PROCEEDINGS OF THE 33rd

MAFF CONFERENCE OF

RIVER AND COASTAL ENGINEERS

1 to 3 July 1998

Organised by:

Ministry of Agriculture, Fisheries and Food
Flood and Coastal Defence Division
Eastbury House
30-34 Albert Embankment
London SE1 7TL

Tel: 0171-238 6742
Fax: 0171-238 6665

Further copies of these Proceedings may be obtained from Room 412 at the above address priced £25.00 each.

NOTE:
The views and opinions expressed in this document do not necessarily reflect the policy of the Ministry of Agriculture, Fisheries and Food, or other Government Departments.

ANALYSIS OF UNCERTAINTIES IN THE FLOOD DAMAGES OF THE MEUSE FLOODS IN 1993 AND 1995

Herman Wind and Chris de Blois
Twente University, Dep. Civil Engineering and Management

1. INTRODUCTION

In 1993 and 1995 large parts of the Meuse basin were flooded. In the Dutch part of the basin in 1993 an area of 17,000 ha was inundated and in 1995 this area was in the order of 15,500 ha. The direct and indirect economic damage in 1993 amounted to kDfl 253.800. Although both flood volumes are comparable in magnitude, the economic financial damage in 1995 was kDfl 165.000, which is considerably lower than the damage in 1993. In order to investigate mitigation actions, the Dutch governments commissioned an extensive study in 1993 to Delft Hydraulics. Delft Hydraulics developed the MaasGis model (Delft Hydraulics, 1994) and formulated recommendations to reduce future flood damage. The causes of the differences between the 1993 and 1995 flood damages are investigated in Nierop, 1997, Blois, 1997 and Wind, Nierop and Blois (1998).

In addition to flood damages also the uncertainty in flood damage is an important aspect. For instance, in order to determine whether the Meuse flood damages in 1993 and 1995 were two realisations from the same statistical distribution, knowledge of the uncertainty in the estimates of the flood damage is required. Also in a cost-benefit analysis the uncertainty becomes an important factor when costs and benefits are of the same order of magnitude. Finally uncertainty may be an important guide in determining the appropriate spatial resolution and the selection of factors that influence the flood damage significantly.

This paper focuses on the factors determining the uncertainty in a flood damage model. First the flood damages resulting from a flood damage assessment model, Inunda, will be compared with the observed damages. Next the uncertainty in the flood damage results will be presented and will be analysed from the point of view of co-variance. An analysis of the structure of Inunda will be used in section three to determine the major factors leading to the observed uncertainty in flood damage results. This leads to the conclusion that for the Meuse the river discharge is a major source of uncertainty and not, for instance, the uncertainty in the damage per unit land use. This result will be verified in Section 4 using a simplified flood damage model. The same model will also be used in Section 5 to study the factors determining the major contribution to the value of expected flood damage. In Section 6 an overview will be given on the major sources of flood damages and their related uncertainty. The paper is concluded with a discussion.

2. FLOOD DAMAGE DATA OF THE 1993 AND 1995 MEUSE FLOODS AND RESULTS OF THE INUNDA MODEL

The flood damage data for the 1993 and 1995 Meuse floods (Delft Hydraulics, 1993 and Nierop,1997) have been obtained by damage experts. In order to investigate the causes of the differences between the 1993 and 1995 Meuse floods and the related uncertainty, Blois (1997) developed a model for the assessment of flood damages based on a raster GIS with squares of

150m * 150m. The total flood damage S equals the sum of the flood damages s_i of eight land use types: private houses, trade catering recreation, services, green house catering, other agriculture, manufacturing, horticulture, private institutions, building. The damage to these eight land use types is a function of five variables: x_1 embankment height; x_2 damage per unit land use per inundation class; x_3 land height; h water level; q discharge. This leads to the following expression for flood damage S:

$$S = \sum_{i=1}^{8} s_i = \sum_{i=1}^{8} s_i(x_1, x_2, x_3, h, q) \quad [2.1]$$

The distribution of the flood damages over the damage categories is shown in Table 2.1.

	Data 1995	Data 1993	Model 1993	Contribution to total damage	Standard deviation	Contribution to Stand. Dev. Independent	Contribution to Stand. Dev. Dependent
Private houses	40.0	80.8	79.8	0.46	7.5	0.44	0.32
Industry	10.2	12.0	10.0	0.06	2.1	0.03	0.09
Building		2.6	2.1	0.01	0.7	0.005	0.03
Trade, catering recreation	38.7	47.2	43.9	0.26	7.6	0.45	0.32
Retail and remainder	7.3		-		-		
Services	5.9	9.0	14.4	0.08	2.5	0.05	0.11
Agriculture	15.8	10.5	12.3	0.07	0.8	0.006	0.04
Greenhouse gardening	5.1	8.8	7.4	0.04	1.5	0.02	0.07
Institutions		2.7	2.6	0.02	0.3	0.00	0.02
Total	123.0	173.6	172.5	1.00	22.9	1.00	1.00

Table 2.1: Flood damage data and model results as obtained for the 1993 Meuse floods (in 1000 kDfl) relevant for flood damage modelling (From Delft Hydraulics,1993 and Wind 1995).

