Underground Travel of Renovated Wastewater

By Robert C. Rice[1] and Peter Raats[2]

Introduction

Land application of sewage effluent and other liquid waste is becoming more popular as evidenced by the increasing number of land treatment facilities. The Clean Water Act of 1977 places land treatment in the innovative and alternate technology (I&A) category that qualifies for 85% funding from the U.S. Environmental Protection Agency (EPA) instead of the 75% funding for conventional plants. Also, I&A projects receive 100% funding of startup costs, 15% credit for cost effectiveness, and 100% cost of modification if the system does not do an adequate job (10). New EPA policy also requires applicants for construction grants to provide complete justification for the rejection of land treatment. In view of these factors and high cost and energy requirements of in-plant treatment (4), land treatment can be expected to play a larger and more important role in waste treatment in the future. An important aspect of land treatment systems, and particularly high-rate infiltration systems, is the underground flow system as it relates to the quality of the renovated water and to restricting the spread of renovated water into the aquifer (2,4). The purpose of this paper is to apply existing flow theory in order to predict minimum underground detention times and attenuation of nitrate peaks for two types of systems: (1) Shallow aquifers where the renovated water is collected with horizontal drains; and (2) deep aquifers where the renovated water is collected with wells.

Bouwer (1,2,4) has presented criteria for design and operation of high-rate recharge systems with respect to the hydraulic properties of the aquifer, ground-water mound buildup, and underground detention time. Adequate under-

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ground detention time is required to obtain high quality renovated water. Under high-rate recharge systems, the first water that infiltrates after a dry period leaches nitrates formed in the upper soil layers during drying. This causes the percolation water to contain a high nitrate concentration (5,9). The nitrogen concentration in this first water when sampled directly below a basin can be several times that of the incoming wastewater (5,8). Under field conditions, however, the nitrate peak will be attenuated as the water moves laterally away from the basins toward drains, wells, or other collection points.

In two recent papers, the second writer (11,12) presented a comprehensive theory for steady, multidimensional, convective transport of solutes. The first paper dealt with the general theory; the second paper with specific flow problems. The time $t - t_0$ required for a parcel of water to move from a point s_0 to a point s along a streamline can be calculated from

$$t - t_0 = \int_{s_0}^{s} v^{-1}\, ds \qquad (1)$$

in which v = the speed along the streamline. For a given geometry and boundary condition, the cumulative transit time distribution function, q, is defined as the fraction of the stream tubes with transit times smaller than τ, in which τ = transit time. The transit time density distribution is defined as the derivative of q with respect to τ. The function q can be determined by measuring the concentration of an ideal tracer in the output following a step change of the concentration in the input. The function of $dq/d\tau$ can be determined by measuring the output resulting from a pulse distributed uniformly in the input. For input distributed uniformly over the stream tubes, the transit time density distribution, $dq/d\tau$, may be regarded as a transfer function of the flow system; thus, the general relationship between the input, P, and the output, O, can be written as

$$O[t] = \int_{t_0}^{t} \frac{dq}{d\tau} P[t - \tau]\, d\tau \qquad (2)$$

in which the square brackets denote functional dependence (11,12); and the term $dq/d\tau$ corresponds to the relative concentration of a unit pulse input.

Shallow Aquifers

Travel Time.—When the water table is relatively close to the surface because of a shallow, impermeable layer, tile drains or open ditch drains can be used to collect the renovated water. The water moves through three zones (Fig. 1): (1) The unsaturated zone between the infiltration basin and the water table; (2) the saturated zone below the water table within the recharge area; and (3) the saturated zone from the edge of the recharge basin to the drain. Given the width of the recharge area, W, the distance between the edge of the recharge area and the drain, L, the depth to the impermeable layer, d, and the infiltration rate, I, the distributions of the transit times for the three zones will be determined.

Let x be a horizontal coordinate with its origin at the point of maximum travel to the drain. Let $h[x]$ be the height of the water table above the impermeable layer at x. Ignoring the capillary fringe, the volumetric water content, θ, in the unsaturated zone is assumed to be constant. Therefore, the transfer velocity, v_1, in zone 1 is given by

$$v_1 = \frac{I}{\theta} \quad \text{(3)}$$

Introducing Eq. 3 into Eq. 1 and integrating between the limits $s_0 = 0$ and $s = d - h[x]$ gives

$$\tau_1 = \frac{\theta(d - h[x])}{I} \quad \text{(4)}$$

The distribution of the velocity of the water in zones 2 and 3 is determined from horizontal flow theory using the Dupuit-Forchheimer assumptions. The integrated mass balance for the water in zone 2 implies that the velocity, v_2,

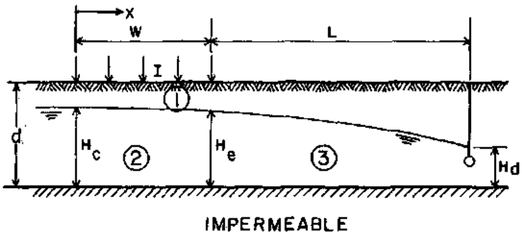

FIG. 1.—Geometry and Symbols for Shallow Aquifer Recharge System

in zone 2 is given by the ratio of the flow rate, Ix, and the cross-sectional area, fh:

$$v_2 = \frac{Ix}{fh[x]} \quad \text{(5)}$$

in which f = the porosity. Introducing Eq. 5 into Eq. 1 gives

$$\tau_2 = \frac{f}{I} \int_x^W \frac{h[x]}{x} dx \quad \text{(6)}$$

Similarly, for zone 3:

$$v_3 = \frac{IW}{fh[x]} \quad \text{(7)}$$

and $$\tau_3 = \frac{f}{IW} \int_W^{W+L} h[x] dx \quad \text{(8)}$$

The expressions for the travel times, τ_1, τ_2, and τ_3, given by Eqs. 4, 6, and 8, all involve the water table height, $h[x]$. The function $h[x]$ is determined by using the Dupuit-Forchheimer version of Darcy's law. For zone 2:

