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THE ELABORATION OF THE PF-CURVE BY GRAPHICAL METHOD

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THE DESORPTION CURVE

The soil moisture conditions in the unsaturated zone are described by two characteristics, the desorption curve, representing the relation between the soil moisture stress and the soil moisture content, and the conductivity curve, which gives the relation between the capillary conductivity and the soil moisture content.

The soil moisture stress is used for calculating evaporation, storage, capillary rise, capillary infiltration, soil aeration and for determining wilting point, field capacity and soil moisture profile connected with water balance terms in the soil. Because further the crop reaction depends on the air and water content of the soil, the equation for the desorption curve represents the link between the hydrology of the soil and the plant response with respect to this hydrological condition.

Therefore, the determination of the function for the desorption curve is very important for the study and application of the unsaturated soil water management research. Also, the equation for the desorption curve is evolved to make available a check on the accuracy of separate soil moisture stress - moisture content observation. So, in this note the main aim is to explain the elaboration of the desorption curve by a few graphical methods, which elaborations have the object of producing the best values for the soil moisture content - moisture stress relation at any value of these two variates.

The desorption curve is represented by the formula:

$$\left(\frac{G}{\psi}\right)^{\ell} = \frac{v^{p}}{\left\{P + \Delta P - v\right\}^{1-p}} \quad \text{or} \quad \frac{G}{\psi} = \frac{v^{\frac{p}{\ell}}}{\left\{P + \Delta P - v\right\}^{\frac{1-p}{\ell}}}$$

$$\text{or} \quad \frac{G}{\psi} = \frac{v^{m}}{(P + \Delta P - v)^{n}}$$

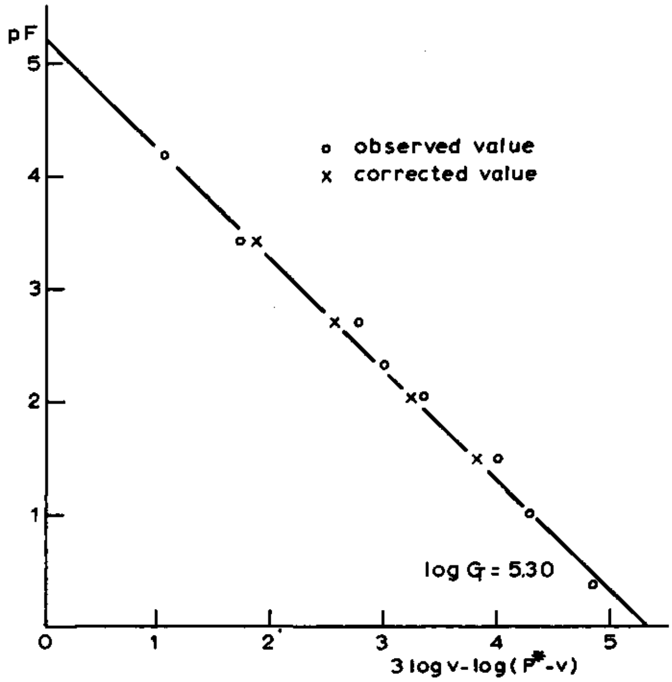


Fig. 1. Example of adjustment according to the method of 3 log v - log (P*-v). Data are shown in table 1 (a) and corrected values for pF 3.4, 2.7, 2.0 and 1.5 calculated by curve fitting procedures are shown in table 1 (b)

Table 1

(a) (data)

pF	0.4	1.0	1.5	2.0	2.3	2.7	3.4	4.2
v	48.5	45.2	42.7	34.3	28.5	25.2	13.2	8.0
log v	1.6857	1.6551	1.6304	1.5353	1.4548	1.4014	1.1206	0.9031
P^*-v	1.5	4.8	7.3	15.7	21.5	24.8	36.8	42.0
I log (P^*-v)	0.1761	0.6812	0.8633	1.1959	1.3324	1.3945	1.5658	1.6232
II 3 log v	5.0571	4.9653	4.8912	4.6059	4.3644	4.2042	3.3618	2.7093
II - I	4.8810	4.2841	4.0279	3.4100	3.0320	2.8097	1.7960	1.0861

(b) (corrected)

pF	1.5	1.5	1.5	2.7	2.7	2.7	2.0	2.0	3.4	3.4
v	42.7	40.7	39.7*	25.2	23.2	22.2*	34.3	32.3*	13.2	14.2*
P^*-v	7.3	9.3	10.3	24.8	26.8	27.8	15.7	17.7	36.8	35.8
log v	1.6304	1.6096	1.5988	1.4014	1.3655	1.3464	1.5353	1.5092	1.1206	1.1523
II 3 log v	4.8912	4.8288	4.7964	4.2042	4.0965	4.0392	4.6059	4.5276	3.3618	3.4569
I log(P^*-v)	0.8633	0.9685	1.0128	1.3945	1.4281	1.4440	1.1959	1.2480	1.5658	1.5539
II - I	4.0279	3.8603	3.7836	2.8097	2.6684	2.5952	3.4100	3.2796	1.7960	1.9030