In Table 2.1 only those flood damages are included which are relevant for damage modelling. Damages which are not included in Table 2.1 are: damage to public property, damages to cars, caravans and gardens, the mining sector as well as damages outside Limburg. In 1993 these damages amounted to kDfl 80.2 and in 1995 to kDfl 42.0 respectively, leading to the total flood damages in 1993 of kDfl 253.8 and kDfl 165.0 in 1995. For the analysis of the 1993 flood data the reader is referred to Delft Hydraulics 1993. Details of the 1995 flood data can be found in Nierop (1997).

The uncertainty analysis of the flood damage data has been carried out for each of the eight economic sectors in Table 2.1, using the five uncertainty sources mentioned above. The discharge q is the discharge at Borgharen, near the border between the Netherlands and Belgium. The water level h is the water level along the river, not including the shape of the flood wave. For each of the uncertainty sources an estimate of the uncertainty has been made. The uncertainty has been assumed to be normally distributed. For details of the uncertainty analysis is referred to Blois (1997) and Blois (1998).

The results of the uncertainty analysis of Inunda are presented in Table 2.2. and in Figure 2.1. For the summation of uncertainties per sector or per uncertainty source, it is important whether the uncertainties are correlated or uncorrelated. If uncertainties are fully correlated, than the resulting uncertainty equals:

$$\sigma_{corr} = \sum \sigma_i \qquad [2.2]$$

If uncertainties are uncorrelated, than the resulting uncertainty can be calculated from;

$$\sigma_{uncorr} = \sqrt{\sum \sigma_i^2} \qquad [2.3]$$

As it is not always clear to which extent uncertainties are correlated, it is important to know the error this may lead to. It can be shown that the "real" uncertainty always lies between σ_{uncorr} and σ_{corr} and that

$$\sigma_{uncorr} \leq \text{"real"} \leq \sigma_{corr} \leq \sqrt{n}\,\sigma_{uncorr} \qquad [2.4]$$

where n is the number i of uncertainties σ_i. In case of Table 2.2 n = 5 for the columns and n = 8 for the rows. This implies that in case it is not known to what extent uncertainties are correlated, the error is less than $\sigma_{corr} - \sigma_{uncorr}$.

	X1	X2	X3	H	Q	Corr.	Inunda	Uncorr.
Private houses	643	1055	1255	1714	4541	9208	7473	5163
Industry	183	1743	492	197	1400	3997	2091	2304
Building	30	516	124	142	303	1115	650	628
Trade, hotel recreation	517	1819	1560	2370	5185	11451	7560	6206
Services	1207	1433	1057	2456	1199	7352	2468	3478
Agriculture	14	785	25	131	442	1397	838	911
Horticulture	937	219	798	772	687	3413	1510	1622
Institutions	42	215	105	50	159	571	338	294
Total								
Dependent	3573	7785	5416	7814	13915	38503	22928	18905
Inunda	2293	3094	3291	6041	13704	28423	19347	15810
Independent	1747	3239	2456	3906	7189	18537	11269	9301

Table 2.2: Standard deviations (kDfl) in damage categories and in the overall flood damage (Blois, 1997).

The data in columns 2 to 6 in Table 2.2 have been obtained by varying one parameter at the same time. The figures in Column 8 result when all 5 parameters are varied simultaneously. It follows from Table 2.2 that the uncertainties are generally weakly correlated, except for the discharge. This is what would be expected, as a change in discharge affects the whole Meuse valley and hence the related damage categories in the same way.

3. ESTIMATING UNCERTAINTIES BY MEANS OF A DETERMINISTIC MODEL

The variation in the flood damage S due to a variation in the discharge q follows from:

$$\Delta S_q = \frac{\partial S}{\partial q}\Delta q = \sum_{i=1}^{8}\{\frac{\partial S_i}{\partial x_1}\frac{\partial x_1}{\partial q}\Delta q + \frac{\partial S_i}{\partial x_2}\frac{\partial x_2}{\partial q}\Delta q + \frac{\partial S_i}{\partial x_3}\frac{\partial x_3}{\partial q}\Delta q + \frac{\partial S_i}{\partial h}\frac{\partial h}{\partial q}\Delta q + \frac{\partial S_i}{\partial q}\Delta q\} \qquad [3.1]$$

In this simple analysis the assumption will be made that the partial derivatives of the water level h with respect to the discharge q and the partial derivatives of the variables x_1, x_2, and x_3 with respect to h and q equal zero. This results in:

$$\Delta S_q = \Delta q \sum_{i=1}^{8}\frac{\partial S_i}{\partial q} = \Delta q \frac{\partial S}{\partial q} = \frac{\partial S}{\partial q}\frac{\Delta q}{q}q \qquad [3.2]$$

A similar result would have been obtained by carrying out a statistical error analysis for the total flood damage, while assuming maximum correlation between the variables x_1, x_2, x_3 and q only and zero correlation for all other combination of variables, in agreement with the results shown in Table 2.1.

