Development of a Stochastic Model of Rainfall for Radar Hydrology

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Contents

Scientific report

Title of project

Development of a stochastic model of rainfall for radar hydrology (Contract No. ENV4–CT96–5030).

Summary of results arising out of this research

- The arrival rate fluctuations of the raindrops which contribute most to rain rate and radar reflectivity behave, for moderate rain rates, according to Poisson statistics. This implies a rejection of the (multi-)fractal hypothesis for rainfall, at least at the raindrop scale. Hence, Poisson statistics may be used as a starting point for the development of a theory of sampling fluctuations in surface rainfall observations.

- The sampling distribution of the estimator of any rainfall integral variable converges to a Gaussian probability density function for large values of the expected sample size. The approach to normality is slower for the higher order moments of the raindrop size distribution. As a result of the asymmetry of the sampling distributions, the median always underestimates the population value of a rainfall integral variable. These results have important practical implications for the estimation of radar reflectivity – rain rate relationships from surface raindrop observations.

- Von Smoluchowski's (1916) stochastic model of density fluctuations for intermittent observations provides a reasonable first approximation of the *spatial* rainfall process at the droplet scale, i.e. the fluctuations in the rainfall integral variables in a given volume of air. Such a model may be used to investigate the additional amount of rainfall information which may be hidden in the reflectivity fluctuations of high resolution radar observations.

- A reasonable first approximation of the *temporal* rainfall process at the droplet scale (including both sampling fluctuations and natural variability) is provided by a doubly stochastic Poisson process (Cox process), where the rate process follows a log-transformed, normal (Gaussian), first-order autoregressive (AR(1)) process. Such a model may be used to determine over what time period surface rainfall observations should be accumulated to reduce the sampling fluctuations as much as possible without loosing an unacceptable amount of natural variability.

- A statistical model of the natural variability of the rainfall process provides a direct physical interpretation of the scaling exponents of a recently proposed general formulation for the raindrop size distribution as a scaling law. These exponents can be expressed in terms of the variances of and the covariances between the parameters of the raindrop count and size distributions. The values of the scaling exponents indicate whether it is the raindrop concentration or the characteristic raindrop size which controls the variability of the raindrop size distribution.

List of publications (with authors) and patents related to this research

- Porrà, J. M., R. Uijlenhoet, D. Sempere Torres and J.-D. Creutin (2000). Sampling effects in drop size distribution measurements: estimation of bulk rainfall variables. *Journal of the Atmospheric Sciences* (submitted).
- Serrar, S., G. Delrieu, J.-D. Creutin and R. Uijlenhoet (2000). Mountain reference technique: Use of mountain returns to calibrate weather radars operating at attenuating wavelengths. *Journal of Geophysical Research – Atmospheres* 105, 2281–2290.
- Uijlenhoet, R. (1999). Raindrop size distributions and the *Z-R* relationship. In I. D. Cluckie, editor, *Radar Hydrology for Real Time Flood Forecasting*, European Commission, Luxembourg (in press).
- Uijlenhoet, R. (2000). Raindrop size distributions and radar reflectivity-rain rate relationships for radar hydrology. *Hydrology and Earth System Sciences* (accepted for publication).
- Uijlenhoet, R. and J. N. M. Stricker (1999). Dependence of rainfall interception on drop size – a comment. *Journal of Hydrology* 217, 157–163.
- Uijlenhoet, R. and J. N. M. Stricker (1999). A consistent rainfall parameterization based on the exponential raindrop size distribution. *Journal of Hydrology* 218, 101–127.
- Uijlenhoet, R., J. N. M. Stricker, P. J. J. F. Torfs and J.-D. Creutin (1999). Towards a stochastic model of rainfall for radar hydrology: testing the Poisson homogeneity hypothesis. *Physics and Chemistry of the Earth (Part B)* 24, 747–755.
- Uijlenhoet, R., H. Andrieu, G. L. Austin, E. Baltas, M. Borga, M. Brilly, I. D. Cluckie, J.-D. Creutin, G. Delrieu, P. Deshons, S. Fattorelli, R. J. Griffith, P. Guarnieri, D. Han, M. Mimikou, M. Monai, J. M. Porrà, D. Sempere Torres and D. A. Spagni (1999). HYDROMET Integrated Radar Experiment (HIRE): experimental setup and first results. In Proceedings of the *29th International Conference on Radar Meteorology*, American Meteorological Society, Boston, 926–930.

Participation in conferences

- Fourth International Symposium on Hydrologic Applications of Weather Radar. San Diego, California, USA, 5–9 April 1998. Active participation (paper title: Uijlenhoet, R., G. Delrieu and J.-D. Creutin. Radar rainfall attenuation correction algorithms revisited).
- XXIII General Assembly of the European Geophysical Society. Nice, France, 20 – 24 April 1998. Active participation (paper title: Uijlenhoet, R. and J.-D. Creutin. A stochastic model of rainfall at the raindrop scale).

- Advanced Study Course on Radar Hydrology for Real Time Flood Forecasting. Water and Environment Management Research Centre, University of Bristol, UK, 24 June – 3 July 1998. Active participation (lecture title: Uijlenhoet, R. Raindrop size distributions and the Z–R relationship).

- Sixth International Conference on Precipitation: predictability of rainfall at the various scales. Mauna Lani Bay, Hawaii, USA, 29 June – 1 July 1998. Active participation (paper title: Uijlenhoet, R., J.-D. Creutin, J. N. M. Stricker and P. J. J. F. Torfs. Stochastic models of rainfall at the raindrop scale).

- Crues de la Normale a l'Extrême: Précipitations – Infiltrations – Ruissellements – Entraînements. Lyon, France, 10 – 11 March 1999. Passive participation.

- XXIV General Assembly of the European Geophysical Society. The Hague, The Netherlands, 19 – 23 April 1999. Active participation (chairperson of session NP1.03: Scaling vs. non-scaling methods in rainfall modelling; paper titles: (1) Uijlenhoet, R., J.-D. Creutin, J. N. M. Stricker and P. J. J. F. Torfs. Use of the variance function in rainfall downscaling; (2) Uijlenhoet, R., H. Andrieu, G. L. Austin, I. D. Cluckie, J.-D. Creutin, G. Delrieu, R. J. Griffith, J. M. Porrà and D. Sempere Torres. HYDROMET Integrated Radar Experiment (HIRE): experimental setup and first results; (3) Uijlenhoet, R., J. N. M. Stricker and H. R. A. Wessels. Physical interpretation and experimental validation of a scaling law for the raindrop size distribution; (4) Uijlenhoet, R., J. N. M. Stricker and J.-D. Creutin. Rainfall modelling in terms of raindrops: a review of scaling vs. non-scaling methods).

- 29th International Conference on Radar Meteorology. Montreal, Canada, 12 – 16 July 1999. Active participation (paper title: Uijlenhoet, R., H. Andrieu, G. L. Austin, E. Baltas, M. Borga, M. Brilly, I. D. Cluckie, J.-D. Creutin, G. Delrieu, P. Deshons, S. Fattorelli, R. J. Griffith, P. Guarnieri, D. Han, M. Mimikou, M. Monai, J. M. Porrà, D. Sempere Torres and D. A. Spagni (1999). HYDROMET Integrated Radar Experiment (HIRE): experimental setup and first results).

Assessment by the scientist in charge of the project

This fellowship gave an invaluable opportunity to the grant holder of making the synthesis of different studies made in his home institute. The main achievement of the grant holder was to put a considerable amount of existing experimental results into a coherent theoretical framework based on a new formulation of the microphysical properties of rain. He also visited via simple analytical solutions the implications of this theory on the concrete application of radar detection to hydrology. The grant holder brought us a considerable experience on the subject of rain microphysics and its hydrological applications. The fellowship allowed our laboratories in France and Holland to deepen existing collaboration.

Preface

This is the Final Report to the European Commission of the research I carried out as a Marie Curie Postdoctoral Fellow at the Institut National Polytechnique de Grenoble (INPG) in Grenoble, France. The research described in this report deals with *the development of a stochastic model of rainfall for radar hydrology.* The project has been carried out between 1 May 1997 and 30 April 1999 within the Equipe Hydrométéorologie of the Laboratoire d'étude des Transferts en Hydrologie et Environnement (LTHE). The scientist in charge of the project was Dr. Jean-Dominique Creutin, vice-director of LTHE and former head of the Equipe Hydrométéorologie.

First and foremost, I need to thank Dominique Creutin for providing me with the perfect environment to carry out my research and develop myself as an independent researcher. His guidance, both in matters of science and project management, has been extremely helpful. Secondly, I thank Guy Delrieu, with whom I shared an office. The many discussions we had have been very stimulating and have helped me to set my research priorities for the future. I also acknowledge the support of Michel Vauclin, director of LTHE, for having welcomed me as a researcher in 'his' laboratory. Odette Nave impeccably handled the administrative side of the project. My colleagues, both of the Equipe Hydrométéorologie and of the other research groups, have made me feel at home during my stay at LTHE. Merci à tous!

A particular acknowledgment goes to my Catalan colleagues and friends Josep M. (Pep) Porrà (formerly at the Departament de Física Fonamental, Universitat de Barcelona, Barcelona, Spain) and Daniel Sempere Torres (Departament d'Enginyeria Hidràulica, Marítima i Ambiental, Universitat Politècnica de Catalunya, Barcelona, Spain). The many discussions we had have truly shaped my mind. The shorter and longer visits to Barcelona over the past couple of years have been unforgettable.

Finally, I acknowledge the support of Prof. Ian Cluckie (Water and Environment Management Research Centre, University of Bristol, UK), coordinator of the HYDROMET Project. The association of my Marie Curie Fellowship with the HYDROMET Project has been very beneficial to me. In particular, having been the Coordinator of the HYDROMET Integrated Radar Experiment (HIRE) has been a rewarding experience. Moreover, it has provided a way of continuing the Dutch participation in European research on radar hydrology.

Grenoble, October 1999 — Remko Uijlenhoet

Chapter 1

Introduction

1.1 Background

Knowledge of the spatial and temporal variability of rainfall over a wide range of scales is indispensable in a variety of disciplines. Examples range from the study of hydrology, soil erosion and cloud and precipitation physics to the design and operation of water management, telecommunication and atmospheric remote sensing systems. The recent attention for the role of land surface processes in the climate system has stimulated research in this direction as well. The interest of this project is focussed on *radar hydrology*, more specifically on the application of ground-based weather radar for the estimation of rain rates on different spatial and temporal scales.

Accurate measurement and prediction of the spatial and temporal distribution of rainfall is a basic problem in hydrology because rainfall constitutes the main source of water for the terrestrial hydrological processes. As a result of the gradual development of radar technology over the past 50 years, ground-based weather radar is now finally becoming a tool for quantitative rainfall measurement. The advantages of ground-based weather radar over the traditionally used raingauge networks are: (1) they cover extended areas while measuring from a single point; (2) they allow rapid access for real-time hydrological applications; (3) their spatial and temporal resolution is generally high. Formerly, such results could only be achieved by very dense and therefore impractical raingauge networks. Potential areas of application of ground-based weather radar systems in operational hydrology include storm hazard assessment and flood forecasting, warning and control (Collier, 1989). The current attention for the role of land surface hydrological processes in the climate system has stimulated research into the spatial and temporal variability of rainfall as well. A potential area of application of ground-based weather radar in this context is the validation and verification of sub-grid rainfall parameterizations for atmospheric mesoscale models and general circulation models (Collier, 1993).

RADAR is the acronym for "RAdio Detection And Ranging". According to Battan (1973), radar can be defined as 'the art of detecting by means of radio echoes the presence of objects, determining their direction and range, recognizing their character and employing the data thus obtained'. The principle of radar remote sensing is based upon the transmission of a coded radio signal, the reception of a backscattered signal from the volume of interest and inferring the properties of the objects contained in that volume by comparing the transmitted and received signals. In the case of radar meteorology, the objects in the scattering volume are in principle hydrometeors (precipitation particles), although occasionally the ground surface may be detected as well. Hydrometeors can be raindrops, but snow flakes and ice crystals as well. The main interest in this report lies obviously in the raindrops.

A fundamental problem before radar derived rainfall amounts can be used for hydrological purposes is to make sure that they provide accurate and robust estimates of the spatially and temporally distributed rainfall amounts. The branch of hydrology dealing with this problem is now starting to be known as *radar hydrology*. The fundamental conversion associated with radar remote sensing of rainfall is that from the radar reflectivities measured aloft to rain rates at the ground. This so-called observer's problem is generally tackled in two main steps (e.g. Smith and Krajewski, 1993): (1) conversion of the reflectivity measured in the atmosphere to surface reflectivity; (2) conversion of surface reflectivity to rain rate. The exact manner in which these conversions are carried out will obviously affect the precision of the obtained radar rainfall estimates. Various aspects of the associated assumptions, errors and uncertainties are discussed among others by Battan (1973), Wilson and Brandes (1979), Sauvageot (1982), Doviak (1983), Zawadzki (1984), Clift (1985), Austin (1987), Joss and Waldvogel (1990), Jameson (1991), Andrieu et al. (1997) and Creutin et al. (1997). In this project, the main interest lies in the errors associated with the second conversion.

It has been common practice for more than half a century now (e.g. Marshall and Palmer, 1948) to take for the second of the mentioned conversions a simple power law relationship between radar reflectivity factor Z ($\mathrm{mm^6\,m^{-3}}$) and rain rate R ($\mathrm{mm\,h^{-1}}$), at attenuated wavelengths accompanied with a power law relationship between specific attenuation coefficient k ($\mathrm{dB\,km^{-1}}$) and rain rate. In an ideal situation, i.e. one in which all other possible error sources would be negligible, the main uncertainty in rainfall estimates by (conventional, i.e. single parameter) weather radar would be due to uncertainty in the Z–R relationship. In practice, this would mean a situation where a non-attenuated, pencil beam weather radar is observing nearby homogeneous rainfall close to the ground. In reality, these requirements are hardly ever met. Therefore, in any practical situation the uncertainty in the Z–R relationship will provide a lower bound to the uncertainties associated with radar rainfall estimation.

Establishing Z-R and to a lesser extent k-R relationships has captured the attention of radar meteorologists since the early days of weather radar more than five decades ago. From the point of view of instrumentation, there exist two approaches. The relationships are either calibrated in real time using simultaneous observations from a radar and a network of rain gages (e.g. Wilson and Brandes, 1979) or determined in advance on the basis of observations of raindrop size spectra obtained from disdrometers or optical spectrometers (e.g. Marshall and Palmer, 1948). In both cases it is, apart from errors directly related to the operation of the radar, the limited representativeness of the surface rainfall observations which affects the precision of the radar estimates of rainfall.

