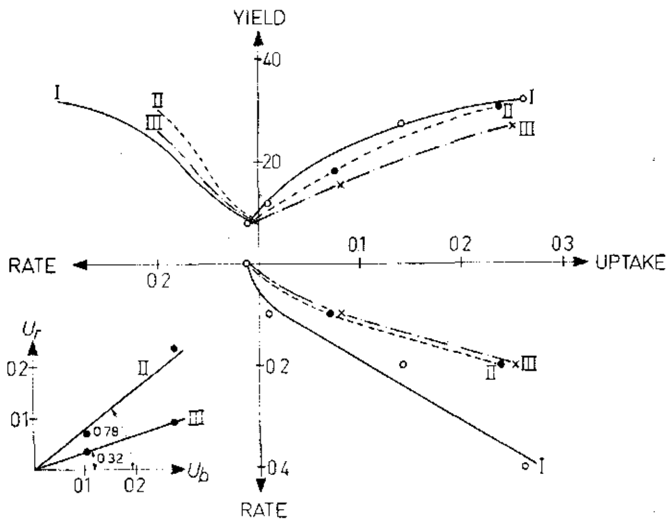


FIGURE 8*b*.
The relationship between total uptake and dry weight of roots in the experiment of figure 8*a*. This relationship is independent of the fertilizer pattern.

respectively. The slope of the line from the origin through these points gives the ratio U_r/U_b. This ratio is the same for both concentrations. Experiment 5 with phosphate gives analogous results.

GOEDEWAAGEN (1942) *repeated the experiments of* GILE *and* CARRERO, using oats grown in containers filled with sand, where dibasic calcium phosphate was mixed either with a part of the sand or with the entire amount of sand in the containers. The results of this experiment are summarized in figure 8. The values of U_r/U_b for different concentrations are plotted in the inset-graph. The points fit a straight line through the origin, which shows that the ratio U_r/U_b is independent of the concentration of the fertilizer. The average value of U_r/U_b is 0.67. Since the ratio X_r/X_b was 0.5 in this experiment, the value $U_r/U_b = 0.67$ agrees with GILE and CARRERO's results. GOEDEWAAGEN *expressed the opinion that in general such a compensation of uptake will occur where fertilizers are distributed nonuniformly.* By comparing root growth in the complete and incomplete nutrient solutions of GILE and CARRERO's experiments and in the fertilized and unfertilized parts of his containers, he found that this compensation results at least partly from a more vigorous root development. From experiments of

FIGURE 9.
Rate of fertilization, phosphorus uptake and crop yield in a phosphorus-fertilization experiment.
Compiled from GOEDEWAAGEN (1942).
Crop: Vicia Faba (broad beans). *Fertilizer:* dibasic calcium phosphate. *Experiment:* in containers, filled with poor sand. *Scale units:* Rate in z g P_2O_5 per container. Uptake in g P_2O_5 per container. Yield in g dry matter per container. *Fertilizer patterns:* I: fertilizer mixed with an upper layer of 20 cm thickness $((X_r/X_b) = 1)$. II: fertilizer mixed with an upper layer of 10 cm thickness $((X_r/X_b) = 0.5)$. III: fertilizer mixed with an upper layer of 5 cm thickness $((X_r/X_b) = 0.25)$. The uptake by the unfertilized plants is negative, since some phosphate migrated from the seed into the soil. The ratio U_r/U_b (U in g P_2O_5) is calculated in the inset-graph. *Results:* $(X_r/X_b) = 0.5$; $(U_r/U_b) = 0.78$ and $(X_r/X_b) = 0.25$; $(U_r/U_b) = 0.32$. Example of case III (figure 5).



other investigators, GOEDEWAAGEN concluded that in addition to this morphological compensation a „physiological" compensation must exist. The nature of this „physiological" compensation is not known.

Another pot experiment of GOEDEWAAGEN (1942) on broad beans and phosphate fertilizer is reproduced in figure 9. In this experiment, different amounts of fertilizer were mixed with a soil layer 5, 10 or 20 centimeters in thickness. The rate-uptake curves are given in the fourth quadrant. Equal amounts of fertilizer were mixed with layers of different thickness, which means that the concentration of the fertilizer in the fertilized soil volume was not the same for the different application patterns. The uptake may be found for each rate from the free-hand curves drawn to fit the data, and the uptake from broadcast and placed fertilizer at equal concentrations of fertilizer in the soil can then be found by interpolation. In the inset-graph the uptakes for two concentration levels are compared, and the ratios U_r/U_b are calculated. The results are as follows:

$$X_r/X_b = 0.5 \quad \text{and} \quad X_r/X_b = 0.25$$
$$U_r/U_b = 0.78 \qquad\qquad U_r/U_b = 0.32$$

The value of U_r/U_b for the pattern with $X_r/X_b = 0.25$ is very low compared with the values in GILE and CARRERO's experiments. In this particular case, the fertilizer was supplied in the upper 5 centimeters of the soil. It is possible, therefore, that the low ratio of U_r/U_b may have resulted from positional unavailability of a part of the fertilizer since the beans were planted some centimeters deep and the water supply was not entirely regular (pers. comm. GOEDEWAAGEN, 1951). At any rate, the experimental conditions differ so much from the conditions supposed in figure 1 that it seems justified to give little weight to this result.

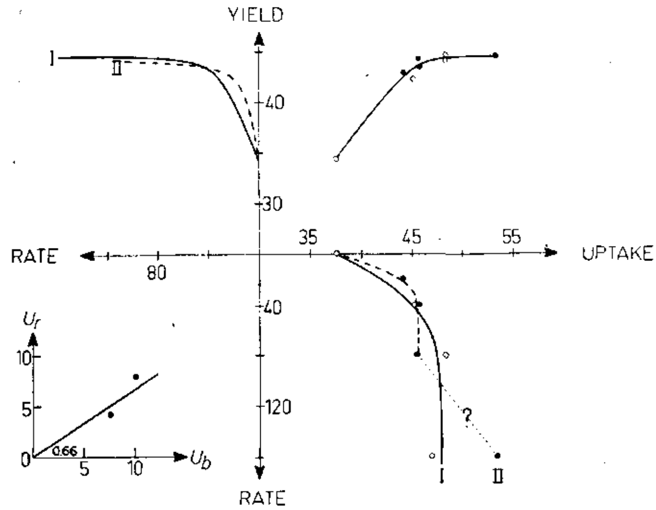


FIGURE 10.
Rate of fertilization, phosphorus uptake and crop yield in a phosphorus-fertilization experiment.
Compiled from PRUMMEL (1950). (With some personally communicated data).
Crop: oats. *Fertilizer:* superphosphate. *Experiment:* field experiment 1058 on sandy soil. *Scale units:* Rate in kg P_2O_5 per ha. Uptake in kg P_2O_5 per ha. Yield in 100 kg seed per ha. *Fertilizer patterns:* I: fertilizer broadcast $((X_r/X_b) = 1)$. II: fertilizer placed in bands 4 centimeters to the side of the seed $((X_r/X_b) = 0.26)$. The ratio U_r/U_b (U in kg P_2O_5) is calculated in the inset-graph. The high uptake for 160 kg placed P_2O_5 cannot be explained. *Results:* $(X_r/X_b) = 0.26$; $(U_r/U_b) = 0.66$. Example of Case IA (figure 5).

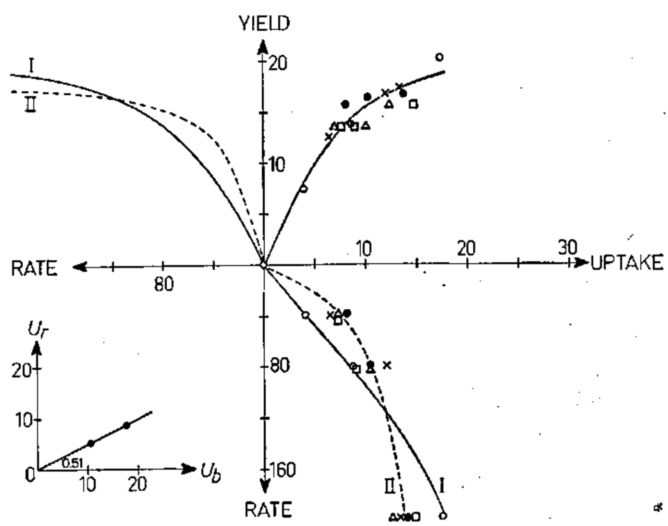


FIGURE 11.
Rate of fertilization, phosphorus uptake and crop yield in a phosphorus-fertilization experiment.
Compiled from PRUMMEL (1950), (with some personally communicated data).
Crop: oats. *Fertilizer:* superphosphate. *Experiment:* field experiment 1052 on reclaimed heath soil. *Scale units:* Rate in kg P_2O_5 per ha. Uptake in kg P_2O_5 per ha. Yield in 100 kg seed per ha. *Fertilizer patterns:* I: fertilizer broadcast $((X_r/X_b) = 1)$. II: Average curve for fertilizer placed at 0 (•), 2 (×), 4 (△) and 6 (□) centimeters from the seed $((X_r/X_b) = 0.26)$. The ratio U_r/U_b (U in kg P_2O_5) is calculated in the inset-graph. *Results:* $(X_r/X_b) = 0.26$; $(U_r/U_b) = 0.51$. Example of Case IB (figure 5).

PRUMMEL (1951) *carried out some field experiments* with oats that permit the calculation of the ratios X_r/X_b and U_r/U_b (figure 10–13). The ratio X_r/X_b for PRUMMEL's experiment has been derived from a root study of GOEDEWAAGEN (pers. comm., 1951). The width of the fertilizer band (X_r) was about 6.5 centimeters, and the distance between the crop rows (X_b) was about 25 centimeters. The fertilizer band lay near the

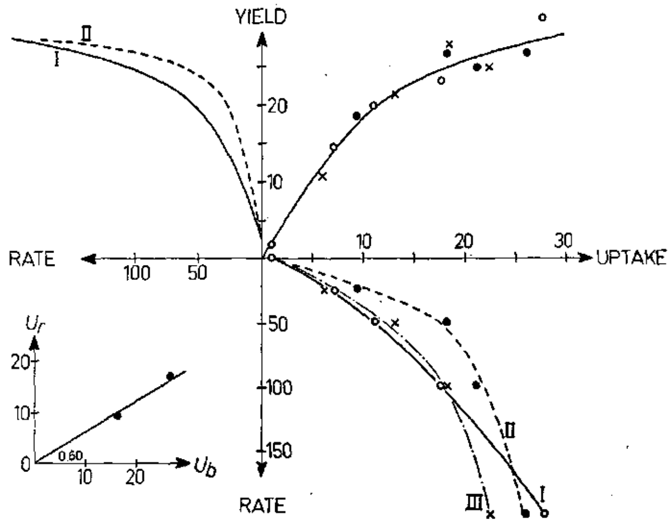


FIGURE 12.
Rate of fertilization, phosphorus uptake and crop yield in a phosphorus-fertilization experiment.
Compiled from PRUMMEL (1950). (With some personally communicated data).
Crop: oats. *Fertilizer:* superphosphate. *Experiment:* field experiment 1001 on reclaimed heath soil. *Scale units:* Rate in kg P_2O_5 per ha. Uptake in kg P_2O_5 per ha. Yield in 100 kg seed per ha. *Fertilizer patterns:* I: fertilizer broadcast ($(X_r/X_b) = 1$). II: fertilizer placed 2 centimeters to the side of the seed ($(X_r/X_b) = 0.26$). III: fertilizer placed 8 centimeters to the side of the seed ($(X_r/X_b) = 0.26$). The ratio U_r/U_b (U in kg P_2O_5) for patterns I and II is calculated in the inset-graph. *Results:* Band placement 8 centimeters to the side of the seed was too far away. For the other patterns: $(X_r/X_b) = 0.26$; $(U_r/U_b) = 0.60$. Example of Case IA (figure 5).

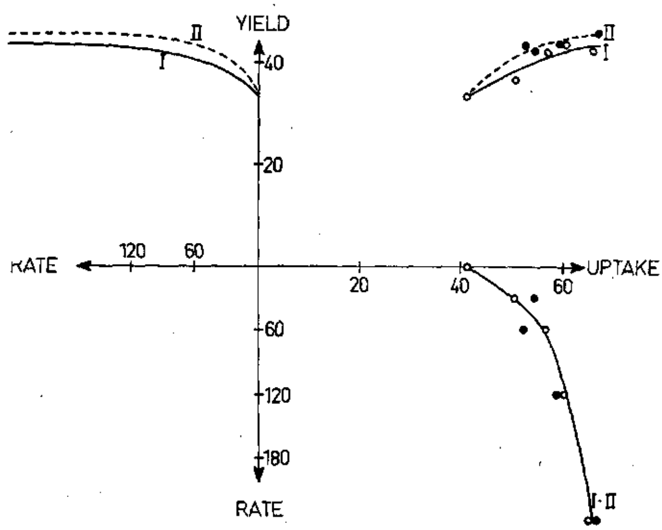


FIGURE 13.
Rate of fertilization, phosphorus uptake and crop yield in a phosphorus-fertilization experiment.
Compiled from PRUMMEL (1950) (With some personally communicated data).
Crop: oats. *Fertilizer:* superphosphate. *Experiment:* field experiment 1066 on loess soil. *Scale units:* Rate in kg P_2O_5 per ha. Uptake in kg P_2O_5 per ha. Yield in 100 kg seed per ha. *Fertilizer patterns:* I: fertilizer broadcast ($(X_r/X_b) = 1$). II: fertilizer placed in bands 4 centimeters to the side of the seed ($(X_r/X_b) = 0.26$). *Results:* The rate-uptake curves in case of broadcasting and placement are not significantly different. Theory and experiment do not agree in this respect. The uptake-yield curve has been affected favourably by placement: this part of the experiment is an example of case II (figure 5).

soil surface, and the thickness was about 6 centimeters. As the thickness of the soil layer mixed with fertilizer is about 5 centimeters where the fertilizer is applied broadcast (TINNEFELD, 1930), the conditions in PRUMMEL's experiments correspond approximately to the conditions assumed in figure 1. The rate-uptake curves are given in the fourth quadrant[1] of the figures. The ratio U_r/U_b has been derived in the inset-graphs. The results are summarized in table 4.

TABLE 4. Values of X_r/X_b and U_r/U_b derived from field experiments with oats fertilized with superphosphate (compiled from PRUMMEL, 1951)

Experiment	X_r/X_b	U_r/U_b
PRUMMEL, fig. 10 . . .	0.26	0.66
,, fig. 11 . . .	0.26	0.51
,, fig. 12 . . .	0.26	0.60

The results of a fourth experiment (figure 13) do not agree with the theory.

The results of a field experiment of COOKE[2] (pers. comm., 1951) are given in figure 14. The estimated value of X_r/X_b is 0.075, and the calculated value of U_r/U_b is 0.33.

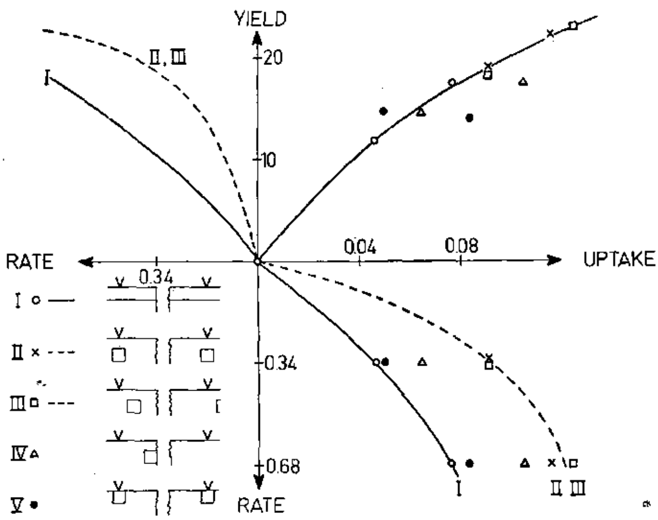


FIGURE 14.
Rate of fertilization, phosphorus uptake and crop yield in an experiment with compound fertilizer.
Compiled from COOKE (pers. com., 1951).
Crop: Swedes. *Fertilizer:* N, P, K-compound fertilizer (9-7-4.5). *Experiment:* field experiment on heavy, phosphate-deficient loam. *Scale units:* Rate in cwt P_2O_5 per acre. Uptake in roots in cwt P_2O_5 per acre. Yield in tons roots per acre. *Fertilizer patterns:* I: fertilizer broadcast ($(X_r/X_b) = 1$). II, III and IV: fertilizer placed in bands at 0, 1 and 3 inches to the side of the seed, respectively. ($(X_r/X_b) = 0.075$). V: fertilizer placed in contact with the seed ($(X_r/X_b) = 0.075$). *Results:* Contact placement damaged the seed. Band placement 3 inches to the side of the seed was too far away. A comparison of pattern I with patterns II and III resulted in: $(X_r/X_b) = 0.075$; $(U_r/U_b) = 0.33$. Example of case I (figure 5).

FREDRIKSSON and WIKLANDER (1950) *carried out a field experiment* with potatoes grown on a loamy soil and fertilized with superphosphate (figure 15). The superphosphate was labelled with P^{32} so that the uptake from soil and fertilizer could be distinguished. It was found that where the fertilizer was applied broadcast, the uptake of phosphorus from the soil increased with increasing fertilizer rate, but where the fertilizer was placed in bands the uptake of phosphorus from the soil decreased with increasing fertilizer rate.

[1] The chemical analyses of the yields in these experiments are published here for the first time, by kind permission of Ir PRUMMEL.

[2] The results of this experiment are published here for the first time, by kind permission of Mr COOKE.

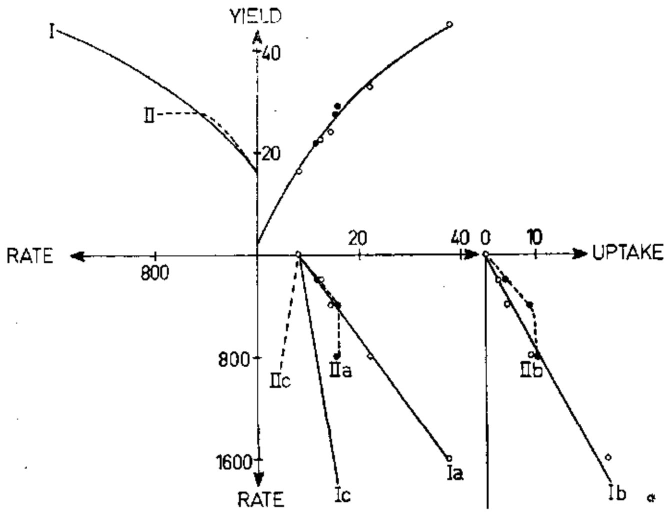


FIGURE 15*a*.
Rate of fertilization, phosphorus uptake and crop yield in a phosphorus-fertilization experiment.
Compiled from FREDRIKSSON and WIKLANDER (1950). *Crop:* potatoes. *Fertilizer:* granulated superphosphate, labelled with P^{32}. *Experiment:* field experiment on loamy sand; harvested at blossoming. *Scale units:* rate in kg superphosphate per ha. Uptake in kg P_2O_5 per ha. Yield in 100 kg dry matter per ha. *Fertilizer patterns:* I: fertilizer broadcast and harrowed in before ridging. II: fertilizer placed about 5 centimeters below the tubers. *Rate-uptake curves:* the curves marked „a" represent the total uptake, the curves marked „c", represent the uptake from the soil and the curves marked „b", represent the uptake from the fertilizer, as determined by means of P^{32}. *Estimated results:* $(X_r/X_b) = 0.05$. $(U_r/U_b) = 0.09$ (see III, 2). Example of case IB (figure 5).

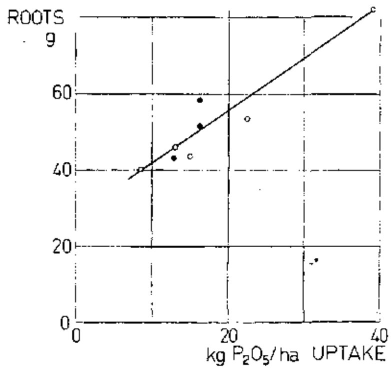


FIGURE 15*b*.
The relationship between the total uptake in kg P_2O_5 per ha and the root weight of twelve plants in the experiment of figure 15*a*. There are no indications that this relationship is not the same for the two fertilizer patterns.

Since the broadcast fertilizer was applied before ridging, this fertilizer was mixed with a large and indefinite volume of soil. The width and thickness of the fertilizer band were not reported; however, since the radioactive fertilizer was carefully placed by hand 5 centimeters below the potatoes, it is probable that the width of the band was not in excess of 3–4 centimeters and that the thickness was small (pers. comm. WIKLANDER, 1952). A rough estimate of the ratio X_r/X_b is 0.05. In this instance, the ratio U_r/U_b is calculated from the uptake from the fertilizer and not from increase in uptake from fertilization (see III,1). The derived value $U_r/U_b = 0.09$ is small compared with the value that can be derived from figure 2.

Russian investigators also have conducted certain field and pot experiments that permit calculation of the nutrient uptake ratio U_r/U_b. The results of the calculations are inaccurate, however, since the application methods were not described in detail, and since for each curve only three observations were made. For these reasons, only the final results based on estimates of the ratio X_r/X_b are given. The results may be summarized as follows:

Experiments of DEMIDENKO and BORINOVA (1946):

Pot experiments with phosphate fertilizer.

Millet: $X_r/X_b = 0.075$, $U_r/U_b = 0.29$.
Buckwheat: $X_r/X_b = 0.10$, $U_r/U_b = 0.43$.
Field experiments with phosphate fertilizer.
Millet: $X_r/X_b = 0.075$, $U_r/U_b = 0.48$.
Buckwheat: $X_r/X_b = 0.10$, $U_r/U_b = 0.28$.

Experiments of ULYAKOV (1936):

Pot experiment with phosphate fertilizer and oats.
„Red soil": $X_r/X_b = 0.05$, $U_r/U_b = 0.17$.

Although the experiment on „black soil" of ULYAKOV permits an estimation of the ratios X_r/X_b (0.05) and U_r/U_b (0.12), the data are not used here since the shape of the rate-uptake curve calculated from the curve for broadcasting with the aid of these values differs too much from the actual curve. As the curves are based on only three observations, no large weight can be given to this difference.