G = soil constant

v = soil moisture content at moisture tension ψ

P = pore space

Δ P = reduction factor

$\left.\begin{matrix} P \\ \Delta P \end{matrix}\right\} P + \Delta P = P^*$

m, n = soil moisture parameters

The formula can also be written as:

$$\log G = \log \psi + m \log v - n \log (P^* - v)$$

$$\text{or} \quad \log G = pF + m \log v - n \log (P^* - v)$$

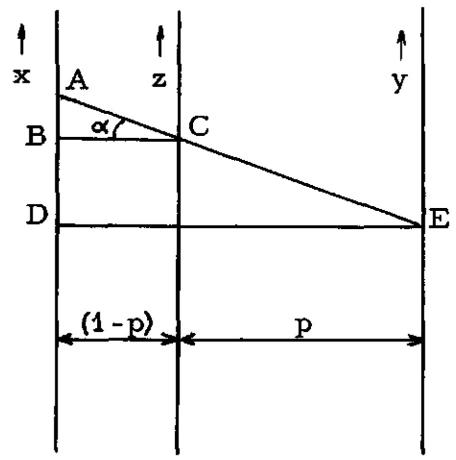
Given magnitudes are pF and v, unknown are log G, m, n and P^*. The formula allows curve fitting procedures rather easily because the unknowns log G, m, n are represented as linear functions. Only P^* is taken up in a non-linear way. The number of determinations of 8 to 9 is, however, small and the accuracy of the determination with such a restricted number of observations becomes rather dependent on somewhat large deviations in the v-values. The problem is to find the observation which deviates strongly. These observations are canceled, or if from a large number of samples a correction factor can be derived, then it is also allowed to apply such a correction. The numerical solution of the pF-value by a given v or of v by a given pF-value is accurate but laborious. In cases where the accuracy is not of prime importance, a graphical solution of the formula of the desorption curve is possible.

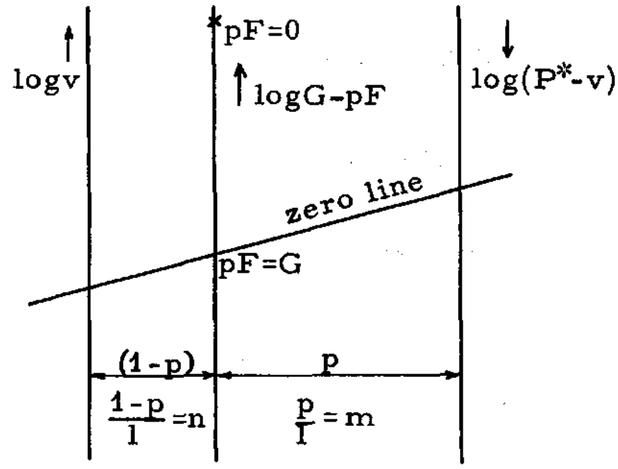
METHOD OF 3 log v - log (P^* - v)

The first graphical method uses the assumption that the ratio between m and n is constant and is of the order of three. The value of m/n is taken to be 3 and 3 log v - log(P^*-v) is calculated. In this case ΔP is considered to be zero. Now 3 log v - log(P^*-v) is plotted against the pF (fig. 1). It may be observed that 4 points deviate slightly from the line. These are points for pF 3,4; 2,7; 2,0 and 1,5. The corrected values for pF 3,4; 2,7; 2,0 and 1,5 are calculated by curve fitting procedures. These results are shown in the table 1 and crosses in fig. 1. Where 3 log v - log(P^*-v) = 0, there log G = pF.

The log G can therefore be read from fig. 1. The value of log G is 5.3. The slope of the straight 3 log v-line should be unity. In fig. 1 this is with good accuracy the case. Often, however, the slope deviates from the value 1,000. In that case the values 3 and 1 in the calculation of 3 log v - log(P^*-v) should have been 3 x α and α. These are the more accurate values of m and n. If α would have been 0.95, then instead of 3 log v - log (P^*-v) one should have calculated 0.95 x 3 x log v - 0.95 log(P^*-v). The ratio m/n may be incorrect, but the correct ratio is difficult to be observed in the figure. If for a certain geographical area the values of m and n are repeatedly determined, then an average value instead of 3 can be calculated and used for curves which are not yet adjustment.
If the ratio of 3 to 1 would not have been correct then the dots would have indicated a slightly curved line. This correction can only be found by trial and error with this technique and is far easier to be found with the 3 parallel axes nomogramme. It is, however, of importance to try this out in order to get some idea about the difference in sensitivity with which in any adjustment technique the different unknowns can be assessed. So is m/n to be determined quite accurately, but m and n separately are far more difficult to assess.

THE THREE PARALLEL AXES PF-NOMOGRAMME





x, y and z represent the parallel axes. The distance between x and y axes is taken to be 1, while the distance between y and z axes is p. Therefore the distance between x and z axes is (1-p). For the gradient of the line AE may be written:

$$\frac{x-z}{1-p} = \tan\alpha = \frac{x-y}{1}$$

$$x-z = (1-p)(x-y)$$

$$x-z = x - px - y(1-p)$$

$$z = px + (1-p)\,y$$

This compares with desorption curve by putting

$$x = \log v, \quad -y = \log(P^{*}-v), \quad z = l\,(\log G - pF)$$

$$l\,(\log G - pF) = p \log v - (1-p) \log(P^{*}-v)$$

$$\log G - pF = \frac{p}{l} \log v - \frac{1-p}{l} \log(P^{*}-v)$$

The nomogramme represents:

$$\log G - pF = m \log v - n \log(P^{*}-v)$$

CONSTRUCTION OF THE THREE PARALLEL AXES NOMOGRAMME

The construction of this nomogramme is carried out as follows, see fig. 2:

1. The AB and CD axes are drawn at 10 cm mutual distance.
2. Mark along the AB axis a log v scale, increasing in the right hand direction. The zero point (log v = 0) is in A.
   Mark along the CD axis a $\log(P^{*}-v)$ scale, increasing in the left hand direction. The zero point ($\log(P^{*}-v) = 0$) is in C.
   Both scales are taken identical.
3. Along the AB axis the values of log v are plotted and along the CD axis the values of $\log(P^{*}-v)$ are plotted. At each point the values of log v or $\log(P^{*}-v)$ are not indicated but the value of the pF.
4. The same pF-values on the two axes are linked with straight lines. These lines are called moisture content line (v-line).

