augustus 1984

nstituut voor Cultuurtechniek en Waterhuishouding
Wageningen

PROGRAM FEMSAT

Part 1 – Calculation Method for Steady and Unsteady Groundwater Flow

E.P. Querner

Nota's (working papers) of the Institute are a means of internal communication and not a publication. As such their contents vary strongly, from a simple presentation of data to a discussion of preliminary research results with tentative conclusions. Some working papers are confidential and when so indicated are not available to third parties.

Table of Contents

1. INTRODUCTION

The computerprogram FEMSAT (van BAKEL, 1978) has been extended, modified, and extensively tested in the last year.
The extensions and modifications as given in this report are the change from an explicit to an implicit calculation scheme, options to include different type of interactions with the surface water system, capillary rise and net rainfall. For the output of the results one can choose between results per node or per sub-region and the posibility to limit the output in various ways.

For steady groundwater flow the program FEMSATS is available and for unsteady flow calculations the program FEMSAT. Both programmes require virtually the same input data, so one can easily change from steady to unsteady flow calculations.

This report will describe the adopted calculation method in Chapter 2, the derivation of the general equation of motion for groundwater flow in Chapter 3, and the boundary conditions such as surface water systems, and how they are treated, in Chapter 4.

Part 2 of this report will contain the user's manual (QUERNER, 1984). It deals with the required input data for the programmes, a worked out example, plotting of results, and the estimation of computertime (VAX-11/750). With a separate contouring program, one can plot the calculated hydraulic heads for any time or the change in waterlevel from the beginning.

2. ADOPTED CALCULATION SCHEME

The program was originally based on an explicit calculation scheme (van BAKEL, 1978) which means that all external flows imposed on a layer used for the calculations at a certain timestep were taken from the previous timestep (see figure 1). The external flows such as a discharge to the surface water system or capillary rise are actualy dependeble of the unknown hydraulic heads at the present timestep.

Explicit methods turn out to be stable if the speed with which information from some point spreads in the computational scheme is smaller than the physical speed of propagation of a disturbance. This often means a severe restriction on the timestep to be used.

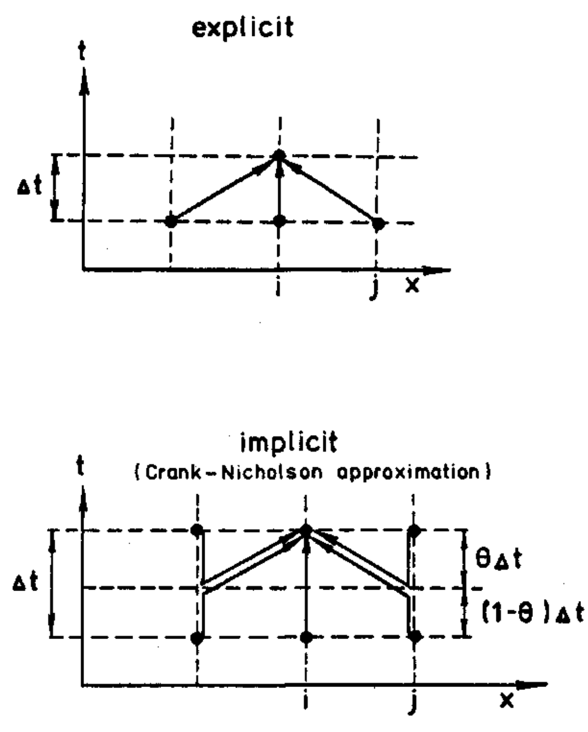

Figure 1 - Explicit and implicit calculation scheme

By an implicit method all factors affecting the flow at a certain timelevel are used to calculate the flow at that particular timelevel. With this method the timestep can be chosen independently of the node spacing. The Crank-Nicolson approximation introduces a central time difference or weighting parameter. It uses in addition to the flows from the present timelevel, the flows from the previous timelevel (see figure 1). This method is in general unconditional stable and will not impose restrictions on the timestep to be used. If the weighting parameter θ is 0.5, it means a straightforward average between the two time levels.

The Crank-Nicolson approximation is preferred over the other methods, because it has the smallest truncation error (error that derives from the truncation of the Taylor series). The reader is referred to REMSON et al. (1971) for a detailed description on the various calculation schemes and their advantages.

To include the unkown boundary flows, it will be required to include their contributions in the equation of motion. Therefore the equation for a boundary condition must be written as a function of the unknown hydraulic head (see Chapter 4).