$$Ix = -Kh \frac{dh}{dx} = -T_r \frac{dh}{dx} \quad \text{(9)}$$

and for zone 3: $$IW = -Kh\frac{dh}{dx} = -T_r\frac{dh}{dx} \quad (10)$$

in which $T_r = Kh$ = the effective transmissivity of the recharge aquifer. For region 2, Eq. 9 is linearized by approximating T_r by:

$$T_r = \frac{K(H_c + H_e)}{2} \quad (11)$$

in which H_c and H_e = the values of h at the center and at the edge of the recharge area, respectively. Substituting Eq. 11 into Eq. 9 and integrating the result subject to the boundary condition $h = H_c$ at $x = 0$ gives:

$$h[x] = H_c - \frac{Ix^2}{K(H_c + H_e)}; \quad 0 < x < W \quad (12)$$

Integration of Eq. 10 subject to $h = H_e$ at $x = W$ gives:

$$h[x] = \left\{H_e^2 + \frac{2IW(W - x)}{K}\right\}^{1/2}; \quad W < x < L \quad (13)$$

Introducing Eq. 12 into Eq. 4, and 6, and Eq. 13 into Eq. 8 gives:

$$\tau_1 = \frac{\theta(d - H_c)}{I} + \frac{\theta x^2}{K(H_c + H_e)} \quad (14)$$

$$\tau_2 = \frac{fH_c}{I}\ln\frac{W}{x} - \frac{f(W^2 - x^2)}{2K(H_c + H_e)} \quad (15)$$

$$\tau_3 = \frac{fK}{3I^2W^2}\left\{H_e^3 - \left(H_e^2 - \frac{2IWL}{K}\right)^{3/2}\right\} \quad (16)$$

The heights H_c and H_e in Eqs. 14, 15, and 16 can be expressed in terms of the known height H_d and parameters W, L, I, and K as follows. Evaluating Eq. 13 at the drain and solving for H_e gives:

$$H_e = \left(H_d^2 + \frac{2ILW}{K}\right)^{1/2} \quad (17)$$

Evaluating Eq. 12 as the edge of the recharge area, solving for H_c, and eliminating H_e by using Eq. 17 gives:

$$H_c = \left[H_d^2 + \frac{IW(W + 2L)}{K}\right]^{1/2} \quad (18)$$

On the right-hand sides of Eqs. 14 and 15, the first terms represent the travel times in zones 1 and 2 in the limit of a flat water table at height H_c above the impermeable base. The term $\theta(d - H_c)/I$ in Eq. 14 is the amount of water, $\theta(d - H_c)$, in a column of height $(d - H_c)$ divided by the rate of infiltration, I. The term (fH_c/I) ln (W/x) in Eq. 15 is the distribution of the travel times for the apparently well-mixed rectangle of height H_c and width W subject to a uniform rate of infiltration I. This distribution was examined in detail by the second writer (12). On the right-hand sides of Eqs. 14 and 15, the last terms represent corrections needed because the water table is not flat. The sum of Eq. 16 and Eq. 14 evaluated at $x = W$ gives the travel time from the edges of the recharge area to the drain. This time is the shortest travel time in the system and, therefore, is the breakthrough time.

The turnover time, $\bar{\tau}$, of the entire system is defined as the total volume

TABLE 1.—Ratio $\tau_2/\bar{\tau}_A$ for Different H_e/H_c and x/W Ratios

x/W (1)	Ratio τ_2/τ_A When H_e/H_c Is 0.0 (2)	0.2 (3)	0.5 (4)	0.8 (5)	1.0 (6)
0.9	0.01	0.03	0.06	0.09	0.10
0.7	0.10	0.15	0.23	0.31	0.36
0.5	0.32	0.39	0.51	0.62	0.69
0.3	0.75	0.84	0.98	1.11	1.20
0.1	1.81	1.91	2.06	2.20	2.30

TABLE 2.—Comparison of $\tau_2/\bar{\tau}_A$ Value, Calculated from Eq. 21 with and without Correction Factor and Flow Nets

					$\tau_2/\bar{\tau}_A$		
I/K (1)	W, in feet (meters) (2)	H_c, in feet (meters) (3)	H_e/H_c (4)	x/W (5)	Eq. 21 (6)	Flow net (7)	Without correction (8)
0.56	10 (3.05)	9 (2.74)	0.55	0.9	0.06	0.06	0.11
				0.7	0.24	0.23	0.35
				0.5	0.53	0.54	0.69
				0.3	1.00	1.05	1.20
				0.1	2.08	2.20	2.30
.0109	50 (15.24)	11.4 (3.47)	0.88	0.9	0.09	0.09	0.11
				0.7	0.33	0.35	0.35
				0.5	0.65	0.71	0.69
				0.3	1.15	1.23	1.20
				0.1	2.24	2.37	2.30

of water in the system, or pore volume, divided by the total flux through the system:

$$\bar{\tau} = \frac{1}{IW}\left\{\theta \int_0^W (d-h)\,dx + f\int_0^W h dx + f\int_W^{W+L} h dx\right\} \quad \text{. . . . . (19)}$$

The last term on the right-hand side of Eq. 19 is exactly equal to τ_3 given by Eq. 16. Introducing Eq. 12 into Eq. 19 gives:

$$\bar{\tau} = \frac{\theta d}{I} + (f-\theta)\left\{\frac{H_c}{I} - \frac{1}{3}\frac{W^2}{K(H_c+H_e)}\right\} + \tau_3 \quad \text{. . . . . (20)}$$

Eq. 15 can be rewritten as:

$$\frac{\tau_2}{\bar{\tau}_A} = \ln\frac{W}{x} - \frac{1}{2}\left(1 - \frac{x^2}{W^2}\right)\left(1 - \frac{H_e}{H_c}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (21)$$

in which $\bar{\tau}_A = fH_c/I$ or the turnover time of the apparently well-mixed system when $H_e/H_c = 1$. The comparison of $\tau_2/\bar{\tau}_A$ for different H_e/H_c and X/W ratios is shown in Table 1. Error introduced by ignoring the correction term is greatest at low H_e/H_c ratios.

