Ch. 4.5 Vis.

- 62 - pages

FOURTH POST-GRADUATE TRAINING CENTRE ON LAND DRAINAGE

September 13 - December 17, 1965

Wageningen - The Netherlands

Ir. W.C. VISSER

Institute for Land and Water Management Research - Wageningen

Syllabus of the lectures
on chapter 4.5 : MOISTURE RELATIONS IN THE UNSATURATED ZONE (ICW-nota N°319)

Not for publication,
distribution among participants only.

MOISTURE RELATIONS IN THE UNSATURATED ZONE (Ir. W.C. Visser)

1. The missing link in the hydrological chain

For the correct management of the movement of water between the atmosphere and the open watercourses, the transfer of water in the four parts of the flow path must be well understood and must be predictable.

The four parts are:

a) The open watercourses. Already for a century this part of the water transfer has been studied and it is an important subject in teaching. The use in practical problems is entirely worked out and generally applied.
b) The groundwater range. The theory on groundwater flow was more recently evolved and got its proper place in project planning at a later date. The use of the knowledge on groundwater flow is not yet generally accepted, as a subject in teaching it lags behind, compared with open channel hydraulics, but it is rapidly obtaining the interest in project planning which it deserves.
c) The transfer of water through the atmosphere is still less generally studied and is given an even more restricted position in teaching. It is a still younger branch of hydrology that is gradually acquiring its position in practical design.
d) Between the last two parts of the flow path the flow through the unsaturated zone has to be fitted. The application of this part of hydrology lags behind the application of the theory with respect to the three other parts, its position in teaching is more that of specialized physics than that of practical engineering. The development of the theory is recent, the theory itself is not yet a consistent set of relations. With respect to the correct design of water management practices it is more or less the missing link.

It cannot be denied that in the chain of hydrologic relations the flow of water in the unsaturated zone cannot be missed. The chain is no stronger than its weakest links. An insufficient hold upon the flow of moisture in an important part of the flow path will invalidate much of the value of correct application of the knowledge about the other parts.

It is customary to brand any less generally accepted part of the knowledge of project design as theoretical. The touchstone of such an allegation is, that the errors due to omitting this knowledge are reasonable and that the exertion, needed to take this less generally applied knowledge into consideration is not matched by the savings in investment or the gains due to better designs. It is necessary to have a correct estimate of the kind and magnitude of the shortcomings of the projects which can be overcome by applying the theory on moisture relations in the unsaturated soil. Knowledge of this theory may as well serve to improve design work as to decide on the acceptability of neglecting these effects.

2. Processes determined by unsaturated flow

The conditions in the unsaturated zone often concern the moisture relations which constitute the link between processes, which are part of different disciplines.

The agriculturalists and the drainage and irrigation specialists find the point of connection between their specialized knowledge in the unsaturated zone. The same holds for the climatologist and the river discharge and flood routing engineer. The hydrologist, by using the storage capacity in his formulae, applies a property of the unsaturated zone in his considerations. The aims of many water management practices are especially directed to the management of moisture conditions in the unsaturated zone. This purpose stresses the more that a correct water management design needs the theory on unsaturated movement as its keystone. The points for which the hydrology of the unsaturated zone is of importance are the following:

a) Infiltration and ponding

Rain or artificially applied water may penetrate into the soil or may run off along the surface. What will occur depends on the quantity of water, the permeability and the pressure gradient. This pressure gradient is strongly dependent on the moisture potential in the unsaturated zone. This moisture potential is related to the energy with which the more or less unsaturated soil attracts the soil moisture. In a wet soil

this attraction is small, but it becomes larger the less water a soil contains. The moisture stress - the consequence of this hygroscopic property of the soil - is in reality a negative pressure, but is usually expressed in positive units, in cm or bar. The rate of penetration of the moisture in the soil depends on the gradient and the unsaturated conductivity.

Slow penetration of water means ponding by high rain intensities. This will concentrate far more water on low lying areas than compares with the actual rainfall. In these lower spots the drainage requirements will have to be taken much higher. The ponding will last longer and the damage to the crop will be larger than on soils with a higher rate of penetration.

If ponds become larger, surface runoff will take place. Surface runoff delivers the water far quicker in the water courses than subsurface runoff. The infiltration rate therefore will excert a large influence on the discharge of open water courses. The capillary properties are of considerable importance in the assessment of the design runoff of rivers.

b) Groundwater flow

The flow of groundwater in the saturated zone is governed by the saturated conductivity of the soil, the saturated cross-sectional area of flow and the storage capacity of the soil.

The saturated cross-sectional area is often assumed to be defined by the distance between an impervious layer and the groundwater table. The saturated conductivity is assumed to hold below the groundwater table, the unsaturated conductivity above the groundwater plane. This latter conductivity is generally considered too small to be of importance.

For coarse grained soils this assumption will approximately be true. For fine soils a larger or smaller part of the capillary zone nearest to the groundwater plane, however, will be entirely saturated, and irrespective of the tension of the groundwater being positive or negative, the layers at both sides of the groundwater plane will convey water in relation to its saturated permeability. Through the saturated

capillary zone of fine textured soils a considerable part of the water flowing through the combined saturated and capillary zones, may be discharged. The gradient is the same for the two zones. The cross-sectional area of flow, however, may be considerably larger than the zone between the groundwater level and the impervious zone.

A certain part of the discharge will take place in the unsaturated capillary zone. No large error is made, if this part of the discharge is neglected. The unsaturated permeability decreases rapidly the more air enters into the soil pores and blocks them for the flow of moisture. The additional unsaturated flow will therefore be of limited importance.

The design runoff in drainage schemes depends on rainfall intensity frequencies and on the storage capacity of the soil. Storage capacities of a magnitude well in excess of the seldomly occuring high rainfall intensity will lower the runoff requirements virtually to the level of the average rainfall excess.

The storage capacity in hydrologic formulae is often assumed to be constant. The magnitude of the storage per unit change in groundwater depth, however, depends not only on the properties of the soil, but also on the depth of the groundwater table, the intensity and direction of the unsaturated flow and the earlier climatic conditions, determining the overall moisture content of the soil.

Insertion of these quantities in the flow equations often prohibits their solution, so the formulae available have been obtained by omitting the capillary effects on storage. With regard to agricultural problems, where the description of the conditions of the unsaturated zone are the purpose for which the formulae are set up and used, the assumption that these points can be neglected are clearly not valid. Ways have to be found to deal with moisture flow with magnitudes of the storage capacity which depend on the flow conditions instead of being constant.

The design constant for the discharge capacity of a drainage system will have to incorporate some aspect of the influence of the unsaturated condition in the rate of penetration of the rainfall, in the cross-sectional area of flow and in the influence of the storage capacity. The design constant should be defined in accordance with the requirements of plant

growth instead of with climatological conditions.

In this respect as a drainage criterion the rate of fall of the groundwater level, equal to the product of discharge in mm and the storage capacity is a considerably better design constant than the rate of discharge itself.

c) Moisture extraction and evapotranspiration

The attention devoted to the transpiration has given more insight into the potential- than **into** the actual magnitude. Often this actual evapotranspiration has been calculated for practical purposes, by multiplying an accurately defined potential avapotranspiration by a rather weakly founded reduction factor. This reduction factor limits in these cases the accuracy of the results, and reduces considerably the value of the highly reliable formulae for the potential evaporation.

This reduction factor depends on the slowing down of the extraction of water from the root zone. This decrease in availability of the soil moisture is dependent on the increased flow resistance of water in the soil layers around the root zone.

The extraction of soil moisture by the roots has much in common with the extraction of water by a well. There is, however, a difference. For the well, the permeability of the soil is independent of the drawdown of the well, but the cross-sectional area of flow decreases with increased drawdown. For the plant root the area of flow remains constant, but the permeability in the unsaturated zone reduces rapidly with increasing depletion of the soil moisture content.

The gradient, lowering the flow potential in the direction of the root, reduces the moisture content which is linked to the moisture potential. In a thin cylinder of soil around the root, this depletion of the moisture content brings about a layer of high flow resistance at the soil-root interface. The degree of flow resistance depends on the relation between the unsaturated conductivity and the soil moisture content, and further on the rate of moisture extraction and the root density. A number of soil, flow and plant factors work together. At lower soil moisture content the reduced availability of the moisture may harm plant growth. At increasing moisture contents the availability of the moisture increases rapidly

and may exceed the evaporative capacity of the atmosphere. Under these circumstances, evapotranspiration is no longer limited by the soil moisture content, but by the atmospheric conditions. It is of importance to know where the point of transition is situated.

The availability of the soil moisture in each layer of the profile will be related to the rate of extraction, the area of the root-soil interface and the capillary conductivity. The plant can maintain its water consumption on the required level by shifting its zone of extraction to the layer, where this relation is most favourable, whereby the extraction may be assisted by the growth of extra roots. The theory of unsaturated flow can give a quantitative description of the reduction of the potential to the actual evapotranspiration under the influence of the soil moisture profile and a description of change in root activity with increase in depth.

d) Aeration and plant growth

Land drainage has as an important purpose the regulation of the air content of the soil. The plant roots breath and require oxygen. This oxygen has to penetrate into the soil and CO_2 has to be removed. The diffusion of gas in the soil moisture may be neglected compared with the gas transfer in the soil air. Drainage has not the ultimate purpose to interfere with the soil moisture, but to manage the soil air. Due to the interrelation between moisture and air content the management of the soil air as objective is carried out by influencing the soil moisture as means.

The conductivity of the soil for air is related to the magnitude of the air filled pore space, irrespective of the soil texture, pore size or other structural parameter. It is assumed that an air content of the soil, surpassing 10% will be sufficient to afford the plant an unhampered availability of oxygen or riddance of CO_2. Also with respect to the flow resistance of gas in the soil as function of the soil air content indications are becoming available. However, the knowledge of the influence of the soil conditions on aeration and of aeration on crop growth lags behind what is known with respect to the influence of soil conditions on availability of soil moisture.

With respect to drainage the influence of soil moisture availability and aeration on crop growth has to be combined. Intensification of aerations tends to evoke drought, intensification of the availability of water tends to evoke waterlogging.

The agricultural purpose of water management is to keep the soil moisture content between the transition point, above which the availability of water is only a function of the atmospheric conditions, and the point above which the aeration is hampered.

The formulae expressing the yield as a function of the availability of soil moisture and the transferability of soil air bring the agricultural problems to the level of objective quantitative calculation. These relations constitute the reliable basis for hydrologic measures. This kind of calculations proves to be valuable due to the knowledge with regards to water and air transfer in the unsaturated zone.

e) Salinity problems

The problems of salinization and leaching is a part of the theory of soil moisture flow in the unsaturated zone, to which the influence of the behaviour of salt is added. The quantity of salt that is transported by capillary action depends on the unsaturated flow and the salt content in the layer from which the flow started. The problem of salinization and leaching can be solved if the unsaturated flow can be described accurately. No new transfer problems are created if the transfer of salt is added to the transfer of water.

It should, however, be taken into account that one aspect of capillary flow shows up in another way with a freshwater-salt water front as with fresh water alone. The moisture velocity in thin capillaries is far smaller than in coarse capillaries. Now one has to consider an unsaturated flowby which the air containing pores are gradually filled with water and air is expelled. The large pores convey the water with the greatest velocity, but from the coarser pores the adjacent finer pores are filled. Due to this deflection of the moisture flow to the finer pores the moisture front moves at the same pace over all the capillaries.

The other aspect is the transfer of moisture through capillaries, filled over their full length without any expelling of moisture from air

filled pores. Here no deflexion sideways takes place from the coarser to the finer pores, because the finer ones are already filled with water. The movement of water in the coarse pores is quick and the salt front in the finer pores will lag behind. The influence of this movement on salinity is that the first traces of rising salt water will reach the surface of the soil in a short time through the coarser pores. The finest pores, however, will contain fresh water even after a long time. With leaching the same relation is found. The salt content at a certain level will change only gradually. Leaching mainly causing flow in the coarse capillaries will have removed the salt water already for a long time, at the time that the removal of the salt water in the fine pores has not progressed much.

The coarser the soil pores the more water is wasted in flushing the already cleansed pores. Flushing therefore should be done with a small quantity of moisture per unit time and no more water shall be used than is needed to move water through the pores which contain water, without filling the capillaries containing air.

The theory of unsaturated flow improves the insight in how to counteract salinity and improves the possibility of finding the most appropriate techniques for leaching.

3. a) The required analyses

The determination of the properties of the unsaturated zone for engineering projects is not nearly as generally accepted as the determination of flow resistances in open water courses or permeabilities of the soil layers. It should, however, be tried to make a number of properties available as basis for design decisions.

The analyses of the desorption curve is one of the first determinations to be carried out. Often these analyses are restricted to field capacity (± pF 2.5) and wilting point (pF 4.2). This is, however, insufficient. The determinations should cover the whole range of moisture stresses between pF 6.0 and 0.5. This enables to calculate any intermediate pF-value by interpolation.

The next point of importance is the determination of the air entry point. This is the moisture tension at which the air just starts to enter

into the soil sample. It marks the transition between saturated and unsaturated conditions. It therefore defines the range of stresses for which the saturated permeability is still valid.

In this respect the saturated permeability of the soil is of importance as the upper limit of the unsaturated permeability. It will be valuable to determine this saturated permeability for the same layers of the profile as for which the desorption curve and air entry point are obtained.

It would also be of importance to determine the unsaturated permeability at different levels of saturation but this determination is still of such a difficulty that it does not seem probable that these values will be obtainable for practical work. It will be discussed that the desorption curve gives possibilities of determining the unsaturated conductivity mathematically.

b) The way in which the analyses should be used

The project planner will need a number of formulae which-by inserting the proper constants- enables him to calculate the moisture content, the moisture stress or the velocity of moisture flow for a given unsaturated soil layer under given boundary conditions. The complexity of the phenomenon of unsaturated flow makes it necessary to simplify in a rigorous way the principles, underlying these formulae. In general it will be advisable to use steady formulae, because the unsteady state flow conditions which may agree better with the actual situation are rather unwieldy and often even not accessable for exact solutions.

A second simplification is the neglect of hysteresis effects, the effect which is the cause that the moisture content, at which an equilibrium is established, will depend on the direction from which the equilibrium is approached. The water content of the soil may attain an equilibrium by extraction orby suppletion of water. The equilibrium moisture content in case of extraction will be lower than the content, in equilibrium with the same moisture stress, but under influence of uptake of water.

This hysteresis effect is not yet sufficiently studied to be accounted for in applied work. A first approximation could be to assume that at the same moisture content the adsorption curve shows moisture

stresses which are 2.5 times larger than the stresses which make up the desorption curve. Because the larger part of the problems for which the unsaturated moisture conditions are of importance deals with extraction of water, the desorption curve is sufficiently accurate.

4. Basic principles of capillarity

In porous material one may distinguish a pore diameter d_p, a particle diameter d_k, a capillary suction Ψ, an air entry stress Ψ_a a saturated permeability k_s, an unsaturated permeability k_c and several pore space values P. (see fig. 5 page 23).

The shape of the capillaries is rather irregular in cross section and ramified and not very accessable to a mathematical description. Still everywhere in the world scientists are trying to solve the relation between the conditions of moisture in straight circular capillaries of constant dimensions and the system of pores and holes in a porous soil (SCHEIDEGGER). Up to now the results of these studies have not yet been very successful. Still this is the only way open to applied science to use the desorption curve as an indication for the magnitude of the unsaturated conductivity.

The capillary conditions in the soil are governed by three relations. These relations are:

1) The relation between the pore size and the soil moisture stress. This relation is given by the equation:

$$\Psi = \alpha_1 \frac{C_p}{F_p}$$

Ψ = soil moisture stress
α_1 = constant of capillary rise
C_p = wetted perimeter of pore
F_p = cross-sectional area of pore

The description of the pore by the parameters F and O can be used for pores of any shape and size.

2) The relation between pore size and conductivity. This relation is given by the equation:

$$K_c = \alpha_2 \frac{F_p^3}{C_p^2 O_p}$$

K_c = capillary conductivity
α = constant of capillary flow
O_p = cross-sectional area of bundle of capillaries

The equation holds for a bundle of straight, not ramified pores of random but constant shape. In the formula a factor F_p/O_p recalculates the amount of flow through the cross sectional area of the pores to the cross-sectional area of the bundle including the thickness of the walls of the pores. The remaining factor $\kappa_2 F_p^2/C_p^2$ represents the equation of Poisseuille for the capillary conductivity for a capillary of random but constant shape.

3) The relation between the soil moisture stress and the soil moisture content.

This relation is given by the equation:

$$\Psi = A \frac{(P_s - V_s)^n}{V_a^m}$$

A= constant of capillary adsorption
P_s= volume of pores allowing capillary flow
V_s= moisture content of these pores
V_a= content of adsorbed non capillary moisture

The equation distinguishes between moisture subject to capillary movement in the uninterrupted body of water in the capillary pores indicated by V_s and the moisture absorbed of capillary bound in contact rings of moisture between adjacent soil particles, indicated by V_a. The latter moisture constitutes an interrupted body of water, not subject to free capillary flow and partly moving in the vapour phase. It should be born in mind that the total pore space P is equal to the capillary pore space P_s plus the pore space of the adsorbed water P_a. Also is the capillary moisture content V_s plus the pore space of the adsorbed water P_a equal to the total moisture content V or

$$P_s - V_s = (P_s + P_a) - (V_s + P_a) = P - V$$

The moisture content V_s may be equated with the total moisture content V.

The relation between the desorption eqation and the conductivity equation has to be situated in the coherent space as far as occupied by the moisture indicated with V_s.

The principle behind the three formulae will first be explained for the simplest case of soil particles of constant size without much regard for the curved and ramified shape of the pores.

a. The relation between wetted perimeter of the pore and the moisture stress

In general the moisture stress depends on the size and shape of the pore as given in the expression:

$$\psi = 0.075 \frac{\text{wetted perimeter}}{\text{cross-sectional area}} \qquad (1)$$

For circular tubes the generally accepted relation between suction and pore diameter is:

$$\psi = \frac{4\, T \cos \Theta}{\rho\, g} \; \frac{1}{d_p} \qquad (2)$$

and for 20° C

$$\psi = \frac{c}{d_p} = \frac{0.30}{d_p} \qquad (3)$$

d_p = pore size

T = surface tension on liquid air interface

Θ = angle of contact between liquid and solid

ρ = density of liquid

g = acceleration of gravity

c = constant, combining the four values given above for 20° C

The value $1/d_p$ clearly comes from $\pi\, d / \frac{\pi d^2}{4}$, the wetted perimeter and cross-sectional area of formula 1.

