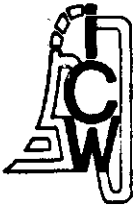


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SIMULATION OF FLOW IN SURFACE WATER SYSTEMS

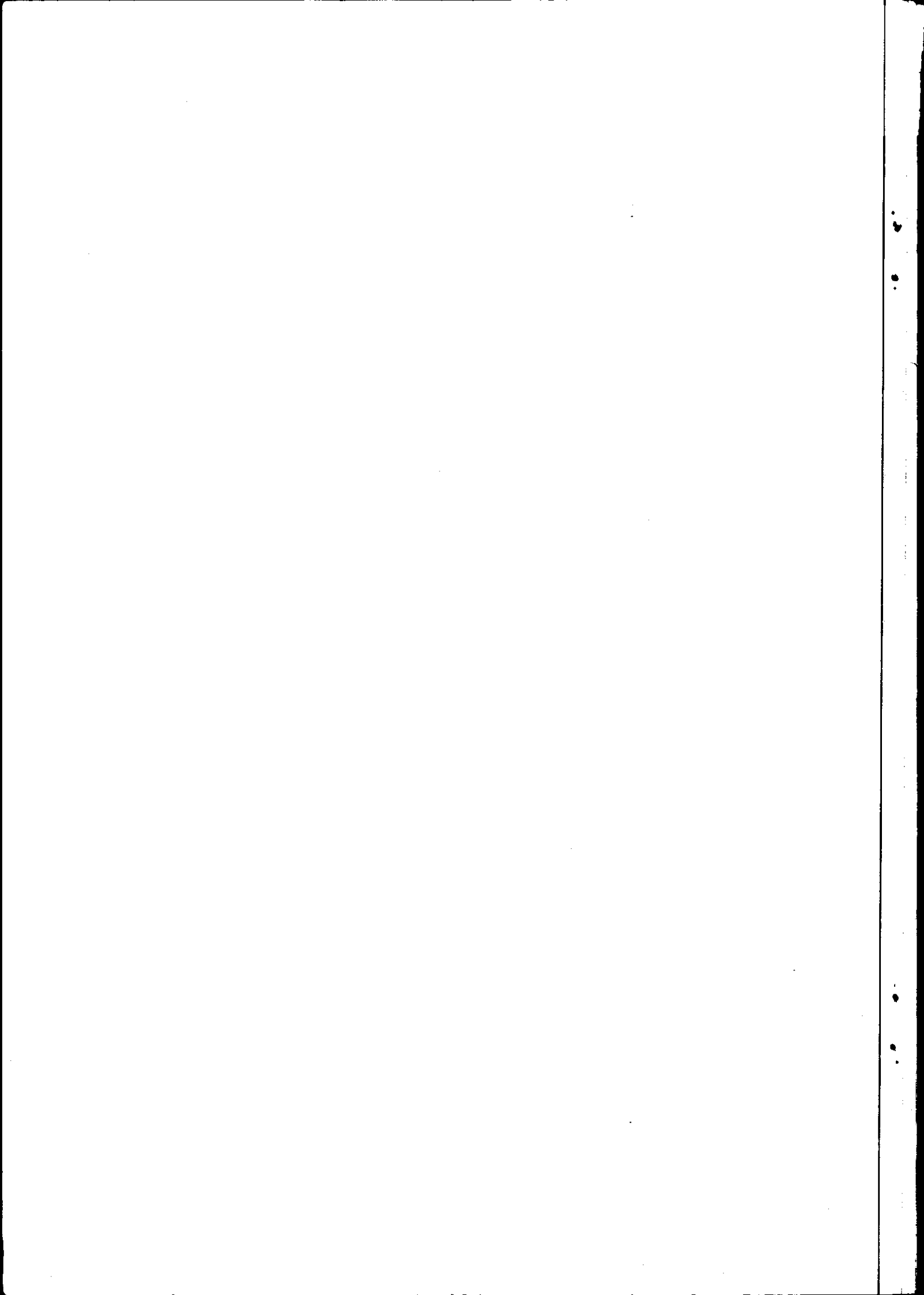
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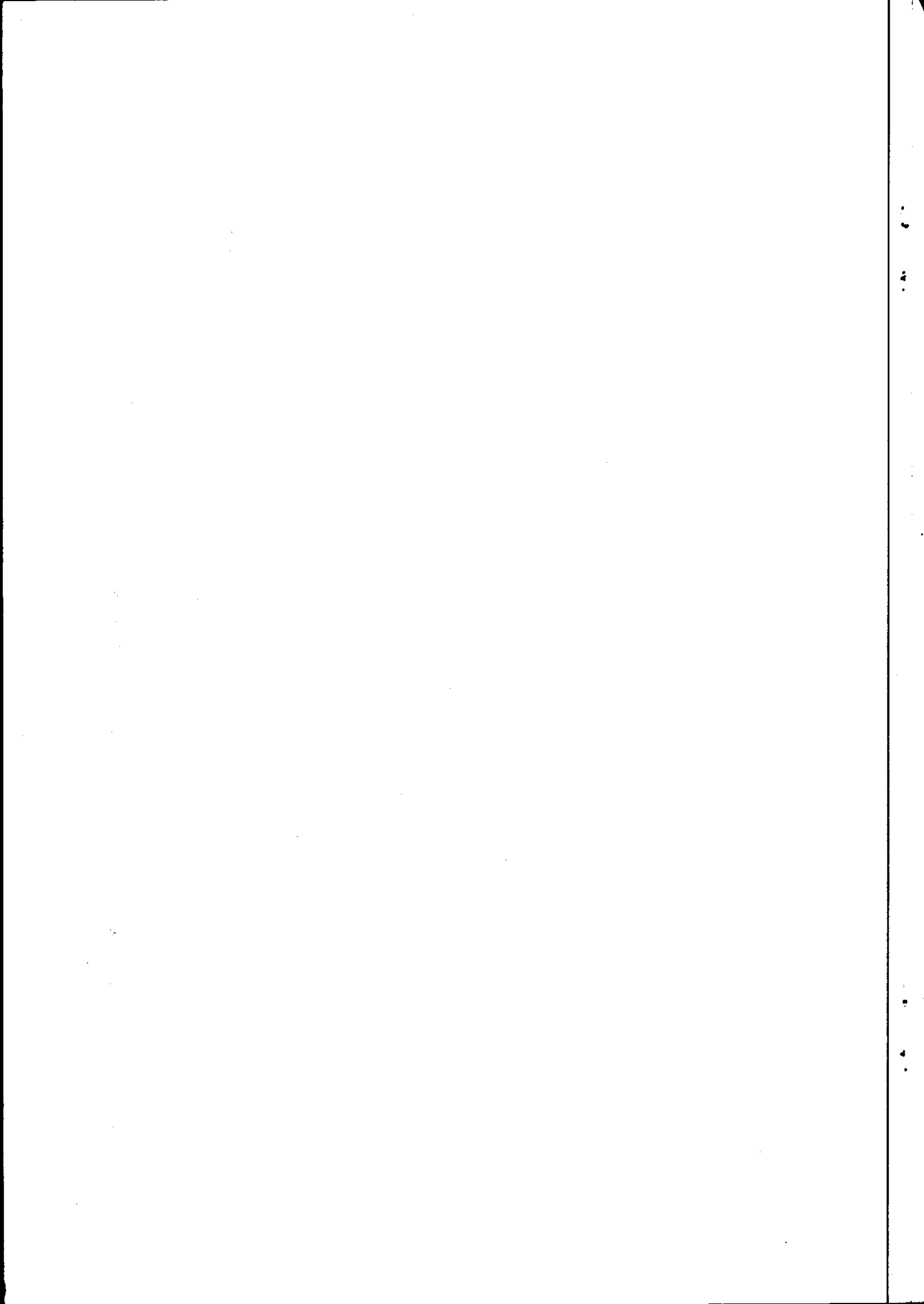