Finally, brief mention will be made of the *pot experiments of* RAUTERBERG (1937) with oats, grown on soil treated with phosphate and potash fertilizers. The results are shown in figures 16 and 17. Although the methods of mixing soil and fertilizer do not permit calculation of the ratio X_r/X_b, it is nevertheless evident from the nature of the rate-uptake curves that compensation has occurred where the fertilizer has been unevenly distributed.

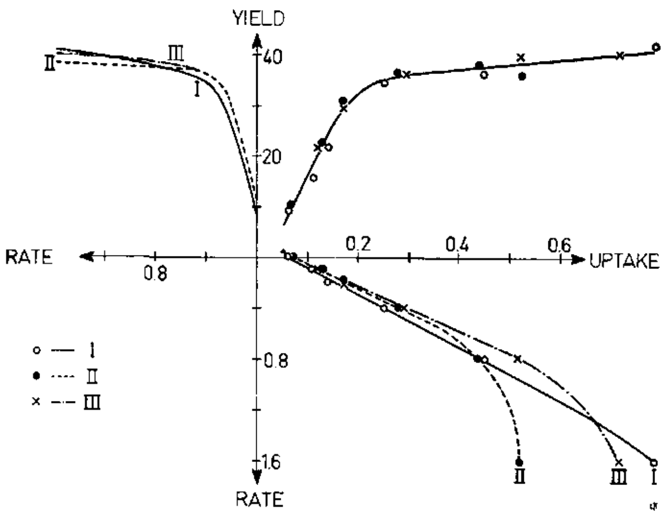


FIGURE 16.
Rate of fertilization, phosphorus uptake and crop yield in a phosphorus-fertilization experiment.
Compiled from RAUTERBERG (1937).
Crop: oats. *Fertilizer:* monobasic calcium phosphate.
Experiment: in Mitscherlich pots filled with sandy loam.
Scale units: Rate in g P_2O_5 per pot. Uptake in g P_2O_5 per pot. Yield in g seed per pot.
Fertilizer patterns: I: Phosphate solution absolutely evenly distributed. II: phosphate solution applied with a pipette. III: phosphate salt evenly distributed. The ratios X_r/X_b cannot be estimated in this experiment. *Results:* The rate-uptake curves of patterns II and III cross the curve of pattern I. Example of case IA and IB (figure 5).

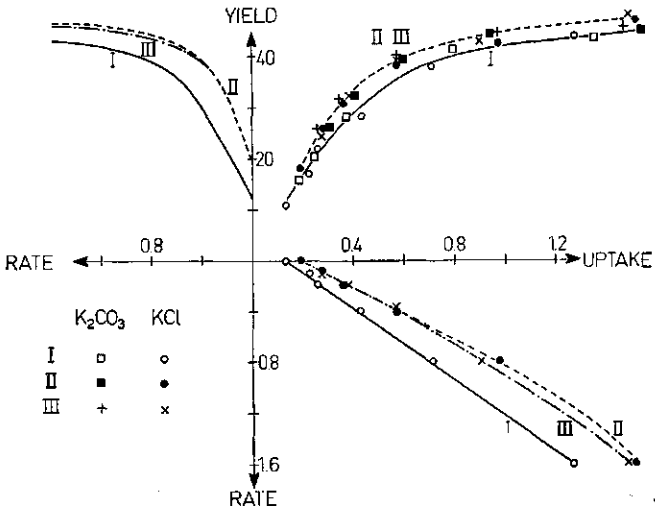


FIGURE 17*a*.
Rate of fertilization, potassium uptake and crop yield in a potassium-fertilization experiment.
Compiled from RAUTERBERG (1937).
Crop: oats. *Fertilizer:* potassium chloride and potassium carbonate. *Experiment:* in Mitscherlich pots, filled with sandy loam. *Scale units:* Rate in g K_2O per pot. Uptake in g K_2O per pot. Yield in g seed per pot. *Fertilizer patterns:* I: potassium solution absolutely evenly distributed through the soil. II: Potassium solution applied with a pipette. III: Potassium salt evenly distributed. The ratios X_r/X_b cannot be estimated in this experiment.
Results: It is evident that the absolutely even distribution of potash was inferior to the other methods. The uptake-yield curve is unfavourably affected by mixing the fertilizer solution thoroughly with the soil, but is independent of the fertilizer material. Example of case II (figure 5).

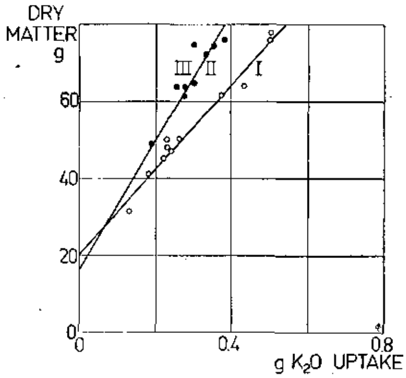


FIGURE 17*b*.
The relationship between total uptake and total dry matter in the experiment of figure 17*a*. The curve for patterns II and III cross the ordinate between the origin and the point of intersection of the ordinate and pattern I.

3. THE DERIVATION OF THE COMPENSATION FUNCTION

All the corresponding values for X_r/X_b and U_r/U_b that were calculated in III, 2 have been represented in figure 18. The points in the figure have thus been derived from experiments in the field, in containers and in water cultures, with the elements N, P, K and Fe, the crops rice, corn, oats, broad beans, swedes, potatoes, millet and buckwheat and in the U.S.A., U.S.S.R., England, Sweden and Holland. Nevertheless, the deviations from an average line are comparatively small. It is therefore justified to suppose that *as a first approximation* the relationship between these two ratios is independent of the conditions, e.g. fertilizer material, concentration of the fertilizer, kind of crop, type of soil and weather conditions as long as these are the same for broadcast and placed fertilizer.

In the reduction of data, neither the absolute values of X_r and X_b nor the distribution of fertilizer with depth have been taken into account. Hence, if a ratio $U_r/U_b = b$ is obtained for a placement pattern with $X_r/X_b = a$, a ratio $U_r/U_b = b^n$ must be expected for a ratio $X_r/X_b = a^n$. The following relation may then be written:

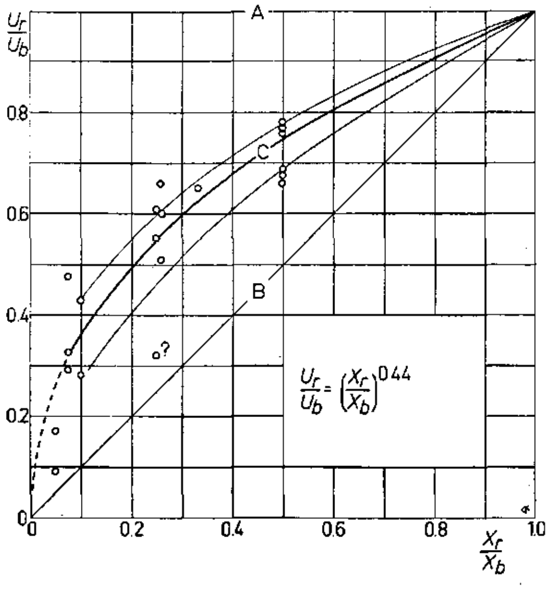


FIGURE 18.
The compensation function (curve C). The relationship between the ratio of the uptake of the nutrient from the placed fertilizer U_r to that from the broadcast fertilizer U_b and the ratio of the width of the band of placed fertilizer X_r to the distance between the crop rows X_b as computed from experiments encountered in literature. The point with ? refers to the experiment of GOEDEWAAGEN on broad beans (figure 9). The confidence region of the compensation function (with confidence level $p = 0.042$; two sided symmetrical) calculated according to HEMELRIJK (1951), lies between the lightly-ruled curves on either side of curve C. See also the subscription of figure 2.

$$n = \log (U_r/U_b) \ / \log b = \log (X_r/X_b) \ / \log a$$
$$\log (U_r/U_b) = (\log b/\log a, \log (X_r/X_b) = c \log (X_r/X_b)$$

This relationship is checked in figure 19, in which the data are plotted on logarithmic paper. It will be noted that the data may be represented fairly well by a straight line passing through the point $[X_r/X_b = 1;\ U_r/U_b = 1]$. This behaviour indicates that ($\log b/\log a = c$) is constant. In obtaining the slope $c = 0.44$, the data for the two values for $X_r/X_b = 0.05$ have not been fully considered. These points fall far below the line that fits the remaining data. Omission of these two points may be justified on the basis that the coefficient of variation of estimates of X_r/X_b should increase with decreasing values of X_r/X_b because of the difficulty of obtaining a precise estimate of small values of X_r.

The relationship between the two ratios in figure 18 may thus be represented by $U_r/U_b = (X_r/X_b)^{0.44}$. This function is called the compensation function. The confidence

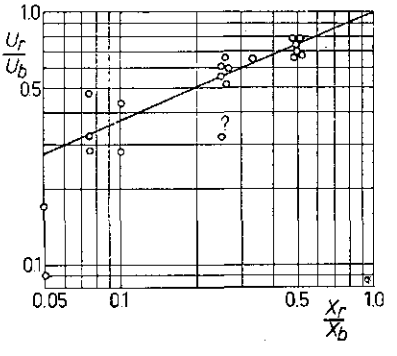


FIGURE 19.
The evaluation of the constant c (= 0.44) in the relationship $\log (U_r/U_b) = c \log (X_r/X_b)$ on double logarithmic paper. It has been assumed that the points are distributed symmetrically on both sides of the true line. The point marked ? refers to the experiment of GOEDEWAAGEN with broad beans (figure 9). Only the points with $(X_r/X_b) = 0.05$ deviate markedly from the straight line.

region, which has been calculated according to the method of HEMELRIJK (1949), gives an impression of the precision of the function.

The compensation function can now be used to calculate the relationship between rate of fertilizer and total uptake for any method defined in figure 1, if this relationship is known for one method. The main results of such a calculation, shown in figure 3 and discussed in II, 2, are as follows:

1. For low rates of fertilizer the uptake increases with decreasing width of the fertilizer band.
2. For high rates of fertilizer the uptake decreases with decreasing width of the fertilizer band.

Examples of calculations based on experimental data are discussed in detail in IX.

4. COMPENSATION WITH HIGH RECOVERY PERCENTAGES

A theoretical consideration of the relationship between the uptake rate from fertilizer and the width of the fertilizer band was given in II, 2. This consideration holds for the total uptake from fertilizer as well, as long as this uptake is small compared with the amount of fertilizer applied. Only under these circumstances can differences in concentrations caused by differences of the uptake rate from broadcast and placed fertilizer be neglected. If the total uptake from the fertilizer represents a substantial proportion of the amount applied, the concentration of the nutrient in the soil decreases more rapidly with placed fertilizer than with broadcast fertilizer. As a result, the compensation function is no longer applicable. An attempt to apply it in such cases results in an overestimate of the uptake from placed fertilizer.

In GILE and CARRERO's experiment, the nutrient solutions were renewed frequently. The concentration of the nutrient was thus substantially independent of the rate of uptake. In this case, therefore, the uptake can be used to calculate for the two „placement" methods both the ratio of the absorption rates ($\dot{U}_r/\dot{U}_b$) and the ratio of the total absorption (U_r/U_b). In the other experiments the maximum recovery[1] of placed fertilizer was about 30 to 40 % (figures 12 and 14). Nevertheless, the compensation factors did not differ significantly from those calculated from GILE and CARRERO's data. It appears, therefore, that the compensation function can be used to calculate the uptake in case of placement in conditions where the recovery of placed fertilizer does not exceed 30 %. It should be noted, however, that where the recovery percentage of placed fertilizer is calculated by applying the compensation function to the total uptake from broadcast fertilizer, the calculated values are much too high if the recovery of the nutrient applied in the broadcast fertilizer is greater than about 15 %.

The recovery percentage of placed fertilizer R_r can be estimated from the recovery percentage of broadcast fertilizer R_b by means of the equation

$$R_r = \frac{100\,(X_b/X_r)^{0.56}\,R_b}{100 + \{(X_b/X_r)^{0.56} - 1\}\,R_b} \qquad (10)$$

This equation incorporates a correction for the fact that because of absorption of the nutrient by the plant, the concentration of placed fertilizer is lower than that of broadcast fertilizer. When R_b is small, this equation may be simplified to

[1] The recovery percentage R is defined by the expression $R = 100\,(U/M)$, in which U is the increase in uptake resulting from fertilization and M is the quantity of fertilizer applied.

$$R_r = \left(\frac{X_b}{X_r}\right)^{0.56} R_b \cdot$$

Since $$R_b = \frac{100\ U_b}{M_b} \quad \text{and} \quad R_r = \frac{100\ U_r}{(X_r/X_b)M_b}$$

the foregoing expression may be transformed to

$$U_r = \left(\frac{X_r}{X_b}\right)^{0.44} U_b$$

which is the compensation function.

It is evident that the fraction

$$\frac{100}{100 + \{(X_b/X_r)^{0.56} - 1\}\ R_b}$$

governs the difference between the estimates of R_r obtained from equation 10 on the one hand and from the compensation function on the other. If, for purposes of illustration, X_r/X_b is set equal to 0.25, the fraction will have the values 1.0, 0.95, 0.90 and 0.85 where R_b is equal to 0, 5, 10 and 15 %, respectively. These results show that inadmissible deviations are not introduced by replacing the ratio of the uptake rates ($\dot{U}_r/\dot{U}_b$) by the ratio of the uptakes from fertilizer (U_r/U_b) as long as the recovery percentage of broadcast fertilizer is low. Further consideration of the fraction shows that if R_b is held constant, the error involved in substituting total absorption for absorption rate will increase as X_b/X_r increases, i.e., as the placed fertilizer occupies a smaller and smaller proportion of the soil volume.

Equation 10 may be derived as follows:

The quantity of fertilizer available to the plant after a lapse of time t is determined by three terms. The first is the quantity of fertilizer introduced into the region in which roots are present. The second is the quantity of fertilizer that has become unavailable by leaching, fixation, uptake by microorganisms or by any other process. The third is the quantity of fertilizer taken up by the plant. Let the rates at which the processes occur be denoted by $m(t)$, $w(t)$ and $\dot{U}(t)$, respectively. Then if $q(t)$ is the quantity of fertilizer present in available condition at an arbitrary instant t,

$$q(t) = \int_0^t m(\tau)\mathrm{d}\tau - \int_0^t w(\tau)\mathrm{d}\tau - \int_0^t \dot{U}(\tau)\mathrm{d}\tau \qquad (1)$$

Although the functions on the right hand side of the equation may vary greatly with time, certain conclusions can be drawn without knowing anything about the exact shape of the functions.

For the broadcast pattern as defined in figure 1, the subscripts b are added to the functions in eq. 1 to indicate that these functions depend on the application pattern:

$$q_b(t) = \int_0^t m_b(\tau)\mathrm{d}\tau - \int_0^t w_b(\tau)\mathrm{d}\tau - \int_0^t \dot{U}_b(\tau)\mathrm{d}\tau \qquad (2)$$

The amount of broadcast fertilizer available under an area of the same width as the band of placed fertilizer may then be represented by

$$\left(\frac{X_r}{X_b}\right) q_b(t)\,.$$

Exactly the same availability of the placed fertilizer would be obtained, if an additional amount of fertilizer were added at the proper rate and distributed in the soil in the proper manner during the growth of the crop to compensate for the difference in rate of nutrient absorption between the placed fertilizer and the broadcast fertilizer occupying the equivalent soil zone. The equation giving the quantity of placed fertilizer available in the soil at any arbitrary time t under the circumstances just described is thus

$$\left(\frac{X_r}{X_b}\right) q_b(t) = \int_0^t m_r(\tau)\mathrm{d}\tau - \left(\frac{X_r}{X_b}\right)\int_0^t w_b(\tau)\mathrm{d}\tau - \int_0^t \dot{U}_r(\tau)\mathrm{d}\tau \qquad (3)$$

The subscripts r are added to indicate that the functions m and $\dot{U}$ are different from those given in equation 2 for broadcast fertilizer. The function w is the same in both cases, since the concentration and distribution of the fertilizer in the soil are identical in the placed and broadcast application methods.

For the same reason, the rate of uptake from placed fertilizer can be calculated at any instant from the rate of uptake from broadcast fertilizer with the compensation function

$$\dot{U}_r(t) = \left(\frac{X_r}{X_b}\right)^{0.44} \dot{U}_b(t)$$

so that [1]

$$\int_0^t \dot{U}_r(\tau)\mathrm{d}\tau = \int_0^t \left(\frac{X_r}{X_b}\right)^{0.44} \dot{U}_b(\tau)\mathrm{d}\tau = \left(\frac{X_r}{X_b}\right)^{0.44} \int_0^t \dot{U}_b(\tau)\mathrm{d}\tau \qquad (4)$$

The supply of fertilizer necessary to maintain the conditions in the volume of soil receiving placed fertilizer exactly the same as that in the equivalent volume of soil receiving broadcast fertilizer can now be calculated by equating eq. 3 to eq. 2, after the latter has been multiplied by X_r/X_b, by simplifying and by substituting eq. 4 into the resulting equation. These operations yield equation 5:

$$\left(\frac{X_r}{X_b}\right)\int_0^t m_b(\tau)\mathrm{d}\tau - \left(\frac{X_r}{X_b}\right)\int_0^t \dot{U}_b(\tau)\mathrm{d}\tau = \int_0^t m_r(\tau)\mathrm{d}\tau - \left(\frac{X_r}{X_b}\right)^{0.44} \int_0^t \dot{U}_b(\tau)\mathrm{d}\tau \qquad (5)$$

At the end of the uptake period t_o the total applied quantity of broadcast fertilizer and the total uptake from broadcast fertilizer are given by M_b and U_b, respectively. Thus

$$\int_0^{t_0} m_b(\tau)\mathrm{d}\tau = M_b \qquad (6)$$

$$\int_0^{t_0} \dot{U}_b(\tau)\mathrm{d}\tau = U_b \qquad (7)$$

[1] During the main uptake period X_r/X_b is independent of time (V).

After substituting eq. 6 and 7, in eq. 5, the total applied amount of placed fertilizer M'_r, can be expressed in X_r, X_b, M_b and $R_b = 100\ U_b/M_b$ as follows:

$$M'_r = \int_0^{t_0} m_r(\tau)\mathrm{d}\tau = \left(\frac{X_r}{X_b}\right) M_b - \left(\frac{X_r}{X_b}\right) U_b + \left(\frac{X_r}{X_b}\right)^{0.44} U_b =$$

$$= \left(\frac{X_r}{X_b}\right)\left(\frac{M_b}{100}\right)\left[100 + R_b\left\{\left(\frac{X_b}{X_r}\right)^{0.56} - 1\right\}\right] \tag{8}$$

The prime is added to M'_r to indicate that a part of this amount is applied continuously during the uptake period. The total uptake from the placed fertilizer can be written

$$U_r = \int_0^{t_0} \dot{U}_r(\tau)\mathrm{d}\tau = \left(\frac{X_r}{X_b}\right)^{0.44} U_b \tag{9}$$

Equations 8 and 9 may then be combined to give the recovery percentage of the placed fertilizer in equation 10′:

$$R'_r = \frac{100\ U_r}{M'_r} = \frac{100\ (X_b/X_r)^{0.56}\ R_b}{100 + \{(X_b/X_r)^{0.56} - 1\}\ R_b} \tag{10'}$$

The prime is added to R'_r to indicate again that a part of the fertilizer is applied continuously.

Thus, to provide the same fertilizer supply in the soil zone containing placed fertilizer as in an equivalent soil zone containing broadcast fertilizer requires initial addition of $(X_r/X_b)\ M_b$ units of fertilizer and gradual addition of

$$M'_r - \left(\frac{X_r}{X_b}\right) M_b = \left\{\left(\frac{X_r}{X_b}\right)^{0.44} - \left(\frac{X_r}{X_b}\right)\right\} \frac{R_b\ M_b}{100}$$

units of fertilizer during the period of plant absorption. With $X_r/X_b = 0.25$ and R_b equal to 0, 10, 20, 40 and 60 %, this additional amount of fertilizer is equal to 0, 0.03 M_b, 0.06 M_b, 0.12 M_b and 0.18 M_b, respectively and if M_b is equal to 100 kg/ha, the additional amount of fertilizer is equal to 0, 3, 6, 12 and 18 kg per hectare, respectively. These calculations illustrate once more that up to recoveries of about 15 % the compensation function can be applied to the total uptake from the broadcast fertilizer.

If the recovery percentage of broadcast fertilizer is higher, an estimation of the recovery percentage R_r of fertilizer placed in the normal way, i.e. at the same time as the broadcast fertilizer, can be obtained by assuming that this recovery percentage is the same as the recovery percentage R'_r of fertilizer placed in the hypothetical manner described above.

It can be proved that the recovery of fertilizer placed in the above hypothetical way and at the same time of broadcast fertilizer is exactly the same if, for instance, the following conditions are met:

a. The growing conditions are the same during the main uptake period.

b. No available fertilizer is left at the end of the uptake period.

c. The uptake rate and the rate of leaching, fixation, etc., are proportional to the concentration of fertilizer in the soil.

If the recovery percentage of broadcast fertilizer is high (about 40 % or greater), these conditions are fulfilled reasonably well as long as the (weather) conditions do not change markedly during the main uptake period.