3. EQUATION OF MOTION

3.1. Unsteady flow

For the derivation of the equation of motion the solution domain is divided into a finite number of elements. Each element can have a triangular or quadrilateral shape. The nodes are representative for a certain area surrounded by it.

The model assumes that groundwater is flowing horizontal in waterbearing layers and vertical in less-permeable layers. The finite element network is taken the same for each layer. The nodes are assumed to lay in the middle of each layer.

The equation of motion can be obtained by considering an aquifer layer with a node i and applying the principle of conservation of mass and momentum.

Unsteady flow conditions will indicate that during a time interval from t to t + Δt a quantity of water will flow to or from node i. The amount of water involved will result in a rise or fall of the hydraulic head. Therefore one can write the continuity equation as :

$$B_i \frac{\Delta h_i}{\Delta t} = \theta \left| \sum_j Q_{ij} + Q_e \right|^{t+\Delta t} + (1-\theta) \left| \sum_j Q_{ij} + Q_e \right|^{t} \qquad (1)$$

where θ is a weighting parameter between the timelevels t and t + Δt and B_i is the storage factor for node i. The storage factor has been defined elsewhere (van BAKEL, 1978 and NEUMAN et al., 1974). The term Q_{ij} is the flow from node j to node i and Q_e is the external flow.

Equation (1) can be written as :

$$B_i \frac{\Delta h_i}{\Delta t} = \sum_j Q_{ij}^{t} + Q_e^{t} + \theta \left| \sum_j \Delta Q_{ij} + \frac{dQ_e}{dh_i} \Delta h_i \right| \qquad (2)$$

assuming that :

$$Q^{t+\Delta t} = Q^{t} + \Delta Q \qquad (3)$$

and

$$h^{t+\Delta t} = h^{t} + \Delta h \qquad (4)$$

The first two terms on the right hand side of equation (2) represents the flows to or from node i at time t and the third and fourth term are the actual change in flow over the considered timestep. The linearization of the equations have been done to avoid re-evaluating certain level dependeble parameters after each iteration.
For the external flow Q_e imposed on a layer it has been assumed that it depends on the hydraulic head h_i . If it is just a function of time (discharge or recharge) then Q_e should be specified at time $t + \theta \Delta t$ and $dQ_e / dh_i = 0$.

The equation of motion for the flow between node i and adjacent nodes j in the same layer, can be written as :

$$\sum_j Q_{ij} = \sum_j A_{ij} (h_j - h_i) \qquad (5)$$

where the matrix A_{ij} contains the conductivity parameters for horizontal flow in a waterbearing layer. For the definition of the conductivity matrix see NEUMAN et al. (1974). Between two nodes the flow is linear related to the difference in hydraulic head. Equation (5) can therefore also be used to define a change in flow given the changes in hydraulic head between two adjacent nodes. The flow towards a node is assumed as positive and from a node as negative.

The term Q_e is the summation of all boundary flows of the node under consideration. This can be a discharge/recharge, leakage, discharge to/from the surface water system or a flux to/from the unsaturated zone.
For equation (2) one can write the boundary flows as a function of the phreatic waterlevel h_i, except for the recharge,discharge or nett rainfall. The level dependeble relations (boundary conditions) are often non-linear, but linear relations are required for the general equation of motion, otherwise the adopted solution procedure is not valid. A linearization technique has been applied, where a non-linear relation is replaced by a series of linear relations. In figure 4 an example is given of such a linear relation. More details on the particular equations for all the boundary flows is given in Chapter 4.

The external flow is composed of the following flow terms :

$$Q_e = A_r (q_t + q_s + q_l + q_c) + Q_{in} \tag{6}$$

In which A_r is the area allocated to node i. The linear relation for the tertiary surface flux can be written as :

$$q_t = \beta_t (h_t - h_i) \tag{7a}$$

or

$$q_t = \alpha_t + \beta_t (h_g - h_i) \tag{7b}$$

For the secondary surface flux as :

$$q_s = \beta_s (h_s - h_i) \tag{8}$$

For the leakage as :

$$q_l = \frac{h_{i,l+1} - h_i}{0.5\ c_{i,\ l+1}} + \frac{h_{i,l-1} - h_i}{0.5\ c_{i,\ l-1}} \tag{9}$$

For the capillary rise as :

$$q_c = \alpha_c + \beta_c (h_g - h_i) \tag{10}$$

where the layer under consideration is given the subscript l. The leakage term is written for a layer enclosed by two aquitards. All the flows are given per unit area of the node considered.

