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A tutorial on bio-economic SDP modeling: an illustration of fisheries policies *

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Stochastic dynamic programming (SDP) is a useful tool for analyzing policy questions in fisheries management. In order to understand and reproduce solution procedures such as value function iteration, an analytic elaboration of the problem and model characteristics is required. Because of the increased use of numerical techniques, our aim is to develop a tutorial of a specific case that uses this tool. We describe and analyze a bi-level SDP model to study fisheries policies. At the first level, a policy maker decides on the fish quota to be imposed, keeping in mind fish stock dynamics, capital stock dynamics and long-term social benefits. At the second level, fishermen react on this quota and on current states of fish stock and capital stock by deciding on their investments and fishing effort. An analysis of the behavior of the model is given and a method is elaborated to obtain optimum strategies based on value function iteration. Bi-level decision making enables us to present the model in an understandable manner, and serves as a basis for extension to more complex settings.

Key words: stochastic dynamic programming, value function iteration, fisheries management, bi-level

1. Introduction

Policy makers in fisheries management face uncertainty related to dynamics in fish stock due to environmental variability. Moreover, policy makers have to take into account human behavior when setting policies. Quota are not only based on known or unknown dynamics in fish stock and fleet capacity of the fishery sector, but also based on how fishermen behave in an unregulated setting. The implementation of these interactions in a model is a challenge, because policy makers and fishermen have different and partly conflicting objectives. The policy maker seeks a long-term sustainable fish stock as well as maximization of social benefits, while fishermen have their own priorities and may behave myopically. There is thus a need to account for fishermen behavior in the quota decision making process.

Fisheries policies are often analyzed in the literature in a setting of dynamic optimization, where a sole owner determines optimal harvest and investment levels for a specific fishery [Charles(1983), Boyce(1995), Sethi et al.(2005), Singh et al.(2006)]. Such models have been developed in deterministic and stochastic settings, often assuming uncertainty in fish stock dynamics

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due to environmental variability. The assumption of sole ownership, however, ignores behavior of fishermen and the effect of a policy on this behavior. Models that do account for behavior of different decision makers are frequently presented in a game theoretic framework. Most game theoretic studies simplify the problem by using a single-stage approach where each player makes a decision at the beginning of the game, given observed states that change deterministically over time [Bailey et al.(2010)]. Fewer studies use a multi-stage approach, where players make a decision at stage one that is independent of the other player's behavior. At stage two, each player uses decisions from stage one to decide on the best strategy [Ruseski(1998), Kronbak and Lindroos(2006)]. More often, fisheries models are sequential games where players make decisions one after another. The setting typically consists of two sectors, regions or countries that compete for multiple species [Fischer and Mirman(1992), Sumaila(1995), Munro(2009), Wang and Ewald(2010)]. For example in [Fischer and Mirman(1992)], two regions each determine their optimal level of consumption, based on two deterministically interacting species. The assumption of biological externality, which stems from interaction between species, justifies the game theoretic approach. A similar approach is used in [Wang and Ewald(2010)], where a number of fisheries harvest two interacting species. The assumption of ecological uncertainty makes it a complex model, which in turn makes the interpretation of bio-economic dynamics and long-run Nash equilibrium strategies not straightforward. See [Bailey et al.(2010)] for an overview of the application of game theory to fisheries over the last 30 years.

In these studies, where policy questions are analyzed with numerical solution methods such as stochastic dynamic programming, an elaboration of model characteristics and implementation of the solution procedure is often not provided. Complex models, such as [Wang and Ewald(2010)], are therefore not easily understood and reproduced. Because of the increased use of numerical solution methods, our objective is to develop a tutorial for a specific case of a stochastic dynamic programming model with multiple players, that takes the reader step-wise through the model and solution procedure of value function iteration. The following research questions are answered: (i) what are characteristics of the model that are used in the solution procedure, and (ii) how is the solution procedure implemented in a deterministic and stochastic setting.

We develop a model setting in which the objective of a policy maker is to determine levels of quota that maximize social benefits, subject to dynamics in fish stock and capital stock and based on behavior of fishermen. We study a single-species fishery, where fish stock growth is considered stochastic. From this perspective, it is useful to analyze fisheries policies in a framework of stochastic dynamic programming. The assumptions of myopic fishermen and a single-species fishery furthermore allow us to use a game theoretic framework, which we call a bi-level model. At the first level of the bi-level stochastic dynamic programming model, the policy maker determines the optimal quota for known values of fish stock and capital stock. At the second level, fishermen make their harvest and investment decisions, where they are restricted by available fish stock, capital stock and the quota set by the policy maker at level one. Bi-level decision making enables us to present the model in a simple manner such that it can be reproduced. We show how the model can be solved by means of value function iteration in order to find the optimal strategy for determining the quota. To check robustness of the model, we analyze how the model performs under increased uncertainty and what the impact is of a continuous versus a discrete decision space, i.e. a grid-search procedure.

The paper is organized as follows. Section 2 describes the model and Section 3 analyses mathematical properties of the model that can be used to derive optimum solutions. In Section 4, a description is provided of a solution approach based on value function iteration. In Section 5 we investigate, along numerical experiments, the behavior of an optimal path of quota towards a steady state when uncertainty increases. Section 6 summarizes our findings.