The pore size d_p is with regard to the intricate shape of the pore rather undefined and a direct measurement is not carried out easily. For a soil, consisting of uniform particles and a pore space of about 40%, the relation was found:

$$\psi = \frac{c}{d_k} = \frac{0.83}{d_k} \qquad d_k = \text{grain size} \qquad (4)$$

b. Geometry of the soil pores

The geometry of the soil pores may be calculated for an arrangement of 8 spheres in a prismatic arrangement with 6 side planes as depicted in fig. 1. The sharp angle has for all side planes the value α. This angle can be varied between 90 and 60 degrees. In fig. 1 number of geometric length are given. The volume I of the prism can be calculated as:

I = AD x BE x FI.

$$I = d_k^3 \left\{ \sin \alpha \sqrt{1 - \frac{2 \cos^2 \alpha}{1 + \cos \alpha}} \right\}$$

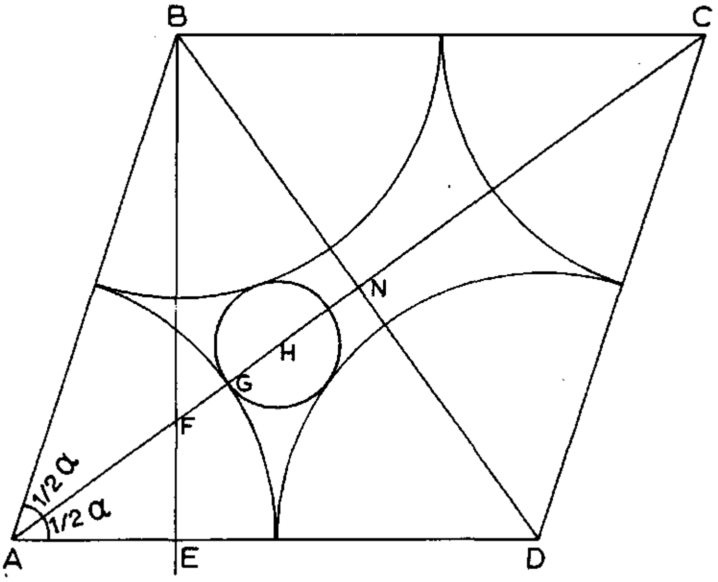


fig 1^a

$AB = AD = d_k$

$BE = d_k \sin\alpha$

$AE = d_k \cos\alpha$

$EF = d_k \dfrac{\sin\alpha \cos\alpha}{1+\cos\alpha}$

$AF = \sqrt{\dfrac{2\cos^2\alpha}{1+\cos\alpha}}$

$AC = d_k \sqrt{2(1+\cos\alpha)}$

$BD = d_k \sqrt{2(1-\cos\alpha)}$

$GH = d_k \sqrt{\dfrac{1}{2(1+\cos\alpha)}} - \dfrac{1}{2}$

$\cos\frac{1}{2}\alpha = \sqrt{\dfrac{1+\cos\alpha}{2}}$

$\sin\frac{1}{2}\alpha = \sqrt{\dfrac{1-\cos\alpha}{2}}$

$tg\frac{1}{2}\alpha = \dfrac{\sin\alpha}{1+\cos\alpha}$

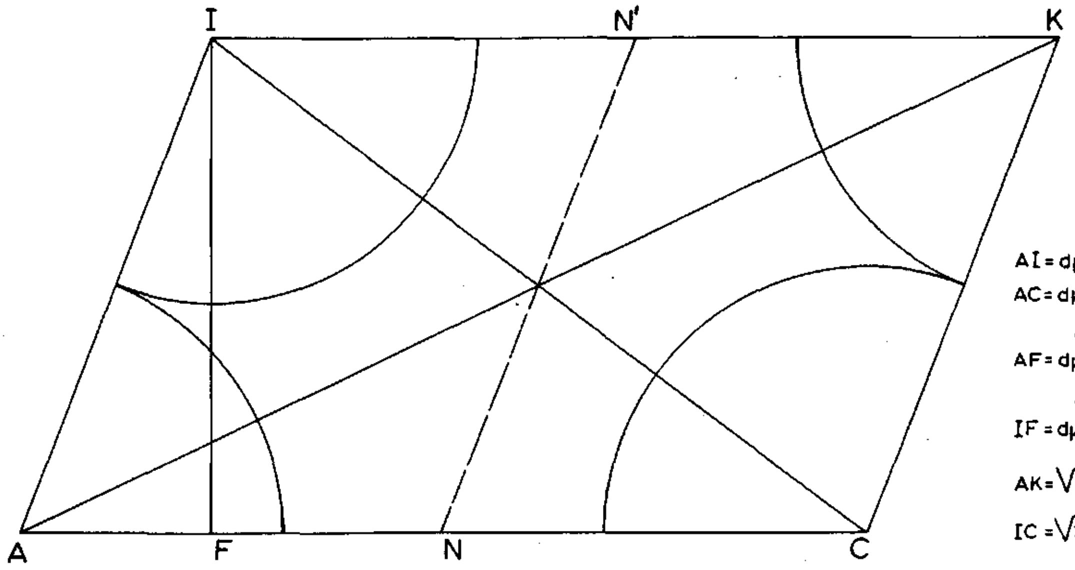


fig 1^b

$AI = d_k$

$AC = d_k \sqrt{2(1+\cos\alpha)}$

$AF = d_k \sqrt{\dfrac{2\cos^2}{1+\cos\alpha}}$

$IF = d_k \sqrt{1 - \dfrac{2\cos^2\alpha}{1+\cos\alpha}}$

$AK = \sqrt{3(1+2\cos\alpha)}$

$IC = \sqrt{3(1-2\cos\alpha)}$

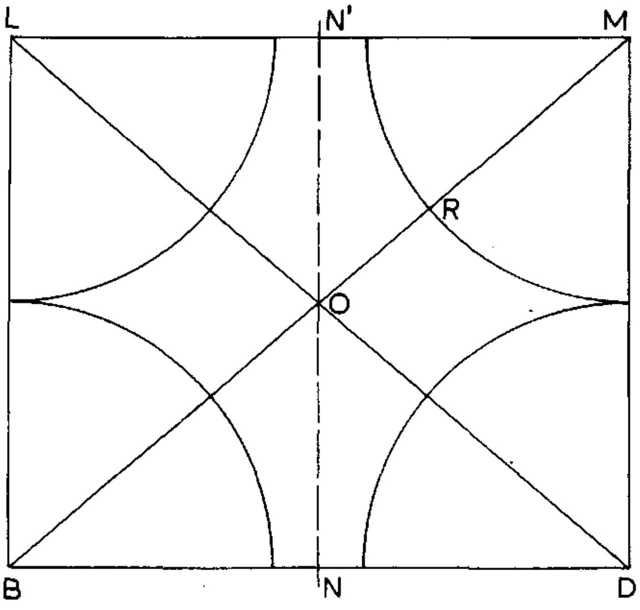


fig 1^c

$BD = d_k \sqrt{2(1-\cos\alpha)}$

$BM = d_k \sqrt{3-2\cos\alpha}$

$OR = d_k \sqrt{3-2\cos\alpha} - \dfrac{1}{2}$

$I = d_k^3 \left\{ \sin\alpha \sqrt{1 - \dfrac{2\cos^2\alpha}{1+\cos\alpha}} \right\}$

$V = d_k^3 \left\{ \sin\alpha \sqrt{1 - \dfrac{2\cos^2\alpha}{1+\cos\alpha}} - \dfrac{\pi}{6} \right\}$

$P = 1 - \dfrac{\frac{\pi}{6}}{\sin\alpha \sqrt{1 - \dfrac{2\cos^2\alpha}{1+\cos\alpha}}}$

The pore volume v is obtained by subtracting the volume of the spheres within the prism, equal to the volume of one sphere from the volume of the prism. The pore space in percentage is equal to 100v/I or:

$$P = 1 - \frac{\pi/6}{\sin\alpha\sqrt{1 - \frac{2\cos^2\alpha}{1+\cos\alpha}}}$$

The following values are calculated.

Table 1

α =	90	85	80	75	70	65	60
P =	0.476	0.471	0.454	0.427	0.387	0.332	0.260
I =	1.000	0.989	0.959	0.913	0.854	0.784	0.707

The ratio of wetted perimeter over cross-sectional area of the pore depends on the position of the cross section.

In fig. 1a this ratio is: $fd_k = \frac{\pi}{\sin\alpha - \pi/4}$ (5)

In fig. 1b this ratio is: $fd_k = \frac{\pi}{\sqrt{2(1+\cos\alpha - 2\cos^2\pi)} - \pi/4}$ (6)

In fig. 1c this ratio is: $fd_k = \frac{\pi}{\sqrt{2(1-\cos\alpha)} - \pi/4}$ (7)

The following values for $f(d_k)$ are obtained:

Table 2

α	90	85	80	75	70	65	60
Fig. 1a formula 5	14.64	14.90	15.75	17.39	20.37	25.97	38.91
Fig. 1b formula 6	2.59	2.44	2.38	2.35	2.38	2.46	2.59
Fig. 1c formula 7	2.59	2.79	3.04	3.36	3.85	4.28	5.00

The values according formula 5 have to be inserted in formula 1 in case of moisture extraction, to which a shape factor α has to be added. This shape factor α allows for the discrepancy between the actual situation and the calculation for uniform spherical soil particles in a homogeneous

pile. Therefore the formula has to be written as:

$$\Psi = 0.075\alpha \ f(d_k) \frac{1}{d_k} \qquad (8)$$

For P = 40% we obtain: $\Psi = 0.075\alpha\, 19.47 \frac{1}{d_k} = 1.46\alpha \frac{1}{d_k}$

Compared with formula 4, the value of α must be of the order of 0.57. This means that in a normal soil the wetted perimeter is smaller or the cross-sectional area of the pore is larger than in a geometrical arrangement of soil particles. The formula becomes therefore:

$\Psi = 0.043 \ f(d_k) \frac{1}{d_k}$ with for d_k the values of table 2 according to formula 5.

The meniscus of the largest size is calculated according to formula 6. For a cubicle arrangement the largest meniscus is 5.66 times as large as the smallest. In the tetrahedronal arrangement this ratio is 1 to 15. The calculation of formula 6 does not however describe a real pore in its longitudinal direction, but more at right angles. The largest pore size will be more near the solution of formula 7, and this formula shows that the largest pore is 5.66 to 7.78 times as large as the smallest.

The properties of a geometric arrangement differ from the properties of an actual soil because of irregularities in the shape of the particles and in the arrangement. The pore spaces above 47.6% or below 26% have no simple comparison in the geometrical arrangement.

Problem 1. A soil has a pore space of 45% and is compacted to 38%. The particle size is uniform, the diameter is 80μ.

Determine the size of the smallest and largest pores before compaction. Determine the reduction in percentage for the largest and smallest pore due to compaction.

Example 2. Make an estimate of the difference in the height of capillary rise between a medium consolidated soil of 40% pore space and a soil at maximum consolidation. The particle size is 100μ.

The difference in height of capillary rise for maximum and normal consolidation is:

$$\Psi_{mc} - \Psi_{c} = \frac{0.075\alpha}{d_k}\left(f(d_k)_c - f(d_k)_{mc}\right)$$

$$= \frac{0.075.057}{0.01}(38.91 - 1940)$$

$$= 83.5 \text{ cm}$$

Example 3. A plastic drain tube of 5 cm diameter and 5 gram weight per cm length has slits of 3 cm length and 0.6 mm width. The capillary force of the meniscus in these slits is able to exclude water from the tube in case this is dry. Will the drain tube float on the water in the trench or will it sink? What width should a rectangular hole of 1 cm length be given to make sure that water will not be excluded and floating will be prevented?

A cylinder of radius r, floating in water with its axis of rotation parallel to the water surface, will per cm length displace a volume of water, dependant on the ratio of immersion h/r or F/r^2, F being the circle segment in cm^2 and h the depth of immersion (see fig. 2)

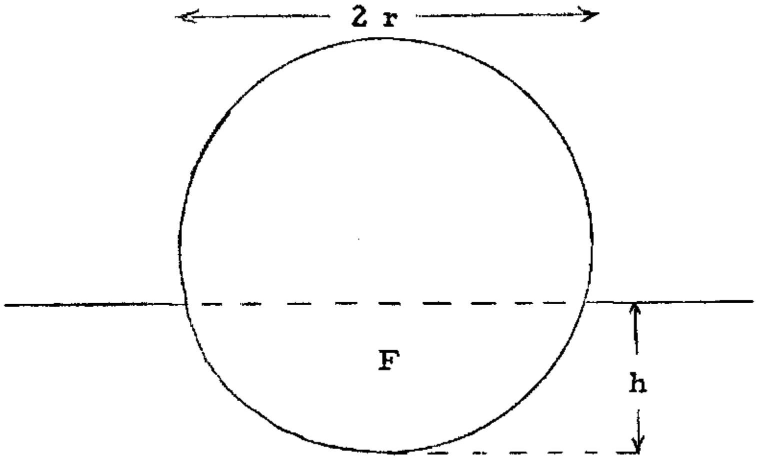


fig. 2

Table 3 on circular dimensions, to be taken from any mathematical table, shows the following relation between h/r and F/r^2.

Table 3

h/r	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
F/r^2	0.06	0.16	0.30	0.44	0.61	0.79	0.98	1.17

The depth of immersion h depends on the equilibrium between displaced water and weight. The displacement of a tube of 5 gr, equal to 5 cc of water, is calculated as:

$$d^2\left(\frac{F}{d^2}\right) = 5^2\left(\frac{F}{d^2}\right) = 5cc\,\frac{F}{d^2} = \frac{5}{5^2} = 0.20\;\frac{F}{r^2} = 4 \times 0.20 = 0.80$$

The corresponding value for $F/r^2 = 0.80$ is according table 3 $h/r = 0.60$. The drain therefore will float with a depth of immersion of 0.60×2.5 cm $= 1.5$ cm.

The slits in the tube with a length $l = 3$ cm and a width $w = 0.06$ cm can resist a capillary tension of

$$\Psi = 0.075\;\frac{2(l+w)}{lw} = 0.15\left(\frac{1}{l} + \frac{1}{w}\right) = 0.15 \times 17$$

The menisci can hold the pressure head of 1.5 cm and then has still 1 cm in reserve. The drain tube will float in the trench. For a rectangular slit of 1 cm length and a width w, which cannot hold more than 1.5 cm of pressure the width is calculated as:

$$\Psi = 0.15\left(\frac{1}{l} + \frac{1}{w}\right) \quad w = \frac{0.15\,l}{l\Psi - 0.15} \quad w = \frac{0.15}{1.5 - 0.15} = 0.111 \text{ cm.}$$

A slit of 10 mm long and 1.11 mm wide at every few meters is sufficient to keep this drain tube from floating.

5. a) The conductivity of porous material of uniform particle size

The conductivity of a straight, circular, not ramified capillary of uniform cross-section is expressed by the formula of Poisseuille. This formula shows a clear similarity with the flow formula of Darcy.

$$q = \frac{\pi r^4}{8\eta}\;\frac{d\Psi}{dl} = \frac{1}{2\eta}\left(\frac{\pi r^2}{2\pi r}\right)^2 \pi r^2\;\frac{d\Psi}{dl} \quad (9) \quad \text{compare with } q = k\,f\,i$$

$f = \pi r^2$ = cross-sectional area of capillary

$i = \frac{d\Psi}{dl}$ = gradient

$k = \frac{1}{2\eta}\left(\frac{\pi r^2}{2\pi r}\right)^2$ = conductivity constant

r = radius of tube

η = viscosity of fluid in ρg units

$k = r^2/8\eta$ = conductivity constant

ρ = specific gravity

g = acceleration of gravity

This formula is adapted to non-circular cross-sections of pores by expressing the conductivity constant in terms of the wetted perimeter Cd_k and the cross-sectional area of flow Fd^2_k. Further for soils the intensity of flow should be expressed in terms of the area taken

in by the capillaries - pore plus soil Od^2_k - instead of by the pore alone.

The formula for the conductivity of a unit area of soil k expressed in the area of flow and the wetted perimeter of a single capillary now becomes:

$$k = \frac{\beta}{2\eta} \frac{(\text{mean area of flow})^3}{(\text{mean wetted perimeter})^2 (\text{soil area})} = \frac{\beta}{2\eta} \frac{F^3}{C^2 O} d^2_k \qquad (10)$$

β = shape factor

The cross-sectional area of the tube in this case, however, differs from that of the capillary suction. In this case an average value for F and C should be taken instead of the largest or the smallest.

In this case F is obtained by dividing the total pore volume $(I - \pi/6)dx^3$ by the pore length d_k or

$$F = d^2_k \left\{ \sin\alpha \sqrt{1 - \frac{2\cos^2\alpha}{1+\cos\alpha}} - \frac{\pi}{6} \right\} \qquad (11)$$

The wetted perimeter is found by dividing the surface of the spheres πd^2_k by the length d_k of the pore, so that:

$$C = \pi d_k$$

The area of soil particles plus pore area is:

$$O = \sin\alpha \, d^2_k$$

For k is obtained the expression:

$$k = \frac{\beta}{2\eta} \frac{\left\{ \sin\alpha \sqrt{1 - \frac{2\cos^2\alpha}{1+\cos\alpha}} - \frac{\pi}{6} \right\}^3}{\pi^2 \sin\alpha} d^2_k \qquad (12)$$

For the successive values of the term comprising this variable becomes:

Tabel 4

α	90	85	80	75	70	65	60
$f(\alpha) \times 10^{-6}$	1095	1026	850	620	388	198	73
$\frac{1}{2\eta} f(\alpha) \times 10^{-6}$	464	435	360	263	165	84	31

For $\frac{1}{2\eta}$ is used: 49050 for k and d_k in cm. sec

or: 0.424 for k in m/24 hours and d_k in micron

Experimentally the relation between k and d^2_k for a pore space of 40% and the rather homogeneous particle size of graded samples of sand was found as

$$k_s = 0.000370\ d^2_k$$

From pore space geometry is derived:

$$k_s = 0.000197\ d^2_k$$

The factor β in formula 12, accounting for the variations in shape of the capillary cross sections, the divergence from the geometrical arrangement of the particles and of the spherical form has apparently the value of 370/197 or 1.88.

The saturated conductivity of an arrangement of spheres of uniform diameter becomes

$$k = 0.4905 \times 1.88 \times (f(\alpha)\ d^2_k$$

$$k = 0.92\ f\ (\alpha)\ d^2_k \qquad (13)$$

b) The indicative value of the capillary tension for capillary conductivity

The capillary tension was described by $\alpha \cdot \frac{C}{F}$. The capillary conductivity was given by $\beta\ (\frac{F}{C})^2 \cdot \frac{F}{O}$. One may assume that the factor F/O will not be influenced very much by the irregularities of the shape and size of the soil particles. The discrepancy between calculated and observed capillary tension and conductivity are accounted for by the shape factors α and β

The possibility to calculate the capillary conductivity from the capillary tension is based on the fact, that the value $(\frac{1}{\alpha}\ \frac{F}{C})^2$ should be equal to $\beta\ (\frac{F}{C})^2$ or

$$\frac{1}{\alpha^2} = \beta$$

In reality $\frac{1}{\alpha^2} = \frac{1}{0.57^2} = 3.08 = 1.73 \times 1.88 = 1.73\ \beta$

This means that the shape factors, which were not accounted for as tortuosity and ramification decrease the conductivity by a factor of 1.73 . As long as no values, originating from direct observations concerning this factor can be made available the value of 1.7 may be the best estimate for practical use.

Example 4. Calculate the percentage of change in permeability, due to maximum consolidation of a soil as compared with average consolidation.

Assuming average consolidation at $P = 40\%$ and maximum consolidation at $P = 26\%$, the percentage ratio is expressed by:

$$\frac{100\, k_{18}}{k_{40}} = 100 . \frac{0.92\ .\ 0.000031}{0.92\ .\ 0.000197} \frac{d^2k}{d^2k} = 100 . \frac{31}{197} = 15.6\%$$

The percentage of reduction of the permeability does not depend on the pore size. By sonsolidation the permeability is reduced considerably.

Example 5. A soil is covered with a thin film of water, kept at constant level. The soil is of uniform pore size, no sealing occurs, no capillary suction is assumed in the soil. The particle size is 100μ, the pore space is 40%. How much water will infiltrate under these steady conditions ? Also determine the rate of infiltration, assuming that a thin layer of fine material with a particle size of 10μ, due to sealing, covers the soil. The pore space is 35%.

Using the formula $k_s = 0.00037\, d^2k$ the rate of infiltration is calculated as $k_s = 3.70$ m/24 hours

For the pore space of 35% the formula to be used reads

$$k_s = 0.00011\, d^2k$$

$$k_s = 0.011 \text{ m/24 hours}$$

The decrease in particle size due to separation of the coarse and fine fraction has a larger influence on permeability than the decrease in pore space due to compaction.

6. The moisture retaining capacity

The capacity of a soil to retain moisture against suction depends on the negative pressure, exercised by capillary forces. When they

balance against the suction forces, no pressure gradient is present and no moisture flow will appear. The capillary forces depend on the wetted perimeter and the area of the meniscus.

By increasing the moisture stress first the meniscus in the smallest pore, described by formula 5, will give way and only the moisture, present in the circular wedges at the contact points of two spheres will remain in the soil. In this case the capillary force in the moisture of the circular wedges is higher than the suction force described by formula 5, which emptied the pore. The moisture content will decrease to a small extent by increasing moisture stress from zero to a first small value due to the larger capillaries, for which the tension necessary to empty them, is described by formula 6 and 7. The decrease in moisture content, however, cannot be large because these larger pores are only interconnected by the pores of formula 5. Is the moisture stress according to formula 5 reached, then a considerable part of the volume of the pores is emptied.

A further increase of the tension will extract the moisture of the circular wedges, see fig. 3, which withdraw to a position more and more near to the contact point.

Fig. 4 shows the type of relation that may be expected between ψ and the moisture content v. The range AB depicts the tension at which all pores still can hold their water. The part BC is the range of emptying of the larger pores. Between C and D the small pores loose their moisture and beyond point D the wedges of different contact points are no longer in mutual contact. In the range between points D and E the moisture withdraws to the narrowest part of the slit between the particles.