Equations (7) to (10) are discussed in more detail in Chapter 4.

For the derivation of the general equation of motion it has been assumed that there is present a waterbearing layer with a tertiary surface water system and a semi-impervious layer below it (see figure 2). All other flow terms not included in the general equation of motion can be treated in a similar way, but are not included here for simplicity.

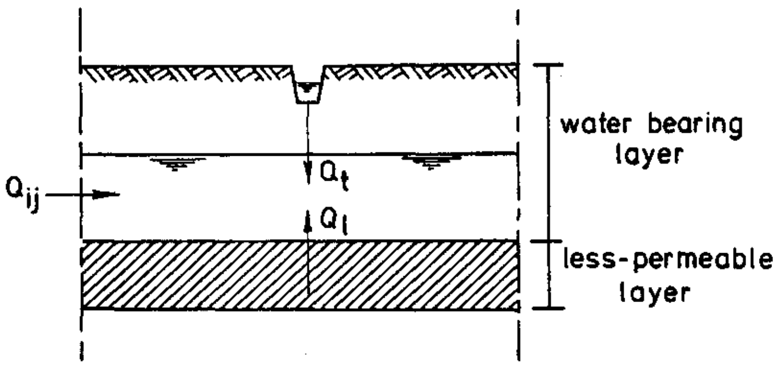

Figure 2 - Schematization of terms for waterbalance

Substituting equation (5), (7a) and (9) into equation (2) it becomes :

$$B_i \frac{\Delta h_i}{\Delta t} = \sum_j Q^t_{ij} + Q^t_e + \theta \left| \sum_j A_{ij} (\Delta h_j - \Delta h_i) \right|$$

$$+ \theta \left| - \beta_t \Delta h + \frac{\Delta h_{i,l+l}}{.5\, c_{l+l}} - \frac{\Delta h_i}{.5\, c_{l+l}} \right| \qquad (11)$$

The actual change of discharge from the tertiary surface water system is calculated as the derivative of the tertiary discharge for the considered groundwater depth (see equation 21).

If the actual change in hydraulic head is not too great or the change in surface discharge for changing depth is also small, it is possible to keep the derivative constant during the iteration process. A marginal error will occur in the calculation of the contribution, of the surface discharge, on the change in hydraulic head for a node i. In figure 3 this effect is shown. With the derivative of the discharge kept constant, the contribution of it on the hydraulic head (Δh^*_l) is for instance q_l, but it should have been q_2 (see figure 3). If the change in hydraulic head over the timestep is Δh^*_2, which is in fact smaller then Δh^*_1, but the relation is in that particular zone strongly non-linear, then the result would be an substantial error in the discharge (q_3 against q_4 in figure 3). If this occurs, the timestep is taken as half the current timestep and the calculations are repeated for two timesteps, after which the old timestep is used again for further calculations.

The test performed to check the relations on its non-linear behaviour is done by calculation of the discharge by $h_i + \Delta h$ and $h_i + 0.5\Delta h$. If the condition :

$$\frac{\left| q^{t+\Delta t} - q^{t+.5\Delta t} \right|}{q^{t+\Delta t}} > 0.20$$

is true, then the timestep used is to big, and requires to be adjusted. This test and possible adjusment of timestep is done by the relations for the discharge to the surface water system and for the capillary rise. If the drainage resistance is calculated as a relation of the groundwater depth, such as equation (20), then the drainage resistance is calculated at time t + Δt and t + .5 Δ t. If this change is also more then 20 percent, then the timestep will be decreased. The factor 0.20 will be standard in the programme, but can be changed by inserting it in the input data.

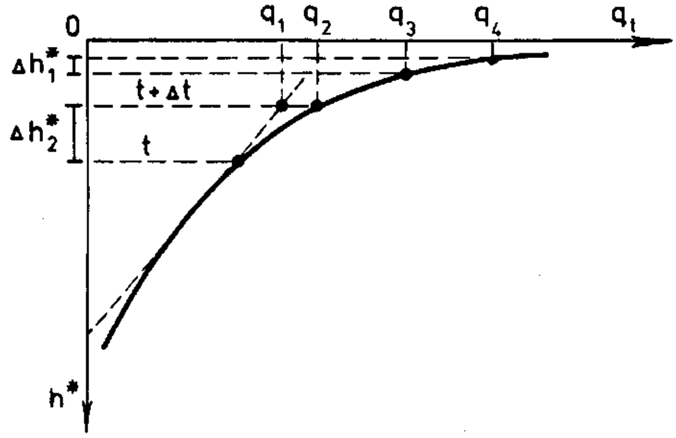

Figure 3 - Error in contribution of surface discharge on the hydraulic head

From equation (11) the change in hydraulic head between time t and t + Δt is then :

