On the road to a universal equation for the UHI

Within the urban climate community, the urban heat island (UHI) effect, here defined as the difference in temperature between the urban and rural environment at the pedestrian level, is a longstanding research topic. The UHI causes adverse effects for human thermal comfort, urban energy consumption, and urban air quality. While the UHI has been regarded of significant importance, simple models for estimating the UHI intensity within urban areas are still lacking (Arnfield, 2003). In addition, computer power to run global weather prediction models is increasing. Yet this is not enough to couple the global models to relatively complex urban energy balance parameterizations. In order to produce accurate temperature predictions for urban areas it can be beneficial to have a simple method to forecast the UHI as an operational tool.

Despite several attempts to derive a uniform formula for the UHI, the complexity of the system so far has inhibited this. Considering the amount of factors that govern the UHI, this is no wonder. In general, we can distinguish between urban and rural governing factors. The UHI magnitude depends not only on urban properties such as areal green and water cover, street geometry (aspect ratio) and thermal properties of building materials, but also on wind speed and direction, incoming radiation, cloud cover, urban soil moisture, season and the location of the city itself (e.g. elevation, latitude, etc.). The geographical location of the city also includes properties of its rural surroundings, i.e. whether it is a forest, grassland, desert, or any other land use type. Of course, many more factors affect the UHI magnitude. Possibly too many factors are involved to capture the UHI in one model and the dependencies are unique for each city or town. Hence, combining all these variables poses a great challenge. Oke (1998) made a first attempt to combine known relations between the UHI and some of these variables (street geometry, cloud cover and wind). This resulted in a simple model for the diurnal cycle of the UHI.

We will attempt to approach this problem from a different angle. Within atmospheric boundary-layer research, dimensional analysis (e.g. Langhaar, 1951) is a widely used method to tackle these kinds of complex systems. A well-known example of this approach is Monin-Obukhov similarity theory (Monin and Obukhov, 1954). This theory relates the shape of the vertical profiles of wind speed and temperature to the turbulent fluxes of momentum and heat within the atmospheric surface layer. In this study we use dimensional analysis in search of a uniform equation for the UHI. However, due to the amount of variables influencing the UHI, the problem would be mathematically too complex to solve. Therefore, selecting the most important variables is the first step of our approach. The variables will be selected using the statistical principle component analysis method, a way of identifying patterns and data reduction. Using this analysis we are able to find the variables which are most important in influencing the UHI magnitude. On this compressed dataset the dimensional analysis will be performed.

About dimensional analysis

Dimensional analysis is a method to estimate the dependence of one variable, in this case the UHI, on other variables, e.g. clouds, wind speed, vegetation in the city, soil moisture, etc. The analysis can be carried out using three steps.

1. Selecting variables

First, all quantities that affect the studied variable need to be selected. As mentioned before, the amount of variables influencing the key variable can make for a complicated mathematical problem. Therefore, we use a principle component analysis and limit the amount of variables.

2. Define dimensionless groups

Once the right quantities have been selected, one can derive dimensionless groups. Depending on the amount of selected variables (m) and their basic S.I. dimensions (n), the number of dimensionless groups (r) to be made is m-n=r. For example, if we consider six variables with a total of 3 basic dimensions, such as temperature (K), length (m), time (s), etc., 3 dimensionless groups can be made (6-3 = 3). The dimensionless groups should be unique.

3. Use data to determine a universal function between dimensionless groups

Once the dimensionless groups have been defined, the mutual dependencies between the dimensionless groups have to be quantified. In order to do so, an experimental dataset is needed, either from observations, or model output. With this dataset, the dimensionless groups are plotted as a function of each other. Consequently, we derive an equation describing the functional relation between the dimensionless groups, i.e. by a suitable regression analysis method. Finally, rewriting this equation will provide an expression for the studied variable, i.e. UHI. As the number of variables increases, the number of dimensionless groups increases as well, and it becomes more complex to fit an equation through all the groups.

