The hillslope-storage Boussinesq model for variable bedrock slope

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Abstract
The recently introduced hillslope-storage Boussinesq (hsB) model is cast in a generalized formulation enabling the model to handle a locally varying bedrock slope. This generalization extends the analysis of hydrological behaviour to hillslopes of arbitrary geometrical shape, including hillslopes having curved profile shapes. The generalized hsB model performance for a free drainage scenario is evaluated by comparison to a full three-dimensional Richards equation (RE) based model. In addition, comparison of both models to a storage based kinematic wave (KW) model enables us to assess the relative importance of diffusion processes for different hillslope shapes, and to analyse the influence of profile curvature on storage and flow patterns specifically.

Model development
The mass balance equation for describing subsurface flow along a unit-width hillslope reads:

$$\frac{\partial}{\partial t}(fh) = -\frac{\partial q}{\partial x} + N \qquad (1)$$

where h = h(x,t) is the elevation of the groundwater table measured perpendicular to the underlying impermeable layer, f is drainable porosity, x is distance to the outlet measured parallel to the impermeable layer, q = q(x,t) is subsurface flux along the hillslope bedrock, t is time, and N is a source term. This equation can be generalized by mapping the three-dimensional hillslope shape onto a one-dimensional soil pore space (Fan & Bras, 1998):

$$S(x,t) = fw(x)h(x,t) \qquad (2)$$

where S(x,t) is the actual storage, w(x) is the hillslope width, and h(x,t) is the water table height averaged over the width of the hillslope. Moreover we define subsurface flux as (Boussinesq, 1877]):

$$q = -kh\left(\frac{\partial h}{\partial x}\cos i(x) + \sin i(x)\right) \qquad (3)$$

where k is the saturated hydraulic conductivity, i(x) is the local bedrock slope at x. Upon combination of Eqs. 1, 2, and 3, we obtain a generalized hsB equation that can handle spatially variable bedrock slope:

$$f\frac{\partial S}{\partial t} = \frac{\partial}{\partial x}\left(\frac{kS}{f}\cos i(x)\frac{\partial (S/w)}{\partial x}\right) + \frac{\partial}{\partial x}\left(kS\sin i(x)\right) + fNw \qquad (4)$$

Experiment set-up
We have applied the generalized hsB model, the KW model as described by (Troch et al., 2002), and the RE models as described by (Paniconi & Wood, 1993) to a set of nine characteristic hillslopes (see Fig. 1).

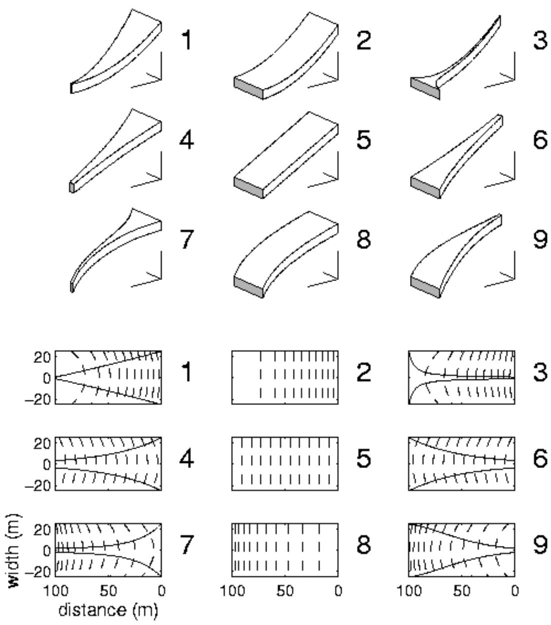

Figure 1. Nine characteristic hillslopes.

The nine characteristic hillslopes consist of three divergent, three straight and three convergent hillslopes with profile curvature varying from convex to straight to concave. For 5 and 30% slopes free drainage experiments are inter-compared, starting from 40% saturation. In order to generalize the results a dimensional analysis is conducted. A dimensionless representation will give us scaled model results. We define the following dimensionless variables:

$$\tau = \frac{tki}{fL}, \chi = 1 - \frac{x}{L}, \phi = \frac{Qcum(t)}{V}, \sigma = \frac{h(x,t)}{d}$$

where L is hillslope length, $Q_{cum(t)}$ is cumulative flow up to time t, V is the initial volume of water stored in the hillslope, and d is soil depth. The dimensionless variables define (in order) kinematic time, flow distance as a fraction of the total flow path, dimensionless flow, and dimensionless storage.