$$\Delta h_i = \frac{\sum_j Q_{ij}^{t} + Q_e^{t} + \theta \left(\sum_j A_{ij} \, \Delta h_j + \Delta h_{i,l+1} / .5 \, c_{l+1} \right)}{B_i / \Delta t + \theta \left(\sum_j A_{ij} + \beta_t + 1/.5 \, c_{l+1} \right)} \qquad (12)$$

All the coefficients (not containing Δ) can be specified for time t. With equation (12) the change in hydraulic head over the considered timestep can be calculated, and then with equation (4) the total head.

3.2. Steady flow

For the calculation of steady flow the storage factor is not included. The hydraulic head for node i can also be written in a similar manner as for the unsteady flow situation, except that now the final hydraulic head as such is calculated and not a certain change over a timestep.

The continuity equation becomes now :

$$\sum_j Q_{ij} + Q_e = 0 \tag{13}$$

All flows to or from node i from the surrounding nodes j in the same layer must be equal to the external flow. Equation (5) can be substituted into the above equation, which results in :

$$\sum_j A_{ij} (h_j - h_i) + Q_e = 0 \tag{14}$$

For the boundary flows exerted on a node one can use the relations such as given in equation (6) to (10). The same situation as for the unsteady flow is taken for the derivation of the equation of motion (figure 2).

The hydraulic head for node i can now be written as :

$$h_i = \frac{\sum_j A_{ij} h_j + \alpha_t + \beta_t h_g + h_{i,l+1} / .5\ C}{\sum_j A_{ij} + \beta_t + 1 / .5\ C_{l+l}} \tag{15}$$

Equation (15) can be solved with the Gauss-Seidel iterative method in the same manner as will be outlined for the unsteady flow situation in Chapter 5.

4. SPECIAL BOUNDARY CONDITIONS

4.1. Drainage or sub-irrigation

The interaction between the surface water system and the groundwater system is commonly modelled by means of socalled tertiary and secondary systems.

The tertiary system consists of shallow ditches, sometimes filled with water and only present in the toplayer of the groundwatermodel. The secondary system consists of channels, commonly filled with water and they can protude into deeper layers. The relations which govern the flows for these two types are discussed below.

4.2. Tertiary surface water system

For the tertiary surface water flux one can use two type of relations :

1 - discharge dependeble on groundwaterdepth only

2 - discharge dependeble on difference between hydraulic head and the open-water level in the ditch

As stated in Chapter 3 the general equation of motion requires linear relations for the boundary flows. A non-linear relation such as shown in figure 4 must be linearised. Between two successive depth points a linear relation is defined as :

$$q_t = \alpha_t + \beta_t \, h^* \qquad (16)$$

in which h* is the groundwaterdepth.

The two conditions for this linear relation are :

$$h_1 < h^* <= h_2$$

$$q_1 < q_t <= q_2$$

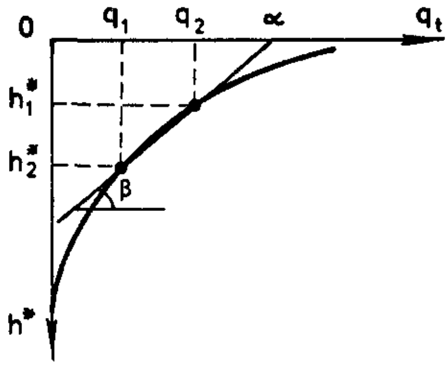

Figure 4 - Typical relation between groundwater discharge and groundwater depth

Equation (16) can be written in a generalized form as the relation for the discharge dependent on the hydraulic head as :

$$q_t = \alpha_t + \beta_t (h_g - h_i) \tag{17}$$

where h_g is the groundlevel for node i. The relation which is determined for the particular area to be analysed, can also include the discharge from the secondary surface water system.

The relation as proposed by ERNST (1978) and given in figure 5 can be selected as a relation in the computerprogram by using the appropriate option. Other relations can be defined by given N-times a q_t and h* value, resulting in (N-1) linear relations.

