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Different sediment mixtures at constant flow conditions can produce the same celerity of bed disturbances

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ABSTRACT
We present a theoretical analysis and a numerical test of the effect of a change in bed sediment composition on sediment transport rate and celerity of bed level perturbations. Usually sediment transport rate and celerity are correlated, but we theoretically reveal conditions under which removal of the coarsest grain-size fractions from the sediment in the bed increases the sediment transport rates but does not alter the celerities. We identify an area of the Dutch Rhine branches where these conditions occur. Numerical simulations for this area using a 2D morphological model confirm the findings of the theoretical analysis.

Keywords: Graded sediment, river morphology, 2D mathematical modelling

1 INTRODUCTION

Complex two-dimensional models for river morphology involve several calibration parameters. This leads to problems of equifinality in the sense that different combinations of calibration parameter values can produce the same bed topography. It is difficult to decide which combination is the right one. The current practice in the Netherlands is therefore to calibrate models on as much different morphological phenomena as possible: the longitudinal bed profile, the pattern of bars and pools, the sediment transport rates and the celerities of bed level perturbations. Often, however, these different phenomena are correlated and hence not independent. That is why we seek parameter ranges for which the correlations are weak and, hence, the different morphological phenomena are independent. This might clarify the individual contributions of different parameters to the results of morphological computations. The insight thus obtained should render calibrations less ambiguous.

It is against this background that we analyze the effect of a change in bed sediment composition on two phenomena: sediment transport rate and celerity of bed level perturbations. An analysis of characteristics reveals conditions under which removal of the coarsest grain-size fractions from the sediment in the bed increases the sediment transport rates but does not alter the celerities. We identified an area of the Dutch Rhine where these conditions occur. We numerically simulated the transport rates and bed level celerities for this area using a 2D morphological model based on a multi-fraction approach and the Hirano active-layer concept. The simulations confirm the findings from the analysis of characteristics.

2 THEORETICAL ANALYSIS

The celerities of infinitesimal perturbations in a river morphology system can be estimated using an analysis of characteristics along the lines of Ribberink (1987). The starting point is a one-dimensional set of equations for water motion and morphology with graded sediment.

The 1D quasi-steady flow equations read

$$u\frac{\partial u}{\partial x} + g\frac{\partial z_b}{\partial x} + g\frac{\partial h}{\partial x} + \frac{gu|u|}{C^2 h} = 0 \tag{1}$$

$$h\frac{\partial u}{\partial x} + u\frac{\partial h}{\partial x} = 0 \tag{2}$$

where

C	=	Chézy coefficient for hydraulic roughness ($m^{1/2}/s$)
g	=	acceleration due to gravity (m/s^2)
h	=	flow depth (m)
u	=	cross-sectionally averaged flow velocity (m/s)
x	=	streamwise co-ordinate (m)
z_b	=	bed elevation (m + datum)

The friction term in Eq. (1) can be neglected when focusing on short spatial scales. Substitution of Eq. (2) into Eq. (1) then leads to the simplified flow equation:

$$\frac{\partial u}{\partial x} = \frac{u}{h\left(1-Fr^2\right)}\frac{\partial z_b}{\partial x} = 0 \tag{3}$$

in which the Froude number, Fr, is defined by

$$Fr = \frac{u}{\sqrt{gh}} \tag{4}$$

The sediment balance for the complete sediment mixture can be written as

$$\left(1-\varepsilon\right)\frac{\partial z_b}{\partial t} + \frac{\partial q_s}{\partial x} = 0 \tag{5}$$

where

q_s	=	total volumetric transport rate per unit width, excluding pores (m^2/s)
t	=	time (s)
ε	=	porosity (-)

The sediment transport is a function of both flow velocity and average sediment grain size:

$$\frac{\partial q_s}{\partial x} = \frac{dq_s}{du}\frac{\partial u}{\partial x} + \frac{dq_s}{dD_m}\frac{\partial D_m}{\partial x} \tag{6}$$

where

D_m	=	average sediment grain size (m)

Substitution of this equation into Eq. (5) and elimination of $\partial u / \partial x$ by applying Eq. (3) lead to

$$\frac{\partial z_b}{\partial t} + \frac{u}{\left(1-\varepsilon\right)\left(1-Fr^2\right)h}\frac{dq_s}{du}\frac{\partial z_b}{\partial x} = -\frac{1}{\left(1-\varepsilon\right)}\frac{dq_s}{dD_m}\frac{\partial D_m}{\partial x} \tag{7}$$