2. Model description

The model describes dynamics of a biological system that interacts with dynamics of economic behavior. We describe the biological submodel, the economic submodel and optimization. In the used symbols, we distinguish between model parameters (exogenous in lower case letters) and decision variables (capitals) that include direct decision variables, dependent variables and stock variables. Without loss of generality, the dynamics are modeled in discrete time using an index t.

2.1. Biological model; growth of fish stock

The development of one species of fish is based on the Gordon-Schaeffer model [Gordon(1954), Schaefer(1954)]. Used parameters are

Data

- m carrying capacity of the species in kton
- r intrinsic growth rate
- ξ lognormal random variable with cumulative distribution function $G(\xi)$ based on parameters μ and σ, with $\mu + \frac{1}{2}\sigma^2 = 0$, such that $\mathbb{E}(\xi) = 1$

The random variable describes a random multiplicative effect. A lognormal distribution is a common assumption in fisheries economics literature. Let the stock variable be fish stock X_t in kton and H_t harvest in kton. Then dynamics of fish stock is given by

$$X_{t+1} = X_t + \xi r X_t \left(1 - \frac{X_t}{m}\right) - H_t. \tag{1}$$

2.2. Economic submodel; harvest and investment decisions

The economic part of the model includes capital stock dynamics depending on investment in fleet equipment and all costs to harvest fish. A fixed selling price p is assumed. Used parameters are

Data

- p selling price in euro/kton
- γ yearly depreciation rate of capital
- c^e cost of effort in euro/hpd
- c^i investment cost in euro/hpd
- c^s crew cost in euro/euro
- q catchability coefficient in Schaeffer harvest function

Let K_t describe capital stock and I_t investment. Then following neoclassical investment theory we have

$$K_{t+1} = K_t(1-\gamma) + I_t, \tag{2}$$

where the fishery sector is confronted with investment costs $c^i I_t$.

To describe cost of harvesting, a decision variable E_t representing fishing effort (intensity) is introduced that makes harvest H_t in fact a dependent variable. The variable E_t is expressed in horse-power-days (hpd). The relation between harvest H_t and effort E_t is one of the elements where the two sub-models are linked. Harvest not only depends on effort, but also on the size of fish population. We assume the following Spence harvest function [Spence(1973)]

$$H_t = X_t\left(1 - e^{-qE_t}\right) \to E_t = \frac{1}{q}\ln\frac{X_t}{X_t - H_t}. \tag{3}$$

Effort is limited by capital

$$E_t \le K_t \to H_t \le X_t\left(1 - e^{-qK_t}\right), \tag{4}$$

which implies that harvest is always less than fish stock:

$$H_t < X_t. \tag{5}$$

Profit of the fishery sector is sales from harvest, pH_t, minus effort cost, c^eE_t, and crew cost, $c^s pH_t$. Effort cost c^eE_t can be expressed in the harvest function, H_t, by substituting variable E_t: $c^eE_t = \frac{c^e}{q}\ln\frac{X_t}{X_t-H_t}$. Direct profit for the fishery sector,

$$\pi_2(Q_t, X_t, K_t) = pH_t - c^s pH_t - \frac{c^e}{q}\ln\frac{X_t}{X_t - H_t},$$

depends on harvest decision $H_t = H(Q_t, X_t, K_t)$ and, via (4), on the investment decision $I_t = I(Q_t, X_t, K_t)$ taken by the fishery sector as described in Section 2.3.

2.3. Optimization model

Objectives to be optimized depend on the players such as different groups of fishing companies and authorities like countries and the European Union. In this paper, we focus on a policy maker (EU authority) that sets a quota in order to maximize the discounted stream of future social benefits, given levels of fish stock X_t and capital stock K_t. The fishery sector reacts on that by deciding on investment level $I(Q_t, X_t, K_t)$ and harvest $H(Q_t, X_t, K_t)$, given levels of fish stock, capital stock and quota decision Q_t.

2.3.1. Decisions at level 2 At the second level, the harvest decision is restricted by quota Q_t, capital stock via equation (4) and the marginal cost of harvesting. Harvest is set to zero if the effort cost $c^e/q\ln X_t/(X_t - H_t)$ is higher than the return $(1-c^s)pH_t$. The fish stock at which $c^e/q\ln X_t/(X_t - H_t) > (1-c^s)pH_t$, is in fact the level below which it is not profitable to harvest. Profit for the fishery sector, given quota Q_t and stock levels X_t and K_t, is

$$\pi_2(Q_t, X_t, K_t) = \max_H \left\{ pH - c^s pH - \frac{c^e}{q}\ln\frac{X_t}{X_t - H} \right\}, \tag{6}$$

subject to $0 \le H \le Q_t$ and $H \le X_t\,(1 - e^{-qK_t})$. If the optimization problem in equation (6) has an interior solution, the analytical expression follows from the first order condition

$$\frac{d}{dH}\left\{ pH - c^s pH - \frac{c^e}{q}\ln\frac{X_t}{X_t - H} \right\} = 0. \tag{7}$$

Given upper and lower bounds in (6), the solution is given by

$$H(Q_t, X_t, K_t) = \min\left\{ \left(X_t - \frac{c^e}{pq(1-c^s)} \right)^+, Q_t, X_t\left(1 - e^{-qK_t}\right) \right\}, \tag{8}$$

where y^+ stands for $\max\{0, y\}$ and where $\frac{c^e}{pq}$ has been identified in [Conrad and Clark(1987)] as the bioeconomic equilibrium escapement in the Spence model.

