Identifiability of Non-Linear Models from a Parametric Output-Sensitivity Perspective

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1 Abstract

Local structural identifiability is a well-researched topic that has lead to various methodologies that address the question from different angles. Among these are (i) Pohjanpalo's method [1] that involves a Taylor-series approximation (ii) the similarity transformation approach (iii) non-linear observability, where the state vector is augmented with the (constant) parameters in the model (e.g. [2]).

In this presentation we will show an alternative viewpoint that considers local structural identifiability as an output controllability question, meaning that we study an accessible controllability algebra associated with the parametric output sensitivities $\frac{\partial y}{\partial \theta}$. Hereto, the original system model in control-affine format, i.e.

$$\begin{aligned} \dot{x}(t) &= f(x,\theta) + g(x,\theta)\,u(t) \\ y(t) &= h(x,u,\theta) \end{aligned}$$

is viewed as the seed for the dynamics of the parametric output sensitivities, i.e.

$$\begin{aligned} \dot{x}_\theta(t) &= \frac{\partial\{f(x,\theta)+g(x,\theta)\,u(t)\}}{\partial x}\,x_\theta + \frac{\partial\{f(x,\theta)+g(x,\theta)\,u(t)\}}{\partial \theta} \\ y(t) &= \frac{\partial h(x,\theta)}{\partial x}\,x_\theta + \frac{\partial h(x,\theta)}{\partial \theta} \end{aligned}$$

For each parameter in the original system model (or output equation) a corresponding coupled system can be defined that can be studied in a non-linear control framework. The outputs of the $p(=\dim(\theta))$ coupled systems (which are the parametric output sensitivities) can be compared with one-another and identifiability properties can be derived. The dynamics of the parametric output sensitivities are part of a space that can be calculated on basis of the combined controllability distributions $\{\Delta_i^\theta, i=1,\ldots,p\}$ that arise from the vector fields (drift- and control vector fields) of each (augmented) system associated with parameter θ_i. A sufficient condition for the parameters to be locally structurally identifiable is that a rank test needs to be satisfied. In the presentation this is demonstrated in detail.

Viewing identifiability as an output controllability property nicely fits in the theory of a range of parameter estimation methods – e.g. the recursive prediction error method – where parametric output sensitivities are key to minimizing the sum of squared prediction errors.

In the presentation I will elaborate on these insights and show you a few examples in which the method is demonstrated in more detail.

References

[1] H. Pohjanpalo, 'System Identifiability Based on the Power Series Expansion of the Solution', Mathematical Biosciences, vol. 41, pp 21–33, 1978

[2] E.T. Tunali and T. Tarn, 'New Results for Identifiability of Nonlinear Systems', IEEE Transactions on Automatic Control, vol. AC-32 (2), pp 146–154, 1987

[3] J.D. Stigter and R.L.M. Peeters, 'On a Geometric Approach to the Structural Identifiability Problem and its Application in a Water Quality Case Study', Proceedings of the European Control Conference 2007, Kos, Greece, pp 3450–3456, 2007