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1. INTRODUCTION

For the physical modelling of groundwater, one of the boundary conditions that must be given are the waterlevels in the surface water system. These levels are in fact dependent on the boundary flow. Because with an increase in drainage water the waterlevel in the channel will rise, depending on the size and roughness of the channel. To include the surface water system in a hydrological model it can then describe the interaction between groundwater and surface water more accurately. Therefore a model was developed to describe the water-movement in a channel network. The model was set up in such a way that it can be integrated in a hydrological (groundwater) model. Another important criteria for the model was a fairly simple calculation scheme, so that the overall modelling system is not complicated, easy to use and that it requires not too much input data. Using a computational method for unsteady flow one can for instance model the water movements in an area for controlling the watertable and water levels in the channel system.

The modelling of water movements can be for situations such as short and high discharge rates (e.g. rain storm) and for long term slowly changing discharge rates (drainage). It is evident that a computational method should be capable to simulate these extreme situations. But for a specific case the solution procedure is adapted to allow a justified calculation scheme (e.g. acceptable timestep). Neglecting certain terms of the equation of motion for certain situations still gives accurate results, but it saves computational time. This can be explained as follows. For the simulation of a short and high discharge rate in a system a small timestep is required to calculate the quickly changing flow process in time. Inertia effects are very important in these type of calculation and a timestep of 10 seconds up to

some minutes are the reality. In the case of a slowly changing process, such as the simulation of drainage and/or subsurface-irrigation during the year, a relative large timestep would be preferable. Inertia effects are not significant and can be neglected. A timestep of one to a couple of hours is now possible. The model SIMWAT (simulation of flow in surface water systems) has been designed to cope with these slow and fast changing flow processes.

In the following two chapters the governing equations are discussed for ordinary open channel flow and special structures such as weirs, culverts and pumps. The user's manual for the programme SIMPRO, which includes the groundwater and surface water module, is described by QUERNER (1986b).

2. DERIVATION OF EQUATION FOR UNSTEADY FLOW

2.1. Equation of motion

For a prismatic channel one can write the continuity equation as :

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \quad (2.1)$$

with Q as the discharge rate and A as the cross-sectional area of a channel section.

The equation of motion, often referred to as the Saint-Venant equation can be written as :

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left(\alpha \frac{Q^2}{A} \right) + g A \frac{\partial h}{\partial x} - g A I + g \frac{Q |Q|}{C^2 R A} = 0 \quad (2.2)$$

where α is a coefficient to express the non-uniform velocity distribution in the section, g is the acceleration due to gravity, I is the bottom slope, C is Chézy coefficient for boundary roughness and R is the hydraulic radius. The first term of equation (2.2) is the effect of inertia, the second term is the effect of non-uniform flow, the third term is the effect of hydrostatic pressure, the fourth term is the gravity component and the fifth term is the bottom friction term. The friction term is here written in a form as derived from the Chézy equation. The Manning formula could have been taken as well. The above equation is valid for turbulent flow where the vertical velocity component can be neglected.

With equation (2.1) and (2.2) one can model the water movements at any location and time within a network of channels with given boundary conditions. They cannot be solved analytical and therefore an approximate solution is necessary in the form of a solution at a number of specified locations and at certain time intervals. From the differential equations (2.1) and (2.2) one can obtain the difference equations

by considering a section of the channel and applying the principles of conservation of mass and momentum.

An amount of water stored by a change in water level from h^t to $h^{t+\Delta t}$ over a storage area S can be written as :

$$S \frac{h^t - h^{t+\Delta t}}{\Delta t} = \theta \left| \sum_j Q_{ij} + Q_e \right|^{t+\Delta t} + (1-\theta) \left| \sum_j Q_{ij} + Q_e \right|^t \quad (2.3)$$

in which Q_e is the external inflow, Q_{ij} the flow from node j to node i and S the storage area at time t . The above equation can apply to a channel-junction (node) as shown in figure 1.

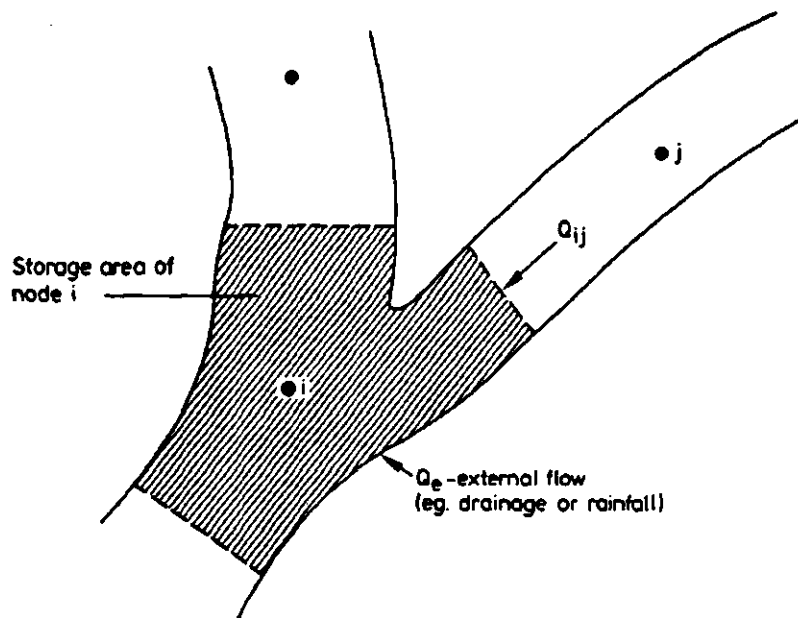


Fig. 1. Water balance of junction

The storage of water is considered to be concentrated in the nodes and the transportation of water to occur in the branches between two nodes. Flow from a node is assumed as positive.

The weighting parameter θ is necessary to describe the variable hydraulic parameters as a function of time. For $\theta = 0.5$ the parameters are calculated as the average between two successive time levels. The closer the weighting parameter is to 0.5 the greater is the accuracy but for numerical stability it should be greater than 0.5. Therefore the weighting parameter is given in general the value 0.55.

