A STUDY OF UPSTREAM BOUNDARY CONDITIONS FOR OVERLAND FLOW

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ABSTRACT

The solution of equations of continuity and motion for overland flow, whether analytical or numerical, requires specification of initial and boundary conditions. A laboratory experiment was carried out to evaluate the influence of upstream boundary conditions on overland flow, under rainfall. The experiments were conducted in an impermeable plane surface using a rainfall simulator. Results of this study show the importance of considering, for gentle slopes, other upstream boundary conditions rather than the most frequently used h(0,t)=0 for $t \ge 0$, where h(x,t) is the flow depth as function of position x and time t.

The method of characteristics is used to solve the kinematic wave equations for overland flow on an impermeable plane under time dependent upper boundary conditions. An application is worked out for a cascade of two planes. Typical characteristic contour plots and hydrographs of discharge and depth of flow are presented.

NOTATION

The following symbols are used in this report:

С	- constant	(m/s)
C1	- parameter	(m)
C2	- parameter	(-)
C3	- parameter	(m)
C4	- parameter	(s/m)
D	- backwater distance	(m)
Di	- domain i of characteristic map	(-)
f	- Darcy-Weisbach friction factor	(-)
h(x,t)	- flow depth function of x and t	(m)
L.	- lenght measured along the plane	(m)
L *	- horizontal length of application of rainfall	(m)
m	- parameter for the type of flow	(-)
Р.	- rainfall per unit area of the plane	(m∕s)
q	- volumetric water flux per unit plane width	(m ² /s)
Q	= discharge	(1/s)
Qo	- discharge at x-0	(l/s)
Q_{L}^{-}	- discharge at x-L	(1/s)
R	- effective rainfall rate	(m/s)
S	- slope gradient	(-)
t	= time	(s)
Т	- water temperature	(°C)
Τ _R	- duration of effective rainfall	(s)
ບົ	- mean overland flow velocity	(m/s)
х	- distance along the plane (x-0 is defined as the	
	upslope limit of the applied vertical rainfall)	(m)
α	- hydraulic coefficient	(-)
β	- surface tension contact angle	(degrees)

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1. INTRODUCTION

The solution of equations of continuity and motion for overland flow, whether analytical or numerical, require specification of initial and boundary conditions.

The upstream boundary condition most frequently used in overland flow modelling is h(0,t) = 0 for $t \ge 0$ (Fig. 1a), where h(x,t) is the flow depth as function of position x and time t. For vertical rainfall and for steep slopes and watersheds this condition is valid, but for moderate to gentle slopes its validity is questionable (Singh, 1978). Mathematical tractability is perhaps one reason for its use.

Robertson et al, 1966, Kilinc & Richardson, 1973, and many other researchers, assumed an overland flow profile as presented in Fig. la. Shen and Li, 1973, carried out experiments on overland flow over smooth surfaces caused by a constant base flow and constant rainfall They studied the effect of different boundary conditions (one rate. is shown in Fig. 1b, for a fully supercritical regime). They also concluded that the flow regime may change from supercritical to subcritical and vice versa under the influence of rainfall. Lima. 1989, studied the effect of oblique rainfall on the overland flow process and verified that the impact of inclined raindrops and the shear stress caused by wind (blowing up-slope) at the water surface may create a discharge at the upstream boundary of the plane (depending strongly on slope, rainfall intensity and wind speed). This implies the existence of a non-zero water depth at x-0 (Fig. 1c).

A laboratory experiment, described in Chapter 2, was carried out to evaluate the influence of upstream boundary conditions on the overland flow process for vertical rainfall. Case d (Fig. 1) was chosen for the laboratory set-up. It tries to represent saturation overland flow (Dunne & Leopold, 1978) where excess rainfall only begins at a certain distance from the top of the slope. The experiments were carried out on an impermeable plane under the conditions described in detail in Chapter 2. In fact 1d is a particular case of 1b (Shen and Li, 1973) with zero inflow discharge, Q_0 , at x-0.

Case d (Fig. 1) is rather difficult to model because: (1) resistance coefficients for steady uniform turbulent flow are not applicable; at x=0 Q₀ may be zero, but there exists a non-zero water depth; (2) surface tension phenomena are not negligible on small slope gradients (as observed in the laboratory experiments described in Chapter 2); (3) the initial flow is not uni-directional as water will also flow upslope at x=0; (4) rainfall is space dependent (upstream of x=0 rainfall intensity is zero); (5) raindrop impacts on the overland flow sheet further complicates the analysis. An empirical relation is suggested in Chapter 2.