In case of particles of different size, the properties of the pore arrangement becomes less distinct. However, certain general aspects may show up as the presence of a saturated capillary zone (fig. 4, range AB) or the large variation in v at small variation of ψ (fig 4, range CD). In fig. 5 the general shape of the moisture retention curve or desorption curve is given. It is habitual to plot these curves as log ψ against v.

fig. 3

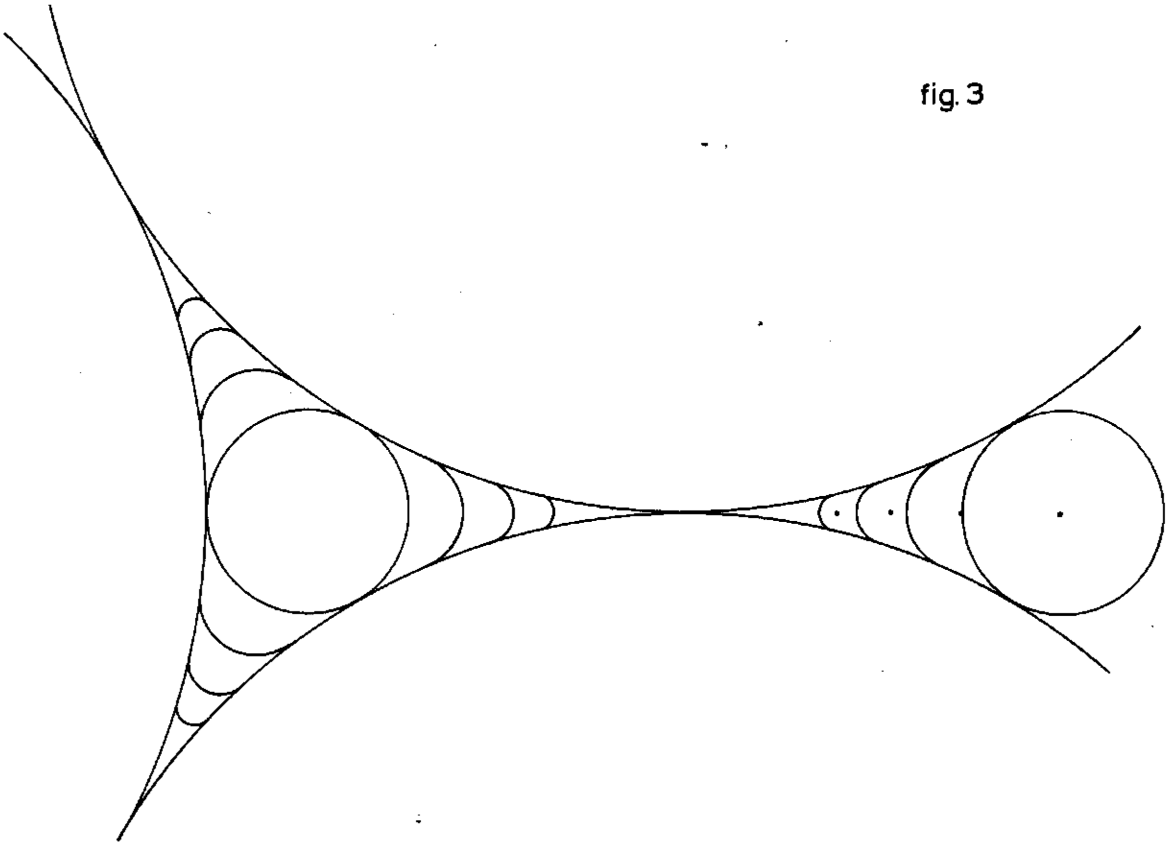


fig. 4

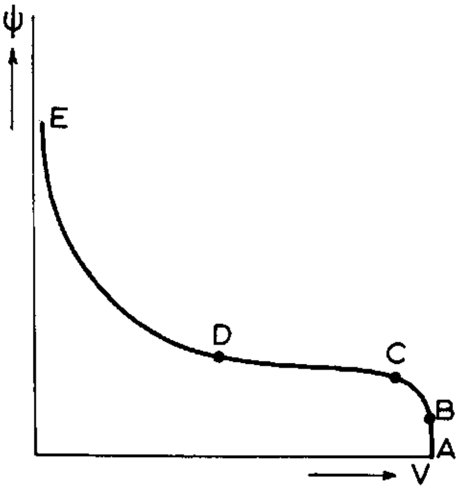
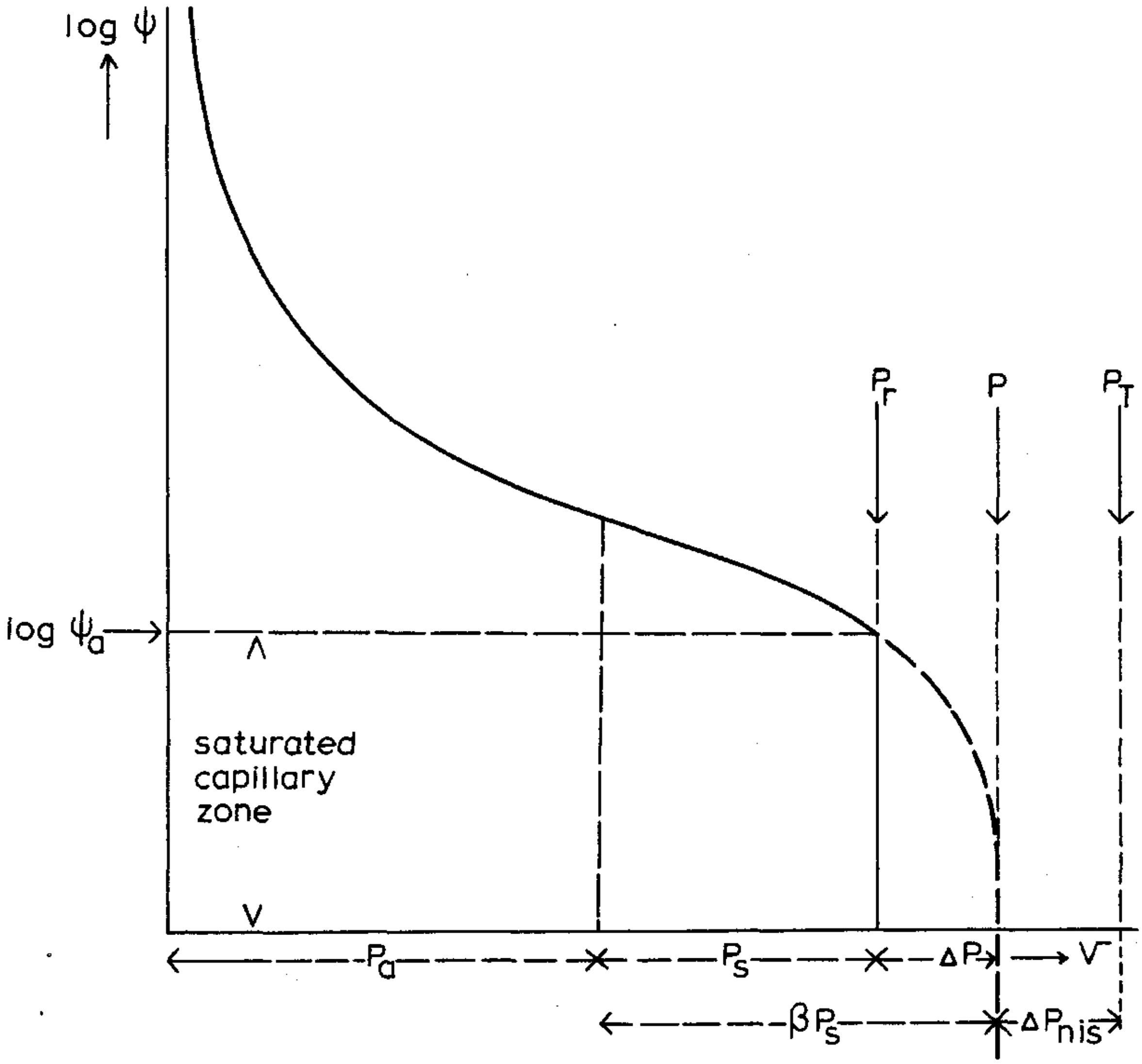


fig 5


log ψ
log ψ_a
Λ
saturated capillary zone
V
P_r
P
P_T
P_a
P_s
ΔP
V⁻
βP_s
ΔP_nis

The origin of the desorption curve explains that the range ABC will depend strongly on both particle size and arrangement, the range DE depends on the size of the wedge between any two particles and therefore on the particle size only. In this part the influence of the arrangement is small. The part of low moisture content is mainly dependent on the soil texture. The range of the high moisture contents is mainly dependent on the soil structure.

a. The mathemetical expression of the pF-curve

The two aspects of the desorption curve, the influence of adsorption processes and the influence of soil structure have to be incorporated in the formula for the stress-volume relation.

Giving the moisture tension influenced by adsorption by Ψ_a and the zero point of this scale by Ψ_{ao} then adsorption processes are rendered by:

$$\frac{\Psi_r}{\Psi_{ro}} = v^{-m} \quad \text{or} \log \Psi_r - \log \Psi_{ro} = -m \log v$$

The influence of the soil structure is more difficult to describe. It will be of the nature of a skew probability distribution for which no formula is clearly indicated. The wet end of the desorption curve, however, determines the unsaturated permeability and the curve for this property enables to draw conclusions with respect to the pF- curve.

From the formula for the unsaturated permeability (§ 9) for the wet end of the desorption curve an equation may be derived. The part of the moisture stress influenced by soil structure is indicated by Ψ_s, the zero point by Ψ_{so}. The structure indication in the desorption curve (for the derivation see formula 23) can be described by:

$$v_s = \frac{P_s}{2} \left(\alpha^2 \psi^2 + 2\alpha\psi + 2\right) e^{-\alpha\psi} \qquad (14)$$

The formula v - of importance for the calculation of the unsaturated flow - is rather unwieldly for other purposes. It was found that the formula may quite satisfactory be used as a apart of a formula with wider applicability:

$$\frac{\Psi_s}{\Psi_{so}} = \left(\frac{P_s}{P_s - v_s}\right)^{-n} = \left(\frac{P_s}{P - v}\right)^{-n}$$

Combining the two effects on Ψ, the relation between Ψ and v is given by:

$$\begin{array}{llcl}
\log \Psi_{ro} & -\log \Psi & = & m \log v \\
\log \Psi_{so} - n \log P_s & -\log \Psi & = & -n \log (P - v) \\
\hline
\log \left(\dfrac{\Psi_{ro}\, \Psi_{so}}{P_s{}^n} \right) & -\log \Psi & = & m \log v - n \log (P-v)
\end{array}$$

and by lumping the
constants together:

$$\log A \quad - \log \Psi = m \log v - n \log (P-v) \tag{15}$$

$$\frac{A}{\Psi} = \frac{v^m}{(P - v)^n} \tag{16}$$

By inserting the formula derived from the relation for the unsaturated permeability whilst subtracting P_a from P and v, and taking $P - P_a = \beta P_s$ the formula becomes:

$$\frac{A}{\Psi} = \frac{v^m}{P_s{}^n \left(\beta - \frac{\alpha^2 \Psi^2 + 2\alpha\Psi + 2}{2} e^{-\alpha\Psi} \right)^n} \tag{17}$$

The value of β is inserted in the formula in order to detach the real maximum for the pore size from the assymptotical maximum. The assymptotical maximum P governs the pore size distribution. The real maximum P_r determines, what the actual largest pore will be. Because a largest pore size of necessity must exist, this means that $P - P_r = \Delta P$ must have a positive value and $\beta = \frac{P_s + \Delta P}{P_s}$ must be somewhat larger than 1.00.

Formula 15 is best suited for nomographical elaborations, formula 17 is used to link the desorption curve to the curve for unsaturated conductivity. In formulae 15 and 16 the value of P which gives a good fit of the observations, often is larger than the real pore space P_r, indicating in the same way as mentioned for P_s and βP_s that an air entry stress of some magnitude is present. In those cases, instead of P_r or P_s the value $P_r + \Delta P$ or $P_s + \Delta P$ should be inserted in each of the formulae 14 - 17. In fig. 5 the experimental pF-curve is depicted. For

a number of samples the pore space, moisture contents at successive stages of moisture stress and the constants of formulae 15 or 16 are given in table 5. In fig. 6 - 9 the mutual relation between the constants are shown for the formula

$$\left(\frac{A}{\Psi}\right)^{b} = \frac{v^{p}}{(P - v)^{1-p}} \tag{18}$$

which will be encountered in the assessment of the constants. For the values of ΔP negative, an important indication of non-intersphere pore space is encountered, showing that cracks and holes of a nature, differing from the normal pore structure are present.

This non-intersphere pore space may be small enough to exert some capillary pull and should not be understood as non-capillary pore space. The distinction is made with regard to the origin and not to the size. The most general description of the total pore space P_T as the sum of the real pore space P_r, the pore space excess ΔP and the non-intersphere pore space ΔP_{nis} is:

$$P_T = P_r + \Delta P + \Delta P_{nis}$$

see fig. 5

Example 6. From a desorption curve is known that the following constants are valid

$p = 0.75$ $\log A = 6$ $\Psi_a = 50$ cm
$b = 0.25$ $P_T = 54\%$ $P_r = 48\%$

a) Calculate the non-intersphere pore space and the pore space excess
b) Calculate the soil moisture stress at one third saturation or $v = 18\%$
c) Calculate the moisture content at wilting point

Table 5

No:	Moisturecontent in volume %: pF:										Constants: (calculated)				Soil-properties		
	Pore-space	0.4	1.0	1.5	2.0	2.3	2.7	3.4	4.2	6.0	A	b	p	P	Silt %	Humus %	U
6	50.4	47.9	47.3	46.1	40.2	34.7	29.1	15.9	9.2	1.6	5.47	0.31	0.78	48.-	9	5	60
12	52.9	53.2	50.1	47.1	40.6	33.3	29.2	15.7	9.4	2.4	4.95	0.28	0.71	53.5	14	9	75
16	43.7	43.6	40.2	37.3	27.-	22.-	20.5	10.2	6.1	1.5	5.50	0.28	0.82	43.7	8	3	75
22	48.1	46.3	42.3	40.7	33.4	29.6	23.-	14.2	8.1	2.3	4.94	0.27	0.72	46.6	12	6	80
26	34.4	33.1	32.7	31.6	21.7	18.5	12.-	1.9	1.7	0.7	3.08	0.49	0.65	33.6	1	1	80
28	59.9	61.-	56.7	57.1	54.4	52.5	49.7	42.3	23.8	2.-	1.80	0.30	0.29	61.3	23	10	99
	51.3	53.1	51.3	47.7	39.7	37.5	34.4	23.-	16.4	3.6	3.77	0.19	0.55	58.7	21	6	95
35	48.6	49.7	49.-	46.9	43.4	40.7	34.9	26.7	18.8	3.3	3.80	0.22	0.52	52.5	30	5	100
40		45.5	42.6	41.9	37.-	31.3	26.6	12.1	7.-	1.8	4.21	0.34	0.67	44.9	24	5	110
43		81.3	77.1	74.9	71.2	68.8	56.6	28.1	18.9	2.9	5.77	0.34	0.78	78.2	12	32	127
44	58.6	45.7	44.-	43.-	40.1		36.1	27.4	19.4	4.1	3.28	0.22	0.46	47.3	18	3	120
45		37.6	33.1	32.4	20.3	8.1	3.3	1.9	1.1	0.4	1.94	0.87	0.47	36.-	3	1	123
55	51.4	49.6	48.9	48.5	40.2		29.7	15.7	11.3	3.3	6.99	0.27	0.90	49.4	0	8	90
62	80.8	84.1	82.8	79.8	66.4			40.8	29.4	5.5	4.30	0.20	0.58	90.8	1	88	237
63	72.3	71.9	71.1	70.8	67.7			40.9	22.5	4.8	5.80	0.30	0.71	71.8	4	29	125
64	84.8	82.3	77.6	74.5	61.9			46.3	27.5	4.4	-5.81	0.13	0.16	106.-	1	88	315
69	33.1	33.3	32.5	32.4	31.3			15.4	10.3	1.8	6.60	0.27	0.87	32.9	10	1	150
71	60.4	59.2	57.8	56.8	50.8		39.5	30.-	21.8	4.7	6.83	0.21	0.75	60.2	12	14	117
73	55.-	54.-	52.6	51.-	49.4		44.9	31.5	16.9	3.6	5.04	0.31	0.65	53.-	17	5	210
76	48.-	44.3	44.3	42.7	45.-		29.1	20.5	14.6	3.2	6.81	0.21	0.78	45.5	15	5	136
82	47.3	48.7		46.9	45.6	44.3	42.7	40.1	25.4	9.4	5.14	0.28	0.58	47.8	78	2	70
83	47.8	50.8		50.7	45.5	43.9	39.6	25.6	22.7	6.4	7.51	0.21	0.80	51.4	22	7	75
85	42.4	43.4		42.6	40.-	38.3	36.6	33.2	24.1	9.5	2.70	0.22	0.37	45.-	51	2	75
89	35.5	32.1		26.4	23.6	16.-	14.-	7.-	6.-	1.5	5.25	0.29	0.63	32.2	8	0	85
90	36.9	37.5		34.5	30.2	28.5	23.6	20.3	13.1	2.5	-4.80	0.13	0.14	49.4	23	1	95
100	33.1	34.6	32.8	32.-	27.9	21.9	20.1	7.6	4.8	2.7	4.30	0.34	0.72	34.2	0	0	113
101	47.9	38.-	37.6	36.2	24.4		8.1	2.5	1.8	0.8	3.50	0.63	0.82	37.8	0	0	125