2.2. Boundary friction formula

For the calculation of the wall resistance, one can choose out of three options, nl: Chézy equation; Manning equation and flexible vegetation concept. The choice for either Chézy or Manning equation depends on the type of boundary, the user's preference, etc (QUERNER, 1985). For simulation of a maintenance work programme the flexible vegetation concept is introduced. Weed growth (stem length) is then converted to Mannings resistance coefficient depending on flow velocity (bending of vegetation).

When selecting the Chézy equation one should specify the length (height) of the resistance elements. When using the Manning equation the programme requires the coefficient n. The flexible vegetation concept requires various parameters for the calculation of growth rate and bending moment of the weed. At present the method as reported by KOUWEN, LI and SIMONS (1981) is used in the programme. Research on these type of relations will be undertaken in the near future and therefore the latest version of the user's manual should be consulted in this respect.

The cross-section of a channel must be of trapezoidal shape. A bottom width, side slopes and resistance coefficient is the minimum required input data (one section). If necessary the side slopes can have two different angles. In this case the height above the bottom must be given where the angle of the slope changes and for all five sections the resistance coefficient (see figure 2).

When using only one section the conveyance factor is calculated as:

$$\text{Chézy} \quad : \quad K = C A \sqrt{R} \quad (2.4)$$

$$\text{Manning} \quad : \quad K = 1/n A R^{2/3} \quad (2.5)$$

When using five sections the total conveyance factor is calculated as (CHOW, 1959):

$$K_t = \sum_{m=1-5} K_m \quad (2.6)$$

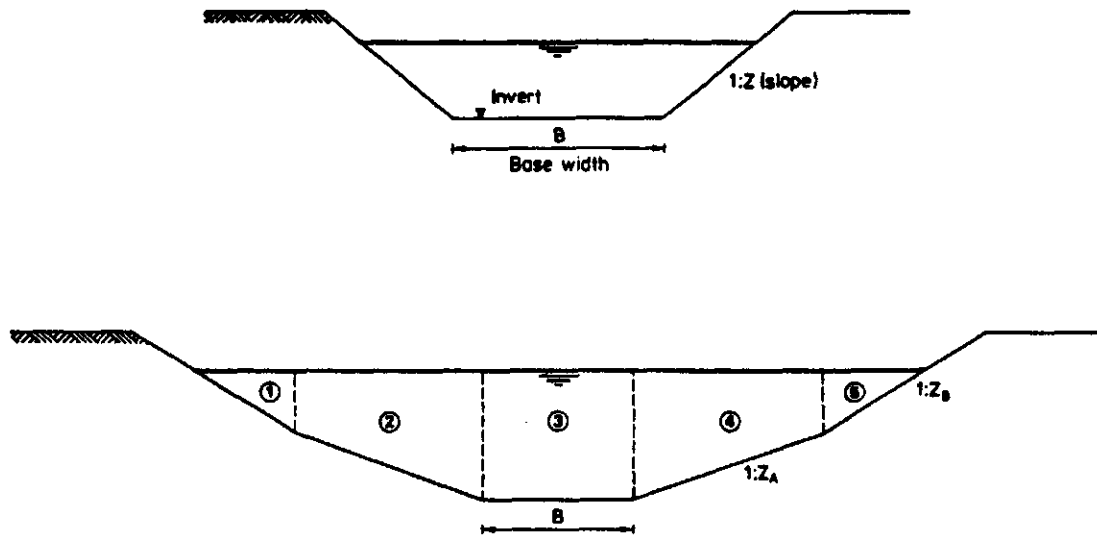


Fig. 2. Schematization of typical cross-sections

2.3. Wave type

With a fast changing discharge rate the inertia term in equation (2.2) plays an important role. This aspect can be observed, for a certain discharge there are two water depths possible. For a rising stage the discharge is greater than with a decreasing stage, as shown in figure 3.

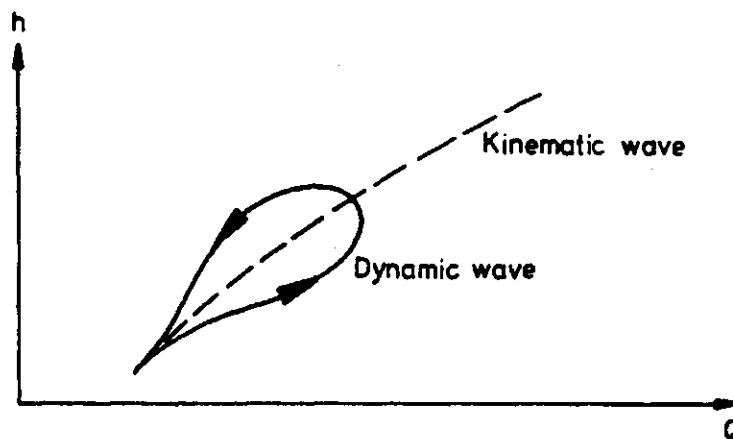


Fig. 3. Discharge curve

With a slowly changing discharge rate the inertia and non-uniformity term have a marginal effect on the properties of the flood wave and can therefore be neglected. With these terms omitted in equation (2.2) we obtain the so-called kinematic wave equation instead of the dynamic wave equation, then we consider the complete equation without any terms neglected.

Whether to use the kinematic or dynamic wave equation has been given by GRIJSEN and VREUGDENHIL (1976). Physical properties of the flood wave can be determined giving a classification to choose the wave type. Two dimensionless parameters are used for the classification :

$$Fr = \frac{Q}{(g A^3/B)^{1/2}} \quad (2.7)$$

and a factor:

$$E = \left| \frac{g^3 A B T_w^2}{B^2 C^4 R^2} \right|^{1/2} \quad (2.8)$$

where Fr is the Froude number, B is the channel width at the water surface, E is a factor to indicate the importance of unsteadiness and non-uniformity and T_w is the wave period. For equation (2.7) and (2.8) one can use the extreme values at the highest water level. With the use of figure 4 the difference in effective velocity of propagation can be determined between a solution based on the kinematic wave equation and the dynamic wave equation. From figure 4 it can be seen that if E is large the kinematic wave approach gives correct velocities of propagation, the same as with the dynamic wave. For small values of E the kinematic wave has no damping and the wave front travels too fast. If the ratio c_d/c_k is greater than 0.95 both wave types can be used to solve the problem. For these cases a kinematic wave type will be preferred, because it gives more freedom to choose the timestep to be used.