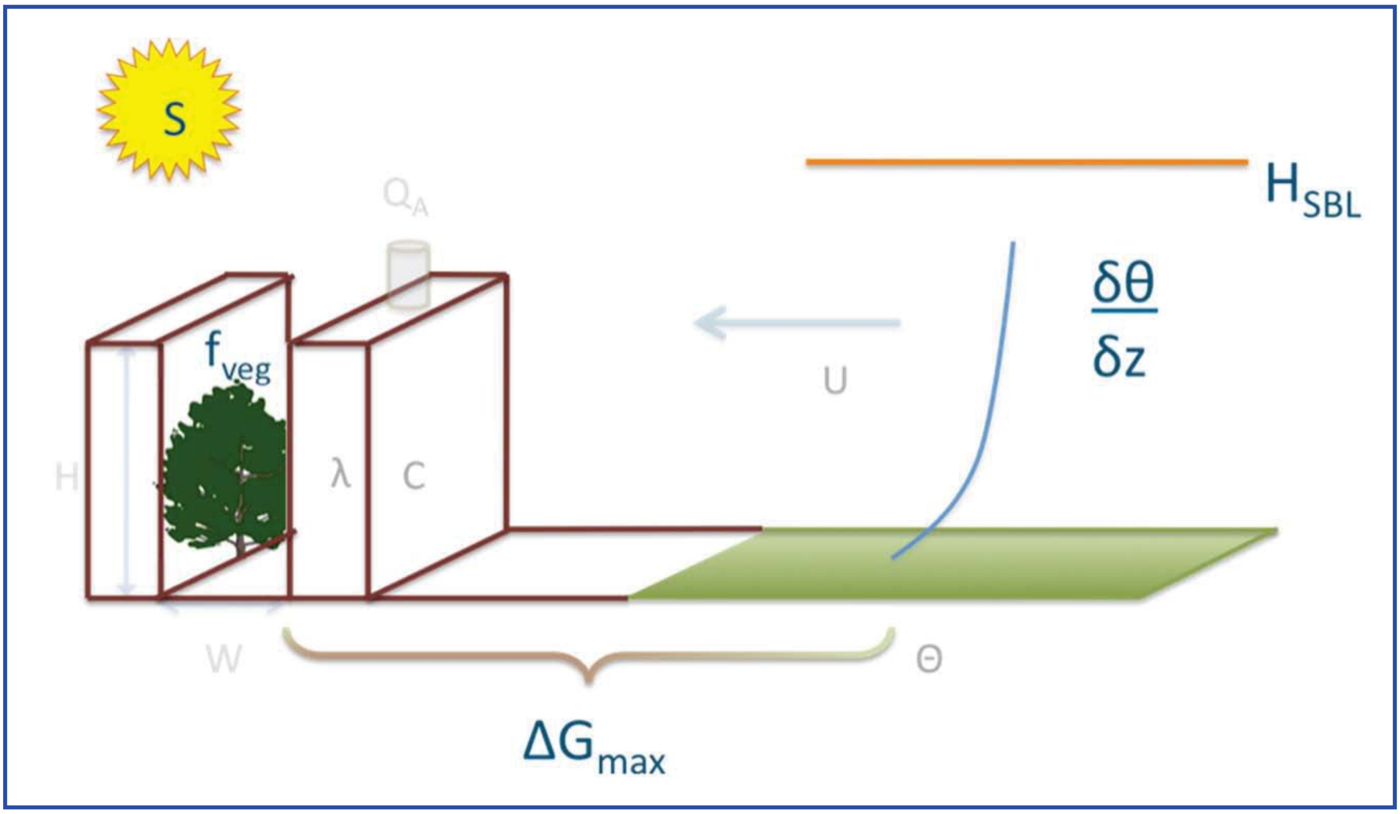

Figure 1. A schematic overview of the variables taken into account with the dimensional analysis.

Application to the UHI

We illustrate the described procedure for the UHI, using preliminary results obtained from output of the Weather Research and Forecasting (WRF) model (Skamarock *et al*., 2008) in single column mode, version 3.2.1. The model is coupled to the single-layer urban canopy model (Kusaka *et al*., 2001). As a reference for the UHI, a second column was run separately with surface properties of grassland. This column represents the rural environment. The MYJ planetary boundary-layer scheme was used.

The analyzed case has an academic setup, initialized with idealized profiles and large-scale forcings, i.e. with a uniform mixed layer temperature and specific humidity, a logarithmic wind speed profile, a temperature and humidity jump at the boundary-layer top, and a constant lapse rate above the boundary layer. This case was run while varying soil moisture content, geostrophic wind speed, vegetation fraction, anthropogenic heat flux, building heat capacity and thermal conductivity, season and latitude.