fig. 6

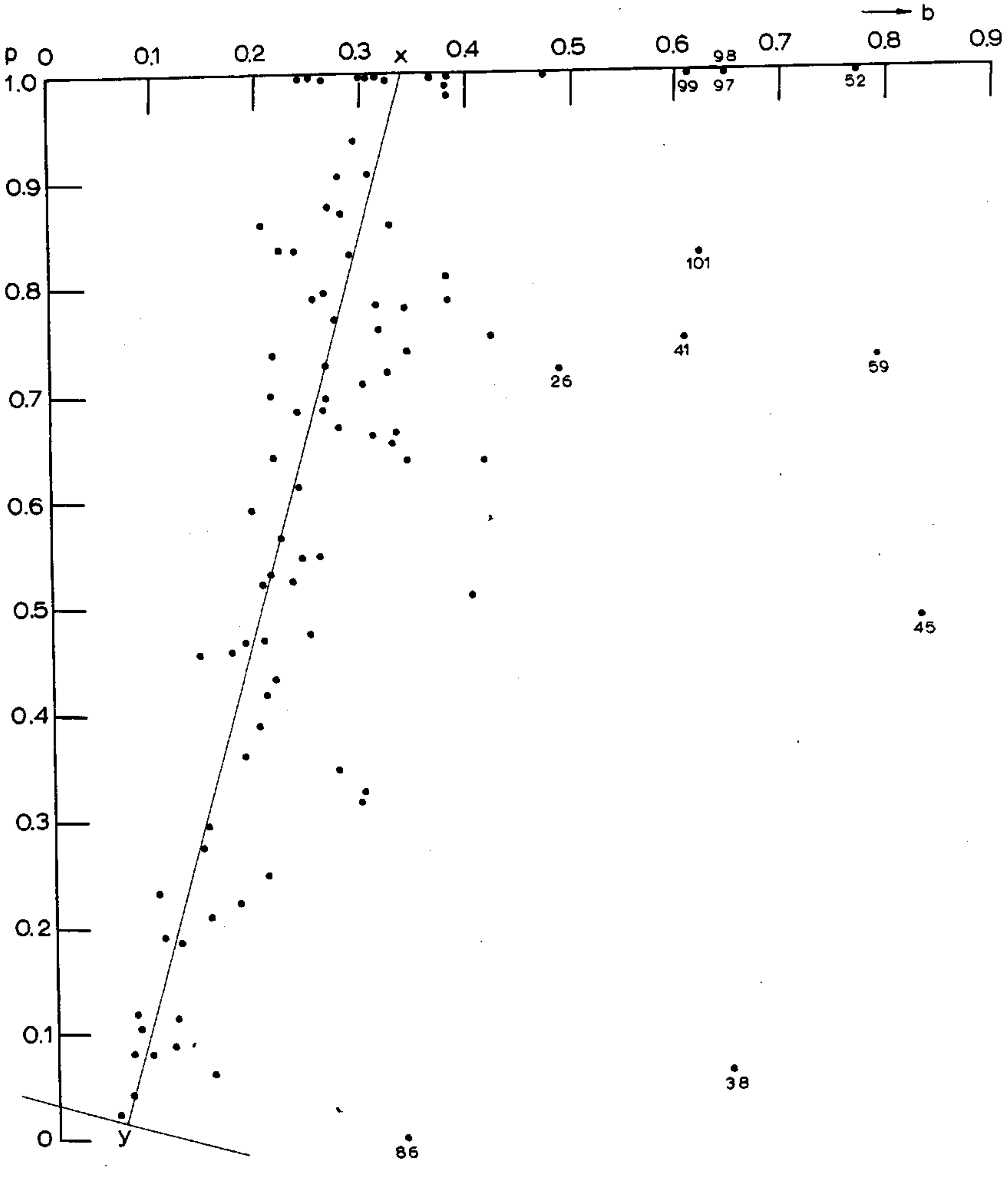


fig. 7

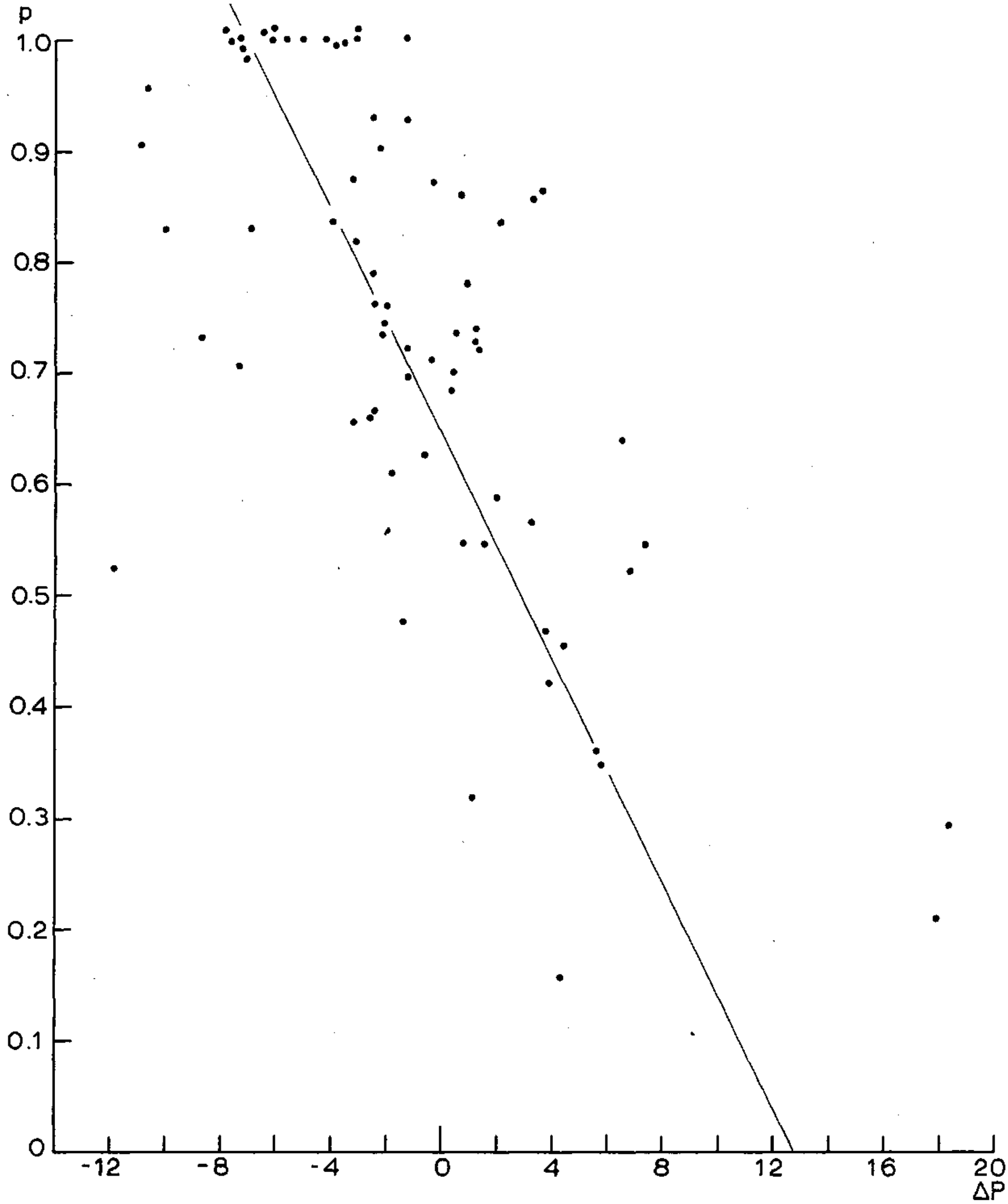
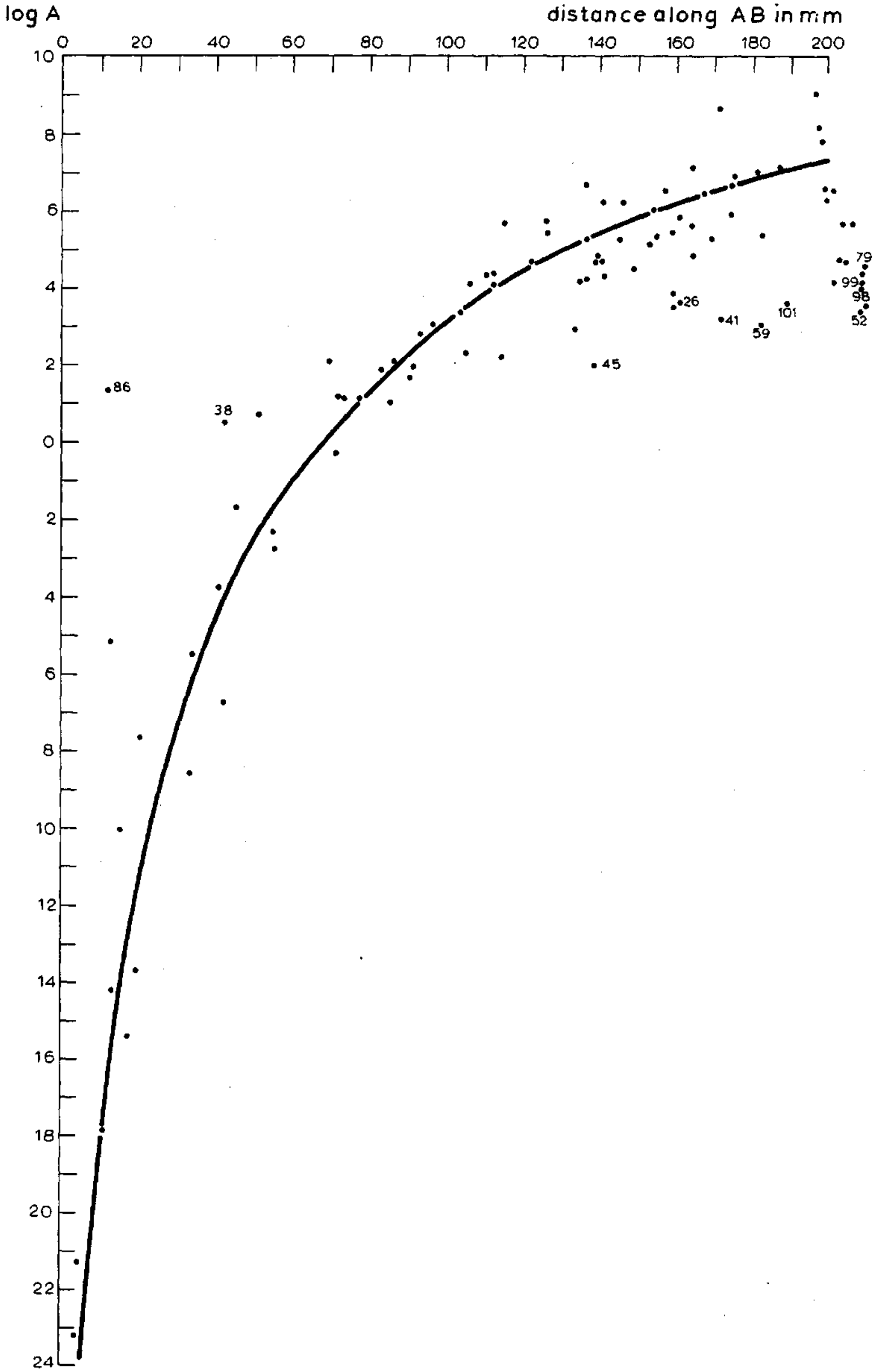
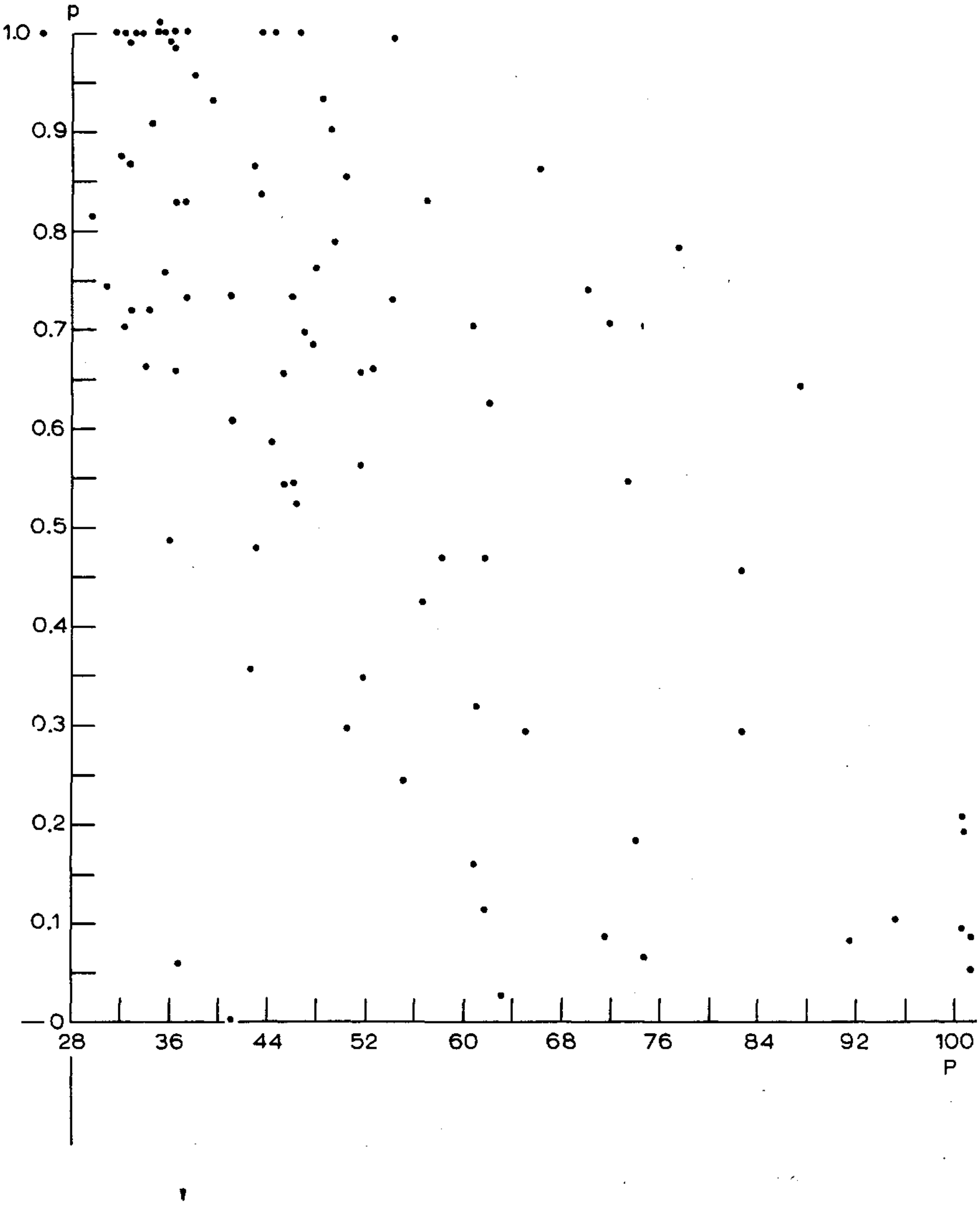


fig 8



64a127 2/3.4

fig. 9



a) In the formula

$$b(\log A - pF) = p \log v - (1 - p)\log (P_r + \Delta P - v)$$

the data are inserted:

$$0.25(6 - \log 50) = 0.75 \log 48 - 0.25 \log (48 + \Delta P - 48)$$

$$0.25 \times 4.30 = 0.75 \times 1.681 - 0.25 \log \Delta P$$

$$\Delta P = 5.53$$

$$\Delta P + \Delta P_{nis} = P_T - P_r = 54 - 48 = 6\%$$

$$\Delta P_{nis} = P_T - P_r + \Delta P = 54 - (48 + 5.53) = 0.47\%$$

b) In the formula

$$b(A - pF) = p \log v - (1-p)\log (P_r + \Delta P - v)$$

the data are inserted:

$$0.25(6 - pF) = 0.75 \log 18 - 0.25 \log (53.53 - 18)$$

$$6 - pF = 3 \times 1.255 - 1.551$$

$$pF = 3.786$$

$$\Psi = 6110 \text{ cm}$$

c) After inserting the data in the formula, the following equation has to be solved:

$$3 \log v = 1.8 + \log(53.53 - v)$$

This has to be done by successive steps. A solution in one turn is not possible. A scheme of calculation is given below.

v	10	13	13.50	13.60	13.607
53.53-v	43.53	40.53	40.03	39.93	39.923
log(53.53-v)	1.6388	1.6078	1.6024	1.6013	1.60122
+ 1.8	1.8	1.8	1.8	1.8	1.8
sum	3.4388	3.4078	3.4024	3.4013	3.40122
sum/3=log v	1.1463	1.1359	1.1341	1.1338	1.13374
v	14.01	13.67	13.62	13.608	13.6063

b. The first derivative of the pF-formula

Problems will be met, for which the change in the soil moisture stress should be compared with the accessory change in moisture content. The soil moisture stress, for instance, is related to the availability of

moisture for plant growth and the quantity of moisture with the same availability may be of interest. This can be calculated with formula 15 or 16, but it is also possible to use the first derivative. From the formula

$$b \log \frac{A}{\Psi} = p \log v - (1 - p) \log (P - v)$$

can be derived.

$$- b \frac{d\Psi}{\Psi} = \left(\frac{p}{v} + \frac{1 - p}{P - v} \right) dv \qquad (19)$$

In other cases the desire is to know how the value of $\frac{1}{\Psi}$ changes with the moisture content. In that case it should be remembered that

$$\frac{d \frac{1}{\Psi}}{dv} = \frac{- d\Psi}{\Psi^2 \, dv}$$

Using formula 19, in such a case the value of $\frac{d \frac{1}{\Psi}}{dv}$ should be determined with

$$\frac{-d\Psi}{\Psi^2 dv} = - \frac{1}{\Psi} \frac{d\Psi}{\Psi \, dv} = + \frac{1}{\Psi} \left(\frac{p}{v} + \frac{1 - p}{P - v} \right) \frac{1}{-b} \qquad (20)$$

This formula can be useful. The inverse of the first derivative given in formula 19 for instance can be used to describe the skew probability distribution of the pore diameters.

Problem 7. Calculate the pore size distribution curve for the soil constants: $p = 0.25$ $\quad A = - 2.5$ $\quad \Delta P = 0$ $\quad \Psi_a = 0$
$b = 0.125$ $\quad P = 50$ $\quad \Delta P_{nis} = 0$
and plot the results against the log of the pore size r.

Insert formula 3 in formula 19 and use instead of formula 19:

$$b \frac{dr}{r} = \left(\frac{p}{v} + \frac{1-p}{P-v} \right) dv \text{ and } \frac{dr}{dv} = \frac{1}{b} \frac{0.3}{A} \frac{v^{p/b}}{(P-v)^{(1-p)/b}} \left(\frac{p}{v} + \frac{1-p}{P-v} \right)$$

Calculate $\frac{dv}{dr}$ for a number of successive values of v.

Example 8. Calculate the effect of the soil moisture content on the rate of change of the availability of the soil moisture to the plant, using formula 20 and the constants of problem 7.

In the eight h row the pF is given. The calculation shows that at a moisture content of some 17% wilting point is reached and evaporation by the plant will come to a stop. At some 35% of moisture the availability of the soil moisture will no longer determine the rate of moisture extraction. The climate will be the limiting factor with respect to evapotranspiration. From 40% moisture content upward waterlogging will appear. Between these upper and lower limits the calculation makes sense. In the last row the resistance which is experienced against moisture extraction shows that for every decrease of the moisture content of 5% the resistance increases to 5 to 10 fold. The lower the moisture content, the longer the plant makes the remaining store last.

v	10	15	20	25	30	35	40	45	49
P-v	40	35	30	25	20	15	10	5	1
log v	1.000	1.176	1.301	1.398	1.477	1.544	1.602	1.653	1.690
log(P - v)	1.602	1.544	1.477	1.398	1.301	1.176	1.000	0.699	0.000
-2logv	2.000	2.352	2.602	2.796	2.954	3.088	3.204	3.306	3.380
+6 log(P-v)	9.612	9.264	8.862	8.388	7.806	7.056	6.000	4.194	0.000
- 2.5	2.500	2.500	2.500	2.500	2.500	2.500	2.500	2.500	2.500
sum = pF	+5.112	+4.412	+3.760	+3.092	+2.352	+1.468	+0.296	-1.612	-5.880
$1/\Psi$	$7.73.10^{-6}$	$3.87.10^{-5}$	$1.74.10^{-4}$	$8.09.10^{-4}$	$4.45.10^{-3}$	$3.40.10^{-2}$	$5.06.10^{-1}$	41.0	$7.60.10^{-5}$
2/v	0.2000	0.1333	0.1000	0.0800	0.0667	0.0571	0.0500	0.0444	0.0408
6/P-v	0.1500	0.1714	0.2000	0.2400	0.3000	0.4000	0.6000	1.2000	6.0000
sum	0.3500	0.3047	0.3000	0.3200	0.3667	0.4571	0.6500	1.2444	6.0408
sum/Ψ = rate of extraction	$2.71.10^{-6}$	$1.18.10^{-5}$	$5.22.10^{-5}$	$2.59.10^{-4}$	$1.63.10^{-3}$	$1.55.10^{-2}$	$3.29.10^{-1}$	51.0	$4.59.10^{-6}$
resistance to becoming available			19200	3860	610	65			

7. Graphical solution of the log Ψ and v values

The numerical solution of log Ψ by given v or of v by given log Ψ is accurate but laborious. In cases where the accuracy is not of prime importance, a graphical solution of the equation of the desorption curve is possible. A nomogram with three parallel axes can easily be made from which for each value of v the value of log Ψ may be derived or for which the log value may be used to solve the value of v. The nomogram may also be used to calibrate a scale for v, matching the linear scale for log Ψ. The log Ψ values are marked on one side of the line and the non-linear v scale on the other side.

The nomogram is not meant to be used many times for different desorption curves. The drawing is so simple that for every desorption curve a new sheet can be constructed.

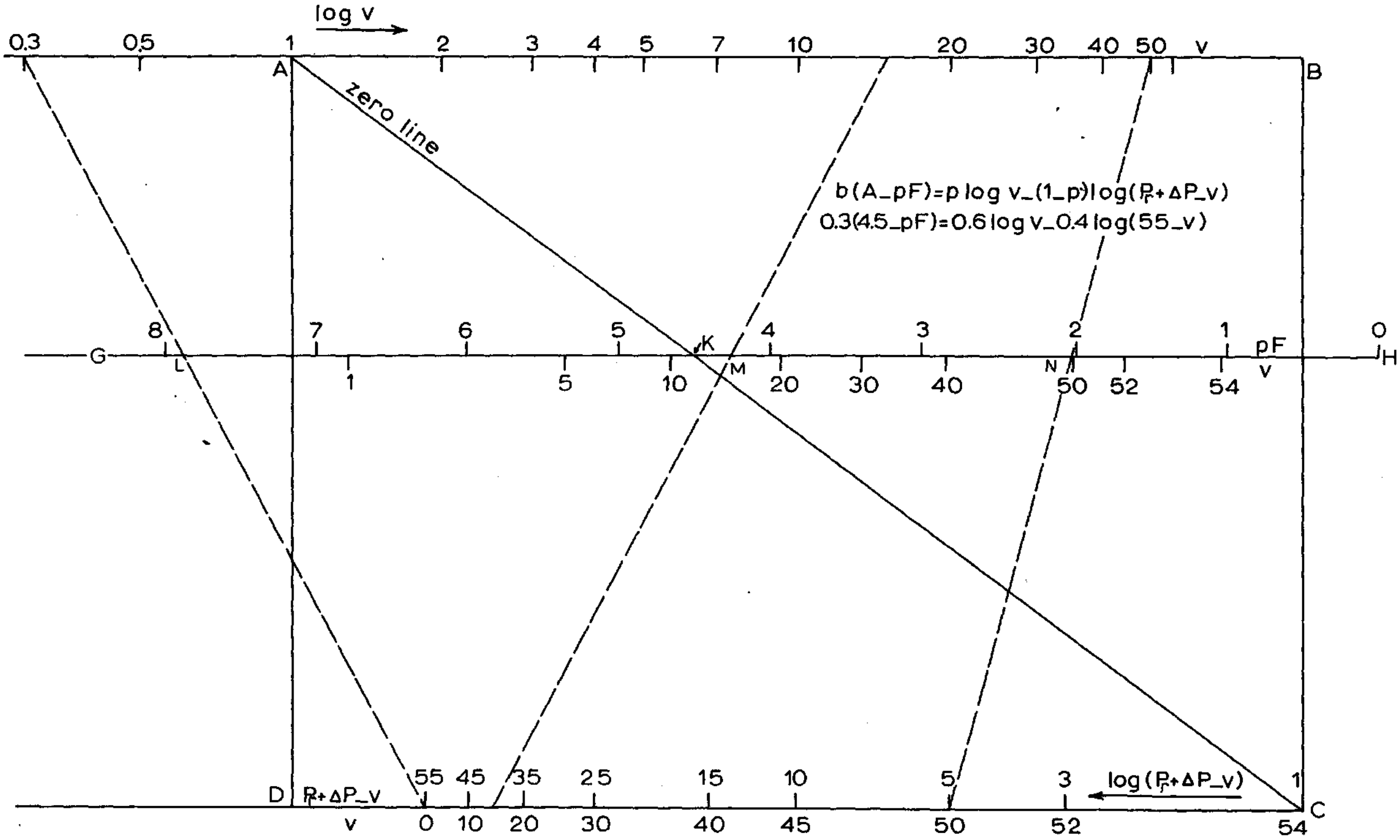
Example 9. Construct a scale, calibrated at one side with pF values and at the other side with moisture content values for given constants

$$A = 4.5 \qquad b = 0.3 \qquad p = 0.6 \qquad P_r + \Delta P = 55$$

The construction is carried out in 8 steps:

1) Draw a rectangle with sides 20 x 15 cm as in fig. 10 and indicate the corners clockwise with ABCD.
2) Mark along the AB axis a log v scale with log v = 0 or v = 1 in A and write the values of v at the marks for log v.
3) Mark along the CD axis a log $(P_r + \Delta P - v)$ scale with log $(P_r + \Delta P - v) = 0$ in C. Number the scale for log $(P_r + \Delta P - v)$ with the values of $P_r + \Delta P - v$ as well as v.
4) Construct the zero line AC on which line log v = 0, log $(P_r + \Delta P - v) = 0$ and $A - \log \Psi = 0$ are situated.
5) Draw the line GH, parallel to AB and CD, at a distance 15 p cm from the line CD, agreeing with a distance of 15(1-p) from line AB.

fig.10


log v
0.3
0.5
1
2
3
4
5
7
10
20
30
40
50
v
A
B
zero line
b(A_pF)=p log v_(1_p)log(P_r+ΔP_v)
0.3(4.5_pF)=0.6 log v_0.4 log(55_v)
G
8
L
7
6
5
K
M
4
3
N
2
1
pF
0
H
1
5
10
20
30
40
50
52
54
v
D
P_r+ΔP_v
v
55
45
35
25
15
10
5
3
log(P_r+ΔP_v)
1
C
0
10
20
30
40
45
50
52
54

6) Mark the point of intersection K of the zero line AC and the parallel line GH with the value A of the pF-scale.
7) Calibrate the line GH at the upper side with a linear pF-scale with scale units of 10 b cm and number the scale - with increasing pF-values in the direction of small values of v - with pF = A in point K.
8) Connect the identical values of v on the axis AB and CD in fig. 10 - the dashed lines for v = 0.3 and 15 are given as an instance - and callibrate and number axis GH at the lower side with the v-values.
9) The pF value for v = 0.3% appears to be 7.9, for v = 15% is 4.25.