The validity of Eq. 15 can be checked by comparisons of travel times computed from flow nets presented by Kirkham (7). The geometric parameters and travel times, $\tau_2/\bar{\tau}_A$, computed from Eq. 21, and the flow nets are shown in Table 2. The comparison is good with the differences being less than 7%. Ignoring

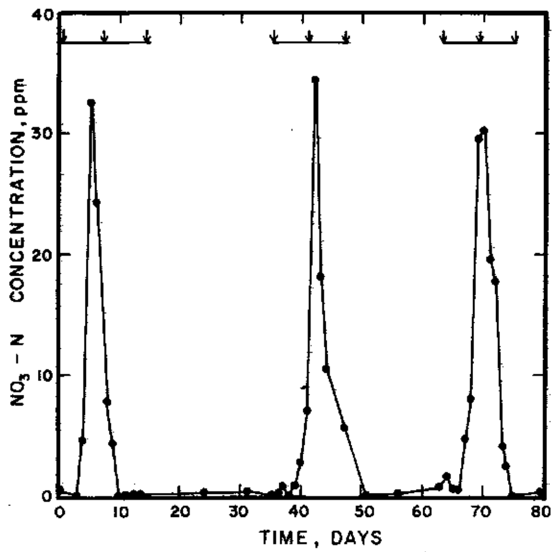

FIG. 2.—Nitrate Concentration in Collection Well of Field System after 30 ft (9.1 m) of Vertical Travel (Inundation Periods are Indicated by Arrows and Lines)

the correction term overestimates $\tau/\bar{\tau}_A$ by as much as 50% when $H_e/H_c = 0.55$.

Underground Movement of Attenuated Nitrate Peak.—The first water that infiltrates after a dry period leaches nitrate formed by nitrification of adsorbed ammonium (5). When measuring the nitrate concentration directly under the recharge area, a definite nitrate peak is noted, as Fig. 2 shows for a field system. The maximum nitrate-nitrogen concentration in the peak can be greater than the concentrations of nitrogen in the wastewater. However, the time for parcels of solute to travel from different locations in the recharge area to the drain are different, (resulting in attenuated nitrate peaks in the drainage water). In addition to this convective mixing, dispersion causes each nitrate peak, i.e., observed directly under the recharge area to be attenuated as it moves to the collection point. The theoretical shape of the peak at the drain can be calculated from the travel times assuming plug flow and no dispersion (11).

To determine the shape of the attenuated nitrate peak, the distribution of the travel times between different points on the field and the drain is needed. Since parcels of water introduced at $x = W$ are the first to appear at the

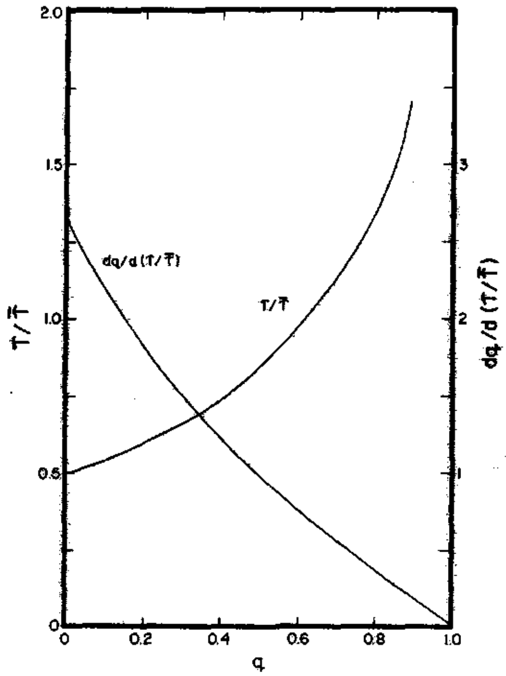

FIG. 3.—Value q Versus $\tau/\bar{\tau}$ and $dq/d(\tau/\bar{\tau})$ for Drain Collection Example

drain, the cumulative transit time distribution function, q, can be expressed by (11):

$$q = 1 - \frac{x}{W} \quad (22)$$

Rearranging Eq. 22 gives:

$$\frac{x}{W} = 1 - q \quad (23)$$

Introducing Eq. 23 into Eqs. 14, 15, and 16, and adding the results gives:

$$\tau = \tau_1 + \tau_2 + \tau_3 = \frac{\theta\,(d - H_c)}{I} + \frac{\theta W^2}{K(H_c + H_e)}(1 - q)^2 - \frac{(fH_c)}{I}\ln(1 - q)$$

$$- \frac{f\,W^2}{2K(H_c + H_e)}(2q - q^2) + \frac{fK}{3I^2\,W^2}\left\{H_e^3 - \left(H_c^2 - \frac{2I\,WL}{K}\right)^{3/2}\right\} \quad (24)$$

Differentiating Eq. 24 with respect to q gives:

$$\frac{d\tau}{dq} = \frac{f H_c}{I(1-q)} - (f + 2\theta)\frac{W^2}{K(H_c + H_e)}(1-q) \quad \ldots \ldots \ldots \ldots \ldots \ldots (25)$$

The transit time density distribution function, $dq/d\tau$, is obtained by taking the reciprocal of Eq. 25:

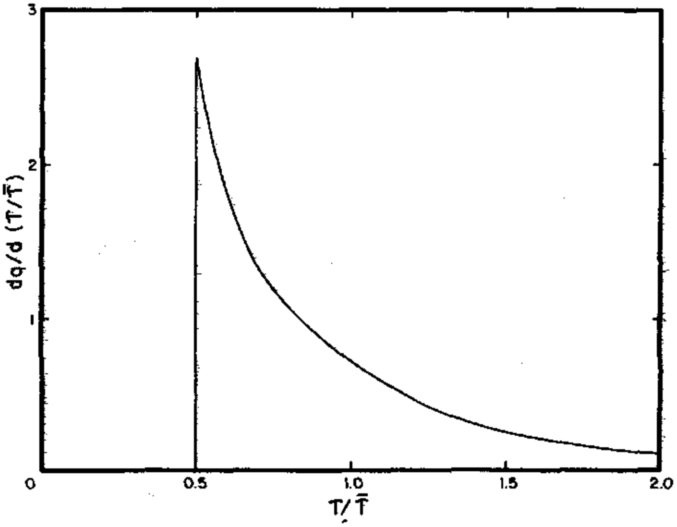

FIG. 4.—Graphical Relationship of $\tau/\bar{\tau}$ and $dq/d(\tau/\bar{\tau})$ Determined from Fig. 3

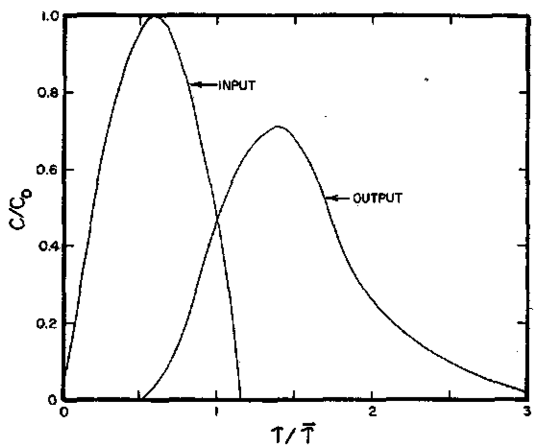

FIG. 5.—Relative Concentration, C/C_0, at Output Drain of Parabolic Input Nitrate Peak

$$\frac{dq}{d\tau} = \frac{I(1-q)}{fH_c - \dfrac{(f+2\theta)W^2 I(1-q)^2}{K(H_c+H_e)}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (26)$$

Eq. 24 gives τ as a function of q and Eq. 26 gives $dq/d\tau$ as a function of q. Thus, the dependence of $dq/d\tau$ upon τ can be found graphically, e.g., Fig. 3 shows $\tau/\bar{\tau}$ and $dq/d(\tau/\bar{\tau})$ as functions of q for a system with I = 1 ft/day (0.30 m/day); W = 10 ft (3 m); L = 10 ft (3 m); H_d = 4 ft (1.2 m); d = 12 ft (3.7 m); K = 4 ft/day (1.2 m/day); θ = 0.15; and f = 0.25. The breakthrough time is 2.2 days and from Eq. 20, $\bar{\tau}$ = 4.3 days. Fig. 4 shows the relationship between $dq/d(\tau/\bar{\tau})$ and $\tau/\bar{\tau}$ determined graphically from Fig. 3. Assuming that the nitrate moves with the water, $dq/d\tau$ corresponds to the relative concentration, C/C_0, of a unit pulse input having a concentration of C_0. The shape of the

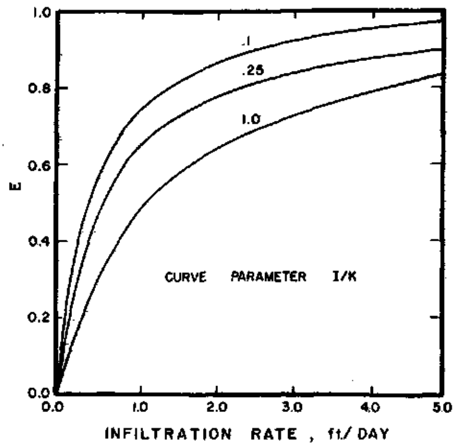

FIG. 6.—Relative Amount of Nitrate Peak Arriving at Drain, *E*, in 3.5-day Period in Relation to Infiltration Rate and *I*/*K* Ratio

nitrate input is usually not a square pulse but rather triangular or parabolic shaped, as shown in Fig. 2 for a field system. The input function can be approximated by a series of unit pulse inputs. The resulting output function is determined from the superposition of the individual transit time density distribution functions, e.g., a parabolic input function is approximated with a series of 0.01 day pulses with the resulting output shown in Fig. 5. The shape of the output function would not change as it moves from the edge of the recharge area to the drain when analyzed using the horizontal flow theory and ignoring longitudinal dispersion. Essentially all of the input nitrogen appears at the drain by $\tau/\bar{\tau}$ = 3.03 or 13 days.

The height and width of the nitrate peak depends on the concentration of nitrogen in the effluent and the amount of denitrification that occurs during the recharge process. The effluent containing nitrogen in the ammonium form is continuously applied to the soil until the oxygen is depleted and the cation exchange capacity is nearly saturated. During drying, oxygen enters the soil and the absorbed ammonium is nitrified. Part of the nitrate thus formed is then denitrified in micro-anerobic zones. The rest of the nitrate is leached during the next inundation. Laboratory and field studies have indicated that with a hydraulic loading rate of 270 ft/yr–400 ft/yr (90 m/yr–120 m/yr), approx 30% of the nitrogen can be removed by denitrification. The nitrate peaks in the reclaimed water can contain one to three times the nitrogen concentration of the wastewater (5,8). The duration of the nitrate pulse is shorter than the inundation period. Lowering the hydraulic loading to 200 ft/yr (60 m/yr) reduces nitrogen by 65%. Denitrification has been maximized in laboratory studies by recycling the high nitrate water back through soil columns (9). As much as 75% to 80% of the nitrogen was removed at higher loading rates by recycling the nitrate peak, which makes up approx 25% of the total amount of the water draining from the column. To detemine the feasibility of recycling the nitrate peak in a field system, the efficiency of recovery must be known. Efficiency of recovery