Table 2 (a)

$l = \frac{0.8}{3.2} = 0.2500$

$p = 0.73$

$1-p = 0.27$

$m = \frac{0.73}{0.25} = 1.080$

$\frac{m}{n} = 2.7037$

$\log G = 5.16$

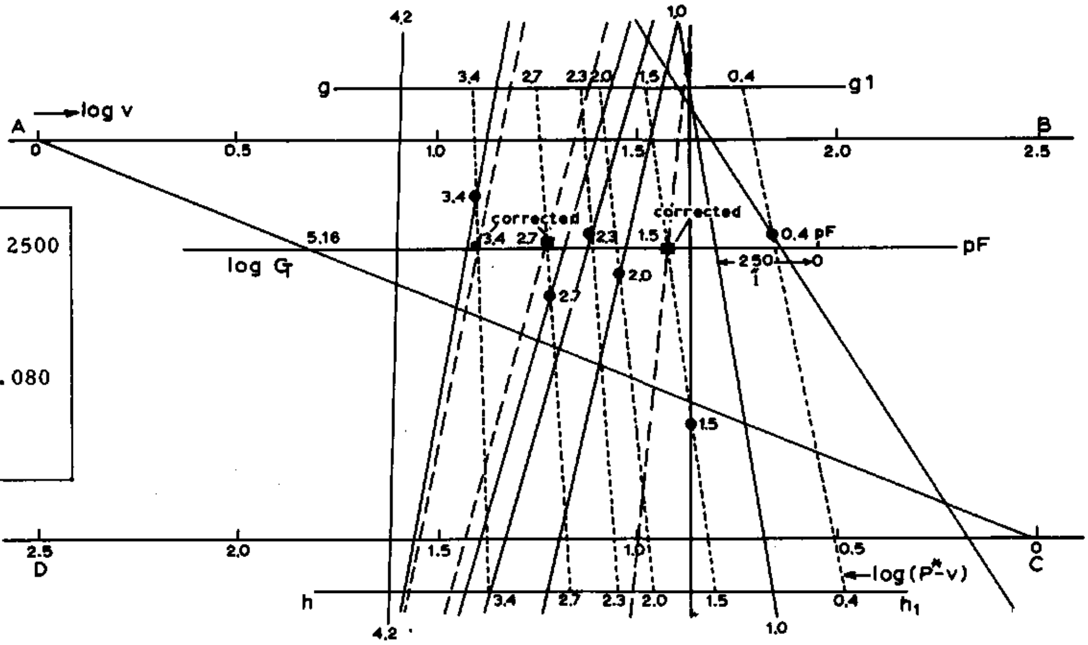


Table 2 (b)

$l = \frac{0.78}{3.2} = 0.2437$

$p = 0.67$

$1-p = 0.33$

$m = \frac{0.67}{0.2437} = 1.3541$

$\frac{m}{n} = 2.0302$

$\log G = 4.44$

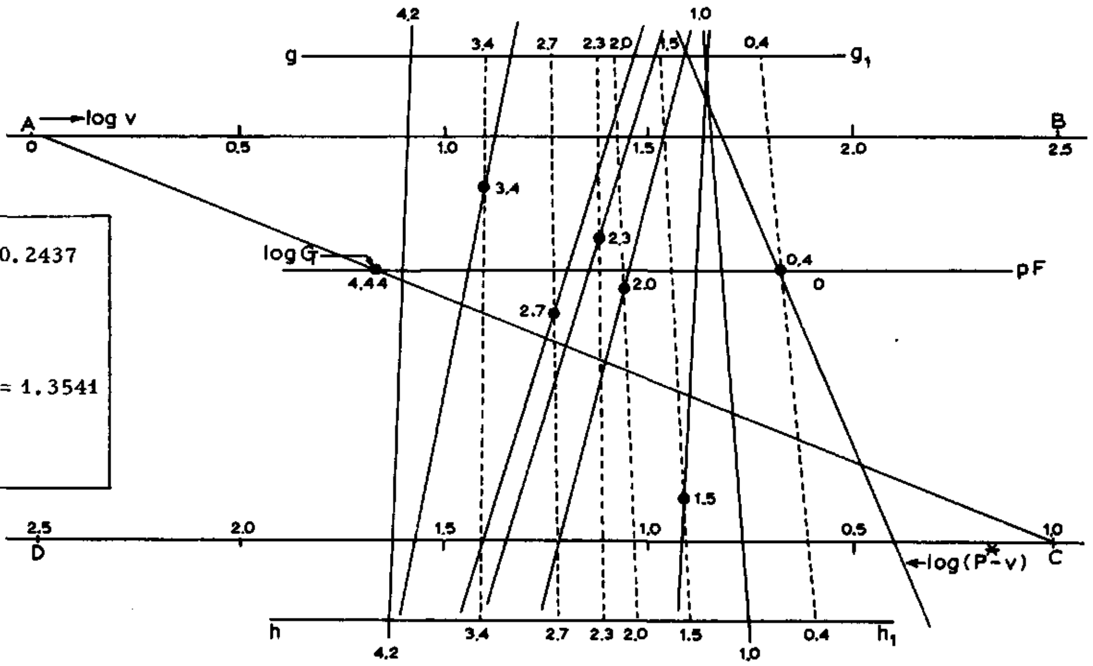


Fig. 2. Examples of construction of the three parallel axes pF-nomogramme. Data are the same as in fig. 1. The points with symbol ⊙ are obtained using values by the method of 3 log v - log (P^*-v). The values of graphical solution of m, n and log G are shown in table 2.