The nomogram,if used for the construction of the double scale line GH, should be calibrated with a reasonably dense division so that the interpolation along the v-scale with units of varying length does not become too inaccurate. The nomogram may also be used to determine the pF-value from a given value of v by just connecting only this particular value of v on the AB and CD axes and reading the matching value of the pF. Only a scale for the pF-values along line GH is sufficient for carrying out this simpler solution.

8. Graphical solution of the constants A, B, p and ΔP of the desorption curve

The constants of the desorption curve may be used in a number of calculations that solve problems that are directly and simply related to the desorption curve, but with these constants also a wider field may be covered as will be shown in § 9 and in the practical applications. The The computation of the constants may be done by curve fitting or by solving four equations with four unknowns. The least square solution of the curve fitting may lead to any required degree of accuracy, but the solution by iteration of the non-linear relation of the value of $P_r + \Delta P$ from the observations is time-consuming.

The graphical solution - with limited accuracy - has the advantage that the Ψ_a-value of the air entry stress - of which the determination often shows a restricted accuracy - need not be known beforehand. Often the moisture contents at still lower pF - values, which should remain constant, do not clearly indicate the constancy due to errors of determination. The mathematical solution does not easily show, which values have to be included in the curve fitting and which should not.

Two solutions will be worked out below. Both solutions present with respect to the determination of $P_r + \Delta P$ identical difficulties as the mathematical solution. The first solution uses to a large extent the method explained in example 9 with respect to the solution of log Ψ and v-values. The problem is to situate the scale GH of fig. 7 at the correct place in the rectangle ABCD so that the readings on axis GH compare as well as possible with the determined values.

Exercise 10. Construct the pF-axis at the correct place in rectangle ABCD of fig. 10 with the use of observed pF and v data of example 11 and determine the constants of the desorption formula with the aid of the nomographic representation of the desorption curve of § 7.

1) Carry out the first four steps of example 9 with the use of what is considered the best value for $P_r + \Delta$ P.
2) Select the lowest value for Ψ that certainly is higher than Ψ_a
3) Connect the values for v on the axes AB and CD for the highest value of Ψ that is available and for the lowest value of Ψ mentioned in step 2.
4) Connect for an arbitrary value of Ψ , between the highest and lowest Ψ value of point 3, the values of v on axes AB and CD.
5) Determine the place between axes AB and CD where a line parallel to these axes, is divided by the three lines for highest, lowest and middle v in two parts that stand in the same relation as the difference of the pF-values. Are the pF values 7.9, 4.25 and 2.0 then the ratio between the two parts just mentioned - LM and MN in fig. 10 - has to be 3.65 to 2.25.

6) Do the same for all the other ψ values between the highest and lowest ψ of point 2.
7) Take the average of the distances to the lines AB or CD of the position for the line GH, obtained for the successive middle value of ψ and draw the line GH through this average position.
8) Do the same as in point 5 for the successive values of ψ below the lowest ψ value of point 2 and check whether they still agree sufficiently with the average position of line GH obtained without these lower ψ values. The values which compare well may be added to the values used in point 6 and be used to determine the average position of the GH-line.
9) Read from the nomogram for the low ψ values which do not fit well in the nomogram, what the values would have been if they belonged to the same relation as the higher ψ values and calculate the difference with the observed values.
10) Conclude from the calculated differences mentioned in step 9 whether they originate from random errors, from the influence of the saturated capillary condition or from a wrong estimate of the value of $P_r + \Delta P$.
11) Repeat the construction for an improved value of $P_r + \Delta P$ and conclude again on the ratio between the influences of the three sources of errors.
12) Read the value of A as the pF value at point K. The value of b as the length of one unit of the pF-scale and the value of p as the ratio between the distance of the log (P-v) scale to the pF - and the log v - scale.

Example 11. Solve the constants A, B, p and $P_r + \Delta P$ of the desorption equation according the second method for the following data for log ψ and v:

pF	0	1	2	3	4	5	6	7
v	54.81	54.05	50.25	37.67	19.29	7.39	2.49	0.80
$P_r + \Delta P - v$	0.19	0.95	4.75	17.33	35.71	47.61	52.51	54.20

The observations conform with the desorption curve used in § 7 and

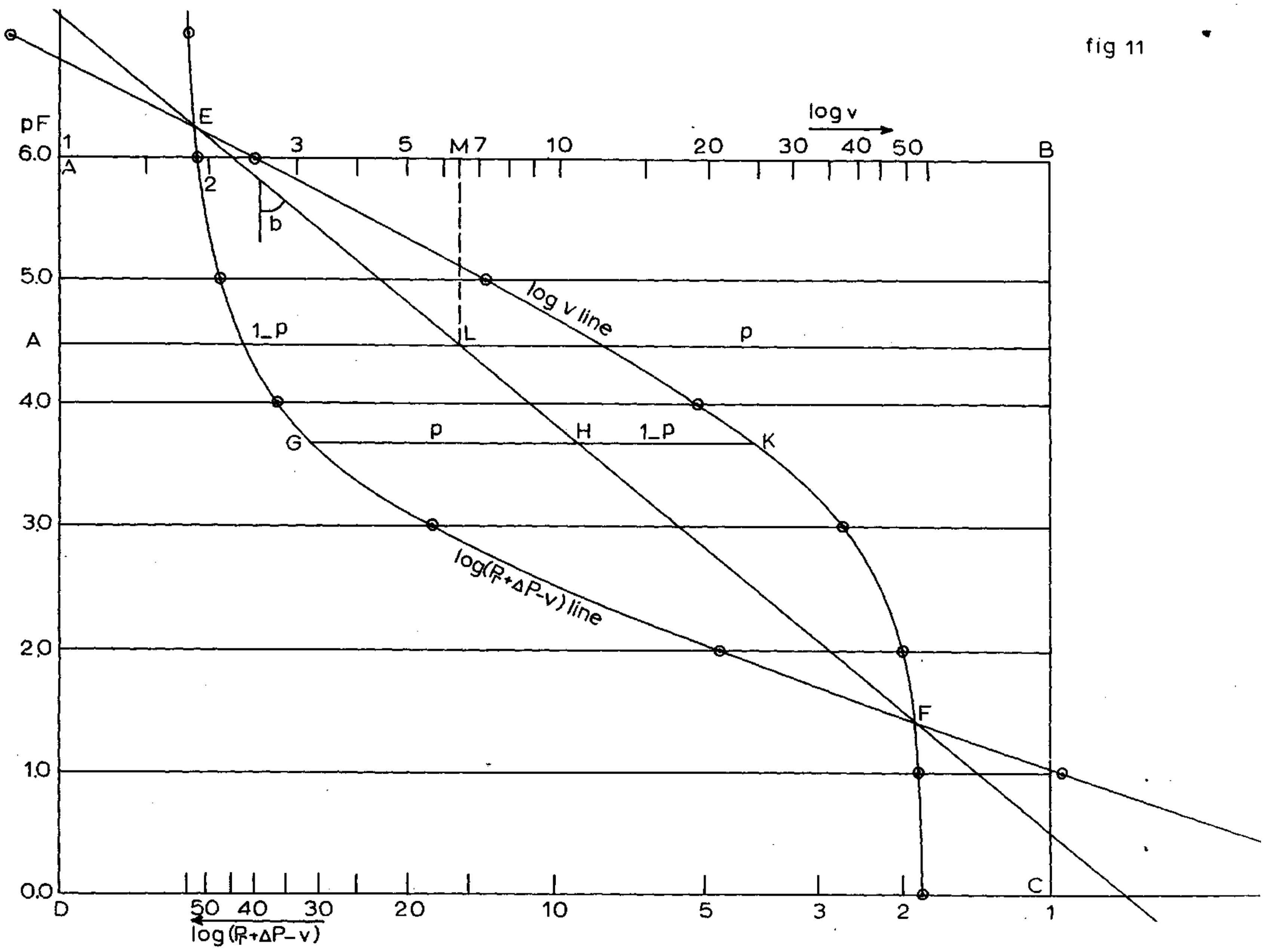
fig. 10 to show how log Ψ and v are solved for known values of the constants.

1) Carry out the first three steps of example 9 for the best estimate of $P_r + \Delta P$ that can be made.
2) Mark along the AD axis a metric scale for log Ψ with 2.5 cm units with the zero point in D (see fig. 11).
3) Plot the points for log Ψ and log v and draw a freehand curve through the dots. Do the same for the log Ψ - log(P_r +ΔP-v) curve. The shape of the log v-curve and the log (P_r + Δ P-v)-curve is - as given in fig. 11 - approaching a vertical asymptote at the high values and an oblique asymtote at the lower values.
4) Determine the points of intersection E and F of the two freehand curves and connect these points by the line EF. The tangent of line EF opposite the axis AB renders the constant b.

   In measuring b allowance should be made for the scales along the horizontal and vertical axes. Are these scales not taken into account the value of b would have been found as 20/16.7 =1.2. This however would have been the wrong answer. The 20 cm along the log scales are equal to two 10 cm units. The 16.7 cm along the pF-scale is equal to 6.67 units of 2.5 cm. The value of b is therefore 2/6.67 = 0.3.
5) Measure at a number of arbitrary pF levels the distance GH and GK and express GH in terms of GK. This doing at the pF level 3.68 yields for p the value: p = 5.35/8.90 = 0.6.

   Often with normal slightly inaccurate data the freehand curves will not be entirely accurate. Therefore for different pF levels the value of p - for which the same value should be found - may vary to a certain extent. A number of ratios at different pF levels should be established and averaged.
6) Divide the distance AB in a part p x AB = BM and a part

fig 11


log v
pF
6.0
5.0
A
4.0
3.0
2.0
1.0
0.0
1
3
5
M
7
10
20
30
40
50
B
A
E
2
b
log v line
1_p
L
p
p
G
H
1_p
K
log(P̄+ΔP-v) line
F
C
D
50
40
30
20
10
5
3
2
1
log (P̄+ΔP-v)

(1-p)AB = AM. Draw a vertical line downward from M and determine the point of intersection L. Read the pF value for the point L. This pF value is the value for A or log G to be inserted in the formula. In fig. 11 the value for A is found to be 4.5.

Problem 12. The reader should carry out the first 3 steps of example 11 for a value of $P_r + \Delta P$ of 50, 53, 57 and 60 and take notice of the change in the shape of the part for the lower values of the log $(P_r + \Delta P - v)$ line.

The deviation from the straight line at the lower $(P_r + \Delta P - v)$ values is an indication of the error, made in the estimation of $P_r + \Delta P$. The direction of the curving - upward or downward - indicates the sign of the error.
Determination of the value of $P_r + \Delta P$ can only be carried out by repeating the construction of the graphical solution for different values of $P_r + \Delta P$ and checking the straightness of the lower part of the $(P_r + \Delta P - v)$curve. The errors of determination and the influence of the saturated capillary zone may influence the accuracy of the determination of $P_r + \Delta P$ - under ideal circumstances already rather restricted - very unfavourable.

9. Pore size distribution and permeability

The distribution of the pore sizes means - because according to the Poisseuille formula for each pore size the flow resistance may be calculated - also a distribution of the resistance coefficients of each individual pore. By integrating the amount of flow as function of the pore size over all existing pore sizes below a certain maximum value, the total flow through the soil between zero pore diameter and the selected maximum value, agreeing with the flow between zero moisture content and any degree of saturation, may be calculated.

The calculation is based on a number of assumptions. It is assumed that the formula of Poisseuille holds only for inpores, continuously filled with water. No non-capillary pore space of the nature of cracks or

root holes is supposed to be present. Moisture transport is taken not to occur in the vapour phase or any such vapour transfer is considered to be negligeable. It is assumed that no discontinuities in the desorption curve occur which diverge so considerably from the desorption function that these divergences should be inserted in the formulae.

a) The relation between the pF-curve and the conductivity curve

The relation between the amount of flow dk_c and the pore size r has to be multiplied by the number of pores dN to obtain the amount of flow through all the pores of that size. This relation can be calculated as follows. According to Poisseuille the amount of flow d_k is related to the pore radius by the formula

$$dk_c = \alpha_1 r^4 dN$$

α_1 = constant of Poisseuille equation

From the volume of one pore follows:

v = moisture content
l = length of pore

$$dN = \frac{dv}{\pi r^2 l} = \alpha_2 \frac{1}{r^2} \; dv$$

$\alpha_2 = \frac{1}{\pi l}$ = constant

The relation between the moisture stress - as an indication of the pore size - and the volume, in case the formula is given by $v = f(\Psi)$, is rendered by the equation for the distribution of pore sizes with capillary properties:

$$dv = f'(\Psi) d\Psi$$

The relation between the pore diameter and the moisture stress is given by the equation for the capillary rise:

$$r = \frac{\alpha_3}{\Psi}$$

α_3 = constant

Combining the formulae and lumping the constants together, the following relation is obtained:

$$dk_c = \alpha \frac{f'\Psi}{\Psi^2} \, d\Psi \qquad \alpha = \alpha_1 \alpha_2 \alpha_3 \tag{21}$$

The first derivative of the desorption curve, divided by the first derivative of the curve for the unsaturated conductivity is proportional to the second power of the moisture stress or:

$$\frac{dv}{d\Psi} \Big/ \frac{dk_c}{d\Psi} = \frac{1}{\alpha} \Psi^2$$

The importance of the unsaturated conductivity is largest at high moisture contents. But in this part of the desorption curve the determinations cannot be carried out with an accuracy, sufficiently high to determine an accurate value of the unsaturated conductivity. A function describing the relation between k_c and ψ has to be used. The desorption curve then only serves to determine the constants.

At high moisture contents RIJTEMA AND WESSELING observed that the following relation holds well in the range of lower values of log ψ:

$$\frac{k_c}{k_s} = e^{-\alpha_1 (\Psi - \Psi_a)} \tag{22}$$

This clearly is the integral of

$$\frac{dk_c}{k_c} = - \alpha \, d\Psi \text{ or } dk_c = - k_c \alpha \, d\Psi = \frac{-\alpha k_s}{e^{-\alpha \Psi_a}} e^{-\alpha \Psi} d\Psi = -Be^{-\alpha \Psi} d\Psi$$

This leads to an equation for the wet branch of the desorption curve, reading:

$$f' \Psi \, d\Psi = -B \Psi^2 e^{-\alpha \Psi} d\Psi$$

$$f \Psi = B(2 + 2\alpha \Psi + \alpha^2 \Psi^2) e^{-\alpha \Psi} + C$$

Nor for $\Psi = \infty$ the value of $v = f(\Psi)$ must be zero, so that $C = 0$
For $\Psi = 0$ the value of $v = f(\Psi)$ is equal to βP_s
For $\Psi = \Psi_a$ the value of v is equal to P_s

From this follows that

$$B = \beta P_s$$

and

$$P_s = \left(1 + \alpha \Psi_a + \frac{\alpha^2 \Psi_a^2}{2}\right) e^{-\alpha \Psi_a}$$

Inserting this in the equation for P-v it follows that:

$$P\text{-}v = P_s \left\{ \beta - \left(1 + \alpha \Psi + \frac{\alpha^2 \Psi^2}{2}\right) e^{-\alpha \Psi} \right\} \tag{23}$$

In this way formula 14 was obtained. This formula relates directly to formula 22 for the unsaturated conductivity. The value of P_s is discussed in the next paragraph.

b. Determination of the constants for the wetter range

The determination of the constants α and Ψ_a from formula 17 is best done using the observations of the wet part of the moisture range. Further it should be born in mind that the pore size determined by the desorption curve is the smallest spot of the pore, whilst the pore size in the flow equation is the mean size of the pore. The value of α , obtained by fitting formula 17 to the observations has to be reduced by a certain factor to obtain the constant to be used as α_1 , in formula 22.

The last point of importance is that the capillary movement of water takes place in the continuous pores in which the structure defined moisture with volume P_s is stored. The moisture in the contact rings which do not form a continous volume of water and through which liquid cannot flow from one ring to the other, is stored in the texture defined pores. This volume of absorbed water will be called P_a. The values of P_s and P_a are constant and together constitute the total pore space P. The actual moisture content of the soil is made up of the space of absorbed water plus the space of water that is transferable in the liquid state, to be called V_t. Therefore

$$P = P_a + P_s$$

$$v = P_a + V_t$$

and according formula 23

$$V_t = P_s \left(1 + \alpha\Psi + \frac{\alpha^2\Psi^2}{2}\right) e^{-\alpha\Psi} \tag{24}$$

Equation 17 can be written

$$v^m = \frac{AP_s^{\,n}}{\Psi} \left\{\beta - \left(1 + \alpha\Psi + \frac{\alpha^2\Psi^2}{2}\right) e^{-\alpha\Psi}\right\}^n \tag{25}$$

In the range of high values of $\alpha\Psi$, the part of the equation 25, given in formula 23 rapidly approaches zero. In the range of lower moisture contents therefore holds

$$v_{low}^{\,m} = \frac{A\beta^n P_s^{\,n}}{\Psi} \tag{26}$$

The solution of the constants of formula 25 is obtained by first calculating the value of m and $AP\beta$ from the low values of v according

$$m \log v_{low} + \log \Psi = \log A \beta^n P_s^{\,n} \tag{27}$$

Then formula 25 is regrouped:

$$\frac{v^m}{\beta^n P_s^{\,n}} \; \frac{\Psi}{A} = \left(1 - \frac{\left(1 + \alpha\Psi + \frac{\alpha^2\Psi^2}{2}\right) e^{-\alpha\Psi}}{\beta}\right)^n \qquad (28)$$

With the use of the values of m and $A\beta^n P_s^{\,n}$ the left hand side of formula 28 can be calculated. The mathematical derivation of the constant α of the equation for the unsaturated conductivity can be made acceptable for practical use if a simplified method is found to solve the unknown parameters α, β and n of the right hand side of equation 28 from the known values for the left hand side and the values of Ψ.

c. Determination of the unsaturated conductivity for the dryer range

In the previous paragraph the formula for the unsaturated conductivity in the wetter range was given and the method was discussed, to calculate the constants α and Ψ_a, which govern this conductivity function from the observations of the desorption curve.

