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On quality equilibrium in continuous location-design competition with binary supplier choice

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On quality equilibrium in continuous location-design competition with binary supplier choice

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We focus on a market situation where two firms attempt to attract as many clients with their demand as possible by deciding on the location in the plane and investing in a competing quality against investment cost. Clients choose one of the suppliers; i.e. deterministic patronizing behavior. To study this situation, a game theoretic model is formulated. We show that for the modelled situation no Nash equilibrium exists. However, when one of the firms (the leader) is aware of what the other is going to do (follower) a so-called Stackelberg equilibrium exists. The questions under study are whether co-location is a natural phenomenon in this case and in which situation one of the firms will leave the market. The study requires a bi- or tri-level thinking where the decisions on location follow from the known quality investment behavior and the actions of the leader take the decisions of the follower into account.

Key words: Game Theory, location-design, leader-follower, Global Optimisation, competition

1. Introduction

In facility location competition, the main instrument is the choice of location. Since the first model of Hotelling (1929), many extensions have been studied in competitive location science where firms basically decide on locating one or more facilities at location(s) x trying to attract market share. The strategic choice of location can be complemented by tactical decisions a such as the price of the product in location-pricing or location-quantity competition (see Sáiz and Hendrix (2008)), the quality in location-design problems like the original Huff model (Huff (1964)) or capacities of the facilities. The objective for a firm depends for a large part on the market capture of demand often represented by so-called demand points. We first sketch the role of bi-level thinking and then describe the market situation we would like to analyse.

Thinking in terms of decisions on several levels may help to analyse a model and its behavior for various model studies such as one firm maximizing its profit, studying the existence of stable market situations (Nash equilibria) and finding so-called Stackelberg equilibria, where one firm is a leader and others react on that.

Specifically, we focus on the location-design problem where the objective $\Pi(x, a)$ is to capture market demand by attracting customers choosing location x and setting a quality a. Bi-level thinking allows us to analyse first the second level optimisation of Π over a when location x is fixed. In fact, literature (e.g.Fernández et al. (2007)) shows that for the classical Huff model with convex investment cost this implies a concave problem . Therefore the optimum quality level $a^*(x)$ is relatively easy to find. Substitution of the second level decision provides a first level problem $\max_x \Pi(x, a^*(x))$ in location space.

In Sáiz et al. (2011) the same model was used to derive Nash equilibria in continuous location-design where two competing firms enter a new market without active suppliers. The analysis provides explicit analytical expressions for the optimal (Nash) values of the quality to be set by the two competitors, facilitating studying the existence of location Nash equilibria on the first level. If customers choose for one of the suppliers only, i.e. deterministic patronizing behavior, the analytical study is hindered as the market capture depends in a discontinuous way on the location and quality decision. The situation we are interested in is when one of the suppliers knows how the other is going to react. This is called a leader-follower situation and a solution is called a Stackelberg equilibrium. The research questions are, whether such an equilibrium exists for deterministic customer behavior and what are its characteristics. Insight in the characteristics of this model facilitates predicting the behavior in the corresponding market situation.

An interesting aspect is that a Stackelberg problem already has a bi-level aspect even if the firms compete only by location. Sáiz et al. (2009) describes a Stackelberg location (without design) problem where a leader competitor locates one facility at x taking into account the reaction of a follower locating at y. The bi-level structure allows substitution of the solution $y^*(x)$ of a Global Optimization (GO) problem into a GO problem $\max_x \Pi(x, y^*(x))$. The elegance of the approach described in that paper is that it provides a guarantee to find the global optimum solution for both leader and follower.

The market situation to be analysed has both bi-level aspect; that of deciding on location and quality level and that of a leader taking the follower actopns into account. Two companies enter a new market and try to win as many customers with their demand (buying power) as possible by deciding on their location x and y, but also on their quality a and b. This defines an optimization problem in six dimensional space, where the leader has to take into account the optimal actions of the follower. The research question is to predict the behavior of both actors. For instance, is co-location a natural tendency in this market situation and under which circumstances is one of the firms going to retreat from the market? Sáiz et al. (2011) found that in probabilistic patronizing behavior the optimal (Nash) locations for each firm is found at a demand point. Is it natural in deterministic patronizing behavior that firms locate at a customer?