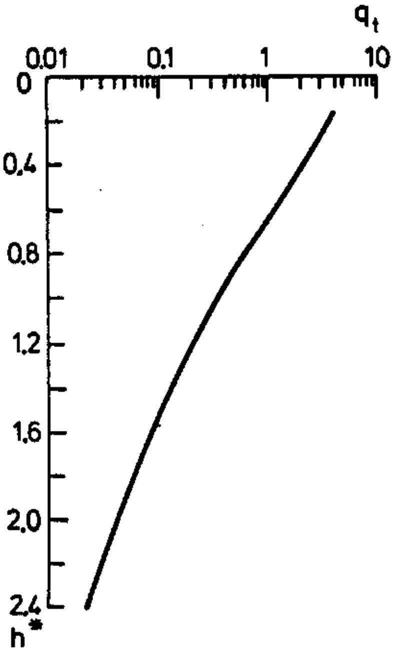

Figure 5 - Relation between groundwater discharge and groundwater depth (ERNST, 1978)

For the second type of relation the equations become :

$$q_t = \beta_t (h_t - h_i) \quad (18)$$

where

$$\beta_t = - 1/\alpha \, . \, T \quad (19)$$

in which h_t is the water- or invertlevel of the ditch in the tertiary system, α is a geometry factor to convert the hydraulic head midway between two ditches to the average hydraulic head, and T is the drainage resistance.

The geometry factor α can vary between 0.65 and 1.0 . The lower limit is for a pure parabolic change in phreatic waterlevel near the ditch. A value of 0.8 is used in the program.

ERNST gave a general relation for the drainage resistance dependent on the groundwater depth as :

$$T = 60 + 100 (h_g - h_i) + 350 (h_g - h_b)^{4.75} \qquad (20)$$

For the calculation of the drainage resistance the hydraulic head from the previous timestep is used. During the iterations for one timestep the drainage resistance is therefore kept constant. If the drainage resistance will change more then 20 percent over half the timestep, then the estimation of the discharge to the surface water system would be over- or under estimated, due to the non-linear effect and the assumption of linear relations by the program. The calculations are then repeated with half the current timestep in the procedure as outlined in paragraph 3.1.

Wether there is water in the ditch one can have two conditions for the factor β_t of equation (19) :

- free draining ditch : $h_t = h_b$

$h_i > h_b \qquad \beta_t = - 1 / \alpha \, . \, T$

$h_i <= h_b \qquad \beta_t = 0.$

- open-water level in ditch : $h_t = h_{wl}$

$h_{wl} > h_b \qquad \beta_t = - 1 / \alpha \, . \, T$

The change of discharge per timestep for the tertiary surface water system can now be written from equation (17) or (18) identical as :

$$\frac{dq_e}{dh_i} \Delta h_i = \Delta q_t = - \beta_t \Delta h_i \tag{21}$$

Equation (21) can be substituted into equation (2), as given in equation (11).

4.3. Secondary surface water system

For the secondary surface water system the equation per unit length of ditch is :

$$q_s = \beta_s (h_s - h_i) \tag{22}$$

where :

$$\beta_s = - \frac{1}{R_r + R_e} \tag{23}$$

The radial resistance R_r can be written as :

$$R_r = \frac{1}{\pi k} \ln \frac{\alpha_s d}{\pi P} \tag{24}$$

in which h_s is the waterlevel in the channel, k is the hydraulic conductivity, d is the thickness of the saturated layer close to the drainage layer, P is the wetted perimeter of the channel and α_s is a coefficient depending on size of the channel. If the channel is small (depth less then 2.50 m) then α_s is 4.0 - 5.0, and for larger channels α_s is 6.0 - 7.0. The upper limit can be selected for empty stage and the lower limit for bankfull channels.

The wetted perimeter P can be calculated from the cross-setions of the ditches, or with the emperical relationship :

$$P = B_b + 2 y \sqrt{.09 D^2 + 1} \tag{25}$$

where B_b is the bottomwidth, y is the waterdepth in the channel and D is the depth of the channel.

The entrance resistance R_e is strongly dependent on local conditions and cannot be calculated explicitly, but must be measured on site and required as input for the computerprogram.