For many problems, this conductivity of the wetter range will be of main importance. The extraction of soil moisture by the plant however is mainly of importance in the dryer range, where the e-function for the unsaturated conductivity looses its importance. The formulae, on which this dry range conductivity **depends are:**

$$\frac{dv}{d\Psi} \Big/ \frac{dk_c}{d\Psi} = \frac{\Psi^2}{\alpha} \quad (29) \quad \text{and} \quad \frac{\Psi}{A} = \frac{(\beta P_{so})^n}{v^m}$$

From $v = \dfrac{(\beta P_{so})^{n/m} A^{1/m}}{\Psi^{1/m}} = \dfrac{\gamma}{\Psi^{1/m}}$ we derive $\dfrac{dv}{d\Psi} = \dfrac{-\gamma}{m} \dfrac{1}{\Psi^{1/m+1}}$ (30)

Inserting formula 30 in formula 29 leads to:

$$\frac{d_{kc}}{d\Psi} = -\frac{\alpha\gamma}{m} \frac{1}{\Psi^{1/m+3}}$$

$$k_c = \frac{-\alpha\gamma}{m(\frac{1}{m}+2)} \frac{1}{\Psi^{1/m+2}} + C$$

The value of c must be zero. Therefore the value of kc can be calculated as

$$\frac{k_c}{k_a} = \left(\frac{\psi_a}{\psi}\right)^{2+1/m} \qquad (31)$$

The formula 31 is based on the supposition that in the dry range the soil moisture still flows through circular pores. In case this flow in reality was a film flow - a flow through channels in which the width is far in excess of the shallowness, this means that instead of formula 29 a formula of the type:

$$\frac{dv}{d\psi} / \frac{d_{kc}}{d\psi} = \frac{\psi}{\alpha}$$

at still lower moisture content will be more acceptable. This renders a formula:

$$\frac{k_c}{k_a} = \left(\frac{\psi_a}{\psi}\right)^{1+1/m} \qquad (32)$$

From the foregoing elaborations it follows that the relation between the capillary conductivity and the soil moisture stress gradually changes with changing values of the moisture stress.

For high stresses: $k_c = \alpha_1 \psi^{-(1+1/m)}$

For medium stresses: $k_c = \alpha_2 \psi^{-(2+1/m)}$

For low stresses: $k_c = \alpha_3 e^{-\alpha(\psi - \psi_a)}$

Inspection of the desorption curve provides - as long as no direct determinations of the unsaturated conductivity can be carried out - the only way to make an estimate at which values of ψ the demarcation between the three ranges has to be expected.

d. The calculation of the constants for the unsaturated conductivity

As was mentioned already the determination as well as the calculation of the unsaturated conductivity is rather difficult and time consuming. Laboratory facilities for carrying out the necessary determinations will not always be available. In case the determination of the desorption curve is well within reach, consideration should be given to the possibility to derive the constants which define the unsaturated conductivity from the

observations for the desorption curve.

The formula

$$\frac{V^m\,\Psi}{\beta^n P_s{}^n A} = \left(1 - \frac{\left(1 + \alpha\Psi + \frac{\alpha^2\Psi^2}{2}\right) e^{-\alpha\Psi}}{\beta}\right)^n \qquad (28)$$

simplifies for high values of Ψ, because then the right hand side of the equation becomes the value of 1.000. For values of Ψ of 2.7 and higher the magnitude of the two unknowns m and log $(\beta^n P_s{}^n A)$ can be determined directly.

Then it is possible to calculate the value of the left hand side of the equation for any value of Ψ the log of which will be called x.

Table 6

Example 13. Calculate the value of m and $(A\beta^n F_s{}^n)$

pF	0.4	1.0	1.5	2.0	2.3	2.7	3.4	4.2	6.0	
Ψ	2.51	10.0	31.6	100	200	501	2512	15848	1000000	
v	53.2	50.11	47.1	40.6	33.3	29.2	15.7	9.4	2.4	
log v	1.72591	1.69984	1.67302	1.60853	1.52244	1.46538	1.19590	0.97313	0.38021	
6-pF = p					3.7	3.3	2.6	1.8		
log v-0.38021=q					1.14223	1.08517	0.81569	0.59292		
m=p/q					3.23923	3.04100	3.18749	3.03585		√
3.08810 log v+pF	5.72978	6.24928	6.66645	6.96330	7.00145	7.22524	7.09306	7.20512	7.17413	√
log $A\beta^n P_s{}^n$	7.17439	7.17439	7.17439	7.17439	7.17439	7.17439	7.17439	7.17439	7.17439	
$x = n \log\left(1 - \frac{\left(1-\alpha\Psi + \frac{\alpha^2\Psi^2}{2}\right) e^{-\alpha\Psi}}{\beta}\right)$	$^{-}1.44461$	$^{-}0.93511$	$^{-}0.50794$	$^{-}0.20709$	$^{-}0.17294$	$^{+}0.05085$	$^{-}0.08133$	$^{+}0.03073$	$^{-}0.00026$	

$\sqrt{m} = 3.08810$ $\quad \sqrt{\log A\beta^n P_s{}^n} = 7.17439$

The determination of the unknowns of the right hand side of the equation α, β and n are calculated by eliminating n:

$$n = \frac{x_1}{\log\left(1 - \frac{\left(1 + \alpha\Psi_1 + \frac{\alpha^2\Psi_1^2}{2}\right) e^{-\alpha\Psi_1}}{\beta}\right)} = \frac{x_2}{\log\left(1 - \frac{\left(1 + \alpha\Psi_2 + \frac{\alpha^2\Psi_2^2}{2}\right) e^{-\alpha\Psi_2}}{\beta}\right)}$$

Writing $\left(1 + \alpha\Psi + \frac{\alpha^2\Psi^2}{2}\right)e^{-\alpha\Psi}$ as $f(\alpha\Psi)$ the solution can be found by calculating for 3 values of Ψ for a given value of α the value of β for the two pairs Ψ_1, Ψ_2 and Ψ_1, Ψ_3 according:

$$\left(1 - \frac{f(\alpha\Psi_1)}{\beta_1}\right)^{x_2/x_1} = \left(1 - \frac{f(\alpha\Psi_2)}{\beta}\right) \tag{33}$$

The value of x has to be selected in such a way that for the two combinations $\Psi_1\,\Psi_2$ and $\Psi_1\,\Psi_3$ the same value of β is obtained.

In many cases β will be near 1.000. The solution is obtained quickest by taking the lowest Ψ and x combination as Ψ_1 and X_1 so that the exponent x_2/x_1 becomes smaller than unity. Further the value of β_1 in the left hand side of the equation is taken 1.000. The value of β from the right hand side of the equation is then calculated as:

$$\beta = \frac{f(\alpha\Psi_n)}{1 - \left(1 - f(\alpha\Psi_1)\right)^{x_2/x_1}} \tag{34}$$

Gives the preliminary calculation an idea of the value of β then this value is used as β_1 in the left hand side of the equation.

The solution is obtained as follows:

Example 14. Calculate the value of α and β

Table 7

x	$\bar{1}.4446$	$\bar{0}.9351$	$\bar{0}.5079$	$\bar{0}.2071$	$\bar{0}.1729$
x_n / x_1		0.6473	0.3516	0.1434	0.1197
$f(\alpha\Psi_1)$	0.99998				
$1-f(\alpha\Psi_1) = y_1$	0.00002				
$-\log Y_1$	$\bar{4}.6990$				
$x_n/x_1 \log y_1$		6.9583	8.3478	9.3262	9.4375
$y_1^{\,x_n/x_1}$		0.00091	0.02227	0.2119	0.2738
$1 - y_1^{\,x_n/x_1} = z_{1n}$		0.99909	0.97773	0.7881	0.7262
$f(\alpha\Psi_n)$ for $\alpha = 0.02$	0.99998	0.99885	0.97362	0.67669	0.23811
$\beta = f(\alpha\Psi_n)-z_{1n}$		0.99976	0.99580	0.85863	0.32788
$f(\alpha\Psi_n)$ for $\alpha = 0.01$	0.99998	0.9998	0.9958	0.9171	0.6767
$\beta = f(\alpha\Psi_n)/z_{1n}$		1.0007	1.0185	1.1637	0.9318

In the upper part of the table the denominator of formula 34 is calculated. Only $f(\alpha \psi_1)$ depends on the value of α chosen, but because of the low value of ψ_1 the influence of α is sufficiently small to allow the use of the same value for different estimates of α . The first part of the calculation only needs to be done once for every desorption curve to be treated.

In the second and third part of table 7 the value of $f(\alpha \psi)$ is used for $\alpha = 0.02$ and 0.01. These values of $f(\alpha \psi)$ can be calculated in advance for general use.

Trying out the correct value of α means only that the division of the values of $f(\alpha \psi)$ for that value of α by the values of z_m from the upper part of the table has to be carried out.

The example shows that $\alpha = 0.02$ is too high so that the value of β deminishes for higher values of ψ . The value of $\alpha = 0.01$ however is too low. By the judgement of the degree of constancy of the value of β it should be remembered that at higher values of ψ divergence of the value of the unsaturated conductivity from the e-function has to be expected. The low value of β for $\psi = 2.3$ may be an instance of this deviation. Further the accuracy of the observations for ψ and v of the desorption curve is not very high, so that the accuracy of the calculated value of α is never to be expected to be high.

e. Influence of structure factors of other origin

A rather difficult problem in calculating the capillary conductivity is the presence of non-intersphere porosity, pores due to cracks and roots or wormholes. The larger sized cracks escape determination by desorption measurements, but may add quite considerably to the saturated conductivity. This non-intersphere conductivity does not follow from the desorption curve and cannot be contemplated as included in the exponentional relation.

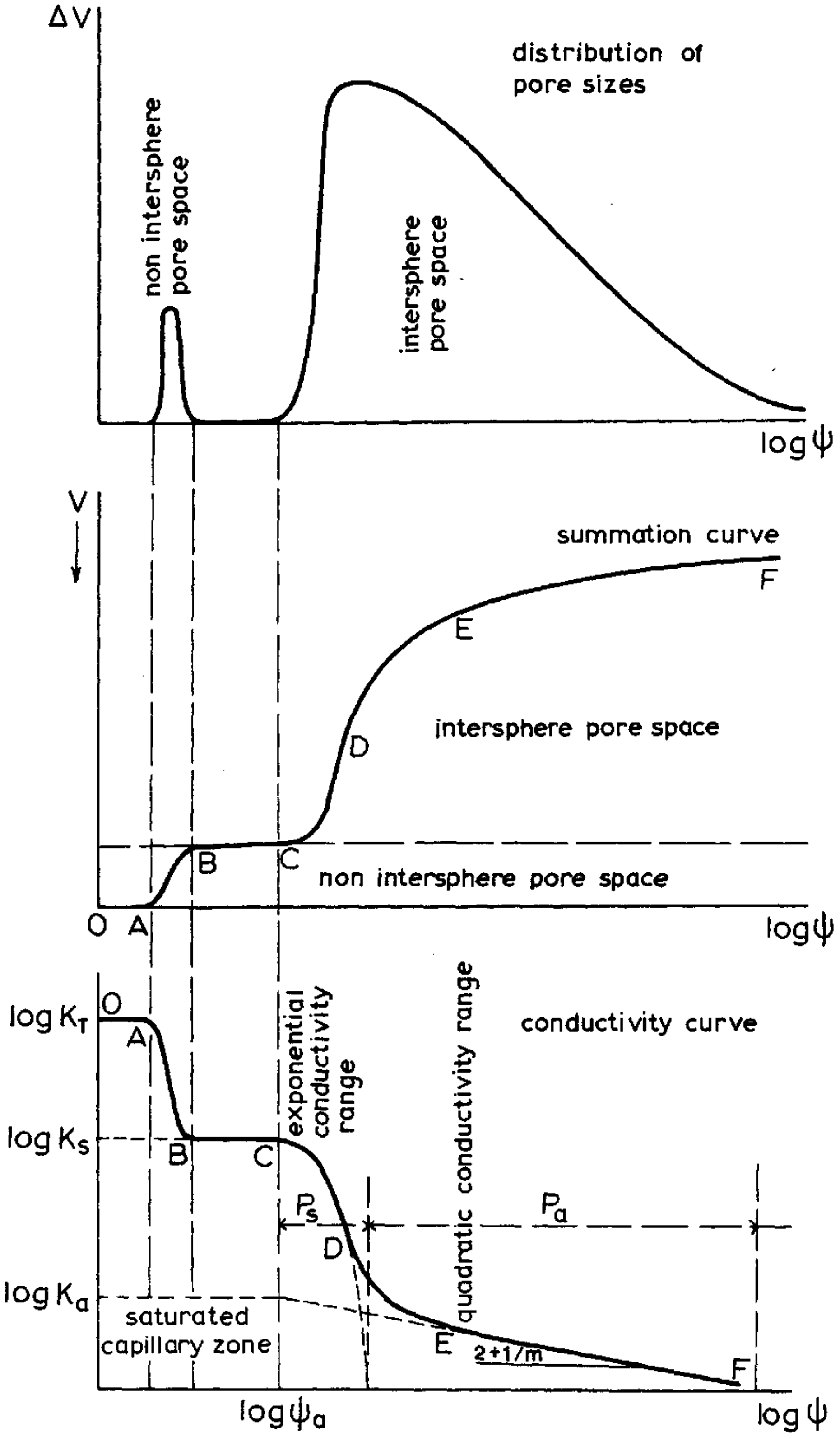
In cases where this macro-porosity is present, the ratio k_c/k_s as some simple function of ψ and ψ_a no longer holds. There seems no other

way to assess the unsaturated conductivity - safe by determining this property directly - than to calculate the conductivity of these pores by assessing the volume and size of these macro pores given by fig. 5 as ΔP_{nis} by other means and subtracting this part of the conductivity from the total of macro and micro porosity. There are, however, situations that a more detailed examination of the desorption curve shows what size and volume these macro pores have.

The way in which the macro-porosity shows up is visualized in fig. 12. The pore size distribution curve is of the double apex type, due to pores of two different origins, the intersphere pores and the cracks. The second curve gives the summation curve and is identical with the desorption curve.

The range in moisture tensions OA corresponds with a larger crack size than present in the soil. The range AB depicts the variation in crack sizes and the part of the total conductivity dependent on the cracks. The part BC shows by its horizontal course that no pores of this size are present and that this is the saturated capillary zone. The moisture stress at c is ψ_a. In the range CD the conductivity is governed by the e-function of formula 22. Passing the transition range DE, at E the range of the exponential nature, given by formula 31 or 32, takes over. On a double log graph the latter relation shows as a **straight line.** The intercept of this line at ψ_a represents the component of the saturated conductivity k_a according to the quadratic formula. The tangent of this line is $1 + 1/m$ to $2 + 1/m$.

Now it is possible that the ratio of intersphere and non-intersphere pore space varies. The latter can even be zero. Further they may merge so that the range BC disappears. Then it should be remembered that a large value of α in formula 22 decreases the exponential conductivity range CD so that it may disappear. Also k_a can decrease or increase the importance of the exponential conductivity component so that the range of transition DE may turn out wider or narrower, at higher or at lower value of ψ.


ΔV
distribution of pore sizes
non intersphere pore space
intersphere pore space
log ψ
V
summation curve
F
E
intersphere pore space
D
B
C
non intersphere pore space
O
A
log ψ
log K_T
O
A
exponential conductivity range
quadratic conductivity range
conductivity curve
log K_s
B
C
P_s
P_a
D
log K_a
saturated capillary zone
E
2+1/m
F
log ψ_a
log ψ

Example 15. Calculate the capillary conductivity of a soil with data for the desorption curve:

$\log \psi$	0.4	1.0	1.5	2.0	2.3	2.7	3.4	4.2	6.0
v	53.2	50.1	47.1	40.6	33.3	29.2	15.7	9.4	2.4
$\left(1+\alpha\psi+\frac{\alpha^2\psi^2}{2}\right)e^{-\alpha\psi}$	0.972	0.892	0.697	0.334	0.163	0.127	0.000	0.000	0.000

if it is known that the air entry point is situated at ψ_a = 20 cm and the saturated conductivity is 1.00 m/day.

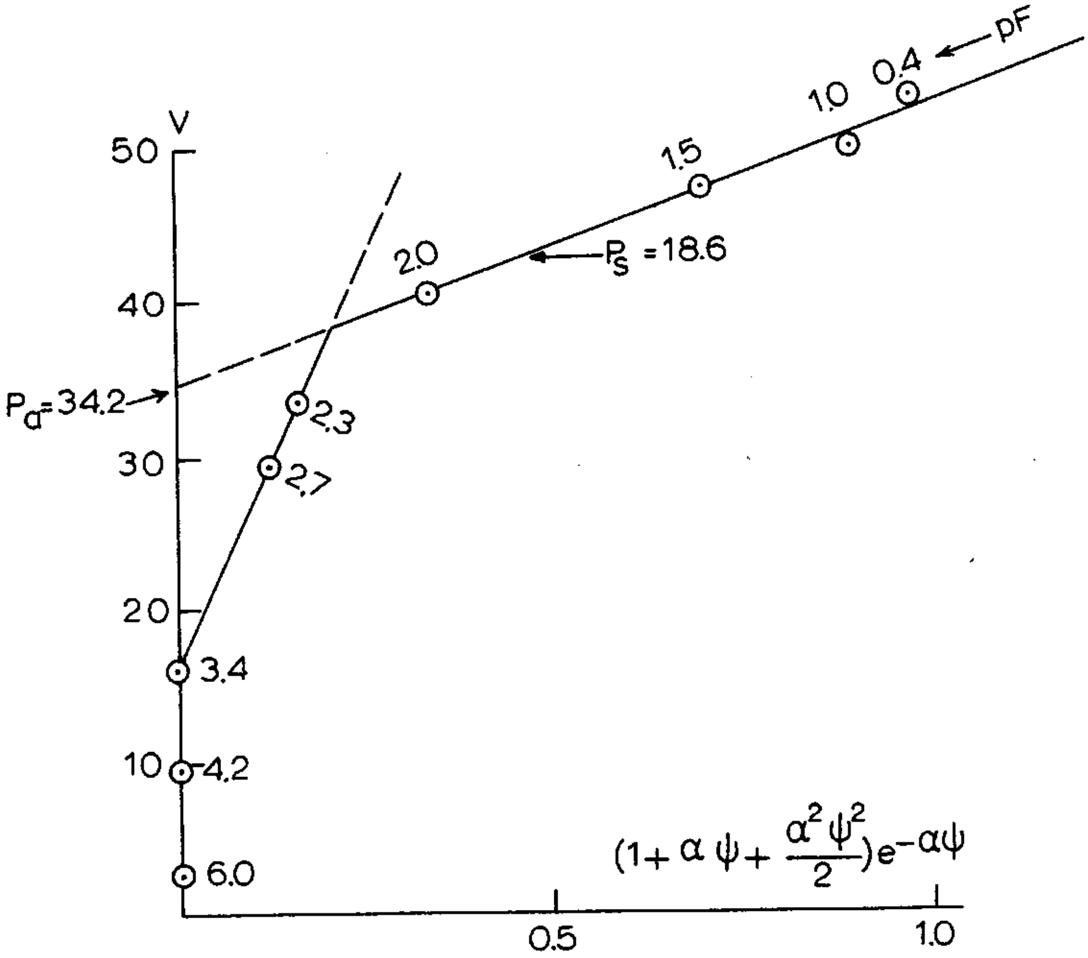
With some experiments the impression is obtained that the value of $\alpha = 0.0055$. This provides one with values of $\left(1+\alpha\psi+\frac{\alpha^2\psi^2}{2}\right)e^{-\alpha\psi}$ equal to the 3rd row in the table above. In fig. 13 the value of the ψ-function is plotted against v. It is obvious that the value of α brings the points for pF 0.4-2.0 correctly on a straight line which proves that α is acceptable. The points for pF 3.4-6.0 are not dependent on the ψ-function as they should not for low moisture contents.

10. Vertical water transfer through the unsaturated zone

The solutions for steady vertical flow - be it due to infiltration into the soil from the soil surface downward or due to capillary rise - are available due to their easy mathematics. These solutions are useful in those cases where the flow of moisture is constant over a prolonged time. Also the amount of water, taking part in the movement of moisture has to be large compared with eventual changes in the storage capacity. These conditions will more often be satisfied in cases of upward flow due to evaporation than in cases of infiltration due to rainfall or irrigation. The complexeties with regard to the permeability of the soil and the influence of root holes and cracks cause more difficulties in calculating the infiltration than the capillary rise.

In this respect the process of infiltration can easily be studied by infiltrometer tests. Often these tests will be advisable and they are not too complicated to be carried out as routine investigation of large numbers of measuring spots and may be part of the group of generally accepted analyses for design purposes, as saturated conductivities, desorption curves, and air entry stresses.

fig 13


V
50
40
30
20
10
0.5
1.0
$(1+\alpha\psi+\frac{\alpha^2\psi^2}{2})e^{-\alpha\psi}$
pF
0.4
1.0
1.5
2.0
2.3
2.7
3.4
4.2
6.0
P_s =18.6
P_a=34.2

The formulae are founded on the Darcy equation:

$$v_c = k_c\left(\frac{d\psi}{dz} - 1\right) \qquad \text{for upward flow}$$

$$v_c = k_c\left(\frac{d\psi}{dz} + 1\right) \qquad \text{for downward flow}$$

v_c = moisture flow in mm/24 hours

Within the saturated capillary zone, between the tensions $\psi = 0$ and $\psi = \psi_a$ the conductivity $k_c = k_s$ is constant and the formula holds:

$$\frac{k_s d\psi}{v_c \pm k_s} = dz \qquad z = \frac{k_s \psi_a}{v_c + k_s} \qquad \text{for upward flow} \qquad (35a)$$

$$z = \frac{k_s \psi_a}{v_c - k_s} \qquad \text{for downward flow} \qquad (35b)$$

In the range where the P-v term of the desorption curve has no influence anymore or in cases that the exponent (1-p) is zero, there the formula because of $k_c = k_s \frac{\psi_a{}^s}{\psi^s}$ holds:

$$\frac{d\psi}{\frac{v_c}{k_s \psi_a{}^s}\psi^s \pm 1} = dz$$

For s = 2:

upward flow $\psi = \sqrt{\frac{k_s \psi_a{}^2}{v_c}}\ \mathrm{tg}\left(z\sqrt{\frac{v_c}{k_s \psi_a}}\right)$ (36a)

downward flow $\psi = \sqrt{\frac{k_s \psi_a{}^2}{v_c}}\ \mathrm{tgh}\left(z\sqrt{\frac{v_c}{k_s \psi_a}}\right)$ (36b)

In formula 36b the hyperbolic tangent can be replaced by an e-function and can be calculated with a normal log-table in case no table on hyperbolic functions is available. The formula then reads:

$$\psi = \sqrt{\frac{k_s \psi_a{}^2}{v_c}}\left(\frac{1 + e^{-2z\sqrt{\frac{v_c}{k_s \psi_a{}^2}}}}{1 - e^{-2z\sqrt{\frac{v_c}{k_s \psi_a{}^2}}}}\right) \qquad (36c)$$

In the formula for upward flow the tangent is expressed in radians. The customary geometric tables can be used by multiplying $z\sqrt{\frac{v_c}{\psi_a{}^2 k_s}}$ by $360/2\pi$.

