

Futures Market Depth: A Price Pattern Model

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ABSTRACT

The lack of sufficient market depth in many newly initiated futures markets results in a relatively high cost of hedging, thereby inhibiting contract growth. In this article the underlying structure of futures market depth has been analyzed, from which a market depth measure has been derived. Understanding the underlying structure will provide the management of the exchange with a framework to improve their market depth, while hedgers will be able to better understand their market depth risk. The managerial implications of our findings have been demonstrated empirically.

A key aspect of futures market performance is the degree of liquidity in the market (Cuny, 1993). The relation between market depth and futures contract success has thoroughly been investigated in the literature (Black, 1986). The futures exchange management will be interested to improve their market depth. A futures market is considered liquid if traders and participants can quickly buy or sell futures contracts with little price effect as a consequence of their transactions. However, in thin markets, the transactions of individual hedgers may have a significant price effect and may therefore result in substantial 'transaction costs' (Thompson, Waller and Seibold, 1993).

These transaction costs are the premiums that traders are forced to pay or the discounts they are forced to accept in order to establish or close out futures positions (Ward and Behr, 1983). Although, to some extent, hedgers can take positions that offset each other, a futures market must normally attract more market depth, in the form of additional traders, to become truly successful. In the literature width is often synonym with liquidity costs and is represented by the bid-ask spread for a given number of futures (Berkman, 1993). The bid-ask spread as a measure of liquidity has some limitations. The price may change between the time at which the market maker buys and sells, and the trader may earn much more or less than the spread quoted at the time of the first transaction. Therefore, the currently quoted bid-ask spread is not a very precise measure of the cost of trading immediately compared to that when delaying the order, particularly when the order is large. Yet that cost is the essence of market liquidity (Grossman and Miller, 1988). The concept of market depth represents market liquidity without suffering from the limitations the bid-ask spread has in this respect. Market depth refers to the number of futures contracts that can be traded at given bid and ask quotes. We have developed a market depth price path model, which reveals the underlying structure of market depth. Subsequently a market depth measure have been derived.

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The article is organized as follows: Section I describes the concept of market depth. Section II gives a brief review of liquidity measures. Section III presents the underlying structure of market depth from which a market depth price path model is derived. The remainder of the article is concerned with an application. Section IV describes the dataset along with some data transformations. Section V presents the results of market depth for three selected futures contracts. The results are summarized in section VI which also presents our main conclusions.

I. Market Depth in Futures Markets

Kyle (1985) defines market depth as the volume of unanticipated order flows in order to move prices by one unit. Market depth risk is the risk the hedger faces when there is a sudden price fall or rise due to order imbalances; this risk seems extremely important to systematic hedgers. Such price changes may occur in the case of a long hedge as well as a short hedge. If a market selling (buying) order arrives, the transaction price will be the bid (ask) price. For a relatively large market selling (buying) order, several transaction prices are possible, at lower and lower (higher and higher) values, depending on the size of the order and the number of traders available. If the selling order is large, the price should keep falling to attract additional traders to take the other side of the order. Given a constant equilibrium price, a deeper market will be one, in which relatively large market orders produce a smaller divergence of transaction prices from the underlying equilibrium price. The price fall (rise) in the futures market is caused by the selling (buying) pressure in the futures market when large amounts of contracts enter at a specific point of time. According to Lippman and McCall (1986) the thickness of the market for a commodity increases with the frequency of offers. The generally known economic factors that determine market depth include: the amount of trading activity¹ or the time rate of transactions during the trading period; the ratio of trading activity by speculators and scalpers to overall trading activity; equilibrium or real price variability; the size of a market order (transaction); expiration-month effect and market structure² (Black, 1986, Thompson and Waller, 1988, Christie and Schultz, 1994, Chan and Lokanishok, 1995). Hasbrouck and Schwartz (1988) report the relation between market depth and the trading strategies that market participants apply. Passive participants wait for the contra-side of the market to arrive, but the active ones seek immediate transaction. Passive participants may avoid depth costs or may even profit from the execution costs that others have to pay, whereas active ones generally incur depth costs. Some exchanges monitor temporary order imbalances i.e. market depth risk, and slow down the trade process if these are present (Affleck-Graves, Hegde and Miller, 1994). For example, an order book official issues warning quotas when trade execution results in price changes that are larger than minimums predescribed by the exchange, and halt trading when order execution would result in price changes that exceed exchange-mandated maximums (Lehmann and Modest, 1994).

II. Liquidity Measures: A Brief Review

In previous research measures of liquidity have been suggested on the basis of indices, where most of these indices represent some weighting of trading activity (Working, 1960, Larson, 1961, Powers, 1979, Ward and Behr, 1974, Ward and Dasse, 1977). An important element in these measures is the proportion between the volumes generated by hedging and by speculation. In these measures market liquidity has been estimated in terms of trader composition and level of transactions. Several researchers (Roll, 1984, Gloston and Milgrom, 1985, Thompson and Waller, 1987, Stoll, 1989, Smith and Whaley, 1994) have proposed methods for indirect estimation of liquidity costs. Roll's measure is a frequently used measure of the bid-ask spread. The other accepted proxy for the bid ask-spread was proposed by Thompson, who suggested the average absolute value of price changes being a direct measure of the average execution cost of trading in a contract. Thompson and Waller (1988) report that the most appropriate measure of liquidity costs in commodity futures markets is the average of the absolute value of the price changes. Smith and Whaley (1994) use the method of moments estimator as a method to determine the bid-ask spread. This estimator uses all successive price changes in the time-and-sales price file. Observed futures transaction prices are assumed to occur at either a bid or an ask level, with equal probability.