In this formula the uncertainty in flood damage ΔS_q is calculated from the stiffness coefficient $\partial S/\partial q$, the variation coefficient $\Delta q/q$ and the volume of the discharge q. The variation coefficient is a best guess of σ/μ for the discharge q. Estimating the value of the variation coefficient is one of the difficult tasks in any uncertainty analysis and, as can be seen in 3.2, directly affects the resulting uncertainty in flood damage ΔS_q. Estimates of the variation coefficient for the damage categories in Table 2.1 can be found in Wind and Blois (1998).

There are various ways in which the stiffness coefficient can be obtained. In this section first use will be made from the results of the Inunda model to derive this coefficient. In the next paragraph a simple flood damage model will serve the same purpose. The damage-discharge curve obtained with Inunda is shown in Figure 3.1.The stiffness coefficient for a discharge q = 3000 m^3/s is in the order of 192 kDfl/m^3. The standard deviation in flood discharge in Inunda is estimated at 60.4 m^3/s. The resulting standard deviation in the flood damage is 192 * 60.4 ≈ 11.600 kDfl, which is in fair agreement with the Inunda result of 13.915 kDfl or 8% of the flood damage.

The conclusion is that the uncertainty in the flood damage due to the uncertainty in one of the variables can be estimated with in 3.2, if the variables are uncorrelated.

4. A SIMPLE FLOOD DAMAGE MODEL

As the focus of this paper is on the factors determining the uncertainty in models like Inunda, an approach should be found for calculating the stiffness coefficient which is independent of Inunda. For this aim a simple one dimensional flood damage model has been derived in Appendix A. In that model the river valley has been approximated by a triangular shape and the valley is uniform in length direction. The flow is steady and the bottom roughness (not the flow resistance) is constant. The flood damages are partially constant and partially depend on the inundation depth. Finally it has been assumed that the type of land coverage is homogeneous in type and value, thus either agriculture or housing etc. The resulting flood damages are expressed in A12 as:

$$S = \frac{2Ac_1q^{0.4} + c_2A^2q^{0.8}}{a} \qquad [4.1]$$

Where:

S : flood damage per unit length of the valley (kDfl/m).
c_1, c_2 : damage coefficients; c_1= constant and c_2= related to inundation depth
q : discharge [m^3/s]

$$A = \{5a/(4C\sqrt{I})\}^{0.4} \qquad [4.2]$$

a : slope of the valley related to inundated area
C : Chezy coefficient [$m^{0.5}/s$]
I : hydraulic gradient [-]

Along the Meuse there are a number of land cover types and hence damage categories. In order to apply the analytical model to this situation, the river valley with a length L will be subdivided into subsections with a length l_i. Each section contains one damage category and the flood damage to the damage category in that river section is s_i. The equation for a sequence of j river sections reads:

$$S = \sum_{i=1}^{j} \frac{l_i}{aL}\{2c_{1i}Aq^{0.4} + c_{2i}A^2q^{0.8}\} \qquad [4.3]$$

For the analysis in this paper the dependence of the flood damage flood on the river discharge will be generalised to q^n, resulting in:

$$S = \frac{2Aq^n}{a}\sum_{i=1}^{j}\alpha_i c_{1i} \qquad [4.4]$$

Where:
$\alpha_i = l_i / L$ (fraction of the area of the river valley with land cover type i)

As the factor A contains a number of parameters, the original parameters will be substituted. This leads to the following expression for the flood damage:

$$S = 2*(\frac{5}{4})^{0.4}a^{-0.6}C^{-0.4}I^{-0.2}q^n\sum_{i=1}^{j}\alpha_i c_{1i} \qquad [4.5]$$

One of the properties of these algebraic equations is that the derivatives of the flood damages can be expressed in terms of flood damage S. For instance:

$$\frac{\partial S}{\partial q} = \frac{n}{q}S \qquad [4.6]$$

And e.g. for the terms after the summation sign:

$$\frac{\partial S}{\partial \alpha_i} = \frac{n}{\alpha_i} \beta_i S \qquad [4.7]$$

Where:
$\beta_i = S_i / S$ (fraction of the flood damages in section j due to land cover type i)

This implies that the variation in flood damage in 3.2 due to a change in one of the parameters can be expressed in terms of the overall flood damage S. Using the stiffness coefficients derived above in 3.2 this leads to the following expressions for the variation in flood damage:

$$\Delta S_q = n \frac{\Delta q}{q} S \qquad [4.8]$$

If n=0.4, as in 4.3 and the variation coefficient for the discharge is 0.02, as used in Inunda, then the contribution of the uncertainty in the discharge to the uncertainty in the flood damage is 0.4 * 0.02 = 0.008 S. This would mean that the discharge is only a marginal uncertainty source, which contradicts the results in section two, where has been found that the variation in discharge is the major source of uncertainty and is in the order of 0.08S. At first sight, the Inunda results are somewhat surprising, as one would expect that with increasing flood volume, the river becomes wider and the surface over which the increase in flood volume can be distributed becomes larger resulting in a smaller increase in inundation depth with increasing flood volume. Furthermore, the rate of increase of flood damage tends to reduce with increasing inundation depth. Hence the rate of increase in flood damage should decrease with increasing discharge and not increase! However as can be seen in Figure 3.1 the flood damage increases rapidly with increasing flood volume. The flood damage in figure 3.1 is related to q^3, which is much higher than is $q^{0.4}$ or $q^{0.8}$ which is found in the model in Appendix A12. An explanation for this strong dependency of flood damage on the discharge could be that the economic values following a trajectory perpendicular to the flow of the river are not uniformly distributed, as is assumed in the analytical model, but increase rapidly with the distance from the river. It seems realistic to position higher economic values in areas with a lower flooding risk, but from aerial photographs this it is not immediately evident.