1.2 Rainfall measurement: a historical perspective

1.2.1 The raingauge era

Traditionally, information on the atmospheric component of the hydrological cycle has been gathered from raingauges. A basic problem with raingauges, however, is the fact that they are point measurements. This means that their limited spatial representativeness can only be increased indirectly, through temporal accumulation. Even then, the spatial representativeness of raingauges remains unclear, as it will depend on the dynamics of the rainfall process. Moreover, accumulation of raingauge measurements reduces their ability to capture the temporal structure of rainfall. This trade-off between spatial representativeness and temporal resolution is a fundamental problem associated with raingauges. Additional difficulties are related to all kinds of practical issues associated for instance with wind effects and maintenance.

The application of raingauges in networks has long been considered a solution to the problem. All kinds of procedures have been proposed over the years to interpolate spatially between the raingauges and fill in the gaps. However, the density of the network (in the form of the mean inter-gauge distance), together with the dynamical properties of the rainfall process (its characteristic advection velocity), dictate a lower limit to the temporal resolution of the spatially interpolated raingauge measurements. The result is that, from a hydrological point of view, most operational raingauge networks are too sparse to provide information on the rainfall process at a satisfactory spatial and temporal resolution. Denser networks, on the other hand, would generally be very impractical.

An additional problem is that even the most sophisticated spatial interpolation procedures (such as the geostatistical procedure know as kriging) generally lack the ability to capture the extreme rainfall variability found in nature. The interpolated rainfall fields are simply too smooth. Recent advances in (multi)fractal descriptions of rainfall fields may provide opportunities in this direction, although they will never be able to overcome the fundamental shortcomings of raingauges.

1.2.2 Weather radar

The remote sensing of rainfall using ground-based radar is a technology which has been in continuous development since World War II. It is currently reaching a state of maturity which renders its hydrological application a feasible enterprise. Radars can provide complete spatial and temporal coverage of an area from one single measurement site and as such they are ideally suited to hydrological applications.

Already since the early 1970s, attempts have been made to use weather radar to estimate the spatial and temporal distribution of rainfall. For almost three decades, radar has been a promise to hydrology. A promise however, which until recently it has not been able to keep. This has been due to both the material and the methods used at the time. First of all, most weather radars which have been used for hydrological applications until recently, were part of existing meteorological radar networks. These instruments were not designed with the hydrological application in mind. For instance, their spatial and temporal resolutions and sampling capabilities were generally insufficient. Secondly, the manner in which the radar data were used was generally not suited to the problem at hand. The hydrologists who were tackling the problem tried to avoid looking at the principle of radar measurements as much as possible.

During the 1980s, all kinds of more or less sophisticated statistical schemes were devised to combine the information from radars with that from networks of raingauges, the type of information hydrologists were used to working with (e.g. Collier, 1986a,b; Collier and Knowles, 1986; Krajewski, 1987; Creutin et al., 1988; Delrieu et al., 1988; Azimi-Zonooz et al., 1989; Seo et al., 1990a,b; Seo and Smith, 1991a,b; Smith, 1993b; Uijlenhoet et al., 1994, 1995, 1997). The idea was that raingauges were providing the 'ground truth' at various points in the area of interest. The radar data were then used in a sense to interpolate between the raingauges. Besides the fact that it remains to be seen to what extent raingauges represent the truth (since 'ground truth is the amount of rain that would have reached the ground if the raingauge had not been there'), the lack of attention for the principle of radar measurements proved to work counter-productive. After adjustment of the radar data using the raingauge measurements (erroneously called 'calibration' at the time), all kinds of errors and inconsistencies remained which this approach was not able to solve.

1.2.3 Radar hydrology

Since the early 1990s, hydrologists working on the problem of radar rainfall estimation have begun to take a different, more physical approach. They are revisiting the established theory of weather radar developed in the 1950s and 1960s by their meteorological and radar engineering colleagues. However, this is done using today's radar technology and, moreover, from a hydrological perspective. The objective is to apply ground-based weather radar to estimate the spatial and temporal distribution of rainfall at the ground.

As opposed to the largely statistical approach of the 1980s, the current physical approach considers the principle of radar measurements and the microstructure of rainfall in quite some detail. Another new aspect is that raingauges are no longer used to 'calibrate' the radar images, but for verification purposes only. This new approach, now starting to be known as *radar hydrology*, is currently starting to provide its first results (e.g. Smith et al., 1996a,b; Andrieu et al., 1997; Creutin et al., 1997; Sempere Torres et al., 1999a; Serrar et al., 2000; Uijlenhoet et al., 1999a). Radar is finally starting to redeem the promise it has been to hydrology for almost three decades.

1.3 Radar rainfall estimation: an overview

1.3.1 An inverse problem

Because radar is a remote sensing technique, it does not provide direct measurements of rainfall, but only indirect ones via the interaction with electromagnetic waves. Radar is a so-called active microwave technique, in which a radio signal with known properties (amplitude, frequency and polarization state) is sent into the scattering medium. In this case, the scattering medium is rainfall and the scatterers are raindrops. Part of the radio signal received by the raindrops is scattered back into the direction of the radar and received by its antenna. The difference between the properties of the transmitted and the received signal provides information on the dielectric properties of the scattering medium. It is the objective of radar hydrology to devise accurate and reliable methods to convert this information into rainfall rates at the ground for hydrological applications.

In order to be able to perform the conversion of the scattering properties of rainfall in the air into rainfall rates at the ground, some model of the microstructure of rainfall and its interaction with the radar signal has to be used. Since rainfall consists of individual raindrops with different sizes and hence different scattering properties, such a model should necessarily comprise a parameterization of the *raindrop size distribution*. The model should be simple, however, as one should be able to invert it on the basis of radar measurements.

More specifically, the number of model parameters should not exceed the number of variables estimated ('measured') by the radar system in question. Otherwise, the inversion problem would be underdetermined. The algorithms used to invert the model and estimate the model parameters on the basis of the available radar measurements are known as retrieval algorithms.

Conventional weather radars are able to estimate only one property of the backscattered signal, namely its mean power. This mean power is commonly expressed in terms of a so-called *radar reflectivity factor* Z. The inversion model to be used with such one-parameter radar systems is therefore necessarily a one-parameter model. The classical Z-R model provides a direct relationship between the radar reflectivity factor Z and the rainfall rate R. Because in reality there is much more uncertainty than can be captured in this one-parameter model, the Z-R model is necessarily statistical in nature. It is a regression model.

Over the past 20 years, ground-based radars have become capable of measuring, apart from the mean power, the Doppler and polarization properties of the backscattered signal as well. These multi-parameter radar systems have created the possibility of using inversion models with more than one parameter. Such models are able to capture more aspects of the microstructure of rainfall than the simple Z-R model. It is the hope that with such models, a larger fraction of the uncertainty is captured and that, as a result, the rainfall estimates become more reliable. The development of retrieval algorithms for multi-parameter radar systems is seen as a big challenge.

1.3.2 Fundamental problems

Up to this point, we have assumed that radars are able to measure the scattering properties of rainfall perfectly and that, as a consequence, the only remaining problem is the conversion of these properties to rainfall rates at the ground. This would imply that *uncertainty in the raindrop size distribution* would be the main error source. Nothing is less true, however. A series of additional problems remains to be tackled before the objective of radar hydrology can be considered achieved.

Perhaps the most fundamental problem of all is that of *calibration*. If a radar system is not well calibrated, then the measured powers do not correspond to true powers. This will introduce a bias in the radar power measurements which greatly affects the corresponding rainfall estimates. Hence, for hydrological applications, it is very important to have a well-calibrated radar system and to control its stability over time.

Additional problems associated with the quantitative use of weather radar can be more easily appreciated if the geometrical configuration of radar measurements is considered in some more detail. Although the radar antenna can in principle be pointed in any direction, the greatest spatial coverage can of course be obtained if it is used in a rotating fashion at a low elevation angle. This is the preferred configuration for operational meteorological and hydrological applications. In this configuration, the radar is providing the user with circular images with fixed resolutions in distance ('range' in radar terminology) and angle.

Although any subdivision of additional problems associated with the quantitative use of weather radar is necessarily arbitrary, we have made an attempt by identifying two classes of problems: (1) instrumental effects, i.e. effects associated purely with the principle of radar measurement; (2) environmental effects, i.e. effects associated with the interaction of the radar signal with its environment (the atmosphere and the ground).

Instrumental effects

A first instrumental effect in weather radar measurements is the range effect caused by the *spatial expansion* of the radar beam. This expansion, associated with the radar's fixed angular resolution, has the effect that the spatial resolution of the radar, both in the horizontal and in the vertical, decreases with range. Hence, the further away from the radar, the worse the spatial variability of the rainfall field is captured. An extreme example of this occurs when, at appreciable distances from the radar (say 100 km), the volume of the resolution cells becomes so large (of the order of a km^3) that situations of *partial beam filling* result. Unless corrected for, this may lead to serious underestimations of radar reflectivities and the corresponding rain rates.

Apart from its spatial expansion, it should be recognized that the weighting of the scatterers inside the radar beam is not done uniformly, neither in range nor in angle. In range, this *non-uniform weighting* is such that the centre of a range resolution cell receives more weight than the front or tail ends. In angle, the radar beam consists of a main lobe with several side lobes. Again, the centre of the resolution cell receives the heaviest weight. In conclusion, the reflectivity associated with a particular resolution cell is the convolution of the true spatial variability of the rainfall field at the cell's location with the radar's range and angular weighting functions.

An additional range effect can be associated with the fact that, unless the elevation angle of the radar is 0° (horizontal), the *height of the beam axis* increases with range. Hence, the further away from the radar, the less representative the measurements are for rain rates at ground level. A particular example of this range effect is the problem of *beam overshooting.* In this case, the radar beam completely overshoots the precipitation area of interest.

Environmental effects

The first environmental effect to be discussed here could just as well have been grouped under the instrumental effects in the previous section. It is the range effect associated with the *attenuation* of the radar signal as it propagates through the atmosphere, particularly at shorter wavelengths. Part of the radiation transmitted by the radar is absorbed or scattered (part of which back to the radar antenna) by the constituents of the atmosphere. Only a fraction of the total energy flux remains travelling away from the antenna. The problem with attenuation is that it is caused to a large extent by the very phenomenon radar hydrologists are interested in, rainfall itself. This renders attenuation a highly nonlinear effect which it is troublesome to correct for.

Other environmental effects are associated with the vertical structure of the atmosphere. For radar meteorological and hydrological purposes this vertical structure is often summarized in terms of a so-called *vertical profile of reflectivity,* the vertical profile of the radar reflectivity factor Z. Because melting snowflakes are seen by conventional (single parameter) weather radars as huge raindrops, the melting layer of precipitation (characteristic of stratiform conditions) causes a *bright band* on the radar screen. The problem with the vertical profile of reflectivity is that it is very difficult to correct for, because its actual structure at any location is unknown. It can have a strong spatial and temporal variability. Moreover, it generally causes a range effect due to the spatial expansion of the radar beam and its increasing height with range.

Even in the absence of precipitation (i.e. in clear air), the vertical structure of the atmosphere may influence the performance of radar systems. During favorable meteorological conditions (particularly temperature and water vapor inversions), the vertical profile of the refractive index of the atmosphere may be such that the electromagnetic waves transmitted by a radar are bent towards the earth's surface. In that case, we speak of *anomalous propagation*, or simply *anaprop*. As a result of anaprop, at some distance the radar signal hits the surface and causes a so-called *ground clutter*. Large errors in rain rate estimates result if these clutters are erroneously interpreted as rainfall.

Finally, it may be the earth's surface itself which causes problems. We have already encountered ground clutter as a result of anaprop. However, in mountainous terrain, ground clutter may even occur during standard propagation conditions. At the same time, the relief may cause partial or complete *obstruction of the radar beam*. Recent research has shown that, under particular conditions, ground clutter caused by relief may be used to estimate total attenuations and to test the stability of the radar calibration. This is one of the few occasions where it is advantageous to use radars in mountainous terrain.

1.4 Objectives of this project

At the spatial and temporal scales associated with rain gages and disdrometers, rainfall can no longer be considered a continuous process. Rather, it is a discrete process describing the arrival of raindrops of different sizes and fall speeds at the ground. As a result, observed rain rate fluctuations are partly caused by actual changes in the meteorological character of the rainfall process and partly by random changes in the arrival rate of raindrops dictated by the laws of chance. The terminology generally adopted for these types of fluctuations are *natural variability* and *sampling fluctuations*, respectively.

In hydrological applications, sampling fluctuations are generally disregarded because the spatial and temporal scales at which these fluctuations play a role are thought to be insignificant as compared to the characteristic scales of typical hydrological processes such as rainfall-runoff transformations. However, it would be of practical importance to be able to distinguish between both sources of variability, because the coefficients of Z-R and k-R relationships should represent the properties of the type of rainfall to which they pertain as much as possible and the properties of the raindrop sampling device from which they are derived as little as possible. A stochastic model of rainfall which explicitly considers discrete nature of rainfall may be employed to investigate over what time period surface rainfall observations need to be aggregated in order to reduce the sampling fluctuations to an acceptable level without loosing an unacceptable amount of information regarding natural variability.

The objective of this project has been to develop a stochastic model of rainfall to investigate to what extent rainfall fluctuations observed with different types of instruments reflect the properties of the rainfall process itself and to what extent they are merely instrumental artefacts. Knowledge of the relative contributions to these fluctuations may lead to improved Z-R and k-R relationships and ultimately to improved radar rainfall estimates. Moreover, a stochastic model such as the one proposed will provide a more realistic framework for simulation studies dealing with problems in radar hydrology (e.g. Chandrasekar and Bringi, 1987; Krajewski et al., 1993).

1.5 Rainfall microstructure

1.5.1 A static picture of rainfall

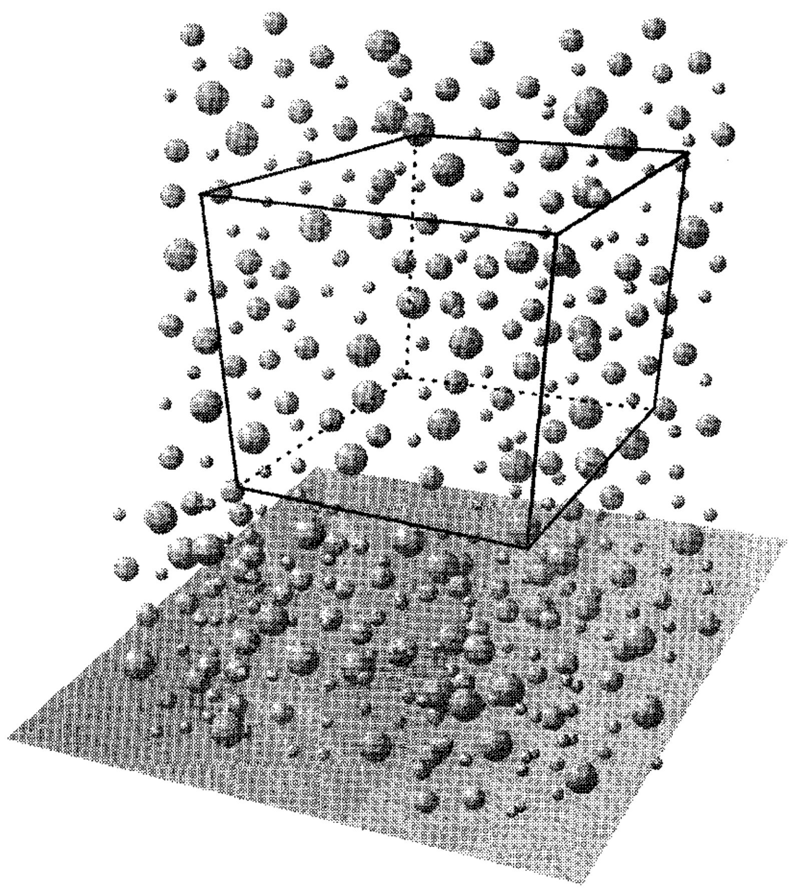

Figure 1.1: Schematic representation of the subject of this report: the spatial distribution of raindrops in a volume of air and the distribution of their sizes (Courtesy of J. M. Porrà).