The proof is as follows:

If the conditions are the same during the main uptake period, the uptake rate from placed fertilizer at the instant t_r can be calculated from the uptake rate from broadcast fertilizer at the instant t_b by the compensation function. Thus,

$$\dot{U}_r(t_r) = \left(\frac{X_r}{X_b}\right)^{0.44} \dot{U}_b(t_b)$$

since

$$\mathrm{d}t_r = \varphi(t_b)\mathrm{d}t_b$$

the total uptake from the placed fertilizer is given by

$$U_r = \int_0^{t_0} \dot{U}_r(\tau_r)\mathrm{d}\tau_r = \left(\frac{X_r}{X_b}\right)^{0.44} \int_0^{t_0} \dot{U}_b(\tau_b)\, \varphi(\tau_b)\mathrm{d}\tau_b .$$

The amount of the nutrient lost from the placed fertilizer by leaching and fixation phenomena is given by

$$W_r = \int_0^{t_0} w_r(\tau_r)\mathrm{d}\tau_r = \left(\frac{X_r}{X_b}\right) \int_0^{t_0} w_b(\tau_b)\, \varphi(\tau_b)\mathrm{d}\tau_b .$$

As no fertilizer is left at the end of the uptake period, the recovery percentage of placed fertilizer can be calculated by

$$R_r = \frac{100\, U_r}{U_r + W_r} =$$

$$= \frac{100}{(X_b/X_r)^{0.56} + \int_0^{t_0} w_b(\tau_b)\, \varphi(\tau_b)\mathrm{d}\tau_b \Big/ \int_0^{t_0} \dot{U}_b(\tau_b)\, \varphi(\tau_b)\mathrm{d}\tau_b} \left(\frac{X_b}{X_r}\right)^{0.56} \qquad (11)$$

As the uptake rate and the rate of leaching, fixation and so on are proportional to the concentration, $m_b(t)$ of the fertilizer, the following equations hold:

$$\dot{U}_b(t_b) = \alpha m_b(t_b) . \qquad w_b(t_b) = \beta m_b(t_b)$$

$$\int_0^{t_0} \dot{U}_b(\tau_b)\varphi(\tau_b)\mathrm{d}\tau_b = \alpha \int_0^{t_0} m_b(\tau_b)\, \varphi(\tau_b)\mathrm{d}\tau_b \qquad \int_0^{t_0} w_b(\tau_b)\varphi(\tau_b)\mathrm{d}\tau_b = \beta \int_0^{t_0} m_b(\tau_b)\, \varphi(\tau_b)\mathrm{d}\tau_b$$

Therefore,

$$\frac{\int_0^{t_0} w_b(\tau_b)\, \varphi(\tau_b)\mathrm{d}\tau_b}{\int_0^{t_0} \dot{U}_b(\tau_b)\, \varphi(\tau_b)\mathrm{d}\tau_b} = \frac{\beta}{\alpha} = \frac{100 - R_b}{R_b} \qquad (12)$$

Substitution of eq. 12 in eq. 11 gives

$$R_r = \frac{100\, (X_b/X_r)^{0.56}\, R_b}{100 + \{(X_b/X_r)^{0.56} - 1\}\, R_b} \qquad (10'')$$

This equation has the same form as eq. 10′. Under these conditions, therefore, R_r' is equal to R_r.

5. Compensation under some extreme conditions

a. The absorption of ions is an energetic process that involves oxygen consumption by the root (HOAGLAND, 1947). For this reason the uptake of ions decreases when the oxygen concentration of the air stream in culture solutions is lower than 10 % (HOAGLAND, 1948). Although it is only in exceptional cases that the oxygen concentration in the air of upper layers of soil is markedly lower than that of the air (BLANCK, 1930; DERAJENKO (see SCHUYLENBORGH, 1947)), it is possible that oxygen supply or other environmental conditions are limiting factors when the fertilizer is placed in an exceedingly small volume of soil because the activity of the roots may be much greater in a fertilizer placement zone of restricted volume than in the remainder of the soil. This is one reason why the compensation function has not been extrapolated to very small ratios of X_r/X_b (figure 18). Additional information is needed to determine whether with wider placement bands the compensation function fails to apply to wet, compact soils where the root environment is unfavourable.

b. The maximum uptake encountered in most experiments with broadcast fertilizer is usually not the absolute maximum uptake. If the proportion of the soil profile that receives fertilizer is increased, the nutrient absorption will probably increase likewise in most cases. Ultimately, the maximum uptake will not be limited by the fertilizer pattern but by the capacity of the plant to store the elements concerned. However, the maximum possible storage in plant tissues is higher than the uptake under normal field conditions, since the absolute uptake by plants grown in favourable conditions is much greater than the uptake in the field (e.g., VAN ITALLIE, 1938). Thus, in general, the uptake is limited by the availability of the fertilizer and not by the storage capacity of the plants. Nevertheless, attention must be paid to this latter possibility if the uptake of the element from the broadcast fertilizer is extremely high.

IV. THE RELATIONSHIP BETWEEN UPTAKE AND YIELD

1. The independence of the uptake-yield curve of the fertilizer pattern

From chemical analyses of plants grown with increasing rates of fertilizer, it has been concluded that a close relationship exists between the total nutrient uptake from the fertilizer and the yield of the entire plant or its individual parts. The type of relationship found in most experiments is reproduced in figure 4. Whether and in what way this relationship is influenced by the fertilizer pattern must be known before it is possible to use the uptake as a measure with practical value.

For this purpose, the uptake-yield curves have been calculated from experiments in which the fertilizer was applied in different ways. These curves are reproduced in figures 7 to 17 and 20. It is evident that in most cases the relationship is the same for different fertilizer patterns. In these experiments, therefore, plants fertilized in different ways grew in the same way, which indicates that the distribution and use of ions in the plant do not depend markedly on the fertilizer pattern.

It has not been reported in the literature that supplying only a portion of the root system with nutrients causes an uneven development of different parts of the plant, apparently because the nutrients present in one part of the plant may be rapidly translocated to other parts of the plant (e.g., van Wiersum, 1947; Hoagland, 1948; de Wit and Talsma, 1952).

Some experiments (figure 8, 9, 13 and 17) have been reported, however, where the uptake-yield curve varied with the fertilizer pattern. It will be pointed out in the fol-

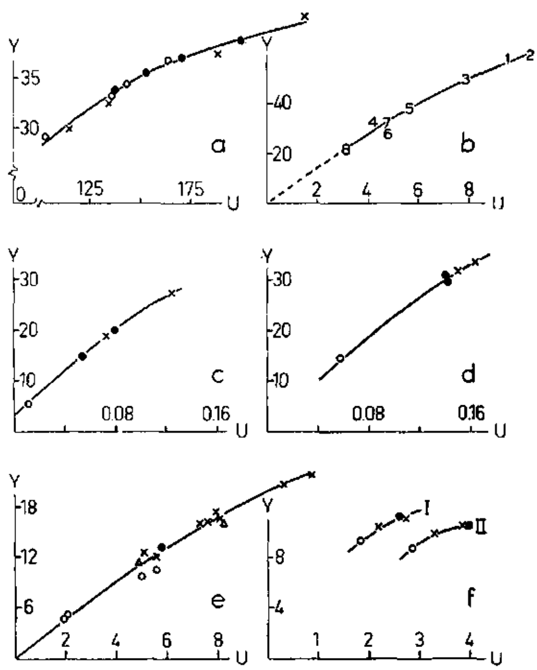


FIGURE 20.
Uptake-yield curves for various phosphorus-fertilization experiments.
a. Compiled from van der Paauw (1950). *Crop:* oats. *Fertilizer:* phosphate. *Experiment:* Vpr 56 in containers filled with sand. *Scale units:* Uptake in mg P_2O_5 per pot. Yield in g seed per pot. *Application patterns:* an equivalent of 150 kg P_2O_5 per ha placed in an upper layer of 2 (x), 6 (.) and 18 (o) centimeters thickness. *Treatments:* Each placement method tested at a relative soil moisture content of 35 % and 50 % and with the fertilizer materials superphosphate and basic slag. *Results:* Uptake-yield curve independent of the placement pattern, the moisture content of the soil and the fertilizer material.
b. Compiled from van der Paauw (1950). *Crop:* oats. *Fertilizer:* phosphate. *Experiment:* field experiment pr 391 (1938) on reclaimed heath soil. *Scale units:* Uptake in kg P_2O_5 per ha, yield of seed and straw in 100 kg per ha. *Application pattern:* the same amount of fertilizer was placed in different ways, as follows: (*see page 28*)

lowing sections that such behaviour can be accounted for by differences between placement patterns as regards the time rate of change of availability of the added nutrient, or by differences between placement patterns as regards possible fertilizer-soil-plant interactions (antagonistic effects) that may affect plant growth.[1]

2. THE INFLUENCE OF THE TIME OF AVAILABILITY OF THE FERTILIZER ON THE UPTAKE-YIELD CURVE

If the fertilizer is present at some distance from the seed, it is positionally unavailable as long as the roots have not reached the bands. Contact placement, on the other hand, assures a comparatively high availability of fertilizer from the beginning of the growing period. This large or small availability during the beginning of the growing period may cause a large or small uptake during this time, and may influence subsequent growth sufficiently to change the final uptake-yield relationship.

Since in most known experiments the availability of the fertilizer has changed considerably during the beginning of the growing period without changing the uptake-

[1] It is easily understood that if the above conclusions hold, the uptake-yield curve must also be the same for different fertilizer materials, although exceptions may be caused by secondary effects of the fertilizer material. The uptake-yield curves of figure 20a, 20b, 21, 43, 44 and 45 show the similarity of the uptake-yield curves obtained with different fertilizer materials supplying a given nutrient.

FIGURE 20, *continued*

Treatment		Treatment number	
Time of application	Incorporation of fertilizer in the soil	Super phosphate	Basic slag
Fall	Yes	1	5
Fall	No	2	6
Spring	Yes	3	7
Spring	No	4	8

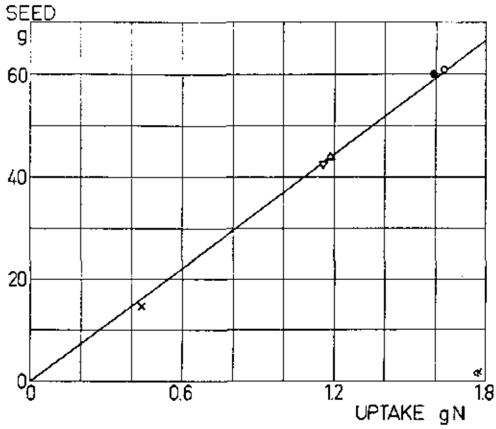
Results: Uptake-yield curve independent of application pattern and fertilizer material.
c. and *d.* Compiled from ULYAKOV (1936).
Crop: oats. *Fertilizer:* calcium phosphate. *Experiment:* in containers filled with red soil (c) and black soil (d). *Scale units:* Uptake in mg P_2O_5 per container. Yield in g dry matter per container. *Application patterns:* Without fertilizer (o), fertilizer evenly distributed (•) and fertilizer placed in a layer of 11 centimeters (×). *Results:* Uptake-yield curve independent of the fertilizer pattern.
e. Compiled from WOLTZ (1949).
Crop: tobacco. *Fertilizer:* superphosphate. *Experiment:* field experiment on Enon sandy loam. *Scale units:* Uptake in lb per acre. Yield in 100 lb tobacco per acre. *Application patterns:* Without fertilizer (o), 40 and 80 lb in rows (×), 40 lb placed in bands (•) and 40 lb side dressed (△). *Treatments:* Each placement method tested on a soil treated or not treated with phosphate and calcium several years previously (+Ca, +P; +Ca, –P; –Ca, +P; –Ca, –P). *Results:* Uptake-yield curve independent of the placement pattern and the fertility level of the soil.
f. Compiled from DEMIDENKO and BORINOVA (1946).
Crop: buckwheat (I) and millet (II). *Fertilizer:* superphosphate. *Experiment:* field experiment. *Scale units:* Uptake in kg P_2O_5 per ha and total yield in 100 kg per ha. *Application patterns:* Without fertilizer (o), 45 kg P_2O_5 as superphosphate broadcast (•), 5 and 10 kg P_2O_5 as superphosphate placed (×). (One apparently incorrect datum changed). *Results:* Uptake-yield curve independent of the application pattern.

FIGURE 21.

Uptake-yield curve in a nitrogen-fertilization experiment.

Compiled from RUYTER DE WILDT and BERKHOUT (1913).

Crop: buckwheat. *Fertilizer:* nitrogen. *Experiment:* in containers filled with sandy soil. The uptake-yield curve is independent of the fertilizer materials employed (Chilean sodium nitrate (o); ammonium sulphate (•); Ca $(CN)_2$ (△); Ca $(CN)_2$ and iron earth (▽) and old Ca $(CN)_2$ (x)).



yield curves, it appears that only under particular conditions does the uptake-yield relationship depend on these changes of availability. In Chapter V this problem will be considered in detail.

3. THE INFLUENCE OF ANTAGONISTIC REACTIONS ON THE UPTAKE-YIELD CURVE

Of much greater importance are changes in environmental conditions caused by the application of the fertilizer. The relationship between uptake and yield depends to a considerable extent on the environmental conditions. With the same uptake, the production of dry matter increases as the environmental conditions become more favourable. This behaviour is exemplified by figure 22, in which the rate-uptake and the uptake-yield curves have been calculated for phosphorus applied with different basic

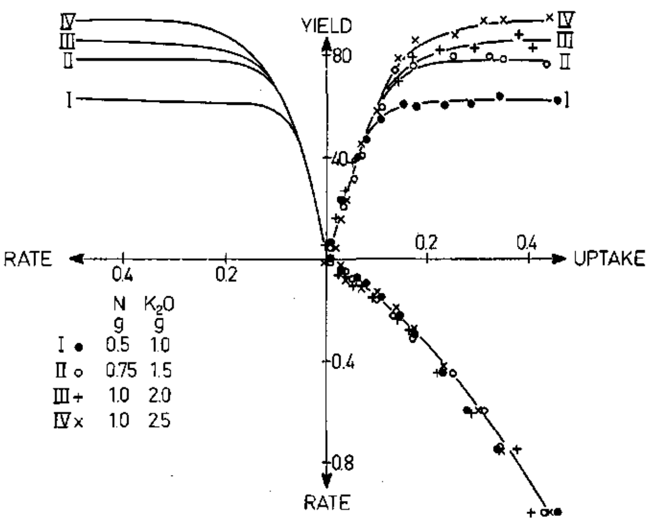


FIGURE 22.

Rate of fertilization, phosphorus uptake and crop yield in a phosphorus-fertilization experiment involving different basal dressings of potassium and nitrogen.

Compiled from BLANCK and SCHORSTEIN (1936).

Crop: oats. *Fertilizer:* phosphate evenly distributed. *Experiment:* in containers filled with sand. *Scale units:* Rate in g P_2O_5 per container. Uptake in g P_2O_5 per container. Yield in g dry matter per container. *Basal dressings:* Four different rates of N and K_2O. *Results:* The rate-uptake curve is independent of the basal dressing. The uptake-yield curve depends highly on the basal dressing.

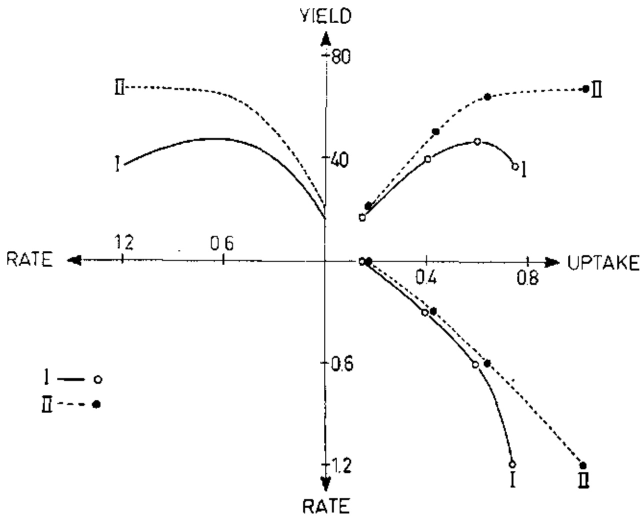


FIGURE 23.
Rate of fertilization, nitrogen uptake and crop yield in a nitrogen-fertilization experiment involving different soil moisture levels.
Compiled from PFÜTZER (1933).
Crop: oats. *Fertilizer:* potassium nitrate, evenly distributed. *Experiment:* in containers. *Scale units:* Rate in g N per container. Uptake in g N per container. Yield in g dry matter per container. *Availability of water:* I: Availability maintained at 30 % of the water-holding capacity. II: Availability maintained at 60 % of the water-holding capacity.
Results: Increased water supply improved both the rate-uptake curve and the uptake-yield curve.

dressings of nitrogen and potash in an experiment of BLANK and SCHORSTEIN (1936). The fertilizer rate-uptake curves are not influenced. However, higher basic dressings caused more favourable uptake-yield curves. In an experiment of PFÜTZER (1933), shown in figure 23, a better supply of water caused not only a better uptake-yield curve, but also a better rate-uptake curve. More examples illustrating this effect may be derived from the literature.

It is well known that the application of an element to the soil may influence the availability of other nutrients initially present in the soil. In some cases, the availability of these other nutrients is suppressed. Reactions of this kind are defined here as antagonistic reactions, whatever the cause may be. The effects of these antagonistic reactions are discussed in the following paragraphs. The same considerations hold, however, *mutatis mutandis*, when the availability of other nutrients is improved by the application of an element, but that does not occur very often.

If the application of an element decreases the availability of a second nutrient not available in abundant amounts, the resulting uptake-yield curve will be less favourable, than the curve obtained without this decrease. It will be shown below that the unfavourable effect of antagonistic reactions decreases with decreasing width of the fertilizer band.

For this purpose, figure 1 is again considered. In figure 1c, the availability of some soil nutrient not applied with the fertilizer (a „second" nutrient) has been represented diagrammatically. The area below line *l* represents the availability of this second nutrient in the soil without fertilizer. It is supposed further that the availability of this second nutrient is decreased by the application of the fertilizer. The distance h_b represents the intensity of the antagonistic reaction caused by the broadcast application of M_b kg of fertilizer per hectare. In general, this intensity increases with increasing rates of fertilizer. It is evident that the decrease in availability of the second nutrient is $X_b h_b$ units if M_b kg of fertilizer per hectare is broadcast and $X_r h_b$ units if

$(X_r/X_b)\ M_b$ kg of fertilizer per hectare is placed in bands with a width of X_r centimeters. To find the influence of this difference on the uptake-yield curve, it is necessary to make the comparison under the circumstances where the uptake of the element applied in the fertilizer is the same with the two application patterns. The uptake can remain the same only if $(X_r/X_b)\ (U_b/U_r)\ M_b = (X_r/X_b)\ M_z$ kg of fertilizer per hectare is placed (see figure 6). The decrease in availability is equal to $X_r h_z$ units. In these expressions, h_z represents the intensity of the antagonistic reaction if M_z kg of fertilizer per hectare is applied broadcast.

In general, the relationship between the intensity of an antagonistic reaction (h) and the concentration of the applied fertilizer in the soil is not known. It may be assumed, however, that at most the antagonistic effects are proportional to the concentration of the fertilizer in the soil. (See for instance table 4 in SCHUFFELEN, 1946).) Hence,

$$h_z \leqq \left(\frac{M_z}{M_b}\right) h_b = \left(\frac{U_b}{U_r}\right) h_b$$

or

$$X_r h_z \leqq X_r \frac{U_b}{U_r} h_b$$

Thus, for the ratio of the decreases in availability,

$$\frac{X_r h_z}{X_b h_b} \leqq \frac{X_r (U_b/U_r) h_b}{X_b h_b} = \left(\frac{X_r}{X_b}\right)\left(\frac{U_b}{U_r}\right) = \left(\frac{X_r}{X_b}\right)^{0.56} < 1$$

This latter coefficient is the coefficient of equivalence (figure 6), which is always smaller than one. Since this coefficient decreases with decreasing width of the fertilizer band, the same is true for the decrease in availability caused by antagonistic reactions.

Hence, if the shape of an uptake-yield curve is unfavourably affected by antagonistic reactions, this effect is smaller, according as the fertilizer is placed in narrower bands.

Since the relationship between the intensity of antagonistic reactions and the concentration of fertilizers in the soil has not been studied quantitatively, it is not possible to formulate exactly the influence of the application pattern on the uptake-yield curves. If the conditions and the results of a fertilizer experiment are known, however, it can be determined fairly well whether antagonistic effects have influenced the shape of the uptake-yield curve and whether other curves would have been obtained had the fertilizer been applied in another way.

4. ANTAGONISTIC REACTIONS UNDER FIELD CONDITIONS

The uptake-yield curves obtained from application of nitrogen or phosphate fertilizers are usually independent of the fertilizer application pattern, indicating that antagonistic effects seldom occur with these elements. Because of competition among cations, however, antagonistic effects are to be expected more frequently with potassium fertilizers. In general, it is to be expected that the most marked antagonistic effects will occur where the plants have a shallow root system and where nutrient availability in the subsoil is low, because under these conditions the greater part of the effective root system is confined to that portion of the soil to which the fertilizer is applied.

The foregoing considerations have been restricted to antagonistic phenomena in the common sense of the word. Under the occasional conditions where root distribution differs markedly between fertilizer placement patterns, however, „antagonism" assumes the broader meaning employed in this paper. The difference in root distribution between fertilizer placement patterns may affect the availability of other nutrients (including water) that would not normally be involved in antagonistic reactions (see VI and VII).

An independent answer to the question of whether or not the shape of the uptake-yield curve is influenced by antagonistic phenomena can be obtained in some cases by studying the shape of the curve that results when the fertilizer is applied broadcast. For this purpose, the part of the uptake-yield curve defined in figure 4 as the region of increase is considered. In many figures reproduced in this paper, the uptake-yield curve crosses the axes at or near the origin [1] when it is extrapolated to zero uptake. Within the region of increase, therefore, the linear uptake-yield relationship produced by passing a line through the origin and the observation obtained without fertilizer should be followed by the observations with increasing quantities of fertilizer provided no antagonistic effects occur. Thus, if the extrapolated value obtained from observations with increasing amounts of fertilizer crosses the ordinate near the origin, the results support the view that antagonistic effects are absent. It may then be expected that the uptake-yield curve is independent of the fertilizer pattern.

On the other hand, if antagonistic effects do occur, the observations with increasing amounts of fertilizer will normally fall below the line extended from the origin through the observation without fertilizer. The extrapolated value obtained from the observations will thus cross the ordinate above the origin. But since this behaviour would result either from antagonistic effects or from failure of the observations to fall within the region of increase, it cannot be regarded as a suitable test for the existence of antagonistic effects, and it cannot be decided without further information, concerning soil, fertilizer and crop, whether the uptake-yield curve should be the same for different fertilizer patterns.

Because of lack of information, the antagonistic effects observed in practice can seldom be attributed unequivocally to any particular cause. The antagonistic effect shown in figure 8 may, according GOEDEWAAGEN, have resulted from N-deficiency as a result from applying phosphate to the soil. The effect on the uptake-yield curve shown in figure 9, on the other hand has perhaps resulted from an effect of phosphate reducing the toxicity of some constituent present in the soil.[2] The basis for this interpretation that fertilization induced a nutrient deficiency in figure 8 and decreased a toxicity in figure 9 is the relative position of the uptake-yield curves for broadcast and placed fertilizer in the two figures. The uptake-yield curve for placed phosphate is above that for broadcast phosphate in figure 8 and below it in figure 9. The experimental conditions connected with the data in figure 13 are not sufficiently well known to permit formulation of a hypothesis.