Market depth measures are rather scarce, although, as already indicated, market depth takes the cost of trading immediately into account, this in contrast to the bid-ask spread. Brorsen (1989) used the standard deviation of the log price changes as a proxy for market depth. Lehmann and Modest (1994) studied market depth by examining the adjustment of quotas to trades and the utilization of the chui kehai trading mechanism, where the chui kehai are warning quotas when a portion of the trade are executed at different prices. Utilizing the chui kehai trading mechanism can give an indication of market depth, but it is not able to measure market depth in the sense of Kyle's (1985) definition. Other researchers such as Bessembinder and Seguin (1993) used price volatility and open interest as a proxy for market depth.

These market depth measures do not provide insight into the underlying structure of market depth. In our view, a market depth measure should not only measure the price change which is caused by the thinness of the market, but should also reveal the underlying price path. The measures reviewed do not reflect this price path, whereas it is this price path that provides insight into the underlying structure of market depth. We argue that it is necessary to recognize the underlying dimensions of market depth in order to compare markets with respect to market depth. Understanding these dimensions will provide a framework for the management of the futures exchange to improve their market depth, while the users of futures will be able to better understand their market depth risk and hence, take proper actions to avoid them (for example, using another futures contract or decide to enter the market in another period).

III. Market Depth Model

This section examines the underlying price path describing market depth. It is assumed that the price path will be downward-sloping in the case of a selling order imbalance and upward-sloping in the case of a buying order imbalance (Working, 1977, Kyle, 1985, Admati and Pfleiderer 1988, Bessembinder and Seguin, 1993). A model that describes market depth has been developed. The characteristic set of parameters describing the model can be interpreted as a measure of market depth.

Figure 1 depicts the price path of a large selling order in a thin market. The first contract is sold for the bid price quoted, but the following contracts are sold for less due to order imbalances. Small order differentials have underproportional and large order ones overproportional effects on the futures price. Note that this price path is caused by a lack of market depth, not by fundamental economic factors.

[INSERT FIGURE 1]

The market depth price path has been divided into four phases for both upward- and downward-sloping price paths. We will discuss these phases for a downward-sloping price path. In our mathematical model both upward- and downward-sloping price paths are examined.

A. Sustainable Phase (I)

In the first phase the first contracts will be sold at the bid price because of outstanding bids in the order book. Phase I could also be called the stock phase, the already existing bids are almost or completely equal to the first bid price. However, after these bids have been 'used', the price has to fall sharply in order to match the next bid in the order book; this point will be called 'breaking point'. If the price path depicted in Figure 1 is denoted by $f(i)$, then the breaking point is located where the curvature ($\frac{d^2 f}{di^2}(i)$) is maximized over i .

B. Lag Adjustment Phase (II)

In Figure 1 phase II, the so-called 'lag adjustment phase' is located between the breaking point and the point of inflexion, where the point of inflexion is located where the slope ($\frac{df}{di}(i)$) is maximized over i . In this phase it is not possible to find enough market depth at a justifiable price. The price will fall sharply because now the bids that have been in the order book for some time (and thus relatively low price bids) are matched. This gives rise to opportunity costs, gains forgone, because the hedger cannot execute the order at the first bid price (Wagner and Edwards,

1993). Important for the length of this interval and for how far the price will fall is the information acquisition by the market participants. Two dimensions of information acquisition are of interest in this respect: whether or not the information acquisition is endogenous and how accurate the private information is. In many models, price impacts are inversely related to market depth, either because market makers' inventory control costs decrease with trading frequency, or because asymmetric information costs are less high (Keim and Madhavan, 1995).

C. Restoring Phase (III)

During the 'lag adjustment phase' the traders will process the information about the price decrease and will be more inclined to enter the market when the price has fallen sufficiently. At that moment the 'restoring phase' begins. In this phase the prices will fall further but its speed decreases. Phase III is located between the point of inflexion and the point where the curvature is minimized over i . The participants do not expect the price to fall any further. However, should the price fall further, then the participants do not expect this price change to exceed the minimum tick size, since the speed of the price decrease is slowing down (Chordia and Subrahmanyam, 1995).

D. Recovery Phase (IV)

The recovery phase starts at the point where the curvature is minimized over i . At some point in the recovery phase the price has decreased so much, to the so-called resistance price level, that agents (speculators or scalpers) enter the market, because they expect to make a profit due to order imbalance. This will occur under the condition that:

$$PF_t \leq E_j[CP_T | I_t] \quad (1)$$

where PF_t is the futures price and $E_j[CP_T | I_t]$ the rational expectation of agent j concerning the cash price at maturity CP_T of the futures contract conditional on his information set I_t , i.e., the information set that contains all available information up to and including time t , necessary to obtain the optimal prediction of CP_T at time t , such that the rational expectation of the price of agent j is the largest of all other rational expectations of the participants. After this resistance price level the price will not decrease any more because the orders are now balanced.