An example of a more detailed description of the microstructure of rainfall is provided by Fig. 1.1. Although it is merely a schematic representation of reality, it serves to show some of the features of the microstructure of rainfall which are relevant to this research project. First of all, although the raindrops are distributed homogeneously in space *on the average*, their local concentration is not everywhere the same. For a volume of a given size, the numbers of raindrops it contains will therefore fluctuate in space and in time. On the average, 1 m^3 of air typically contains of the order of 10^3 raindrops. Closely related to the numbers of raindrops in a volume of air are the distances between them. Again, these will be subject to statistical fluctuations, but a typical mean distance would be of the order of 10 cm. A third and very prominent feature is that raindrops have different sizes. Their diameters range typically from 0.1 to 6 mm. Although Fig. 1.1 does not show this very clearly, in reality there are many more small raindrops than large ones. The majority of the raindrops encountered in nature are smaller than 3 mm (e.g. Rogers and Yau, 1996).

A fundamental property of rainfall in this respect is its so-called *raindrop size distribution*. In its traditional definition, it represents the expected (mean) number of raindrops per unit of raindrop diameter interval and per unit volume of air. According to this definition, the notion of a raindrop size distribution is a mixture of two different concepts, namely that of the spatial distribution of raindrops in a volume of air (which governs the raindrop concentration) and that of the probability distribution of their sizes. A fundamental but seldom explicitly mentioned hypothesis with regard to the existence of the raindrop size distribution is that it is independent of the size of the reference volume under consideration. This assumes a certain amount of spatial *homogeneity* and temporal *stationarity* of the rainfall process. See Porrà et al. (1998) for a review of the hypotheses on which the concept of the raindrop size distribution is based.

A comparison of the definition of the raindrop size distribution with Fig. 1.1 shows that it is in fact a *parameterization* of the actual microstructure of rainfall within the reference volume. Its definition neglects the exact numbers, positions and sizes of the individual raindrops in the reference volume and merely provides an idea of the *average* conditions. The minimum spatial scale for which it can be considered an accurate representation of the *instantaneous* conditions is the scale for which the field approximation of rainfall breaks down. This *representative elementary volume* would roughly be a few tens of cubic meters. According to Orlanski's (1975) rational subdivision of scales for atmospheric processes, this corresponds to the micro-γ scale.

With regard to the shapes of raindrops, those in the figure are perfect spheres. This is a very good approximation to their true shapes. Only raindrops larger than 2 mm deviate significantly (i.e. more than 10%) from the perfect spherical shape. In contrast to common belief, these larger raindrops do not have 'teardrop' shapes, but more closely resemble oblate spheroids (Pruppacher and Pitter, 1971; Pruppacher and Klett, 1978; Beard and Chuang, 1987). Therefore, the raindrop diameter D actually represents an *equivalent spherical* raindrop diameter, i.e. the diameter of a sphere with the same volume as that of the raindrop under consideration. Here, raindrops will be assumed perfect spheres. This has the additional advantage that the influence of wind and turbulence on the orientation of raindrops ('canting') (e.g. Brussaard, 1974; 1976) does not have to be considered.

1.5.2 A dynamic picture of rainfall

Fig. 1.1 provides a rather static picture of rainfall, in the sense that it suggests that the raindrops are not moving. However, nothing is less true. In still air, raindrops have terminal fall speeds which range from about 0.1 $\mathrm{m\,s^{-1}}$ for the smallest raindrops to more than 9 $\mathrm{m\,s^{-1}}$ for the largest raindrops. At altitudes well above sea level, the fall speeds tend to be somewhat higher (e.g. Foote and du Toit, 1969; Beard, 1976). However, in practical situations this effect of air density is likely to be small compared to the influence of wind (updrafts, downdrafts), turbulence and raindrop collisions.

Consider the flux of raindrops through part of the bottom of the reference volume indicated in Fig. 1.1. If the corresponding rain rates would be calculated on the basis of the volumes of the raindrops which pass that surface during subsequent time intervals of one second, then the resulting time series of rain rates might look like that provided by Fig. 1.2. This is actually a time series of rain rates with a temporal resolution of 1 s collected using a capacitor type raingauge with a surface area of 730 $\mathrm{cm^2}$ (Semplak and Turrin, 1969). For reference, a line corresponding to the 20 s moving average has been indicated in the figure.

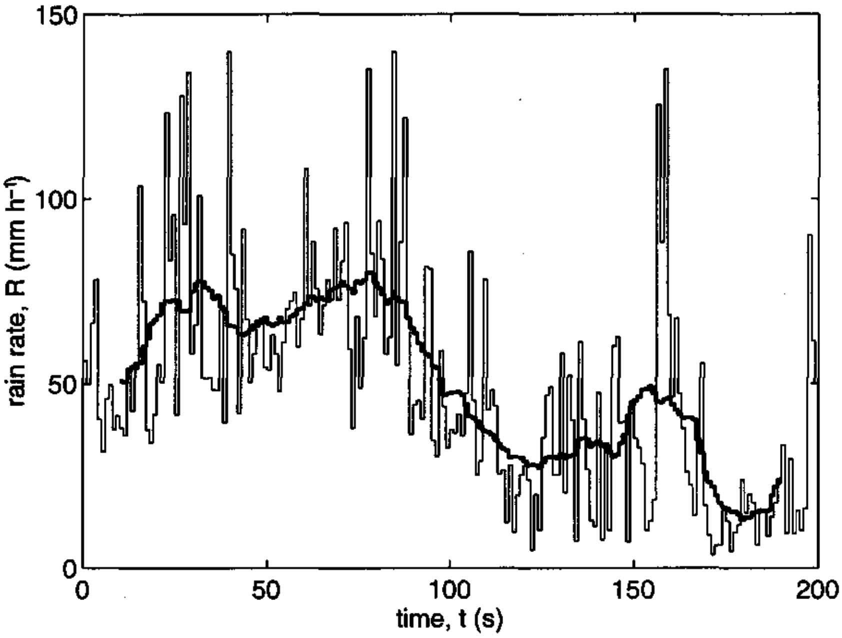

Figure 1.2: Thin line indicates 200 s time series of 1 s mean rain rates collected with a 730 cm^2 capacitor type raingauge at Bell Laboratories, New Jersey on July 21st, 1967 (Semplak and Turrin, 1969). Bold line indicates 20 s moving average.

It will be clear that at least part of the fluctuations in the 1 s observations about the 20 s moving average must have been caused by purely random fluctuations in the numbers and sizes of the raindrops arriving at the raingauge. Note that there are rain rate differences from one second to the next of close to 100 $\mathrm{mm\,h^{-1}}$. The arrival of only one 6 mm raindrop at the raingauge during a 1 s time interval would already produce a mean rain rate of 5.6 $\mathrm{mm\,h^{-1}}$. Hence, the arrival of only a few large raindrops is able to cause the extreme rain rate differences observed at this time scale.

This is an example of a time scale for which the field approximation of rainfall breaks down (Rodriguez-Iturbe et al., 1984; Fabry, 1996). As a result, the observed rain rate fluctuations must be due 'both to statistical sampling errors and to real fine-scale physical variations which are not readily separable from the statistical ones' (Gertzman and Atlas, 1977). The terminology generally adopted for these two types of fluctuations is *sampling fluctuations* and *natural variability*, respectively. In this case, the 20 s moving average may be considered a first rough estimate of the natural variability for the considered time series and the deviations from this moving average consequently as an estimate of the sampling variability. Results of applications of (multi)fractal analysis techniques to study the fluctuations in rain rate time series with comparable resolutions (e.g. Rodriguez-Iturbe et al., 1989; Rodriguez-Iturbe, 1991; Georgakakos et al., 1994) should therefore be interpreted with care.

1.6 Summary of results

1.6.1 Application to the study of sampling fluctuations

The starting point of this research project has been the marked point process model of rainfall developed by Smith (1993). An adapted version of this mathematical model provides a convenient framework for generalizing previous work on sampling fluctuations in rainfall observations (e.g. Sasyo, 1965; Cornford, 1967; Cornford, 1968; Joss and Waldvogel, 1969; de Bruin, 1977; Gertzman and Atlas, 1977; Stow and Jones, 1981; Wirth et al., 1983; Wong and Chidambaram, 1985; Chandrasekar and Bringi, 1987; Hosking and Stow, 1987; Chandrasekar and Gori, 1991; Smith et al., 1993; Bardsley, 1995). Based on the assumptions made by these investigators regarding the counts and sizes of the raindrops, exact analytical solutions have been obtained for all moments of the univariate and bivariate sampling distributions of rainfall integral parameters (such as Z, k and R). These moments have been employed to obtain approximative series expansions for the corresponding probability density and distribution functions. In some cases it has even been possible to obtain the exact functional forms. Similar results have been obtained for the sampling distributions of related properties such as the mean and the variance of the raindrop size and the minimum and maximum raindrop sizes encountered in a sample. Analytical solutions to sampling distributions will allow hydrologists to provide all kinds of parameters derived from surface rainfall measurements with confidence limits representing the expected magnitude of the fluctuations resulting from instrumental considerations. These confidence intervals will be very useful in separating sampling fluctuations from natural rainfall variability.

1.6.2 Application to the study of natural variability

Smith and Krajewski (1993) have demonstrated that, in the absence of sampling fluctuations, the coefficients of power law Z-R relationships can be expressed in terms of the variances of and the covariances between the parameters of the probability distributions of raindrop count and size. It has been demonstrated that their approach can be extended to any pair of rainfall integral parameters, provided these parameters are proportional to moments of the raindrop size distribution (which is approximately the case for Z, k and R). In this manner, statistical power law relationships have been obtained which are in a sense complementary to the deterministic power laws pioneered by Atlas and Ulbrich (1974) and Ulbrich and Atlas (1978). The statistical approach, however, does not only lead to theoretical expressions for the coefficients of power law relationships between rainfall integral parameters, but to theoretical expressions for the goodness of fit of such power laws as well. Moreover, this approach can be readily extended to include higher order power laws, such as double and triple power laws. In this manner, theoretical expressions for the coefficients of power laws used in multi-parameter radar observation of rainfall have been derived. Another interesting feature of the developed approach is that it provides a theoretical confirmation of the scaling law formulation for the raindrop size distribution as proposed by Sempere-Torres et al. (1994, 1998). It provides a direct physical interpretation of the scaling exponents in terms of the variances of and the covariances between the parameters of the raindrop count and size distributions.

1.6.3 Innovative aspects

- The description of rainfall in terms of a stochastic point process where the points represent the arrivals of individual raindrops is new and will be of great importance in separating instrumental artefacts from the actual rainfall properties hydrologists are interested in. The point process framework for modeling the phenomenology of rainfall at the ground has a long history in hydrology (e.g. Le Cam, 1961; Waymire and Gupta, 1981a,b,c; Smith and Karr, 1983; Rodriguez-Iturbe et al., 1984; Waymire et al., 1984; Smith and Karr, 1985; Rodriguez-Iturbe, 1986; Rodriguez-Iturbe et al., 1986; Rodriguez-Iturbe and Eagleson, 1987; Rodriguez-Iturbe et al., 1987; Smith, 1987; Rodriguez-Iturbe et al., 1988). In all previous models, however, the points have been considered to represent the arrivals of entire rainfall events rather than individual raindrops;
- Capturing the influence of natural rainfall variability on the coefficients of relationships between rainfall integral parameters in terms of a consistent set of statistical rather than deterministic power laws is a novel approach as well;
- The proposed model of rainfall will provide a theoretical interpretation of the experimentally based scaling law formulation for the raindrop size distribution as proposed by Sempere Torres et al. (1994, 1998).

Chapter 2

Detailed description of results

2.1 Introduction

Detailed knowledge of the microstructure of precipitation is important both from a fundamental and from an applied point of view. The spatial and temporal distributions of precipitation particles (hydrometeors) determine the manner in which the concept of a particle size distribution should be interpreted (e.g. Jameson and Kostinski, 1998), have important implications for the microphysical processes involving interactions between hydrometeors (e.g. Jameson and Kostinski, 1999d) and strongly influence the sampling characteristics of both in situ and remote sensing measurement devices (e.g. Jameson and Kostinski, 1999a; Kostinski and Jameson, 1999b).

With regard to raindrops (but the same holds for cloud droplets), the classical hypothesis is that they behave according to Poisson statistics, i.e. that they are as homogeneously distributed in space and time as randomness allows. This hypothesis forms the basis of the classical sampling theory of in situ rainfall observations (e.g. Cornford, 1967, 1968; Joss and Waldvogel, 1969; Gertzman and Atlas, 1977; Smith et al., 1993) and can be considered one of the cornerstones of the classical theory of weather radar (Marshall and Hitschfeld, 1953; Wallace, 1953). Although rainfall observations may occasionally behave according to Poisson statistics during rare periods of exceptional stationarity (see Kostinski and Jameson (1997) and Uijlenhoet et al. (1999b) for experimental evidence), it becomes now more and more clear that rainfall exhibits significant spatial and temporal drop clustering. Since the homogeneous Poisson process is not able to cope with these types of clustering, a more versatile description of raindrop statistics is needed.

There exist basically two "schools" with regard to tackling this problem. The first consists of those who propose to generalize the restrictive homogeneous Poisson process (which has a constant mean) to a Poisson process with a randomly varying rate of occurrence, i.e. a so-called doubly stochastic Poisson process or Cox process (see e.g. Cox and Isham (1990) for a summary of the properties of this type of stochastic point process). This approach has been pioneered by Sasyo (1965) and has later been applied by Smith (1993). More recently, it has been put in an entirely new perspective in a remarkable series of articles by Kostinski and Jameson (1997, 1999a), Jameson and Kostinski (1998, 1999b,c) and Jameson et al. (1999). The second school consists of those who propose to abandon the Poisson process framework altogether and replace it with a (multi-)fractal approach. Examples of the latter are Lovejoy and Schertzer's (1990) analysis of the spatial distribution of raindrops and

Zawadzki's (1995) and Lavergnat and Golé's (1998) analyses of the temporal distribution of raindrops.