For other values of the exponent s, differing from 2, the equations become more complicated. To simplify the use of these formulae, a nomogram has been constructed, see § 13.

In the range of the moisture tension, where the formula 22

$$k_c = k_o e^{-\alpha(\Psi - \Psi_a)} \qquad \text{holds}$$

the formula becomes:

$$\frac{k_s e^{-\alpha(\Psi - \Psi_a)}}{v_c \pm k_s e^{-\alpha(\Psi - \Psi_a)}}\, d\Psi = dz \qquad z = \frac{1}{\alpha} \ln\left(\frac{k_s}{v_c + k_s e^{-\alpha(\Psi - \Psi_a)}}\right) \quad \text{for upward flow (37a)}$$

$$z = \frac{1}{\alpha} \ln\left(\frac{k_s}{v_c - k_s e^{-\alpha(\Psi - \Psi_a)}}\right) \quad \text{for downward flow (37b)}$$

It is not difficult to calcultate with these formulae, if a slide rule is available on which scales for e^{-x}, $e^{-0.1x}$ and so on are given (for instance the Aristo Multilog rule with scales LL00, LL01, LL02 and LL03 of DENNERT and PAPE, Aristowerke, Hamburg Altona).

Example 16. Calculate the capillary rise of water in a soil - with $\Psi_a = 0$, $k_s = 30$ cm/day - from the groundwater table to the centre of the root zone at 70 cm above the groundwater table where the tension is 500 cm. For α take 0.05.

Use the equation in the shape

$$v_c = k_s \left(e^{-\alpha z} - e^{-\alpha(\Psi - \Psi_a)}\right)$$

$$v_c = 30 \left(e^{-3.50} - e^{-25}\right)$$

$$v_c = 30(0.03 - 0.00) = 9 \text{ mm/day}$$

Problem 17. Calcultate the value of k_s which gives the same value for v_c in case the value of $\alpha = 0.01$ or 0.10.

Problem 18. Derive a formula for the capillary rise for $k_c = \frac{k_s}{\psi_a{}^2+1}$ - the solution of Gardner - and compare with formula 37.

11. The storage capacity

The storage capacity has two aspects. In a homogeneous soil with a certain moisture profile the increase or decrease of the groundwater depth d will shift the moisture profile over the same height of Δ h. The amount of water, contained between these two moisture content curves is equal to the change in air content L in the soil in the surface layer, multiplied by Δ h (see fig. 14).

In the case of water infiltrating in the soil from above, a moisture front will penetrate into the soil. The amount of water needed for the penetration from a depth d to a depth d + Δd below the soil surface will be equal to Δ d times the air content L at the depth d (see fig. 15). The way to determine the storage capacity depends on the character of the problem that is studied. Generally, the first aspect -where the storage capacity is equal to the air content in the top layer - will be encountered.

The storage capacity depends partly on the equilibrium moisture profile of the soil as given by the desorption function, partly on the amount of moisture flowing up - or downward through the soil profile. Upward flow means that the soil moisture content will be lower than the equilibrium value, downward flow means that the moisture content will be higher than in case of zero flow.

The formulae are of a steady state type and will hold best if the calculations are based on the average amount of moisture under influence of the excess of rain or evaporation on the previous days. For summer conditions, the difference of evaporation minus rainfall over a sufficiently long interval should be taken, as to exclude non-steady situations.

The storage capcity is calculated with the formulae for the capillary flow, in which the formula for the desorption curve is substituted, for example:

$$\alpha z = \ln \frac{k_s}{\left(v_c + k_s e^{-\alpha(\psi - \psi a)}\right)} \quad (37a) \qquad \frac{A}{\psi} = \frac{v^m}{(P-v)^n} \quad (16)$$

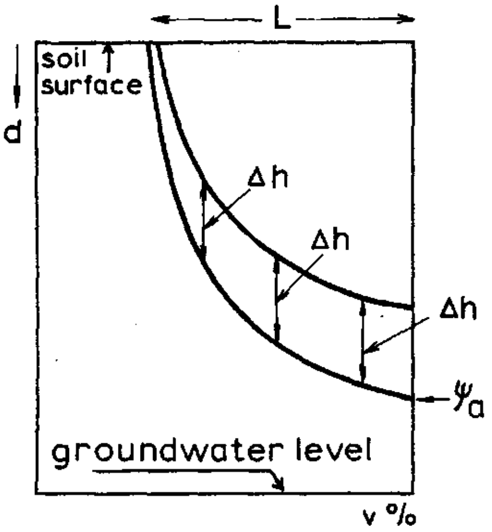


fig.14
Influence exerted by groundwater

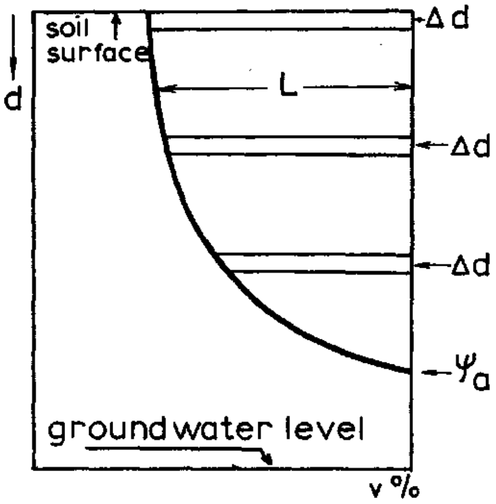


fig.15
Influence exerted by surface application

In these formulae z and ψ are values for the upper layer and not as in the formula for the capillary movement, free variables, describing any height z or moisture stress ψ between the groundwater table and the soil surface.

Example 19. Calculate the storage capacity of a soil with a downward flow v_c with constants:

$\alpha = 0.06$	$z = 80$ cm	$\psi_a = 30$ cm	$v_c = 0.5$ dm/day	
$A = 6.3$	$p = 0.8$	$b = 0.3$	$P_r = 50\%$	$k_s = 5.0$ dm/day

The formula for the downward capillary flow

$$z = \frac{1}{\alpha} \ln \left(\frac{k_s}{v_c + k_s e^{-\alpha(\psi - \psi_a)}} \right)$$

is written as:

$$\psi - \psi_a = \frac{1}{\alpha} \ln \frac{k_s}{v_c - k_s e^{-\alpha z}} = \frac{1}{0.06} \ln \left(\frac{50}{0.5 - 50 e^{-4.8}} \right)$$

$$\psi - \psi_a = \frac{1}{0.06} \ln \frac{50}{0.5 - 50.0.008} = \frac{1}{0.06} \ln 500 =$$

$$\frac{0.434}{0.06} \log 500 = 7.25 \times 2.7$$

$$\psi = 30 + 19.6 = \pm 50 \text{ cm}$$

In order to calculate the moisture content that goes with $\psi = 50$ cm first the value of ΔP should be calculated by inserting $v\% = P_r = 50\%$ and $\psi_a = 30$ cm

$$b(A - \log \psi_a) = p \log v - (1 - p) \log (P_r + \Delta P - v)$$
$$0.3\,(6.3 - 1.477) = 0.8 \log 50 - 0.2 \log \Delta P$$
$$\Delta P = 0.39$$

The next step is the calculation of the value of v for $\psi = 50$ cm with the formula

$$b(A - \log \psi) = p \log v - (1 - p) \log (P_r + \Delta P - v)$$
$$0.3(6.3 - 1.7) = 0.8 \log v - 0.2 \log (50.39 - v)$$
$$6.9 = 4 \log v - \log (50.39 - v)$$

Choose for v a value of 49% and calculate corrected values from (P-v)

v assumed	49	49.67	49.624
log v	1.690	1.696	1.6957
4 log v	6.760	6.784	6.7828
- 6.9	6.900	6.900	6.9000
difference = log (50.39-v)	9.860	9.884	9.8828
50.39-v	0.72	0.766	0.7626
v	49.67	49.624	49.6274

The storage capacity is of the order of 0.76 - 0.39 = 0.37%

12. Mass infiltration and infiltration rate

The velocity with which water infiltrates into the soil and the amount of infiltrating water after a certain time is an important parameter determining drainage capacity, ponding or surface run-off. Several formulae have been proposed and the formula of PHILIPS is outstanding. The formula, however, appears to be too complicated for practival use. *)**)
A simpler formula is the VAN DUIN - PHILIPS equation ***):

$$t = \frac{S}{k}\left\{ z - \Psi \ln \frac{z+\Psi}{\Psi} \right\} \text{ or } t = \frac{1}{k}\left\{ I - S\Psi \ln\left(1 + \frac{I}{S\Psi}\right) \right\} \qquad (38)$$

t = time

S = Storage capacity

z = depth of water front below soil surface

Ψ = capillary potential, assumed constant over profile

I = total amount of infiltrated water

k = permeability corresponding with the average degree of saturation

---

* E.G. Youngs. 1957. Moisture profiles during vertical infiltration. Soil Sci. 84, pp. 283.

** K.K. Watson. 1959. Note on field use of theoretically derived infiltration equation. Journ. Geophys. Res. 64, pp. 1611.

*** R.H.A. van Duin. 1955. Tillage in relation to rainfall intensity and infiltration capacity of soils. Neth. J. Agric. Sci. 3, pp. 182.

The first formula relates the time and the depth of the moisture front, the second formula relates time to the total amount of infiltrated water. The premise of the formula that the storage capacity S, the permeability k and the moisture potential Ψ should be constant over the profile shows that the formula is designed to treat the problem of infiltration in dry soils with a deep water table. The applicability of the formula may, however, be wider than is indicated, because in a profile with a higher groundwater table the decrease of the storage capacity and the soil moisture tension at increasing depth will neutralize the effects of each of both variables separately.

Example 20. The soil is characterized by:

Ψ = 100 cm S = 0.15 k = 50 cm/24 hours

Calculate the time of penetration of the moisture front for successive 20 cm intervals.

The formula relating t to z reads:

$$t = \frac{S}{k}\left\{ z - \Psi \ln \frac{z+\Psi}{\Psi} \right\} = 0.003 \left\{ z - 100 \ln \frac{100+z}{100} \right\}$$

z	$\frac{z+\Psi}{\Psi}$	$\ln\frac{z+\Psi}{\Psi}$	$z - \Psi \ln \frac{z+\Psi}{\Psi}$	t in days	t in hours
20	1.2	0.182	1.8	0.0054	0.13
40	1.4	0.337	6.3	0.0189	0.45
60	1.6	0.470	13.0	0.0390	0.94
80	1.8	0.588	21.2	0.0636	1.53
100	2.0	0.693	30.7	0.0921	2.21
120	2.2	0.789	41.1	0.1233	2.96

Problem 21. The rate of penetration may appear to be very great in case the gross permeability is used to determine the moisture flow. In practice the penetration may be less easy. This may be due to the fact that sealing of the surface or plow pans or other less pervious layers in reality determine the rate of infiltration.

The reader is requested to devise a method of calculating the rate of penetration in a profile where by sealing the

upper layer has a reduced permeability. Attention should be given to the unsaturated flow and its reduced permeability and available storage in the unsaturated zone below the sealed layer, as described in the next paragraph.

13. The use of the nomogram for exponential capillary flow

The nomogram consists of two sheets A and B. The sheets are used by placing them upon each other so that the lines AB and A'B' coincide. Then the parallel lines, numbered with the exponents n = (2-D) going from 1.2 to 3.5 also coincide. Select the line for the appropriate value of the exponent. Crossing the lines AB and A'B' at various angles, lines are present numbered on sheet A with a value d equal to ak_s in cm, on sheet B with v_c in mm. Select values of ak_s and v_c and mark the point of intersection of these lines with the lines for the selected value of the exponent.

Shift sheet B over sheet A till the two points of intersection and the lines AB and A'B' coincide. In this situation the position of the curved lines on sheet A in the coordinate system of sheet B is correct and in accordance with the constants selected. The curved lines mark the relation between $\log \psi$, $\log z$ and v_c which constitute the solution of the two formulae for the selected values of (2-D), ak_s and v_c.

Example 22. Determine the relation between $\log \psi$ and z for upward and downward flow for an exponent n = 1.8, ak_s = d = 100 and v_c = 2 mm by using the nomogram.

Upward flow:

$\log \psi$ =	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
z in cm =	3	10	25	40	48	51	54	56

Downward flow:

z in cm =	5	10	20	50	100	200	500
$\log \psi$ =	0.70	0.98	1.24	1.44	1.49	1.50	1.50

Repeat determination for v_c = 1, 3, 5 and 10 mm

Example 23. Calculate $d = ak_s$ from $k_s = 0.50$ m/24 hours $\Psi_a = 20$ cm and $n = 1.8$.

The value of d is found as the intercept of the curve for unsaturated conductivity on the log Ψ axis, through the point log k_s and log Ψ_a.
The formula for this curve reads in general:

$$\frac{\log k_s - \log k_c}{\log \Psi_a - \log \Psi} = -1.8 \text{ and becomes here}$$

$$\frac{\log k_s - \log d}{\log \Psi_a} = -1.8$$

$$\log d = 1.8 \log \Psi_a + \log k_s = 1.8 \times 1.301 + 9.699 = 2.041$$

d= 110 cm/24 hours

14. Tile drainage and capillary properties

One of the important drainage design constants is the design runoff. The requirements in this respect are expressed in mm per 24 hours. This, however, is a constant which does not account for the storage capacity of the soil nor for the amount of air in the soil which is needed for good growth. A better criterium would be the requirement that after a certain number of days an air content in the soil of a certain magnitude should be restored. Such a criterium of a more plant physiological nature requires the introduction of the storage capacity into the design calculations.

This storage capacity should not be calculated for zero flow but for an average intensity of flow equal to the excess of the rainfall in the preceding period over the average evaporation. The actual storage capacity will be less than the storage capacity as given by the desorption curve.

The introduction of the air entry point in the design considerations is here also of importance. The saturated capillary zone has the same permeability as the groundwater zone and will take an equal part in the conveyance of water to the drain. Due to the absence of air in the

saturated capillary zone the plant will only grow well if the root zone is situated in the unsaturated zone. This will require a certain depth of the water table in excess of the magnitude of the saturated capillary zone. The two properties, the storage capacity and the height of the saturated capillary zone are basic properties for the determination of both drain depth and spacing.

Example 24. A soil with a homogeneous profile is characterised by the following constants of the desorption curve:

$p = 0.75 \quad b = 0.25 \quad A = 5.5 \quad P_r = 45 \quad \Delta P = 5$

1) Calculate the moisture stress at air entry point

2) Calculate the air content with increasing moisture stress

The formula $b(A-\log \Psi) = p \log v - (1-p)\log(P_r + \Delta P - v)$

is reduced to $5.5 - \log \Psi = 3 \log v - \log(50-v)$

Table 8

v	45	42.5	40	38.5	37.0	36.0	35.0	34.0	33.0
log v	1.653	1.628	1.602	1.585	1.568	1.556	1.544	1.531	1.519
3 log v	4.959	4.884	4.806	4.755	4.704	4.668	4.632	4.593	4.557
log 50-v	0.699	0.875	1.000	1.061	1.114	1.146	1.176	1.204	1.230
difference	4.260	4.009	3.806	3.694	3.590	3.522	3.456	3.389	3.327
log Ψ	1.240	1.491	1.694	1.806	1.910	1.978	2.044	2.111	2.173
Ψ	17.4	31.0	49.4	64.0	81.3	95.1	110.1	129.1	148.9
L	0	2.5	5.0	6.5	8.0	9.0	10.0	11.0	12.0

The moisture stress at air entry point appears to be 17.4 cm. The values of L and Ψ show how the air space reacts on changes of Ψ.

3) Determine the height above the groundwater table at which the calculated values of Ψ will be found under the influence of the infiltration of 4 mm of rain water per 24 hours. The constants for unsaturated flow are: k_s = 30 cm α= 0.035.

The formula is used in the shape:

$$e^{-\alpha(\Psi - \Psi_a)} - \frac{v_c}{k_s} = \left(1 - \frac{v_c}{k_s}\right) e^{-\alpha(z - z_a)}$$

or

$$e^{-0.035(\Psi - 17.4)} - \frac{0.4}{30} = \left(1 - \frac{0.4}{30}\right) e^{-0.035(z-17.4)}$$

Table 9

Ψ	31.0	49.4	64.0	81.3	95.1	110.1	129.1	148.9
$\Psi - \Psi_a$	13.6.	32.0.	46.6.	63.9	77.6	93.3	111.7	131.5
$\alpha(\Psi - \Psi_a)$	0.476	1.120	1.631	2.237	2.716	3.265	3.910	4.603
$e^{-\alpha(\Psi-\Psi_a)} = (p)$	0.622	0.326	0.196	0.107	0.066	0.038	0.020	0.010
$(p) - v_c/k_s = (q)$	0.609	0.313	0.183	0.094	0.053	0.025	0.007	
$q/(1 - v_c/k_s) = e$	0.716	0.317	0.185	0.0952	0.0537	0.0253	0.0071	
αz_1	0.483	1.150	1.69	2.35	2.96	3.68	4.95	
z_1	13.8	32.9	48.3	67.1	84.6	105.1	141.4	
$z = z_1 + z_a$	31.2	50.3	65.7	84.5	102.0	122.5	158.8	

The quantity of moisture V, expressed in cm^3/cm^2 that has to be drained away in order to bring $(L_2 - L_1) cm^3/cm$ air into the soil and lower the groundwater table from z_1 to z_2 cm depth is calculated as:

$$V = \sum \left(\frac{z_2 - z_1}{100}\right)\left(\frac{L_2 + L_1}{2}\right)$$

Table 10

z	17.4		31.2		50.3		65.7		84.5		102.0		122.5		158.8
L	0		2.5		5.0		6.5		8.0		9.0		10.0		11.0
$z_2 - z_1$		13.8		19.1		15.4		18.8		17.5		20.5		36.3	
$(L_2 + L_1)/2$		1.25		3.75		5.75		7.25		8.5		9.5		10.5	
V			0.173		0.889		1.774		3.137		4.625		6.572		10.384

4) Define the drainage requirements and select a drainage formula accounting for the existing boundary conditions.

a) The formula of Rothe is taken here because of its simplicity:

$$NL^2 = k_s \left\{ (H-z)^2 - h^2 \right\}$$

N = discharge of drain

L = half of drain distance

k_s = saturated conductivity

H = depth of drainage

h = height of saturated capillary zone between drains

z = depth of saturated capillary zone between drains

For a good drainage the discharge capacity and the costs of drainage have to be compared with each other. The drain depth and spacing will therefore be calculated for a series of suppositions and the best combination will be selected by inspection. We assume that with a prolonged rainfall of 4 mm an air content of 8% exists. Further we assume that a water table of 30 cm below soil surface will be lowered in a short period t to such a depth that by a constant rain of 4 mm the air content is restored to at least 5%.

The 8% air content is obtained by bringing the groundwater table at a depth of 84.5 cm (see table 10). The mean groundwater depth, necessary to restore the 5% air content is taken as the average of the depth for 2.5 and 5.0% air content or 31.2 and 50.3 cm groundwater depth or 40.7 cm. For k_s the value of 30 cm is taken. Three formulae can be set up:

$$\begin{aligned} &1)\quad 0.4\,L^2 = 30 \left\{ (H - 84.5)^2 - 17.4^2 \right\} \\ &2)\quad nL^2 = 30 \left\{ (H - 40.7)^2 - 17.4^2 \right\} \\ &3)\quad n = (v_2 - v_1)/\,t + 0.4 \end{aligned}$$

Formula 1 gives the relation between drainspacing L and the depth of drainage H for 4 mm constant rain and a groundwater depth of 84.5 cm.

Formula 2 gives the same relation for a groundwater

depth of 40.5 cm and a discharge n.