In Chapter 3 the method of the characteristics is used to solve the kinematic wave equations for overland flow on an impermeable plane under time dependent upper boundary conditions. This application of the kinematic wave theory is best illustrated for a cascade of planes (Kibler and Woolhiser, 1970, 1972), where the discharge leaving the downstream boundary of one plane establishes

the upstream boundary for the next plane. An application is worked out for a cascade of two planes. Typical characteristic contour plots and hydrographs of discharge and depth of flow are presented.



Fig. 1 - Upper boundary conditions commonly used for overland flow.

2. LABORATORY EXPERIMENT ON IMPERVIOUS SURFACE UNDER CONDITIONS OF SIMULATED RAINFALL

2.1. INTRODUCTION

Under field conditions, overland flow is extremely complex and is not readily amenable to analytical treatment or description. In order to evaluate the influence of upstream boundary conditions a laboratory experiment was undertaken. In this study the following simplifications were introduced:

1. The applied rainfall was vertical, uniformly distributed, of constant intensity, and limited to a horizontal length L^* (along the inclined plane from x=0 to x=L; see Fig. 2).

2. Sufficient time had elapsed for the establishment of equilibrium conditions.

3. The surface was an impervious plane with uniform width and slope.

2.2. LABORATORY SET-UP

The laboratory experiment was carried out in the Hydraulic Laboratory of the Department of Hydraulics & Catchment Hydrology of the Agricultural University Wageningen.

The laboratory set-up (Fig. 2) is mainly composed of two units:

1. A channel, with an uniform rectangular cross section of 1.02 m width. The slope of the channel can be adjusted with the aid of a jack. The impermeable surface was smooth concrete.

2. The rainfall simulator consisted of circular plates with conveyors to the edges of the plates where the drops are formed. It generates a uniformly distributed and time invariant geometrical rainfall pattern. Drop shape differed significantly from the equilibrium raindrop shape at terminal velocity due to oscillations after release from the drop formers and small fall height. The equivalent diameter of the drops (defined as the diameter of a spherical drop with the same mass) was rather large at the drop formers (up to 8 mm). However, most simulated drops broke up in falling.



Fig. 2 - Layout of the experiment.

The rainfall intensity was measured with a flowmeter. At steady state the rainfall intensity was checked against a volumetric discharge measurement (i.e. a weighing drum and a stopwatch). At steady state, the water depth at L (h_L) and the backwater distance measured upwards from x-0 (D) were measured (Fig. 2). The experiment was repeated for different combinations of lengths (L^{*}= 3.27 and 4.72 m), slopes (S = 0.1, 0.2, 0.3, 0.4, 0.5, 0.7, 1 and 4%), and rainfall intensities (ranging from 0.0207 to 0.1528 mm/s).

2.3. RESULTS

Primary data collected at steady state for each simulation run (defined as the experiment conducted with fixed length, slope and rainfall intensity) were (Table 1): length (L), slope (S), rainfall intensity (P), water temperature (T), mean depth of flow at flume outlet (h_L), and backwater distance measured upwards from x-0 (D). The measurements were taken after a time long enough to reach equilibrium (steady state).

The overland flow sheet observed during the experimental runs, both with and without upstream splash barrier, could be divided into the following sections (Fig. 3a and 3b):

(1) Highly disturbed, downslope overland flow with direct impact of raindrops and of raindrop splash droplets. Injections of dye both in the raindrops and in the overland water sheet were rapidly dispersed.

(2a) Disturbed, mainly radial flow with small circular wave formation due to drop impact in section (1).

(2b) Disturbed, mainly radial flow with small circular wave formation due to drop impact in section (1). Direct impact of raindroplets also existed.

(3) Stagnant water (horizontal water surface). After injection of dye, no water movement observed (except gradual dye diffusion).

(4) Adhesive water. Surface tensions in the solid (bottom of the flume)-liquid interface with a certain contact angle β .

(5a) Wetted flume bottom (pre-wetted with a moisted cloth).

(5b) Wetted flume bottom with scattered water bubbles.

(6) Air dry flume bottom.