The cited doubly stochastic Poisson process models tend to produce clustering of raindrops on certain distinct, predefined spatial and/or temporal scales. The implications of this type of rainfall behavior for sample-to-sample radar echo fluctuations are discussed by Jameson and Kostinski (1999b). (Multi-)fractal processes on the other hand are associated with clustering of raindrops on *all* scales. If rainfall would indeed exhibit such a strong clustering behavior, the implications for radar remote sensing of rainfall would be profound, as pointed out by Lovejoy and Schertzer (1990). For instance, there would no longer be a simple proportionality between the expected number of raindrops in a radar sample volume and the size of that sample volume. Due to increased coherent scattering, it would affect the sample-to-sample echo fluctuations as well. In short, it would essentially be necessary to revise the currently accepted theory of weather radar.

2.2 Experimental verification of the Poisson homogeneity hypothesis in stationary rainfall

Due to the profound implications for radar rainfall estimation and rainfall sampling theory, it is important to investigate experimentally whether the raindrop arrival process at the ground can at times be considered a homogeneous Poisson process or whether it systematically exhibits clustering (or possibly even scaling) behavior. Kostinski and Jameson (1997) find indications for Poisson behavior during 'a time of unusually constant flux'. The same authors argue that 'evidence of nonclustering, Poissonian structure conflicts with any ubiquitous fractal description of rain' (Jameson and Kostinski, 1998). It would not conflict with the doubly stochastic Poisson process description of rain, however. The latter namely contains the homogeneous Poisson process as a limiting case. In view of these arguments, this section reports on the analysis of an exceptionally stationary dataset with mostly sampling fluctuations and very little natural variability. Acceptation of the Poisson homogeneity hypothesis would then automatically imply a rejection of the (multi-)fractal hypothesis (at least at the raindrop scale).

The available dataset consists of raindrop counts in 16 diameter intervals of 0.21 mm width for 1066 consecutive time intervals of about 10 s duration, i.e. almost 3 h in total. The data have been collected as part of the NERC Special Topic HYREX, a large hydrological radar experiment organized in the United Kingdom, at the Bridge Farm Orchard site on 14 February 1995. The instrument used is an Illingworth-Stevens Paired-Pulse Optical Disdrometer, which has an area presented to the rain of 50 cm^2 (Illingworth and Stevens, 1987). Rain rates calculated using the observed raindrop counts vary from 0 to 9 $\mathrm{mm\,h^{-1}}$. The average wind speed during the event amounts approximately 3 $\mathrm{m\,s^{-1}}$.

As mentioned in the previous chapter, observed rain rate fluctuations are caused both by sampling fluctuations and by natural variability, which are not readily separable from each other. That is why experimental studies intended to test the Poisson homogeneity hypothesis in rain are often bound to fail. Unless of course there are strong indications that the amount of natural variability present in a particular time series is negligible as compared to the amount of sampling fluctuations. This rare situation happens to be the case in the dataset at hand during a period of 35 min. This period contains 210 consecutive 10 s raindrop size distributions (comprising a total of 6281 raindrops) and is roughly

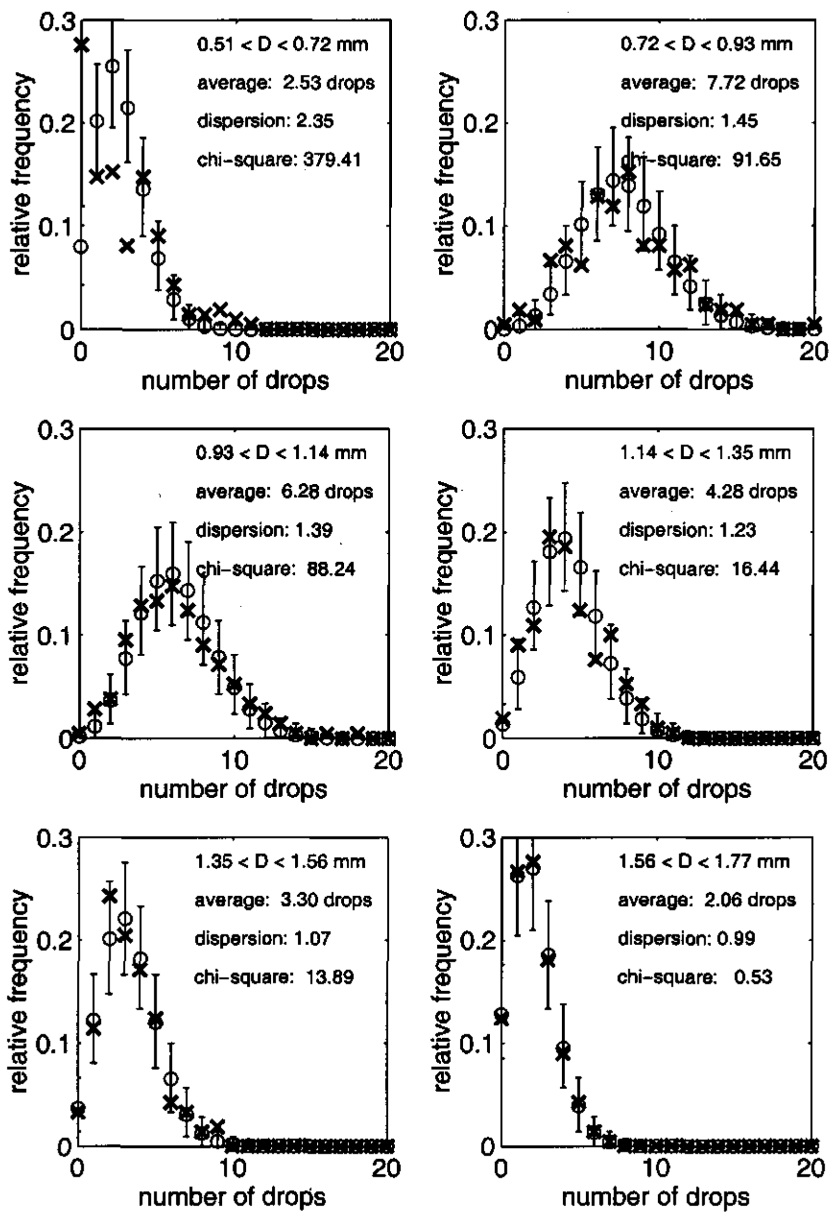

Figure 2.1: Empirical (crosses) and theoretical Poisson (circles) frequency functions of raindrop counts for diameters between 0.51 and 1.77 mm diameter (24 degrees of freedom). Error bars indicate 95% confidence limits. Also indicated are the average number of raindrops per 10 s interval, the Poisson dispersion index and the χ^2 goodness-of-fit statistic.

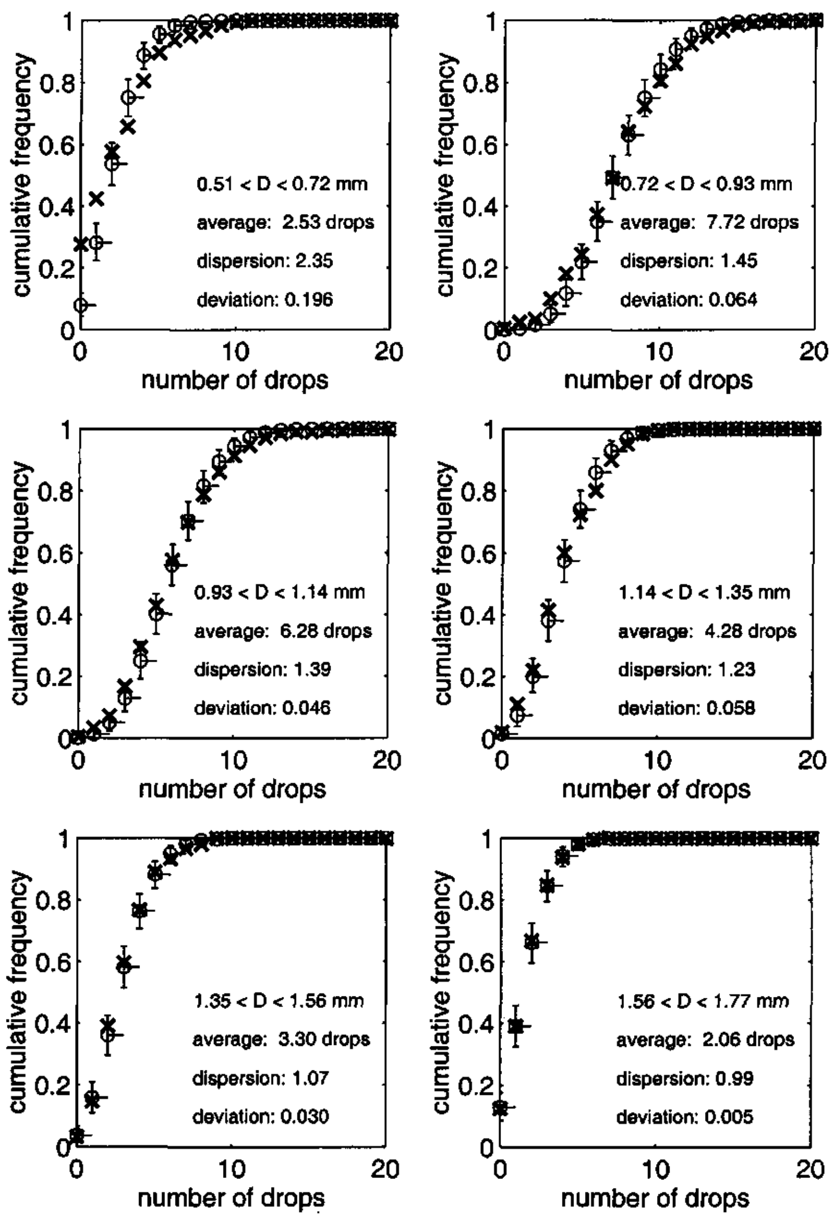

Figure 2.2: Empirical (crosses) and theoretical Poisson (circles) cumulative frequency functions of raindrop counts for diameters between 0.51 and 1.77 mm diameter. Error bars indicate 95% confidence limits. Also indicated are the average number of raindrops per 10 s interval, the Poisson dispersion index and the maximum absolute deviation between the empirical and the theoretical cumulative frequency function.

characterized by uncorrelated fluctuations around a constant mean rain rate of about 3.5 $\mathrm{mm\,h^{-1}}$.

The empirical frequency function calculated from the 210 observations has been compared for each raindrop diameter interval with the theoretical frequency function expected for a homogeneous Poisson process with the same mean. Fig. 2.1 shows the results for the first 6 intervals, corresponding to diameters from 0.51 mm to 1.77 mm. The error bars in this figure represent 95% confidence limits. Fig. 2.1 also provides the mean raindrop count, the value of the Poisson dispersion index and the value of the χ^2 goodness-of-fit statistic for each diameter interval. Fig. 2.2 gives the corresponding results for the empirical *cumulative* frequency function. This figure also provides the maximum absolute deviation between the empirical and the theoretical cumulative frequency function for each diameter interval.

A visual inspection of Figs. 2.1 and 2.2 reveals that only the first diameter interval shows major deviations from Poisson behavior. For all other intervals the relative frequencies more or less correspond to what can be expected on the basis of Poisson statistics. The fit with the Poisson frequency function becomes nearly perfect for the last diameter intervals (which are not shown here as they provide little extra information). A closer look at the values of the Poisson dispersion indices and the χ^2 statistics shows that at the 95% confidence level, the hypothesis that the raindrop counts can be considered random samples from Poisson distributions is only rejected for the first three diameter intervals, containing raindrops with diameters less than 1.14 mm.

In summary, the analysis demonstrates that the Poisson homogeneity hypothesis is only rejected for raindrops with diameters smaller than 1.14 mm. Although these raindrops account for 66% of the raindrop concentration in the air and 55% of the raindrop arrival rate at the ground, they only account for 14% of the rain rate and 2% of the radar reflectivity factor (on the basis of the mean raindrop size distribution during the experiment). In other words, although clustering may be a significant phenomenon for the smallest raindrops, the analyzed data seem to indicate that *for moderate rain rates the arrival rate fluctuations of the raindrops which contribute most to rain rate and radar reflectivity factor behave according to Poisson statistics.*

2.3 An explanation for the apparent fractal correlation dimension of homogeneously distributed raindrops

(Multi-)fractal models have originally been used to describe turbulence. Since rainfall is intimately related to the (turbulent) wind field in the atmosphere, it seems natural to use the same approach for modeling rainfall (e.g. de Lima, 1998). However, Fabry (1996) argues that, since raindrops are not passive tracers of the wind field, the analogy between wind and rain may break down at the smallest spatial and temporal scales. The fact that raindrops have different sizes and therefore different fall speeds would tend to filter out the scaling properties of the wind field at those scales. A "white noise" (i.e. homogeneous) regime would be the result.

Additionally, it has recently been demonstrated that one of the strongest empirical arguments in favor of the (multi-)fractal hypothesis at the raindrop scale available to date (the results reported by Lovejoy and Schertzer (1990)) may not be as convincing as it seems (Jameson and Kostinski, 1998). Lovejoy and Schertzer (1990) report on a box counting analysis of blotting paper observations of the spatial distribution of raindrops. They find

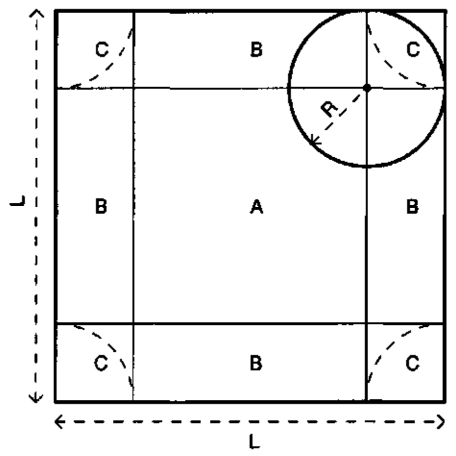

Figure 2.3: Schematic representation of a $L \times L$ (m^2) piece of chemically treated blotting paper (bold square) with a raindrop stain (black dot) and a circle with radius R ($\leq L/2$) (m) around the center of the stain (bold circle). The surface of the blotting paper can be divided into three separate regions (as indicated by the thin lines) according as to whether the circle surrounding a raindrop center will fall entirely inside (region A) or partly outside the boundary of the blotting paper (regions B and C).