(a) On the assumption that P^* is equal to 50.0 ($\Delta P = 0$)

(b) On the assumption that P^* is equal to 51.0 ($\Delta P = 1.0$)

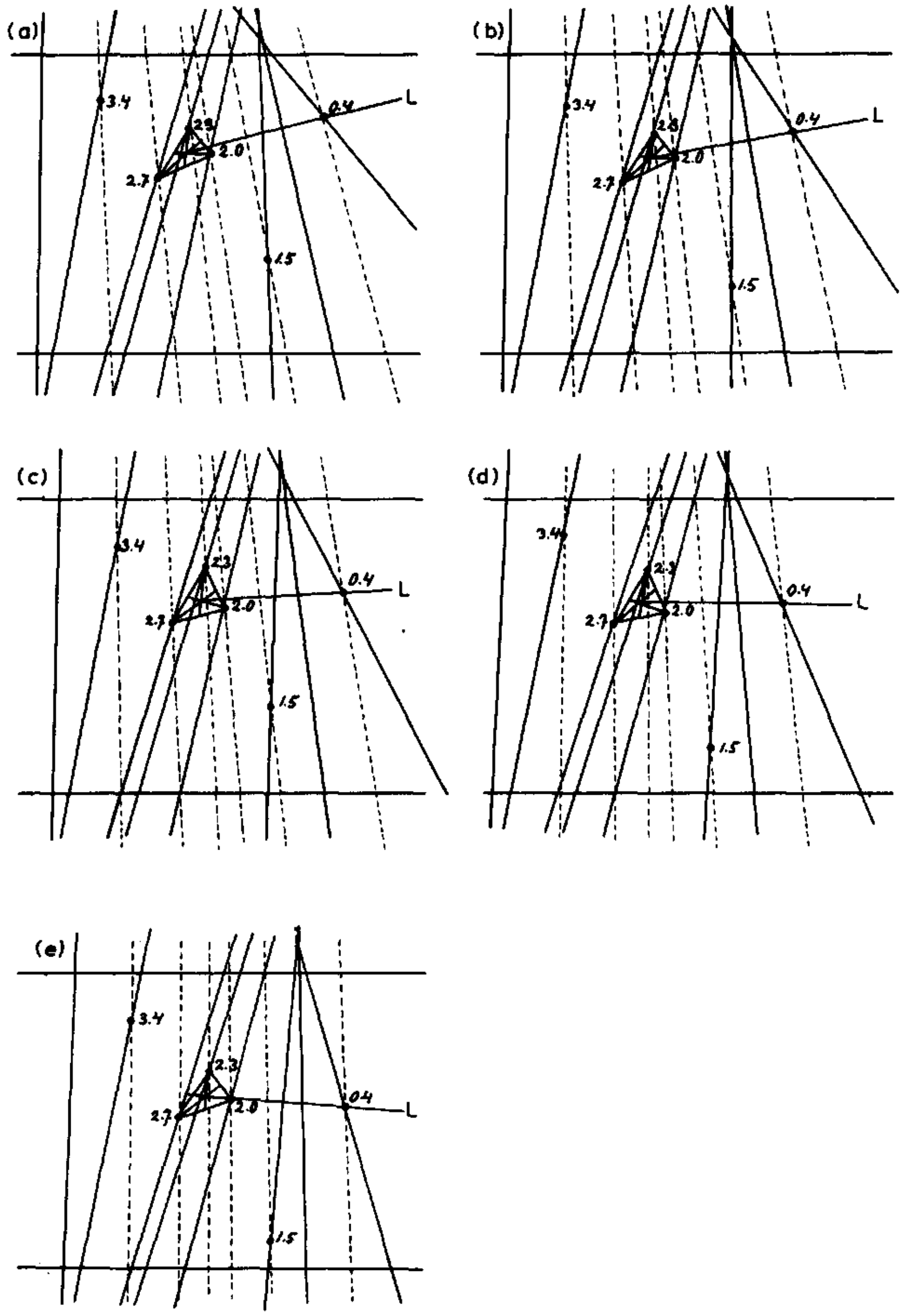


Fig. 3. Determination of P + ΔP according to the pF-nomogramme

(a) on the assumption that P + ΔP is equal to 49.5 (ΔP = 0.5)

(b) on the assumption that P + ΔP is equal to 50.0 (ΔP = 0)

(c) on the assumption that P + ΔP is equal to 50.0 (ΔP = 0.5)

(d) on the assumption that P + ΔP is equal to 51.0 (ΔP = 1.0)

(e) on the assumption that P + ΔP is equal to 52.0 (ΔP = 2.0)

The L-line is the nearest to horizontal for

(d). P + ΔP = 51.0. So this is the best approximation for the value of P + ΔP

5. Two lines $g-g_1$ and $h-h_1$ parallel to the log v axis are drawn and the distance between the two points of intersection with v-line for pF 4.2 and 1.0 is measured and divided proportionally in distance related to the pF differences (0.4, 1.5, 2.0, 2.3, 2.7, 3.4). These fixed points - here pF 4.2 and 1.0 - should have only small deviations. This should be checked first by the method of $3 \log v - \log (P^{*}-v)$.
6. The points of the same pF-value on two axes ($g-g_1$, $h-h_1$) are linked with straight lines. These lines are called the pF-lines.
7. The points of intersection of the v-line with the pF-line for the same pF-value are marked. A straight line is drawn through these points parallel to the AB and CD axes. This line is easy to draw if the observations are correct. This line is the pF-axis.
8. The distance between the AB- and the pF-axis is equal to 10(1-p) cm and the distance between the pF-axis and CD is equal to 10p cm.
9. The value of l is given the length of one unit of the pF-scale on the pF-axis.
10. The point of intersection of the zero line with the pF-axis represents the value of log G.
11. The values of m and n are calculated as follows:

$$m = \frac{p}{l} \quad , \quad n = \frac{1-p}{l}$$

An example of construction of the nomogramme is shown in fig. 2. The data are the same as in fig. 1. The results of the graphical solutions of m, n and log G are given in table 2.