This equation can be interpreted as a kinematic bed topography wave forced by gradients in sediment composition. The corresponding celerity is

$$c_{bed} = \frac{u}{\left(1-\varepsilon\right)\left(1-Fr^2\right)h}\frac{dq_s}{du} \tag{8}$$

This relation assumes a simpler form by introducing the degree of nonlinearity, b, of $q_s = q_s(u)$, defined by $b = (u / q_s)(dq_s / du)$. The result reads

$$c_{bed} = \frac{bq_s}{\left(1-\varepsilon\right)\left(1-Fr^2\right)h} \tag{9}$$

The sediment balance for individual sediment size fractions i is given by

$$\left(1-\varepsilon\right)\left[\frac{\partial p_{i,a}\delta}{\partial t} + p_i\left(z_0\right)\frac{\partial z_0}{\partial t}\right] + \frac{\partial q_{si}}{\partial x} = 0 \tag{10}$$

where

$p_{i,a}$	=	relative occurrence of sediment size fraction i in active layer (-)
$p_i(z_0)$	=	relative occurrence of sediment size fraction i at level z_0 (-)
q_{si}	=	volumetric transport rate of sediment size fraction i per unit width, excluding pores (m^2/s)
z_0	=	upper level of substratum (m + datum)
δ	=	thickness of active layer (m)

The active-layer thickness is assumed constant, so that $\partial z_0 / \partial t = \partial z_b / \partial t$. The transport rate per fraction is written as

$$q_{si} = p_{iT}q_s \tag{11}$$

where

p_{iT}	=	relative occurrence of sediment size fraction i in the bedload (-)

Subsequently Eq. (5) is used to eliminate $\partial q_s / \partial x$. The result is

$$(1-\varepsilon)\left[\delta\frac{\partial p_{i,a}}{\partial t}+\left(p_i(z_0)-p_{iT}\right)\frac{\partial z_b}{\partial t}\right]+ \quad (12)$$
$$q_s\frac{\partial p_{iT}}{\partial x}=0$$

Multiplication of all terms by D_i and summation over all size fractions gives, assuming that the substratum has the same composition as the active layer (P_i (z_0) = $P_{i,a}$) and noting that the values of D_i are constants for the selected size fractions:

$$(1-\varepsilon)\left[\delta\frac{\partial D_m}{\partial t}+\left(D_m-D_{mT}\right)\frac{\partial z_b}{\partial t}\right]+ \quad (13)$$
$$q_s\frac{\partial D_{mT}}{\partial x}=0$$

with

$$D_m=\sum_i p_{i,a}D_i \quad (14)$$

$$D_{mT}=\sum_i p_{iT}D_i \quad (15)$$

Assuming a constant ratio of average bedload grain size to average active-layer grain size, Eq. (13) can be written as

$$\frac{\partial D_m}{\partial t}+\frac{\mu q_s}{\delta(1-\varepsilon)}\frac{\partial D_m}{\partial x}=\frac{D_{mT}-D_m}{\delta}\frac{\partial z_b}{\partial t} \quad (16)$$

with

μ = ratio of average bedload grain size to average active-layer grain size: $\mu = D_{mT} / D_m$ (-)

This equation can be interpreted as a kinematic bed sediment composition wave forced by bed level changes and the difference between the sediment compositions of the bedload and the bed. The corresponding celerity is

$$c_{mix}=\frac{\mu q_s}{\delta(1-\varepsilon)} \quad (17)$$

The two kinematic wave equations Eq. (7) and Eq. (16) are coupled through their right-hand forcing terms. This causes the actual celerities to deviate from the celerities c_{bed} en c_{mix}. The actual celerities, c = dx/dt, figure in the equations for the total differentials or material derivatives:

$$\mathrm{d}z_b=\frac{\partial z_b}{\partial t}\mathrm{d}t+\frac{\partial z_b}{\partial x}\mathrm{d}x=\left(\frac{\partial z_b}{\partial t}\mathrm{d}t+c\frac{\partial z_b}{\partial x}\right)\mathrm{d}t \quad (18)$$

$$\mathrm{d}D_m=\frac{\partial D_m}{\partial t}\mathrm{d}t+\frac{\partial D_m}{\partial x}\mathrm{d}x= \quad (19)$$
$$\left(\frac{\partial D_m}{\partial t}\mathrm{d}t+c\frac{\partial D_m}{\partial x}\right)\mathrm{d}t$$

