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The calculation of non-parallelism of gamma access tubes, using soil sampling data

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1. Introduction

It has been shown by RYHINER and PANKOW (1968) that the gamma transmission method gives good opportunities to measure the variation in soil moisture content at various depths in the soil profile with time. Absolute values of the soil moisture content can be obtained with separate determinations of dry bulk density and organic matter, so some sampling in the direct environment of the access tubes will be necessary in that case.

It is also clear from the discussion presented by Ryhiner and Pankow, that the results of the gamma transmission technique are very sensitive to small errors in distance between source and detector.

An indication of non-parallelism of both access tubes can be obtained when comparing the data of soil sampling with the results obtained with the gamma transmission method. However, the main difficulty remains the separation between the systematical error, due to non-parallelism and the random variation due to sampling errors and soil heterogeneity. This random variation will often be greater than the systematic error due to non-parallelism. It is the purpose of this paper to present a method to calculate the systematic errors due to non-parallelism of the access tubes.

2. Theory

When inserting the access tubes in the soil, non-parallelism shall mainly appear as a crossing of the tubes in a three-dimensional space, rather than as a deviation in a plane.

Denoting the co-ordinates of the centre-line of the source tube by x, y, z and the co-ordinates of the centre-line of the detector tube by x', y', z', gives the following equations of the centre-lines of the tubes.

$$\text{Source tube} \quad : \frac{x-x_o}{a_1} = \frac{y-y_o}{a_2} = \frac{z-z_o}{a_3} \tag{1a}$$

$$\text{Detector tube} \quad : \frac{x'-x'_o}{a'_1} = \frac{y'-y'_o}{a'_2} = \frac{z'-z'_o}{a'_3} \tag{1b}$$

where x_o, y_o, z_o and x'_o, y'_o, z'_o are the co-ordinates of the respective centre-lines at the soil surface; a_1, a_2, a_3 and a'_1, a'_2, a'_3 are the direction coefficients of the respective centre-lines.

The distance L travelled by the mid-point of the source and by the mid-point of the detector in the corresponding access tubes is presented by the following expression

$$L^2 = (x-x_o)^2 + (y-y_o)^2 + (z-z_o)^2 = (x'-x'_o)^2 + (y'-y'_o)^2 + (z'-z'_o)^2 \tag{2}$$

Combination of the equations (1) and (2) gives:

$$x-x_o = \frac{a_1 L}{\sqrt{a_1^2 + a_2^2 + a_3^2}} \tag{3a}$$

$$x'-x'_o = \frac{a'_1 L}{\sqrt{(a'_1)^2 + (a'_2)^2 + (a'_3)^2}} \tag{3b}$$

Systematic exchange of the coefficients gives the following expressions of the co-ordinates of the mid-points of source and detector.

$$\text{Source} \quad : x=x_o + \frac{a_1 L}{\sqrt{\sum(a)^2}},\ y=y_o + \frac{a_2 L}{\sqrt{\sum(a)^2}},\ z=z_o + \frac{a_3 L}{\sqrt{\sum(a)^2}} \tag{4a}$$

$$\text{Detector} : x'=x'_o + \frac{a'_1 L}{\sqrt{\sum(a')^2}},\ y'=y'_o + \frac{a'_2 L}{\sqrt{\sum(a')^2}},\ z'=z'_o + \frac{a'_3 L}{\sqrt{\sum(a')^2}} \tag{4b}$$

As the choice of the origin of the coordinate system and the direction of one of the axes is still arbitrary, it is useful to take the origin at the point of intersection of the centre-line of the source tube with soil surface and to take the direction of the x-axis, through the point of intersection of the centre-line of the detector tube with soil surface. In that case the point (x_o, y_o, z_o) has the co-ordinates (o, o, o) and the point (x'_o, y'_o, z'_o) has the co-ordinates (x'_o, o, o).