Rainfall intensity (P) plotted against distance D for a fixed slope (S) is strongly subjected to hysteresis due to surface tension effects on the flume bottom (Fig. 4). The measuring procedure was as follows: (1) start initial rainfall on dry surface; (2) wait until equilibrium (steady state) was reached; (3) measure primary data; (4) increase or decrease discharge and repeat steps (2) to (4). Hysteresis effect was observed to decrease with increasing slope gradient. The importance of surface tension and surface wettability characteristics with respect to the feasability of scale modelling of the rainfall - surface runoff process, on impermeable planes, was investigated by Graveto, 1970.

Table 1 - Primary data collected

		splash	run	Т	. • • •				
No.	date	barrier	type	(water)	L×	S	Q	h_L	D
				°C	 m		1/s	 mm	cm
1	30/3/88	Y	IN	15.6	4.72	0.04	0.3	3	0
2	30/3/88	Y	IN	15.6	4.72	0.04	0.51	4	0
3	30/3/88	Y	IN	15.6	4.72	0.01	0.1	3	0
4	30/3/88	Y	IN	15.6	4.72	0,01	0.25	4	0
5	30/3/88	Y	IN	15.6	4.72	0.01	0.50	5.5	0
6	30/3/88	Y	IN	15.6	4.72	0.002	0.1	4	10
7	30/3/88	Y	IN	15.6	4.72	0.002	0.2	5	14
8	30/3/88	Y	IN	15.6	4.72	0.002	0.3	6	30
9	30/3/88	Y	IN	15.6	4.72	0.002	0.4	6.5	43
10	30/3/88	Y	IN	15.6	4.72	0.002	0.51	7	54
11	30/3/88	Y	DE	15.6	4.72	0,002	0.4	6.5	52
12	30/3/88	Y	DE	15.6	4.72	0.002	0.3	6	52
13	30/3/88	Y	ĐE	15.6	4.72	0.002	0.2	5	52
14	30/3/88	Y	DE	15.6	4.72	0.002	0.1	4.5	52
16	31/3/88	Y	IN	15.7	4.72	0.003	0.1	4	0
17	31/3/88	Y	IN	15.7	4.72	0.003	0.2	4.5	5
18	31/3/88	Y	IN	15.7	4.72	0.003	0.3	5.5	9
19	31/3/88	Y	IN	15.7	4.72	0.003	0.4	6	15
20	31/3/88	Y	IN	15.7	4.72	0.003	0.51	6	35
21	31/3/88	Ŷ	DE	15.7	4.72	0.003	0.35	5.5	30
22	31/3/88	Y	DE	15.7	4.72	0.003	0.09	4	5
23	31/3/88	Y	IN	15.7	4.72	0.004	0.1	4	0
24	31/3/88	Y	IN	15.7	4.72	0.004	0.2	4.5	3.5
25	31/3/88	Y	IN	15.7	4.72	0.004	0.3	5	8
26	31/3/88	Y	IN	15./	4.72	0,004	0.4	5.5	13
27	31/3/88	Ŷ	IN	15./	4.72	0.004	0.51	6	15
28	31/3/88	Y	DE	15.7	4.72	0.004	0.3	5	14
29	31/3/88	Y	DE	15.7	4.72	0.004	0.1	4	5
30	31/3/88	Y V	IN	15./	4.72	0.005	0.1	4	0
31	31/3/88	Y V	IN	15./	4.72	0.005	0.2	4.5	3
32	31/3/00	I V	IN	15.7	4.72	0.005	0.3	2	0 7
32	31/3/88	I V	IN	15.7	4.72	0.005	0.4	2.2	/
24 25	31/3/00	ı v	IN DE	15./	4.72	0,005	0.51	0 6	12
30	31/3/00	I V		15.7	4.72	0.000	0.50	5	12
20 70	31/3/00	I V	DE TN	15.7	4.72	0.005	0.1	4 2 5	5
30	31/3/00	I V		15.7	4.72	0.007	0.1	5.5	25
20	31/3/00	v	TN	15 7	+./2	0.007	0.2	4 1, 5	2.5
72	31/3/00	v	TN	15 7	+./2	0.007	0.30	4.J 5	د.د ۲
40	31/3/00	$\dot{\mathbf{v}}$	TN	15 7	4.72	0.007	0.40	55	9
-+⊥ //0	31/3/00	v	DD 10	15.7	4.72	0.007	0.31	J.J 4 5	75
42	31/3/00	v ·	מע סע	15.7	4.72	0.007	0.30	4.J २ ६	2.2
43	21/3/00 K//./80	v	TN	15 Q	4.72	0.007	0.10	5.5	3. 1.4
44	0/4/00	1	ти	17.0	4.72	0.001	0.10	4.2	14 1

Table 1 - Primary data collected (cont.)