The set of equations Eqs. (7), (16), (18) and (19) has non-trivial solutions if:

$$\begin{vmatrix} 1 & c_{bed} & 0 & \frac{1}{1-\varepsilon}\frac{\mathrm{d}q_s}{\mathrm{d}D_m} \\ \frac{D_m-D_{mT}}{\delta} & 0 & 1 & c_{mix} \\ 1 & c & 0 & 0 \\ 0 & 0 & 1 & c \end{vmatrix}=0 \quad (20)$$

The solution reads

$$c_{1,2}=\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)_{1,2}=\frac{c_{bed}+c_{mix}+\xi}{2}+ \quad (21)$$
$$\pm\frac{1}{2}\sqrt{c_{bed}^2-2c_{bed}c_{mix}+c_{mix}^2+2\xi\left(c_{bed}+c_{mix}\right)+\xi^2}$$

with

$$\xi=\frac{\left(D_{mT}-D_m\right)}{\delta(1-\varepsilon)}\frac{\mathrm{d}q_s}{\mathrm{d}D_m} \quad (22)$$

For $\xi = 0$, the celerities are equal to the values c_{bed} en c_{mix} of the uncoupled system. For $\xi \neq 0$, the celerities are affected by the interactions between bed topography and bed sediment composition.

3 NUMERICAL SIMULATIONS

We used an existing 2D morphological model of the Dutch Rhine branches between Emmerich (Rhein or Boven-Rijn, km 853), Doesburg (IJssel, km 903), Driel (Nederrijn, km 891) and Woudrichem (Waal, km 953) (Yossef et al, 2006; Van Vuren et al, 2006; Mosselman et al, 2007). The area is shown in Fig. 1. The model is based on the Delft3D modelling system with a multi-fraction approach and the Hirano active-layer concept for graded sediment. We applied the discharge hydrograph of Fig. 2 at the upstream boundary (Boven-Rijn). We schematized the bed sediment composition using 10 fractions with initial values as shown in Table 1.

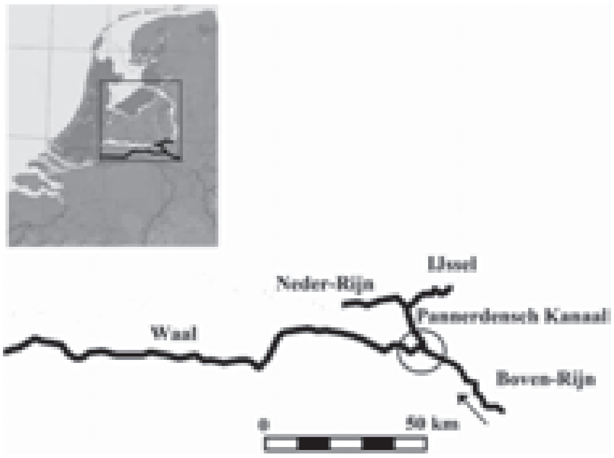

Figure 1. Map of Dutch Rhine branches.

We compared a simulation for the complete set of 10 grain-size fractions with a simulation for the set of 8 grain-size fractions that remain after removal of the two coarsest fractions. In the simulation with 10 fractions, the two coarsest fractions were never set into motion by the flow. Yet they did influence the results. Removal of the two immobile fractions increased the sediment transport rates by increasing the relative occurrence of finer fractions in the sediment mixture and by decreasing the effects of hiding. In principle, this increase in transport rate increases the bed level celerity as well. At the same time, however, the removal of the coarsest fractions decreases the bed level celerity by decreasing the grain size differences between the sediment in transport and the sediment in the bed.

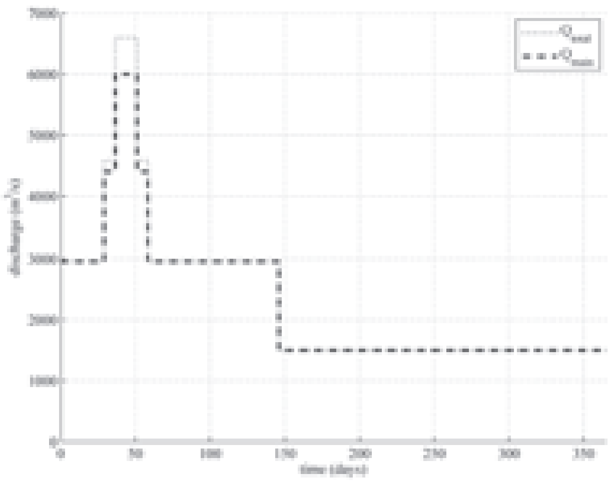

Figure 2. Schematized annual discharge hydrograph.