DETERMINATION OF P + Δ P

In the elaboration according the three parallel axes previously described, m, n and log G were determined but the value of ΔP was considered to be zero. Now this is impossible. The value of ΔP must differ from zero, because it means that a $\psi = 0$ will occur in the sample and $\psi = 0$ represents a pore radius $r = \frac{0.3}{\psi}$ of infinite size, therefore much larger than the sample ring. As Δ P is taken up in the formula in a non-linear way, the adjusted value has to be calculated by an iterative method. This calculation is carried out by constructing the nomographic adjustment for a number of P + Δ P values.

For example, the values used are:

$P + \Delta P$	49.5	50.0	50.5	51.0	52.0
ΔP	-0.5	0	+0.5	+1.0	+2.0

The nomogrammes are reproduced in fig. 3. It is obvious that the point of intersection for pF 0.4 will change most. The influence of ΔP on the points of intersection for pF 2.0, 2.3 and 2.7 is not large. For the three points for pF 2.0, 2.3 and 2.7 the mean value is calculated (shown on the graph) and it is assumed that this mean point is a point on the pF-axis. From this point to the ΔP-sensitive point for pF 0.4, a line is drawn. This line is nearest to horizontal for $P + \Delta P = 51.0$. So this is the best approximation for the value of $P + \Delta P$.

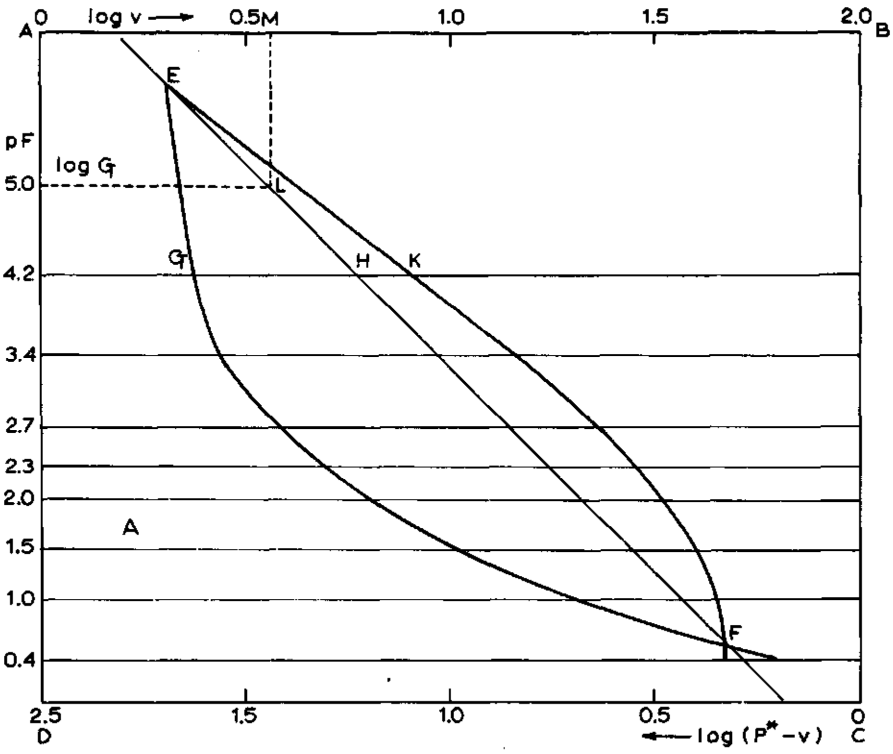
THE THIRD GRAPHICAL METHOD

The construction of the third graph is carried out as follows (fig. 4).

1. Mark the log v-data along the AB axis, increasing in magnitude in the right hand direction. Mark along the CD axis the values for $\log (P^* - v)$, increasing in the left hand direction.
2. Mark along the vertical axis a metric scale for the pF with 2.5 cm unit.
3. Plot the points for pF and log v and draw a freehand curve through the dots. Do the same for the pF against $\log (P^* - v)$ curve.
4. Determine the points of intersection E and F of the two freehand curves and connect these points by the line EF. The tangent of EF opposite the axis AB renders the constant l.
5. Measure at a number of arbitrary pF-levels the distance GH, KH and GK. It will be proved that:

$$p = \frac{GH}{GK}, \quad 1 - p = \frac{KH}{GK}$$

Often with normal slightly inaccurate data the freehand curves will not be entirely accurate. Therefore for different pF-levels the value of p - for which for every pF the same value should be found - may vary to a certain extent.



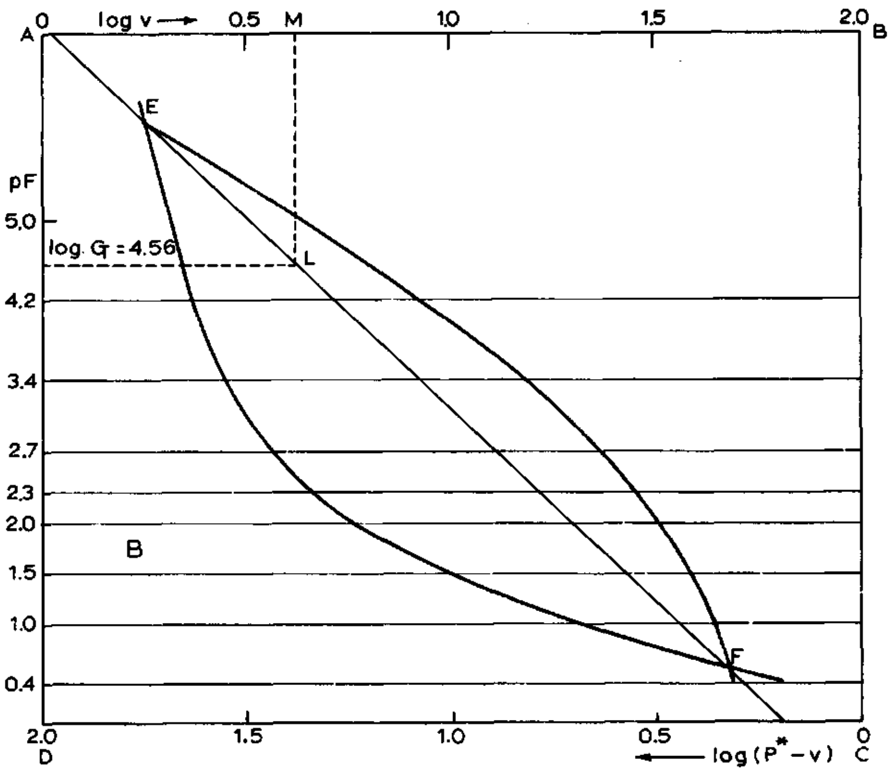


Fig. 4. Examples of construction of the third graphical method. Data are the same as in fig. 1. The values of the graphical solution of m, n and log G are shown in above column.