[1] Slight deviations may occur when the ordinate represents the yield of a part of the plant instead of the total yield, since these two quantities are not always proportional to each other.

STEENBJERG (1944, 1951) found that the uptake-yield curve may have an S-form if the availability of the given nutrient is very low. Most field experiments encountered in the literature are not sufficiently accurate to take this form into account.

[2] Prof. C. A. BLACK (Iowa State College, U.S.A.) remarked that the antagonistic effect in figure 8 may possibly have resulted from iron deficiency induced as a result from applying phosphate to the sand medium and the effect shown in figure 9 perhaps from the effect of phosphate reducing aluminium toxicity.

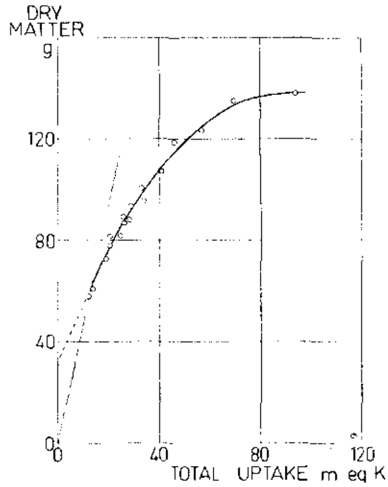


FIGURE 24.
Uptake-yield curve in a potassium-fertilization experiment. Compiled from METZ (1923).
Crop: potatoes. *Fertilizer:* potassium, evenly distributed. *Experiment:* in containers. *Results:* The uptake-yield curve crosses the ordinate above the origin, although the yield obtained without fertilizer is very small. Under these conditions a more favourable uptake-yield curve in case of placement is expected. The most favourable uptake-yield curve that could be found is represented by the lightly-ruled line.

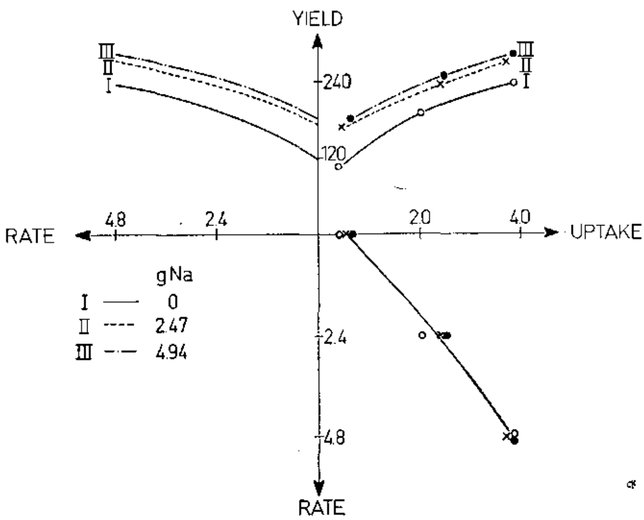


FIGURE 25.
Rate of fertilization, uptake of potassium and crop yield in a potassium-fertilization experiment involving different basal dressings of sodium.
Compiled from DORPH-PETERSEN and STEENBJERG (1950).
Crop: sugar beets. *Fertilizer:* potassium and sodium evenly distributed. *Experiment:* in containers (1944). *Scale units:* Rate in g K per container. Uptake in g K per container. Yield in g dry matter per container. *Treatments:* different amounts of sodium. *Results:* The rate-uptake curve is independent of the amount of sodium applied. The relationship between uptake and yield is favourably affected by the application of sodium. The uptake-yield curves cross extrapolated to zero uptake the ordinate above the origin.

Antagonistic effects are commonly observed in experiments with potassium fertilizers. Figures 24 and 25 show the failure of the uptake-yield curves to extrapolate linearly to zero yield at zero uptake and figure 17 is an example showing this failure, too, and a difference in uptake-yield curves resulting from potassium placement.

V. THE DISTANCE BETWEEN CROP ROW AND FERTILIZER BAND

1. THE AVAILABILITY OF THE FERTILIZER DURING EARLY GROWTH

In the foregoing chapters, placement and broadcast patterns have been compared in the situation where the roots are developed throughout the soil. In this situation, the ratio X_r/X_b is the same as the ratio of the fertilized soil volumes explored by plant roots. Before this condition has been established, the ratio of the fertilized soil volumes penetrated by roots varies. The influence of this varying ratio during early growth on the final uptake and yield and on the development of the plants are dealt with in this chapter.

Figure 26a represents schematically the circumstances that obtain with young plants whose roots have not yet permeated the soil. The distance to which the roots have developed in a horizontal direction where the fertilizer is broadcast, and the distance to which the roots have penetrated into the fertilized soil section where the fertilizer is placed are called Y_b and Y_r, respectively. During the period in which the root system is expanding horizontally into previously unoccupied soil, the value of Y_b changes from zero to X_b, and the value of Y_r changes from zero to X_r. The ratio Y_r/Y_b is thus variable until the ratio X_r/X_b is reached. The constant ratio X_r/X_b is reached at the moment when the fertilized areas in the case of broadcasting and placement are penetrated by roots.

In addition, there can be distinguished two basically different placement patterns, namely, contact placement and side-band placement. The relationship between Y_r/Y_b and time after germination for contact placement has been represented in figure 26b. The ratio is unity for a short time and then gradually decreases to the final value X_r/X_b. The function in the case of side-band placement on one side of the seed is represented by line 1 in figure 26c. Here the ratio first remains at zero for a short time, then increases to a maximum value and finally decreases gradually until the value

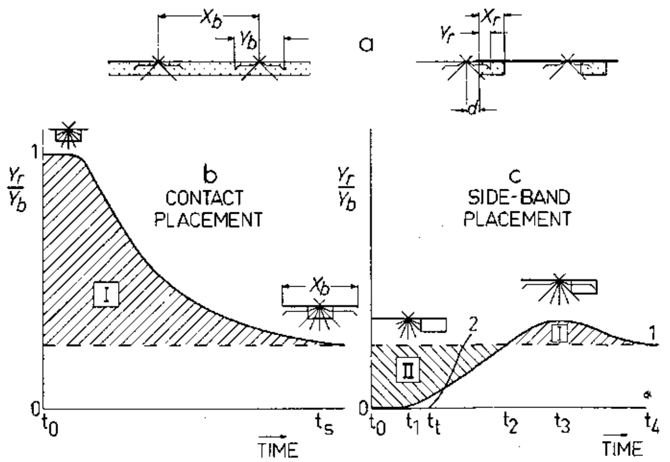


FIGURE 26. Availability of placed fertilizer Y_r and broadcast fertilizer Y_b at different times after emergence. The quantities Y_r and Y_b are represented in figure 26*a*. In figure 26*b*, the relationship between Y_r/Y_b and time in case of contact placement is represented schematically by the curve. The shaded area I is a measure of the greater availability of fertilizer placed in contact with the seed. The relationship in case of side-band placement is represented in figure *c* by line 1. The difference between the shaded areas I and II is a measure for the greater availability of fertilizer in case of side-band placement. Line 2 in figure *c* represents the ratio of the availability of fertilizer broadcast before emergence to that of fertilizer top-dressed at the moment t_t.

X_r/X_b is reached. The root development at some characteristic moments has been given in the figure. If the fertilizer is placed on both sides of the seed row, a curve similar to line 1 in figure 26c is obtained.

2. Damage by contact placement

It is evident that the relative availability of the fertilizer during early growth is greater with contact placement than it is with side-band placement. To obtain the same yields, however, the concentration of placed fertilizer must, in general, be higher than the concentration of broadcast fertilizer (II, 5).

This higher concentration may prevent germination of the seed or may damage the seedling if the fertilizer is placed close to the seed. It has been shown abroad that injury to the crop can be prevented by placing the fertilizer a certain distance to the side of the seed, since during the interval of time before the roots have reached the fertilizer, the fertilizer concentration gradually decreases. To determine which placement method can be recommended for conditions in Holland, it is necessary to know whether damage will occur and if it does occur, what distance between fertilizer and seed is necessary to prevent it.

In Holland, many field experiments have been carried out in which large amounts of fertilizer were tested. Nevertheless, injury caused by high concentrations of the fertilizer has hardly ever been reported. Since the maximum applications per hectare have often been 400 kg of P_2O_5, 400 kg of K_2O and 200 kg of N, seedling injury would not be expected to occur if applications in bands with $X_r/X_b = 0.25$ do not exceed 100, 100 and 50 kg/ha, respectively, and if the fertilizer is mixed with the soil in the same way as with broadcasting.

Further information can be obtained from experiments abroad. Salter (1938), summarizing American experiments, reported that placement in contact with or directly above or below the seed often causes injuries, although it may be advantageous where the rate of application is low and where plenty of rain falls soon after planting. In the United States, the best pattern has been found to be side-band placement. The same has been found by some English investigators (Cooke, 1949, 1951; Miles, 1947), who state that contact placement can be recommended only in the case of cereals fertilized with phosphate, and where the rates do not exceed about 60 kg of P_2O_5 per ha.

The minimum distance between crop and fertilizer row depends on the crop, the fertilizer, the soil and the environmental conditions. The distance must be higher when tender crops are fertilized (sugar beet, mangolds, peas, beans), when highly-soluble fertilizers are used (nitrogen, potash), when the soil is sandy, when the soil is dry and when high rates of fertilizer are applied in concentrated bands (Truog et al., 1925, and Sayre and Clark, 1935). A safe distance in England and the U.S.A. has been found to be about 5 centimeters, even under adverse conditions.

Compared with the U.S.A. and England high amounts of fertilizer are used in Holland. These high amounts must be placed, however, in wider bands (see IX). So the concentration of the fertilizer in the soil will be about the same. As the wheather conditions in Holland and at least a great part of England do not differ very much, also in Holland no damage is to be expected if the fertilizer is placed 5 centimeters to the side of the seed.

3. Side-band placement and top dressing

If the fertilizer is placed at some distance from the seed, the relationship between Y_r/Y_b and the time is represented by line 1 of figure 26c. The length of time (t_1-t_0) between seed germination and the instant that the ratio Y_r/Y_b becomes greater than zero is determined by the time required for the roots to cross the distance d between the crop row and the fertilizer band (figure 26a). At the instant t_3, the roots enter the unfertilized soil on the other side of the fertilizer band. At that moment, the roots of the broadcast-fertilized plants are usually not yet present under the whole soil surface. Thus, the ratio Y_r/Y_b at time t_3 will be greater than X_r/X_b. At the moment t_4, the roots are developed under the entire soil surface and the final ratio X_r/X_b has been reached.

Since during the period (t_1-t_0) the crop grows without fertilizer, the early growth may be influenced to such an extent that the rate-uptake curve and the uptake-yield curve for placed fertilizer cannot be derived from the corresponding curves for broadcast fertilizer by the calculations in the preceding chapters. A careful examination of this influence of the availability of fertilizer during the early growth of the plant is therefore necessary.

This examination is made possible by elaborate experiments on top dressing carried out by several investigators. It is evident that if the effects of fertilizer broadcast before germination and broadcast at the time t_t are compared, the ratio of the availabilities of the fertilizer in both cases change with time in the manner represented by line 2 in figure 26c. The similarity between this line and line 1 for side-band placement permits the determination of the influence of the distance between the fertilizer band and the crop row on yield by means of experiments on top dressing.

4. The influence on the uptake

Many investigations show that the rate of nutrient uptake is highly correlated with the rate of top growth, and that both are small when the plants are young. A characteristic set of data is given in table 5.

It is evident that the uptake during the first weeks of growth is negligible compared with the total uptake. The total uptake is therefore independent of the uptake at the beginning of the growing season if the uptake rate during the main growing period is independent of the uptake during early growth. It has been concluded from extensive experiments by Remy and co-workers that the rate of uptake during rapid growth is in general not influenced by the availability of the fertilizer before that time. The results of one of the experiments compiled from a summary paper of Remy (1938) are given in table 6.

Experiments have been reported in which a delayed supply has caused a markedly smaller total uptake (V, 5). However, analyses have shown that in such experiments the supply with nutrients during emergence and early growth was so small that starvation of the plants took place. It has been shown by Remy (1938) that a smaller uptake does not result if some of the nutrient is available in the soil or if a small dressing is given before emergence.

5. The influence on the uptake-yield curve

Nutrients taken up in the later stages of growth are not always used efficiently by the plant, but may be stored in the tissue or may cause secondary growth. By plotting

TABLE 5. Growth and nutrient uptake of spring-sown oats (HALLIDAY, 1948). (Periods of maximum growth and uptake are denoted in italics.)

Period	Daily increases per acre during period				
	Dry weight	Nitrogen	Phosphorus	Potash	Calcium
	lb	lb	lb	lb	lb
April	1	0.0	0.0	0.0	0.0
May:					
1st week	3	0.1	0.1	0.2	0.0
2nd week	5	0.1	0.1	0.2	0.0
3rd week	10	0.4	0.1	0.4	0.0
4th week	*45*	*1.3*	*0.5*	*2.1*	0.2
June:					
1st week	*75*	*2.3*	*0.6*	*2.6*	*0.3*
2nd week	*75*	*1.3*	*0.4*	*1.8*	0.2
3rd week	*85*	*0.7*	*0.4*	*1.0*	0.2
4th week	*85*	0.2	*0.3*	*0.9*	0.2
July:					
1st week	*80*	0.1	*0.3*	0.5	0.2
2nd week	*75*	0.1	0.2	0.1	0.2
3rd week	*50*	0.0	0.1	–0.1	0.2
4th week	40	0.0	0.1	–0.1	0.1
August:					
1st week	30	0.0	0.1	–0.1	0.1
2nd week	7	0.0	0.0	–0.1	0.0

TABLE 6. Summarized results of pot experiments on time of application of different nutrients for oats (REMY, 1938)

Time of application	Early	Medium early	Late	Divided
First application	At seeding	At emergence	At beginning of stalk growth	At seeding
Second application	At beginning of stalk growth	At beginning of stalk growth	At beginning of blossoming	At heading time
Uptake as percentage of applied amount:				
Nitrogen	94	91	87	90
Phosphorus	52	62	63	67
Potash	94	96	90	93

the total uptake and the yield against each other, it can be determined whether this inefficient use of delayed absorbed nutrients has taken place. As long as the observations for normal and late applications lie on the same curve, the application of the fertilizer has not been too late. This method of comparison has the advantage that treatments in which the uptake is not the same can be compared.

First, the influence of delayed supplied nutrients on the uptake-yield curve of small grains will be discussed.

SIMON (1927) conducted experiments with oats in containers in which nitrogen, phosphorus and potassium were applied at seeding time and at different stages of

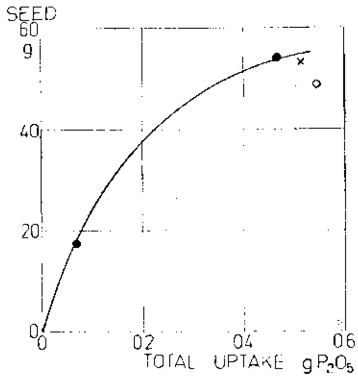


FIGURE 27.
Uptake-yield curve in a phosphorus-fertilization experiment involving different times of application.
Compiled from SIMON (1927).
Crop: oats. *Fertilizer:* phosphate. *Experiment:* in containers. *Treatments:* fertilizer applied before seeding (•); fertilizer applied during the tillering stage (x); fertilizer applied during stalk growth (o). *Result:* The uptake-yield curve is unfavourably affected by applying the fertilizer during stalk growth.

development. Although in each experiment only a few data that can be used in the calculation of the uptake-yield curve have been given, it is possible to determine the deviations caused by differences in the time of application of the nutrient. The concordant results of analogous experiments indicate that the observed deviations are significant. The data of one experiment are reproduced in figure 27.

The experiments show that if the first application of large amounts of N, P or K was delayed until the stalk growth stage, the uptake-yield curve was unfavourably affected. The nutrient absorbed from the late application was in part either stored in the plant tissue or used in the development of secondary shoots. The results of an application during tillering were not the same for all fertilizers. In the case of nitrogen, the reduction in yield of the uptake-yield curve was on the average 10 %. The yield without fertilizer was only 32 % of the yield obtained on pots with normal dressing at seeding. In the case of phosphate, the yield depression of the curve was on the average 6 %; the yield on the unfertilized pot was 40 % of the yield on the one fertilized at seeding. In the case of potash the respective values were 0 % and 76 % on the average. Thus, the unfavourable effect of a delayed application increases with increasing time between emergence and application. The results indicate further that the yield depression resulting from delayed application is smaller with fertile than with infertile soils.

The experiments of SELKE (1941) on nitrogen fertilization of small grains are in agreement with the latter observation. In his work it was found that applications as late as the beginning of stalk growth did not reduce the uptake-yield curve. In all

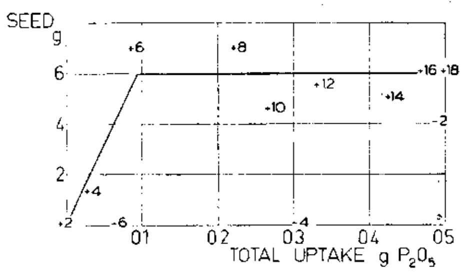
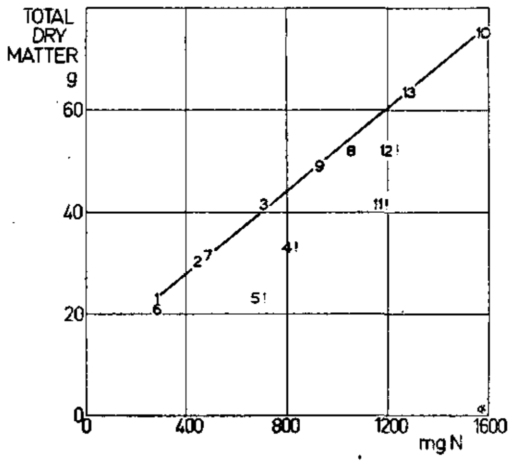


FIGURE 28.
Uptake-yield curve in an experiment with phosphorus available at different times in nutrient solution.
Compiled from BRENCHLEY (1929).
Crop: barley. *Fertilizer:* phosphate. *Experiment:* in culture solution. *Treatments:* Numbers +2, +4, +6, ..., +16 and +18: phosphate supplied during the first 2, 4, 6, ... 16 and 18 weeks of growth. Numbers –2, –4, –6, ... –16 and –18: no phosphate supplied during the first 2, 4, 6, ..., 16 and 18 weeks of growth. *Results:* The uptake-yield curve is unfavourably affected by applying no phosphate at all during the first weeks of growth. The plants with treatments, –8, ..., –18 died completely.

FIGURE 29.
Nitrogen uptake-yield curve with culture solutions differing in concentration at different intervals.
Compiled from AVDONIN (1941).
Crop: oats. *Fertilizer:* „normal" culture solution of AVDONIN (see original paper). *Experiment:* in culture solution.



Treatments:

No	Treatment
1	1/2 normal at the beginning of the experiment. Solution not renewed.
2	1 ,, ,, ,, ,, ,, ,, ,, ,, ,, ,,
3	2 ,, ,, ,, ,, ,, ,, ,, ,, ,, ,,
4	3 ,, ,, ,, ,, ,, ,, ,, ,, ,, ,,
5	5 ,, ,, ,, ,, ,, ,, ,, ,, ,, ,,
6	1/2 ,, (four 1/8 normal increments). ,, ,, ,,
7	1 ,, (four 1/4 normal increments). ,, ,, ,,
8	3 ,, (twelve 1/4 normal increments). ,, ,, ,,
9	3 ,, (twenty four 1/8 normal increments). ,, ,, ,,
10	0–10 days: 1/4 normal; 10–15 d: 1/2 n; 15–20 d: 1 n; 20–35 d: 2 n; 35 d–end: 3 n.
11	3 n „stable" (solution renewed every 5 days).
12	0–10 days: 1/4 n; 10–15 d: 1/2 n; 15–20 d: 1 n; 20–25 d: ? n; 25–35 d: 2 n; 35–45 d: 3 n; 45 d–end: 5 n.
13	0–50 days: the same as no. 10; 50–60 d: 2 n; 60 d–end: 1/2 n.

Result: The uptake-yield curve is unfavourably affected by high concentrations of the nutrient solution.

these experiments, some fertilizer was always applied at or before seeding so that the lowest yield was 50 % or more of the highest yield.

SCHLIESZUS (1942) also has conducted some experiments that permit the calculation of uptake-yield curves for different times of application. These experiments likewise showed that the unfavourable effect of a delayed application on the curve disappeared in the presence of a relatively small supply of the nutrient during early growth.

The uptake-yield curve calculated from an experiment of BRENCHLEY (1929) with barley grown in culture solution is represented in figure 28. The treatments are described in the subscription. The absence of phosphate from the nutrient solution during later stages of growth reduced only the luxury consumption and not the yield. The absence of phosphate from the solution during the first 2 weeks probably affected the uptake-yield curve unfavourably, and the absence of phosphate during the first 4 weeks suppressed the yield much more than the uptake. (The tillering stage began in the third week.) The plants with no phosphate during the first 6 or more weeks made

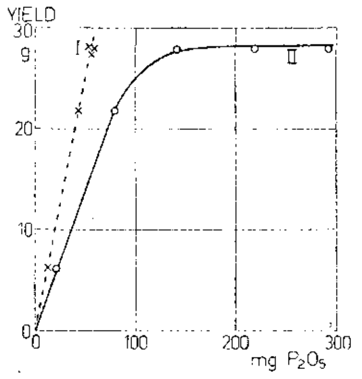


FIGURE 30.

Yield of corn seedlings versus quantities of organic and inorganic phosphate contained therein.
Compiled from SOKOLOV (1945).
Crop: corn, young seedlings. *Fertilizer:* phosphate in different amounts, evenly distributed. *Experiment:* in containers with podsolic soil. *Curves:* I: Relationship between yield and amount of organic phosphate in the seedlings. II: Relationship between yield and total amount of phosphate in the seedlings. *Result:* Large amounts of inorganic phosphate in the seedlings do not improve the yield.

exceedingly poor growth. It appears, therefore, that the availability of nutrients during the first weeks of growth is important for the production of maximum final yields.