Table 1. Initial distribution of grain size fractions.

Fraction number	Grain size fraction limit (mm)		Percentage (%)
	lower	upper	
1	0.063	0.5	8.8
2	0.5	1	11.3
3	1	2	16.5
4	2	2.8	9.0
5	2.8	4	10.1
6	4	8	19.2
7	8	16	15.2
8	16	32	9.9

We found that the conditions in the Boven-Rijn reach between km 854 and 862 are such that the increases and decreases in bed level celerity cancel out each other, so that the bed level celerity remains the same. This is illustrated in Figs. 3 and 4. Figure 3 shows the uncoupled celerities, c_{bed} and c_{mix}, according to Eqs. (9) and (17). Figure 4 shows the coupled celerities, $c_{1,2}$, according to Eq. (21). We repeated the simulations with humps of sediment supplied at two locations along the river. Figures 5 and 6 show the migration and the attenuation of the two humps for 8 and 10 sediment fractions respectively. The results confirmed that the celerities were the same in both cases, despite marked differences in sediment transport rates.

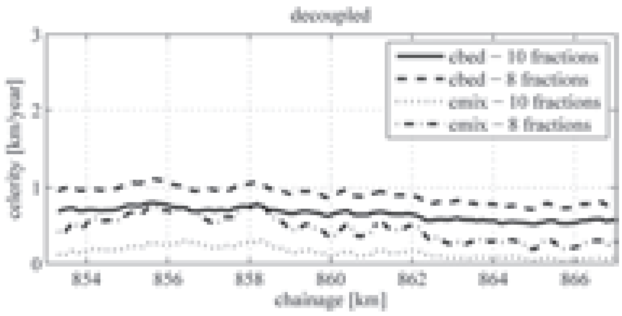

Figure 3. Celerities of uncoupled perturbations in bed topography and bed sediment composition.

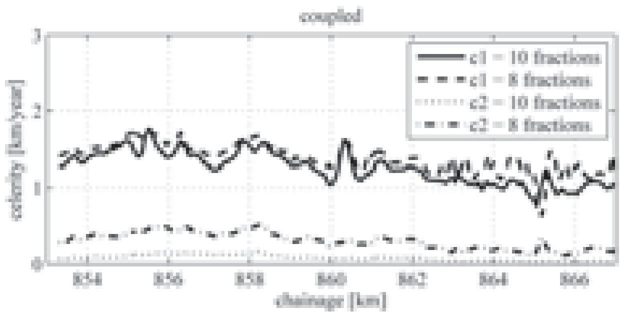

Figure 4. Celerities of coupled perturbations in bed topography and bed sediment composition.

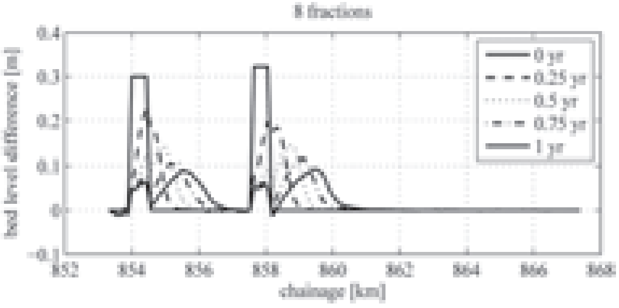

Figure 5. Difference in bed levels between simulations with and without humps for 8 sediment fractions.

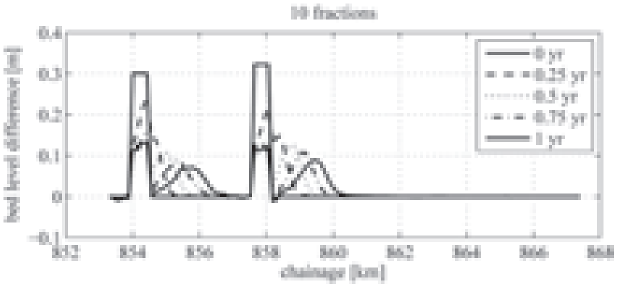

Figure 6. Difference in bed levels between simulations with and without humps for 10 sediment fractions.

4 CONCLUSIONS

The theoretical analysis has revealed conditions under which removal of the coarsest grain-size fractions from the sediment in the bed increases the sediment transport rates but does not alter the celerities. The numerical simulations have confirmed the theoretical analysis. The findings provide more insight into how different parameters affect the results of morphological computations. They also provide a means to define field tests or laboratory experiments to distinguish between the effects of grain size distribution, sediment transport formula and hiding-and-exposure correction.

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