To investigate these questions, first a game theoretic model is defined in Section 2 describing the market situation. Section 3 then analyses the behavior on the level of the quality choice. Section 4 investigates several properties of the location behavior. Section 5 summarizes the findings on the research questions.

2. Model of a Stackelberg location-design market situation

To depict the situation, one can think of two competing news vendors or street bars located at x and y in a new neighbourhood that attract customers located in points p_i to come over every morning and to buy there desired newspaper or coffee. Customers may have a different buying power w_i although this does not influence the characteristics of the problem a lot. The analysis becomes more cumbersome when already several suppliers exist. To keep notation as simple as possible, we leave this aspect out at the moment, although tendencies may be similar. Like in the Huff model, the two competitors can also invest in their attractiveness by increasing their quality. The situation under investigation is that in the decision process, the leader supplier (firm 1), knows what the follower supplier (firm 2) is going to do. The question is what is the corresponding behavior of the two players. The following notation describes the situation formally:

Indices

i index of demand points, $i = 1, \ldots, n$

Variables

$x = (x_1, x_2)$	location of firm 1,	$y = (y_1, y_2)$	location of firm 2
a	quality facility firm 1,	b	quality facility firm 2

Data

p_i	location of the i-th demand point (customer)
w_i	demand (buying power) of customer i, $w_i > 0$
α, β	cost parameters for firm 1 and 2 respectively
S	location space, in fact the convex hull of the set of demand points

Miscellaneous

$W = \sum_i w_i$	total demand
$d_i(z)$	distance between p_i and $z = x$ or $z = y$
$c_1()$, $c_2()$	cost functions for firm 1 and firm 2 with respect to quality expressed in number of customers

Sáiz et al. (2011) describe the effect of a linear and quadratic investment cost function, but we consider the linear relation $c_1(a) = \alpha a$, $c_2(b) = \beta b$, where the coefficients are expressed in the same units as the demand. The patronizing behavior of the customers is to choose for one of the suppliers according to a Huff-like attractiveness $\frac{a}{d_i(x)}$ for firm 1 and $\frac{b}{d_i(y)}$ for firm 2. In our study we assume that the customer goes for the follower (firm 2) in case the values are the same; e.g. the firm 2 salesperson has the most pleasant smile. Formally, the market capture of the follower is given by

$$M(x, y, a, b) = \sum w_i, bd_i(x) \geq ad_i(y) \tag{1}$$

and $W - M$ demand goes to the leader (firm 1). The resulting objective functions are well defined. The profit function of firm 1 is given by

$$\Pi_1(x, y, a, b) = W - M(x, y, a, b) - c_1(a) \tag{2}$$

and for firm 2

$$\Pi_2(x, y, a, b) = M(x, y, a, b) - c_2(b). \tag{3}$$

Another assumption of the market situation is that if one of the firms manages to capture all the market against a nonnegative pofit, it will follow this strategy to make the other firm retreat from the market, after which no investment in quality is necessary anymore.

On the strategic location level (x, y), both firms have to take the optimal decision on the tactical level (a, b) into account. Moreover, the leader knows what the follower is going to do on the tactical level, $b^*(x, y, a)$ as well as on the strategical level: $y^*(x, a, b^*(x, y, a))$. This situation makes the supplier behavior hard to analyse. Thinking in more than one level helps to consider the situation.

We first focus on the question of a possible Nash and Stackelberg equilibria with respect to the choice of the quality of the facilities (a, b) given the facility locations (x, y). Then in Section 4, we focus on properties of location equilibria given the optimum levels on quality choice.

3. Equilibria on the level of the quality choice

Firm 2 is choosing its quality level b given the position of both competitors (x, y) and the quality a set by firm 1. It is convenient to order the advantage in location that the leader has expressed as the ratio $\frac{d_i(y)}{d_i(x)}$. The order of this ratio provides the order in which firm 2 is going to attract the customers on increasing the value for the quality b. A complication is that the relative distance $\frac{d_i(y)}{d_i(x)}$ of two or more customers i and j can be the same. Actually, analysing forward, it is in the interest of firm 2 that this takes place; increasing his quality he captures several customers at the same time.