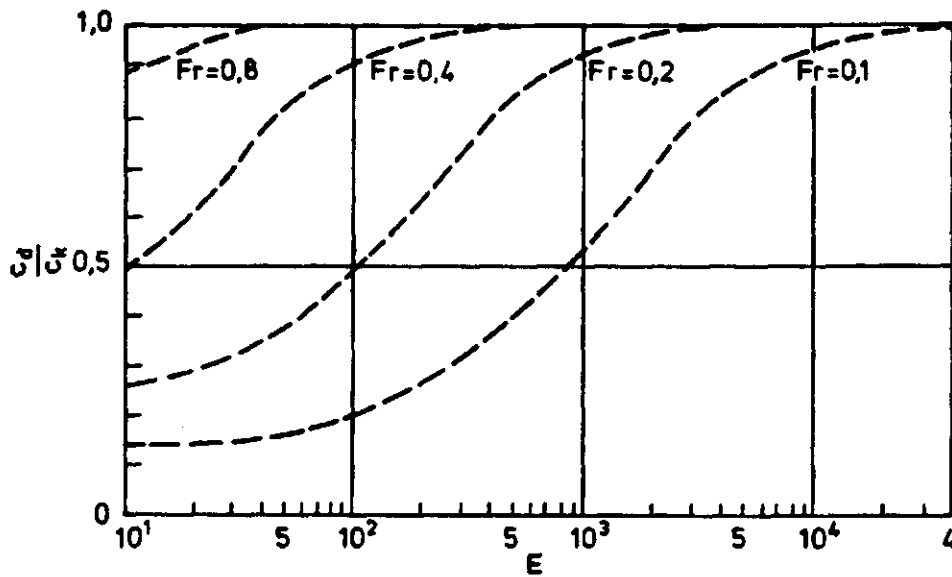


Fig. 4. Velocity of propagation of kinematic waves in relation to dynamic waves

2.4. Equation for kinematic wave

If we consider the equation of motion and neglect the effect of inertia and non-uniform flow, then equation (2.2) becomes:

$$g A \frac{\partial h}{\partial x} - g A I + g \frac{Q|Q|}{C^2 R A} = 0 \tag{2.9}$$

taking the water level at node i as h_i and the adjacent nodes as h_j , then equation (2.8) can be written for a branche length L_{ij} as:

$$- g A \left(\frac{h_i - h_j}{L_{ij}} \right) + g \frac{Q|Q|}{C^2 R A} = 0 \tag{2.10}$$

Equation (2.10) is quadratic in the discharge. For the solution procedure only linear equations are required. This can be achieved by inserting one discharge term as known value from the previous iteration. The discharge between node i and j is then:

$$Q_{ij} = \frac{C^2 R A}{L_{ij} |Q_{ij}^*|} (h_i - h_j) \tag{2.11}$$

where Q_{ij}^* is the discharge calculated in the previous iteration. If we now substitute equation (2.11) into equation (2.3) it can be written as:

$$S_i \frac{h_i^t - h_i^{t+\Delta t}}{\Delta t} = (1 - \theta) Q_o^t + \theta \sum_j \frac{C^2 R A}{L_{ij} |Q_{ij}^*|} (h_i^{t+\Delta t} - h_j^{t+\Delta t}) + \theta Q_e^{t+\Delta t} \quad (2.12)$$

in which:

$$Q_o^t = \sum_j Q_{ij}^t + Q_e^t \quad (2.13)$$

and is the summation of all discharges towards node i at time t and is a known term. Equation (2.12) can be re-arranged to have all known terms on the left hand side of the equal sign and the factors to be multiplied with the water levels h_i and h_j on the right hand side. Equation (2.12) becomes then :

$$(\theta - 1) Q^t + \frac{S_i h_i^t}{\Delta t} - \theta Q_e^{t+\Delta t} = \left| \sum_j \frac{\theta C^2 R A^2}{L_{ij} |Q^*|} + \frac{S_i}{\Delta t} \right| h_i^{t+\Delta t} - \frac{\theta C^2 A^2 R}{L_{ij} |Q^*|} h_j^{t+\Delta t} \quad (2.14)$$

Equation (2.14) can be seen as a set of simultaneous equations. There can be N -equations with N -unknown water levels. Equation (2.14) can be written assembled for all nodal points in matrix form as :

$$\{T\} = [K] \{h\} \quad (2.15)$$

where the vector $\{T\}$ contains all the known terms, the matrix $[K]$ can be seen as a resistance and storage matrix between two connected nodes. The water levels can be solved by taking the inverse of the matrix $[K]$, and multiplied with the vector $\{T\}$ as:

$$\{h\} = [K]^{-1} \{T\} \quad (2.16)$$

The set up of matrix $[K]$ and the inversion is discussed further in paragraph 2.6.

2.5. Equation for dynamic wave

For the calculation scheme of dynamic waves equation (2.2) is now taken without any terms omitted. First the separate terms are looked at. The first term is integrated with respect to x over a full branch L as:

$$\frac{\partial}{\partial t} L \int_0^L Q \, dx = L \frac{dQ}{dt} \quad (2.17)$$

The second term of equation (2.2) is analysed as:

$$\frac{\partial}{\partial t} \left(\alpha' \frac{Q^2}{A} \right) = 2 \frac{Q}{A} \frac{\partial Q}{\partial x} - \frac{Q^2}{A^2} \frac{\partial A}{\partial x} \quad (2.18)$$

The first term on the right hand side of equation (2.18) becomes:

$$2 \frac{Q}{A} \frac{\partial Q}{\partial x} = -2 \frac{Q}{A} \frac{\partial A}{\partial t} = -2 \frac{Q B}{A} \frac{\partial h}{\partial t}$$

The second term on the right hand side of equation (2.18) becomes:

$$\frac{Q^2}{A^2} \frac{\partial A}{\partial x} = \frac{Q^2}{A^2} \frac{\partial (B h)}{\partial x} = \frac{Q^2}{A^2} \left(B \frac{\partial h}{\partial x} + h \frac{\partial B}{\partial x} \right)$$

We assume that the cross-section does not change considerable and so that the term $h(\partial B/\partial x)$ becomes zero. The second term of equation (2.2) becomes now after integration over the branch length:

$$= - \frac{Q^2 B}{A^2} (h_2 - h_1) - \frac{Q B L}{A} \frac{d}{dt} (h_1 + h_2) \quad (2.19)$$

The third term of equation (2.2) is not integrated, but averaged over a branch as:

$$g A (h_j - h_i) \quad (2.20)$$

The result of the five terms from equation (2.2) and divided by $g A$ gives:

$$\begin{aligned} & \frac{L}{g A} \frac{dQ}{dt} - \frac{Q^2 B L}{g A^3} (h_2 - h_1) - \frac{Q B L}{g A^2} \frac{d}{dt} (h_1 + h_2) + (h_j - h_i) + \\ & + \frac{L Q^* | Q}{C^2 R A^2} = 0 \end{aligned} \quad (2.21)$$