With this choice of the co-ordinate system the equations of the mid-points of source and detector can be written as:

$$\text{Source} \quad : x = \frac{a_1 L}{\sqrt{\sum(a)^2}};\ y = \frac{a_2 L}{\sqrt{\sum(a)^2}};\ z = \frac{a_3 L}{\sqrt{\sum a^2}} \tag{5a}$$

$$\text{Detector} : x' = x'_o + \frac{a'_1 L}{\sqrt{\sum(a')^2}};\ y' = \frac{a'_2 L}{\sqrt{\sum(a')^2}};\ z' = \frac{a'_3 L}{\sqrt{\sum(a')^2}} \tag{5b}$$

The distance D between the mid-points of source and detector is given by the expression:

$$D = \sqrt{(x' - x)^2 + (y' - y)^2 + (z' - z)^2} \tag{6}$$

(2) [illegible]

Combination of the equations (1) and (2) gives:

(3a) [illegible]

(3b) [illegible]

Systematic exchange of the coefficients gives the following equations of the coordinates of the mid-points of source and detector:

(4a) Source: [illegible]

(4b) Detector: [illegible]

As the choice of the origin of the coordinate system and the direction of the axes is still arbitrary, it is useful to take the origin at the point of intersection of the source tube with soil surface and to take the direction of the x-axis through the point of intersection of the tube of the detector with soil surface. The point of intersection has the coordinates (0, 0, 0) and the point [illegible] has the coordinates (x'_0, 0, 0).

With this choice of the co-ordinate system the coordinates of the mid-points of source and detector can be written as:

(5a) Source: [illegible]

(5b) Detector: [illegible]

The distance D between the mid-points of source and detector is given by the expression:

(6) [illegible]

Substitution of the equations (5) in (6) gives:

$$D = \sqrt{\left(x'_o + \frac{a'_1 L}{\sqrt{\sum (a')^2}} - \frac{a_1 L}{\sqrt{\sum (a)^2}}\right)^2 + \left(\frac{a'_2 L}{\sqrt{\sum (a')^2}} - \frac{a_2 L}{\sqrt{\sum (a)^2}}\right)^2 + \left(\frac{a'_3 L}{\sqrt{\sum (a')^2}} - \frac{a_3 L}{\sqrt{\sum (a)^2}}\right)^2} \qquad (7)$$

Rewriting of equation (7) gives:

$$D^2 = (x'_o)^2 + 2\,A x'_o L + (A^2 + B^2 + C^2)\,L^2 \qquad (8)$$

where:

$$A = \frac{a'_1}{\sqrt{\sum (a')^2}} - \frac{a_1}{\sqrt{\sum (a)^2}}$$

$$B = \frac{a'_2}{\sqrt{\sum (a')^2}} - \frac{a_2}{\sqrt{\sum (a)^2}}$$

$$C = \frac{a'_3}{\sqrt{\sum (a')^2}} - \frac{a_3}{\sqrt{\sum (a)^2}}$$

3. Experimental procedure

Particularly in long term experiments it is very useful to obtain the necessary information concerning non-parallelism of the access tubes, without breaking down the experimental set up. This information can under field conditions be obtained, by taking undisturbed soil samples at various depths of the profile in the direct environment of the access tubes. During the sampling also measurements with the gamma transmission method are performed at the same depths. From the soil samples the dry bulk density, the moisture content and the organic matter content are determined.

Expressing the results of soil sampling and the data of the gamma transmission method in the equivalent density unit $(0.9\,\rho_s + \rho_w + 1.01\,\rho_o)$ proposed by RYHINER and PANKOW (1968), gives the opportunity to express the value of $100\{\rho_\gamma / \rho_s - 1.00)$ in terms of a deviation in distance Δx, as was shown by the authors mentioned. An example of the soil sampling data, the results of the gamma transmissions method and the derived deviation in distance Δx is given in table 1.