With respect to the investment decision, it is assumed that the fishery sector observes the desired harvest level

$$\hat{h}(Q_t, X_t) = \min\left\{ \left(X_t - \frac{c^e}{pq(1-c^s)} \right)^+, Q_t \right\}, \tag{9}$$

and adjusts its capital stock for next year to have sufficient capital to reach $\hat{h}$

$$\hat{h}(Q_t, X_t) = X_t\left(1 - e^{-qK_{t+1}}\right) \rightarrow K_{t+1} = \frac{1}{q}\ln\frac{X_t}{X_t - \hat{h}(Q_t, X_t)}. \tag{10}$$

Given dynamics of capital stock K_t in (2) and assuming nonnegative investment, this leads to the investment function

$$I(Q_t, X_t, K_t) = \left(\frac{1}{q}\ln\frac{X_t}{X_t - \hat{h}_t(Q_t, X_t)} - K_t(1-\gamma) \right)^+. \tag{11}$$

2.3.2. Decisions at level 1 At the first level, we have

$$\max_{Q(X_t,K_t)} \mathbb{E}\left\{\sum_0^\infty \frac{\pi_{1,t}}{(1+\rho)^t}\right\}, \tag{12}$$

where ρ is the discount rate and by taking $Q_t = Q(X_t, K_t)$,

$$\pi_{1,t} = \pi_2(Q_t, X_t, K_t) - c^i I_t. \tag{13}$$

Typically subscripts 1 and 2 refer to levels 1 and 2 and the investment cost $c^i I_t$ is only accounted for at level 1. Decision $Q_t = Q(X_t, K_t)$ depends on dynamics of fish stock in equation (1) and capital stock in equation (2). Note that this model deals with a stationary system, which means that the optimum strategy consists of a decision rule that tells the policy maker what quota $Q(X, K)$ to set given fish stock X and capital stock K. Furthermore, the optimum strategy depends on behavior of the fishery sector at the second level.

3. Model analysis

In this section we address the question what are characteristics of the model that are used in the solution procedure of value function iteration. We first look at model characteristics that are used to define implicit bounds of decision variables. We then derive steady state values for the deterministic setting, which are later used in the value iteration approach to verify long-term behavior of the system.

3.1. Bounding decision values

Let the following denote the level of fish stock below which it is not profitable to start harvesting:

$$\hat{x} = \frac{c^e}{pq(1-c^s)}. \tag{14}$$

Hence, $H_t = 0$ if $X_t < \hat{x}$. The quota $Q(X_t, K_t)$ has alternative optimal solutions, as a positive quota is not binding in the decision on the harvest level H_t. We consider the minimum level of harvest to be chosen in case Q_t has alternative solutions. Often this means that $Q_t = H_t$.

Fish stock as described in equation (1) increases up to carrying capacity m and decreases if fish stock, would exceed that level. Due to fishing behavior it is also known that for $X_t < \hat{x}$ no fishing takes place and growth is always positive. On one side, $\hat{x}$ is a lower bound on fish stock X_t if initial stock $X_0 > \hat{x}$. On the other side, if the initial stock is higher than the carrying capacity, $X_0 > m$, the stock can only go down from that level. Given an initial stock X_0,

$$X_t \in [\min\{X_0, \hat{x}\}, \max\{X_0, m\}]. \tag{15}$$

This means that the interesting range for harvest H_t and quota Q_t is $[0, \max\{X_0, m\} - \hat{x}]$. This range also provides corresponding values for capital stock K_t and investment I_t. Due to investment cost and depreciation, the level of capital should not exceed what is required to catch the desired level, as specified by equation (10)

$$K_t, I_t \in \left[0, \max\left\{\frac{1}{q}\ln\left(\frac{m}{\hat{x}}\right), K_0\right\}\right].$$

3.2. Steady state values

In a stationary system, X_t, K_t, H_t, I_t, Q_t and $\pi_{1,t}$ are constant over time with steady state values X, K, H, I, Q and π_1. Harvest is a constant fraction of fish stock in equation (1), so that $X_{t+1} = X_t = X$. If for the steady state value of X, $\hat{x} \leq X \leq m$, it can be found that

$$H = rX\left(1 - \frac{X}{m}\right) \rightarrow X = \frac{rm + \sqrt{(rm)^2 - 4rmH}}{2r}. \tag{16}$$

Otherwise, if $X < \hat{x}$, no harvest takes place. In the long run, capital, quota and harvest converge to the same level, so that $Q = H$, $K = \frac{1}{q}\ln\frac{X}{X-H}$, and $I = dK = \frac{d}{q}\ln\frac{X}{X-H}$. The policy maker at level 1 tries to keep stationary social benefits π_1, as high as possible, whereas harvest equals growth;

$$\max_H \left\{\pi_1 := pH - c^s pH - \frac{c^e + \gamma c^i}{q}\ln\frac{X}{X-H}\right\}, \tag{17}$$

subject to

$$H = rX\left(1 - \frac{X}{m}\right).$$

Substitution of growth in the (constant over time) social benefits function gives

$$\pi_1 = r(1-c^s)pX\left(1 - \frac{X}{m}\right) + \frac{c^e + \gamma c^i}{q}\ln\left((1-r) + \frac{r}{m}X\right). \tag{18}$$

For an interior optimum, $\hat{x} < X < m$, the first order condition $\frac{d\pi_1}{dX} = 0$ leads to the equilibrium value

$$X = \frac{3m}{4} - \frac{m}{2r}\left(1 - \frac{1}{2}\sqrt{(r-2)^2 + \frac{8r(c^e + \gamma c^i)}{q(1-c^s)mp}}\right). \tag{19}$$

4. Stochastic Dynamic Programming

Now that we are familiar with characteristics of the model and how these characteristics are used to determine steady state values, we next explain the value function iteration approach and we are concerned with the question how value function iteration is implemented in a deterministic and in a stochastic setting.