The Froude number can be written as:

$$Fr^2 = \frac{Q^2 B}{g A^3} \quad (2.22)$$

The water depth is differentiated to elevation and invert level as:

$$h_1 = h_i - z_i \quad (2.23)$$

and

$$h_2 = h_j - z_j \quad (2.24)$$

Substitution of equation (2.22) to (2.24) into equation (2.21) gives:

$$\frac{L}{g A} \frac{dQ}{dt} - \frac{Q B L}{g A} \frac{d}{dt} (h_1 + h_2) + (1 - Fr^2) (h_j - h_i) - Fr^2 (z_j - z_i) + \frac{L Q^* |Q|}{C^2 R A^2} \quad (2.259)$$

Equation (2.25) integrated in respect of time:

$$\begin{aligned} & \frac{L Q^{t+\Delta t}}{\Delta t g A} - \frac{L Q^t}{t g A} \\ & - \frac{Q B L}{g A^2} \left(\frac{h_1^{t+\Delta t} + h_2^{t+\Delta t}}{\Delta t} \right) + \frac{Q B L}{g A^2} \left(\frac{h_1^t + h_2^t}{\Delta t} \right) \\ & + \theta [(1 - Fr^2) (h_j - h_i)]^{t+\Delta t} \\ & + (1 - \theta) [(1 - Fr^2) (h_j - h_i)] - Fr^2 (z_j - z_i) \\ & + \theta \left[\frac{L Q^* |Q|}{C^2 R A^2} \right]^{t+\Delta t} + (1 - \theta) \left[\frac{L Q^* |Q|}{C^2 R A^2} \right]^t = 0 \quad (2.26) \end{aligned}$$

The known terms from the previous time level t are grouped into the variable F_1 as:

$$\begin{aligned}
 F_1 = & -L \frac{Q^t}{\Delta t g A} + \frac{Q B L}{\Delta t g A^2} (h_1^t + h_2^t) + \\
 & + (1 - \theta) [(1 - Fr^2) (h_j - h_i)]^t + \\
 & + (1 - \theta) \left[\frac{L Q^t Q}{C^2 R A^2} \right]^t - Fr^2 (z_j - z_i)
 \end{aligned} \tag{2.27}$$

and certain other terms grouped into the variable F_2 as:

$$F_2 = \frac{B L}{\Delta t g A^2} (h_1^{t+\Delta t} + h_2^{t+\Delta t}) \tag{2.28}$$

Re-arranging equation (2.26) and written in terms of the discharge Q it follows:

$$Q^{t+\Delta t} = \theta \left[\frac{(1 - Fr^2) (h_i - h_j)}{F_3} \right] - \frac{F_1 + F_2}{F_3} \tag{2.29}$$

where F_3 is:

$$F_3 = \frac{L}{\Delta t g A} - \frac{\theta L |Q^{t+\Delta t}|}{C^2 R A^2} \tag{2.30}$$

Substitute equation (2.29) into equation (2.12) it becomes:

$$\begin{aligned}
 \frac{S_i h_i^t}{\Delta t} - \frac{S_i h_i^{t+\Delta t}}{\Delta t} = & \theta \frac{(1 - Fr^2)}{F_3} (h_i - h_j) - \\
 - \theta \frac{F_1 + F_2}{F_3} + & \theta Q_e^{t+\Delta t} + (1 - \theta) \left[\sum_j Q_{ij} + Q_e^t \right]
 \end{aligned} \tag{2.31}$$

Bringing the known terms to the left hand side gives:

$$\begin{aligned}
 & - (1 - \theta) \left[\sum_j Q_{ij}^t + Q_e^t \right] + \frac{S_i h_i^t}{\Delta t} + \theta \frac{F_1 + F_2}{F_3} - \theta Q_e^{t+\Delta t} = \\
 & = \left[\frac{S_i}{\Delta t} + \theta \sum_j \frac{\theta (1 - Fr^2)}{F_3} \right] h_i^{t+\Delta t} - \\
 & - \left[\theta \sum_j \frac{\theta (1 - Fr^2)}{F_3} \right] h_j
 \end{aligned} \tag{2.32}$$

Similar as for the kinematic wave the above equation can be written in matrix notation as:

$$\{T'\} = [K'] \{h\} \tag{2.33}$$

Now we have the same form of equation as for the kinematic wave type and the solution procedure is identical.

2.6. Solution procedure

If we consider the network of channel sections and a set of equations per nodal point as given by equation (2.16), then it follows that the stiffness matrix is symmetrical. This means that the terms above and below the leading diagonal contain the same values, so that for the program only the diagonal and the coefficients above are stored. In the matrix [K] for node i which is connected to node j in the matrix on the ith row and jth column a value is given. If we choose the numbering of the network in such a way we can arrange the non-zero terms in the matrix to occur near the diagonal. This rearranging of node numbers is done by the program. The (external) numbers defined by the user is changed to an internal number. The assembled roughness and storage matrix contains now many zero terms and in particular there is a distance from the leading diagonal beyond which all terms are zero

(see figure 5a). The semi-bandwidth denoted as BW is only used by the program as shown in figure 5b.

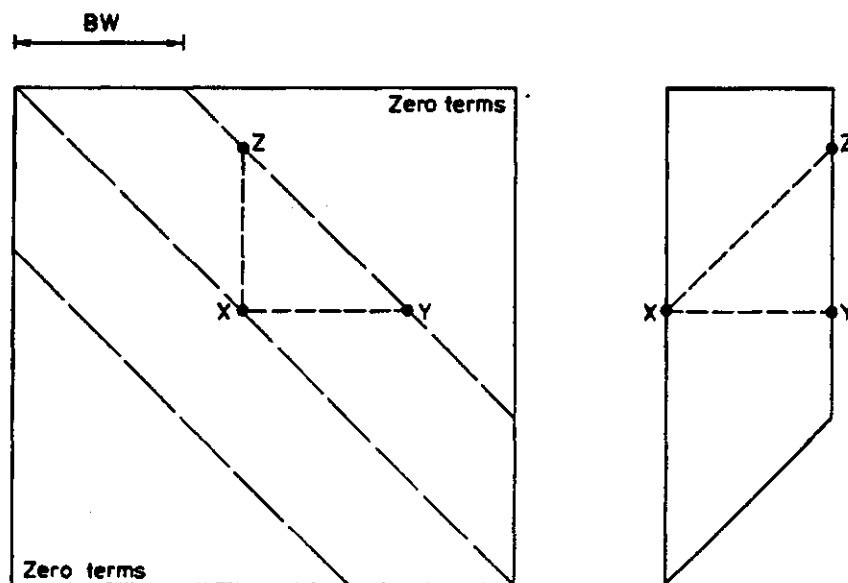


Fig. 5. a full matrix with $M \times M$ terms and semi-bandwidth BW
b banded matrix

Equation (2.16) can now be solved by any one of the matrix inversion techniques. The Crout reduction method was chosen. The essential feature of Crout's reduction procedure is the reordering of the sequence in which the terms of the coefficient matrix are modified. If we consider Gauss elimination, a modification of each term of the reduced coefficient matrix is made every time an equation is eliminated. This procedure is tedious, because it requires that the coefficient matrix be rewritten many times. In Crout reduction it is necessary to rewrite the coefficient matrix only once, because each term is changed directly from its initial value in the unreduced matrix to its final value in the compressed matrix.