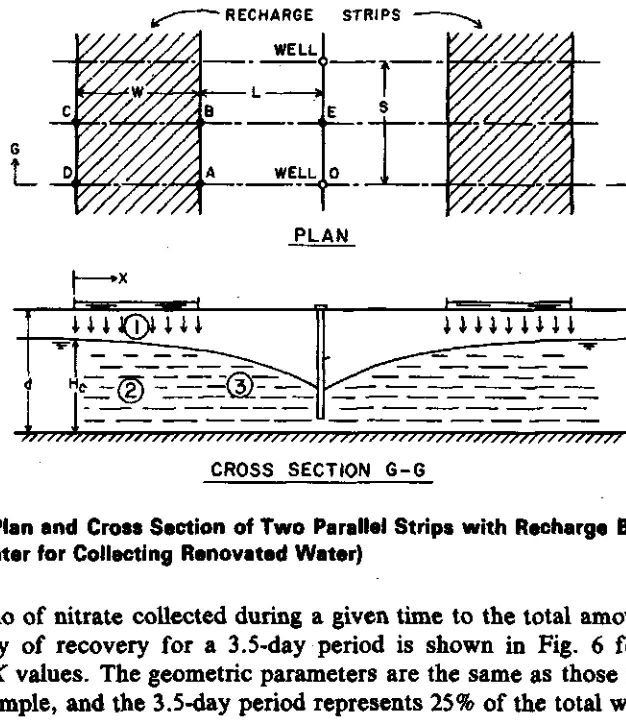

FIG. 7.—Plan and Cross Section of Two Parallel Strips with Recharge Basins (Wells are in Center for Collecting Renovated Water)

is the ratio of nitrate collected during a given time to the total amount applied. Efficiency of recovery for a 3.5-day period is shown in Fig. 6 for different I and I/K values. The geometric parameters are the same as those in the given drain example, and the 3.5-day period represents 25% of the total water applied in a 14-day flooding period. More than 60% of the nitrate is recovered for recycling when I/K is less than 0.6 and I is greater than 30 cm/day. A greater amount of nitrate can be recovered when I is increased or I/K decreases.

Deep Aquifers

Travel Time.—When the water table is relatively deep, the reclaimed wastewater can be collected from wells that are some distance from the recharge area. If the wells are midway between two parallel recharge areas (Fig. 7), the system can be managed so that the wells yield renovated water only and no renovated water moves into the aquifer outside the system (1,2). Again, the flow system can be divided into three zones. For zone 1, the unsaturated zone between the infiltration basins and the water table, and zone 2, the saturated zone below the water table under the recharge area, the analysis in the section on shallow water tables still applies. The flow in zone 3 between the edge of the recharge areas and the wells was previously analyzed by Bouwer using a resistance network analog (1). Results for different geometries are shown in Fig. 8. The equipotentials are expressed in percent head loss between the edge of the recharge basin, point A, and the well, point O. The well was simulated as a point sink, disregarding

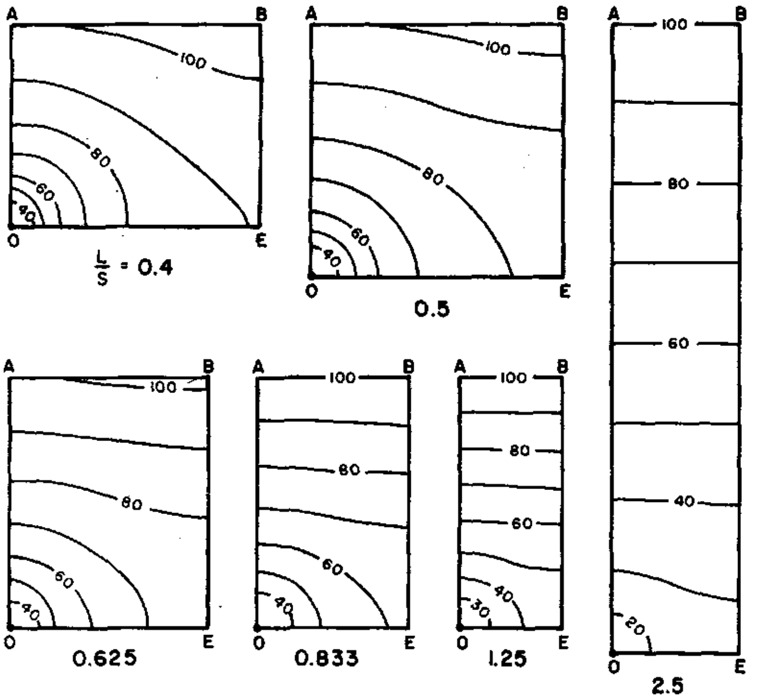

FIG. 8.—Equipotentials in Region ABEO of Fig. 7 for Different L/S Ratios (from Ref. 2)

the actual well diameter (1). The additional drawdown near the well can be calculated with standard well flow equations—see, e.g., Ref. 3. The travel time in zone 3 is determined from a flow system shown in Fig. 9 with a L/S ratio of 0.625 using the finite difference version of Eq. 1. To predict the travel time from line A to the well, point O, the macroscopic velocity, v, of each section of a stream tube is first determined by

$$v = \frac{F}{fa} \quad (27)$$

in which F = the flux; f = the soil porosity; and a = the cross-sectional area of the stream tube. The flux is the infiltration rate, I, multiplied by the recharge area, WL_r, feeding the stream tube:

$$F = IWL_r \quad (28)$$

in which W = width of the recharge basin and L_r = width of stream tube in recharge basin (Fig. 9). The value of I_r has been taken as a constant for each stream tube. The true cross-sectional flow area is the product of the width