Formula 3 expresses this discharge n as the sum of 4 mm constant rain and the runoff of excess moisture stored in the soil resulting from the heavy rain.

The calculation is carried out as follows:

Table 11

H	100	120	140	160	180	200	190
$H-40.7=p$	59.3	79.3	99.3	119.3	139.3	159.3	149.3
$H-84.5=q$	15.5	35.5	55.5	75.5	95.5	115.5	105.5
p^2	3516	6288	9860	14232	19404	25376	22290
q^2	240	1260	3080	5700	9120	13340	11130
$p^2-17.4^2=r$		5985	9557	13929	19101	25073	21987
$q^2-17.4^2=s$		957	2777	5397	8817	13037	10827
$L^2=30/0.4^s \times 10^4$		71.77	20.83	40.48	66.13	97.78	81.20
2 L in m		5.36	9.12	12.72	16.26	19.78	18.02
$n=0.4\ r/s$		2.50	1.38	1.03	0.87	0.77	0.81
$t=\frac{(0.889-0.173)}{n-0.4}$		0.34	0.73	1.14	1.52	1.93	1.75

The value of t shows how many days it takes to lower the water table from 30 to 50 cm and to increase the air content from 25 to 50% (see table 10) during a constant rain of 4 mm/day. The drain spacing 2L becomes very small if the runoff is brought at a high capacity. We assume that the combination 2L = 18 meter and t = 1.75 days is economically the most attractive one. The depth of drainage appears to be 1.90 m. This is checked by calculating what time is required to lower the water table from 0 to maximum depth, taking into account a constant rain of 2 mm.

The formula to be used reads:

$$N = \frac{30}{L^2}\left\{\left(H - \frac{z_1 + z_2}{2}\right)^2 - 17.4^2\right\}$$

The values $(z_1 + z_2)/2$ are taken from table 10.

The average value $\frac{z_1 + z_2}{2}$ is used because for these values the value of v in table 3 is known.

Table 12

$(z_1+z_2)/2=p$	24.4	40.7	58.0	75.1	93.2	112.2	140.3
190-p	165.6	149.3	132.0	114.9	96.8	77.8	49.7
p^2	27423	22290	17424	13202	9370	6053	2470
$p^2-17.4^2=q$	27120	21987	17121	12899	9067	5750	2167
$\frac{30}{L^2} q = N$	1.003	0.812	0.633	0.477	0.335	0.212	0.080
subtract 2 mm rain	0.803	0.612	0.433	0.277	0.135	0.012	
v_2-v_1	0.173	0.716	0.885	1.363	1.488		
$t = v_2-v_1/N-0.2$	0.22	1.17	2.04	4.92	11.02		
Σt	0.22	1.37	3.43	8.35	19.37		
L	2.5	5.0	6.5	8.0	9.0		

From the computation it appears that a drain distance of 18 m and a depth of drainage of 1.90 m is able to bring the air content L of the root zone during a continuous rainfall of 2 mm from 2.5% comparing with a groundwater level of 30 cm below soil surface down at the following rate:

t = 0	2.04	4.92	11.02
L= 2.5	5.0	6.5	8.0
z = 31.2	50.3	65.7	84.5

Agricultural experience will have to show whether these requirements are sufficient or need to be raised. The purpose of the example is, to show that in areas where no experience with relation to drainage requirements exist, it is possible to calculate the design constants, based on the capillary properties of the soil and an assumption regarding the need of the plant of a speed of restauration of the desired air content in the soil after a heavy rain or artificial application of water.

15. Extraction of soil moisture by the plant

The moisture, extracted by the roots, flows through the unsaturated soil surrounding the root. This flow is governed by the law of extraction by a sink, as known in groundwater hydrology, but instead of the saturated conductivity the capillary conductivity has to be inserted in the formula. The equation, governing the extraction was derived (VISSER, 1964) * and by taking the extraction equal to the evaporation, the following equation was obtained:

$$(gE_o - E) \left(\frac{A'}{\psi^{n-1}} - E \right) = B \qquad (39)$$

g = constant of the order 0.90 to 1.10 describing the exposition of the crop to wind, lateral inflow of drier air, density of canopy, etc.

E_o = open water evaporation, calculated or determined with a pan

E = real evaporation

n = exponent of the formula for the capillary conductivity

A', B = constants to be determined by field experiments

The capillary properties are contained in the constant A, but do not need further consideration here because their magnitude depends on the root density, which in general will not be known.

The difficulty of determining ψ values in the field and the comparative ease to determine moisture contents, makes it desirable to express the formula just given in terms of moisture content by substituting ψ by the expression given in the desorption equation. Often, however, one may restrict the desorption formula to the abridged form:

$$\psi = \frac{G(P_r + \Delta P - v)^n}{v^m} \quad \text{can be abridged to} \quad \psi = \frac{G'}{v^m}$$

---

* W.C. Visser. 1964. Moisture requirements of crops and rate of moisture depletion of the soil. Techn. Bull. 32 I.C.W.

Very often an even further simplification is possible by taking instead of

$$\Psi^{n-1} = \frac{G''}{v^{(n-1)m}} \quad \text{the formula} \quad \Psi^{n-1} = \frac{G''}{v^3}$$

In that case as formula for the extraction equation may be used:

$$(gE_o - E)\,(Av^3 - E) = B \tag{40}$$

Because the value of B is small, the most simplified expression for the evaporation is to take the lowest of the following two formulae:

$$E = gE_o \qquad \text{or} \qquad E = Av^{(n-1)m} \simeq A\,v^3$$

with the point of transition:

$$gE_o = A\,v^3 \tag{41}$$

The soil moisture content should be averaged for the upper 50 to 80 cm. A detailed analysis of the importance of the moisture content of separate layers may be made by calculating $A\,v^3$ for the successive layers and using $\sum_{i=1}^{n} A_i v_i^3$ instead of $A\bar{v}^3$. The determination of the A-values for successive layers is rather complicated. The reader is referred to Tech. Bull. 32, mentioned before.

Problem 25. A light clay soil, grown with grass, could be described by the constants: $g = 0.9$ $A = 300$ $(n-1)m = 3.1$ $B = 0.001$

> Calculate the actual evaporation for 10, 20, 30 and 40% of moisture content and 0.2, 0.4 and 0.6 cm of potential evaporation. Plot the results against each other and draw lines through the points for constant moisture content and constant potential evaporation.

For the determination of the constants for given data on evaporation and moisture content the reader is referred to BLOEMEN (1963)*.

16. Influence of the groundwater depth and the moisture content on plant growth

The aim of water management is generally to realize conditions

---

* G.W. Bloemen, 1963. The graphical representation of the relation between real evapotranspiration and open water evaporation or moisture condition of the soil. Nota 218 I.C.W.

under which the plant will grow best. This means that a relation has to be found between the moisture conditions in the unsaturated zone and the productive capacity of the crop. The reaction of the crop yield on the availability of moisture can be expressed as a relation between the yield and the moisture content of the soil. In many cases the relation between the evaporation and the groundwater depth will not be very strong. The relation between the aeration and the groundwater depth is far more direct. Therefore an expression for the relation between crop yield and aeration will most practically be given as the relation between the yield and the groundwater depth.

The formula for the yield, as influenced by the plant, the aeration and the evapotranspiration consists of three terms, each of the general shape:

$$(a - \frac{q}{x})$$

in which a is a reaction constant, q the real yield and x the growth factor. Firstly one has to consider how the growth factor should be expressed.

The expression for the growth factor of the aeration A of soils is of an exponential type:

$$A = \alpha\ (H^{l} - b)$$

H = groundwater depth
l = exponent equal to 4 for the relation between air space and aeration intensity, minus the exponent n of the $(P_r + \Delta P\text{-}v)$ term of the desorption equation.
b = the air entry stress ψ_a raised to the power l, describing the influence of the saturated capillary zone

The expression for the growth factor of the moisture consumption is the sum of evaporation over the growing period divided by the number e of liters of water needed for the production of one kg of dry matter which is often put to be of the order of 300. For the constant c in formula 42, describing the ratio of yield divided by the water consumption, the value should be taken which holds for situations with low availability of water. In the yield equation this constant c equal to 1/e is used in such a way

that the formula allows for the increase of the value of c at higher availabilities of the soil moisture and will be of the nature of $\frac{1}{200}$ kg/l or 5 kg/mm water.

The equation for the crop yield reads:

$$\left(1 - \frac{q}{Q}\right)\left(a - \frac{q}{H^{l} - b}\right)\left(c - \frac{q}{\Sigma E}\right) = D \qquad (42)$$

Q = maximum yield to which the crop may grow under optimal conditions on account of its genetical capacity
a, c = response factors or growth factors
q == real yield
D = constant dependent on root density and flexability of the crop in the adaptation to environmental conditions
Σ E = sum of daily real evapotranspiration over the whole growth period in mm

In case also other growth factors might influence the plant yield, more terms between brackets should be added, all of the same kind of a response factor a minus the ratio of the yield q to the availability x of the growth factor. Would nitrogen be limiting, then a term

$\left(n - \frac{q}{N_{soil} + N_{fertilizer}}\right)$ should be added.

The value of D is of the order of 0.05^{n} with n the number of terms used in the equation.

Exercise 26. Calculate the yield with the following constants:

Q = 5000 kg/ha a = 0.5 b = 20 c = 5 1 = 2
D = 0.000125

Vary H between 20 and 200 cm below soil surface. Vary ΣE between 100 and 1000 mm over the growing period.

Repeat the calculation for a low yielding and a high yielding crop and draw a conclusion about the relations between the

requirements of available moisture and groundwater depth and the productive capacity of the crop variety.

17. Salinisation and leaching

A soil with a moisture profile in equilibrium with the depth of the groundwater table as well as with the rate of capillary infiltration or rise has a fixed ratio of moisture filled capillaries and air filled capillaries dependent on the groundwater depth. The movement of water through the wider pores of such a soil is far more rapid than through the smaller pores. If in such a soil at a certain depth a salt front occurs, then this difference in velocity of the moisture flow will cause a rapid change in the location of the salt front in the wider and a slow change in the smaller pores. In case of capillary rise, the salt will quickly - but in small quantities - reach the soil surface through the widest pores. Somewhat later the somewhat smaller pores will transport the saline water to the surface so that gradually the salinity will increase as a volume of pores with successive smaller sizes no longer contain fresh water, but will be filled with saline water. Further the water evaporates and the salt remains behind so that the salt concentration will rise. With respect to the computation of the mass of transported salt it will be important to have in mind that the finer capillaries may at a certain moment still have not been able to bring the saline water to the surface. Cases may be met where this time lag may be important.

By leaching the problem is the same and a high rate of application of leaching water will flush the wider capillaries many times over, but only leaching of sufficiently long duration will expel the saline water from the smaller capillaries. The leaching efficiency will depend strongly on the rate of application. A high rate means a low leaching efficiency.

Example 27. Calculate the rate with which salt is transported through the capillaries of different size of a soil of known desorption curve.

The calculation of an example is best done in the following way.

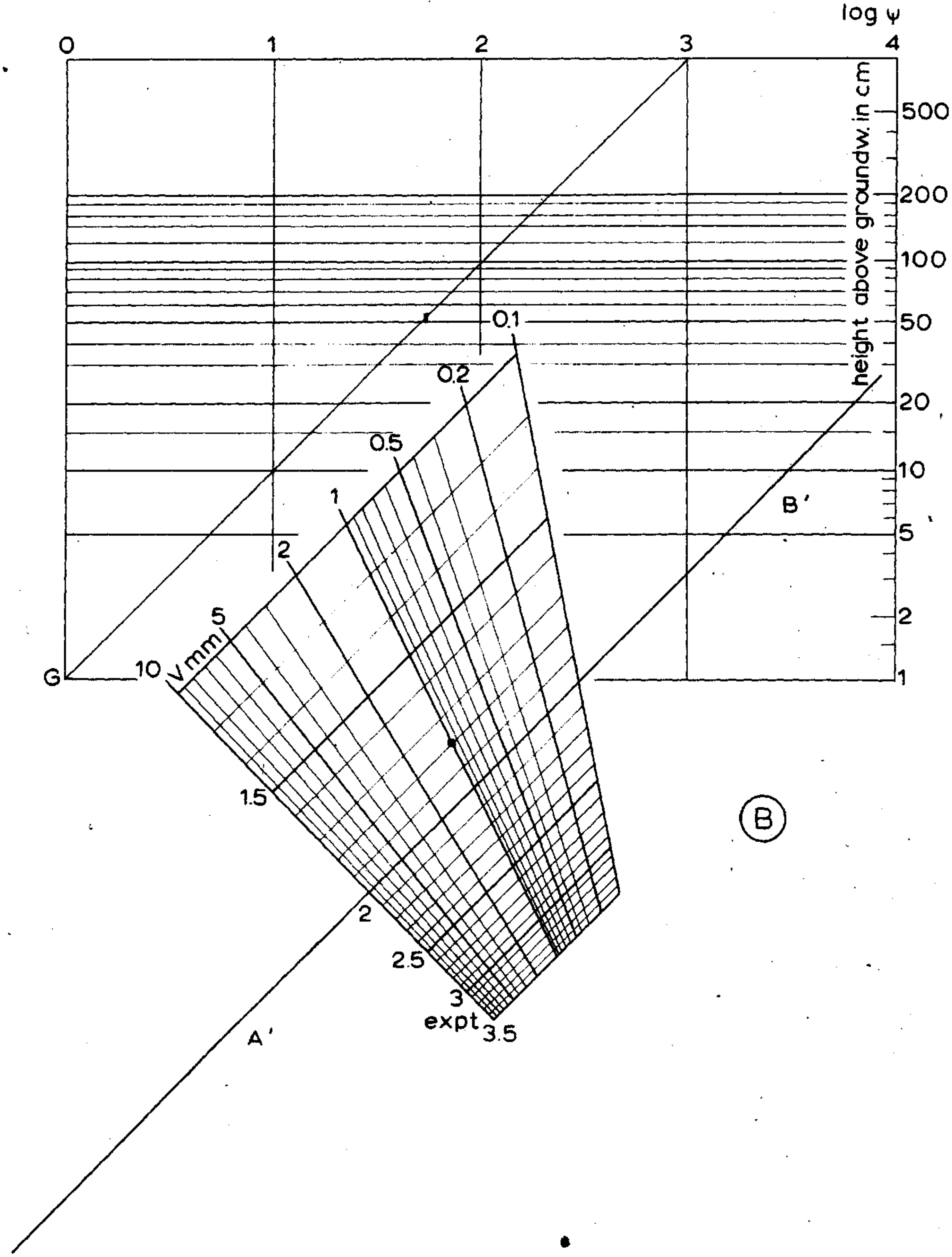
1) With formula 37 for the solution of the formula for capillary rise and infiltration, the rate of infiltration and the profile of moisture stresses are calculated as given on page 44, example 16. This gives the gradient i for the area of unsaturated flow.
2) As discussed in example 9 an axis calibrated for pF and v values is made.
3) Along the lines, discussed in chapter 9c on page 36 , this same axis is calibrated also with the permeability scale k.
4) The profile for the capillary rise, available according chapter 13 on page 48 , example 20, but plotted there against the moisture stress, is in this case plotted against the moisture content.
5) The profile of step 4 is divided in for instance 20 cm layers and 5 moisture content parts. The partitioning points of the successive layers are connected so as to form 5 columns. This splits the moisture profile up into a number of quadrangles.
6) For the centre of each quadrangle the average value for the permeability k, the gradient i and the area of flow F are determined by taking k from the axis prepared in step 3, by measuring for F the difference in moisture content at the upper and lower boundary of the columns, prepared in step 5, and by assessing for i the difference in Ψ at the boundaries of the successive 20 cm layers as prepared in step 1.
7) For each centre of a quadrangle the amount of flow is calculated from $q = k \rho i$ and the velocity from $v = k\ i$.
8) For each layer the line of the integrated amount of flow is plotted against the moisture content axis, by plotting at the centre of each column the sum of the amounts of flow of that column plus that of all the columns at lower moisture contents. The sum over all columns is for each layer the same. The partitioning points are determined where 20%, 40% ...... 100% of the flow is reached. By connecting the partitioning points for successive layers, the lines of flow are constructed.

9) Compare the location of the flow lines between which 20% of the amount of flow occurs with the boundaries of the columns, between which 20% of the moisture content is situated.

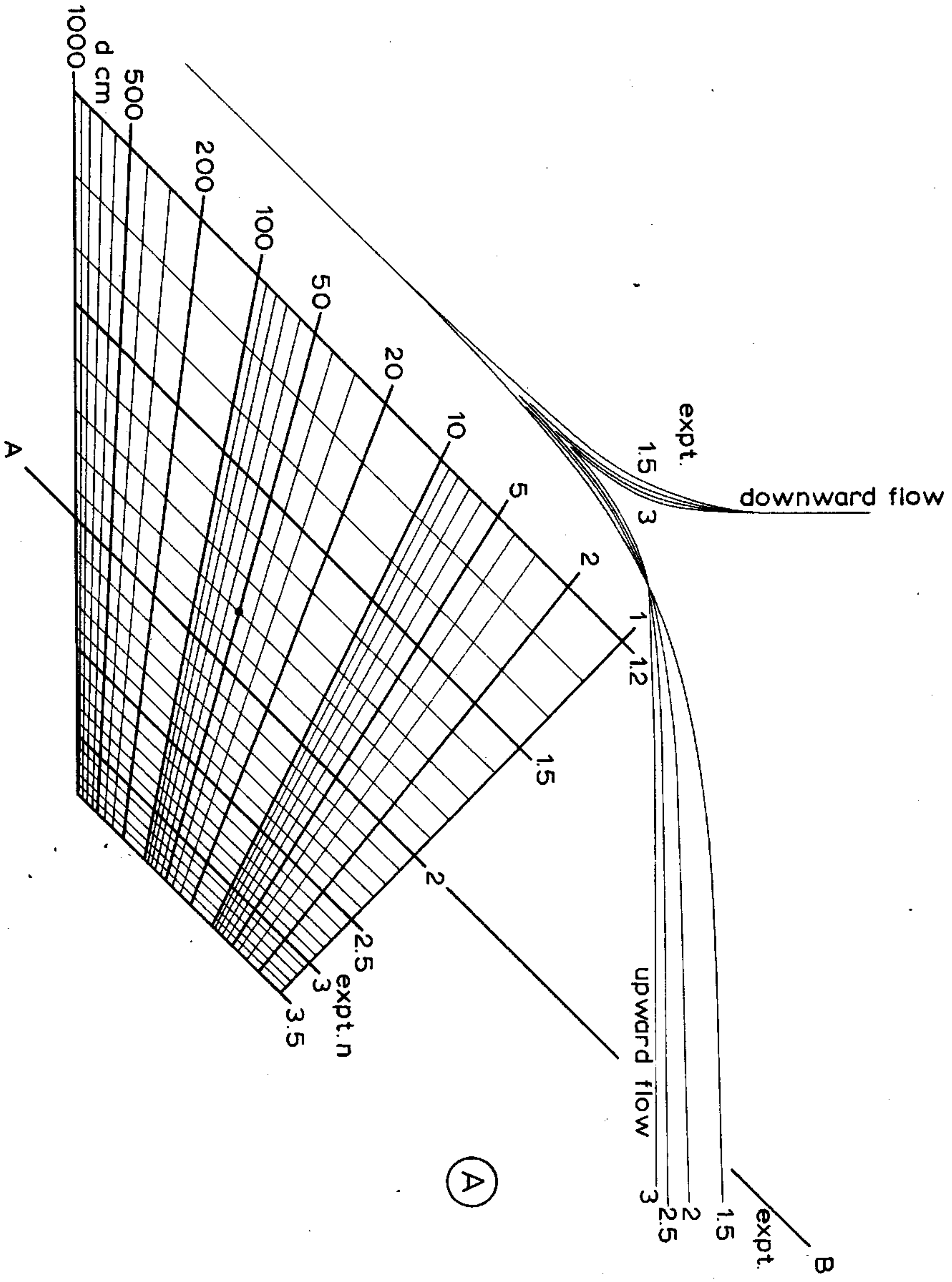
10) Determine for the middle between two successive lines of flow the flow velocity by dividing the constant amount of flow q_k by the area of flow F_{kl}, k denoting the k^{th} flow line, l denoting the l^{th} layer.

11) Compute the time needed by the flowing moisture to cover the distance along a line of flow, by plotting along this line of flow the value for the integrated time, needed to cover the distance between the boundaries of each layer.

12) Draw for each layer the line denoting after what length of time the successive percentages of the moisture content at the layer under observation are reached by the salt front and occupied by the saline water.


log ψ
0
1
2
3
4
height above groundw. in cm
500
200
100
50
20
10
5
2
1
G
0.1
0.2
0.5
1
2
5
10
V mm
1.5
2
2.5
3
expt
3.5
A'
B'
B

62c-148-1/7^{b}


d cm
1000
500
200
100
50
20
10
5
2
1
1.2
1.5
2
2.5
3
3.5
expt.n
A
expt.
1.5
3
downward flow
upward flow
3
2.5
2
1.5
expt.
B
Ⓐ

62 c-148-2/7b