(a) In the case using the original data

(b) In the case using the corrected values calculated by the method of 3 log v - log (P^*-v)

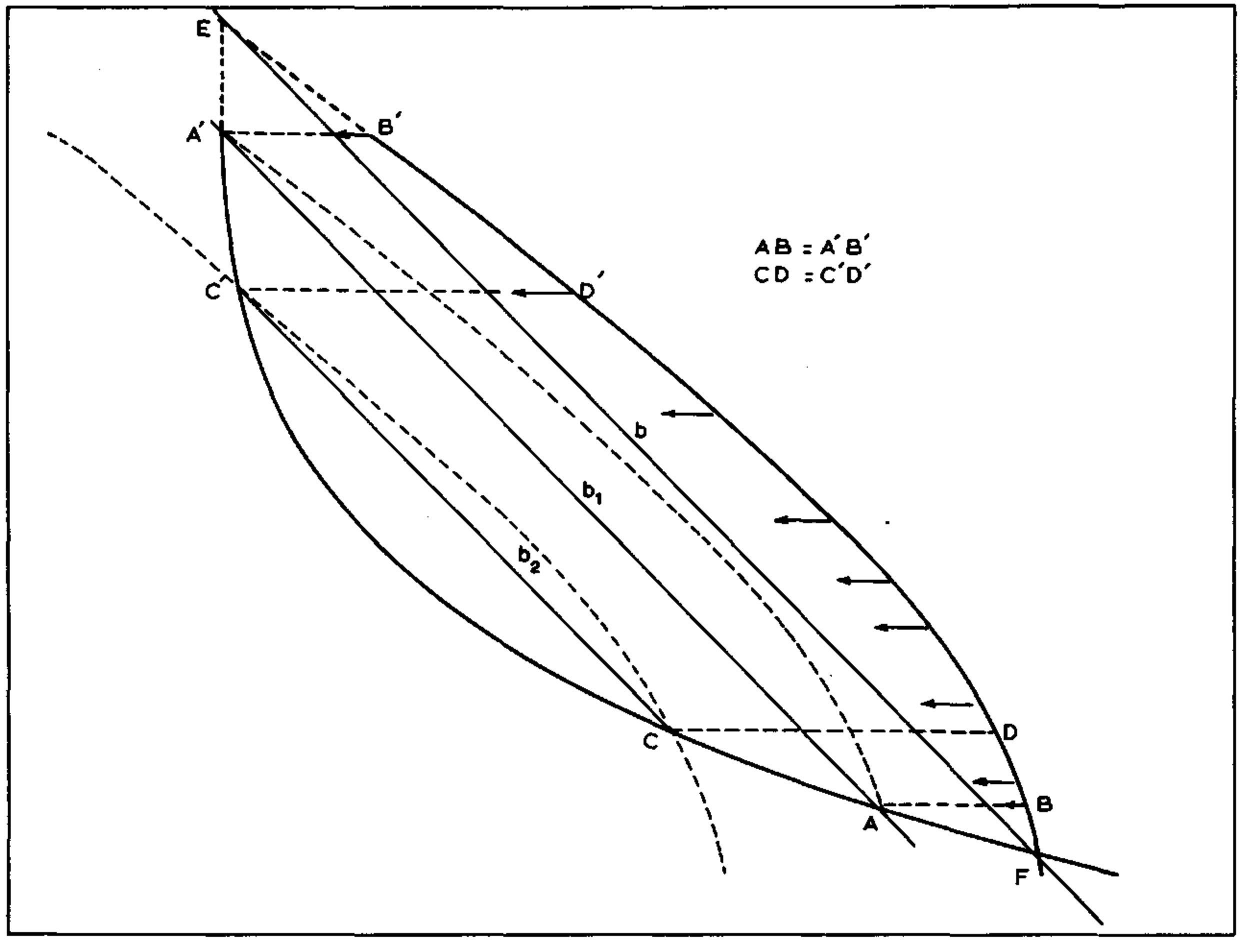
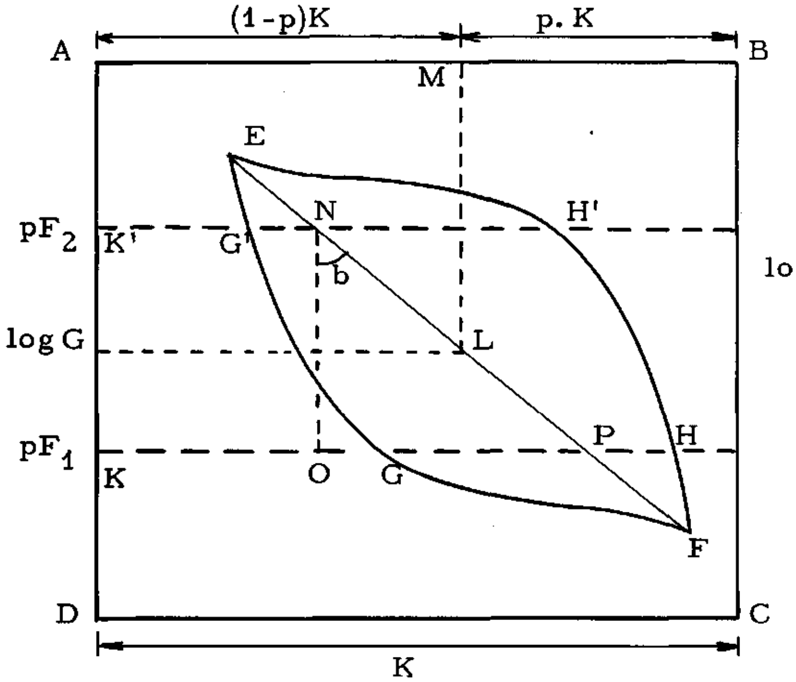