In the literature, the Stochastic Dynamic Programming (SDP) solution follows the Bellman equation [Puterman(1994)] and takes the following general notation

$$V_t(X_t, K_t) = \max_{Q_t}\left\{\pi_t(Q_t, X_t, K_t) - c^i I_t(Q_t, X_t, K_t) + \delta \mathbb{E} V_{t+1}(X_{t+1}, K_{t+1})\right\}, \tag{20}$$

where V_t is the value function that represents the maximized value of the objective function from time t onwards. The expectations operator $\mathbb{E}$ holds the transition probabilities of moving from a given current state of fish stock X_t, to next period's fish stock X_{t+1}.

In our specific model, we are dealing with a system that is stationary and that is subject to discounting. This means that a function V exists such that the optimal solution fulfills the Bellman equation. Behavior of the system depends on the optimum quota rule $Q(X_t, K_t)$ that solves (12), maximizing discounted future social benefits

$$V(X_t, K_t) = \max_Q\left\{\pi_2(Q, X_t, K_t) - c^i I(Q, X_t, K_t) + \delta \mathbb{E} V(X_{t+1}, K_{t+1})\right\}, \tag{21}$$

where X_{t+1} and K_{t+1} follow from dynamic equations (1) and (2), i.e. they depend on values for Q_t, X_t and K_t and the fishermen behavior $H(Q_t, X_t, K_t)$ and $I(Q_t, X_t, K_t)$.

4.1. Value function iteration of the deterministic case

Optimum $Q(X_t, K_t)$ can be found by a value function iteration approach that iteratively approximates the value function V. In this approach the system starts with an arbitrary valuation of function V_1 and determines V_2 iteratively. The iterative process of repeating $V_1 = V_2$ continues until $V_2(X,K) - V_1(X,K)$ converges to an arbitrarily small convergence accuracy ε, for all state values (X,K). This works with a discretization of state space of (X,K) with vectors x, k, repeating for each grid point the iteration

$$V_2(X_t,K_t) = \max_Q \left\{ \pi_2(Q,X_t,K_t) - c^i I(Q,X_t,K_t) + \delta V_1(X_{t+1},K_{t+1}) \right\}, \tag{22}$$

with X_{t+1}, K_{t+1} following from the dynamics and fishermen behavior. Iteratively, we set $V_1 = V_2$.

Algorithm 1 :Pseudo code value function iteration

Funct data, x, k vectors, ε; Q, V matrices
1. $V_1 = 0$ matrix
2. **for** all i, j
3. solve (22) for $X_t = x_i, K_t = k_j$
4. **if** $\max_{ij}(V_2(i,j) - V_1(i,j)) - \min_{ij}(V_2(i,j) - V_1(i,j)) > \varepsilon$
5. $V_1 = V_2$ and go to step 2

We first outline the approach for a deterministic setting, when random variable $\xi = 1$, and illustrate the results of a base case. We show how steady state values are reached if the system starts at arbitrary levels of states of fish stock and capital stock.

Using $X_t \in \{x_1, x_2, \ldots, x_i, \ldots, x_{max}\}$ and $K_t \in \{k_1, k_2, \ldots, k_j, \ldots, k_{max}\}$ in fact discretizes the state space, such that function $V(X_t, K_t)$ is approximated by the matrix $F(i,j) = V(x_i, k_j)$. For each matrix entry (i,j), iteratively the minimum over Q is found of a function

$$f_{ij}(Q) = \pi_2(Q, x_i, k_j) - c^i I(Q, x_i, k_j) + \delta V_1(X_{t+1}, K_{t+1}), \tag{23}$$

where V_2 as well as V_1 are approximated by a matrix F. As a result of the dynamics and decision, X_{t+1} and K_{t+1}, may not be on the grid defined by x, k. The usual approach is to interpolate $V_1(X_{t+1}, K_{t+1})$ from matrix $F(i,j)$. Iterative minimisation of $f_{ij}(Q)$ over Q in (23) can be done by using a grid on a range of $[0, Q_{max}]$, or by using a one-dimensional minimisation algorithm.

The implementation requires considering first appropriate boundaries x_1, x_{max} and k_1, k_{max} of the system and whether all combinations x_i, k_j are feasible. As discussed in Section 3.1, fish stock values $X_t < \hat{x}$ are not considered, as harvest is then zero and quota can be set at $Q_t = 0$. Upper bound x_{max} depends on the possibility of considering starting values $X_0 > m$. We can take $[x_1, x_{max}] = [\hat{x}, \max\{X_0, m\}]$. Following the reasoning in Section 3.1, it is appropriate to take $[k_1, k_{max}] = [0, \frac{1}{q}\ln(\frac{m}{\hat{x}})]$. Grid points x_i and k_j are not necessarily equidistant in their corresponding ranges. A more refined grid, i.e. using more grid points, results in a better approximation of value function $V(X,K)$ and policy decision $Q(X,K)$.