The change of discharge per timestep for the secondary surface water system can be written from equation (22) as :

$$\Delta q_s = - \beta_s \Delta h_i \tag{26}$$

4.4. Leakage

If we consider an water bearing layer enclosed between two less-permeable layers, then the change in vertical flux for layer L can be written as :

$$q_l = \frac{h_{i,l+1} - h_i}{.5\ c_{l+1}} + \frac{h_{i,l-1} - h_i}{.5\ c_{l-1}} \tag{27}$$

where c is the hydraulic (vertical) resistance of the less-permeable layer and defined as :

$$c = \frac{d}{k} \tag{28}$$

where d is the vertical thickness of the layer and k the hydraulic conductivity.

If between two aquifers the aquitard is missing, then equation (27) cannot be used to calculate the flux between the two aquifers. These two layers at node i will then be considered as one layer with adjacent nodes in the two separate layers, such as shown in figure 6.

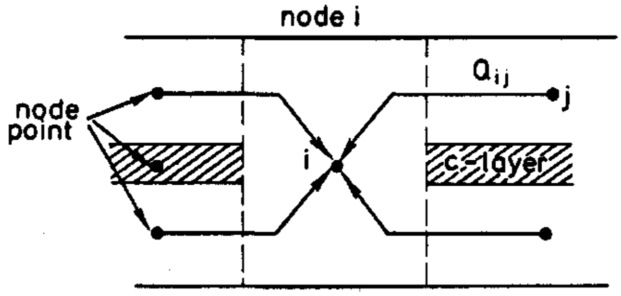

Figure 6 - Solution procedure for nodes where c-layer is not present

The program assumes that the C-layer is not present when the hydraulic conductivity is less then 50.

4.5. Capillary rise

The flux for a specified soil profile depends on the groundwaterdepth and the pressure head in the root zone of the unsaturated zone (de LAAT, 1980). Therefore the relation for the capillary rise is non-linear. Again a linearization technique has been applied and the capillary flux for each depth zone can be written as :

$$q_c = \alpha_c + \beta_c (h_g - h_i) \qquad (29)$$

Figure 7 gives a typical relationship between capillary flux and the groundwater depth.

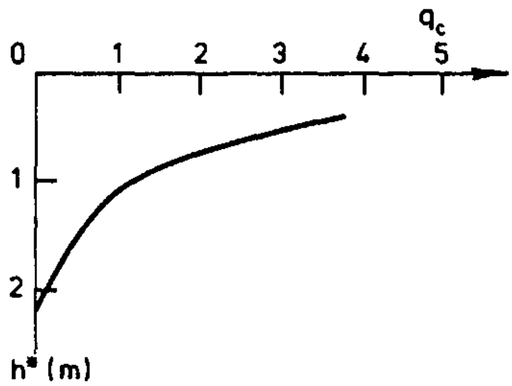

Figure 7 - Typical relationship between capillary flux and groundwaterdepth

4.6. Model boundary

For the boundary of the domain one can impose a constant hydraulic head or a constant flow. In case of a constant head for a nodal point, equation (12) is not performed for those nodes. This results in a zero change in head per timestep and the overall hydraulic head will not change in time.

5. SOLUTION PROCEDURE AND FLOW DIAGRAM

In equation (12) all the flow contributions are written in terms of the unknown hydraulic head. It means that the iteration process is unconditonal stable. Equation (12) can now be solved to obtain the hydraulic heads for each node. The Gauss-Seidel iterative method was used with successive over-relaxation. The Gauss-Seidel method is a point iterative solution method which uses updated information whenever it is possible. This means that the latest calculated value of Δh_i is used for the solution of levels in all other nodes affected by node i in the same iteration.

An over-relaxation factor is used to accelerate the iteration process. The calculated values obtained at the n-th iteration, can be improved by considering a weighted mean of h_i^n and h_i^{n-1} . In this way the hydraulic heads converge in less iterations to their final value. The computerprogram facilitates two methods of over-relaxation. They are the constant multiplication factor per timestep, and the factor dependent on the ratio old-new hydraulic head. The two methods of over-relaxation are dicussed in Chapter 6.

Now the hydraulic heads for time t + Δt can be obtained by using equation (4). With these new levels the actual flows in all nodes can be calculated using equation (7) to (10).

The computerprogram FEMSAT can be devided into a number of modules. Figure 8 gives a flow chart with the main activities. The program is build-up in such a way, that extensions can be included very easily.