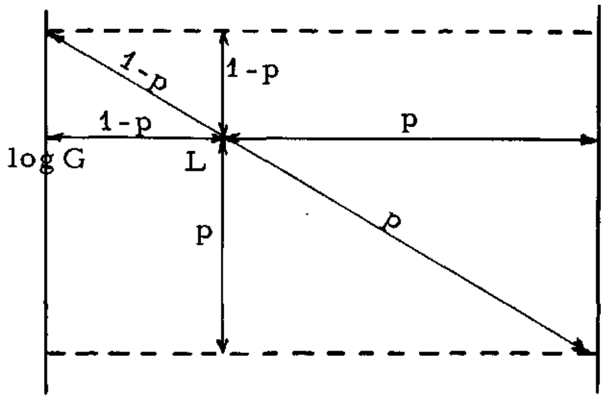
Fig. 5. Example of finding the b-line (EF-line) by shifting of curves when the points of intersection (E and F) of two curves are not found

A number of ratios at different pF-levels should be established and averaged.

6. Divide the distance AB in a part p x AB = BM and a part (1-p) x AB = AM. A is the zero point for log v. B the zero point for log (P^{*}-v). Draw a vertical line downward from M and determine the point of intersection (L) with the line EF. Read the pF-value for the point L. This value is the value for log G to be inserted in the formula.

   The reason that the tangent b is equal to 1 and the pF-value for the intersection point (L) is equal to log G, is proved as follows:





In the formula:

$$1(\log G - pF) = p \log v - (1-p)\log(P^{*}-v)$$

$v_1 \rightarrow pF_1 \qquad 1(\log G - pF_1) = p\log v_1 - (1-p)\log(P^{*}-v_1)$

$v_2 \rightarrow pF_2 \quad - \quad 1(\log G - pF_2) = p\log v_2 - (1-p)\log(P^{*}-v_2)$

$$1(pF_2 - pF_1) = p(\log v_1 - \log v_2) + (1-p) \times \left\{\log(P^{*}-v_2) - \log(P^{*}-v_1)\right\}$$

$$\therefore \quad 1 = \frac{p(\log v_1 - \log v_2) + (1-p)\left\{\log(P^{*}-v_2) - \log(P^{*}-v_1)\right\}}{pF_2 - pF_1}$$

In the graph $\tan b = \frac{OP}{ON}$

$$NO = pF_2 - pF_1 , \quad OP = PK - AK'$$

$$PK = KG + GP , \quad GP = pGH = p(KH - KG)$$

$$\therefore \quad PK = KG + p(KH - KG) = KG(1-p) + p.KH$$

$$NK' = K'G' + G'N \quad G'N = pG'H' = p(K'H' - K'G')$$

$$NK' = K'G' + p(K'H' - K'G') = K'G'(1-p) + pK'H'$$

$$KH = \log v_1 , \quad K'H' = \log v_2$$

$$KG = K - \log(\overset{*}{P} - v_1), \quad K'G' = K - \log(\overset{*}{P} - v_2)$$

$$\therefore \quad PK = p \log v_1 + (1-p)\left\{K - \log(\overset{*}{P} - v_1)\right\}$$

$$-) \quad NK' = p \log v_2 + (1-p)\left\{K - \log(\overset{*}{P} - v_2)\right\}$$

$$OP = p(\log v_1 - \log v_2) + (1-p)\left\{\log(\overset{*}{P} - v_2) - \log(\overset{*}{P} - v_1)\right\}$$

$$\therefore \quad \tan b = 1$$

In the formula, where $p \log v - (1-p)\log(\overset{*}{P} - v) = 0$, there $pF = \log G$

$$\therefore \quad p \log v = (1-p)\log(\overset{*}{P} - v)$$

at the point (L) $\quad \log v = (1-p)K , \quad \log(\overset{*}{P} - v) = pK$

$$\therefore \quad \frac{\log v}{\log(P^* - v)} = \frac{1-p}{p}$$

$$\therefore \quad pF = \log G$$

If one cannot find the point of intersection of the two curves in fig. 4 , then one will find the b-line (EF-line) by reasoning as follows:

One of the curves is shifted parallel with the direction of the vertical axis till this curve intersects with the other curve. In this case, the line between the points of intersection has the angle b (b-line). On an arbitrary place, one always finds a line with a same angle (b_1, b_2-lines). And one can control the value of 1 by this shifting of the curve (fig. 5).

When the $\log(\overset{*}{P} - v)$ curve is shifted to the left vertical axis ($\log v = 0$), then it will mean that when v is small, the value of $\log(\overset{*}{P} - v)$ is almost equal to 2.