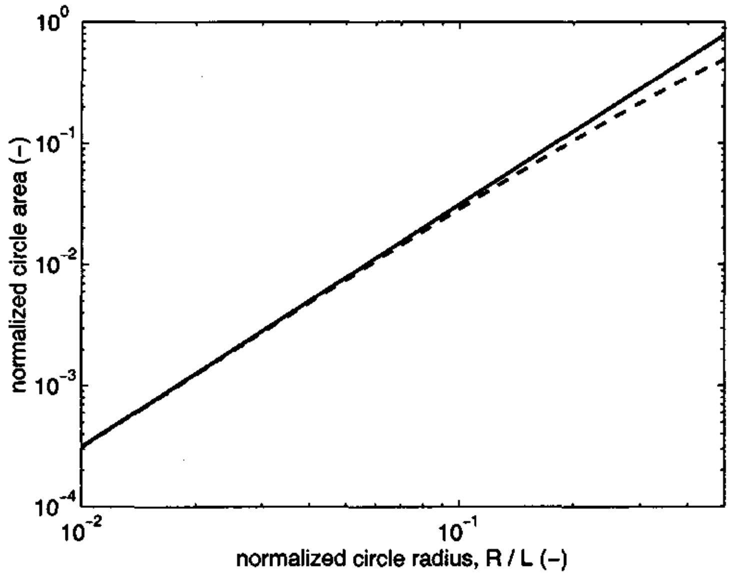

Figure 2.4: Expected surface area of a circle with radius R (m) around the center of a raindrop uniformly distributed on $[0, L] \times [0, L]$ (normalized by the surface area of the blotting paper L^2 (m^2)) versus the normalized radius R/L (–). Solid line: without adjusting for the expected fraction of the circle falling outside the boundary of the blotting paper; dashed line: with boundary correction. Since for a Poisson process the expected number of raindrop stains is proportional to the expected surface area falling inside the blotting paper boundary, multiplication of the vertical axis with the (expected) total number of raindrop stains on the blotting paper yields the expected number of raindrop stains as a function of R/L.

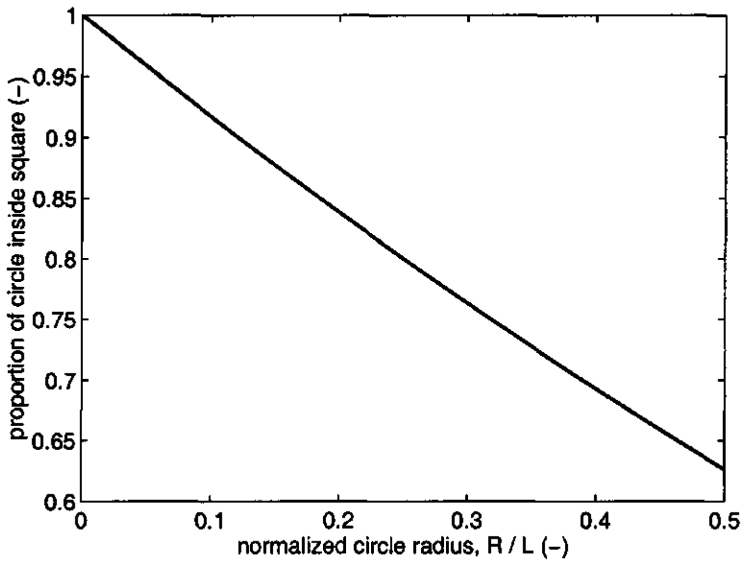

Figure 2.5: Expected proportion of a circle with radius R (m) around a uniformly distributed raindrop center falling inside the boundary of the blotting paper as a function of the normalized radius R/L (–) (i.e. the ratio of the dashed to the solid line in Fig. 2.4).

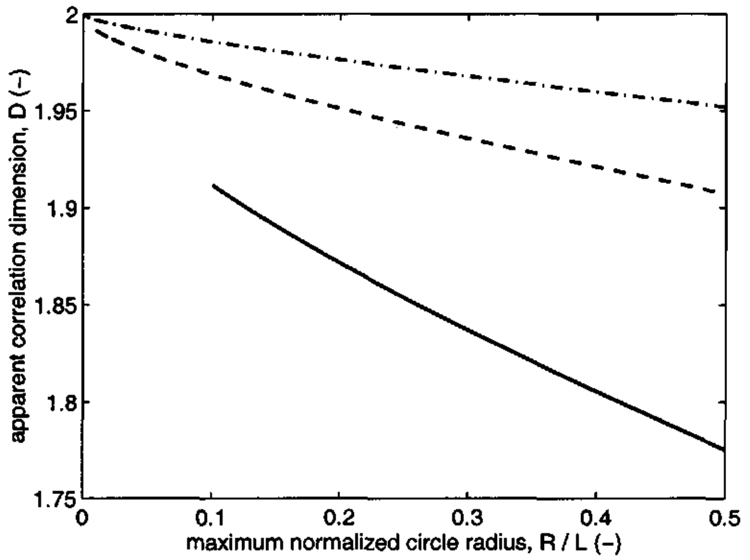

Figure 2.6: Apparent fractal correlation dimension (i.e. the slope of the dashed line in Fig. 2) as a function of the interval used to perform the regression analysis. Solid line: minimum normalized radius $R/L = 10^{-1}$; dashed line: minimum normalized radius $R/L = 10^{-2}$; dash-dotted line: minimum normalized radius $R/L = 10^{-3}$. The slopes have been obtained using linear regression on the logarithmic values, uniformly distributed over the interval in logarithmic space.

evidence for the scaling behavior of raindrops in space. However, first of all the limited size of their sample (comprising only 452 raindrop stains) questions the statistical significance of their results. Moreover, since the sizes of the raindrops are not taken into account in their analysis, it remains unclear whether the reported scaling behavior is exhibited to the same extent by raindrops of all sizes. Perhaps the deviation from homogeneity is largely restricted to particular raindrop sizes. Thirdly, Jameson and Kostinski (1998) present the results of a numerical simulation experiment intended to mimic Lovejoy and Schertzer's box counting analysis. They find exactly the same fractal dimension as Lovejoy and Schertzer, even though their simulation is based on uniformly distributed raindrops, consistent with the Poisson homogeneity hypothesis. This indicates that the fractal dimension reported by Lovejoy and Schertzer may have been a mere sampling artifact.

That this is indeed the case can be demonstrated analytically. Fig. 2.3 provides a schematic representation of the blotting paper employed by Lovejoy and Schertzer (1990) in their box counting analysis. This analysis consisted of drawing concentric circles around the center of each of 452 raindrop stains collected on a 128×128 cm^2 piece of chemically treated blotting paper – using an exposure time of approximately 1 second – 'during a moderately heavy stratiform rain'. The radii of the circles were logarithmically varied from a few mm (i.e. of the order of the size of the stains) to more than 1.5 m (i.e. largely exceeding the size of the blotting paper). For each value of the radius, the numbers of drop stains falling inside each of the 452 circles were averaged. In this manner, Lovejoy and Schertzer obtained an average number of drop stains as a function of the circle radius. This empirical function was subsequently plotted on log-log paper. Apart from a fall-off both at small radii (due to the finite number of drop stains) and at large radii (due to the finite size of the blotting paper), the function was found to be reasonably well described by a straight line for 'the part of the graph [...] that was relatively unaffected by the [...] fall-off' (reported as the range between 2 mm and 40 cm). The obtained slope of the straight line, the 'correlation dimension', was found to be 1.83 (1.79 and 1.93 on two other occasions), as opposed to a value of 2 expected for uniformly distributed drop stains. Lovejoy and Schertzer interpreted this as 'evidence that rainfall is scaling over this range' and implying that 'drops are (hierarchically) clustered over the range'.

For homogeneously distributed raindrops (obeying Poisson statistics), the expected number of raindrop stains falling inside a circle of a given radius would be the product of (1) the expected stain density (in this case $452/1.28^2 = 276$ m^{-2}, i.e. 276 drops per square meter) and (2) the expected surface area of the circle, with a center uniformly distributed over the blotting paper, which falls inside the boundary of that blotting paper. Using the subdivision shown in Fig. 2.3, the latter can be calculated analytically in a straightforward manner for circles with radii up to half the length of a side of the blotting paper. Fig. 2.4 shows the result on a log-log plot. This figure can be directly compared to Fig. 2 of Lovejoy and Schertzer (1990). Clearly, the expected surface area of the circle will eventually become equal to the area of the blotting paper, when the circle will cover the paper entirely no matter where its center will be located. This will be true when the circle radius equals $\sqrt{2}$ times the length of a side of the blotting paper.

Although the difference between the solid and the dashed line in Fig. 2.4 may not seem significant, it should be recognized that this is due to the fact that the y-axis in that figure has a logarithmic scale. Fig. 2.5 shows the ratio of the dashed to the solid line in Fig. 2.4, i.e. the expected proportion of a circle with a given radius falling inside the blotting paper. For a radius which equals half the length of a side of the blotting paper, this proportion is only slightly more than 60%. Even for the largest radius taken into account by Lovejoy and Schertzer (1990) to fit a straight line to their Fig. 2 (corresponding to a normalized circle radius of $40/128 = 0.3125$), this figure is only 75%, indicating that the expected number of drop stains will be underestimated by 25% for circles of this size. It should be noted that this underestimation can be entirely explained as a boundary effect in an otherwise homogeneous rainfall sample and does not require invoking any scaling hypothesis.

Fig. 2.6, finally, shows the influence of this boundary effect on the estimation of the slope of the dashed line in Fig. 2.4. For a maximum normalized radius as employed by Lovejoy and Schertzer (0.3125), this leads to slopes of 1.83, 1.93 and 1.97, respectively, depending on whether the minimum radius used in the regression analysis is a fraction 10^{-1}, 10^{-2} or 10^{-3} of the length of a side of the blotting paper. Apparently, the influence of the blotting paper's finite size remains appreciable even for circles which are orders of magnitude smaller than the paper. The indicated slopes represent "apparent" correlation dimensions, as their fractal values are entirely the result of sampling effects. In conclusion, *Lovejoy and Schertzer's (1990) claim that their box counting analysis provides empirical evidence for the fractal hypothesis that 'inhomogeneity in rain is likely to extend down to millimeter scales' has to be reconsidered. As a result of instrumental artifacts, their test results are not significant enough to reject the Poisson homogeneity hypothesis in favor of a fractal description of rainfall.*

2.4 A stochastic model of stationary rainfall for the study of sampling fluctuations

The results reported in the previous two sections have demonstrated that Poisson statistics may be used as a starting point for the development of a theory of sampling fluctuations in surface rainfall observations. The basis of such a theory would be provided by the choice of an appropriate stochastic model of stationary rainfall, i.e. rainfall in which no natural variability is present. In this section, rainfall is modeled as a so-called *marked point process*, perhaps the simplest model able to account for the raindrop structure of rainfall. The basic hypotheses of the model are (1) spatial homogeneity and (2) the absence of raindrop interaction.

In most previous rainfall studies, raindrop populations have been characterized by the raindrop size distribution only. This quantity provides the average number of drops per unit volume of air and per unit diameter. It is an average because both the number of raindrops in a volume and their diameters fluctuate. Hence, any description of rainfall at the level of raindrops has to include two types of fluctuations: the statistics of the distribution of raindrops in space and the distribution of their sizes. Here, raindrops are assumed to be uniformly distributed (in a statistical sense) in a given volume. In other words, the number of drops is assumed to be distributed according to a Poisson distribution. This is the simplest model that is possible. In the previous sections, the Poisson homogeneity hypothesis has been justified experimentally as a good approximation to reality during stationary rainfall. Additionally, Kostinski and Jameson (1997) find indications for homogeneous Poisson behavior during 'a time of unusually constant flux'. Within this model, the density of raindrops per unit volume of air, the raindrop concentration, is the only parameter required to characterize the spatial distribution of drops. The distribution of drop diameters is characterized by a certain probability density function. For instance, Marshall and Palmer (1948)

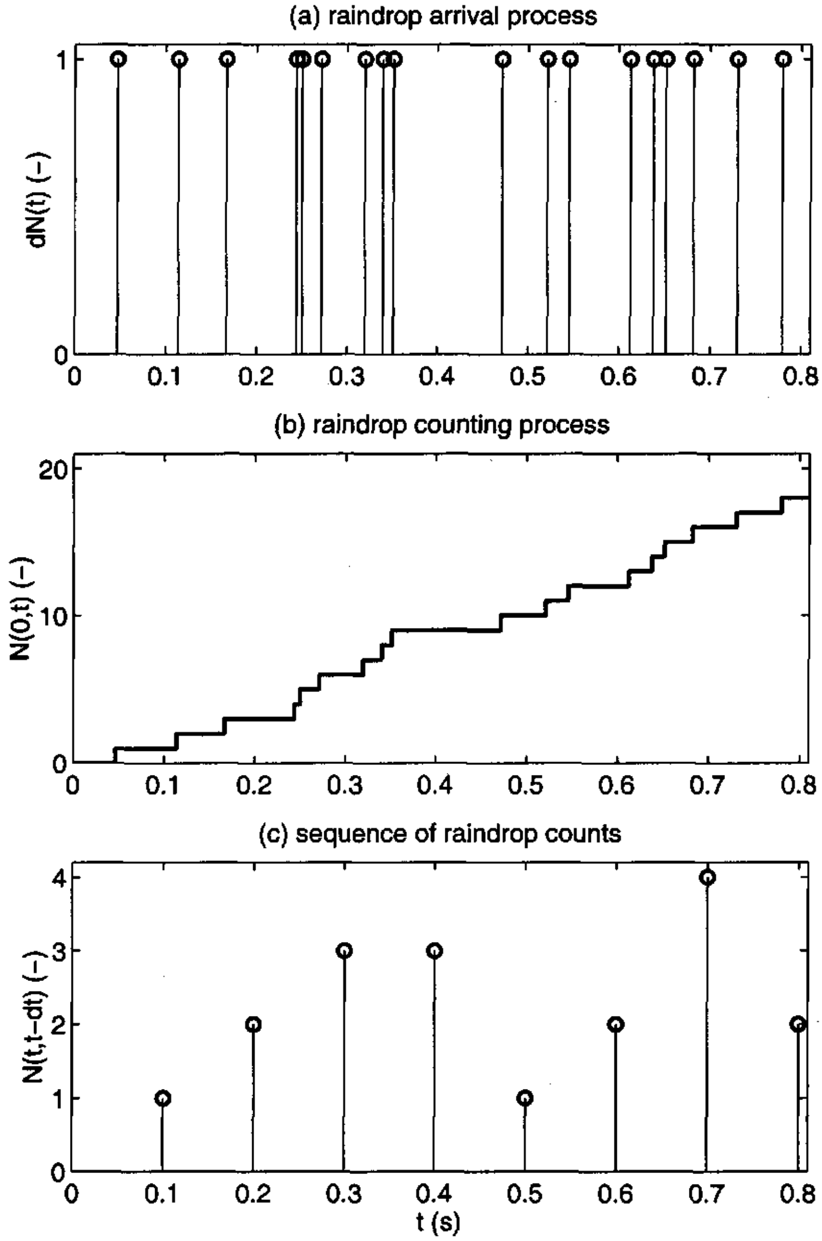

Figure 2.7: Three different representations of the stochastic process describing the arrival of raindrops at the ground: (a) the stochastic point process describing the arrival times of the raindrops; (b) the stochastic counting process, the integral of the point process under (a), describing the number of raindrops which have arrived up to a particular time; (c) the stochastic process describing the sequence of raindrop counts in fixed time intervals.