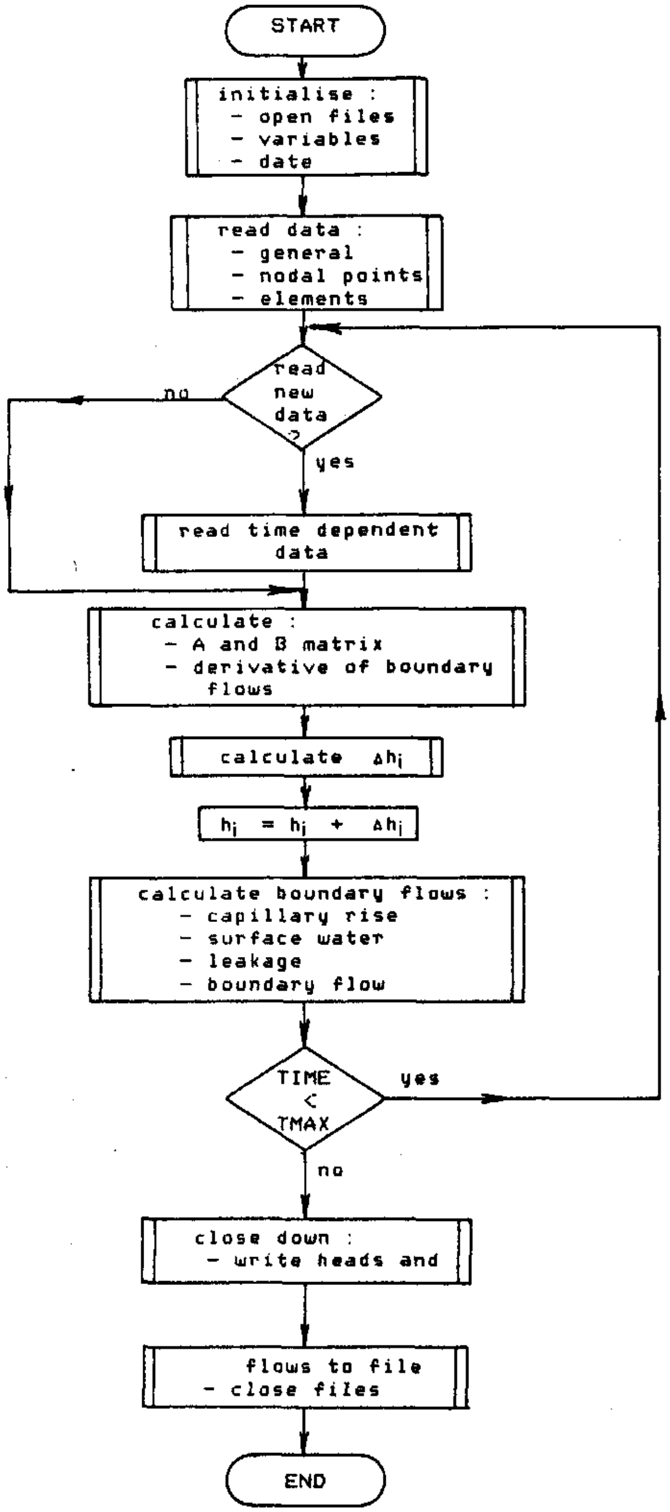

Figure 8 – Flow chart of program FEMSAT

6. OVER-RELAXATION

During the iteration process one can decrease the number of iterations by using an over-relaxation technique. The calculated change in hydraulic head for the considered timestep at iteration number n is modified by considering the change from the previous iteration.

Two common methods of over-relaxation are a constant multiplication factor per timestep or the factor is dependeble on the relation old-new head. The equations can be written as :

$$\Delta h_i^{n} = \Delta h_i^{n-1} + W\,(\Delta h_i^{n} - \Delta h_i^{n-1}) \qquad (30)$$

and :

$$\Delta h_i^{n1} = \Delta h_i^{n} \left| \frac{\Delta h_i^{n}}{\Delta h_i^{n-1}} \right| \qquad (31)$$

Equation (31) is combined with :

$$\Delta h_i^{n} = \theta_h' \,\Delta h_i^{n1} + (1 - \theta_h)\,\Delta h_i^{n-1} \qquad (32)$$

in which n is the iteration at present considered and W is the relaxation factor or acceleration parameter. The value of W depends on the size of the solution domain and the actual maximum changes in the hydraulic heads per iteration and can vary between 0 and 2.