Let $r_j(x, y), j = 1, .., t$ be the ordered values of $\frac{d_i(y)}{d_i(x)}$ from small to big, where customers i with the same relative distance are included in the same ratio value j, such that $t \leq n$. Notice that $r_t(x, y)$ can take the value ∞ if the leader is located at a demand point. We can write the total demand going to the the follower after capturing the customers corresponding to the first j relative distances as

$$m_j = \sum_{\{i | \frac{d_i(y)}{d_i(x)} \leq r_j(x,y)\}} w_i. \tag{4}$$

Notice that due to clustering the demand of the customers with the same relative distance, the series m_j is strictly increasing. Now we can rewrite the profit of firm 2 given the locations x, y and quality a as

$$\Pi_2(x, y, a, b) = m_k - \beta b, k = \max\{j | b \geq ar_j(x, y)\}. \tag{5}$$

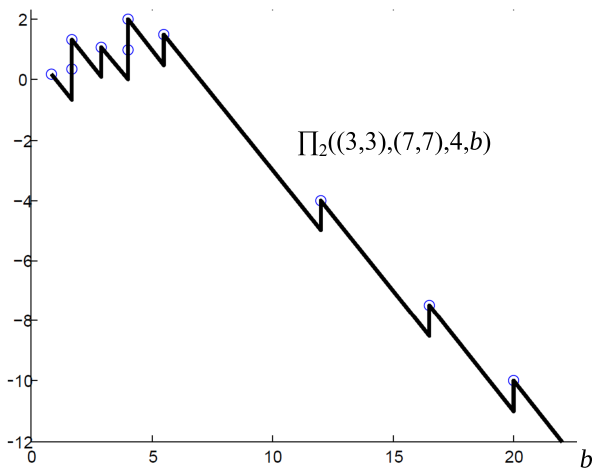

Figure 1 Profit of the follower with increasing quality b

Example 1. An instance consists of 10 customers located at $P = \{(1,4),(4,2),(5,8),(1,9),8,5),(7,4),(6,3),3,7),(8,8),(2,2)\}$ who each spend $w_i = 1$ unit each morning at their most attractive supplier. The leader is located at $x = (3,3)$ and the follower at $y = (7,7)$. The instance is designed such that 2 pairs of customers are at equal relative distance to the two suppliers, i.e. $t = 8$. Investing in quality costs the follower $\beta = 1$ units. The leader already invested $a = 4$ units in its quality. Figure 2 gives the development of Π_2 in (5) as function of b according to the model. The balls correspond to the attracted customers. Notice that following an increasing value of b, the figure reveals $t = 8$ peaks that correspond to the increasing values of r_j. The values of b for which Π_2 is discontinuous do not depend on β, the hight of the function Π_2 itself does. The follower obtains the maximum profit investing $b = 4$.

The figure illustrates several characteristics. Notice that Π_2 is discontinuous and its local maxima are not necessarily first increasing in b and then decreasing. Second, depending on the quality set by firm 1, a badly located firm 2 will decide to stay out of the market if he cannot attract the first client: $\beta a r_1(x,y) > m_1$.

The optimum choice in case of different locations is given in Proposition 1.

Proposition 1. *Given the described game with $r_j(x,y)$ as defined before, $r_0(x,y) = 0$, m_j defined by (4) and $m_0 = 0$. Let $x \neq y$ and quality a be given. The optimum quality investment for firm 2 is $b^* = ar_k(x,y)$ with $k = \arg\max_{j=0,\ldots,t}\{m_j - \beta a r_j(x,y)\}$ and corresponding profit $\Pi_2(x,y,a,b^*) = m_k - \beta a r_k(x,y)$ and market capture $M = m_k$.*

Given the ordered relative distances $r_j(x,y)$ and $r_0 = 0$, firm 2 requires a minimum quality level $b = ar_j(x,y)$ to attract customers with a relative distance up to $r_j(x,y)$. The corresponding profit is $m_j - \beta a r_j(x,y)$ over which the maximum over j should be taken. Notice that if this value is negative for $j = 1,\ldots,n$, the follower stays out of the market and chooses $b^* = 0 = ar_0$.