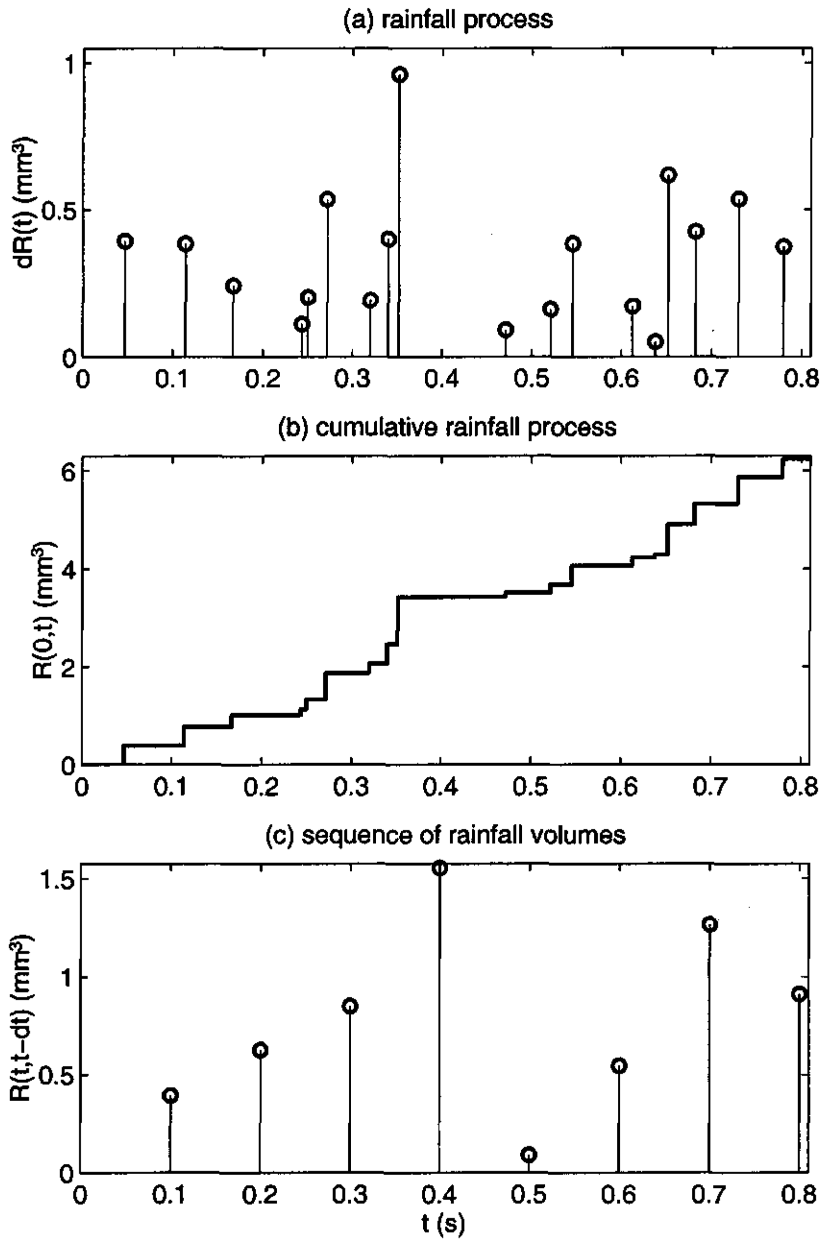

Figure 2.8: Three different representations of the stochastic process describing rainfall at the ground: (a) the marked point process describing the arrival times and the sizes of the raindrops; (b) the stochastic process describing the cumulative amount of rainfall which has arrived up to a particular time; (c) the stochastic process describing the sequence of rain rates in fixed time intervals.

have suggested that drop diameters are distributed according to an exponential density (see Uijlenhoet and Stricker (1999a) for a recent review of the exponential raindrop size distribution). The model just introduced is a marked point process, in which the point process represents the positions of drops in the sample volume and the mark associated with a drop is its diameter (Smith, 1993).

The sample-volume process described above can be transformed into an arrival process by considering the arrival of raindrops at a given surface during a given period of time (Figs. 2.7, 2.8). The connection is established by assuming (1) that raindrops fall vertically downward at a terminal velocity which depends exclusively on their diameter and (2) that drops do not interact with each other. The absence of interaction together with the Poisson distribution of drops in space implies that the inter-arrival times of drops are exponentially distributed (Smith, 1993). The arrival process obtained in this way also becomes a marked point process. The arrival times now constitute the point process (in time; Fig. 2.7) and the diameter of each drop is its mark (Fig. 2.8). The exponential distribution completely determines the statistics of the arrival point process. In fact, this process will also be of the Poisson type because the number of raindrops arriving at the sampling surface during a given period of time is distributed according to a Poisson distribution as well. A final simplification in the proposed model of stationary rainfall is that the random diameters of the individual raindrops are assumed to be independent and identically distributed, independent of their arrival times.

2.5 A theory of sampling fluctuations in properties derived from measurements of raindrop size distributions

Rainfall properties estimated from raindrop size measurements show great variability. Generally speaking, three factors can explain this variability: (1) climatological factors (as different kinds of rainfall have different properties), (2) physical factors (as meteorological conditions change during rainfall events), and (3) instrumental factors (those associated with the device used to measure drop diameters). The latter includes any instrument malfunctioning, device sensitivity, and sample size effect. In this section the focus is on the analysis of the magnitude of the variability caused by the sample size: the sampling fluctuations.

The stochastic rainfall model described in the previous section is used in this section to obtain the sampling distributions of rain rate (Fig. 2.9), the time needed to exceed a given rain water depth (Fig. 2.10) and the minimum and maximum raindrop diameters (Figs. 2.11 and 2.12). For the first two of these distributions, exact forms can only be obtained under the assumption that the volumes of the arriving raindrops are exponentially distributed. If this restrictive assumption is relaxed, only asymptotic expansions can be derived. For the sampling distributions of the minimum and the maximum raindrop diameters, exact solutions can be found for any form of the raindrop size distribution. Notice that, although the sampling distribution of the maximum raindrop diameter shows a pronounced rain rate dependence (Fig. 2.12), that of the minimum drop diameter is almost entirely independent of rain rate (Fig. 2.11).

Both the exact and the asymptotic forms of *the sampling distributions are found to converge to Gaussian distributions* (Fig. 2.9). The relevant parameter controlling this evolution turns out to be the average number of drops in the sample. An advantage of the asymptotic

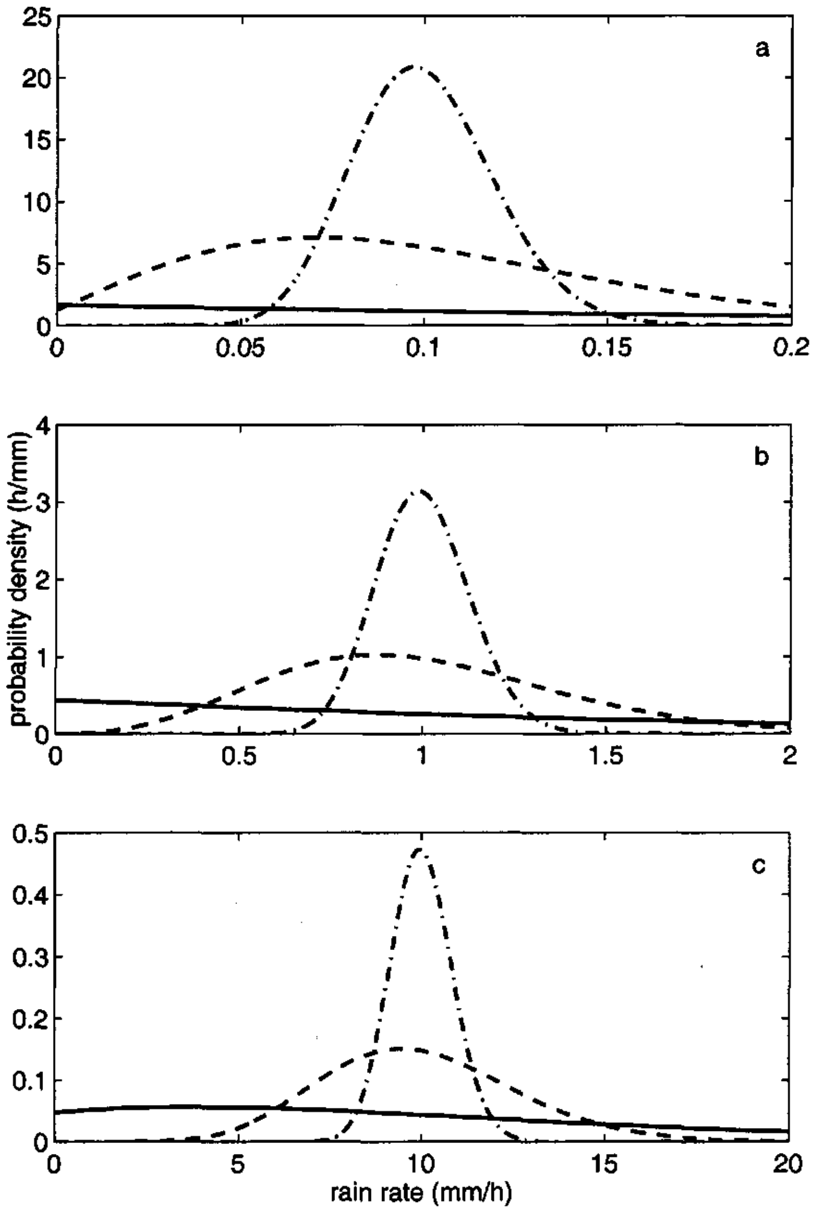

Figure 2.9: Exact form of the continuous part of the sampling distribution of rain rate in case the arrival process of raindrops at a surface obeys a Poisson process and the volumes of the arriving raindrops are exponentially distributed. The rain rate dependence of the arrival rate and the mean volume of the raindrops are those which follow from a combination of the Marshall-Palmer (1948) distribution and Atlas and Ulbrich's (1977) raindrop terminal fall speed parameterization (Uijlenhoet and Stricker, 1999a). (a) mean rain rate is 0.1 $\mathrm{mm\,h^{-1}}$; (b) mean rain rate is 1 $\mathrm{mm\,h^{-1}}$; (c) mean rain rate is 10 $\mathrm{mm\,h^{-1}}$. Solid line: integration time is 0.1 s; dashed line: integration time is 1 s; dash-dotted line: integration time is 10 s. In all cases the sampling surface is 50 $\mathrm{cm^2}$, corresponding to a Joss-Waldvogel (1967) disdrometer.

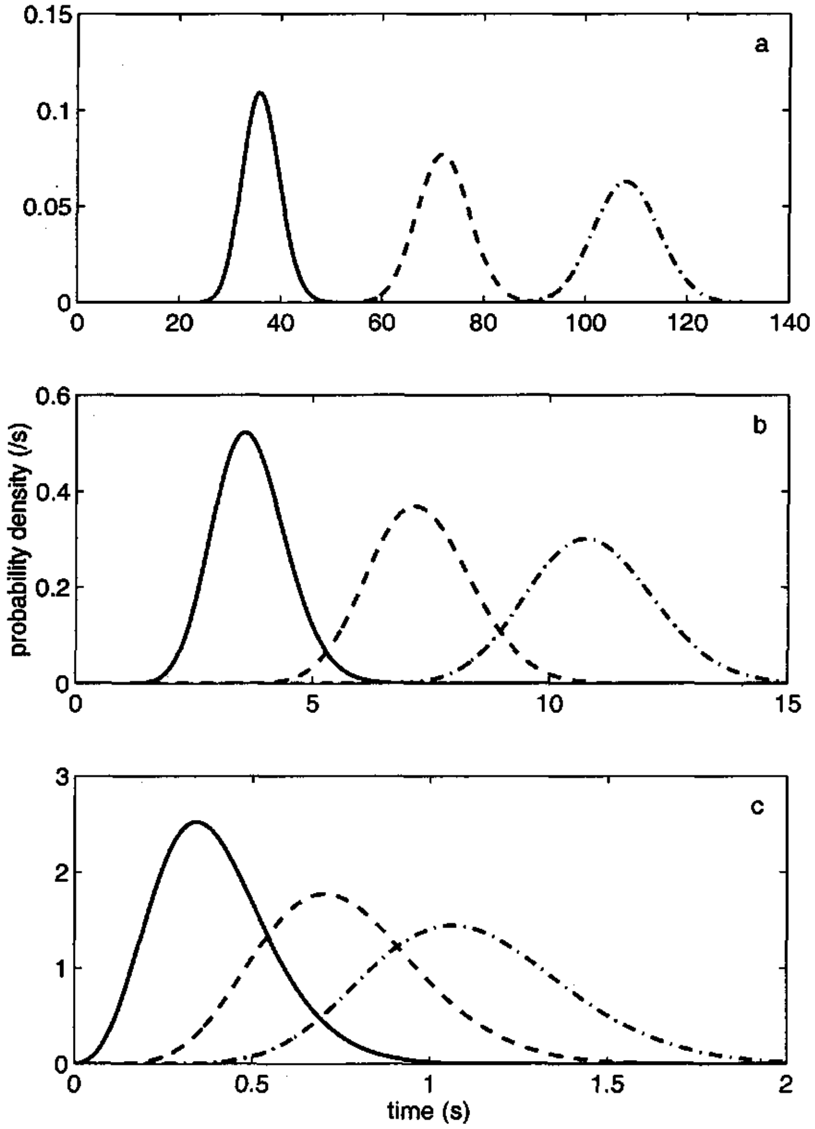

Figure 2.10: Exact form of the sampling distribution of the time needed to exceed a given rain water depth in case the arrival process of raindrops at a surface obeys a Poisson process and the volumes of the arriving raindrops are exponentially distributed. The rain rate dependence of the arrival rate and the mean volume of the raindrops are those which follow from a combination of the Marshall-Palmer (1948) distribution and Atlas and Ulbrich's (1977) raindrop terminal fall speed parameterization (Uijlenhoet and Stricker, 1999a). (a) mean rain rate is 0.1 $\mathrm{mm\,h^{-1}}$; (b) mean rain rate is 1 $\mathrm{mm\,h^{-1}}$; (c) mean rain rate is 10 $\mathrm{mm\,h^{-1}}$. Solid line: exceeded rain water depth is 0.001 mm; dashed line: exceeded rain water depth is 0.002 mm; dash-dotted line: exceeded rain water depth is 0.003 mm. In all cases the sampling surface is 50 $\mathrm{cm^2}$, corresponding to a Joss-Waldvogel (1967) disdrometer.