This analysis of the uncertainty of the flood damages with the simple analytical model leads to two conclusions. The first conclusion is that the estimate of the value of the variation coefficient is at least as important as estimating the stiffness coefficient, where the stiffness coefficient represents the response of the integrated hydraulic and economic system to its driving forces. The second conclusion is that flood damages depend much stronger on the increase in flood discharge than can be inferred from simple models. This dependency on flood discharge along the Meuse could be explained in cases where the economic values along the river are not more or less uniformally distributed, but strongly increase with the distance from the river.

5. FACTORS FOR THE EXPECTED VALUE OF FLOOD DAMAGE IN THE SIMPLE FLOOD DAMAGE MODEL

In a cost benefit assessment of mitigative actions against flooding, the costs of the measures should be smaller than the damage prevented. The damage prevented can be calculated from the expected value of the flood damage E(S) (see A12 or A13). The expression A15 can be used to evaluate the effects of the construction of an embankment. The expected value of the flood damage in absence of an embankment is given in A16. The shape of the expected damage function is such that the largest contribution to the value of E(S) is concentrated around the top. The top of the expected damage function is located where $\partial E/\partial q = 0$. If flood damages are independent of the inundation depth ($c_2 = 0$), then the discharge related to the top equals:

$$q_{top} = 0.4\ q_r \qquad [5.1]$$

where q_r is the discharge with an annual frequency of exceedance of $b_1 e^{-1}$, where b_1 is a constant, explained in Appendix A. If flood damages are only dependent on inundation depth ($c_1 =$ 0), then the top is located at:

$$q_{top} = 0.8\ q_r \qquad [5.2]$$

It is interesting to note that none of the other parameters in the calculation of the expected flood damage are affecting this result. If neither c_1 nor c_2 are equal to zero, then it is expected that the top discharge will be intermediate between these two values. It is interesting to note that the annual frequency of exceedance of $b_1 e^{-1}$, in case $b_1 = 1$, is rather high, implying that the floods with a return period of 3 to 5 years contribute significantly to the expected flood damage. For floods with a higher return period the probability is high but the damages are low, and for exptreme floods the damages are high but the probability of exceedance is low. The results in the Appendix allow for a sensitivity analysis, yielding those parameters which contribute most to the uncertainty in the cost-benefit analysis. In this analysis the finding of the previous section of a rapid increase of economic values with the distance from the river, has not been included and may change this conclusion.

6. EFFECTS OF THE INUNDA RESULTS ON MODEL DESIGN

The Inunda model is based on detailed information. In case one focuses on assessing the flood damages on the expected value of the flood damage and not on a detailed evaluation of the effects of civil engineering works, as was the objective of the Delft Hydraulics study (1993), one may wonder whether a simplified model would have yielded the same results, both in terms of the flood damages as well as in the uncertainty in the flood damages. In order to investigate this question, we have added to Table 2.1 the contribution of each damage source S_i to the flood damages S as they are modelled in Inunda.

The contribution of each sector as a percentage of the flood damage is found by dividing the contribution of that sector by the total flood damage and multiplication of that result by 100%. The contribution of each damage category to the uncertainty for uncorrelated uncertainties has been obtained from:

$$1 = \sum \frac{\sigma_i^2}{\sigma_{tot}^2} \qquad [6.1]$$

And for correlated uncertainties:

$$1 = \sum \frac{\sigma_i}{\sigma_{tot}} \qquad [6.2]$$

It follows from Table 2.1 that 80% of the flood damage is caused by three sources: private houses, trade, catering and recreation and to a minor extent to services. The remaining five sources contribute 20% of the flood damage. As has been outlined in section two, the contribution to the uncertainty standard depends on the degree the parameters are correlated. In Figure 6.1 the contribution of each source to the flood damage as well as to the uncertainty is presented. In this figure the dominant contribution of the damage to private houses and to trade, hotel and recreation is eminent. Furthermore it is clear that the contribution of the last source to the uncertainty is relatively larger than for private houses. Adding smaller sources relatively adds a small part to the flood damage and, as the uncertainty sources are mainly independent Blois (1998), the additional sources reduce marginally the relative uncertainty.

The conclusion is that the dominant sources of flood damage in the Meuse valley are private housing and trade, hotel and recreation. The contribution to the uncertainty is comparable for both sources, yielding trade, hotel and recreation as the relatively most uncertain source.