No.	date	splash barrier	run type	T (water)	L*	S	Q	հլ	D
				°C	m	-	1/s	mm	cm
45	6/4/88	Y	IN	15.8	4.72	0.001	0.20	5	99
46	6/4/88	Y	IN	15.8	4.72	0.001	0.30	6	238
47	6/4/88	Y	IN	15.8	4.72	0.001	0.4	7	325
48	6/4/88	Y	IN	15,8	4.72	0.001	0.51	8	426
49	6/4/88	Y	DE	15,8	4.72	0.001	0.3	7.5	424
50	6/4/88	Y	DE	15.8	4.72	0.001	0.1	5	424
51	7/4/88	N	IN	16.0	4.72	0.002	0.1	4	11
52	7/4/88	N	IN	16.0	4.72	0.002	0.2	5	18
53	7/4/88	N	IN	16.0	4.72	0.002	0.3	6	54
54	7/4/88	N	IN	16.0	4.72	0.002	0.4	6.5	66
55	7/4/88	N	IN	16.0	4.72	0.002	0.5	7	71
56	7/4/88	N	DE	16.0	4.72	0.002	0.3	6	70
57	7/4/88	N	DE	16.0	4.72	0.002	0.1	4.5	68
58	7/4/88	Y	IN	16.1	3.27	0.002	0.1	4	13
59	7/4/88	Y	IN	16.1	3.27	0.002	0.2	4.5	45
60	7/4/88	Y	IN	16.1	3.27	0.002	0.3	5	70
61	7/4/88	Y	IN	16.1	3.27	0.002	0.4	6	98
62	7/4/88	Y	IN	16.1	3.27	0.002	0.51	6.5	144
63	7/4/88	Y	DE	16.1	3.27	0.002	0.3	5	138
64	7/4/88	Y	DE	16.1	3.27	0.002	0.1	4.5	126
65	8/4/88	Y	IN	16.1	3.27	0.002	0.3	5	80

Remarks:

No. = experimental run number Date = day/month/year Splash barrier: Y = splash barrier installed at x=0 N = no splash barrier installed Run type: IN = run executed for increasing rainfall intensity DE = run executed for decreasing rainfall intensity

(See also Notation for list of symbols)









Fig. 4 - Hysteresis effect in the P-D relationship.

In Fig. 5 the effect of the splash barrier is illustrated. Removal of the splash barrier increased D.

In Fig. 6a, D was plotted against slope (S) in linear scales for different rainfall intensities (P). For a horizontal surface (S=0), D tends to infinity at steady state. In Fig. 6b, ln(D) is plotted against 1/S, for the same rainfall intensities. Linear relations have been fitted with high regression coefficients. Thus, for a certain rainfall intensity, the backwater effect (D) can be estimated as a function of S, by:

$$C_2/S$$

D = C₁ e (2.1)

where C_1 (m) and C_2 are parameters. C_2 can be considered independent of slope (S) and rainfall intensity (P) because of the approximately parallel lines in Fig. 6b. Values of C_1 and C_2 are presented in the Table of Fig. 6.



Fig. 5 - Effect of the splash barrier in the P-D relationship.

The plot of $ln(C_1)$ against the rainfall intensity (P) results in a straight-line relationship, defined by the equation (Fig. 7):

$$C_4 P$$

 $C_1 - C_3 e$ (2.2)

where C_3 (m) and C_4 (m/s) are parameters.



Fig. 6 - Relationship between D and S, for different rainfall intensities and with splash barrier: a) linear scales; b) ln(D) against 1/S.