PEMBER (1917) and PEMBER and McLEAN (1925) carried out experiments analogous to those of BRENCHLEY (1929), but failed to find an unfavourable effect of delayed supply of phosphorus. BRENCHLEY supposed that this behaviour resulted from the fact that in PEMBER's experiments some phosphate was available, since the seeds were germinated in a culture solution containing phosphate, and since tap-water was used in making up the culture solutions.[1]

The results of AVDONIN's (1941) experiments with oats and barley in culture solution (an example is given in figure 29) are in agreement with the experiments discussed thus far. The uptake-yield curve was unfavourably affected only when high concentrations were used at some time during the growing period. Large amounts of nutrients during the first weeks of growth did not change the uptake-yield curves.

An experiment of SOKOLOV (1945), reproduced in figure 30, shows that only a small part of the total quantity of P_2O_5 absorbed by corn seedlings was used for the synthesis of organic components. The growth was not increased when the content of inorganic phosphate in the seedlings was increased to high levels by heavy phosphate fertilization.

It is concluded from the foregoing experiments that if plant nutrients are supplied by the soil in more than minimal quantities, it suffices that normal amounts of nutrients are available at the beginning of the tillering stage. Or, more precisely, if without additional fertilizer the yields are in excess of about 50 % of the maximum yield, it suffices that the added fertilizer is first available during the tillering stage.

SELKE (1941) has carried out analogous experiments with potatoes, sugar beets and corn; SCHLIESZUS (1942) with soybeans, white mustard and clover; AVDONIN (1940) with peas and millet and SIMON (1927) with potatoes. These experiments also show that if the yield without fertilizer is about 50 % or more of the maximum yield, it suffices

[1] The opinion is sometimes expressed that it is necessary to supply young seedlings with abundant amounts of phosphate. This opinion is perhaps based upon the observations that (*a*) some phosphate is necessary during early growth, (*b*) the response of plants to phosphate application is commonly most marked in the early growth stages, and (*c*) the plant can absorb enough phosphate during the first quarter of the growing season to fulfill the requirements of the entire season. The author, however, has not encountered experiments in which it was found that high phosphate availability during the entire growing season resulted in greater yields than did medium availability during early growth and high availability some time after emergence (see also V, 9).

that the fertilizer is available some weeks after emergence. Since the times of fertilizer application were not stated in terms of the development of the crops in these experiments, no definite statements can be made regarding the stage of growth at which the fertilizer must be available.

6. Root growth of small grains

It has been concluded from the literature reviewed in the foregoing pages that the rate-uptake curve and the uptake-yield curve are independent of the availability of the fertilizer during some weeks after emergence, as long as some nutrient is available during the first weeks of growth. Thus, the curves are independent of the distance *d* between crop row and fertilizer band (figure 26) if the roots cross this distance within these first weeks of growth. The length of the period $(t_1 - t_0)$ (figure 26c) depends, of course, on the distance *d* and the rate of horizontal growth of roots.

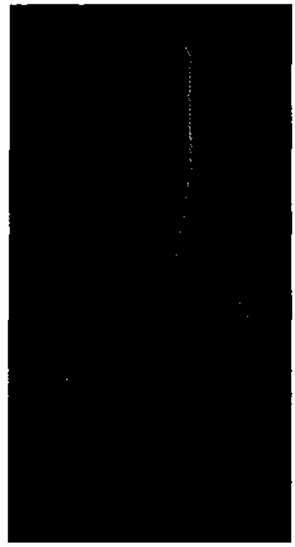


Figure 31.
The primary root system of a wheat seedling. Goedewaagen (1942).

The rate of root growth depends on soil structure, fertility and moisture content of the soil, weather conditions and so on. However, young plants are usually grown in soils in which the upper layer is cultivated shortly before seeding and directly after emergence in such a way as to maintain a good tilth. Therefore it is not necessary to take into account the influence of soil structure on the development of the young roots. As regards soil fertility, it has been found that during early growth the development of the root system in poor soils is not worse but often better than on well-fertilized soils (Goedewaagen, 1942; Sabinin and Minina, 1935). It appears, therefore, that to determine the rate of root growth of young plants receiving placed fertilizer, it suffices to study the rate of root growth under normal conditions.

Since the stages of development of small grains can be distinguished easily, it is advantageous to discuss the root development of these crops in detail. Upon germination of the grain, the primary roots appear. These form the basis of the primary root system (figure 31). After the second or third leaf has been formed, secondary roots appear from the nodes above the primary roots (figure 32). By this time the seed is consumed. The subsequent development depends on the production of organic materials by the leaves. Soon after the

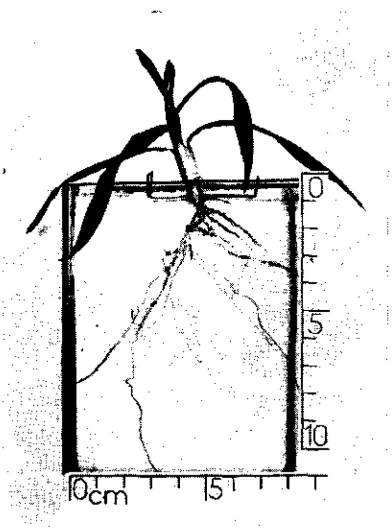


Figure 32.
The root system of a rye plant on a sandy soil at the beginning of the tillering stage.

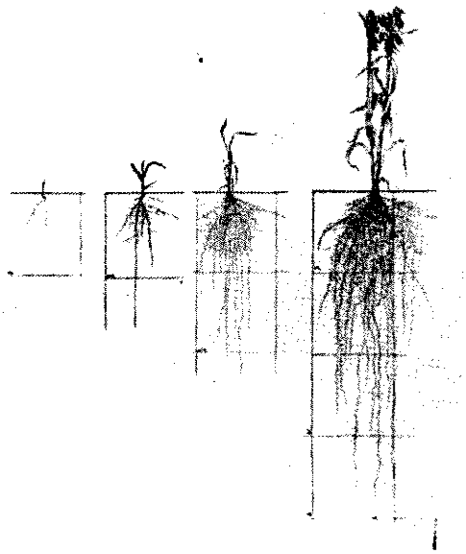


FIGURE 33.
The extent of the root system of oats at different stages of growth. WEAVER (1926).

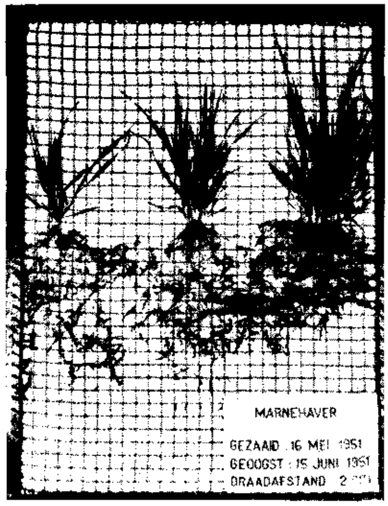


FIGURE 34.
Root system of oats sown in sandy soil May 16 and harvested June 15.

appearance of the secondary roots, the tillering stage begins (GOEDEWAAGEN, 1942). It may be seen from the photographs that by the beginning of tillering the development of the primary root system in the horizontal direction is several centimeters. Since the primary root system does not penetrate the surface layers, it is possible that fertilizer placed some centimeters laterally from the seed is not yet available. If that is the case, the fertilizer is first reached by secondary roots, which grow at a considerable rate in a horizontal direction directly below the soil surface. WEAVER (1926), for instance, observed a growth rate of 12.5 mm/day during 70 days. This value does not seem to be extraordinary high, according to GOEDEWAAGEN (1942). The rapid growth rate is illustrated by figure 33. It is evident, therefore, that during the first stages of tillering the secondary roots penetrate several centimeters in a horizontal direction.

Although the rate of root growth decreases with time, illustrations (figure 33, 34; GOEDEWAAGEN, 1942) indicate that in the second part of the tillering stage roots may be found underneath the entire soil surface. By that time a lateral development of 10 to 15 centimeters (one-half the distance between the rows) is reached, and the ratio Y_r/Y_b is equal to X_r/X_b. It may be seen from the figures that at that time vigourous top growth has just started. Thus, the greater part of the nutrient is taken up during the time when the ratio Y_r/Y_b is equal to X_r/X_b. These conclusions from the literature were verified by observations made on grain crops in the neighbourhood of Wageningen during 1951.

7. Conclusions with respect to small grains

The conclusions with respect to small grains may be summarized as follows:

a. Injury of seed and seedlings is avoided by placing the fertilizer about 5 centimeters laterally from the seed.

b. The root system develops so rapidly that even on soils with a low fertility level, fertilizer placed 5 centimeters laterally from the seed is reached by either the primary or secondary roots during the first stages of tillering. Therefore, the uptake-yield curve is independent of the distance between crop row and fertilizer band.

c. Beyond the second part of the tillering stage, roots are present under the entire soil surface, so that by the time vigourous top growth starts, the constant ratio X_r/X_b has been reached and the compensation function can be used to calculate the uptake.

d. Only on soils exceedingly low in fertility it is necessary to supply fertilizers during the first weeks of growth.

8. Other crops

A summary of the observations on root development of other crops cannot be given so easily since it is more difficult to distinguish different stages of development. During early growth, the roots of corn and potatoes grow rapidly in a horizontal direction, as shown by figures 35 and 36. With these crops, fertilizer placed about 5 centimeters from the crop row is available soon after emergence, and no depression of the uptake-yield curve is expected from placement of side bands within a normal distance. The observations indicate also that before vigourous top growth starts the roots are in general present under the entire soil surface.

Sugar beets, mangold, etc. have quite another type of root system. In most cases, one tap grows rapidly downward while development of the lateral roots is restricted, as may be seen in figure 37. Observations have shown, however, that at the time of thinning the lateral roots have a length of about 5 centimeters, and that in Holland at the end of June roots are developed under the entire soil surface when the beets are grown with the usual spacing (30 to 40 centimeters). This root development is rather late, but it must be taken into account that the period of rapid growth of beets starts at about the same time. Beets are relatively late-developing plants.

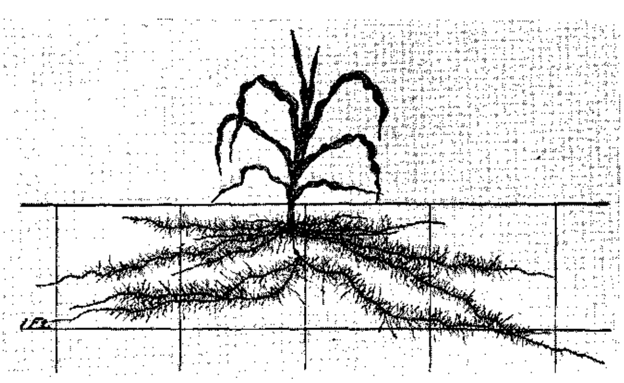


FIGURE 35.
The root system of a corn plant 36 days old. WEAVER (1926).

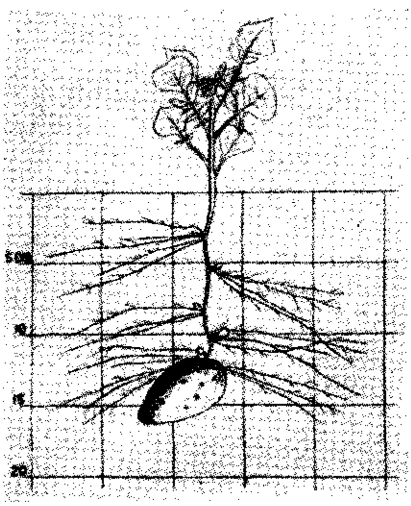


FIGURE 36.
The root system of a young potato plant. GOEDEWAAGEN (1942).

Although peas and beans form tap roots also, the horizontal extension of the root system is so large that fertilizer placed normal distances from the seed is available during early growth (figure 38).

For all these crops it has been shown (V, 5) that withholding the fertilizer during the first weeks of growth does not depress the uptake-yield curve or the final uptake. Thus, the conclusions formulated with respect to small grains hold, *mutatis mutandis*, for other field crops.

9. INSURANCE AGAINST UNFAVOURABLE CONDITIONS BY CONTACT PLACEMENT

Although the distance between crop row and fertilizer band is usually of minor importance with respect to final uptake and yield (apart from damage to the seed or from starvation on poor soils), the influence on the resistance of the crop to unfavourable environmental conditions cannot always be neglected. In this connection, figure 26c is again considered. During the period from t_0 to t_1, the plants grow without

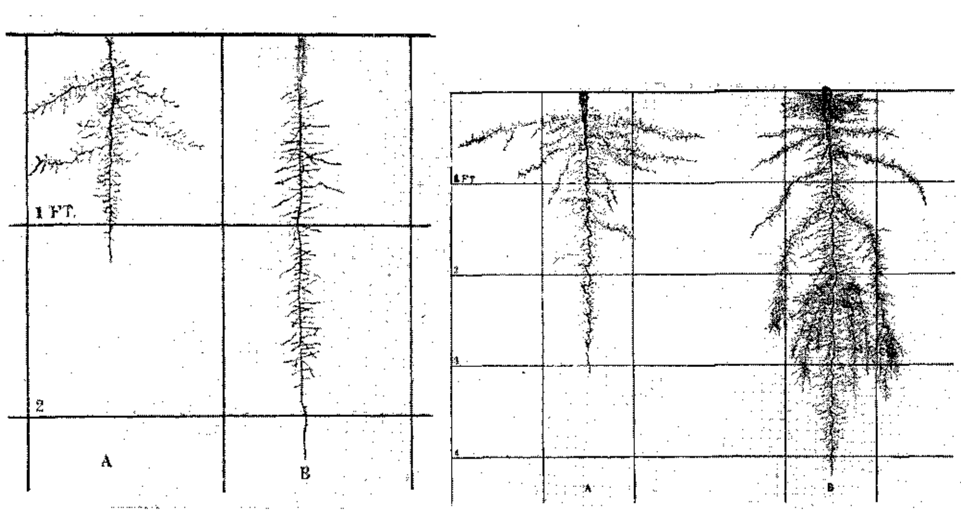


FIGURE 37.
The root system of 2-month-old (left hand figure) and 3-month-old (right hand figure) young sugar beets on dry land (A) and irrigated soil (B). WEAVER (1926).

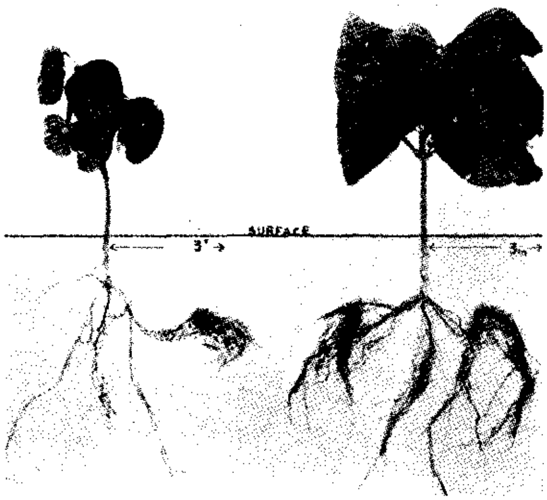


FIGURE 38.
The root system of bean seedlings. Fertilizer placed in one band 2.5 inches to side of seed (left hand figure) and in bands 1.5 inches to each side of seed (right hand figure). SAYRE and CLARK (1935).

fertilizer when the fertilizer is applied in side bands but not when it is applied broadcast. During this time interval, therefore, the nutrient status of the broadcast-fertilized plant is better than that of the side-band-fertilized plant. Except in the case of nitrogen, a high nutrient status favors the development of a sturdy plant that will offer increased resistance to unfavourable environmental conditions such as bad weather and parasites. The probability that the crop will be damaged during early growth is therefore greater where the fertilizer is placed in side bands than where it is applied broadcast. To diminish this risk, it is advantageous to place the fertilizer as close as possible to the seed. The early growth will be best with contact placement, provided no damage occurs (figure 26b).

From the time that the roots have reached the fertilizer band in case of side-band placement, the availability of the fertilizer increases rapidly, so that the plants catch up rapidly, and often pass broadcast-fertilized plants receiving an equal quantity of fertilizer per acre. Because of the rapid changes in development during early growth (figure 39), data on crops harvested at an early growth stage cannot be used to determine the final effect of different placement patterns. Only crops harvested at or near maturity are suitable for this purpose.

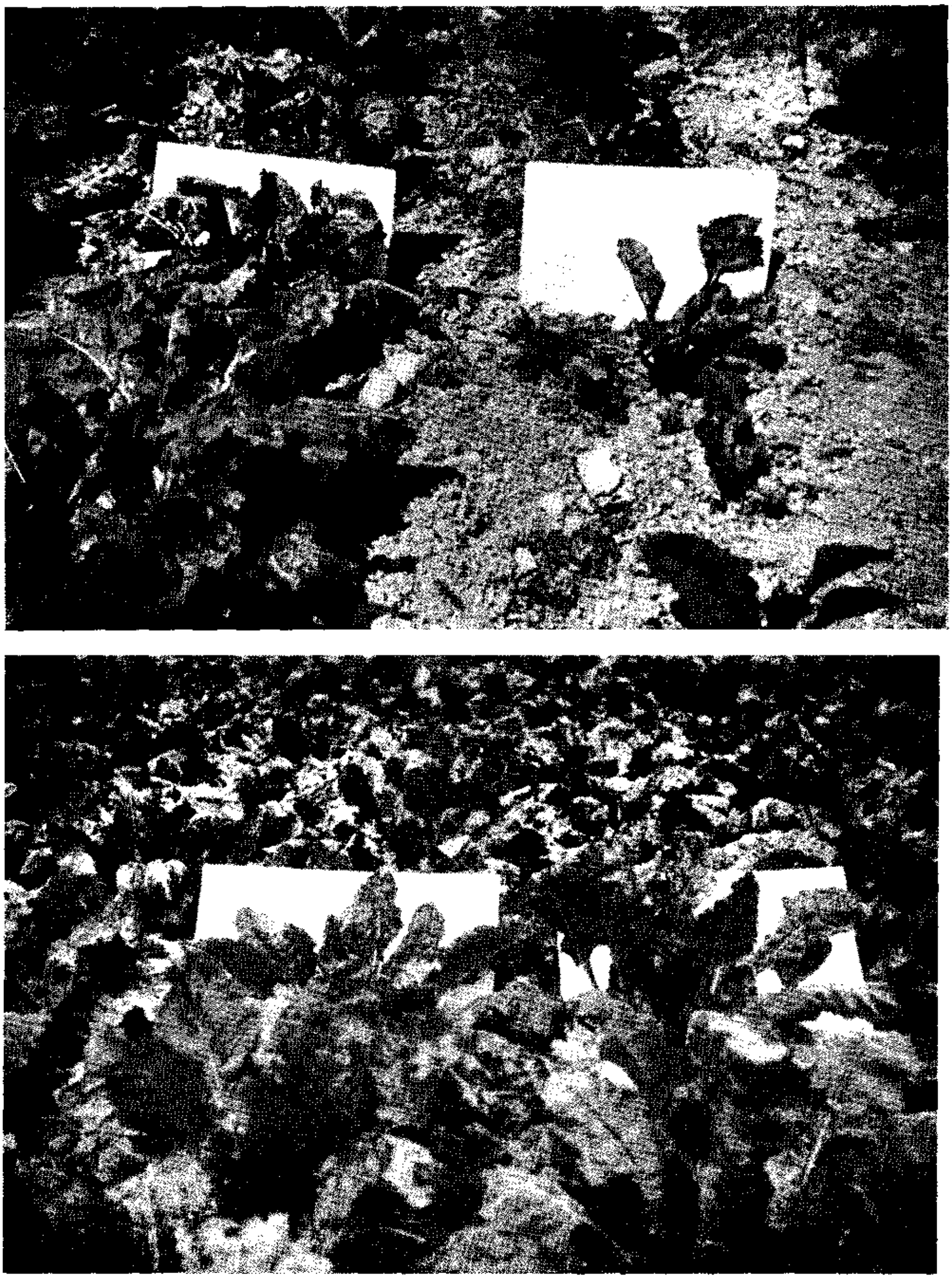


FIGURE 39.
Growth of sugar beets with fertilizer applied in different ways. The left hand row received 7 cwt N-P-K (0–16–13.4) compound fertilizer per acre, placed beside the seed; the right-hand row received the same dressing broadcasted and harrowed in. In July (top figure) there was much better growth with the placed fertilizer, but differences in size of tops and yield had vanished by the end of August (bottom figure). COOKE (1951).

VI. THE INFLUENCE OF PLACEMENT PATTERN ON ROOT DEVELOPMENT

In the preceding chapter, attention was directed to the presence or absence of roots in different parts of the soil, and not to the problem of how the development and the activity of the roots is influenced by the placement pattern.

The relationship between fertilizer rate and root weight is qualitatively the same as the relationship between fertilizer rate and yield, as shown in table 7.

TABLE 7. The influence of nitrogen, phosphate and potash on the development of roots and tops of winter wheat on a poor sandy soil (GOEDEWAAGEN, 1942)

Fertilizer per ha	Dry weight Tops	Dry weight Roots	Uptake
kg	mg	mg	mg
N [1]			N
0	196	232	11
20	737	556	36.3
120	1623	481	100.6
P_2O_5 [1]			P_2O_5
0	333	310	3.7
114	1141	425	28.5
455	1353	452	36.3
K_2O [1]			K_2O
0	824	298	29.5
86	1703	628	106.5
442	1439	522	107.5

Other nutrients applied in sufficient amounts

Beyond a certain nutrient level, the root weight decreases with increasing rate of fertilizer. At this level the amount of roots seems not to be a limiting factor, since in general the decrease in root weight with further addition of the nutrient is not coupled with a decrease in uptake or top growth. In the case of nitrogen, maximum root weight seems to be reached at moderate applications or fertility levels (GOEDEWAAGEN, 1942). This behaviour explains the fact that with nitrogenous fertilizer confusing results have been obtained; sometimes stimulating effects on root growth are reported (TOLLENAAR, 1930), and sometimes suppressing effects are reported (CHRIST and WEAVER, 1924). Since the method of application is of importance only on soils with a low or moderate fertility level (IX), only the effect on root growth in this condition will be discussed.