7. DISCUSSION AND CONCLUSIONS

This paper focuses on the factors determining the uncertainty in a flood damage model.The 1993 and 1995 Meuse floods have served as examples. In order to draw some lessons from these studies, a simple analytical model has been developed and the results have been compared with the field data and the computed results.

From an analytical approach it followed that the uncertainty in the flood damage results from the properties of the integrated economic-hydraulic system leading to a stiffness coefficient, the uncertainty in each parameter and the co-variation between the uncertainties. In the simple approximation of the uncertainty in flood damage [3.2] the uncertainty in both the stiffness coefficient and the model parameters play an equal role. The impression is that during model development often more time is spent on the model characteristics than on the characteristics of the data.

Comparing the total value of the correlated and the uncorrelated uncertainties in Table 2.2, it follows that the degree in which variables are correlated also is important for the aspect for the value of the total uncertainty. An expression has been found for the maximum error if it is not known to which extent the uncertainties are correlated. For the Inunda model of the Meuse it follows from Table 2.2 that uncertainties are generally weakly correlated, except for the discharge.

From the Inunda results it was found that the uncertainty in flood damages was strongly influenced by the river discharge: $S \approx q^3$. The cause of this strong influence is not immediately clear. Arguments have been given why a one might expect that flood damage depends linearly or less on the flood discharge. An explanation for the strong dependency of flood could be that economic values strongly increase with the distance from the river.

Dominant sources of flood damage in the Meuse valley are private housing and trade, hotel and recreation. The contribution to the uncertainty is comparable for both sources, yielding trade, hotel and recreation as the relatively most uncertain source.

The question whether uncertainty must be reduced, depends on the problem context (Green 1993,Wind et al. (1997). An example of the importance of the problem context is given by Peerbolte (1993), where the costs of dike raising in the Netherlands for sea level rise scenario's of respectively 0.3m, 0.6 m and 1.0 m are compared with the capital values at risk. As can be seen in Figure 7.1, the benefits by far outweigh the necessary investments for the higher sea level rise scenario's. This despite the large uncertainty bands. From the point of view of a cost benefit analysis, in the example of Peerbolte there is no need to reduce the uncertainty in the results. The decision context will determine in this case whether action will be taken or not.

A simple analytical model has been used to investigate which flood frequencies contribute most to the expected value of flood damages. It was found that the discharge related to the top is in the order of:

$$q_{top} = q_r \qquad [7.1]$$

where q_r is the discharge with a annual frequency of exceedance of b_1e^{-1}, where b_1 is a constant, explained in Appendix A. It is interesting to note that the annual frequency of exceedance of b_1e^{-1}, in case $b_1 = 1$, is rather high, implying that the floods with a return period of 3 to 5 years contribute significantly to the expected flood damage. The results in the Appendix allow for a sensitivity analysis, yielding those parameters which contribute most to the uncertainty in the cost-benefit analysis. In this analysis the finding of the previous section of a rapid increase of economic values with the distance from the river, has not been included and may change this conclusion.

8. LITERATURE

Blois, C.J. de, *EMBRiO: Evaluatie van methoden voor de berekening van risico en schade bij overstromingen. Fase 3: Onzekerheidsanalyse en analyse van hoogwatergegevens.* Report for the Dutch National Water Board, University of Twente, Enschede (in Dutch), 1997.

Blois, C.J. de, *Dealing with uncertainty and inaccuracy in large-scale models for decision support in water management.* PhD thesis, University of Twente (forthcoming), 1998.

Delft Hydraulics, *Onderzoek Watersnood Maas,* 19 volumes (in Dutch), 1994.

Green, C.H., Parker, D.J., and Penning-Rowsell, E.C., "Designing for Failure". Paper given at the *IDNDR Conference Protecting Vulnerable Communities,* London, 1993.

Nierop, T.M., *Schade in kaart: schadeanalyse van de Maasoverstromingen 1993 en 1995 in Limburg.* MSc thesis, University of Twente, Enschede (in Dutch), 1997.

Peerbolte, E.B., *Sea-level Rise and Safety.* PhD thesis, University of Twente, Enschede, 1993.

Wind, H.G., Blois, C.J. de, Kok, M., Peerbolte, E.B., and Green, C.H., "Uncertainty in flood damage assessment, when does it matter? A European perspective." Contribution to the *RIBAMOD Conference,* Delft Hydraulics, Delft, 1996.

Wind, H.G., Nierop, T.M., and Blois, C.J. de, "Flood damage modelling: analysis of the 1993 and 1995 Meuse floods" in *Water Resource Research* (submitted), 1998.