3. EQUATIONS FOR SPECIAL SECTIONS

3.1. Constant water level

A constant water level is one of the boundary conditions one can specify. If we consider equation (2.15) for these nodal points the terms in the coefficient matrix are known and must be taken out of the roughness and storage matrix and included in the array with the known terms {T}. All node points connected to this node with the prescribed water level must be corrected in the [K] matrix as shown in figure 6. The inflow or outflow, outside the region, to maintain the prescribed level follows from the calculations.

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} & & & & \\ & K_{22} & K_{23} & & & & \\ & & K_{33} & K_{34} & & & \\ & & & K_{44} & K_{45} & & \\ & & & & K_{55} & & \\ & & & & & & \end{bmatrix} \cdot \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \end{bmatrix} = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix}$$

$$\begin{bmatrix} K_{11} & K_{12} & 0 & & & & \\ & K_{22} & 0 & & & & \\ & & 1 & 0 & & & \\ & & & K_{44} & K_{45} & & \\ & & & & K_{55} & & \end{bmatrix} \cdot \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \end{bmatrix} = \begin{bmatrix} T_1 - K_{13} h_3 \\ T_2 - K_{23} h_3 \\ \text{Fixed level} \\ T_4 - K_{34} h_3 \\ T_5 \end{bmatrix}$$

Fig. 6. Correction to [K] matrix and {T} vector for node with prescribed water level (in node 3 a fixed water level is present)

3.2. Pump

A pumping station can be present on the boundary (external) or inside the region to be studied (internal). The pumping rate must be specified in the form of a rating curve. The rating curve must be discretised into sections, each having the relation:

$$Q = \alpha_n H + \beta_n \quad (3.1)$$

where the coefficient α and β apply for a section n of the pump curve as shown in figure 7. The discretization in piecewise linear sections is required for the adopted solution procedure. The pumping head can be written as the delivery head minus the suction level, in the case of a pump as a boundary node. Then equation (3.1) can be written as:

$$Q = \alpha_n h_d - \alpha_n h_i + \beta_n \quad (3.2)$$

For an internal pump or inlet between two nodal points the pumping head is the difference between the head at node i and j . Equation (3.1) can now be written as:

$$Q = \alpha_n (h_j - h_i) + \beta_n \quad (3.3)$$

The known terms of equation (3.2) or (3.3) can be brought to the left hand side and are then in the same form as equation (2.15) and can be substituted in equation (2.16).

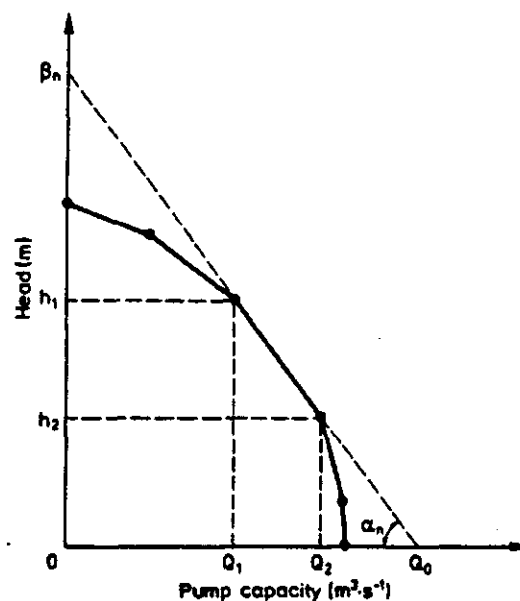


Fig. 7. Typical pump curve and its discretization in piecewise linear sections

3.3. Weir

A weir can be situated on the boundary or internal as a channel section. The discharge over a weir can be written as:

$$Q = c B H^{3/2} \quad (3.4)$$

where c is the discharge coefficient (1.6 - 2.1), B the width and H the head at the upstream side. This equation can be written for a weir on the boundary as:

$$Q = K H \quad (3.5)$$

with: $K_w = c B \sqrt{H}$
 $H = h_i^* - h_w$

where h_i^* is the water level at node i from the previous iteration and h_w is the weir level. Equation (3.5) can now be written as:

$$Q + K_w h_w = K_w h_i \quad (3.6)$$

In this form the equation is written similar as equation (2.15) with known terms on the left hand side of the equal sign. For a weir between two nodal points equation (3.6) becomes:

$$Q = K_w (h_i - h_j) - K_w (h_w - h_j^*)$$

or

$$Q + K_w (h_w - h_j^*) = K_w (h_i - h_j) \quad (3.7)$$

In this form the equation is written similar as equation (2.15) and can be substituted into equation (2.16).

Dependent of the downstream water level the weir can be submerged. If the upstream head above weir level is smaller the 1.5 times the head at the downstream side above weir level then equation (3.4) is not valid any more. The discharge is now also dependent on the downstream level as:

$$Q = c B \sqrt{2 g (h_j - h_w)} \sqrt{(h_i - h_j)} \quad (3.8)$$

This equation can be re-arranged to the same form as equation (3.7).

3.4. Culvert

The discharge thru a culvert section can be written as:

$$Q_{ij} = \frac{C^2 R A^2}{L |Q_{ij}^*|} (h_i - h_j - \Delta H_t) \quad (3.9)$$

where ΔH_t is the entrance and exit loss, dependent on the geometry of entrance and exit. This loss is written in terms of the velocity head in the culvert as:

$$\Delta H_t = \alpha_t \frac{v^2}{2 g} = \alpha_t \frac{Q^* |Q|}{2 g A^2} \quad (3.10)$$

where α_t is the sum of entrance and exit loss coefficient. For circular culverts two hydraulic parameters are defined to calculate the wetted perimeter and area (culverts flowing not full) as:

$$P = F_4 \left\{ \bar{h} / D \right\} D$$

$$A = F_5 \left\{ \bar{h} / D \right\} D$$

where F_4 and F_5 are constants depending on the dimensionless water depth in the culvert, \bar{h} is the average water depth and D is the culvert diameter. Equation (3.10) is substituted into equation (3.9) and re-arranged to the same form as equation (2.15).

3.3. Critical depth

The critical depth occurs at a stage between subcritical and supercritical flow. It is reached for one specific discharge and it requires the least amount of energy head, as shown in figure 8. The critical depth occurs when the Froude number is equal to unity and can be calculated with equation (2.7). A lower value than unity indicates subcritical and a higher value than unity indicates supercritical flow.