Results and conclusion

Fig. 2a, 2b, and 2c show the dimensionless storage patterns for the 5% bedrock slope simulations.

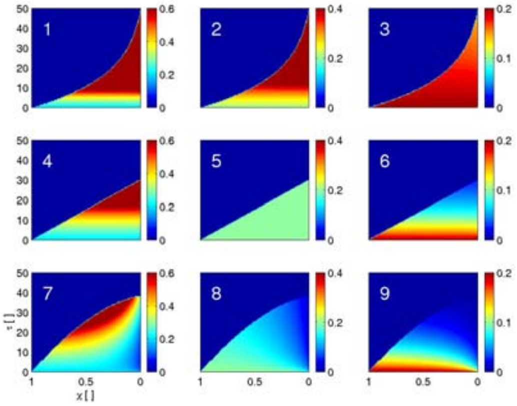

Figure 2a. hsB 5% storage.

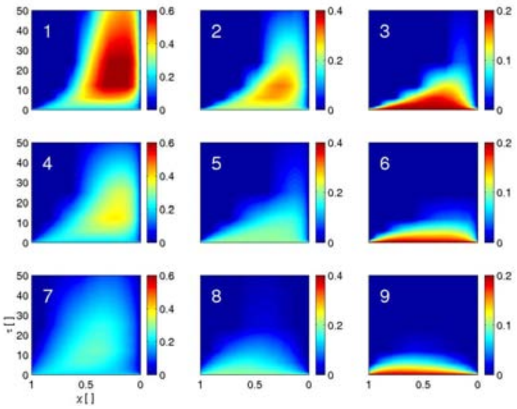

Figure 2b. RE 5% storage.

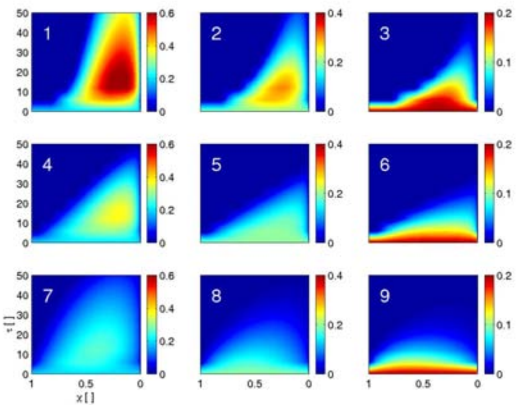

Figure 2c. KW 5%/30% storage.

Fig. 3a and 3b show the storage patterns for the 30% bedrock slope case.

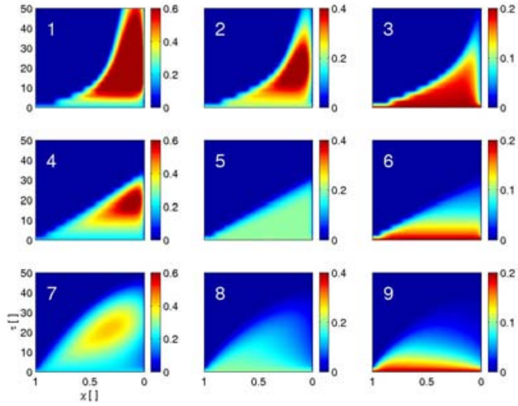

Figure 3a. hsB 30% storage.

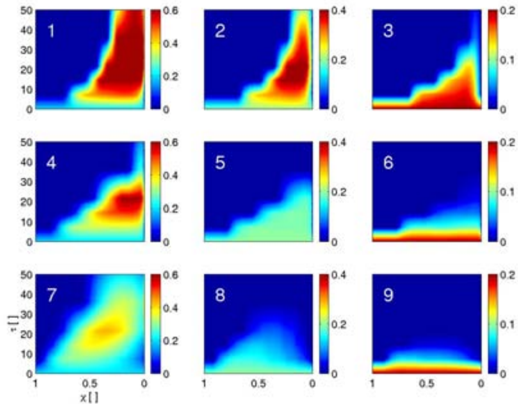

Figure 3b. RE 30% storage.

Fig. 4 displays the dimensionless hydrographs for all hillslopes (k = 1 m/h, 0.22 ≤ f ≤ 0.3).