Despite the non symmetric relation, the re-definition facilitates the proof that there is no Nash equilibrium on the quality level.

Proposition 2. *Given the described game and locations $x \neq y$. No Nash equilibrium a, b exists on the quality level.*

By contradiction, let a,b be a Nash equilibrium with the firms located at $x \neq y$. This means a is optimal for firm 1 and b is optimal for firm 2. According to proposition 1 for firm 2, $\exists k,\ b = ar_k(x,y)$. Now firm 1 by increasing its quality to $a+\varepsilon$ for a value $0 < \varepsilon < \frac{m_k - m_{k-1}}{\alpha}$ increases its profit with $m_k - m_{k-1} - \alpha\varepsilon > 0$. This proofs that a is not an optimal quality for firm 1 and contradicts a,b to be a Nash equilibrium.

For a Stackelberg equilibrium, the line of the proof already shows what the leader (firm 1) should look for: an ε optimal solution. So, what is the optimal quality $a^*(x,y) = max_a \Pi_1(x,y,a,b^*(x,y,a))$ in a leader-follower situation? Let us first consider the co-location case.

Proposition 3. *Given the described game. Let $x = y \neq p_i\ \forall i$. No solution a,b exists where both firms stay in the market, i.e. co-location does not take place.*

In the co-location situation, $x = y$, all distances are the same for the two firms. This means that the follower gets all according to (1), if $b \geq a$ and nothing if $a > b$. The follower at most wants to invest $b = \frac{W}{\beta}$ to have a nonnegative profit. For $\alpha < \beta$ the leader can set his investment $a = \frac{2W}{\alpha+\beta} > \frac{W}{\beta}$ generating the positive profit $\Pi_1 = W - \alpha\frac{2W}{\alpha+\beta} = \frac{\beta-\alpha}{\alpha+\beta}W > 0$ pushing the follower out of the market. For $\beta \leq \alpha$ the follower can invest $b = \frac{W}{\beta}$ generating nonnegative profit $\Pi_2 = 0$ pushing the leader out of the market.

There is a condition in Proposition 3 that the suppliers should not be located at a demand point p_i In that case, the follower will get the demand of customer i, namely. To think about the higher level, therefore it is not in the interest of the leader to locate at a demand point if $\alpha < \beta$.

If both suppliers are not located at the same position, the leader has to maximize $W - M - \alpha a$, where M is defined in Proposition 1. This typically has the character of minimizing the damage caused by the follower:

$$\min_a (M + \alpha a) = \min_a \left(\max_j \{m_j - \beta r_j(x,y)a\} + \alpha a \right) \tag{6}$$

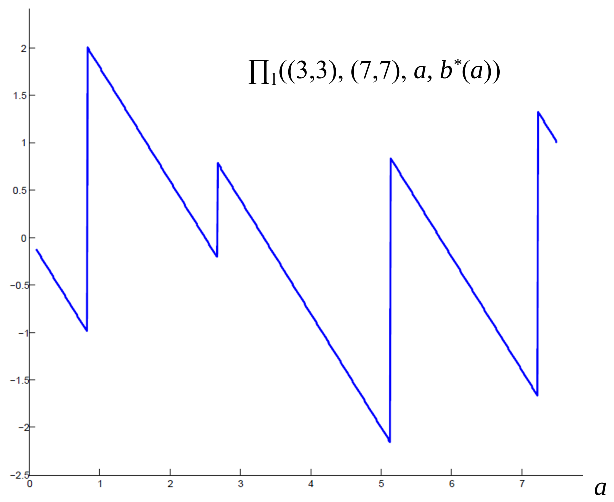

Figure 2 Profit of the leader with increasing quality a; the follower adapts its quality $b^*(a)$