Substituting eq. (2.2) into eq. (2.1) yields an expression for D (m) as a function of S and P (m/s):

$$(C_4P + C_2/S)$$

D - C₃ e (2.3)

The parameters C_2 , C_3 and C_4 can be calculated by regression techniques applied to laboratory data for each type of surface. In these experiments, for the smooth concrete surface, we have:

$$C_2 = 0.00458; C_3 = 0.00502 \text{ m}; C_4 = 22704.7 \text{ s/m}$$
 (2.4)

In Fig. 8, the Darcy-Weisbach friction factor (f) was plotted against D, for measurements with increasing rainfall intensities (P). The following expression was used to calculate the Darcy-Weisbach friction factor (See Notation for list of symbols and units):

A strong reduction of the friction factor (f) is observed for higher values of D, which is in accordance with an increase of the average water depth along the flume. In logarithmic scale the relation is approximately a straight line (Fig. 9). A scattered picture was found when plotting f against Reynolds number, Re.



Fig. 7 - Relationship between $ln(C_1)$ and P.





🗢 without splash barrier





Fig. 9 - The relationship between f and D using logarithmic scales.

3. KINEMATIC MODELLING OF OVERLAND FLOW USING THE METHOD OF CHARACTERISTICS

3.1 INTRODUCTION

The kinematic wave theory can be applied whenever the inflow, free surface slope and inertia terms are all negligible in comparison with those of bottom slope and friction. The friction slope is assumed to be equal to the bed slope. In the laboratory experiment described in Chapter 2, a region of horizontal water surface is created, upstream of x=0, producing therefore a region of invalidity in the solution given by the kinematic wave approximation.

However, the kinematic equations are adequate for general application to overland flow studies. The method of characteristics solution of kinematic wave problems is well known. Although application of this solution is subjected to the computational difficulty known as kinematic shock (Kibler & Woolhiser, 1972), the method has the advantage of giving an analytical solution. A kinematic analytical solution for overland flow under assumed time dependent upper boundary conditions is presented for constant rainfall rates. An example in given for a simple case of a cascade of two planes.

3.2 THEORY

3.2.1. BASIC EQUATIONS

The equation of continuity for shallow water flow may be written as (see Notation for list of symbols and units):

$$\partial h/\partial t + \partial q/\partial x = R$$
 (3.1)

By assuming that the bed slope equals the friction slope (kinematic wave assumption) and by using existing open-channel flow friction equations we can express the discharge at any point and time as a function of the water depth only, as follows:

$$q = \alpha h \tag{3.2}$$

By using the method of characteristics one can show that along the characteristics (Eagleson, 1970), where

$$m-1$$
 dx/dt - amh

(3.3)

we have:

$$dq/dx = R \tag{3.4}$$

$$dh/dt = R \tag{3.5}$$

$$\frac{m-1}{dq/dt = R \alpha mh}$$
(3.6)

$$\frac{m-1}{dh/dx - R/(\alpha mh)}$$
(3.7)

To solve overland flow on a sloping plane subject to a uniform effective rainfall rate, under time dependent upper boundary conditions, the following boundary conditions are assumed (Fig. 10):

$$\begin{cases} h(0,t) = C t \quad (variable) \qquad 0 \leq t \leq t^* \\ h(0,t) = C t^* \quad (constant) \qquad t^* \leq t \leq T_R \end{cases}$$
(3.8)
$$h(x,0) = 0 \qquad 0 \leq x \leq L \qquad (3.9)$$

where C, t^* , and T_R are constants.





3.2.2. CASE I - RAINFALL OF INFINITE DURATION $(T_R \rightarrow \infty)$

The characteristics in the solution domain are shown in Fig. 10. The solution domain can be partitioned into 3 domains: D_1 , D_2 , and D_3 . The solutions for these three domains, obtained by integration of equations 3.3 to 3.7 under the boundary conditions 3.8 and 3.9, are summarized here:

I.1 Domain D₁

Domain D_1 is bounded by t=0, x=L and the characteristic issuing from the origin: t=t(x,0). The solution is given by:

$$h = \left[\frac{R}{\alpha}(x - x_0)\right]$$
(3.10)

$$t - R \qquad [\frac{(1-m)}{\alpha}] \qquad (3.11)$$

where x_0 is a parameter representing the intersection of a characteristic with the x-axis (dotted line in Fig. 10).

Therefore, along such a characteristic:

$$h - Rt$$
 (3.12)

I.2 Domain D₂

Domain D₂ is bounded by x=0, x=L, the characteristic issuing from the origin: t=t(x,0), and the characteristic issuing from time t*: t=t(x,t*). The solution is given by:

$$h = [xR/\alpha + (Ct_0)]$$
(3.13)

$$t = \frac{[xR/\alpha + (Ct_0)] - (C-R)t_0}{R}$$
(3.14)

where $t_0(2)$ is a parameter representing the intersection of a characteristic with the t-axis (dotted line in Fig. 10).