The market depth price path is the result of the behavior of the participants in the market and the way the exchange is organized. The four phases indicate that the underlying structure of market depth can be described by an S-shaped curve. Note that these curves are independent of price changes caused by fundamental economic factors. The S-shaped price paths occur within fundamental price changes and are caused by market depth. In section V these S-shaped price paths are empirically identified.

The shape of individual depth market price paths is determined by situation-specific and futures contract-specific causal factors (Simon, 1989). A priori we do not assume symmetry with respect to the inverse S-shape of selling and buying orders i.e., we do not assume a downward-sloping price path to be exactly the reverse of an upward-sloping one. It is possible, for example, that there are many stop-loss buying orders and almost no stop-loss selling ones and vice versa, thus causing asymmetry. Nor do we assume the length of the four phases to be equal. In a market which, for example, is not able to absorb orders near the equilibrium price, phase I will become rudiment.

Note that we are not interested in the absolute price level, or changes in this, because these price changes are caused by fundamental economic factors. Vertical shifts of these S-shape curves are due to fundamental economic factors³.

E. Mathematical Specification of the Model

Having presented our conceptual model, we will elaborate on the mathematical model. In the model both selling and buying orders (downward- and upward-sloping price paths) are taken into account. An upward-sloping S-shape path may well be approximated by the Gompertz curve, which has a non-symmetrical S-shape and hence, does not impose certain restrictions on the length of the different phases previously described (Franses, 1994a). The Gompertz model is a growth curve and can therefore only be used to describe an upward-sloping price path. However, subtracting the Gompertz curve from an appropriate constant may establish a downward-sloping S-shaped curve which will cover the four phases. Whereas the parameters of interest in the Gompertz growth model can be estimated by standard regression techniques, as will be shown below, the unknown parameters of such a downward-sloping Gompertz curve are difficult to estimate. However, we propose a simple procedure to estimate both the parameters of interest, characterizing both the upward- and downward-sloping curves, by simply subtracting the downward-moving prices from an appropriate constant. As a consequence, after the data transformation, the price path will always be upward-sloping. Furthermore, in order to abstract from fundamental economic factors that determine the general price level, the curve starts at a price transformed to be equal to the minimum tick size, since we assume that each curve ends in an area where the price change is slowing down, such that the participants are continually expecting the price change to alter from sign. If not, it should not be greater than the minimum tick size. In section IV and the appendix more details are given on how the prices have been transformed. Given these transformations, we can describe the transformed price series using the Gompertz model given by

$$TPF_i = \alpha \exp(-\beta \exp(-\delta i)) \quad (2)$$

where TPF_i is the transformed price of the futures contract i ($i = 0, 1, 2, \dots, n$), such that TPF_0 is equal to the minimum tick size, and α , β and δ are positive parameters. The parameters of the Gompertz model represent three dimensions of market depth. The first dimension, represented by parameter α , indicates the distance between the upper and lower bounds, i.e. it indicates how far the price rises (falls) as a consequence of market depth. The second dimension, presented by parameter β , indicates the amount of futures that can be traded at a price near the equilibrium before the price changes dramatically. The third dimension, presented by parameter δ , has a one to one relation to the speed of adjustment rate, which, as we will show below, can be given by $[1 - \exp(-\delta)]$, cf. Chow (1967) and Franses (1994a,b).

We propose to consider the Gompertz curve, i.e. the three parameters which characterize the Gompertz curve, a measure of market depth. As will be shown in section V, a graphical representation of this measure will generate valuable information about the underlying structure of market depth.

Taking natural logarithms (2) yields

$$\ln(TPF_i) = \ln\alpha - \beta \exp(-\delta i) \quad (3)$$

A convenient representation of the Gompertz process is obtained by subtracting TPF_{i-1} from (3) which gives after some rewriting using (2)

$$D\ln(TPF_i) = [1 - \exp(-\delta)] [\ln\alpha - \ln(TPF_{i-1})] \quad (4)$$

where D is the first order differencing filter defined by $Dz_i = z_i - z_{i-1}$. Equation (4) is of particular interest because it can be interpreted as a partial price adjustment model. To be able to see this, notice that $0 < [1 - \exp(-\delta)] < 1$. As a consequence, although α will always exceed TPF_i , $\ln(TPF_i)$ is rising toward $\ln\alpha$ at a constant speed of adjustment rate $[1 - \exp(-\delta)]$. For instance, if $[1 - \exp(-\delta)] = 0.1$, then it will take much more contracts to reach a certain price rise than in the situation where $[1 - \exp(-\delta)] = 0.5$. Similarly, if $\ln\alpha$ exceeds $\ln(TPF_i)$ by one percent of $\ln(TPF_i)$, then $\ln(TPF_i)$ will increase by $[1 - \exp(-\delta)] \times 100$ percent. We can also interpret $\exp(-\delta)$ as the elasticity of TPF_i with respect to TPF_{i-1} .