Example 2. We consider the same instance as that of Example 1 with 10 customers, where the leader has an investment cost coefficient of $\alpha = 1.2 > \beta = 1$. Figure 2 sketches the development of Π_1

as function of a, where the follower continuously adapts its quality $b^*(x,y,a)$. The figure illustrates the multi-modal and discontinuous character of the profit function. Starting at $a=0$, the leader does not have any market capture and increasing quality a leads to negative profit up to the moment that the follower lets go three customers near $a=.83$. We will see where this approximate number comes from. Typically, if the leader increases its quality, then for certain values of quality a the follower pulls back from one or several customers. This provides a higher market share for the leader against a higher cost of investment in quality.
To study the optimal quality choice for the leader, one should first observe that with increasing quality a of the leader, the follower will never go for more customers. He will also increase the quality up to a certain level where he should let go. That "certain level", is exactly the candidate solution for the leader we are interested in.

Starting at $a=0$ where the leader does not attract any customer and where $M=m_t=W$, we study when the follower will give up customers with a relative distance higher than $t-k$. The loss in market share is m_t-m_{t-k}, but the gain in cost reduction is $\beta(r_t-r_{(t-k)})a$. This means that the follower will let go these customers and reduce investments if $a>\frac{m_t-m_{t-k}}{\beta(r_t-r_{t-k})}$. For increasing a, in the example this happens for $k=3$, where the ratio is .827. So, starting with a follower having $q=n$ customers, the first candidate optimal choice for the leader should be an epsilon solution:

$$a=\min_{k=1,..,q}\frac{m_p-m_{q-k}}{\beta(r_q-r_{(q-k)})}+\varepsilon, \tag{7}$$

where ε is an arbitrary small positive number. After finding the number of dropped customers k, one can find more candidates by repeating (7) iteratively setting $q:=q-k$ up to the follower has no more customers left over, $q=0$. The latter is the most interesting option for the leader as long as it provides a positive profit; he can push the follower out of the market. After generating the candidates for a via (7), one has to evaluate Π_1 for them to find the best quality level for the leader. This procedure is described in Algorithm 1.

Algorithm 1 Quality $(a^*,b^*)(x,y,P)$

Determine ordered distance ratios $r_j=\frac{d_j(y)}{d_j(x)}$ $j=1,\dots,t$
$q:=t,\ i:=0$
while $(q>0)$
  $i:=i+1$
  determine a_i according to (7)
  use the corresponding k to set $q:=q-k$
  $b_i:=a_ir_q$ (according to Proposition 1
endwhile
if $\Pi_1(x,y,a_i,b_i)\geq 0$
  $(a^*,b^*):=(a_i,0)$
else
  $k:=\arg\max_i \Pi_1(x,y,a_i,b_i)$
  $(a^*,b^*):=(a_k,b_k)$

Example 3. We consider the algorithm 1 for same instance as that of Example 1 with 10 customers and investment cost coefficients $\alpha=1.1,\beta=1$, locations $x=(3,3)$ and $y=(7,7)$. The algorithm generates the threshold values that can be recognized in Figure 2 being $a\in\{0.8275,2.6712,5.1302,7.2250\}$. Notice that the threshold values derived by (7) do not depend on the value of α, but of course the profit does. Although the highest profit of 2.0 is found at $a=0.8275$, the leader has the alternative to push the follower out of the market at $a = 7.22$ against a positive profit. He will do so, up to the follower disappears and he can reduce the quality investment cost. In the described game of the market situation, this does not mean the follower will disappear completely. As we will see, this means that the location $y = (7,7)$ is not optimal if firm 1 is located at $x = (3,3)$.

The analysis does not yield closed analytical expressions due to the typical discrete character of the problem. Are the results easily extendible? What happens if already several suppliers exist in the market? The follower simply has to focus on the most attractive competitor for customer i, who is not necessarily the leader. For the leader, expression (6) becomes less straightforward, as the other competitors also have to be taken into account.

Concluding, Stackelberg equilibria for the described market situation may occur in contrast to Nash equilibria as shown by Proposition 2 and Proposition 3. The latter result on co-location suggests that the follower can push the leader out of the market if $\beta \leq \alpha$. As the leader knows the follower is going to locate at the same place, he can never beat the follower and practically has to leave the market. The follower gets all.