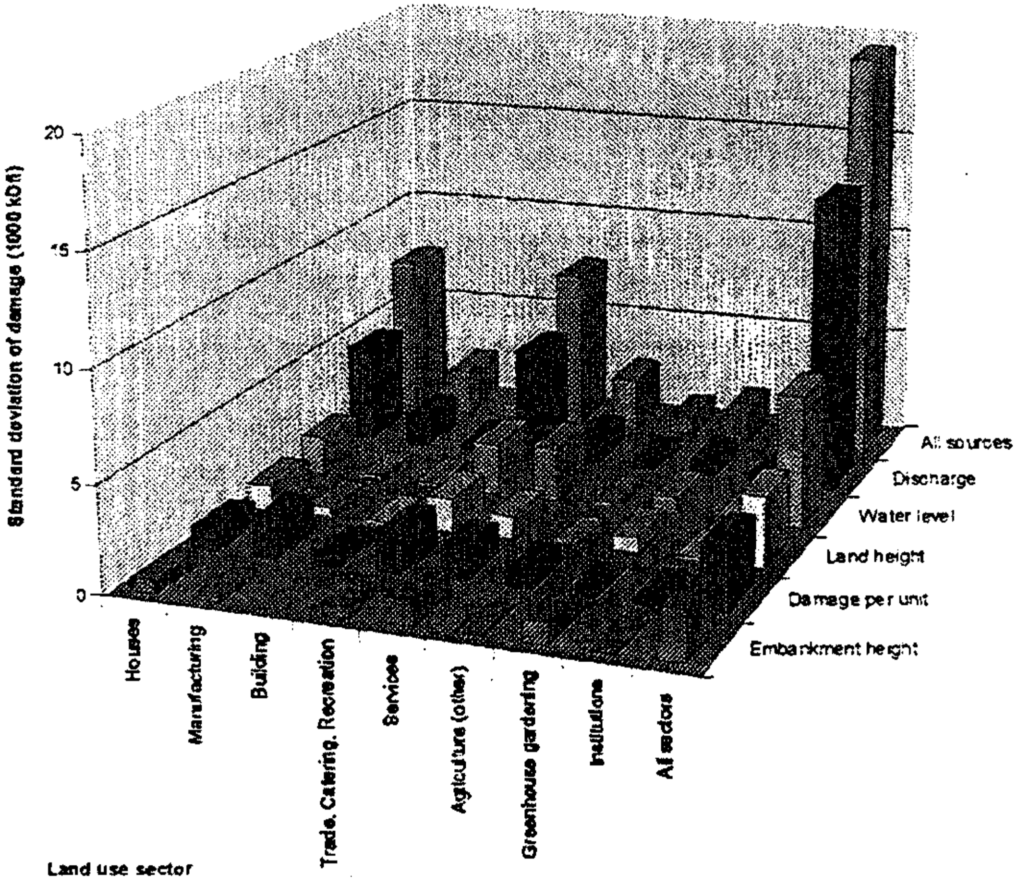

Figure 2.1: The contribution of each uncertainty source and each sector to the uncertainty in the flood damages (Blois 1997).

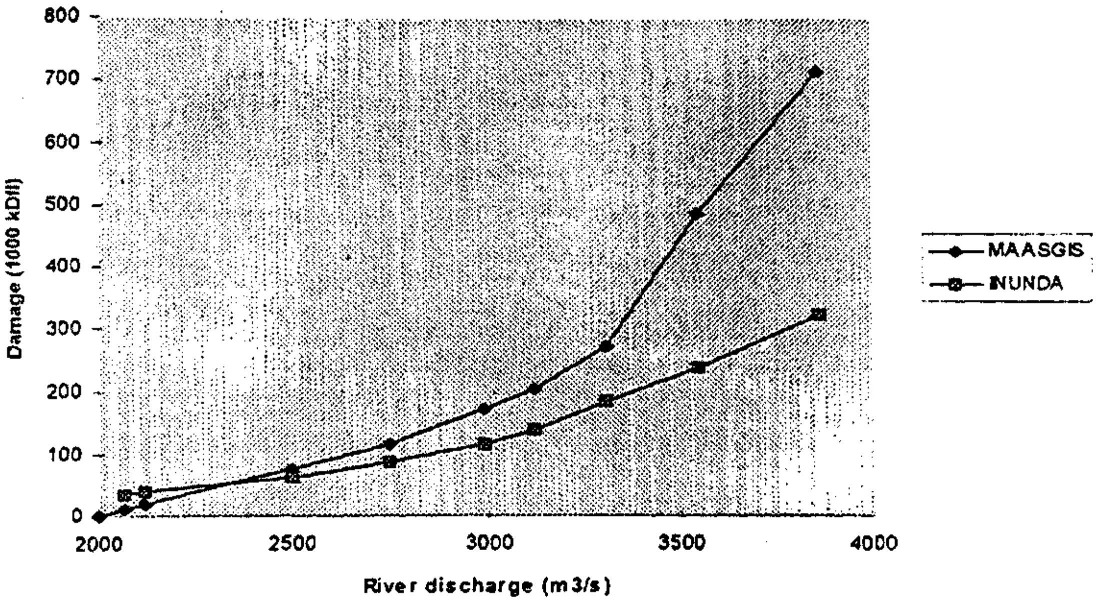

Figure 3.1: Flood damage - discharge curves of Maasgis (Kok, e.a 1994) and Inunda (Blois 1997).