GOEDEWAAGEN (1942) found that fertilization increases the root growth not only in the parts of the soil in which the fertilizer is mixed but also in the unfertilized parts of the soil. In most of his experiments, however, the fertilizer was mixed with a comparatively large part of the soil (broadcast experiments). Some experiments of GOEDEWAAGEN (1942) and experiments published or discussed by TRUOG, et al. (1925), GÖRBING (1930), SAYRE and CLARK (1935) and SALTER (1938) indicate that even if the fertilizer is placed in restricted zones root growth increases with increasing fertilizer rates, both in the fertilized and unfertilized part of the soil. More fibrous roots may of course be developed in the fertilized parts (morphological compensation, III,2). It appears, therefore, that where the fertility level of the soil is not high, the root development of a fertilized plant is better than the root development of an unfertilized plant, whatever the application method may be.

Although only a few experimental results are available, it is possible to get at least an indication of the quantitative effect of placement on root growth. In IV, 1 it was shown that the relationship between uptake and yield is in most cases independent of the fertilizer pattern, which indicates that the use of ions by the plant and the distribution of ions in the plant are independent of the fertilizer pattern. The distribution of ions and of assimilates between tops and roots is thus probably independent of the fertilizer pattern where the comparison is made at constant nutrient uptake. As a consequence it is to be expected that the relationship between root weight and total uptake and between the „activity" of the roots and total uptake also is independent of the fertilizer pattern.

There are yet no experimental tools to determine the relationship between the „activity" of a root system and the total uptake. Only a few experiments are encountered by means of which it can be tested whether or not the root weight-uptake curve is independent of the fertilizer pattern, since accurate determination of the root weight is very difficult. In the experiment of GOEDEWAAGEN (1942), reproduced in figure 8, the relationship between root weight and total uptake is independent of the fertilizer pattern (figure 8*b*). In the field experiment of FREDRIKSSON and WIKLANDER (1950), reproduced in figure 15, the scattering of the observations is greater (figure 15*b*). It may be concluded, however, that the observations for placed-fertilized plants do not differ significantly from the root weight versus total uptake curve for broadcast-fertilized plants. Although GILE and CARRERO (1917) also determined root weights in their experiments (table 3), there are not enough observations to determine whether or not the root weight versus uptake curves for placed and broadcast fertilizer are the same or not (figure 7*b*).

There is little doubt that there are instances in which the root weight versus total uptake curve is affected by the fertilizer pattern. Differences between fertilizer patterns in this respect would be expected in those cases where antagonistic effects are indicated from an analysis of data obtained on yield and nutrient content of the above-ground portion of plants (IV, 3 and 4) or where roots are damaged by excessive concentrations of the fertilizer (V, 2) or where roots grow better with deep than with shallow application patterns in dry years (VII, 4).

VII. APPLICATION OF THE THEORY TO FIELD CONDITIONS

1. A SUMMARY OF THE CALCULATION SCHEME

The theory may be summarized in the following calculation scheme:

From suitable broadcast experiments the following experimental relationships are derived:

$$U_s = \Phi(M_b) + U_o \text{ (rate-uptake curve)} \quad (1)$$

$$Y = \Psi(U_s) \quad \text{(uptake-yield curve).} \quad (2)$$

Where U_s is the total uptake of the nutrient applied in the fertilizer, U_o is the uptake of the same nutrient without fertilizer, Y is the yield and M_b is the quantity of fertilizer applied per hectare. For placement methods, as defined in figure 1, the total uptake can be calculated by

$$U_s = \left(\frac{X_r}{X_b}\right)^{0.44} \Phi\left(\frac{X_b}{X_r} M_r\right) + U_o \quad (3)$$

where U_s and M_r again represent, respectively, the total uptake and the quantity of fertilizer applied per hectare; in this instance, however, they refer to placed fertilizer.

In most cases the uptake-yield curve is independent of the fertilizer pattern. If antagonistic effects occur, a function

$$Y = \Psi'(U_s) \quad (4)$$

must be expected in case of placement. The form of this function may be estimated from the form of the uptake-yield curve for broadcast fertilizer by studying the environmental conditions. Apart from injury or starvation of the seedlings, the distance between crop row and fertilizer band is of minor importance.

2. THE INFLUENCE OF WIDE DISTANCES BETWEEN THE CROP ROWS

The roots are not always present under the entire soil surface during the main uptake period if the crop is cultivated in wide-spaced rows or is harvested at an early stage of growth. These circumstances cause a decrease in efficiency of broadcast fertilizer since the bands can be situated in such a way that full exploration by roots is possible. To calculate the influence of incomplete root occupation on the yield, it is necessary to determine the part of the soil without roots. This can be done by means of observations on the root development in the surface layer of the soil. Although the density of the root system gradually decreases to zero, arbitrary boundaries can be accepted. If the width of the area without roots is x, the comparative explored sections are X_r and $(X_b – x)$, and not X_r and X_b. Equation 3 must, therefore, be modified as follows:

$$U_s = \left(\frac{X_r}{X_b - x}\right)^{0.44} \Phi\left(\frac{X_b}{X_r} M_r\right) + U_o \quad (3a)$$

and the coefficient of equivalence becomes

$$\frac{X_r}{X_b}\left(\frac{X_b - x}{X_r}\right)^{0.44}$$

Thus, for example, if X_b, X_r and x are 60, 10 and 10 centimeters, respectively, a value of 0.33 must be expected for the coefficient of equivalence instead of the value 0.37 under the condition that the roots are developed under the entire soil surface. It is evident that for relatively great values of x, band placement is much more efficient than broadcasting.

3. The influence of thickness of fertilizer band

The theory has been developed for the case in which broadcast and placed fertilizer are distributed in the same way below the soil surface (figure 1). In general, fertilizer applied broadcast is distributed evenly on a more or less rigid soil surface and then harrowed in. Since the tines of the harrow penetrate about 6 centimeters in the soil, this is about the maximum depth of the fertilized surface layer. The actual depth is smaller, since the fertilizer is not evenly mixed through this layer. Tinnefeld (1930) found that about 85 % of the fertilizer is mixed through a surface layer of 4 centimeters.

Although some mechanical distributors place the fertilizer in a thin band in the soil, this is a bad practice, as shown by American investigators (Smith, 1927). Only when small amounts are applied or when plenty of rain falls before the roots reach the fertilizer, will no injury occur. The only equipment that can be recommended, therefore, is the type that mixes the fertilizer thoroughly with a certain soil layer. Particularly when large amounts are applied, as is the case under Dutch conditions, high demands must be made in this respect on placement attachments.

Thus, the thickness of the fertilizer band is usually several centimeters, and is in general not identical with the thickness of the broadcast fertilizer layer. Before the plants use the fertilizer, however, it is often subjected to some leaching, as a result of which the initial differences in vertical distribution disappear. In those cases in which the differences in thickness are large and no leaching occurs, it is appropriate to use the ratio of the fertilized volumes, instead of the ratio of the fertilized surfaces, to calculate the uptake of placed fertilizer. Equation 3 thus becomes

$$U_s = \left(\frac{X_r t_r}{X_b t_b}\right)^{0.44} \Phi\left(\frac{X_b t_b}{X_r t_r} M_r\right) + U_o \qquad (3b)$$

in which t_b and t_r represent the thickness of the soil layer in which the broadcast and placed fertilizer are mixed, respectively.

4. The influence of depth of application

With band placement, it is possible to place the fertilizer at a certain depth below the soil surface. By this method the unfavourable growing conditions that may occur in the upper layer of the soil can be avoided. The main reasons for these unfavourable growing conditions are related to the following points:

a. A favourable activity and development of roots is possible only in layers where air and water are available in sufficient amounts. Except on water-logged soils, air is available in sufficient amounts in the top layer of the soil. In a dry period, however, a dry surface mulch may be formed on the top of the soil by the drying action of the air above the soil. The thickness of the dry layer increases rather slowly with time because of the slow movement of water in vapour form through dry soil. It can be calculated that under Dutch conditions, a thickness of about 3 centimeters is exceeded only in very dry periods. The fertilizer in this layer cannot be taken up by the crop. Athough in Holland a dry layer normally forms only as long as the soil surface is not occupied wholly by the crop, the presence of a pre-formed dry layer may delay root development therein. Moreover, since the major absorption of nutrients by crops takes place after the soil surface is shielded by leaves, there is opportunity for decrease in availability of applied nutrients by fixation during alternate drying and wetting in the spring or early summer (VERVELDE and MEYERMAN, 1950).

b. To promote rapid germination, seeds are often placed some centimeters below the soil surface. Moreover, roots of some plants have a tendency to avoid the upper layer. Under these circumstances, there may be a less intensive root development in the upper layer, although this does not seem serious when the growing conditions are favourable there.

c. The roots in the soil surface between the plant rows can be destroyed by cultivation. Since farmers are aware of this danger, shallow cultivation is practiced, so that the disturbed layer seldom exceeds a thickness of 1 to 2 centimeters.

Apart from the effect of cultivation, it appears that a serious suppression of availability from placing the fertilizer in the upper layer occurs only if this layer dries out before the fertilizer has leached to deeper layers. By placing the fertilizer at a depth of about 4 centimeters, unfavourable conditions are avoided. Deeper placement increases the risks that during wet years the oxygen supply in the fertilized layer is deficient, and that soluble fertilizer constituents leach to deeper layers.

The advantage of deeper placement in actual cases depends greatly on weather and soil conditions that cannot easily be determined beforehand, so that a prediction of the beneficial effect is not possible in general. However, an idea of the maximum beneficial effect can be obtained when the depth of the dry layer is known and when no leaching takes place. The uptake in case of placement below the dry layer (*a* centimeter) may be calculated by the equation:

$$U_s = \left(\frac{tX_r}{(t-a)X_b}\right)^{0.44} \Phi\left(\frac{X_b}{X_r} M_r\right) + U_o. \qquad (3c)$$

in which t is the thickness of the fertilizer band. Thus, for $t = 5$ centimeters and $a = 2$ centimeters, the increase of the uptake from the fertilizer, governed by the term $\left(\frac{t}{t-a}\right)^{0.44}$, is about 25 %.

The uptake-yield curves are different only when the availability of other nutrients is not the same in both cases. If by shallow application the root development before a dry period is increased markedly in the upper layer, the active root system during the dry period will be relatively smaller. Moreover, if other nutrients (including water) are in a minimum, it is possible that a decrease in the uptake-yield curve will occur. However, since in Holland dry upper layers of soil usually occur during the spring, and not during the main uptake period when the entire soil surface is shielded by leaves, this effect need receive little attention.

Since the weather is not known beforehand, it is safest at first to neglect the effect of deeper placement in humid regions. Later on, an estimate of the most suitable depth may be made. It is evident that in humid climates, as in Holland, these corrections are seldom large. In countries having a climate drier than that of Holland, the depth of placement will be much more important. A more detailed consideration of this problem will be necessary there.

VIII. A COMPARISON BETWEEN CONCLUSIONS FROM THE THEORY AND SIMPLE PLACEMENT EXPERIMENTS

In deriving the theory on fertilizer application, use has been made of all experiments, permitting the calculation of fertilizer rate-uptake curves and uptake-yield curves for different placement methods, that have come to the author's attention. Although the conditions in these experiments differed widely, it has been possible to interpret the results from one viewpoint.

A check on the theory can be obtained by comparing the main conclusions with the results of more simple experiments and the experience in different countries. It is unfortunately the case that in simple experiments the conditions are in general not given in sufficient detail, and further that in many experiments only one rate of fertilizer has been applied in several different ways, so that it is not possible to calculate the ratio X_r/X_b or to construct rate-yield curves. This is especially the case in American experiments (for the greater part published in the Proceedings of the Joint Committee on Fertilizer Application), since in that country the stimulating effect on early growth is often an important purpose of placement. In the following pages, the main conclusions of the theory are compared one by one with practical experience and field experiments.

a. In all countries, with all crops and fertilizers, but not under all conditions, it is possible to save fertilizer by proper placement methods, as follows from the generalization of the compensation function in III, 3.
Since in practically all countries having organized agricultural research, placement experiments have been carried out (see list of literature), this conclusion needs no further discussion.

b. The graph of the coefficient of equivalence (figure 6), calculated from the compensation function, shows that 50 to 75 % or more of the fertilizer can often be saved by applying the proper placement pattern. Although this conclusion holds strictly only in Case IA and IB for lower fertilizer rates (figure 5; II, 5), this value of the coefficient has often been found in practice.
GYARFAS (1912, 1913), summarizing the results of experiments on oats and barley carried out in Hungary, stated that by band placement at least 50 % of the fertilizer can be saved. FLOESS (1919) concluded from experiments on sugar beets in Russia, Hungary and South Germany that 50 to 60 % of the fertilizer can be saved by placing. From experiments in England and Scotland, a saving of 50 to 60 % in the case of fall and spring-sown small grains has been found (CROWTHER, 1945; STEWART and REITH, 1945; REITH, 1949). A typical result is given in figure 40. From

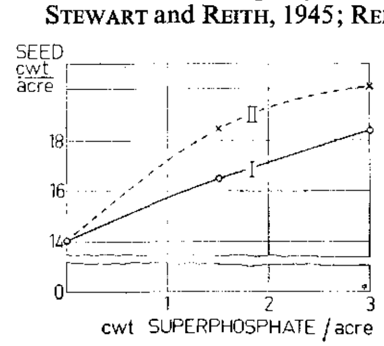


FIGURE 40.
Average results of all superphosphate placement experiments on summer and winter grain in England during 1944. I: Superphosphate broadcast. II: Superphosphate placed. There was no evidence of any difference in behaviour among the cereals. Compiled from CROWTHER (1945).

the main results with swedes and potatoes in England, savings of 66 % and 45 %, respectively, have been derived (COOKE, 1949). Experiments of TRUOG et al. (1925) with corn and cabbage have shown that by band placement 66 % of the fertilizer can be saved. All these experiments were carried out with phosphate fertilizer or with compound fertilizer containing phosphorus in relatively large proportion. Much greater advantages of placement are sometimes reported. Under these conditions, however, the stimulating effect on early growth has been an important factor (e.g. AVDONIN, 1949).

c. Since the recovery of nitrogenous fertilizers is often much greater than 20 %, no large advantages can in general be expected from placement.
This conclusion is illustrated by table 8, which shows the average results on small grains obtained by STEWART, (1949).

TABLE 8. Average yields of small grains in experiments with different amounts of sulphate of ammonia applied broadcast or placed (STEWART, 1949)

Sulphate of ammonia applied per acre	Acre yield of small grains with indicated method of application	
	Broadcast	Placed
cwt	cwt	cwt
0	22.5	22.5
0.75	25.5	25.5
1.5	28.0	28.0

OPITZ, et al. (1936, 1938), who repeated American experiments on potatoes, using nitrogenous fertilizers, concluded that band placement has no advantage over suitable broadcast methods. From American experiments, no conclusive results can be obtained. It seems that placement of small amounts may be advantageous, but this may be a result of stimulating effects on early growth. More attention has been paid to divided or split applications under conditions favourable to loss by leaching (SALTER, 1938).

d. Although the compensation function is independent of the soil type, the absolute savings obtained by placement on soils with a high fixation power are exceptionally large, since the slope of the rate-uptake curve in the case of broadcasting is small under such conditions. This situation is schematically represented for the case where the fixation power is so high that an „S-shaped" yield curve is obtained (figure 41).
The advantage of placement on soils with a high fixation power has been discussed

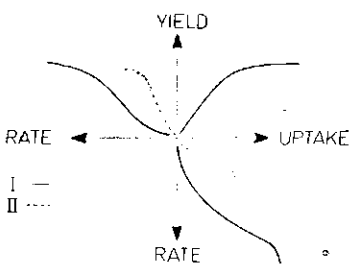


FIGURE 41.
Diagrammatic figure, showing the highly beneficial effect of placement on a soil with high fixation power. The curve for placed fertilizer (II: $(X_r/X_b) = 0.25$) has been calculated from the curve for broadcast fertilizer (I) by means of the compensation function.

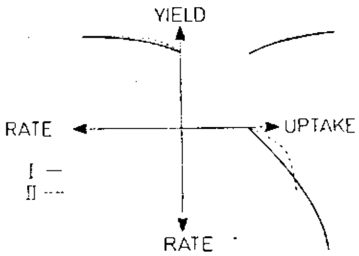


FIGURE 42.

Diagrammatic figure showing the effect of placement on soils giving relatively little yield response to fertilizer. The curve for placed fertilizer (II: $(X_r/X_b) = 0.25$) has been calculated from the curve for broadcast fertilizer (I) by means of the compensation function. The differences in yield are so small that it is hardly possible to find any difference between the two patterns in field experiments.

so often (e.g., SALTER, 1938; WOLF, 1947; AVDONIN, 1949; PRUMMEL 1950) that the subject need be given no further consideration here.

e. Under conditions where no luxury consumption takes place or where large amounts of fertilizer are applied in small bands, placed fertilizer may give lower returns than broadcast fertilizer (Case IB).
American investigators (SALTER, 1938; WOLF, 1947) agree on the point that large amounts must be broadcast or placed in wider bands. They have found also that at high rates of application it is advantageous to divide the fertilizer between two bands, one on each side of the crop row, and to apply the fertilizer in continuous bands rather than in discontinuous bands (SALTER, 1938). Although these effects may result partly from reduction in salt injury or other secondary effects, it is supposed here that a lower maximum uptake in case of placement is the main cause.

f. On soils with a high fertility level the effect of placement is the same as on soils with a low fertility level, as shown schematically in figure 42. At a high fertility level, however, the differences in yield are so small as to elude discovery in most field trials.
Table 9, which gives the comparative effects of broadcast and placed fertilizer on the yield of sugar beets grown on fertile soils, may be cited as an example.

TABLE 9. Mean yields of sugar and tops in ten experiments (1949) on sugar beets with different amounts of a compound fertilizer (0–16–13.4) applied broadcast or placed (COOKE, 1951)

Fertilizer applied per acre	Acre yield of sugar		Acre yield of tops	
	Fert. placed	Fert. broadcast	Fert. placed	Fert. broadcast
cwt	cwt	cwt	ton	ton
0	37.0	37.0	12.6	12.6
3.5	37.5	37.5	13.1	13.8
7.0	38.8	38.7	13.4	13.6

g. Under some conditions the maximum yields obtained with placed fertilizer are higher than those with broadcast fertilizer (Case II). Under these circumstances, a beneficial effect of placement may be found on fertile soils as well as on infertile soils.
This point is illustrated in table 10 by the results of an experiment of COOKE (1951).

h. Within practical limits, the influence of the distance between seed and fertilizer is of secondary importance, apart from injurious effects of high salt concentrations and stimulating effects on early growth on poor soils.
Experiments illustrating this secondary influence of distance between seed and fertilizer on the effect of placement are given in figures 11 and 14 and in table 10.

From the experiments of COOKE (1949), other data illustrating the secondary effect of the distance between seed and fertilizer may be obtained. In an experiment of PRUMMEL (1950), in figure 12, 8 centimeters from the seed was too great; this distance exceeds practical limits for small grains. American and Russian experiments do not give conclusive results in this respect, since in these countries stimulating effects on early growth can be very important.

TABLE 10. Mean yields of threshed peas in three experiments (1947) with different amounts of a compound fertilizer (0–10–20) applied in different ways (COOKE, 1951)

Fertilizer applied per acre	Acre yield of peas with indicated method of fertilizer application				
	Broadcast	Placed			
		In contact	Below seed	1 inch to side	3 inches to side
cwt	cwt	cwt	cwt	cwt	cwt
0	13.2	13.2	13.2	13.2	13.2
3	13.0	16.2	15.8	16.7	15.8
6	13.5	14.5 [1]	16.7	16.1	16.7

[1] Reduction of stand from excessive salt concentration near the seed.

IX. BAND PLACEMENT UNDER DUTCH CONDITIONS

1. Introduction

The developed theory may be applied to the results of existing broadcast experiments. In many such experiments, the record of the conditions has not always been as complete as desirable for the present purpose; nevertheless, it is possible to use the experimental data to make some prediction of the most desirable placement methods and the conditions under which placement will be advantageous.

An elaboration of the theory shows that differences exist not only between different crops and fertilizers but also between different soil types, fertility levels and so on. To avoid detailed discussion of too many problems of restricted importance, the theory is elaborated for only a few main conditions. The discussion is organized in such a way as to help regional advisers to find which aspects are important under their conditions and to apply the theory. Particular attention has been paid to phosphate. Results of broadcast experiments on small grains and potatoes, which are often grown in field trials, have been used in connection with the discussion. In addition analogous discussions of broadcast experiments with potash and nitrogenous fertilizers for beets have been given.

2. Band placement of phosphate fertilizers

It has been shown that placement of fertilizers is in general more efficient than broadcasting only if broadcast fertilizer increases the yield. The only exception to this rule is Case II (figure 5), which occurs when the uptake-yield curve is influenced by antagonistic phenomena.

With phosphate fertilizer, antagonistic phenomena in the chemical sense of the word appear to be rare. One case is given in figure 8. Other possible causes of antagonistic phenomena may be found in the micro-biological activity of the soil (greater activity in the fertilized parts) or in different root development, especially when broadcast and placed fertilizer are not applied at the same depth. For phosphate, the microbiological aspect is of minor importance (Gerretsen, 1939). Determination of the cause of these effects appears to be difficult, if not impossible. Even if a field is found where one of these effects occurs, it is not at all certain that the same effect will be found in other years.

It is therefore practically impossible yet to advise either on an experimental or a theoretical basis to apply phosphate in bands, when the only advantage that can be expected results from a diminution of antagonistic phenomena. On soils with a high fertility level, the method of application is thus no problem at all. These soils can be found with the aid of chemical soil tests. In Holland an extraction with citric acid, giving the „P-citr value" of the soil, has been developed at the Agricultural Experimental Station in Groningen. The directions for use of this value, taken from de Vries and Dechering (1948), are for arable land as follows:

P citr value	Phosphate condition of the soil
10	very bad
10–30	bad
30–40	moderate
40–50	good
50–60	very good
60	rich

As far as examined, these boundaries are for arable land practically independent of the soil type. In general, only on soils with a P-citr smaller than 30 to 40 is there found a marked effect of phosphate (VAN DER PAAUW, 1948). Therefore, only on these soils will placement of phosphate be expected to be appreciably more efficient than broadcasting. Since intensive cropping is usual in Holland, it is in most cases advantageous to place the fertilizer only if the maximum yield obtained in this way is at least as large as the maximum yield with broadcast fertilizer. Thus, only where case IB (figure 5) can be avoided, fertilizer placement can be advised.