Example 1. We introduce a base case with data taken from [van Dijk et al.(2012)], a study on North Sea plaice. This species is one of the main commercially exploited flatfish in the North Sea and is subject to increasing fishing pressure [Kell and Bromley(2004)]. We use the following parameter values: $m = 460, r = 0.74, \xi = 1, q = 0.0139, \gamma = 0.1, \delta = 0.95, p = 1.83, c^i = 2.1, c^s = 0.25, c^e = 3.54$. For this base case, given above parameter values and (14), fishing is not profitable below $\hat{x} = 185.6$. According to (19), the stationary value of fish stock is $X = 349.5$.

The value function iteration algorithm is run with $x_1 = 170, x_{max} = 500, k_1 = 4, k_{max} = 70$ and taking 23 equidistant points x_i and k_j on each axis. So the distance between the grid points

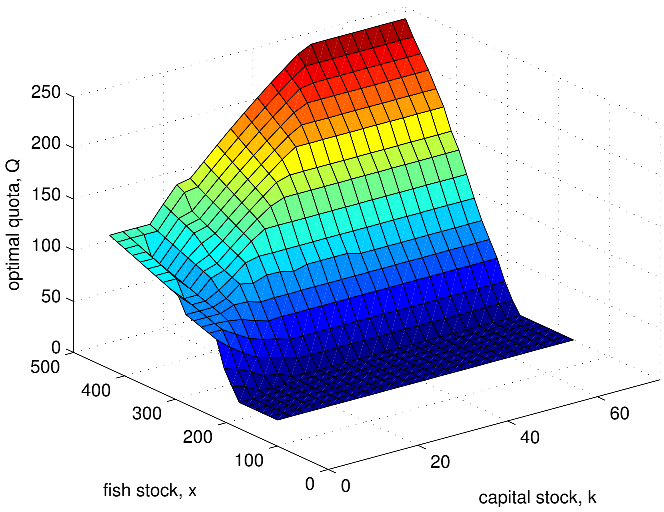

Figure 1 Quota function found by value function iteration, deterministic model

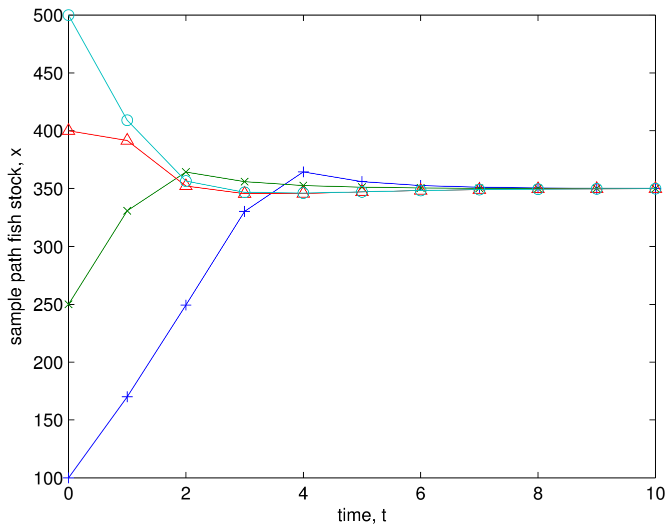

Figure 2 Convergence of stock to stationary state, deterministic model; $K_0 = 9$, $X_0 = 100, 250, 400, 500$

is 15 for X_t and 3 for K_t. Iterative minimisation of Q is done by a one-dimensional minimisation algorithm FMINBND of MATLAB. For the matrices V_1 and V_2 in the algorithm, we have that $\max_{ij}(V_2 - V_1) - \min_{ij}(V_2 - V_1)$ becomes smaller than $\varepsilon = 0.1$ within 20 iterations, indicating that the policy $Q(X,K)$ found at iteration 20 is optimal. Resulting values of $Q(X,K)$ are depicted in Figure 1.

The behavior of the system is sketched in Figure 2 by simulation for four different starting values of the fish stock X_0 and initial capital stock of $K_0 = 9$. The system converges after 7 time steps to the steady state. Stable dynamics of fish stock, due to equation (1), are determined by fishing behavior. For low values of fish stock, no fishing takes place and for values higher than carrying capacity m, harvesting reduces the stock due to low effort cost. The policy maker helps to reach the stable situation. For instance for $X_t = 200, K_t = 9$, the fishery sector would harvest about 14 kton. Notice that the policy maker sets quota at zero, preventing harvest, in order to promote recovery of the fish stock and keeping in mind long term social benefits.

4.2. Value function iteration of the stochastic case

Now consider the model in a stochastic setting. We show how the system behaves when random variable ξ has a variation. We look at stability of the system, deviations of long term average states and how fast the system reacts to deviations from stationary values.

When random variable ξ has a variation, the necessary condition (Bellman equation) for an optimal Q needs to be adjusted, i.e. expected value and probabilities come into play. The Bellman equation (21) for an optimum solution $Q(X_t, K_t)$ is that there exists a value function V such that

$$V(X_t, K_t) = \max_{Q} \left(\pi_{t,1}(Q, X_t, K_t) - c^i I_t(Q, X_t, K_t) + \delta \mathbb{E}_\xi \left[V(X_{t+1}, K_{t+1}) \right] \right), \tag{24}$$

where expected value $\mathbb{E}_\xi$ is taken over future social benefits.