For example, six variables were selected from the WRF simulations to enter the dimensional analysis. These were the UHI [K] (in this case the difference in the *minimum* two meter air temperature between the urban and rural simulation), vegetation fraction (f_{veg}[-]), the maximum downwelling shortwave radiation (S_{max} [W m^{-2}]), the maximum difference in ground heat/storage flux between the urban and rural simulations (ΔG_{max} [W m^{-2}]), the lapse rate in the rural environment during the night ($d\theta/dz$ [K m^{-1}]) and the stable boundary-layer height (H_{SBL}[m]) (Fig. 1). Using these variables we are able create three dimensionless groups. The first group (Π_1) is a function of the vegetation fraction, which has often shown to be a strong governing parameter of the UHI (e.g. Steeneveld *et al*., 2011).

$$\Pi_1 = 1/(1+ f_{veg}) \qquad (1)$$

The physical rationale behind applying a transformed version of f_{veg} rather than f_{veg} itself, is to ensure well behaved limiting behavior of Π_1 for f_{veg}-> 0.

The second group (2) is the ratio of the absolute maxima of the two fluxes (shortwave and storage fluxes) and relates to the amount of energy stored within the buildings and pavement.

$$\Pi_2 = \Delta G_{max}/S_{max} \qquad (2)$$

The rationale behind this group is based on the idea that that the difference in heat storage capacity of the offered solar radiation by the land surface will be reflected in different cooling and cooling rates at night, and thus in the UHI.

The third group (3) includes the remaining variables, (UHI, stable boundary-layer height and lapse rate) and gives an indication of the stability of the atmospheric boundary layer.

$$\Pi_3 = \frac{\mathrm{UHI}}{(\partial\theta/\partial z)H_{SBL}} \quad (3)$$

This group is introduced in order to represent the state of the boundary layer in the rural surroundings. A stronger stability will limit the turbulent transport of relatively warm air from aloft to the surface, and thus maximizing the UHI. Note that the two variables in the denominator are connected, since a stronger stability will reduce H_{SBL} due to the suppressed turbulence.

Plotting these dimensionless groups as a function of each other, an approximately linear relation appears and an equation for the UHI can be formulated. This equation for the UHI is compared to the UHI calculated by the WRF model in Figure 2. Half of the dataset was used to derive the equation and the other half for validation. The derived model has a correlation coefficient to the one to one line of about 0.78.

The simple dimensional analysis described above gives a promising, preliminary result. However, these findings are only based on WRF model results, which has its own limitations. Furthermore, this is only one example with a limited dataset. In order to find whether or not there is a uniform equation for the UHI, a much larger dataset is needed, including field observations from different cities around the globe. Therefore, possible cooperation and contributions from field campaigns from the audience of *Urban Climate News* are highly welcomed and appreciated.

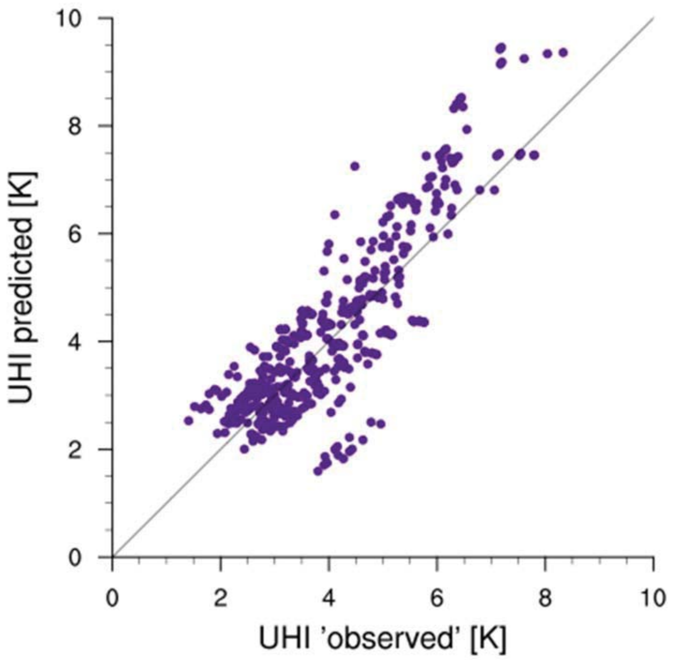

Figure 2. A scatter plot of the predicted UHI by the equation calculated using dimensional analysis and the UHI calculated by the WRF model. (r^2 = 0.78)

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Natalie Theeuwes

Natalie.Theeuwes@wur.nl

Meteorology and Air Quality Section, Wageningen University, Wageningen, Netherlands