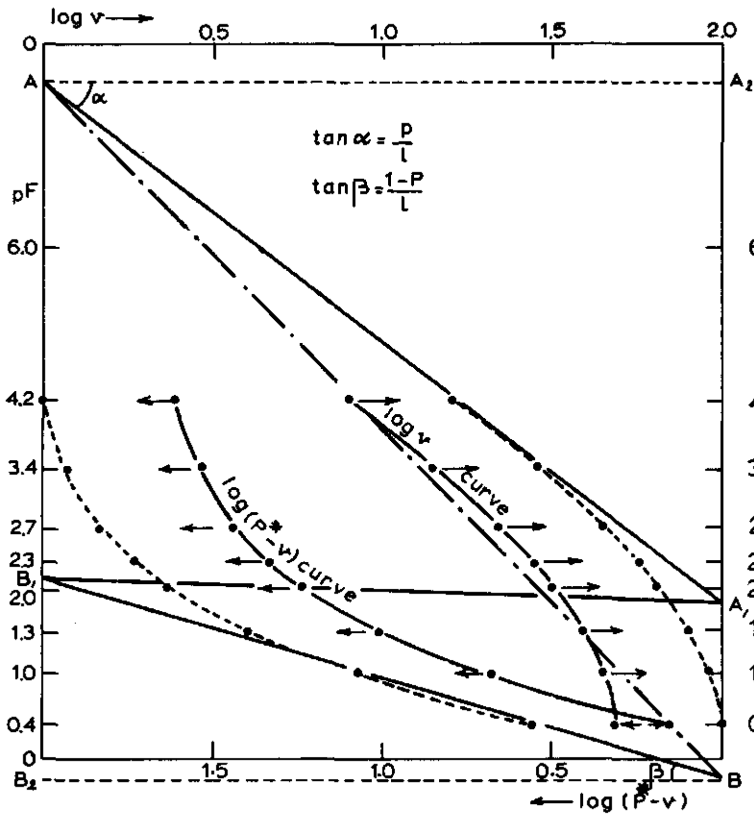


Fig. 6. Method of checking of the observed values using A_1B_1 line obtained by shifting of the curves to the vertical axis

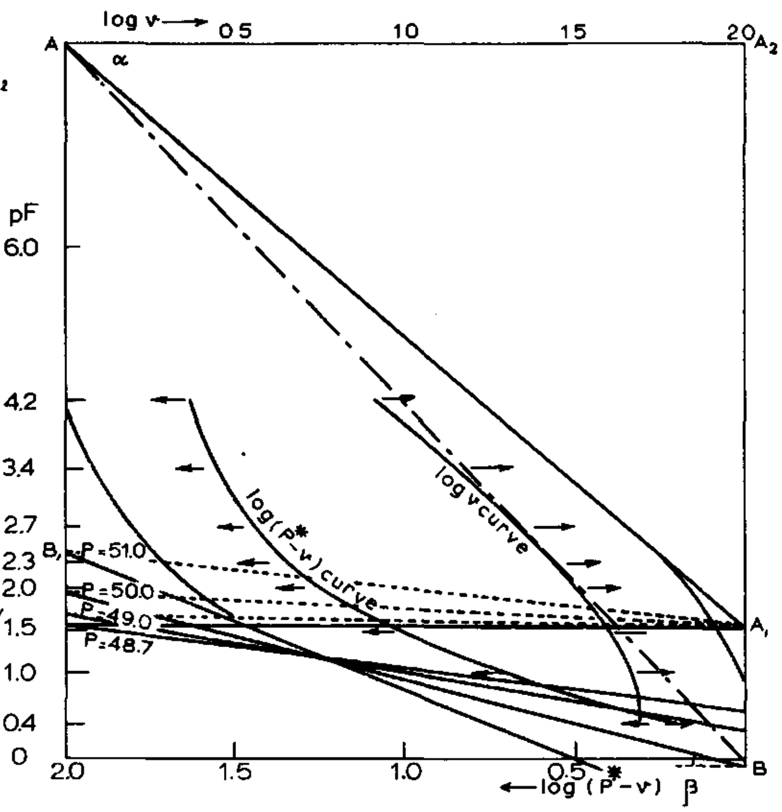


Fig. 7. Example of checking of the observed values using A_1B_1 line obtained by the various assumptions for the P^*-value. The A_1B_1 line is the nearest to horizontal for $P^* = 48.7$

$$\therefore \quad pF = \log G - \frac{p}{l} \log v + \frac{1-p}{l} \log (P^* - v)$$

$$pF = \left[\log G + \frac{2(1-p)}{l} \right] - \frac{p}{l} \log v \qquad (1)$$

And when the log v curve is shifted to the right vertical axis ($\log(P^*-v) = 0$), then it will mean that when v is large, the value of log v is almost equal to 2.

$$\therefore \quad pF = \log G - \frac{p}{l} \log v + \frac{1-p}{l} \log (P^* - v)$$

$$pF = (\log G - \frac{2p}{l} \log v) + \frac{1-p}{l} \log (P^* - v) \qquad (2)$$

From the formulae (1) and (2) it follows that the tangent of the angle, which the lower part of $\log(P^*-v)$ curve has with respect to the horizontal axis, approaches the value $\frac{1-p}{l}$. In the same way, the tangent of the angle, which the upper part of the log v curve has with respect to the horizontal axis, approaches the value $\frac{p}{l}$.

$$\therefore \quad \tan \alpha = \frac{p}{l} \quad , \quad \tan \beta = \frac{1-p}{l} \qquad \text{(fig. 6)}$$

In fig. 6, the line A_1B_1 should be parallel with the horizontal axis, if the observation value is correct. It is proved as follows:

$$AA_2 = BB_2 = N$$

$$B_1B_2 = N \tan \beta = N \frac{1-p}{l}$$

$$A_1B = A_2B - A_1A_2 = N \tan b - N \tan \beta$$

$$= N.1 - N.\frac{p}{l} = N \frac{1-p}{l}$$

$$\therefore \quad B_1B_2 = A_1B$$

$$\therefore \quad A_1B_1 \,//\, AA_2 \,//\, BB_2$$

Therefore one will be able to check the observation value by this line (A_1B_1), as is shown in fig. 7.

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