4. Optimum location

In the former section, we already analysed the situation $\beta \leq \alpha$, i..e. a strong firm (easy access to financing an increase in quality) in a follower situation can push the leader out of the market. The model assumes that therefore the leader does not locate and firm 2 does not have to invest in quality. What happens if the leader is stronger in increasing quality at lower cost, $\alpha < \beta$? Two results follow nearly directly from the analysis. First, as discussed in Proposition 3 co-location (choosing $y = x$,) is not of interest to the follower as the leader can push the follower out of the market by increasing quality in a cheaper way. Second, the follower is always going to participate in the market.

Proposition 4. *Given the described game. The follower is always going to participate in the market and his optimal profit is at least* $\Pi_2(x, y^*, a, b^*) \geq \max_i w_i$.

Given a location x and quality choice a of the leader, a feasible solution of the follower is locating at $y = p_k$ where $k \in \arg\max_i w_i$ and $b = 0$. The corresponding profit is $\Pi_2 = \max_i w_i$, as he attracts customer k without any investment cost. So the follower has no reason to stay out of the market and will at least gain a value of $\max_i w_i$.

When we focus on the profit of the follower $\Pi_2(x, y, (a,b)^*(x,y))$ varying its location y when x is given, one can consider Proposition 4 in the following sense. For most of the area, the profit is 0, as the location is too bad compared to the one of the leader. However, at each demand point p_i, function Π_2 has at least a local maximum of w_i. This means than one can use this value as a bound to detect areas where the optimum cannot be.

Example 4. Consider firm 1 located at $x = (3,3)$ and $\beta = 1$, the 10 customers located at $P = \{(1,4),(4,2),(5,8),(1,9),8,5),(7,4),(6,3),3,7),(8,8),(2,2)\}$, $w_i = 1\ \forall i$. For $\alpha > \beta$, the profit of firm 2 has clearly a peak of $\Pi_2 = 10$ for $y = x$ and also local optima at the customers. These local optima are better visible for the case where the leader has easier access to investing captial, $\alpha < \beta$. For $\alpha = 0.9$, Figure 3 outlines the profit with a maximum of $\Pi_2 = 1.52$ attained near $y = (6.34, 3.96)$. At most of the surface, the follower is not able to capture demand against a low enough quality (profit is zero) and at the customer locations he attracts at least the buying power of one customer w_i without any additional cost.

The example illustrates that the optimum location of the follower is not necessarily at a demand point in contrast to what was found as a property of the equivalent Huff variant of the model in Sáiz et al. (2011). In most of the area, gradient information, i.e. in which direction does the objective improve, is lacking. Finding the best location can be done by a heuristic based on a finite number of function evaluations, like a grid search. Notice that given the solution of the quality level, we

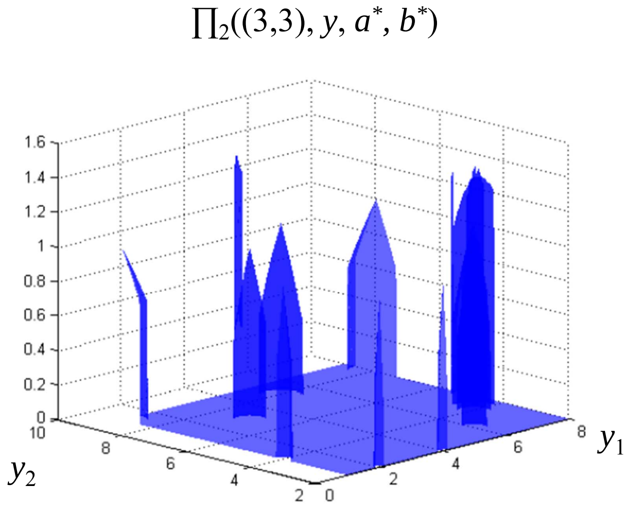

Figure 3 Profit of the follower as function of location y given $x=(3,3)$ and the optimal quality (a^*,b^*) as function of $x, y,\ \alpha=0.9, \beta=1$

are dealing with a problem in only two-dimensional space. Following a grid search procedure as is done to generate Figure 3, we can also get an impression of the location decision for the leader, substituting the level of the follower decision.