From the figures given in this paper, it may be seen that even for high rates of application the yield increases slightly with increasing uptake. However, these slight returns do not pay under most conditions. For instance, with small grains an application of 1 kg of phosphate (P_2O_5) must cause at least an increase in yield of 2 kg of seed. If the recovery of the applied phosphate is assumed to be about 10 % (an average value), the marginal slope of the uptake-yield curve is represented by 20 kg of seed/kg phosphorus. Although this is only a rough estimation, it is evident from the economic viewpoint that the whole part of the uptake-yield curve with a small slope may be reckoned to the region of luxury consumption. By proper placement methods, therefore, it must be possible to avoid Case IB, as is shown in the discussion of some experiments.

First, attention will be paid to an experiment on oats reproduced in figure 43. Superphosphate and basic slag are applied at rates to supply 50, 150 and 300 kg of P_2O_5/ha. The uptake-yield curve is the same for both fertilizers and, when extrapolated, passes through the origin. Therefore, no secondary reactions have taken place, and the same curve can be expected in case of placement (IV, 4). With basic slag, no luxury consumption at all has taken place, so that the maximum yield would have been still lower in case of placement (Case IB). Other data (VAN DER PAAUW, 1940, 1950) show that soon after application the uptake of phosphorus from basic slag is often low, so

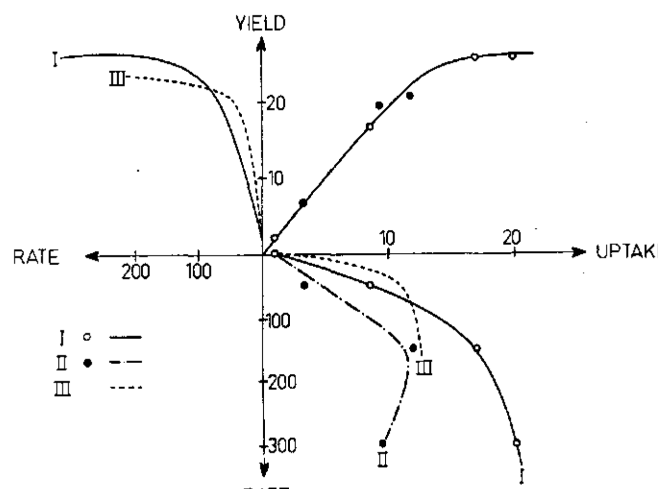


FIGURE 43.
Rate of fertilization, phosphorus uptake and crop yield in a phosphorus-fertilization experiment.
Compiled from VAN DER PAAUW (1950). (With some data obtained from the „Landbouwproefstation, Groningen").
Crop: oats. *Fertilizer:* phosphate. *Experiment:* field experiment Pr 391 (1937) on young reclaimed heath soil. *Scale units:* rate in kg P_2O_5 per ha; uptake in kg P_2O_5 per ha; yield in 100 kg seed per ha. *Treatments:* I: superphosphate broadcast; II: basic slag broadcast. *Results:* The uptake-yield curve is independent of the fertilizer material and crosses the origin. No luxury consumption of phosphate from basic slag. Some luxury consumption of phosphate from superphosphate. Curve III: superphosphate placed in bands with $(X_r/X_b) = 0.25$; calculated from curve I by means of the compensation function.

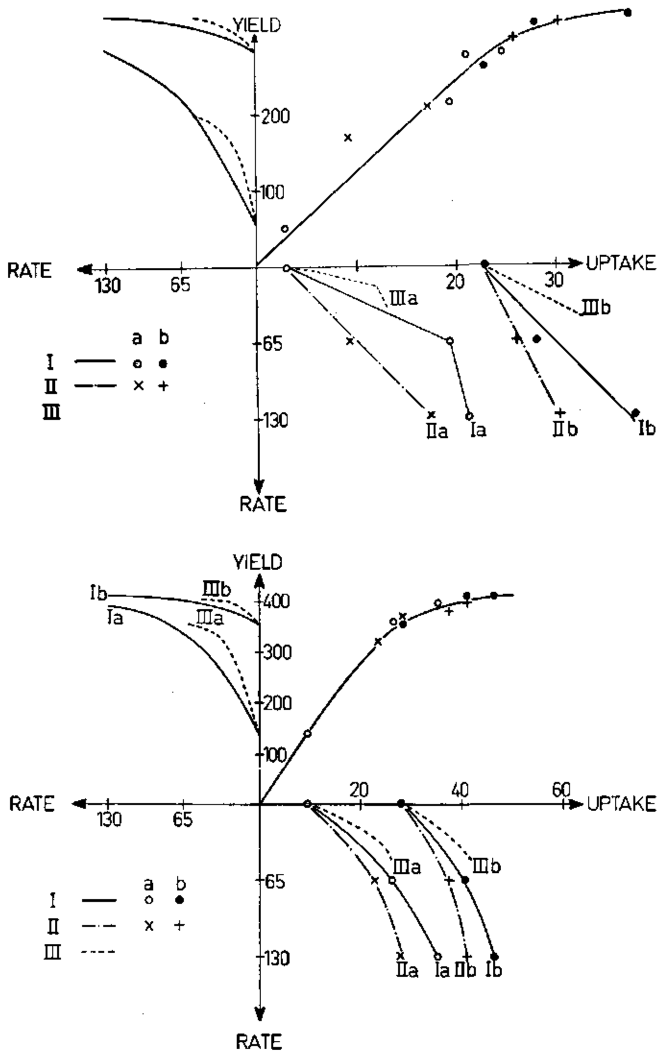


FIGURE 44.

Rate of fertilization, phosphorus uptake and crop yield in a phosphorus-fertilization experiment.

Compiled from VAN DER PAAUW (1950).

Crop: potatoes. *Fertilizer:* phosphate. *Experiment:* field experiment Pr 601 (1940 and 1943) on reclaimed heath soil. *Scale units:* Rate in kg P_2O_5 per ha, uptake in tubers in kg P_2O_5 per ha. Yield in 100 kg potatoes per ha.

Treatments:

Fertilizer	Without farmyard manure	With farmyard manure containing 66 kg P_2O_5 per ha
Superphosphate broadcast	Ia	Ib
Basic slag broadcast	IIa	IIb

The top figure gives the first year's (1940) results. The bottom figure gives the fourth year's (1943) results. It is not known why curve Ia has a shape different than that of the other curves. *Results:* The uptake-yield curve is independent of the fertilizer material and of the application of farmyard manure. Under these conditions the farmyard manure has only a phosphate effect. Luxury consumption of phosphate occurs only in the presence of farmyard manure and superphosphate. Curves III*a* and III*b*: superphosphate placed in bands with $(X_r/X_b) = 0.25$; calculated from curves Ia and Ib by means of the compensation function.

that this conclusion may be generalized. The same holds under many conditions for dibasic and tribasic calcium phosphate.

Heavy application of superphosphate caused some luxury consumption. In the figure, the relationship between fertilizer rate and yield has been calculated for a placement method with $X_r/X_b = 0.25$, by means of the compensation function.[1] It

[1] Only on young reclaimed peat soils have high recovery percentages of phosphate fertilizers been found (VAN ITALLIE, 1939). An estimate of the effect of placement on the recovery percentage in such soils may be obtained by equation 10 in III, 4.

is evident that, although the band is not particularly narrow, a serious depression in maximum yield would have been caused by this placement method. This is not the case when the depression in maximum uptake does not exceed 30 % or for placement methods with $X_r/X_b = 0.4$ to 0.5. From the investigations of VAN ITALLIE (1938, 1939), it may be concluded that if the uptake from the soil is low, there is usually little luxury consumption except on soils where the fertilizer is readily available.

Where some phosphorus is absorbed from sources other than the fertilizer material placed in bands, luxury consumption is often greater as may be seen from figure 44. In this experiment, potatoes were fertilized with superphosphate or basic slag in the presence or absence of farmyard manure containing 66 kg P_2O_5 per ha. From the potatoes cultivated in these experiments only the tubers where harvested since in field experiments the determination of the amount of stalks and leaves at maturity is not feasible. Large mistakes are not made by neglecting the uptake in the leaves, however, since during senescence of the leaves a part of the phosphate migrates to the tubers. The luxury consumption may be underestimated in this way.

In these experiments, the uptake-yield curve is independent of the fertilizer material and is independent of the application of farmyard manure. Since the extrapolated curves pass through the origin, it may be expected that the same curves would have been found if the fertilizer had been placed. Luxury consumption is so small in the absence of farmyard manure that application of superphosphate in bands having the width 0.25 X_b would be expected to result in a maximum yield lower than that obtainable by broadcasting. In the presence of farmyard manure, luxury consumption is much greater and no depression of maximum yield would be expected from applying the superphosphate in bands of width 0.25 X_b instead of broadcast. The same conclusion holds for the experiment reproduced in figure 45 in the case where monobasic calcium phosphate was applied.

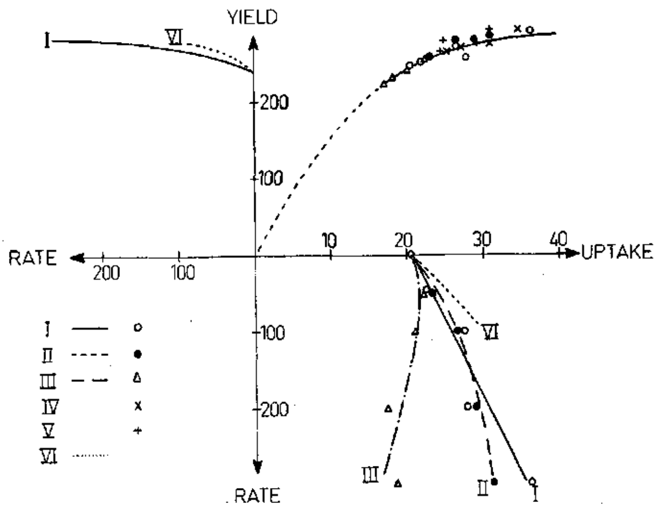


FIGURE 45.
Rate of fertilization, phosphorus uptake and crop yield in a phosphorus-fertilization experiment involving different phosphate compounds. Compiled from VAN DER PAAUW (1940). (With some data obtained from the „Landbouwproefstation, Groningen"). *Crop:* potatoes. *Fertilizer:* phosphate. *Experiment:* field experiment Pr 335 (1938) on reclaimed heath soil; second-year experiment (in the first year there was no response to phosphate). *Scale units:* Rate in kg P_2O_5 per ha. Uptake in kg P_2O_5 per ha. Yield in 100 kg potatoes per ha. *Treatments:* I: monobasic calcium phosphate (superphosphate); II: dibasic calcium phosphate; III: basic slag; IV: monobasic ammonium phosphate; V: dibasic ammonium phosphate. *Results:* The uptake-yield curve is independent of the fertilizer material. Curve VI: monobasic calcium phosphate placed in bands with $(X_r/X_b) = 0.25$; calculated from curve I by means of the compensation function.

The uptake of phosphate on well fertilized soils is roughly 40 to 50 kg of P_2O_5/ha (VAN ITALLIE, 1939). If the uptake from the soil is estimated conservatively at 15 kg, the uptake from the fertilizer is 25 to 35 kg/ha. If the fertilizer is placed in bands having the width 0.25 X_b, the uptake from fertilizer is about 0.55 (25 to 35) = 14 to 20 kg. No depression in maximum yield from band placement will occur, therefore, if the luxury consumption from the broadcast fertilizer is about 10 to 15 kg of P_2O_5/ha. Estimating the total dry matter yield of potatoes or grains at 7500 kg, this represents an increase in P_2O_5 content of about 0.2 %. From the data of VAN ITALLIE (1938, 1939), it may be concluded that such an increase in P_2O_5 content without a corresponding increase in yield often takes place when large amounts of superphosphate are applied. When it is considered further that some of the excess phosphate is often lost before final harvest, it may be concluded that except on soils markedly deficient in phosphate, no decrease in maximum yield will result if readily soluble phosphate fertilizer is placed in bands with a width one-fourth the distance between the plant rows. Thus, for small grains, sugar beets and potatoes, grown with row widths of about 25, 40 and 50 centimeters, respectively, the corresponding fertilizer band widths are about 6, 10 and 13 centimeters.

On soils that fix phosphate strongly, the recovery percentage of broadcast fertilizer is very low, and placement is very advantageous (figure 41). The uptake without fertilizer on these soils is very low, however, and (provided the uptake-yield curves are the same) the maximum yields obtained with placement are smaller than the maximum yields obtained by broadcasting large amounts of phosphate. Since the use of high rates of broadcast fertilizer is not economically profitable on these soils, it is not necessary to place the fertilizer in such wide bands that Case IB is avoided. The optimum width on these soils can be calculated if the results of broadcast experiments are known.

The amounts of fertilizer that must be applied in bands may easily be calculated. In the case of broadcasting, rates of 70 to 130 kg of P_2O_5/ha are used. For band placement methods with $X_r/X_b = 0.25$, roughly half these amounts, or 35 to 65 kg of P_2O_5, are needed. VAN DER PAAUW (1948) has shown that the phosphate availability of soils generally remains constant when about 40 kg of P_2O_5/ha is applied each year. Since methods that cause a decrease of the phosphate level of soils with a P-citr smaller than 40 must be avoided in Holland, at least 40 kg of P_2O_5/ha must be applied annually on these soils. The difference of 25 kg of P_2O_5 is so small that experimentation to ascertain the most profitable quantity of P_2O_5 is hardly worthwile.

3. BAND PLACEMENT OF POTASH FERTILIZERS

On light sandy soils and on peat soils, the recovery of potash fertilizers is much better than the recovery of phosphate, as may be seen from the recovery percentages given in table 11.

On sand and peat soils recovery percentages as high as 60 % have been found (VAN ITALLIE, 1938, 1939). Under these conditions, equation 10 in III, 4 must be used to calculate the recovery percentage of placed fertilizer. With $X_r/X_b = 0.25$, this formula becomes

$$R_r = \frac{100}{100 + 1.2\,R_b}\,2.2\,R_b$$

TABLE 11. Yield of oats, potash uptake and potash recovery on three soils with different amounts of K_2SO_4 under the same climatic conditions (compiled from VAN ITALLIE, 1939)

K_2O applied per ha	Sandy Soil			Sandy clay soil			Clay soil		
	Yield	Uptake of K_2O per ha	Recovery of added K_2O	Yield	Uptake of K_2O per ha	Recovery of added K_2O	Yield	Uptake of K_2O per ha	Recovery of added K_2O
kg	%	kg	%	%	kg	%	%	kg	%
0	87	103	–	97	111	–	87	124	–
120	100	142	32	100	132	17	96	151	22
480	97	203	20	100	181	15	100	194	15

With R_b equal to 50 %, the recovery percentage of placed fertilizer becomes about 69 %.

Thus, the same uptake is obtained by placing 72.5 kg of fertilizer in bands with a width 0.25 X_b as is obtained by broadcasting 100 kg of fertilizer. With present prices, this saving from placement amounts to about 9 guilders. Thus, only under conditions in which placement involves no additional costs is the placement of potash profitable on soils with a high recovery percentage.

The results of two broadcast experiments on clay soil are shown in table 12.

TABLE 12. Yield of sugar and uptake of potash by sugar beets on two clay soils fertilized with different amounts of K_2SO_4 (Compiled from BRUINSMA, 1940)

K_2O applied per hectare	Kruisland		Standdaarbuiten	
	Sugar yield per hectare	K_2O uptake per hectare	Sugar yield per hectare	K_2O uptake per hectare
kg	100 kg	kg	100 kg	kg
0	76.2	302	53.3	151
280	76.2	306	57.0	172
1100	75.5	344	64.5	213

In the Kruisland experiment luxury consumption occurred. Application of increasing quantities of potassium sulfate increased only the K_2O uptake and not the dry matter yield. Thus, apart from possible antagonistic reactions, placement would not have improved the yield. In the Standdaarbuiten experiment, the yield increased about linearly with increasing applications. Since the recovery of added potash is low, a beneficial effect of placement is to be expected. The amounts of K_2O applied are not high enough to determine where luxury consumption will begin. It is not possible, therefore, to determine the minimum width of the fertilizer band.

Luxury consumption of potash varies markedly with the crop, with the soil type and with the climatic conditions. Numerous experiments (VAN ITALLIE, 1939) have shown that it increases with increasing availability of potash in soil and fertilizer, and that it is often extremely high with small grains. It is by no means certain, however, that luxury consumption of potash is more common than that of phosphate. Thus, in general, placement in bands with a width one-fourth that of the crop row can be advised. Therefore, at least a 50 % saving of potash fertilizer can be made on soils with a low recovery percentage. It seems probable that under some conditions smaller band widths are preferable.

In this connection, particular mention should be made of certain river clay soils that show the phenomenon of potash fixation. FERRARI (1952) has found that on these soils crops may produce poor yields and show symptoms of potash deficiency even when they have been fertilized with large amounts of potash. It has been found in the laboratory that these soils react with the potash to such a degree that plants cannot use it. Since the recovery of potash on these soils is very low, highly beneficial effects from placing the fertilizer are to be expected.

There are not enough data available to determine the point at which luxury consumption would occur if large amounts were broadcast on the potash-fixing soils. FERRARI's experiments indicate that the rate of broadcast fertilizer required to obtain maximum yields is so great that it is uneconomical to attain them in practice. The placement problem here is thus identical with that described in IX, 2 for phosphate-fixing soils. If it is the purpose of the farmer to obtain reasonable yields with normal fertilizer rates, the fertilizer band must be narrow. If it is the purpose to obtain yields as high as can be obtained with broadcast application of potash, wider bands must be used. Present evidence is not sufficient to permit calculations of the most profitable width.

In the foregoing discussion of experiments with potash, no reference has been made to antagonistic phenomena. It was shown in IV, 4 that, particularly in the case of potash fertilizer, such effects must be expected. Although these antagonistic phenomena have been studied in the laboratory (for instance, LEHR, 1947) and in the field (for instance, MASCHHAUPT, 1934), it is not yet possible to predict on which soils and under what conditions serious effects on yield can be expected. Since, however, it can be expected that the maximum increase in yield with placed fertilizer that will result from antagonistic phenomena is about 10 %, antagonistic effects may be neglected for the present, except in those cases where it can be proved that they occur. The beneficial effect of band placement is underestimated in this way.

4. BAND PLACEMENT OF NITROGENOUS FERTILIZERS

Several investigators (for instance REMY, 1938; PFÜTZER, 1933; SPINKS and DION, 1949) have shown that recovery of fertilizer nitrogen by the plant is often relatively high compared with that of other fertilizers. For Dutch conditions, it has been found by „Het Instituut voor Rationele Suikerproductie" (unpublished) that recovery of nitrogen by sugar beets is in most cases greater than 50 % (table 13). Recovery percentages of about 50 % have been found for grassland (MULDER, 1949) and also for arable crops (MULDER, pers. comm. 1951) in agreement with table 13.

With the high recovery percentages characteristic of nitrogenous fertilizers, equation 10 in III, 4 must be used to calculate the recovery percentage in case of placement. For a band placement method with $X_r/X_b = 0.25$ this equation becomes

$$R_r = \frac{100}{100 + 1.2\, R_b}\, 2.2\, R_b$$

In table 13 are given the recovery percentages compiled from broadcast experiments on sugar beets.[1] The estimated recoveries of placed fertilizer are calculated by means of the above equation. The detailed results of one experiment are represented [1] in figure 46.

[1] The results of the experiments are published here by kind permission of Ir H. RIETBERG and J. STUMPEL.

FIGURE 46.
Rate of fertilization, nitrogen uptake and crop yield in a nitrogen-fertilization experiment.
Compiled from STUMPEL (pers. comm., 1952).
Crop: sugar beets. *Fertilizer:* nitrogen. *Experiment:* field experiment on clay soil (Strijen, 1943). *Scale units:* Rate in 100 equivalents per ha, uptake in 100 eq. per ha and yield of tops and beets in 1000 kg per ha. *Treatment:* Curve I: fertilizer broadcast. *Results:* The uptake is proportional to the rate of application; the recovery of broadcast nitrogen is 60 %. Curve II: nitrogen placed in bands with $(X_r/X_b) = 0.25$; calculated from curve I by means of equation 10 (III, 4).

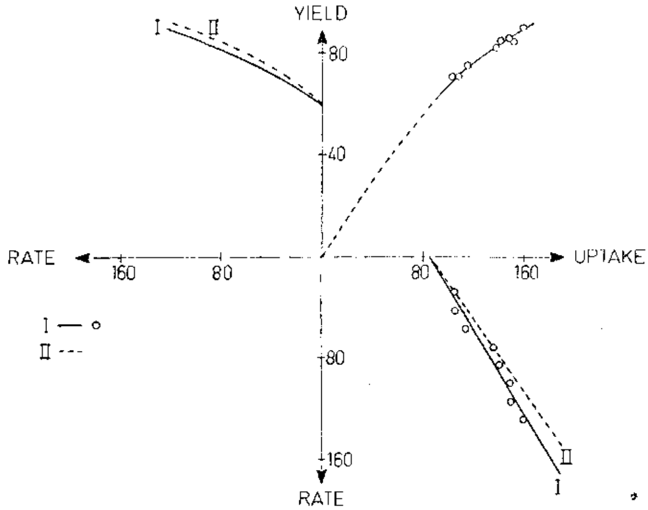


TABLE 13. Recovery percentages of fertilizer nitrogen by sugar beets on fine-textured soil, compiled from broadcast experiments by the „Instituut voor Rationele Suikerproductie". The recovery of band-placed fertilizer has been calculated for the band width $(X_r/X_b) = 0.25$.

Centre	Year	Recovery percentage: Found for broadcast fertilizer	Recovery percentage: Calculated for placed fertilizer
De Heen	1935	76	87
Halsteren	1935	84	92
De Heen	1936	76	87
Halsteren	1936	89	95
De Heen	1937	46	65
Halsteren	1937	52	71
Kruisland	1938	76	87
Heense Molen	1940	76	87
Haarlemmermeer	1940	88	94
De Heen	1941	66	81
Standdaarbuiten	1941	48	67
Klundert	1941	66	81
Strijen	1943	86	93
Wouw	1948	56	74
Wouw	1949	56	74

If the recovery percentage of broadcast nitrogen is 50 %, the recovery percentage of placed nitrogen is about 69 %. Thus, the same uptake is obtained by placing 72.5 kg of nitrogen as is obtained by broadcasting 100 kg. At present prices, the saving amounts to about 22 guilders. The profit would be still higher if the fertilizer could be placed in narrower bands; however, the chance of damaging the seedlings by concentrated band placement is so greatly increased (V,2) that this practice is not feasible.