Therefore, along such a characteristic:

$$h = Rt + (C-R)t_0$$
 (3.15)

I.3 Domain D3

Domain D₃ is bounded by x=0, x=L, and the characteristic issuing from t*: t=t(x,t*). The solution is given by:

$$h = \begin{bmatrix} xR & m 1/m \\ t & t \\ \alpha & t \\ \alpha & t \end{bmatrix}$$
(3.16)

$$t = \frac{m \ 1/m}{(Ct^*)} + t_0 \qquad (3.17)$$

Therefore, along such a characteristic:

 $h = R(t-t_0) + Ct*$

$$t_{0}(5)$$
 D_{4} D_{5} D_{4} D_{6} D_{7} D

(3.18)

Fig. 11 - Characteristic map for time dependent upper boundary conditions and rainfall of finite duration (T_R) .

3.2.3. CASE II - RAINFALL OF FINITE DURATION (T_R)

Depending upon the relative disposition of the boundary characteristic, t=t(x,0), the characteristic issuing from t₀-t*, t=t(x,t*), and the line t=T_R, many cases can be distinguished. Let us consider as an example the case where t=T_R does not intercept the two mentioned characteristics separating the three domains. The characteristics in the solution domain for this case are shown in Fig. 11. The solution domain can be partitioned into 5 sections: D₁, D₂,..., D₅. The solution for domains D₁, D₂, and D₃ is the same as

described in case I. For domains D_4 and D_5 , the following solutions are obtained:

II.4 Domain D₄

Domain D₄ is bounded by x-L, t-T_R and the characteristic issuing from t₀-T_R: t-t(x,T_R). The solution is given by:

$$h = [x_{04}R/\alpha + (Ct^*)]$$
(3.19)

$$t = \frac{(x - x_{04})}{\alpha m} \{ x_{04} R / \alpha + (Ct^*) \} + T_R$$
(3.20)

where x_{04} is a parameter representing the intersection of a characteristic t-t(x, x_{04}) with the line t-T_R.

I.5 Domain D₅

Domain D_5 is bounded by x=0, x=L and the characteristic issuing from T_R : t=t(x,T_R). The solution is given by:

$$h = Ct*$$
 (3.21)

and

$$t = \frac{x}{\alpha m} (Ct^*) + t_0$$
 (3.22)

3.3. APPLICATION - CASCADE OF TWO PLANES

Consider the following cascade of two planes (Fig. 12):



Plane 1: m = 1.5 $\alpha_1 = 5958 \text{ mm}^{1/2}/\text{min}$ $L_1 = 51 \text{ m}$ $R_1 = 2.73 \text{ mm/min}$ Plane 2: m = 1.5 $\alpha_2 = 9486.8 \text{ mm}^{1/2}/\text{min}$ $L_2 = 200 \text{ m}$ (3.26) $R_2 = 1.67 \text{ mm/min}$

The planes are assumed with unit widths; the rainfall stops at time $T_{\rm R}.$

The following domains have to be taken into account: D_{11} , D_{12} , D_{13} for plane 1, and D_{21} , D_{22} ,..., D_{25} for plane 2 (Fig. 13). The solution for plane 1 is given by equations 3.10 to 3.20 making C-0. For plane 2 the same equations are valid, assuming that, at x_1 -L₁ and x_2 -0, the following relations are true:

$$h_2 = (\alpha_1/\alpha_2) \quad h_1$$
 (3.27)

and 1/m $C = (\alpha_1/\alpha_2) R_1$ (C-2 mm/min in this example) (3.28)

However for domain D_{25} equations 3.21 and 3.22 are not valid because besides the ceasing of the rainfall we have to consider the recession limb of plane 1 (Figures 13, 14 and 15).

For plane 1 the solution for domain D_{13} is given by:

$$h_1 - (x_{012}R_1/\alpha_1)$$
 (3.29)

$$t = \frac{(x_1 - x_{012})}{\alpha_{1m}} \{ x_{012}R_1/\alpha_1 \} + T_R$$
(3.30)

where x_{01}^2 is a parameter representing the intersection of a characteristic t-t(x, x_{012}) with the line t-T_R.