An interesting consequence of Proposition 4 is that the profit of the leader is also bounded.

Corollary 1. *Given the described game. If $\alpha<\beta$ the optimal profit of the leader is bounded by $0 \le \Pi_1(x,y,a,b) \le W - \max_i w_i$. For $\alpha \ge \beta$, the leader stays out of the market.*

Example 5. Consider the case of Example 4. For each location of the leader over a grid of 10000 points, the follower decision is evaluated by solving the quantity level and the best location is taken as an approximation of $y^*(x)$. The resulting profit function for the leader is depicted in Figure 4. One can observe very well, that outside the convex hull of the demand points the leader looses profit, as the follower can locate at a more profitable place. The optimum profit value is $\Pi_1 = 6.57$ at a location near $x=(3.66, 4.31)$ if the follower chooses as demand point $y=P_7=(1,9)$ gaining a profit of 1. The figure illustrates the discontinuous character of the objective function.

5. Conclusions and discussion

We analysed a market situation of two firms, firm 1 and firm 2, entering a new market where customers reveal a deterministic patronizing being attracted by low distance and high quality with a game theoretic model. In case of breaking tie, i.e. both firms are as attractive, the customer is going to firm 2. We investigated the behavior of the supplying firms with respect to Nash equilibrium, co-location tendency, tendency to locate at customer locations and strength to push the competitor out of the market.

We found the following results. No Nash equilibria exist for the described situation. Co-location does not occur, as one of the firms has the ability to push the other out of the market. If the second firm (firm 2) behaves as a follower, it will always enter the market with more tendency to locate at

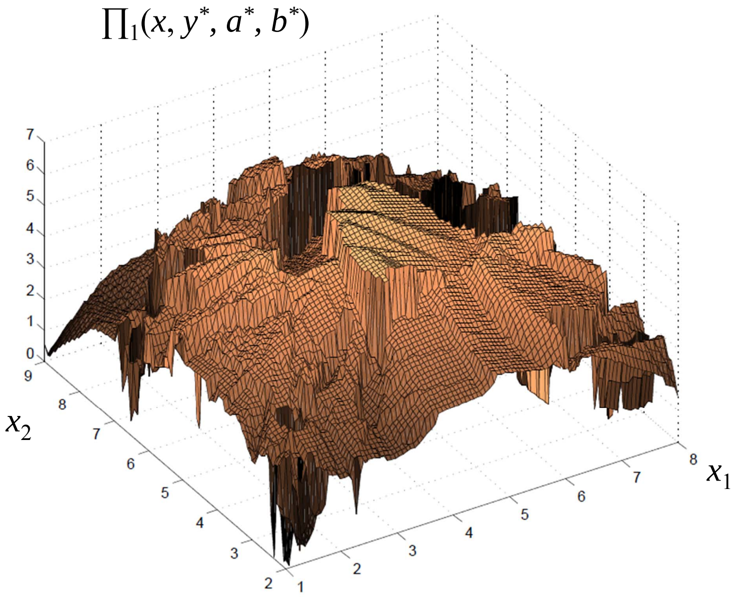

Figure 4 Profit of the leader as function of location x, the optimum follower location y^* and the optimal quality (a^*, b^*) as function of x, y^*, $\alpha = 0.9, \beta = 1$

a customer location (supplying only that customer) if its abilities to invest in quality is less strong. When its ability is stronger than that of the leader, it will push the leader (firm 1) out of the market. The leader (firm 1) does not have the tendency to locate at a customer.

A specific algorithm has been developed to determine the optimal quality of both firms for the case the investment cost coefficient of the leader is lower than that of the follower. The algorithm is based on sytematically following the order in which customers are taken from the competitor depending on the relative distance to both competitors.

The mathematical location problem when substituting the optimal quality levels is discontinuous and derivative information is lacking in most of the area. A heuristic procedure can be used to generate a good, but not necessarily optimal location for the leader and the follower.

Natural research questions following from this study is whether the tendency extends to situations where other suppliers are already in the market. A more mathematical question is what happens if the investment costs are not taken as linear, but as strictly convex, such that a firm has less tendency to temporarily taking a high quality cost to push the competitor out of the market.

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