The model in (4) may be extended on two fronts. First, the Gompertz curve is an approximation to the transformed price series. Hence, we add a disturbance term u_i to (2) as follows

$$TPF_i = \alpha \exp(-\beta \exp(-\delta i)) \exp(u_i) \quad (5)$$

where $u_i \sim \text{IID}(0, \sigma^2 I_n)$. We assume traders to form rational expectations with respect to TPF_i conditional on their information set Φ_{i-1} . Because $E(u_{i+1} | \Phi_{i+1}) = u_{i+1}$, whereas $E(u_i | \Phi_{i+1}) = 0$, it can easily be inferred that the disturbance term of (4) is given by adding u_i to the right-hand side. Second, notice that the price observations per futures contract cannot be described by a single curve like the one depicted in Figure 1, but by a sequence of such curves such that an upward-sloping curve is always succeeded by a downward-sloping one and the other way round. As a consequence, our data series on the transformed price consists of a (not restricted to being balanced) panel of upward-sloping curves in chronological order. Although we have transformed all decreasing prices into rising ones, we have kept in mind which ones were really upward-sloping and which curves were actually (i.e., before transformation) downward-sloping. We may wish to allow the speed of adjustment parameter to vary between really upward- and actually downward-sloping curves. To meet those ends, (4) is modified as in

$$D\ln(TPF_{it}) = \pi_s - \tau_s \ln(TPF_{it-1}) + u_{it} \quad (6)$$

where $\pi_s = [1 - \exp(-\delta_s)] \ln\alpha_s$, $\tau_s = [1 - \exp(-\delta_s)]$, $i = 0, 1, \dots, n_t$ with $c = 1, \dots, H$ and $s = 1, 2$. H denotes the number of curves that forms the graph of the futures price observed per contract. Each time a new curve starts, s switches from 1 to 2 or the other way round, allowing the parameters to differ between really upward-sloping curves and actually downward-sloping ones. Hence, although the parameters of the Gompertz curve may differ between really upward-sloping and actually downward-sloping curves, all really upward-sloping are described by one Gompertz model and all actually downward sloping curves are described by one Gompertz model. We call these models the characteristic Gompertz curves. Notice that our dataset on TPF_{it} consists of $N = \sum_{i=1}^H n_i$ observations. In the next section more details are given on how we obtained these observations. Finally, we assume that $u \sim \text{IID}(0, \sigma^2 I_N)$.

IV. Data and Data Transformation

We have applied our model to data of the Amsterdam Agricultural Futures Exchange (ATA). This exchange is one of the largest agricultural futures exchanges in Europe. The trading system the ATA employs is the open outcry system. There are no scalpers on the trading floor; all orders enter the trading floor via the brokers. The order book plays an important (informational) role. These two characteristics, different markets with respect to market depth and an open outcry without scalpers, made it very interesting to apply our model to data from the ATA. On the ATA potatoes and hogs are traded. The potato futures contract is a relatively successful one in the sense that the volume generated is large relative to competitive potato contracts in Europe, although the annual volume is small compared with agricultural futures traded in the United States. The hogs are not successful regarding their

volume. With the help of transaction-specific data we will have applied our model, which is described below.

We used real time transaction-specific data for three futures contracts: potato contract delivery April 1995, hog contract deliveries August 1995 and September 1995. The average price path length in our sample of the potato futures was 20 contracts, for hogs delivery August 9 contracts and for hogs delivery September 11 contracts.

Our model is a growth curve and therefore it is an appropriate model for describing the price path in case of a selling order. However, the price path in case of a buying order may follow a reverse path. In order to maintain the linearization derived in section III, we had to identify increasing and decreasing price paths in our data. From the data it did not become clear where to identify the exact split between an increasing and decreasing price path when prices are constant for several contracts. Therefore, we followed the following procedure: for an odd number intersecting contracts we used the middle contract, and for an even number of constant contracts a random assignment with equal probabilities was used to determine the split. As soon as the curves were identified, we subtracted the observations of the downward-sloping ones from a curve-specific constant, such that all curves became upward-sloping and finally, we shifted the curves downward, such that each curve started at the minimum tick size. The reader is referred to the appendix for a formal explanation of the data transformation.

V. Empirical Results

In this section we present the estimates of the parameters of our model that were obtained simply by applying ordinary-least squares to (6). From these estimates and the fact that $TPF_0 = \alpha \exp(-\beta)$ is the known minimum tick size, we could simply derive the parameter estimates of the characteristic Gompertz curves. In the case of potatoes the minimum tick size was equal to 0.10 Dutch guilders and for hogs the minimum tick size amounted to 0.005 Dutch guilders.

The regression results for the potato futures contracts are presented in Table I. In Table IV we present the companion parameter estimates of the characteristic Gompertz curves to be discussed later on in this section. In Table I we can see that the estimates of the speed of adjustment parameter are equal to 0.051 and 0.059. Notice that these estimates lie within the (0,1) interval, which is in accordance with our model. The values of the corresponding *t* statistics are high, and are also highly significant when compared with the percentiles that should be considered in the context of Dickey-Fuller tests applied to time series, see e.g. Stewart (1991: 200-203) and Table 8.5.2 in Fuller (1976: 373). In addition, the result for the Durbin-Watson statistic does not indicate any misspecification at all. Finally, the low value of the R^2 also excludes the possibility that the relationship between the dependent and independent variables in Table I is spurious, cf. Granger and Newbold (1974). In spite of its low value, the R^2 is significantly different from zero, as indicated by the *F*

statistic. The intercept of the actually downward-sloping curves is, however, not significant. Nevertheless, according to Table IV this does not lead to insignificance of the characteristic Gompertz curve parameters. To see whether one single market depth path for potatoes suffices, we tested the restriction that the first two coefficients and the last two in Table I were equal. We found that we had to reject this hypothesis. Therefore, the market depth for potato futures contracts, delivery April 1995, significantly differs between periods of price rise and price fall.