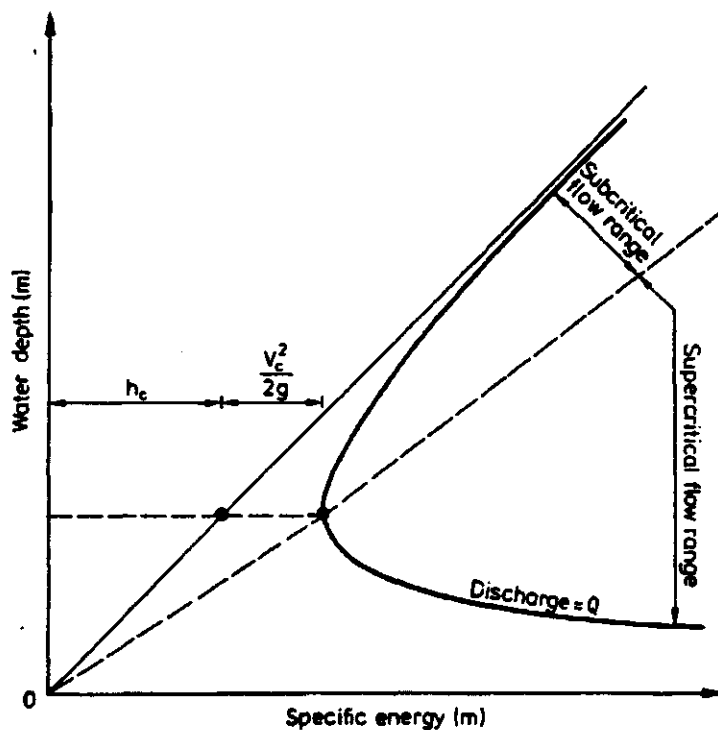


Fig. 8. Specific-energy curve

4. EXAMPLE

For a channel system as shown in figure 9 the programme was run for a winter period with a drainage situation. A weir is situated at the outlet. The larger channels in the region are modelled and shown on the figure, but the smaller channels and ditches are represented in the model as additional storage per nodal point. A typical depth-storage curve (depth minus groundlevel) is given and per nodal point a multiplication factor takes the area to be drained into account. The results of three nodal points and channel sections are shown in figure 10.

Over the calculation period a drainage rate is specified changing in time. This can be a time consuming effort and for these situations a groundwater submodel can be linked, so that the interaction between groundwater and surface water is done automatically (QUERNER, 1986a). The advantage is also that the drainage rate is calculated taken into account the difference in groundwater level and water level in the channel system.