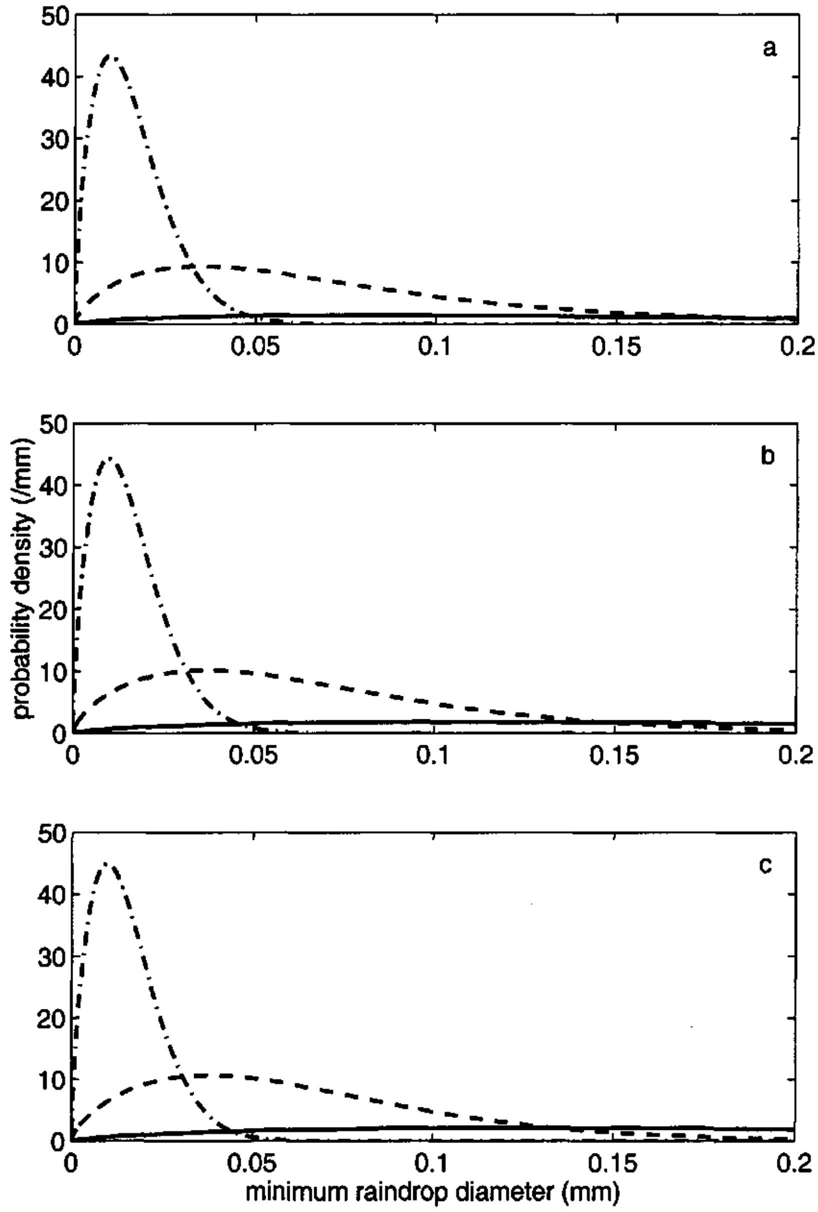

Figure 2.11: Exact form of the continuous part of the sampling distribution of the minimum raindrop diameter in case the arrival process of raindrops at a surface obeys a Poisson process and the diameters of the arriving raindrops are exponentially distributed. The rain rate dependence of the arrival rate and the mean diameter of the raindrops are those which follow from a combination of the Marshall-Palmer (1948) distribution and Atlas and Ulbrich's (1977) raindrop terminal fall speed parameterization (Uijlenhoet and Stricker, 1999a). (a) mean rain rate is 0.1 $\mathrm{mm\,h^{-1}}$; (b) mean rain rate is 1 $\mathrm{mm\,h^{-1}}$; (c) mean rain rate is 10 $\mathrm{mm\,h^{-1}}$. Solid line: integration time is 0.1 s; dashed line: integration time is 1 s; dash-dotted line: integration time is 10 s. In all cases the sampling surface is 50 cm^2, corresponding to a Joss-Waldvogel (1967) disdrometer.

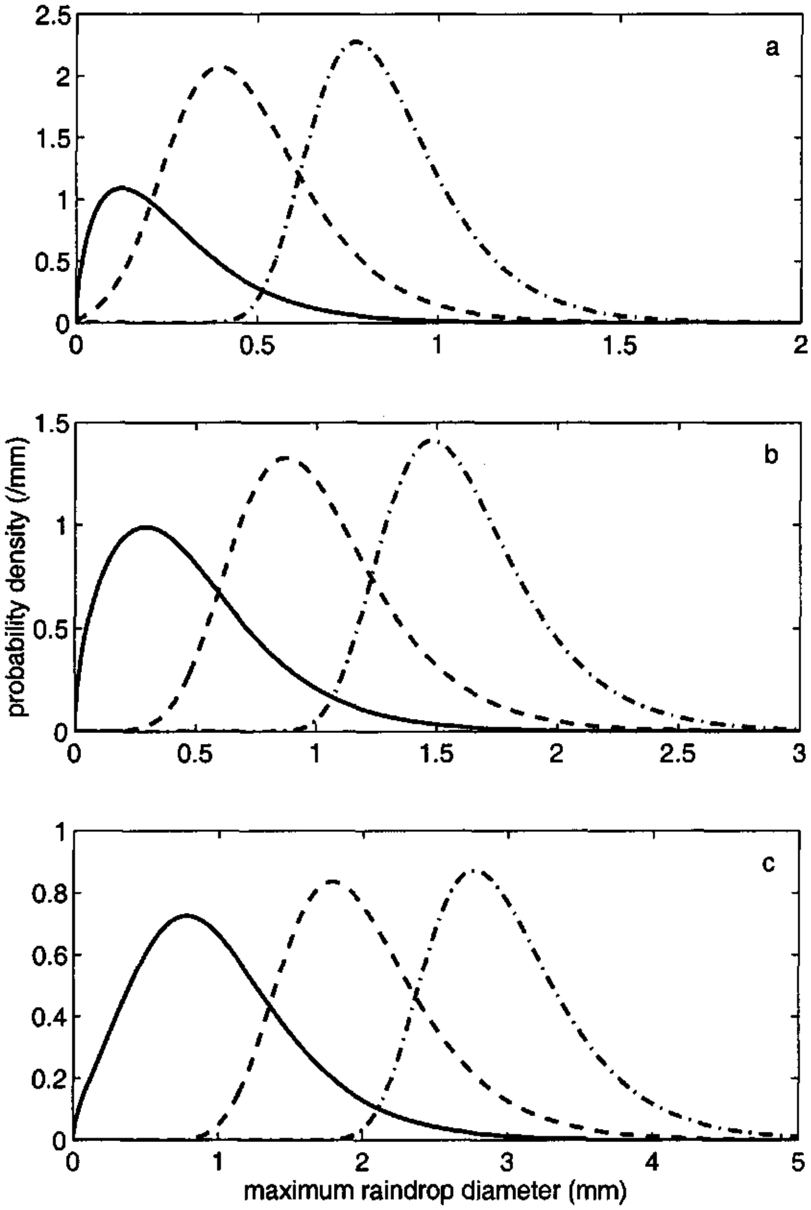

Figure 2.12: Exact form of the continuous part of the sampling distribution of the maximum raindrop diameter in case the arrival process of raindrops at a surface obeys a Poisson process and the diameters of the arriving raindrops are exponentially distributed. The rain rate dependence of the arrival rate and the mean diameter of the raindrops are those which follow from a combination of the Marshall-Palmer (1948) distribution and Atlas and Ulbrich's (1977) raindrop terminal fall speed parameterization (Uijlenhoet and Stricker, 1999a). (a) mean rain rate is 0.1 mm h^{-1}; (b) mean rain rate is 1 mm h^{-1}; (c) mean rain rate is 10 mm h^{-1}. Solid line: integration time is 0.1 s; dashed line: integration time is 1 s; dash-dotted line: integration time is 10 s. In all cases the sampling surface is 50 cm^2, corresponding to a Joss-Waldvogel (1967) disdrometer.

method over the exact solutions is that it provides expansions for the sampling distributions of all rainfall integral variables, not only that of rain rate, but for instance those of kinetic energy flux density and radar reflectivity as well. *The skewness of the distributions is found to be more pronounced for smaller mean sample sizes* (Fig. 2.9) *and for higher order moments of the raindrop size distribution.* For instance, the sampling distribution of the normalized mean diameter becomes nearly Gaussian for mean raindrop counts greater than 10 while the sampling distribution of the normalized rain rate remains skewed for mean raindrop counts as large as 500.

Another result of the analysis is the conclusion that *the median always underestimates the population value of a rainfall integral variable.* The amount of underestimation depends on the raindrop size distribution. This provides a theoretical confirmation and explanation of simulation results presented by Smith et al. (1993).

The practical relevance of the results of this section lies in the possibility of estimating the effect of the sample size associated with a given disdrometric instrument on a particular rainfall estimator. For instance, for the Joss-Waldvogel (1967) disdrometer and the Optical Spectro-Pluviometer (Salles et al., 1998), the average number of raindrops sampled for rain rates below 0.1 mm h^{-1} is less than 100 and the corresponding rain rate and radar reflectivity estimates will therefore be strongly influenced by sampling fluctuations. The proposed methodology for obtaining asymptotic approximations to sampling distributions, provides a manner to better establish the accuracy of estimates of rainfall integral variables obtained from disdrometric measurements. Moreover, the developed method can be generalized to provide estimates of the correlation between estimators of different rainfall variables induced by sampling fluctuations only.

2.6 Extensions of the stochastic rainfall model

This section presents two extensions of the stochastic rainfall model discussed in the previous sections: (1) the generalization of the *temporal* model of stationary rainfall to a model which includes *spatial* dimensions as well; (2) the generalization of the temporal model to include, besides sampling fluctuations, natural variability as well.

With the smaller and smaller sample volumes currently employed, (Doppler) radar observations of precipitation will be subject to sampling fluctuations as well. For example, suppose the rectangular reference volume indicated in Fig. 1.1 (p. 16) represents a radar sample volume of 1 m^3. Fig. 2.13 shows what the temporal (sampling) fluctuations in the raindrop concentration ρ_V (m^{-3}), the rain rate R (mm h^{-1}) and the radar reflectivity factor Z (mm^6 m^{-3}) might look like for this hypothetical sample volume, assuming a Marshall-Palmer raindrop size distribution and a constant (mean) rain rate of 1 mm h^{-1}. This simulation has been based on an adapted version of von Smoluchowski's (1916) stochastic model of density fluctuations for intermittent observations (e.g. Fürth, 1918, 1919; Chandrasekhar, 1943; Smith, 1993a). Although ρ_V remains roughly constant, R and particularly Z are observed to fluctuate appreciably. Note the differences in correlation structure between these three rainfall integral variables as well. Again, in a practical situation, a first estimate of the magnitude and speed of these fluctuations may be obtained on the basis of the Poisson homogeneity hypothesis. A model such as this may be used to investigate the additional amount of rainfall information which may be hidden in the reflectivity fluctuations of high resolution radar observations.

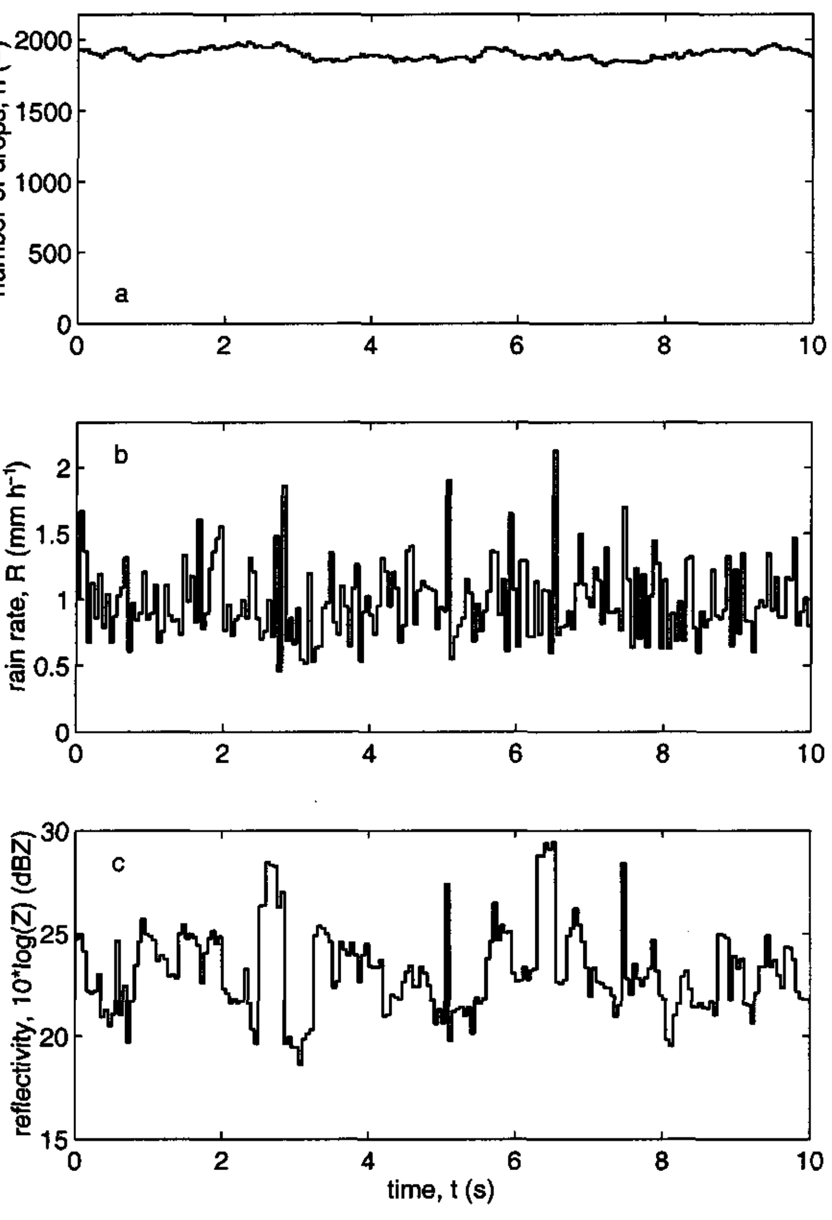

Figure 2.13: Simulation of the temporal evolution of rainfall integral variables in a 1 m^3 sample volume on the basis of the Poisson homogeneity hypothesis: (a) number of raindrops n (–) or raindrop concentration ρ_V (m^{-3}); (b) rain rate R ($mm\,h^{-1}$); (c) radar reflectivity factor Z ($mm^6\,m^{-3}$). Mean rain rate $R = 1$ $mm\,h^{-1}$, Marshall-Palmer raindrop size distribution $N_V\,(D, R)$, raindrop diameter resolution $\Delta D = 0.1$ mm, maximum raindrop diameter $D_{max} = 6$ mm, time step $\Delta t = 0.05$ s.