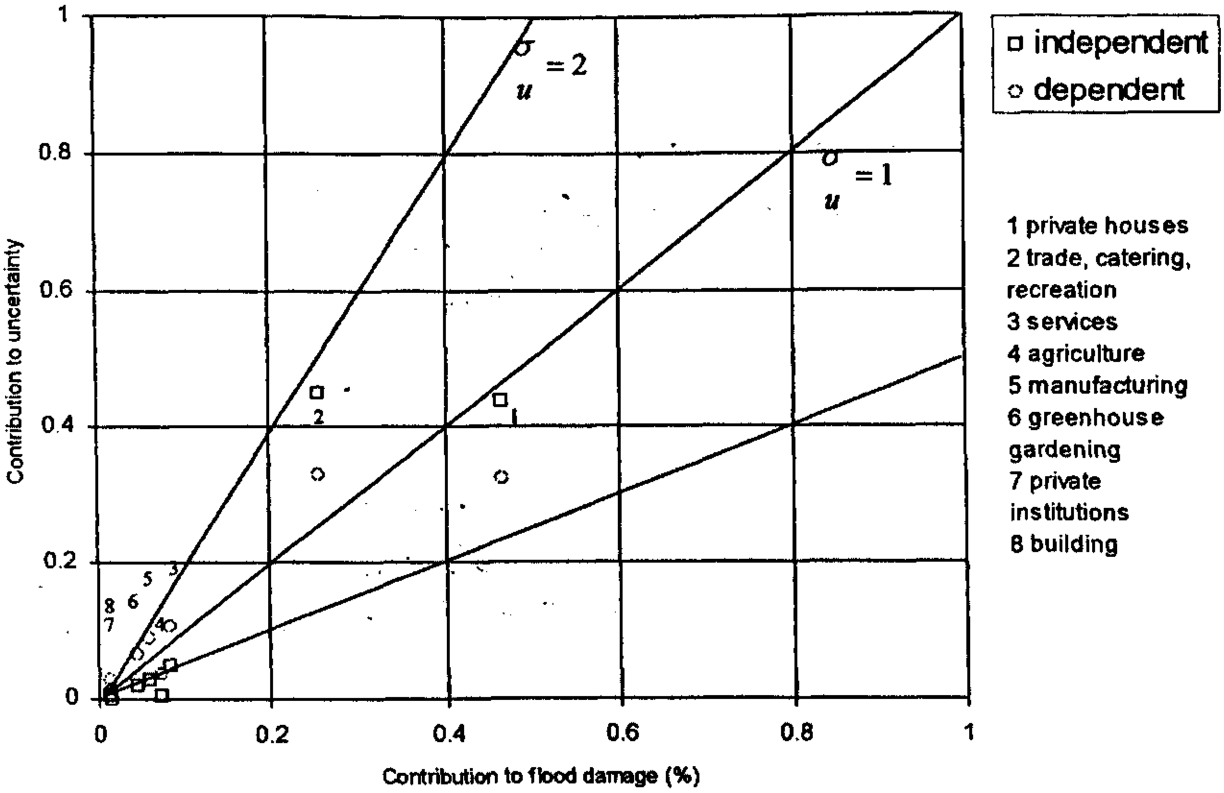

Figure 6.1: Contribution to the flood damages and to the uncertainty in flood damage of eight sources in the Meuse valley for the 1993 flood.

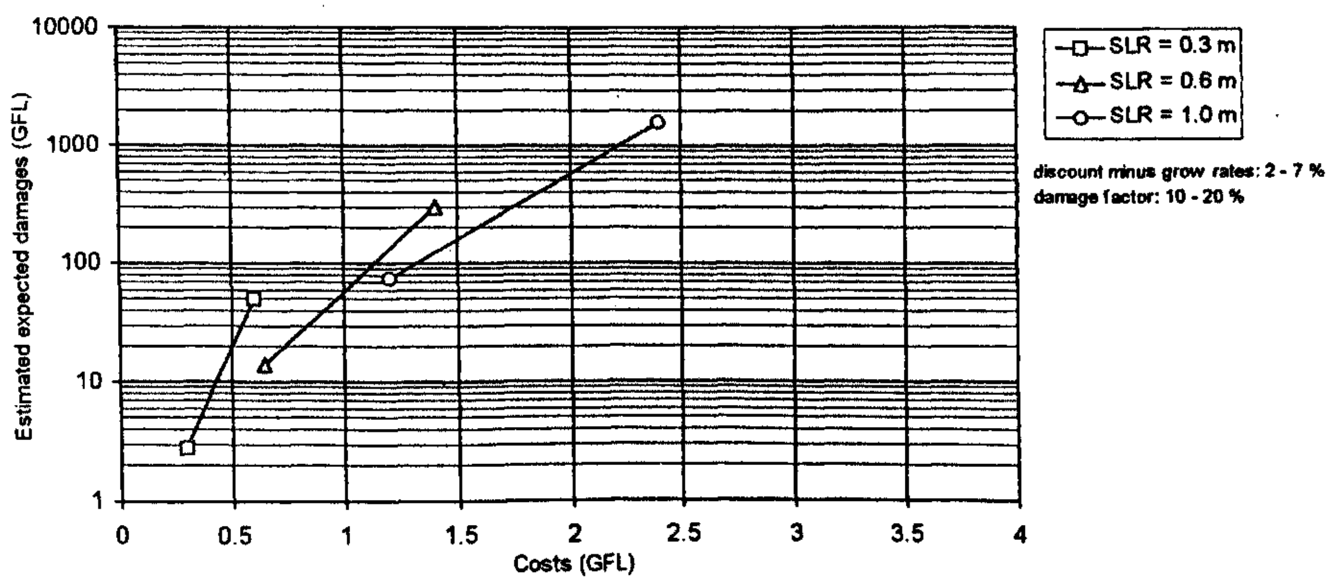

Figure 7.1: Estimated benefits and costs for different sea-level rise scenario's (Peerbolte, 1993)