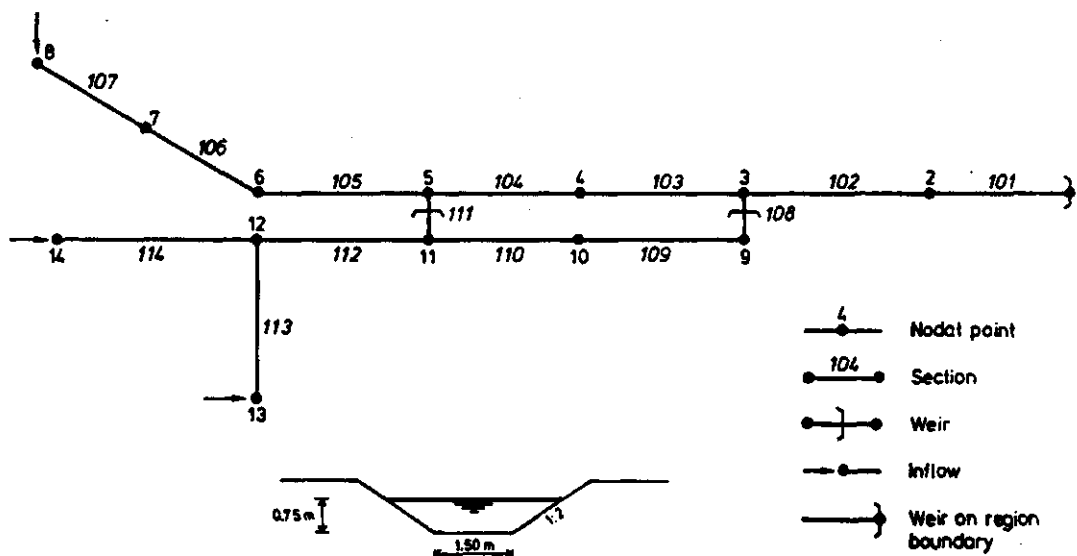


Fig. 9. Channel layout and geometry

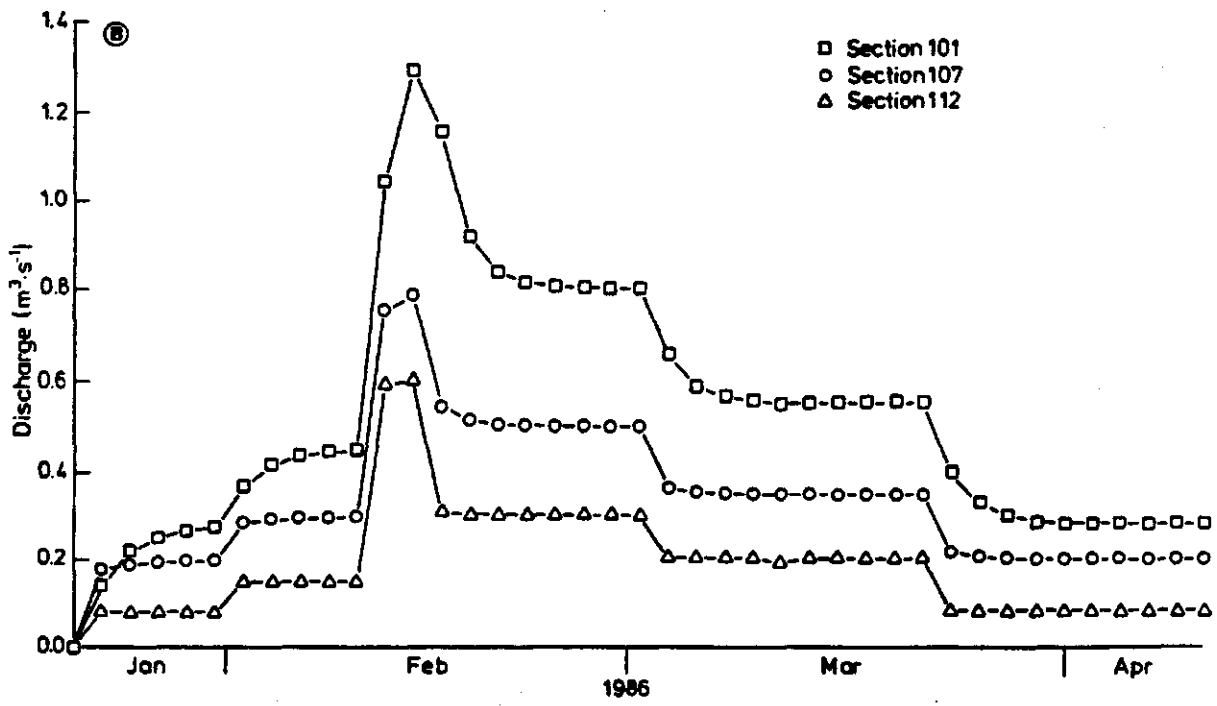
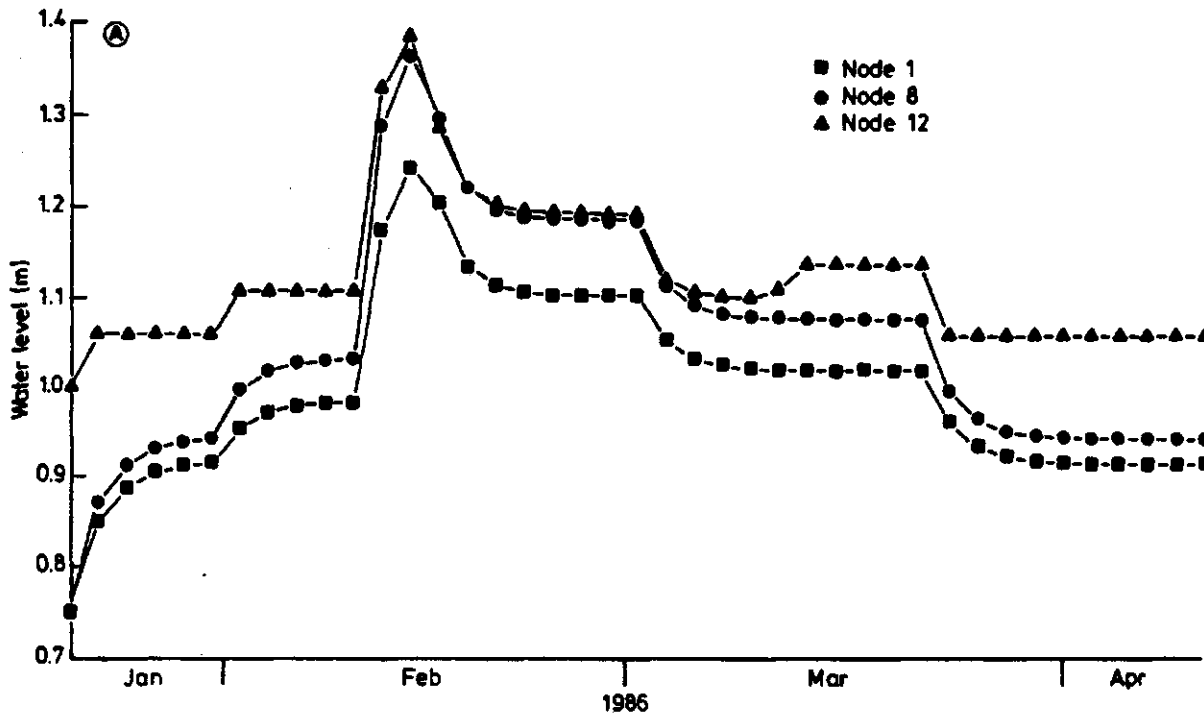


Fig. 10. Results of calculations with kinematic wave equation for the network shown in figure 9

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Appendix A - Derivation of the unsteady gradually varied flow equation

Consider a small channel segment of length dx and write the change in energy terms between upstream and downstream ends of the reach in differential form :

$$\frac{v^2}{2g} + h + z = \left[\frac{v^2}{2g} + d\left(\frac{v^2}{2g}\right) + h + dh + z + dz + dH \right] \quad (a1)$$

where dH is the total head loss over the reach with length dx . It should be noted that $d(v^2/2g)$ and dh may be either positive, negative, or zero (uniform flow). Equation (a1) reduces to:

$$dH = - \left[d\left(\frac{v^2}{2g}\right) + dh + dz \right] = - \left[\frac{1}{2g} dv^2 + dh + dz \right]$$

$$d(v)^2 = 2 v dv$$

$$dH = - \left(\frac{v}{g} dv + dh + dz \right) \quad (a2)$$

The rate of change of total head can be obtained by dividing equation (a2) by dx :

$$\frac{dH}{dx} = - \left(\frac{v}{g} \frac{dv}{dx} + \frac{dh}{dx} + \frac{dz}{dx} \right) \quad (a3)$$

For unsteady flows there is an additional component due to changes of velocity with time. The acceleration a has two components:

$$a = v \frac{\partial v}{\partial x} + \frac{\partial v}{\partial t} \quad (a4)$$

The first term of equation (a4) represents the head loss due to change of velocity in the downstream direction (change in cross-section).

The second term of equation (a4) represents the head loss due to change of velocity in time at a given point, which can be invoked by Newton's second law:

$$F_1 = \rho B \frac{\partial V}{\partial t} \quad (a5)$$

where ρ is the mass density of the fluid and F_1 is the force exerted on a volume B of fluid undergoing local acceleration. The work done, or energy expended in accelerating this volume is the force times the downstream distance dx , so:

$$dF_1 = \rho B \frac{\partial V}{\partial t} dx \quad (a6)$$

where dF_1 represents the energy lost as a result of local acceleration. This energy loss is transformed to a head loss dH_1 by dividing by the weight of water:

$$dH_1 = \frac{\rho B}{\gamma B} \frac{\partial V}{\partial t} dx = \frac{1}{g} \frac{\partial V}{\partial t} dx \quad (a7)$$

where γ is the weight density.

The rate of head loss due to local acceleration is :

$$\frac{dH_1}{dx} = \frac{1}{g} \frac{\partial V}{\partial t} \quad (a8)$$

Introducing this component of head loss into equation (a3) and switching to partial derivative notation throughout gives the equation for the downstream rate of head loss in unsteady gradually varied flows:

$$\frac{\partial h}{\partial x} = - \left(\frac{V}{g} \frac{\partial V}{\partial x} + \frac{\partial h}{\partial x} + \frac{\partial z}{\partial x} + \frac{1}{g} \frac{\partial V}{\partial t} \right) \quad (\text{a9})$$

$$\frac{\partial h}{\partial x} \approx S_e$$

$$\frac{\partial z}{\partial x} \approx - S_o$$

thus:

$$S_o - S_e = \frac{V}{g} \frac{\partial V}{\partial x} + \frac{\partial h}{\partial x} + \frac{1}{g} \frac{\partial V}{\partial t} \quad (\text{a10})$$

Equation (a10) was derived from energy considerations, but one could also arrive at the same relationship with expressions for the conservation of momentum in an unsteady flow. Therefore equation (a10) is usually referred to as the momentum equation. The above equation together with the continuity equation are the system of partial differential equations that describes one-dimensional, gradually varied, unsteady open-channel flows. Equation (a10) was first developed and published by Jean-Claude Barre de Saint-Venant (1797-1886) in France in 1848, and is known as the Saint-Venant equation.

Equation (a10) has two dependent variables (V or Q and h) and two independent variables (x and t). Partial differential equations of this type can only be solved numerically. To do this, we must specify the values of the dependent variables at t=0 (initial conditions) and two boundary conditions.