A practical and simple method for approximating the value of W is given by Carre (REMSON, 1971). For the first iteration use W = 1.0 and for the second iteration W = 1.375. For successive iterations W can be calculated with the equations :

$$W_{opt} = 2 \left| 1 + \left| 1 - \frac{(\lambda_m + W - 1)^2}{W} \right|^{.5} \right|^{-1} \qquad (33)$$

In which λ_m is the ratio of maximum change in hydraulic head between the present iteration n and the previous iteration and given as :

$$\lambda_m = \frac{\max |\Delta h_i^n - \Delta h_i^{n-1}|}{\max |\Delta h_i^{n-1} - \Delta h_i^{n-2}|} \qquad (34)$$

Equation (34) gives the optimum (also maximum) value for the over-relaxation that is posible. Carre suggests then to reduce the maximum value as given in equation (34). The new relaxation factor to be used in the next iteration becomes :

$$W = W_{opt} - \left(\frac{2 - W_{opt}}{4} \right) \qquad (35)$$

Equation (32) has been included after the over-relaxation has been done to smooth the changes, otherwise with great changes in the hydraulic head between the iterations the over-relaxation becomes rather big. The weigthing parameter θ has been set to 0.7 in the programmes FEMSAT and FEMSATS.

The number of iterations performed by the program have been compared. For all the test runs it has been found that the ratio factor uses 10 - 15 % less iterations to converge to the set criterium, than the constant over-relaxation factor. For other cases this may be different.

Also the ratio factor is unconditional stable, which is not allways true when using the constant factor. Here the maximum change in hydraulic head between two successive iterations becomes sometimes a little bit greater instead of smaller. This means a divergence from the exact solution. For those cases the last iteration is repeated again with a relaxation factor which is 0.8 times the factor used last.

The method of over-relaxation to be used in the programmes can be selected by the user.

7. REFERENCES

BAKEL, P.J.T. van, 1978. A numerical model for non-stationary saturated groundwater flow in a multi-layered system. ICW-nota 1077, pp. 39

ERNST, L.F., 1978. Drainage of undulating sandy soils with high groundwater tables. ICW Technical Bulletin No. 106, pp. 50

KOVAR, K., 1981. Model for saturated groundwater flow. Model Study Over-Betuwe, Vol. 2, National Institute for Water Supply, Arnhem, pp. 53

LAAT, P.J.M. de, 1982. Model for unsaturated flow above a shallow water-table, applied to a regional sub-surface flow problem. Agricultural Research Report No. 895, Wageningen, pp. 126

QUERNER, E.P., 1984 Program FEMSAT, Part 2 - User manual. ICW-nota 1558

NEUMAN, S.P., R.A. FEDDES and E.BRESLER, 1974. Finite element simulation of flow in saturated-unsaturated soils considering water uptake by plants. Hydraulic Engineering Laboratory Technion, Israel, pp. 104

PINDER, G.F. and W.G. GRAY, 1977. Finite element simulation in surface and subsurface hydrology. Academic Press, New York, pp. 295

REMSON, I., G.E. HORNBERGER and F.J. MOLZ, 1971. Numerical methods in subsurface hydrology. John Wiley and Sons, New York, pp. 389

VREUGDENHIL, C.B., 1973. Computational methods for open-channel flow, in : Hydraulic Research for Water Management, TNO Proceedings and Informations, No. 18, pp. 38-79

8. LIST OF SYMBOLS

A_{ij}	–	conductivity matrix	m^2/d
A_r	–	area representative for node	m^2
B	–	storage factor	–
B_b	–	bottomwidth of channel	m
c	–	hydraulic (vertical) resistance	d
D	–	depth of channel	m
d	–	thickness of layer	m
h_b	–	invertlevel of ditch	m
h_i	–	hydraulic head for node i	m
h_g	–	groundlevel at node i	m
h_s	–	waterlevel in secondary system	m
h_{wl}	–	waterlevel in ditch	m
h^*	–	groundwater depth	m
Δh	–	change in hydraulic head per timestep	m
J	–	number of nodes connected to node i	–
k	–	hydraulic conductivity	m/d
L	–	layer index	–
N	–	number of points	–
n	–	iteration index	–
P	–	wetted perimeter	m
Q_{ij}	–	flow between node i and j	m^3/d
Q_e	–	external flow	m^3/d
q	–	flow per unit area	m^3/d
R_r	–	radial resistance	d
R_e	–	entrance resistance	d
t	–	time	d
Δt	–	timestep	d
W	–	relaxation factor	–
x	–	length	m
y	–	waterdepth in channel	m
θ	–	weighting parameter	–
α	–	constant or geometry factor	–
β	–	slope of linear relation	–
T	–	drainage resistance	d