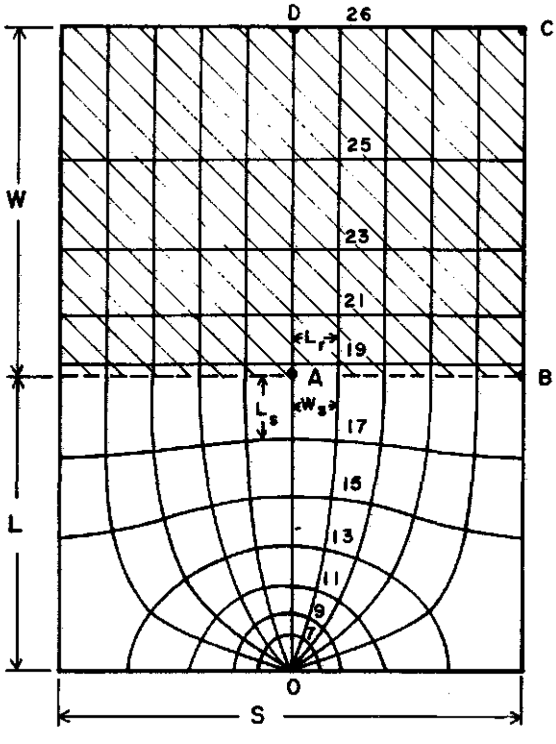

FIG. 9.—Streamlines and Equipotentials, above Water Table Adjacent to Well for Hypothetical System in feet (1 ft = 0.305 m)

of the stream tube, W_s, and the height of the aquifer, H_a. The macroscopic velocity is then

$$v_i = \frac{IWL_r}{H_a W_s f} \quad (29)$$

For a fully penetrating well, H_a is equal to the saturated height of the aquifer. For the recharge system, however, H_a is better defined as the effective height of the aquifer at the well, H_w, plus the total head, H_p. The term H_w is expressed by

$$H_w = \frac{T_e}{K} \quad \text{(30)}$$

in which T_e = the effective transmissivity of the aquifer ; and K = the horizontal hydraulic conductivity for the unconfined aquifers. The effective transmissivity is used rather than the total transmissivity because most of the flow occurs in the upper or "active" region of the aquifer, particularly when the thickness of the aquifer is large compared with W (1). The value of T_e around the well is greater than T_e for the recharge area because the well usually penetrates deeper than the active region for recharge (1). The velocity can now be expressed as

$$v_i = \frac{IWL_r}{fW_s\left(\frac{T_e}{K} + H_p\right)} \quad \text{(31)}$$

The total head, H_p, is the water table drop from the recharge area to the

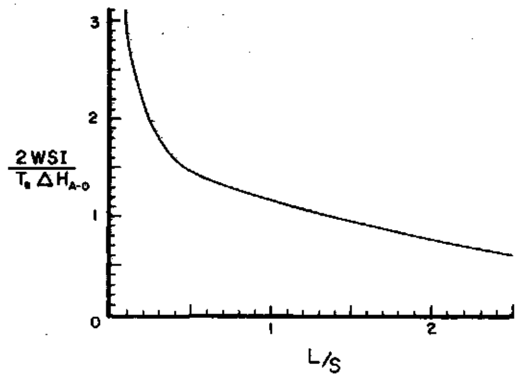

FIG. 10.—Dimensionless Graph of 2 $WSI/(T_e\,\Delta H_{A-O})$

well, ΔH_{A-O}. This relationship has been previously established with resistance network analog studies and is shown in Fig. 10 in terms of L/S and WSI/T_e, in which S = the well spacing, and L = the distance from the well to the recharge basin (1). The horizontal convergence to the well is assumed to be negligible. Along AB the water table is assumed to be horizontal, which is valid if $L/S > 0.5$. The value of H_p for a given equipotential line is then calculated from ΔH_{A-O}.

The travel time for each section, Δt_i, is found from the finite difference version of Eq. 11, or

$$\Delta t_i = \frac{\Delta s_i}{V_i} \quad \text{(32)}$$

Substituting Eq. 31 into Eq. 32 and, from Fig. 9, letting $\Delta s_i = L_s$ gives:

$$\Delta t_i = \frac{f L_{s_i} W_{s_i} \left(\dfrac{T_e}{K} + H_p\right)}{I W L_{r_i}} \quad \text{(33)}$$

The total travel time is the sum of the individual times for each section of the stream tube.

The breakthrough time, τ_b, is the travel time in the stream tube along AO plus the travel time in the unsaturated zone at $x = W$. An expression for the breakthrough time was developed because this represents the minimum

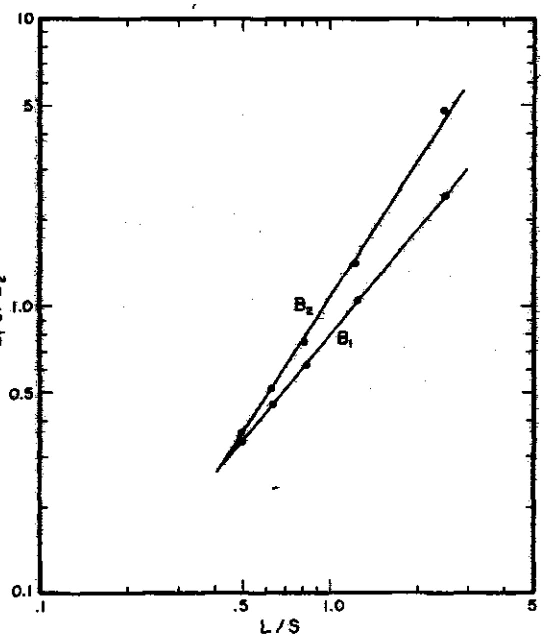

FIG. 11.—Constants B_1 and B_2 as Function of L/S

underground detention time. For a given L/S ratio, L_s, W_s, and L_r can be expressed in terms of S as

$$L_{s_i} = C_{1i} S; \quad W_{s_i} = C_{2i} S; \quad L_{r_i} = C_{3i} S \quad \text{(34)}$$

and from Fig. 10, H_p can be expressed as

$$H_{p_i} = \frac{C_{4i} WSI}{T_e} \quad \text{(35)}$$

in which C_{1i}, C_{2i}, C_{3i}, and C_{4i} are constants for a given stream tube section. Substituting Eqs. 34 and 35 into Eq. 33 and rearranging yields

$$\Delta t_i = \frac{fS}{IW} \frac{C_{1i} C_{2i}}{C_{3i}} \frac{T_e}{K} + f \frac{S^2}{T_e} \frac{C_{1i} C_{2i} C_{4i}}{C_{3i}} \quad \text{(36)}$$