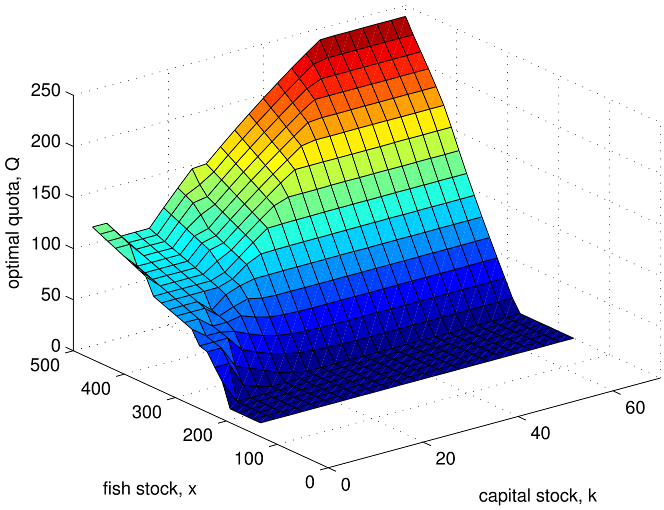

Figure 3 Quota function found by value function iteration, using continuous optimization by FMINBND; stochastic model

For the use of the algorithm, there are several practical ways to discretize the distribution of ξ. One way is to iterate over all possible grid points x_i, k_j for V_1 and assign probabilities to these outcomes given decision Q. This approach is quite cumbersome as it requires re-calculation of V_1 for many values of quota and associated probabilities. A more usual approach is to discretize the space of possible outcomes of stochastic variable ξ. A way to do this is by using quantiles of the lognormal distribution. An equidistant grid is taken over probability range $[0, 1]$ with a step p_ξ and generating a discrete outcome space $\{\theta_1, \theta_2, \ldots, \theta_n\} = \{G^{-1}(p_\xi), G^{-1}(2p_\xi), G^{-1}(3p_\xi), \ldots, G^{-1}(1 - p_\xi)\}$. The consequence of this operation is that the outcome space is truncated by the p_ξ-quantiles and each outcome has the same probability of occurrence. Equation (24) is iteratively approximated by using in the algorithm

$$\mathbb{E}_\xi V_1(X_{t+1}, K_{t+1}) \approx p_\xi \sum_{i=1}^{n} V_1 \left(X_t + \theta_i r X_t \left(1 - \frac{X_t}{m} \right) - H(Q, X_t, K_t), K_{t+1} \right), \tag{25}$$

where $H(Q, X_t, K_t)$ is the harvest level chosen by the fishing sector on level 2. Interpolation is required in the state space to valuate V_1 for every possible outcome θ_i of the growth multiplier.

Ranges for state variables do not change compared to the deterministic model since possible outcomes θ_i are always positive. This means that for $X_t < \hat{x}$, growth is positive and for $X_t > m$ growth is negative, so the same bounds can be used as in the deterministic case. Calculation time for the value function iteration increases as for each evaluation of a suggested quota Q_t, now n values of the value function are interpolated.

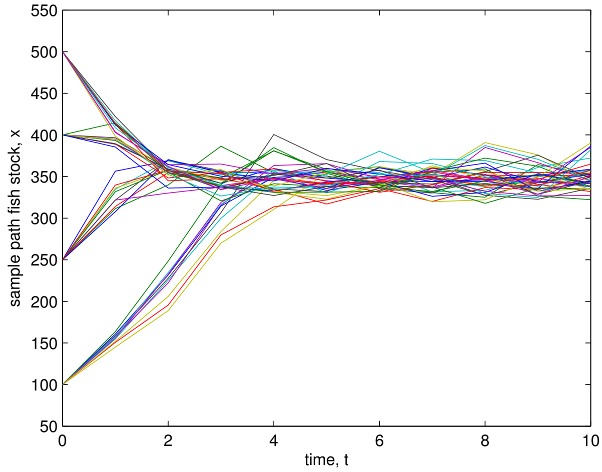

Figure 4 Realisations of stochastic paths for 4 different starting values of fish stock following optimal quota setting; $K_0 = 9, X_0 = 100, 250, 400, 500$

EXAMPLE 2. In example 2, variable ξ is lognormal distributed with parameters $\sigma = 0.159$ and $\mu = -0.0126$. Distributing $n = 40$ points over the outcome space with $p_\xi = 0.025$ we have $\{\theta_1, \theta_2, \ldots, \theta_n\} = \{0.596, 0.716, \ldots, 1.593\}$. We run the value function iteration by solving (24) with (25). $\|V_1 - V_2\| < \varepsilon$ is obtained after 15 iterations. The behavior of the system is sketched for 4 different starting values X_0 and $K_0 = 9$ in Figure 4, showing 10 realisations of sample paths that follow the optimum strategy.

5. Sensitivity analysis

So far illustrations show that the model leads to stable paths. Volatility of fish stock, however, is difficult to assess. We therefore analyze whether higher volatility reduces stability and if it leads to a different long term average solution. Experiments are shown in Section 5.1.

The suggested solution procedure of dynamic programming enhances an iterative nonlinear optimization step for each matrix element F_{ij}. Alternative to this continuous search of the optimal quota is a grid-search [Hendrix and Toth(2010)], where possible quota values are limited to rounded values and where the solution procedure is evaluated for a grid of possible quota values. In Section 5.2 we analyze the quality of the solution of the grid-search and the impact for efficiency of the procedure.