Table I
Regression Results for Potato Futures Delivery April 1995

Dependent variable is $\text{Dln}(TPF_{it})$

Coefficient	Estimate	Standard error	t-value	Prob > t
π_1	0.016	0.002	6.621	0.000
π_2	0.001	0.003	0.253	0.800
τ_1	0.051	0.002	31.805	0.000
τ_2	0.059	0.003	29.992	0.000

Degrees of freedom 46786, from 46790 observations

R^2 0.099

$F(4,46786)$ 1283

Probability of *F* 0.000

Durbin-Watson statistic 1.914

Table II presents the regression results for the hog futures contract, delivery August 1995. Since the hypothesis $H_0: \pi_1 = \pi_2 = \pi$ and $\tau_1 = \tau_2 = \tau$ could not be rejected, the market depth for hog futures, delivery August 1995, is characterized by a single characteristic Gompertz curve, the parameter estimates of which are to be found in Table IV. Compared with Table I, the other statistics in Table II lead to similar conclusions with respect to the performance of the regression.

Table II

Regressions Results for Hogs Futures Delivery August 1995

Dependent variable is $\ln(TPF_{it})$				
Coefficient	Estimate	Standard error	t-value	Prob > t
π	-0.478	0.035	-13.813	0.000
τ	0.147	0.008	18.646	0.000
Degrees of freedom 2739, from 2741 observations				
R^2	0.249			
F(2,2739)	454			
Probability of F	0.000			
Durbin-Watson statistic	1.811			

Table III shows the estimates regarding the market for hog futures contracts, delivery September 1995. The results are quite similar to those in Table II. Again, we could not reject the hypothesis $H_0: \pi_1 = \pi_2 = \pi$ and $\tau_1 = \tau_2 = \tau$. The characteristic Gompertz curve parameter estimates that could be derived from the parameter estimates in Table III are presented in Table IV.

Table III

Regressions Results for Hogs Futures Delivery September 1995

Dependent variable is $\ln(TPF_{it})$				
Coefficient	Estimate	Standard error	t-value	Prob > t
π	-0.339	0.032	-10.581	0.000
τ	0.108	0.007	14.750	0.000
Degrees of freedom 2314, from 2316 observations				
R^2	0.200			
F(2,2314)	288			
Probability of F	0.000			
Durbin-Watson statistic	1.855			

Table IV
Estimates of the Characteristic Gompertz Curves Describing Market Depth

markets	slope of actual curves	parameter estimates characteristic Gompertz curve ⁽¹⁾		
		α	β	δ
Potatoes futures contracts, delivery April 1995	negative	1.013 (0.003)	2.316 (0.053)	0.060 (0.002)
	positive	1.374 (0.012)	2.621 (0.055)	0.053 (0.002)
Hogs futures contracts, delivery August 1995	both negative and positive	0.039 (0.008)	2.048 (0.072)	0.159 (0.007)
Hogs futures contracts, delivery September 1995	both negative and positive	0.044 (0.011)	2.166 (0.098)	0.115 (0.007)

⁽¹⁾ standard errors in parentheses

We have now come to discuss the parameter estimates of the characteristic Gompertz curves, which have been reported in Table IV. All parameters are highly significant. In order to make the interpretation of the parameter estimates easier, we substituted them in (2), by which we obtained the graphs in Figures 2 and 3 that, in fact, visualize our market depth measure.

[INSERT FIGURES 2 AND 3]

Our model offers information about the underlying structure of market depth presented by the three dimensions characterizing the S-shaped market depth price path: 1) the distance between the upper and lower bounds, i.e. how far the price falls (rises) due to insufficient market depth, 2) the amount of futures that can be traded at a price near the equilibrium before the price changes dramatically and 3) the speed of adjustment rate. Insight into market depth structure will provide the user of futures and the management of the exchange with valuable information about how to improve their performance in operating at, respectively managing of, the futures exchange. The application of our model to potato and hogs futures has illustrated these points.

From Figures 2 and 3 and Table IV the following conclusions can be drawn. The upward- and downward-sloping Gompertz curves for the potato futures have dissimilar shapes. The distances between the upper and lower bounds (dimension 1) of the upward-sloping price path and of the downward-sloping price path are not equal. The amount of futures that can be traded at almost the equilibrium price (dimension 2) is larger for the upward-sloping price path than for the downward-sloping one, while the speed of the price change (dimension 3) is higher for former. The upward- and downward-sloping Gompertz curves are similar for both hogs deliveries August and September. The main difference between the two hogs futures with respect to market depth is caused by dimension 3. The speed of price change is significantly higher for hogs delivery August than for that of September. The distance between the upper and lower bounds, indicating how far the price falls or rises due to order imbalances, is larger for the potato futures contract than for both hogs futures contracts. This can be explained by the fact that the participants in the hog markets are relatively large firms (meat packers and mixed feed enterprises) which are able to form accurate expectations quickly with respect to the resistance price level and to recognize the order imbalances. This is in contrast with the potato market, which is characterized by relatively small firms (farmers and private potato cash traders). Note that the ATA has no scalpers on the floor who could absorb temporary order imbalances and that brokers are allowed only to trade by order of a customer. The presence of scalpers does not only have an impact on dimension 1 but also on dimensions 2 and 3. The absorption power of all three futures markets is small, as can be seen in Figures 2 and 3, which show that phase I of the price path is only present in the potato market. This could indicate that in the case of potatoes there is a larger stock of bids and asks available in the order book than in the case of hogs. However, the speed of price change is much higher in the case of hogs than in that of potatoes. So when the bids and asks in the order book are used and the price is going