APPENDIX A: A SIMPLE MODEL FOR FLOOD DAMAGE ASSESSMENT

Rivergeometry

The river cross section is represented by a linear bottom profile, with slope a:

$$z = z_0 + ay \qquad \text{[A.1]}$$

If the water depth h = h_0 at y=0 then, the innundation depth can be expressed as:

$$h = h_0 - ay \qquad \text{[A.2]}$$

Riverhydrology

The flow is steady $\frac{\partial u}{\partial t} = 0$ and the bottom roughness is uniform. With the Chézy formula the depth averaged flow velocity v can be represented by

$$v = C\sqrt{hI} \qquad \text{[A.3]}$$

I: hydraulic gradient
C: Chézy coefficient

From continuity follows the discharge *dq* over a width *dy*

$$dq = vhdy = C\sqrt{hI}\,hdy = -\frac{C}{a}\sqrt{I}\,h^{\frac{3}{2}}dh \qquad \text{[A.4]}$$

In the last step *dy* has been replaced by *dh* by means of A.2

$$q = 2\int_{h_0}^{0} -\frac{C}{a}\sqrt{I}\,h^{\frac{3}{2}}dh = \frac{4}{5}\frac{C}{a}\sqrt{I}\,h_0^{\frac{5}{2}} \qquad \text{[A.5]}$$

or

$$h_0 = Aq^{\frac{2}{5}} \qquad \text{[A.6]}$$

$$A = \left(\frac{5a}{4C\sqrt{I}}\right)^{\frac{2}{5}} \qquad \text{where [A.7]}$$

The inundation depth *h* as a function of the discharge *q* becomes with A.2 and A.7:

$$h = Aq^{\frac{2}{5}} - ay \qquad \text{[A.8]}$$

The frequency per annum that a discharge q_1 exceeds a discharge *q* will be expressed by the exponential distribution:

$$\Pr\{q_1 > q\} = b_1 e^{-b_2 q} \qquad \text{[A.9]}$$

It may be noted that for *q=0* the probability of exceedance equals b_1 and not 1.

The value where $b_2 = \frac{1}{q_r}$ q_r is the reference discharge with the annual

frequency of exceedance. $b_1 e^{-1}$

Economic values and damage function

The economic value will be assumed to be uniformly distributed over the river banks.
The damage due flooding s per unit area consists of a constant fraction c_1 and a part which depends on the inundation depth h:

$$s = c_1 + c_2 h \qquad [A.10]$$

The flood damage S due to a depth h_o at the centre of the river follows from:

$$S = 2\int_{h=o}^{h=h_o} s\,dh = 2c_1 h_o + c_2 h_o^2 \qquad [A.11]$$

With A8 the flood damage can also be expressed in terms of the discharge q:

$$S = \frac{2Ac_1}{a} q^{\frac{2}{5}} + c_2 \frac{A^2}{a} q^{\frac{4}{5}} \qquad [A.12]$$

The expected value of the flood damage

The expected value of the flood damage E(s) can be written as:

$$E(s) = \int_{q=o}^{\infty} s(q) p\{q_1 \geq q\} dq \qquad [A.13]$$

In case of a dike, damage only will occur if the flood volume exceeds a value q_n. The expected value of the flood damage in that case becomes

$$E(s) = \int_{q=q_n}^{\infty} s(q) p\{q_1 \geq q\} dq \qquad [A.14]$$

$$E(s) = \left(\frac{c_2}{2a} A^2 q_n^{\frac{4}{5}} - \frac{c_1}{a} A q_n^{\frac{2}{5}}\right) b_1 e^{-b_2 q_n} + b_1 \left(\frac{c_1 A}{a} - \frac{c_2}{a} A^2 q_n^{\frac{2}{5}}\right) \frac{1}{b_2^{1.4}} \Gamma(1.4; b_2 q_n) + \qquad [A.15]$$

$$+ b_1 \left(\frac{c_2 A^2}{2a}\right) \frac{1}{b_2^{1.8}} \Gamma(1.8; b_2 q_n)$$

If in absence of the dike, q_n equals zero, then the expected value becomes:

$$E(s) = \frac{b_1}{b_2^{1.4}} \frac{c_1 A}{a} \Gamma(1.4) + b_1 \frac{c_2 A^2}{2a} \frac{1}{b_2^{1.8}} \Gamma(1.8) \qquad [A.16]$$

where ?(a) and ?(a,b) are the gammafunction and the incomplete gammafunction respectively.

If the flood damages on both banks are taken into consideration, the expected value of the damages should be multiplied by a factor two.