5.1. Impact of volatility on model outcomes

Because stock growth is unknown, we investigate whether the described solution method is robust with respect to higher or lower volatility in the model, what the impact is on the optimum solution and if the long term average depends on volatility σ. Experiments are based on varying volatility $\sigma \in \{0.05, 0.16, 0.5\}$ and corresponding $\mu \in \{-0.0013, -0.0126, -0.125\}$ to have an average growth of 1. Besides the base case of $\sigma = 0.159$ [van Dijk et al.(2012)], we now look at scenarios of low

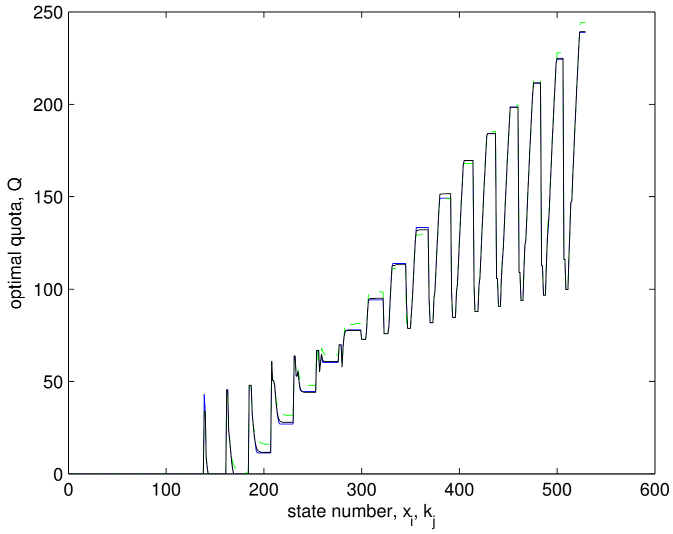

Figure 5 Optimal policy, $Q(x_i, k_j)$, for low volatility (in blue), high volatility (in green) and base case volatility (in black) against state numbers x_i, k_j

volatility and high volatility. For the SDP procedure this means that the range of multiplication factors ranges between $[\theta_1, \theta_n] = [0.85, 1.16]$ for the scenario of low volatility and $[\theta_1, \theta_n] = [0.2, 3.66]$ for high volatility.

The consequence for the solution procedure is that the high growth possibility in the high volatility scenario may lead to fish stock values that exceed the $x_{max} = 500$ value used in the ranges of the algorithm. We know, however, that stock levels exceeding carrying capacity imply a decrease in stock in the next year. In that case, setting a quota is not necessary as harvest is limited by the size of capital stock. Running the algorithm for the three scenarios gives in all cases a convergence of the value function within 20 iterations.

With respect to the optimum policy, the optimum strategy is the same for all values of state variables. It is known from literature on dynamic programming and practical experience that including state values with a low probability of occurrence, slows down convergence of value function iteration. To compare outcomes, in Figure 5 we depict the optimum quota of Figure 1 by putting on the x-axis state numbers that correspond to coordinates of state spaces x_i, k_j, for $i, j = 1, \ldots, 23$. Each curve of optimal quota corresponds to a fixed coordinate of x_j and all coordinates of k_j, where $j = 1, \ldots, 23$. The coordinate of x_j increases as we proceed to the next curve, while coordinates of k_j are always evaluated for $j = 1, \ldots, 23$. The policy of the base case, in black, overlaps both policies of low volatility, in blue, and high volatility, in green. It can thus be concluded that both procedure and optimal solution are robust with respect to the unknown natural variation. We obtain an optimal policy, which is valid for different values of μ and σ.

We now examine the impact on behavior of the system. So far, the model shows stable behavior towards the long term average. It is expected to observe higher and lower volatility in fish stock when feeding the optimum strategy to the system. This is illustrated in Figures 6 and 7 for 10 sample paths from 4 different starting values of X_t. We are interested in finding out whether the long term average is influenced by volatility and whether fish stock and corresponding harvest are higher or lower with increasing volatility. We simulate the system for a period of 10,000 years based on the same random numbers for all scenarios, where realizations of the growth factor are calculated by transforming normal pseudo-random numbers to lognormal random numbers with specified volatility and expected value. When measuring long term average, median, 5% and 95%

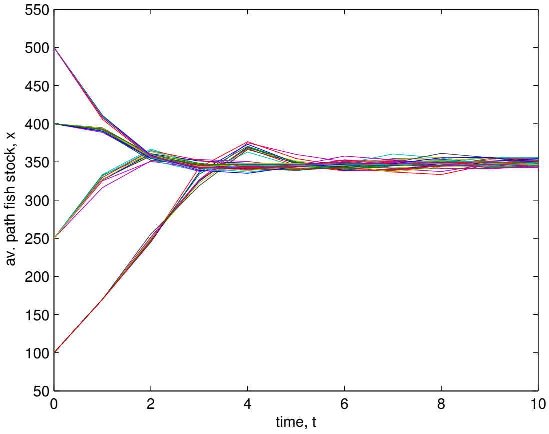

Figure 6 Fish stock X_t under low volatility

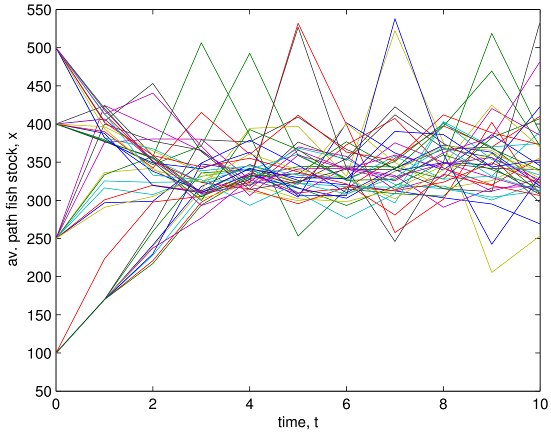

Figure 7 Fish stock X_t under high volatility

percentiles, it can be observed that the interpercentile-range increases with volatility, but the average value slightly changes downward. Results are given in Table 1. The distribution of X_t becomes more skewed with increasing volatility; the median value starts to deviate from the mean.