to change because of market depth, then the speed of adjustment rate (dimension 3) is higher for hogs futures market than for the potato futures market.

From these findings we may derive the following managerial implications for the participants in the futures market. A participant who wishes to trade contracts should recognize that the potato futures market is better able to absorb the order than the hogs futures market. Also the speed of adjustment rate is lower for the potato market than for the hogs market. However, the price fall or rise in the potato market will last longer. In sum, the potato futures market is better able to absorb order imbalances and is better in slowing down the price fall or rise than the hogs futures market. Conversely, depth price changes are more persistent in the potato futures market than in the hogs futures market.

Our empirical results have illustrated how our model can provide valuable information for the futures exchange management. By revealing the underlying structure of market depth our model supplies the exchange some clues on how to improve market depth. Since there are no scalpers at the ATA who are able to absorb temporary order imbalances, the first phase is almost not present in the futures market of the three futures. In order to improve the absorption capacity the ATA might consider to allow scalpers on the floor. The problem of the high speed of (adverse) price changes in the hogs markets might be solved by implementing a mechanism for slowing down the trade process if order imbalances do occur and to attract market depth by reporting these. Also the order book information can be improved. At the ATA the order books of the different brokers are not linked and the customer has no information with regard to outstanding orders. An order book mechanism that allows potential participants to view real time limit orders, displaying the desired prices and quantities at which participants would like to trade, affects the speed of adjustment rate and the distance between the lower and upper bounds, because participants would now be able to make well-founded decisions with respect to the resistance price level.

VI. Summary and Conclusions

In contrast to the existing market depth measures we showed that the market price depth path has an S-shape in which four phases can be distinguished: the sustainable price phase, the lag adjustment phase, the restoring phase and the recovery phase. The S-shaped price path may well be approximated by the Gompertz curve, which has a non-symmetrical S-shape and hence, does not impose certain restrictions on the length of the different phases. The three parameters of our model represent the three dimensions of market depth. The first dimension represents the distance between the upper and lower bounds, i.e. it indicates how far the price falls (rises) due to market depth. The second dimension indicates the amount of futures that can be traded at a price near the equilibrium before the price sharply falls or rises. The third dimension indicates the speed of the price fall or rise. Our market depth measure has convenient characteristics. First, it can be estimated simply. Second, it provides insight

into the underlying structure of market depth and the management of the futures exchange with guidelines to improve market depth. Third, our measure can be used to compare different futures contracts. Fourth, the measure, which can be presented in a graphical way, is relatively easy to interpret.

The empirical results of applying our model to the Dutch potato and hogs futures showed that the characteristic parameters of the model are statistically significant. We found that market depth for potato futures contracts, delivery April 1995, differs significantly between periods with price rise and periods with price fall. This is in contrast to the hogs futures. It appeared that the absorption capacity of orders for the potato futures market is larger than for the hogs. Also the speed of the price fall and rise is lower for potatoes than for hogs. However, the distance between the upper and lower bounds is greater for potatoes. These results are in line with the better performance of the potato futures market with respect to the trading volume compared with the hogs futures market.

Further research, in which our measure is applied to different kinds of futures exchanges regarding their trading system and underlying products in order to find the relation between the dimensions of market depth and the futures exchange characteristics, is clearly called for.

Appendix

In order to maintain the linearization derived in section III, we transformed the price data as follows: The transformation consists of two stages. In the first stage the curves are identified, in the second stage each curve is shifted downward such that the first observation of each curve is equal to the minimum tick size.

First stage:

Let PF_i denote the actual (i.e., before transformation) price of contract i . Here, $i=1, \dots, N$, where N is defined in the last paragraph of section III. If we have a sequence of prices according to $PF_i < PF_{i+1} > PF_{i+2} > \dots > PF_{i+h} < PF_{i+h+1}$, then the transformed price series becomes $PF_i, PF_{i+1} \mid PF_{i+h}, (PF_{i+1}-PF_{i+2}+PF_{i+h}), (PF_{i+1}-PF_{i+3}+PF_{i+h}), \dots, (PF_{i+1}-PF_{i+h-1}+PF_{i+h}), PF_{i+1} \mid PF_{i+h}, PF_{i+h+1}$. Here, the transformed price of contract $i+2$ equals $(PF_{i+1}-PF_{i+2}+PF_{i+h})$ and the notation $z_i, z_{i+1} \mid z', z_{i+2}$ implies that if one take first differences, then $Dz_{i+1} = z_{i+1} - z_i$ and $Dz_{i+2} = z_{i+2} - z'$. The following explanation applies. First, we consider the price of contract $i+1$ to be the last observation of the upward-sloping curve as well as the current upper bound from which we subtract the falling prices $PF_{i+2}, \dots, PF_{i+h}$. As soon as these falling prices reach a minimum, here PF_{i+h} , this minimum price is assumed to be the current lower bound. These upper and lower bounds are used to transform the downward-sloping curve that starts with PF_{i+1} and ends with PF_{i+h} in an upward-sloping curve, such that both curves are the mirror images of each other with reference to the horizontal line drawn in the middle of the upper and lower bounds. The first observation of the upward-sloping curve is equal to $(PF_{i+1}-PF_{i+1}+PF_{i+h}) = PF_{i+h}$, which is the current lower bound. The last observation of the upward-sloping curve is equal to $(PF_{i+1}-PF_{i+h}+PF_{i+h}) = PF_{i+1}$, which is the current upper bound.

Now let us consider the following sequence of prices: $PF_i < PF_{i+1} = PF_{i+2} > PF_{i+3} > \dots > PF_{i+h} < PF_{i+h+1}$. Here, we let a pseudo-random number generator decide which price, PF_{i+1} or PF_{i+2} , is both the last observation of the upward-sloping curve and the first observation of the downward-sloping curve. If PF_{i+1} results, then the transformed price series becomes $PF_i, PF_{i+1} \mid PF_{i+h}, PF_{i+h}, (PF_{i+1}-PF_{i+3}+PF_{i+h}), \dots, PF_{i+1}, PF_{i+h+1}$. Conversely, if PF_{i+2} is selected, then the transformed price series is $PF_i, PF_{i+1}, PF_{i+2} \mid PF_{i+h}, (PF_{i+2}-PF_{i+3}+PF_{i+h}), \dots, PF_{i+2}, PF_{i+h+1}$. In case of the sequence $PF_i < PF_{i+2} = PF_{i+3} = PF_{i+4} > PF_{i+5}$ we consider PF_{i+3} to be the turning price, whereas in presence of the sequence $PF_i < PF_{i+2} = PF_{i+3} = PF_{i+4} = PF_{i+5} > PF_{i+6}$ the pseudo-random number generator assigns whether PF_{i+3} or PF_{i+4} becomes the turning price, etc. We use the same procedure to determine the turning price in case of a minimum.

Second stage:

We assume that each curve ends in an area where the price change is slowing down such that the participants are continually expecting that the price change will alter from sign and if not, that it will not be greater than the minimum tick size. As a consequence, in order to let the Gompertz curve parameter estimates be independent

of the price level, we may shift each price curve downward such that it will start with the minimum tick size. To illustrate how the data of the price regressors in (6) are finally constructed, consider the transformed price series of the first example we gave and let us denote the curve identified in this example by c. Consider also equation (6). Then, $D\ln(TPF_{c1}) = \ln(PF_{i+1} - PF_{i+2} + PF_{i+h} - (PF_{i+h} - M)) - \ln(PF_{i+h} - (PF_{i+h} - M))$ which is related to a constant and the variable $\ln(TPF_{c0}) = M$, where M denotes the minimum tick size. Next, $D\ln(TPF_{c2}) = \ln(PF_{i+1} - PF_{i+2} + PF_{i+h} - (PF_{i+h} - M)) - \ln(PF_{i+1} - PF_{i+2} + PF_{i+h} - (PF_{i+h} - M))$ which is related to a constant and the variable $\ln(TPF_{c1}) = \ln(PF_{i+1} - PF_{i+2} + PF_{i+h} - (PF_{i+h} - M))$, etc. This shows how we constructed the regressors in (6).

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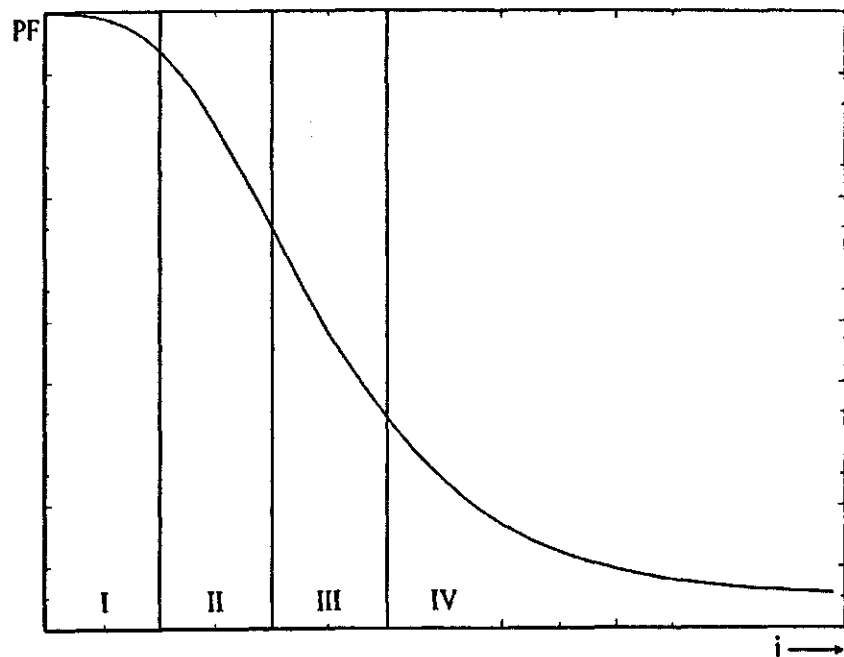


Figure 1. Price pattern of a selling order in a thin market.