Table 1. Values of $0.9\,\rho_s + \rho_w + 1.01\,\rho_o$ as obtained by sampling and by the gamma transmission method. The difference between both methods is expressed as a deviation in distance Δx

Depth cm	$0,9\rho_s + \rho_w + 1.01\rho_o$ sampling a	b	mean	gamma method	$100\{\rho_\gamma/\rho_s - 1\}$ %	Δx cm
10	1.658	1.470	1.564	1.638	+ 4.71	+ 1,20
20	1.651	1.693	1.672	1.574	- 5.87	- 1.60
30	1.624	1.634	1.629	1.563	- 4.03	- 1.15
40	1.573	1.627	1.600	1.521	- 4.96	- 1.37
50	1.656	1.648	1.652	1.587	- 3.95	- 1.12
60	1.702	1.690	1.696	1.653	- 2.55	- 0.76
70	1.650	1.662	1.656	1.648	- 0.50	- 0.18
80	1.638	1.604	1.621	1.631	+ 0.60	+ 0.13
90	1.654	1.600	1.627	1.645	+ 1.11	+ 0.27
100	1.592	1.606	1.599	1.654	+ 3.45	+ 0,84

The depth given in this table represents in fact the distance travelled by the mid-points of the source and of the detector in their respective access tubes.

When the replications of the soil sampling data in the top soil show large differences, which can be very often the case in arable land, it will be wise to measure directly the distance between the two centre-lines at the soil surface, which reduces the problem to the solution of an equation with two unknown constants. When this, due to any reason, has not been performed it might be useful to set into the series of data also the value L = o, combined with the theoretical distance D of 40 cm and to solve the equation as one with three unknown constants using the relation

$$D^2 = aL^2 + bL + c \tag{9}$$

It appears from equation (8) in section 2 that the constant a must be greater than $\frac{1}{4}\,b^2\,c^{-1}$. When a is smaller than this value the random variation due to sampling errors and soil heterogeneity is such that the three dimensional crossing of the tubes cannot be shown. The only remaining possibility in that case is to show a deviation in a plane with intersecting centre-lines. The [illegible] Values of [illegible] γ [illegible] + 1.03 [illegible] obtained [illegible] counting and by gamma [illegible] diffusion method. The difference [illegible] both methods is expressed as a deviation in distance Δx.

Depth	[illegible] counting			gamma method	deviation %	Δx
[illegible]	[illegible]	1.170	1.5[illegible]	[illegible]	[illegible]	+ 1.20
20	[illegible]	1.[illegible]95	1.672	[illegible]	[illegible]	- 1.06
30	[illegible]426	1.634	1.62[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]573	1.627	[illegible]	1.[illegible]	[illegible]	[illegible]
50	[illegible]	[illegible]	1.652	[illegible]	[illegible]	[illegible]
60	1.702	1.690	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	1.650	1.6[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	1.6[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
90	1.[illegible]	1.66[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
100	1.592	1.606	[illegible]	[illegible]654	[illegible]	[illegible]

The depth [illegible] this table [illegible] by the [illegible] and of the [illegible] their respective [illegible] tube.

[illegible] very often [illegible] in practice [illegible] distance between two centre-lines [illegible] problem to the solution [illegible] to any [illegible] has not been [illegible] series of data [illegible] of [illegible] three [illegible] the [illegible]

$$\mu^2 = aL^2 + bL + c \qquad (9)$$

[illegible] from equation (8) [illegible] that the constant [illegible] greater than [illegible] 10^{-4}. When [illegible] small [illegible] this value the [illegible] counting [illegible] and [illegible] is such that the three-dimensional crossing of the tubes cannot [illegible]. The only remaining [illegible] case [illegible] show a deviation [illegible] with [illegible] problem reduces then to a linear relationship between D and L.

In table 2 the data of L and D are given. As a result of the calculations it appeared that the relation between both factors can be given best by the expression:

$$D^2 = 0.05742\, L^2 - 5.15427\, L + 1630.0005 \qquad (10)$$

It follows from the constants that $\frac{1}{4}\, b^2\, c^{-1}$ equals 0.00407, so a is greater than $\frac{1}{4}\, b^2\, c^{-1}$ and the given condition of crossing of the access tubes is fulfilled. The values of D calculated for each level with equation (10) are also given in table 2. From the data presented by Ryhiner and Pankow the corresponding correction factors for each level are calculated by which the original data from the gamma transmission method must be divided. The corrected gamma data are also given in table 2, as well as the mean values of the soil sampling data. The remaining deviation between the corrected data of the gamma transmission data and the soil sampling data is due to random variation.