The breakthrough time along the stream tube is the sum of the times for each section or

$$\tau_b = \sum_{i=1}^{n} \Delta t_i = B_1 \frac{T_e f S}{KIW} + B_2 \frac{f S^2}{T_e} \quad (37)$$

in which $B_1 = \sum_{i=1}^{n} \frac{C_{1i} C_{2i}}{C_{3i}}$ (38)

and $B_2 = \sum_{i=1}^{n} \frac{C_{1i} C_{2i} C_{4i}}{C_{3i}}$ (39)

The value of A and B was calculated from flow nets for L/S ratios between

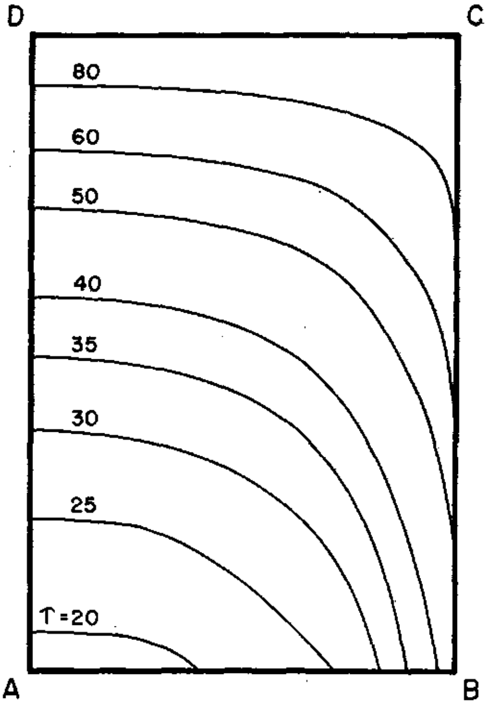

FIG. 12.—Contours of Equal Travel Time in Days for Recharge System Shown in Fig. 9

0.5 and 2.5 and plotted on Fig. 11, and can be expressed by

$$B_1 = 0.79 \left(\frac{L}{S}\right)^{1.21} \quad (40)$$

$$B_2 = 1.05 \left(\frac{L}{S}\right)^{1.57} \quad (41)$$

Substituting Eqs. 40 and 41 into Eq. 37 and adding the unsaturated flow time, τ_1, gives the breakthrough time

$$\tau_b = 0.79\left(\frac{L}{S}\right)^{1.21}\frac{fT_eS}{KIW} + 1.05\left(\frac{L}{S}\right)^{1.57}\frac{fS^2}{T_e} + \tau_1|\,x = W \quad (42)$$

The travel times for zones 1 and 2 are, given by Eqs. 14 and 15 respectively.

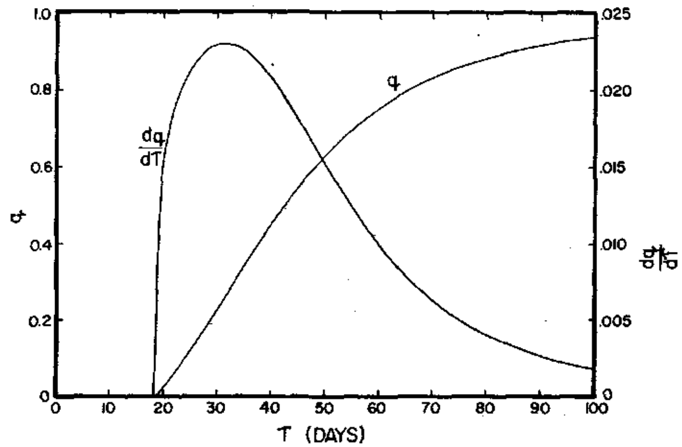

FIG. 13.—Cumulative Transit Time Distribution, q, and Transit Time Density Distribution, $dq/d\tau$, as Function of τ

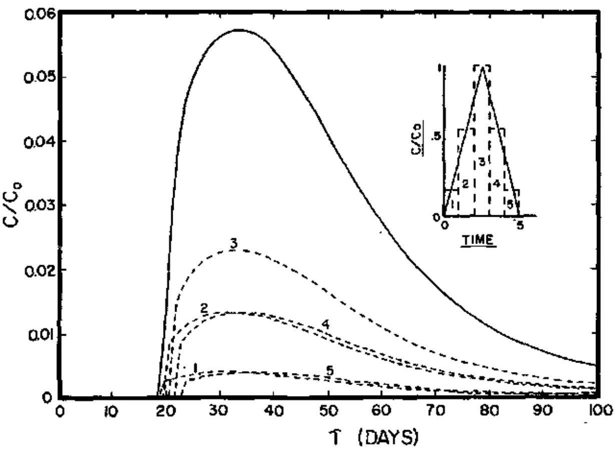

FIG. 14.—Relative Concentrations at Well, C/C_0, of Triangular Input Nitrate Peak [Numbered Curves Represent Corresponding Square Wave Approximation of Nitrate Peak (Inset)]

However, the expressions for H_c and H_e given by Eqs. 17 and 18 are no longer valid. The height H_e at the edge of the recharge area is now given by

$$H_e = H_w + \Delta H_{A-O} \qquad (43)$$

Evaluating Eq. 12 at the edge of the recharge area, solving for H_c, and eliminating H_e by using Eqs. 43 and 11 gives:

$$H_c = H_w + \Delta H_{A-O} = \frac{IW^2}{2T_r} \qquad (44)$$

Underground Movement of Attenuated Nitrate Peak.—The total travel time from any point in the recharge area to the well can be predicted from Eqs. 14 and 15, and the time calculated for the particular stream tube. Contours of equal time are then constructed as shown in Fig. 12. These contours are for the system shown in Fig. 12, where L = 500 ft (152.5 m), S = 800 ft (244 m), W = 600 ft (183 m), $d - H_c$ = 30 ft (9.1 m), T_e = 30,000 sq ft/day (2,790 m²/day), T_r = 20,000 sq ft/day (1,830 m²/day), K = 282 ft/day (86 m/day), I = 0.8 ft/day (0.24 m/day), θ = 0.10, and f = 0.15. From Fig. 10 for L/S of 0.625, then $(2WSI)/(T_e\Delta H_{A-O})$ = 1.3. Substituting then gives ΔH_{A-O} = 19.6 ft (6.0 m). The area between each contour and A-B divided by the total recharge area, ABCD, represents the cumulative transit time density distribution, q, for that particular contour time. The relationship between q and τ for the above system is shown in Fig. 13. The transit time density distribution, $dq/d\tau$, is the slope of this curve and is also shown in Fig. 13. It represents the attenuated output of a unit pulse input. The turnover time, $\bar{\tau}$, for this system is 52 days.