The parameter x_{012} may be eliminated to give an explicit expression for h_1 as a function of x_1 and t.

$$t = \frac{x_1 \quad 1-m}{\alpha_1 m} \quad \frac{h_1}{R_1 m} + T_R$$
(3.31)

Equation 3.30 at x_1-L_1 in combination with equation 3.27, gives the values of h_2 for any time t_0 at x_2-0 for plane 2. Being h_2 constant along a characteristic, in domain D_{25} the solution is given by:

$$t = \frac{x_2}{\alpha_{2m}} h_2^{1-m} + t_{02}$$
(3.32)

Figure 13 shows the characteristic curves for the two slopes in the x-t plane. In this example no shock-wave formation occurs (Kibler and Woolhiser, 1970, 1972). Figure 14 and 15 sketch the depth of water and discharge hydrographs for various values of x.



Fig. 13 - Characteristic map for a cascade of two planes and rainfall of finite duration (T_R) .



Fig. 14 - System of hydrographs of depth of overland flow at various values of x, for the cascade of two planes.



Fig. 15 - System of hydrographs of discharge of overland flow at various values of x, for the cascade of two planes.

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4. CONCLUSIONS

1) In the laboratory work, under vertical rainfall, it was found that a body of nearly stagnant water upstream of x=0 was created for gentle slopes (less than 1%). That body of water led to the existence of a non-zero water depth at x=0, which consequently, radically changes the overland flow process over the slope. The form of the f-Re and f-D relations, which are of fundamental importance to the mathematical modelling of overland flow, are affected. So, for gently sloping impermeable plane surfaces under vertical rainfall, care should be take in using h(0,t)=0 for $t\ge 0$ as a upper boundary condition for overland flow. For oblique rainfall under wind blowing up-slope there also exists a non-zero water depth at x=0, depending strongly on slope, rainfall intensity and wind speed (Lima, 1989). Also in this case the condition h(0,t)=0 for $t \ge 0$ is no longer applicable.

2) As explained in the Introduction, the experiments undertaken in Chapter 2 are difficult to simulate with a conceptual model. So, an empirical formula relating the backwater effect (D) with the slope (S) and the rainfall intensity (P) was suggested (Eq. 2.3).

3) The role of surface tension, particularly for shallow flows shortly after the start of the rainfall requires more experimental attention. The hysteresis effect observed is caused by these surface tension forces. Additional research could be the study of the effects of roughness and surface wettability characteristics (using different surface materials or adding to the surface a wetting agent).

4) The method presented here using characteristics is of easy utilization and may serve as a test for finite difference schemes designed to solve more difficult problems. Furthermore, the method is free of any constraints on the space and time step used.

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ANNEX - Example of derivation of the equations of Chapter 3.

Domain D_2 (case I.2) was chosen as an example of derivation of the equations for the different domains. The reader can easily obtain the solution for the other domains by proceeding in a similar way.

Domain D_2 is bounded by x-0, x-L, the characteristic issuing from the origin: t-t(x,0), and the characteristic issuing from time t^* : $t-t(x,t^*)$. Integrating equation 3.5, gives:

$$h = \int_{0}^{t} R dt + K = R(t - t_0) + K$$
(A1)

where the constant of integration K follows from the boundary conditions: x=0, $t=t_0$ and $h(0,t_0) = Ct_0$ (equation 3.8). The value of K is:

$$\mathbf{K} = \mathbf{Ct}_{\mathbf{0}} \tag{A2}$$

Substituting equation A2 in A1, equation 3.15 is obtained:

$$h = Rt + (C-R)t_0$$
 (A3=3.15)

Integration of equation 3.3, yields:

$$\begin{array}{ccc} t & m-1 & x \\ \int \alpha & mh & dt - \int dx \\ t_0 & 0 \end{array}$$
 (A4)

Subsituting equation A3 in A4 and solving the integral, gives:

$$\frac{\alpha}{R} \{ [Rt + (C-R)t_0] - (Ct_0) \} = x$$
(A5)

and rearranging equation A5, equation 3.14 is obtained:

$$t = \frac{[xR/\alpha + (Ct_0)] - (C-R)t_0}{R}$$
(A6-3.14)

Substituting equation A3 into equation A5, t can be eliminated to give equation 3.13:

$$m \ 1/m \\ h = [xR/\alpha + (Ct_0)]$$
 (A7-3.13)