5. Some methods of applying fertilizer

Many different fertilizer-distributing machines have been developed in America for the purpose of applying the fertilizer in the most desirable spot. In general, it is possible with these machines to vary the fertilizer rate, the distance between seed and fertilizer, and the depth below the soil surface. The width of the fertilizer band is fixed, since in countries where in general only small amounts are applied, the narrow bands of about 3 to 4 centimeters width, obtained with normal shovels, are preferable.[1]

In Holland and other countries with an intensive cropping system, it is desirable to apply the fertilizer in different widths under different conditions. In humid climates, this is much more important than placing the fertilizer below the soil surface. It may be preferable, in such climates, to avoid the difficulty of constructing distributors capable of varying both depth and width of the band by using distributors capable only of placing the fertilizer on the soil surface in bands of varying width. Such machines are easily built, as can be seen from figure 47. If such a distributor is mounted on the same framework with a seed drill, it is possible to place the fertilizer in bands of different width at different distances from the seed. After this operation, the fertilizer may be harrowed in.

In England, potatoes are often fertilized in the manner represented diagrammatically in figure 48. By this method a fairly good placement and a good mixing of soil and fertilizer is obtained.

An effect analogous to that of placement can be obtained by distributing the fertilizer in spots distributed at random. With this method of application, the distance between seed and fertilizer is not constant and uneven early growth may result. When the nutrient level of the soil is not very low, the mean diameter of the spots receiving fer-

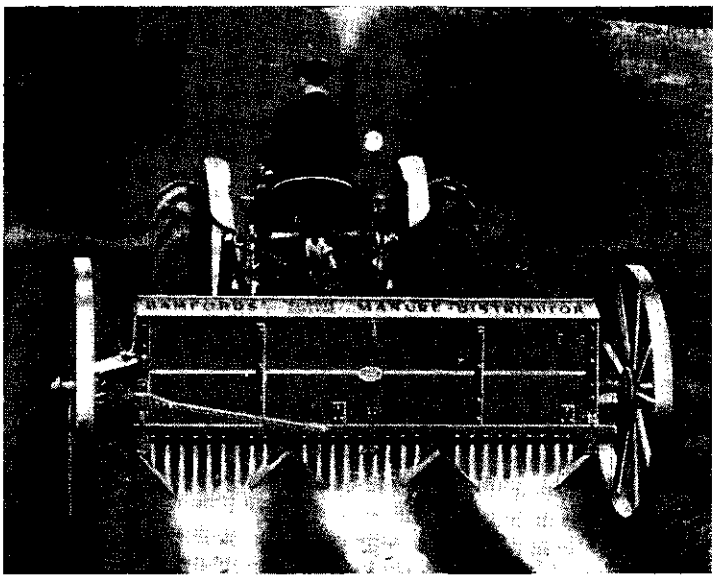


FIGURE 47.
A broadcast-distributing machine can be transformed into a placement machine, by building V-shaped sheets under the machine. The width of the fertilizer band can be regulated by varying the angle of the V.

[1] The width is twice as great if the fertilizer is placed on both sides of the seed row.

FIGURE 48.
Placement of fertilizer for potatoes can be obtained by broadcasting the fertilizer after ridging and before planting the potatoes.

tilizer and the mean distance between these spots is small compared with the row-distance, however, the unevenness is only temporary, and will have disappeared before harvest. Another method is ploughing under the fertilizer using a shallow skincoulter. There are thus many possible means of obtaining the benefits from fertilizer placement without the use of expensive machinery.

SUMMARY

In this paper, a theory has been developed (II) by means of which it is possible to calculate the effect of an arbitrary placement method of artificial fertilizer on yield if the effect of one method is known.

This theory is based on the following points:

a. The reactions between soil and fertilizer are the same for broadcasting and placement methods if the concentrations of the fertilizer in the soil are the same in both cases.

b. A definite relationship exists between yield and uptake of the given nutrient. This relationship is in general independent of the fertilizer pattern.

c. As a first approximation, the ratio of the increase in nutrient uptake with placed fertilizer to that with broadcast fertilizer depends only on the ratio of the volumes of soil fertilized by the respective methods.

The following conclusions were reached:

1. If the relationship between yield, uptake and amount of applied fertilizer is known for one method of placement (e.g. broadcasting), these relationships can be calculated for any other method of placement (II).
2. The width of the fertilizer band is of primary importance in determining the effect of a placement pattern (II).
3. The relative efficiency of placed fertilizer is maximized when it is applied in narrow bands and at low rates. When it is applied in narrow bands and at high rates, crop yields are lower than those obtained by broadcasting (II).
4. With wider bands and low rates, the beneficial effect of placement is smaller; at higher rates, however, the yields may equal those obtained by broadcasting (II).
5. If severe antagonistic effects occur, the maximum yield with placed fertilizer will normally exceed that with broadcast fertilizer (IV, 3).
6. The distance between fertilizer band and crop row is of primary importance where the fertilizer concentration is high enough to cause seedling injury from salt effects, and where the soil fertility level is low (V).
7. On other soils it is only the early growth and not the final yield that is influenced by the distance between seed row and fertilizer band (V).
8. Field experiments that do not last the entire growing season cannot be used to determine the effect of different placement patterns on the mature crop (V, 9).
9. No large benefits of placement can be obtained if recovery of the broadcast fertilizer is high (III, 4).
10. On soils with a high fixation power, large amounts of fertilizer can be saved by placement (VIII, conclusion *d*).
11. Placement of fertilizer is particularly advantageous if the roots are not developed under the entire soil surface (VII, 2).
12. The influence of depth of placement varies markedly with weather conditions and can be predicted only if assumptions are made concerning weather conditions to be experienced during crop growth (VII, 4).

The theory has been applied to broadcast experiments in the Netherlands, with the following conclusions:

PHOSPHATE: (IX, 2).

13. On soils with a phosphate availability of 30 to 40 P-citr units or lower, placement of soluble phosphate fertilizer can be advantageous.
14. The optimum width of the fertilizer band is about one-fourth the distance between the crop rows.
15. About 50 to 60 % of the phosphate can be saved by placement in this way.
16. For lower rates, narrower bands may be advantageous.
17. Placement of basic slag and dibasic or tribasic calcium phosphate is advantageous only when low rates are applied.
18. On young, reclaimed peat subsoils no marked advantage results from placement.

POTASH: (IX, 3).

19. The benefits from placement of potash depend highly on soil, crop and weather conditions.
20. In general, no large advantages from placement are to be expected on coarse-textured soils.
21. On clay soils, it can be advantageous to place the fertilizer in bands with a width of approximately one-fourth the distance between the crop rows. The most desirable band width varies greatly with the conditions.
22. On soils with a high fixation power, band placement of low rates of potash is highly advantageous.

NITROGEN: (IX, 4).

23. By placing nitrogenous fertilizers 25 % of the fertilizer can be saved, on an average.

EQUIPMENT: (IX, 5).

24. Fertilizers can be placed without expensive placement equipment.

Before applying in practice the above conclusions, which are formulated for Dutch conditions, it is necessary for the local adviser to apply the theory to broadcast experiments in his own district to ascertain that his conditions do not deviate from those assumed herein. This procedure will make possible a more precise prediction of the optimum width of the fertilizer band.

NOTATION AND CONVERSION TABLES

Symbol	Definition	Dimension
d	Distance between crop row and the nearest side of the fertilizer band	l
M	Fertilizer rate; applied amount of fertilizer per unit surface . . .	$m.l^{-2}$
R	Recovery percentage, $100\frac{U}{M}$	–
U_s	Total uptake of nutrient per unit surface	$m.l^{-2}$
U_o	Total uptake of nutrient per unit surface by a crop grown without fertilizer	$m.l^{-2}$
U	Total uptake from fertilizer per unit surface; increase in uptake from fertilizer = $(U_s - U_o)$	$m.l^{-2}$
$\dot{U}$	Uptake rate from fertilizer (dU/dt)	$m.l^{-2}.t^{-1}$
X_b	Distance between the crop rows	l
X_r	Width of the fertilizer band	l
Y	Yield	$m.l^{-2}$

The subscripts *r* and *b* are added to the symbols to indicate that the symbol refers to the condition in the case of placement or broadcasting, respectively.

Weight	Gram (g)	Kilogram (kg)	Pounds (lb)
1 gram (g)	1	10^{-3}	$2{,}20.10^{-3}$
1 kilogram (kg)	10^3	1	2,21
1 pound (lb)	454	0,454	1
1 Eng. hundred weight (cwt)	$5{,}08.10^4$	50,8	112
1 ton (metric)	10^6	10^3	$2{,}20.10^3$

Length	Centimeter (cm)	Meter (m)	Inch
1 centimeter (cm)	1	0,01	0,394
1 meter (m)	100	1	39,4
1 inch	2,54	0,0254	1

Area	Square meter (m^2)	Hectare (ha)	Acre
1 square meter (m^2)	1	10^{-4}	$2{,}47.10^{-4}$
1 hectare (ha)	10^4	1	2,47
1 acre	$4{,}05.10^3$	0,405	1

Weight/Area

1 kg/ha = 0,891 lb/acre
1 lb/acre = 1,12 kg/ha

LITERATURE

Agriculture 1945 Farming notes. *Agriculture* **52,** 45.

AVDONIN, N. S. 1936 On the methods of fertilizer application. (Russian) Chemisation of Socialistic Agriculture 6th issue, 3–21.

——— 1941 Problems on the systems of plant nutrition. (Russian) Publications from the Institute on the Cultivation of Grain Crops in „non-black soil" Regions. Moskou, 135 pp.

——— 1949 The application of granulated superphosphate in rows. (Russian) Agrobiologija, 2nd issue, 29–48.

BLANCK, E. 1930 Handbuch der Bodenlehre. Band VI. Berlin.

BLANCK, E. and H. SCHORSTEIN 1936 Über die Einfluss des Nährstoffverhältnisses in der Düngung auf den Hafer. *J. für Landwirtschaft* **84,** 59–88.

BLANCK, E., W. HEUKELSHOVEN and H. SCHORSTEIN 1936 Ein zweiter Beitrag zur Frage nach dem Einfluss des Nährstoffverhältnisses in der Düngung auf Hafer und Gerste. *J. für Landwirtschaft* **84,** 37–58.

BRENCHLEY, W. E. 1929 The phosphate requirement of barley at different periods of growth. *Annals of Botany* **43,** 89–110.

BRUINSMA, J. R. 1940 Kalium, natrium en barium proefvelden en besproeiingsproefvelden te Standdaarbuiten en te Kruisland in 1938. *Med. Instituut voor Rationele Suikerproductie* **10,** 141–168.

COOKE, G. W. 1949 Placement of fertilizers for potatoes. *J. Agr. Sci.* **39,** 96–103.

——— 1949 Placement of fertilizers for row crops. *J. Agr. Sci.* **39,** 359–373.

——— 1951 Placement of fertilizers for sugar beet. *J. Agr. Sci.* **41,** 174–178.

CROWTHER, E. M. 1945 Combine-drilling of phosphate fertilizers for cereals. *Agriculture* **52,** 170–173.

DEMIDENKO, T. T. and R. A. BORINOVA 1946 Effect of granulated superphosphate upon the yield of stubble-field crops. *Comptes Rendus (Doklady) de 'lAcadémie des Sciences de l'URSS* **54,** 247–250.

DORPH-PETERSEN, K. and F. STEENBJERG 1950 Investigations of the effect of fertilizers containing sodium. *Plant and Soil* **2,** 283–300.

FERRARI, TH. J. 1952 Een onderzoek over de stroomruggronden van de Bommelerwaard met als proefgewas de aardappel. Proefschrift Wageningen. *Versl. Landbouwk. Onderz.* no **58**-1.

FLOESS 1919 Die Drill oder Reihendüngung. *Deutsche Landwirtschaftliche Presse* **46,** 595–596 and 605–606.

FREDRIKSSON, L. and L. WIKLANDER 1950 Studier över potatisens fosfatupptaganda med tillhjälp av radioactiv fosfor. *Kungl. Lantsbrucksakademiens Tidskrift* **89,** 446–458.

GILE, P. L. and J. O. CARRERO 1917 Absorption of nutrients as affected by the number of roots supplied with the nutrients. *J. of Agr. Research* **9,** 73–95.

GERRETSEN, F. C. 1939 Bodembacteriologie. 's-Gravenhage.

GOEDEWAAGEN, M. A. J. 1942 Het wortelstelsel der landbouwgewassen. 's-Gravenhage.

GÖRBING, J. 1930 Der Einfluss der Phosphorsäure auf die Wurzelentwicklung. *Superphosphate* **III,** 257.

GYÁRFÁS, J. 1912 Ergebnisse vorjähriger Drilldüngungsversuche in Ungarn. *Deutsche Landw. Presse* **39,** 273–274.

——— 1913 Neue Versuchsergebnisse mit Drilldüngung. *Deutsche Landw. Presse* **40,** 237–238.

HALLIDAY, D. J. 1948 Nutrient uptake by farm crops. *Jealott's Hill Res. Sta. Bull.* **7.**

HEMELRIJK, J. 1951 Statistische bepaling van het lineaire verband tussen twee physische grootheden. *Ned. Tijdschr. v. Nat.* **17,** 147–158.

HOAGLAND, D. R. 1948 Lectures on the inorganic nutrition of plants. Massachusetts, 2nd print.

ITALLIE, TH. B. VAN 1938 De chemische samenstelling van gewassen in verband met landbouwkundige vraagstukken. 's-Gravenhage.

——— 1939 De betekenis van het gewasonderzoek bij phosphorzuur en kaliproefvelden in Nederland. *Versl. Landbouwk. Onderz.* **45,** 679–762.

JACOB, W. C., C. H. VAN MIDDELEN et al. 1949 Utilization of phosphorus by potatoes. *Soil. Sci.* **68**, 113–120.

LEHR, J. J. 1947 Laboratoriumproeven ter bestudering van de invloed van de bemesting op groei en minerale samenstelling van de suikerbiet. IV. De invloed der kationen verhoudingen op de ontwikkeling van de suikerbiet. Potproef op dusariet-zandmengsels 1942. *Med. Inst. v. Rat. Suikerprod.* **17**, 65–109.

MC AULIFFE, C. D., N. S. HALL et. al. 1947 Exchange reactions between phosphates and soils: hydroxylic surfaces of soil minerals. *Proc. Soil Sci. Soc. Amer.* **12**, 119–123.

MASCHHAUPT, J. G. 1934 In hoeverre kunnen K, Na, Ca en Mg elkander in de plant vervangen? *Versl. Landbouwk. Onderz.* **40**, 1025–1096.

METZ 1923 Über die Wirkung des Natrons neben dem Kali als Nährstoff der Pflanzen. Teil IV. Die Kartoffel. *Die Ernährung der Pflanze* **19**, 132–136.

MILES, R. Q. 1947 The placement of fertilizers. *Jealott's Hill Res. Sta. Bull.* **4.**

MULDER, E. G. 1949 Onderzoekingen over de stikstofvoeding van landbouwgewassen. I. Proeven met kalkammonsalpeter op grasland. *Versl. Landbouwk. Onderz.* **55**–7.

OPITZ and GOEPP 1936 Reihendüngung und Pflanzenabstand beim Kartoffelbau. *Mitteilungen für die Landwirtschaft* **51**, 248–250 and 269–270.

OPITZ, K. and W. GROHNWALD 1938 Weitere Versuchergebnisse zur Frage der Reihendüngung und des Pflanzenabstandes beim Kartoffelbau. *Pflanzenbau* **14**, 289–305.

PAAUW, F. VAN DER 1940 Vergelijkend onderzoek over de waarde van ammoniumfosfaten als fosforzuurmeststof. *Versl. Landbouwk. Onderz.* **46**, 111–218.

——— 1948 Fosfaatbemesting in de landbouw. *Landbouw* **1.** s-Gravenhage.

——— 1950 Oorzaken van de verschillende werking van superfosfaat en thomasslakkenmeel. *Versl. Landbouwk. Onderz.* **56**–6.

PEMBER, F. R. 1917 Studies by means of both pot and solution culture of the phosphorus and potassium requirements of the barley plant during different periods of growth. *Rhode Island, Agr. Exp. Sta. Bull.* **169.**

PEMBER, F. R and F. T. MCLEAN 1925 Economical use of nitrogen, phosphorus and potassium by barley, oats and wheat in solution cultures. *Rhode Island, Agr. Exp. Sta. Bull.* **199.**

PFÜTZER, G. 1933 Beitrag zur Frage der Nährstoffaufnahme durch die Pflanzen bei gesteigerten Nährstoffgaben. *Landw. Forschungen* **50**, 1–22.

PROCEEDINGS OF THE NATIONAL JOINT COMMITTEE ON FERTILIZER APPLICATION **1** (1925)–**26** (1950). (mimeographed).

PRUMMEL, J. 1950 Rijenbemesting. *Landbouwk. T.* **62**, 620–627.

RAUTERBERG, E. 1937 Die Wirkung der Nährstoffe bei nesterartige und bei absolut gleichmässiger Verteilung im Boden. *Die Ernährung der Pflanze* **33**, 201–208.

REMY, TH. 1938 Fertilization in its relationship to the course of nutrient absorption by plants. *Soil Sci.* **46**, 187–210.

RUSSELL, E. J. and D. J. WATSON 1940 The Rothamsted field experiments on the growth of wheat. *Techn. Comm. Imp. Bur. of Soil Sci.* **40.**

RUYTER DE WILDT, J. C. and A. D. BERKHOUT 1913 Cyaan amide, dicyaan amide en kalkstikstof. *Versl. Landbouwk. Onderz.* **13**, 61–127.

SABININ, D. and H. MININA 1935 Physiologische Grundlagen der Technik der Einbringung von Düngemitteln. *Zeitschr. für Pflanzenern., Düng. u. Bodenk.* **40**, 1–48.

SALTER, R. M. 1938 Methods of applying fertilizers. Yearbook of U.S. Dept. of Agr. (Soils and Men), 546–562.

SAYRE, C. B. 1934 Root development of beans, cabbage and tomatoes as affected by fertilizer placement. *Proc. Amer. Soc. for Hort. Sci.* **32**, 564–571.

SAYRE, C. B. and A. W. CLARK 1935 Rates of solution and movement of different fertilizers in the soil and the effects of the fertilizers on the germination and root development. New York State (Geneva) Agr. Exp. Sta. Techn. Bull. 231.

SCHLIESSUS, A. 1942 Die Wirkung zeitlich gestaffelter Grunddüngung an P, K und N auf Pflanzenertrag und Nährstoffaufnahme. *Bodenk. u. Pflanzenern.* **28**(73), 31–56.

SCHUFFELEN, A. C. 1946 Over de werkzame concentratie van voedingselementen in de grond. *Landbouwk. T.* **58,** 367–376.

SCHUYLENBORGH, J. VAN 1947 A study on soil structure. Proefschrift Wageningen.

SELKE, W. 1941 Die Wirkung zusätzlicher später N-gaben auf Ertrag und Quälität der Ernteproducte. Bodenk. u. Pflanzenern. **20,** 1–49.

SIMON, E. 1927 Wie beeinflusst die Aufnahmezeit die Wirkung der Nährstoffe. Diss. Cottbus.

Soil Science 1949 Articles on the utilization of phosphorus by different crops as determined with the aid of P^{32}. *Soil Sci.* **68,** 113–202.

SOKOLOV, A. V. 1945 The influence of feeding conditions upon the plant content of various phosphorus compounds. *Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS XLIX*, 123–126.

SPINKS, J. W. T. and S. A. BARBER 1947 Study of fertilizer uptake using radioactive phosphorus. *Scientific Agr.* **27,** 145–156.

——— 1948 Study of fertilizer uptake using radioactive phosphorus II. *Scientific Agr.* **28,** 79–87.

SPINKS, J. W. T. and G. DION 1949 Study of fertilizer using radioactive phosphorus. *J. of the Chem. Soc.* Suppl. issue number 2, 410–415.

STEENBJERG, F. 1944 Om kemiske plantenanalyser oy deres anvendelse. *Tidskrift for Planteavl* **49,** 158–177.

——— 1951 Yield curves and chemical plant analyses. *Plant and Soil* **3,** 97–109.

STEWART, A. B. 1949 Fertilizer placement for arable crops. Dept. of Agric. for Scotland. Leaflet no 3 (New series). Edinburgh.

STEWART, A. B. and J. W. S. REITH 1945 Comparisons of broadcast and drill applications of fertilizers. *Scottish J. of Agr.* **15,** 167–171.

TINNEFELD, L. 1930 Die Düngerverteilung im Boden durch die verschiedenen Ackergeräte. Diss. Halle, Saule (reprint Arch. Pflanzenbau).

TRUOG, E., H. J. HARPER et al. 1925 Fertilizer experiments: etc. Wisconsin Madison, Agr. Exp. Sta. Res. Bull. **65.**

ULYAKOV, I. P. 1936 On the methods of phosphate treatment of red soils (Russian) *Chemisation of Socialistic Agr.* **2/3,** 91–97.

VERVELDE, G. J. and G. C. MEIJERMAN 1950 Onderploegen of ineggen van kunstmest. *Maandbl. Landbouwvoorlichtingsd.* **7,** 12–16.

VRIES, O. DE and F. J. H. DECHERING 1947 Grondonderzoek. 's-Gravenhage.

WEAVER, JOHN E. 1926 Root development of field crops. New York.

WIERSUM, L. K. 1947 Transfer of solutes across the young roots. Diss. Groningen.

WIJK, W. R. VAN and C. T. DE WIT 1951 Een natuurkundige theorie over de wijze van meststof toedienen. *Landbouwk. T.* **63,** 764–775.

WIKLANDER, L. 1950 Kinetics of phosphate exchange in soils. *Ann. Roy. Agr. Coll. of Sweden* **17,** 407–424.

WIT, C. T. DE and T. TALSMA 1952 De bepaling van de werkzaamheid van wortels met behulp van radioactieve isotopen. *Landbouwk. T.* **64,** 398–399.

WOLF, B. 1947 Fertilizer placement. *Amer. Fertilizer* **107,** 7, July 26, no 2.

WOLTZ, W. G., N. S. HALL and W. E. COLWELL 1949 Utilization of phosphorus by tobacco. *Soil Sci.* **68,** 121–128