In this work we have generalized the hsB model to variable bedrock slope and we have conducted an intercomparison of a RE model, taken as a benchmark, with the hsB, and a KW model, in order to investigate within which settings the application of the KW and hsB model is valid.

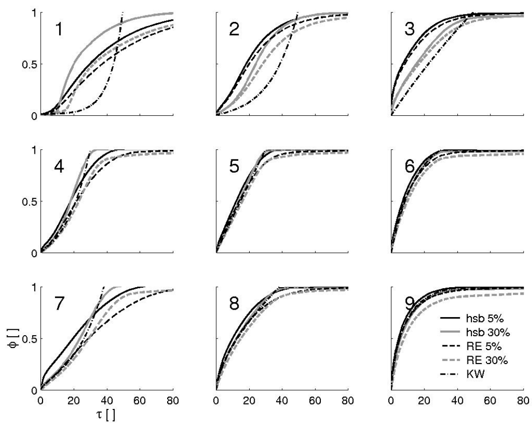

Figure 4. Hydrographs 5%/30%, all models.

Generally one can conclude that (1) the hsB results show a good match with the RE results. (2) In relation to the validity of applying a KW model it is limited to simulation settings in which hydraulic diffusion has a marginal impact on storage. (3) The RE hydrographs appear to have a slight delay compared to the hsB results. This may be explained by the capillarity effects of the soil.

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NWO and Water

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NWO: the organisation

The Netherlands Organisation for Scientific Research (NWO) promotes fundamental and applied research of high quality. An international orientation has high priority. The means to perform this mission are M€ 400 per year from the Ministry of Education, Culture and Science and about M€ 50 per year additional funds from other ministries and enterprises. Via several funding schemes NWO finances over 4,300 positions in scientific research, both at universities and at NWO institutes.

NWO has a variety of funding schemes.
A line of personal grants exists for young talented postdocs up to well-established postdocs with budgets from k€ 200 up to k€ 1,250 ("Vernieuwingsimpuls"). On top of this, annually four Spinoza-prises (M€ 1.5) are awarded to internationally renowned scientists. The research councils fund open programmes and a wide range of dedicated programmes, aimed at research on specific themes. NWO participates in many thematic European research programmes, in particular in the recent so-called EUROCORES, which are coordinated by the European Science Foundation. Moreover, NWO has funding schemes for investments, for centre subsidies and for travel, workshops, and international coordination.

The mode of operation of NWO is characterised by three layers of appraisal of research proposals, which are submitted for funding. At the first level proposals are reviewed by independent peers, for ALW usually scientists from abroad. The reviews can be commented by the applicants of the proposals. The next level is the judgment and prioritising of proposals by the programme committee, which is composed of senior Dutch scientists. Decisions on funding are made by the Research Councils, Boards, or Steering Committees in case of co-funding by external parties. By approximation, about 4,000 – 5,000 external referees are annually involved in the review process and more than 1,000 (mostly Dutch) scientists are involved in committees, boards and councils.

The funding by NWO is mainly organised according to clusters of disciplines in Research Councils ("Gebiedsbesturen"). In addition two foundations exists for research in a variety of disciplines: WOTRO for research in tropical and developing countries and NCF for supercomputing.
Besides, NWO administers a couple of research institutes: on physics (four institutes), sea research (NIOZ), space research and earth observation (SRON) and astronomy (ASTRON). In addition, NWO hosts coordinating offices for managing external funds for research on Genomics, Biopartner, ICT, and the European office for EDCTP (clinical trials of medication in Africa).

The NWO strategy

In the Strategic Plan 2002-2005 of NWO seven interdisciplinary themes were presented, one of them called 'System Earth'. This theme acts as an umbrella over a number of issues related to the large-scale human influences on earth systems, generally indicated as Global Change. This covers a wide variety of research, e.g., on climate change, on development of hydrogen energy systems, on policy mechanisms under the Kyoto-protocol, on the use of tropical coastal zones, and on water management in The Netherlands.

In particular the Research Council on Earth an Life Sciences has identified strategic themes on Global Change: biodiversity, the coupled bio-geosphere, climate variability, continent-ocean boundaries, water and coastal zones, bio- and geo-informatics and monitoring. Some of these themes are shared with WOTRO and the Council on Exact Sciences (EW). In addition, several external parties join in relevant funding programmes. In particular the Ministry VROM has committed funding for a series of issues related with climate change, which are covered by different programmes.
In the near future, the following relevant programmes are open for proposals (see Table).