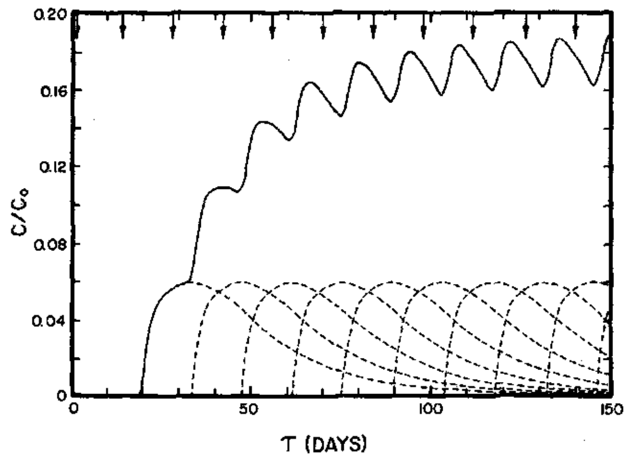

FIG. 15.—Relative Concentration of Nitrate at the Collection Well, C/C_0, after Several Inundation Periods (Arrows Represent Start of Inundation Period)

Using the superposition principle again, a triangular input can be approximated by a series of unit pulse inputs, as Fig. 14 shows. In this example, C_0 is the maximum concentration of the applied peak. Usually an intermittent inundation schedule should be followed, i.e., a flooding period followed by a dry period. Because the nitrate is leached out during the first portion of each infiltration period, a series of nitrate peaks will be superimposed on each other as Fig. 15 shows. In this example it is assumed that the area ABCD is divided into a number of small basins. Half of the basins are flooded and half are dry at any one time. At the end of a 14-day period the flooded and dry basins are interchanged. The steady-state solutions can then be applied with an input nitrate peak every 14 days. The nitrate peak is spread out into a fairly uniform concentration at the well, which varies from 14% to 20% of the input concentration. Recycling the nitrate peak would not be efficient in this case.

Summary and Conclusions

Equations were presented to predict the minimum underground detention time for ground-water recharge facilities utilized in wastewater reclamation systems. Horizontal flow theory was used to analyze shallow aquifers in which the reclaimed water is collected in drains. Flow nets were used to determine the detention times for deep aquifers in which the renovated water is collected from wells. Analysis of the underground flow system for deep aquifers showed that the nitrate peak, associated with the newly infiltrated wastewater, would be considerably flattened by the time the renovated water is collected at the well. Separating the portion of the renovated water that has a high nitrate content for special use or recycling through the infiltration basins to obtain more denitrification will not be practical. Analysis of the flow system in shallow aquifers, however, indicates that more than 60% of the nitrate could be recovered in 25% of the renovated water when the infiltration rate is greater than 1 ft/day (30 cm/day) and I/K is less than 0.6.

Appendix I.—References

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Appendix II.—Notation

The following symbols are used in this paper:

a = cross-sectional area;
B_1, B_2 = constants;
C/C_0 = relative concentration;
C_1, C_2, C_3, C_4 = constants;
d = depth to impermeable layer;
E = efficiency of nitrate recovery;
F = flow rate per unit width of recharge;
f = porosity;
H_a = effective height of aquifer;
H_c = h at center of infiltration strip;
H_d = h at drain;
H_e = h at edge of infiltration strip;
H_p = total head;
H_w = effective height of aquifer at well;
h = height of water table;
I = infiltration rate;
i = subscript for section i;
K = horizontal hydraulic conductivity;
L = distance between edge of recharge basin and well or drain;
L_r = width of strip feeding stream tube;
L_s = average length of stream tube section;
O = output;
P = input;
q = cumulative transit time density distribution;
S = distance between wells;
s = arc length parameter on streamline;
T_e = effective transmissivity of aquifer for well system;
T_r = effective transmissivity of aquifer for recharge;
t = time;
v = macroscopic velocity;
W = width of recharge area;
W_s = width of stream tube in section i;
x = horizontal distance;
θ = volumetric water content in unsaturated zone;
τ = transit time;
$\bar{\tau}$ = turnover time;

$\bar{\tau}_A$ = turnover time of apparently well-mixed system; and
$dq/d\tau$ = transit time density distribution.

Subscripts

0 = subscript for initial condition; and
1, 2, 3 = subscripts denoting zones 1, 2, and 3.

15912 UNDERGROUND TRAVEL OF RENOVATED WASTEWATER

KEY WORDS: **Convection;** Drainage; **Ground water; Groundwater flow;** Infiltration; **Land development;** Nitrates; Recharge; **Travel time; Wastewater**

ABSTRACT: Existing theory is used to decribe the underground movement of renovated wastewater. Equations are presented to predict the minimum underground detention time for ground-water recharge systems. Analysis of the flow in a deep aquifer system showed that a nitrate peak, associated with the newly infiltrated wastewater, would be considerably flattened by the time the renovated water reaches the collection well. In shallow aquifers, the degree of attenuation is much less and a distinct nitrate peak would show up at the collection drain.

REFERENCE: Rice, Robert C., and Raats, Peter, "Underground Travel of Renovated Wastewater," *Journal of the Environmental Engineering Division,* ASCE, Vol. 106, No. EE6, **Proc. Paper 15912**, December, 1980, pp. 1079-1098