Figure 1 depicts the price path of a selling order. On the vertical axis the futures price per contract traded is given where PF is the price realized when entering the futures market, i gives the serial number. On the horizontal axis the serial number of the futures contract is given.

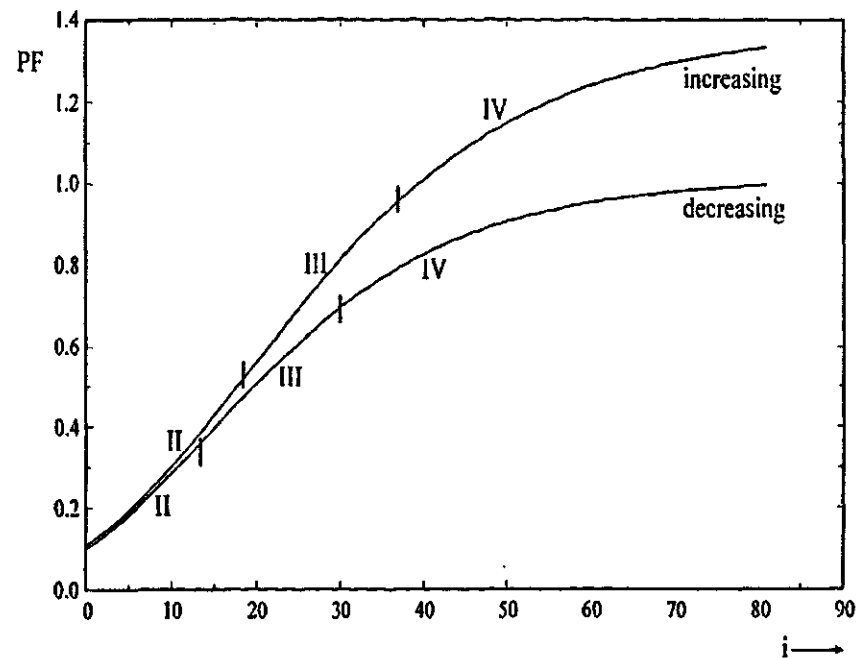


Figure 2. The Gompertz model for the potato futures contract.

The figure depicts the Gompertz curve for the upward-sloping and downward-sloping price paths.

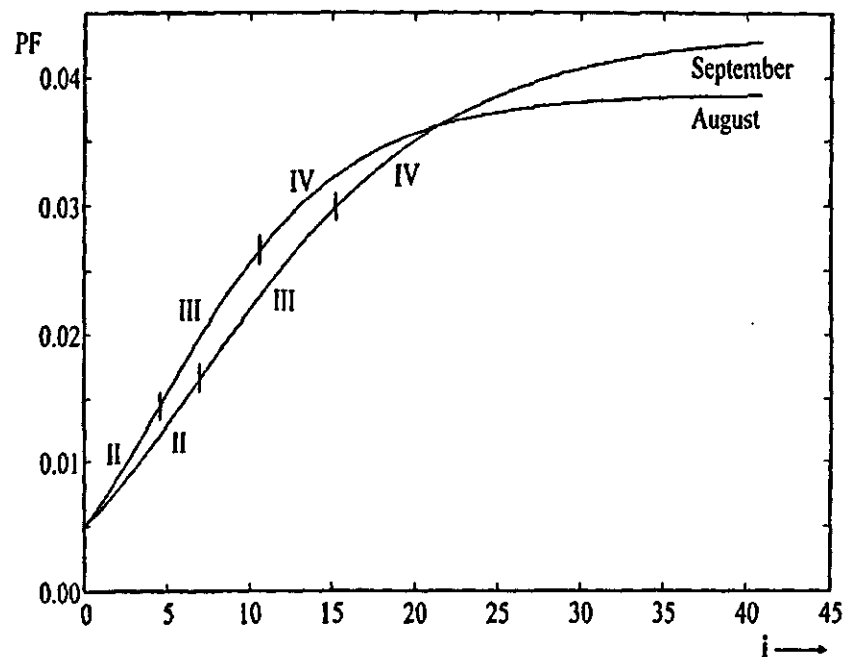


Figure 3. The Gompertz model for hogs futures contracts deliveries August and September.

The figure depicts the Gompertz curves for both hogs delivery August and Hogs delivery September. No distinction is made between upward- and downward-sloping price paths because, as already indicated, they can be described by the same Gompertz curve.

1. In the literature trading activity is often used as an indicator for market liquidity. However, Park and Sarkar (1994) showed that, in the case of the S&P 500 index futures contract, changes in trading activity levels may be a poor indicator of changes in market liquidity.

2. List does not pretend to be exhaustive

3. Note that during the selling or purchase of the futures contracts, which normally takes place within several minutes, the equilibrium price is constant and that the price changes are caused by market depth factors.