Table 1 Stock X_t distribution for varying volatility and optimal quota

σ	0.05	0.16	0.5
Mean	349.4	348.3	347.5
$Perc(5\%)$	343.3	331.1	302.4
Median	349.4	348.0	344.1
$Perc(95\%)$	355.6	367.1	405.9

5.2. Impact of grid-search on the solution procedure

For the base case, we use standard one-dimensional nonlinear optimization to derive iteratively for every grid point (x_i, k_j) the optimum of $f_{ij}(Q_t)$ in equation (23). Quota are determined in terms of rounded numbers, e.g. $Q \in \mathbb{Q} = \{0, 10, 20, \ldots, 270\}$. The grid-search procedure selects the best value of $\mathbb{Q}$ and generates the value function V with that. We evaluate *effectiveness*, i.e. whether the procedure generates accurate results, and *efficiency*, i.e. the impact on computing time.

With respect to efficiency, evaluating $|\mathbb{Q}| = 28$ values at each minimization step requires slightly more time than using the standard FMINBND procedure of MATLAB; 400 seconds versus 360. Using a more accurate grid with 55 points doubles computing time. Despite that, in a grid-search procedure the approximate value of Q_t converges faster due to being fixed to a grid point. The number of iterations up to convergence is similar in both procedures.

As for effectiveness, the best values of quota are sketched in Figure 8. They are in fact rounded values of the continuous optimization. The impact on discounted welfare is more difficult to assess in an SDP context. We consider the value of $f_{ij}(Q_t)$ around deterministic stationary states of the base case, $x_{13} = 350$ and $k_{15} = 46$, with the same matrix V found after convergence. It appears that the optimal (continuous) value of $Q_t(350, 46) = 77.6$ provides the same value as using the value $Q_t = 80$ found by the grid search. The objective function is rather flat at the optimum value, which means that one could use rounded numbers for quota instead.

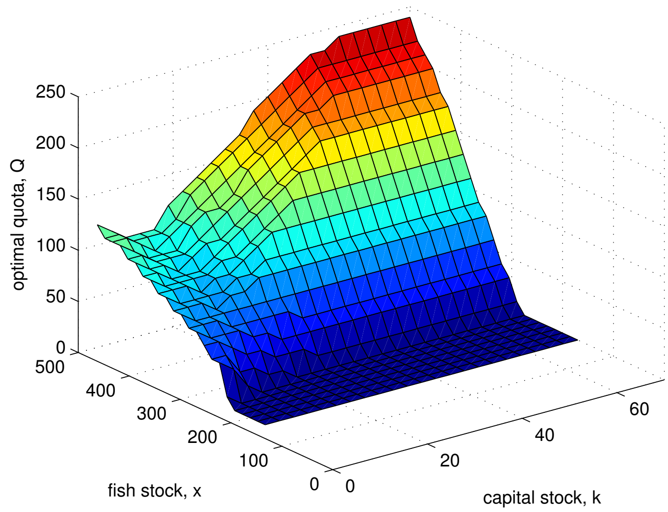

Figure 8 Quota found by using iterative grid search, with grid size 10

6. Conclusions

Stochastic dynamic programming models have shown to be an interesting modeling tool to analyze questions in fisheries management. In order to understand and reproduce such models, we presented a tutorial in which we analyze model characteristics and we show how these characteristics and solution procedure are implemented in a deterministic and a stochastic setting. Robustness of the model is assessed with respect to higher volatility and with respect to the search procedure of the optimal policy.

In our specific case, we study fisheries policies with a bi-level stochastic dynamic programming model that describes dynamics in fish stock and capital stock. At the first level, a policy maker decides on quota to be fished, keeping in mind long term social benefits. At the second level, fishermen react on this quota and on current states of fish stock and capital stock by deciding on their investments and fishing effort.

We find that fish stock dynamics provide stability to the model, with a tendency to converge to a long term steady state. For the derivation of the optimal quota, a careful assessment of boundaries of the system is required to analyse those states that have a high probability of occurrence. Given this analysis, an SDP approach based on value function iteration is a feasible option to derive the optimal quota.

The procedure is robust with respect to parameter variation of the model, but its convergence is sensitive with respect to exact boundaries used in the implementation. Furthermore, the optimal solution is robust with respect to volatility in the model. Moreover, we find that the quota is optimal for both a continuous solution procedure and a grid-search procedure.

The bi-level set up of the model allows us to analytically derive boundary values for state and action spaces and steady states, which is a useful, yet a not always taken step before proceeding to implementation of the solution procedure. In solving the problem, we can make efficient use of standard techniques after having a good understanding of the problem structure. These insights are obtained from a mathematical analysis and numerical experimentation, starting with a simpler version of the problem. The model we studied can be extended in several directions, for example to study multi-annual quota adjustment and non-myopic behavior of fishermen. In further studies the model will be used to answer questions on feasibility of such extensions.

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