Table 2. Values of L and D, as derived from table 1 and the values of D calculated with equation (10). The correction factors due to non-parallelism are also given, as well as the density derived from the original gamma data, the corrected values and the mean values of soil sampling

L (cm)	D (cm)	D (eq. 10) (cm)	Correction factor	$0,9\,\rho_s + \rho_w + 1.01\,\rho_o$		
				original gamma data	corrected gamma data	mean sampling
0	40.00	40.38	1.0169	-	-	-
10	41.20	39.28	0.9943	1.638	1.647	1.564
20	38.40	39.37	0.9797	1.574	1.607	1.672
30	38.85	39.07	0.9681	1.563	1.615	1.629
40	38.63	38.92	0.9613	1.521	1.582	1.600
50	38.82	38.93	0.9615	1.587	1.651	1.652
60	39.24	39.08	0.9685	1.653	1.707	1.696
70	39.82	39.44	0.9816	1.648	1.679	1.656
80	40.13	39.81	0.9945	1.631	1.640	1.621
90	40.27	40.39	1.0171	1.645	1.617	1.627
100	40.84	41.10	1.0435	1.654	1.585	1.599

4. Discussion

The results presented in the previous section show that it is possible to determine factors to correct for non-parallelism of the access tubes. The practical meaning of this procedure with respect to the determination of the change in moisture content between two successive measuring dates, as well as the consequences in the determination of the absolute moisture contents must be further considered. For this reason the original data obtained under dry and wet conditions are given in table 3. Moreover, the corrected values are given, as well as the uncorrected and corrected values of the change in moisture content. This table also presents the original and the corrected values of the absolute moisture content.

Table 3. Values of the original and corrected data, obtained by the gamma transmission method under dry and wet conditions, as well as the change in moisture content and the values of the absolute moisture content

Depth	$0{,}9\,\rho_s + \rho_w + 1.01\,\rho_o$				Change in moisture content		Moisture content			
	original		corrected				dry		wet	
	dry	wet	dry	wet	original	corrected	original	corrected	original	corrected
10	1.492	1.638	1.501	1.647	14.6	14.6	25.9	26.8	40.5	41.4
20	1.489	1.574	1.520	1.607	8.5	8.7	25.5	28.6	34.0	37.3
30	1.403	1.563	1.449	1.615	16.0	16.6	23.2	27.8	39.2	44.4
40	1.374	1.521	1.429	1.582	14.7	15.3	27.9	33.4	42.6	48.7
50	1.477	1.587	1.536	1.651	11.0	11.5	34.0	39.9	45.0	51.4
60	1.546	1.653	1.596	1.707	10.7	11.1	29.3	34.3	40.0	45.4
70	1.591	1.648	1.621	1.679	5.7	5.8	35.7	38.7	41.4	44.5
80	1.594	1.631	1.603	1.640	3.7	3.7	43.5	44.4	47.2	48.1
90	1.628	1.645	1.601	1.617	1.7	1.6	55.9	53.2	57.6	54.8
100	1.634	1.654	1.566	1.585	2.0	1.9	64.4	57.6	66.4	59.5

The change in moisture of the whole profile according to the original data is 88.6 mm, whereas it is after correction 90.8 mm. The error due to non-parallelism is 2.5%. The error in the separate layers varies from o to 4.5%. As the systematic error operates each measuring date in the same direction the effect of non-parallelism on the determination of changes in moisture content is not so very critical.

The errors in the absolute moisture contents are under the presented conditions of non-parallelism in the order of 10 to 20% of the original moisture content. Particularly when these moisture content data are used in a drying cycle to derive the suction profiles in the field, the correction of non-parallelism is very important. In this type of investigations the determination of non-parallelism is always necessary.

5. Summary

An equation has been derived to determine the errors due to non-parallelism of gamma access tubes with the aid of soil sampling data, taken from the direct environment of the measuring spot.

It is shown that the determination of the change in moisture content is not so very sensitive to errors due to non-parallelism. When the absolute values of moisture content are needed, it is always necessary to determine the errors due to non-parallelism of the access tubes.

6. References

RYHINER, A.H. and J. PANKOW. 1968. Soil moisture measurement by the gamma transmission method. I.C.W.-nota 475: 9 p.