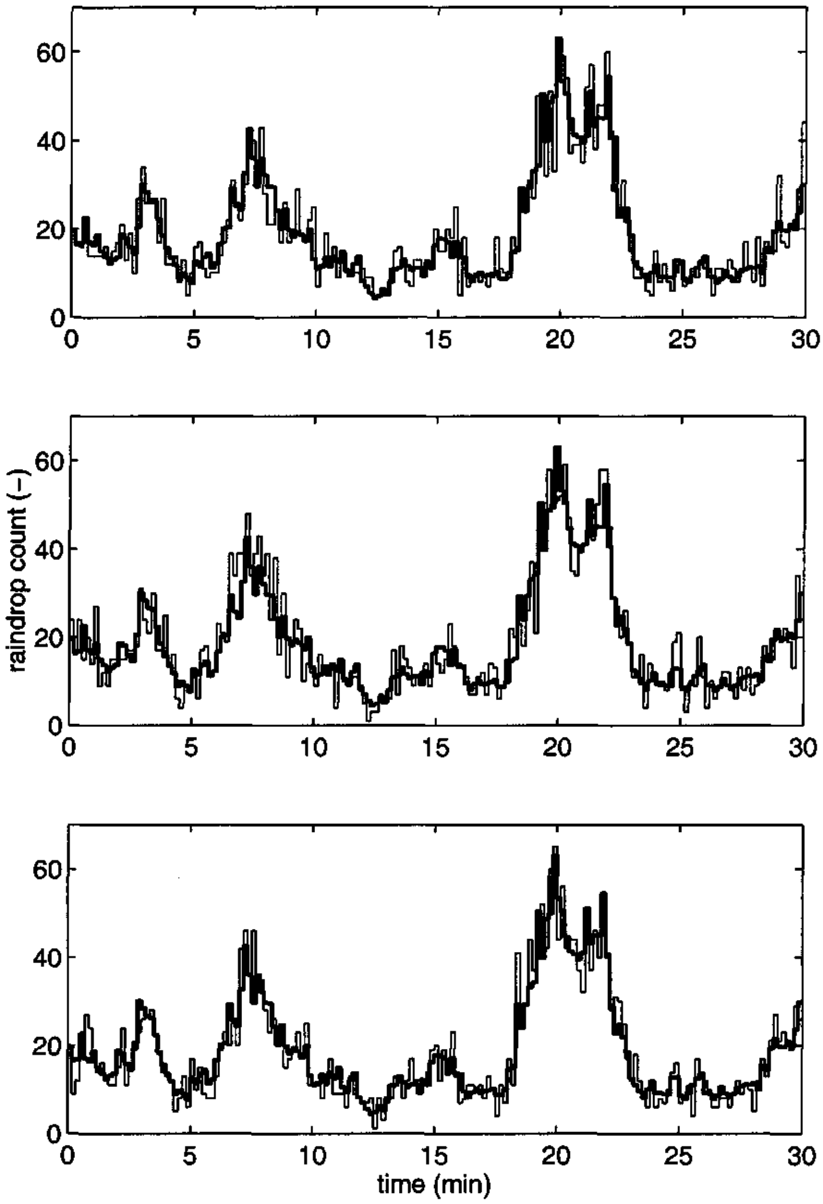

Figure 2.14: The raindrop arrival process, including both sampling fluctuations and natural variability, modeled as a doubly-stochastic Poisson process (Cox process). Bold lines: one realization of an arrival rate process modeled as a logarithmically transformed Gaussian (normal) first-order autoregressive (AR(1)) process with a mean of 25 drops per second, a coefficient of variation (CV) of $1/\sqrt{2}$ and a first-order autocorrelation coefficient of 0.95. Thin line: three different realizations of independent Poissonian fluctuations around the arrival rate process.

Secondly, *a reasonable first approximation of the temporal rainfall process at the droplet scale (including both sampling fluctuations and natural variability) is found to be provided by the doubly stochastic Poisson process (Cox process),* where the rate process follows a logarithmically transformed, normal (Gaussian), first-order autoregressive (AR(1)) process (Fig. 2.14). Such a model may be used to determine over what time period actual surface rainfall observations should be accumulated to reduce the sampling fluctuations as much as possible without loosing an unacceptable amount of natural variability.

2.7 A statistical model of natural rainfall variability

In this final section, the subject of sampling fluctuations is abandoned. The focus is entirely on the natural variability of the rainfall process. Smith and Krajewski (1993) have demonstrated that, in the absence of sampling fluctuations, the coefficients of power law Z-R relationships can be expressed in terms of the variances of and the covariances between the parameters of the probability distributions of raindrop count and size. Their approach can be extended to any pair of rainfall integral parameters, provided these parameters are proportional to moments of the raindrop size distribution (which is approximately the case for Z, k and R). In this manner, statistical power law relationships have been obtained which are in a sense complementary to the deterministic power laws pioneered by Atlas and Ulbrich (1974) and Ulbrich and Atlas (1978). The statistical approach, however, does not only lead to theoretical expressions for the coefficients of power law relationships between rainfall integral parameters, but to theoretical expressions for the goodness of fit of such power laws as well. Moreover, this approach can be readily extended to include higher order power laws, such as double and triple power laws. In this manner, theoretical expressions for the coefficients of power laws used in multi-parameter radar observation of rainfall have been derived.

Another interesting feature of the statistical approach is that it provides a theoretical confirmation of the general formulation of the raindrop size distribution as a scaling law proposed by Sempere-Torres et al. (1994, 1998) (see Uijlenhoet and Stricker (1999b) for a summary of this formalism). *It provides a direct physical interpretation of the scaling exponents in terms of the variances of and the covariances between the parameters of the raindrop count and size distributions.* The values of the scaling exponents indicate whether it is the raindrop concentration or the characteristic raindrop size which controls the variability of the raindrop size distribution (Fig. 2.15).

2.8 Summary of results arising out of this research

- The arrival rate fluctuations of the raindrops which contribute most to rain rate and radar reflectivity behave, for moderate rain rates, according to Poisson statistics. This implies a rejection of the (multi-)fractal hypothesis for rainfall, at least at the raindrop scale. Hence, Poisson statistics may be used as a starting point for the development of a theory of sampling fluctuations in surface rainfall observations.

- The sampling distribution of the estimator of any rainfall integral variable converges to a Gaussian probability density function for large values of the expected sample size. The approach to normality is slower for the higher order moments of the raindrop size

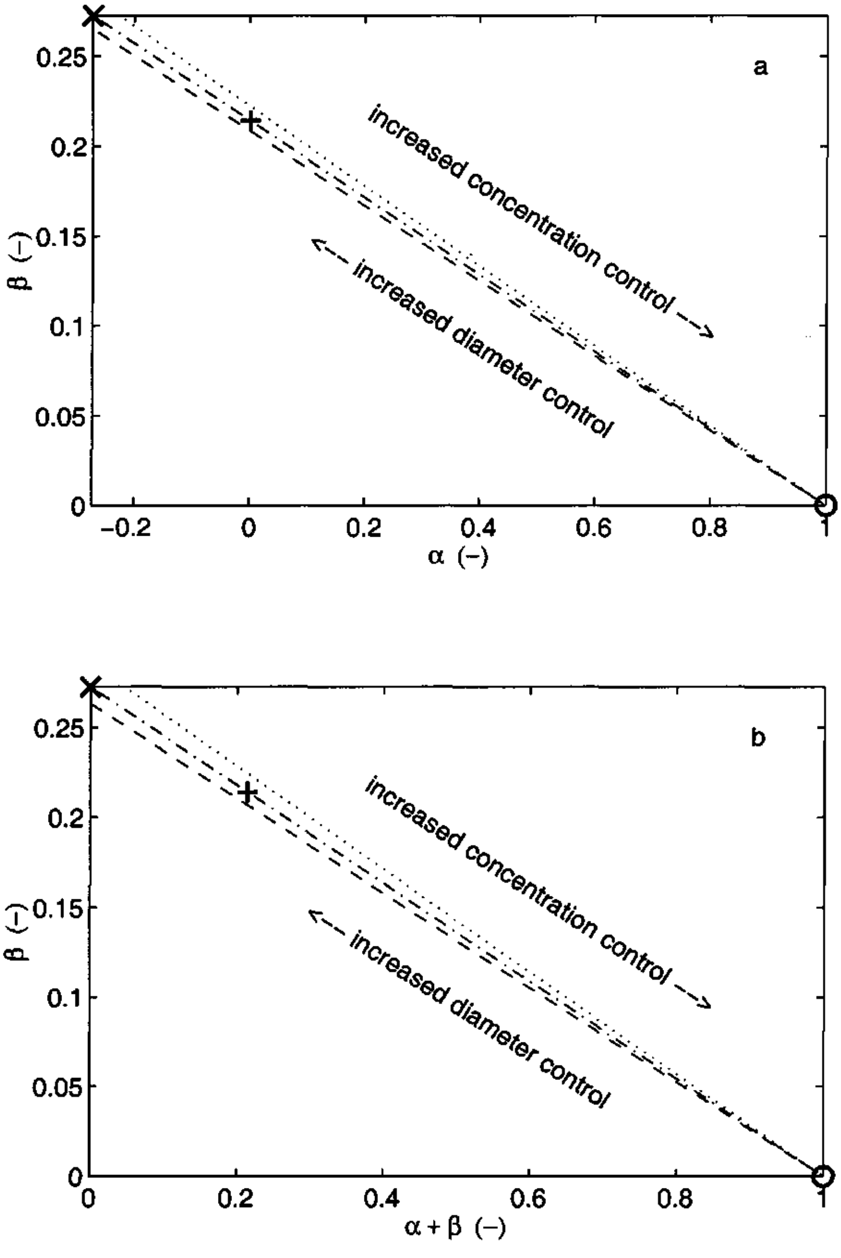

Figure 2.15: (a) Theoretical self-consistency relationship between the scaling exponents α (–) and β (–), $\beta = \frac{1-\alpha}{4+\gamma}$, for three different values of the exponent γ of a power law relationship between raindrop terminal fall speed and equivalent spherical raindrop diameter (dashed line: $\gamma = 0.8$; dash-dotted line: $\gamma = 0.67$; dotted line: $\gamma = 0.5$). The cross at the point with coordinates $(\alpha, \beta) = (-0.27, 0.27)$ corresponds to raindrop size controlled rainfall, the plus at the point with coordinates $(\alpha, \beta) = (0, 0.21)$ to Marshall and Palmer's (1948) exponential raindrop size distribution, the circle at the point with coordinates $(\alpha, \beta) = (1, 0)$ to equilibrium rainfall (raindrop concentration controlled) conditions. (b) Idem in the transformed parameter space spanned by the exponents $\alpha + \beta = \gamma_{\rho_V}$ and $\beta = \gamma_{D_C}$.

distribution. As a result of the asymmetry of the sampling distributions, the median always underestimates the population value of a rainfall integral variable. These results have important practical implications for the estimation of radar reflectivity – rain rate relationships from surface raindrop observations.

- Von Smoluchowski's (1916) stochastic model of density fluctuations for intermittent observations provides a reasonable first approximation of the *spatial* rainfall process at the droplet scale, i.e. the fluctuations in the rainfall integral variables in a given volume of air. Such a model may be used to investigate the additional amount of rainfall information which may be hidden in the reflectivity fluctuations of high resolution radar observations.

- A reasonable first approximation of the *temporal* rainfall process at the droplet scale (including both sampling fluctuations and natural variability) is provided by a doubly stochastic Poisson process (Cox process), where the rate process follows a log-transformed, normal (Gaussian), first-order autoregressive (AR(1)) process. Such a model may be used to determine over what time period surface rainfall observations should be accumulated to reduce the sampling fluctuations as much as possible without loosing an unacceptable amount of natural variability.

- A statistical model of the natural variability of the rainfall process provides a direct physical interpretation of the scaling exponents of a recently proposed general formulation for the raindrop size distribution as a scaling law. These exponents can be expressed in terms of the variances of and the covariances between the parameters of the raindrop count and size distributions. The values of the scaling exponents indicate whether it is the raindrop concentration or the characteristic raindrop size which controls the variability of the raindrop size distribution.

Chapter 3

Additional information

3.1 Training content

With regard to research, this fellowship gave an invaluable opportunity to the grant holder of making the synthesis of different studies made in his home institute. The main achievement of the grant holder was to put a considerable amount of existing experimental results into a coherent theoretical framework based on a new formulation of the microphysical properties of rain. He also visited via simple analytical solutions the implications of this theory on the concrete application of radar detection to hydrology. In addition, the grant holder acted as Coordinator of the HYDROMET Integrated Radar Experiment (HIRE), an international radar-hydrological field experiment organized in the framework of the Community-funded HYDROMET project in Marseille (France) from 1 September to 1 December 1998.

With regard to education, the grant holder acted as a lecturer at the Advanced study course on radar hydrology for real time flood forecasting, held at the Water and Environment Management Research Centre of the University of Bristol, UK from 24 June to 3 July 1998. He also acted as chairperson of session NP1.03 (Scaling vs. non-scaling methods in rainfall modelling) at the XXIV General Assembly of the European Geophysical Society, held from 19 to 23 April 1999 in The Hague, The Netherlands. In addition, he assisted in the supervision of several MSc and PhD students at the universities of Grenoble.

3.2 Unexpected developments

The expected results of this project were:

- A mathematical theory for the stochastic description of rainfall at the raindrop scale;
- Software for the visualization of the theoretical results and for the application of the theory to actual rainfall data obtained from disdrometers and optical spectrometers;
- Scientific publications describing the results of the research project.

All these results have been achieved.

Regarding the expected links with any other Community-financed project, this research project was to have close links with the HYDROMET project funded by the Community under the Environment and Climate Programme. Specifically, a close working relationship was to be established with the following HYDROMET partners: Dr. J.-D. Creutin

(LTHE/INPG, Grenoble, France), Dr. D. Sempere Torres (UPC, Barcelona, Spain) and Prof. I. D. Cluckie (WEMRC, University of Bristol, Bristol, UK). LTHE/INPG has been the host institution for the proposed project, whereas UPC and the University of Bristol have been visited during the course of the research. The former with the purpose of writing a joint scientific article (Porrà, J. M., R. Uijlenhoet, D. Sempere Torres and J.-D. Creutin (2000). Sampling effects in drop size distribution measurements: estimation of bulk rainfall variables. *Journal of the Atmospheric Sciences* (submitted)), the latter to attend and lecture at the Advanced Study Course on Radar Hydrology for Real Time Flood Forecasting. All HYDROMET science meetings and workshops have been attended, which has guaranteed regular scientific interaction with the other partners of the HYDROMET project (Italy and Greece). In summary, *all expected links have been established.*

This project has been complementary to Task 2.3 of the HYDROMET Project (Radar measurement uncertainty due to sampling fluctuations on the drop size distribution). The emphasis has been on the experimental verification of various theories concerning the measurement and parameterization of raindrop size distributions. The proposed stochastic model has provided a convenient theoretical framework for this verification, since it has allowed experimental results to be explained in terms of both instrumental artefacts and actual properties of the rainfall process. Detailed knowledge of the rainfall process at the raindrop scale has aided the accomplishment of Task 4.1 of the HYDROMET Project (Marseille experiment and space state modelling) as well. Again, *all proposed tasks have been accomplished.* No unexpected developments have taken place.

3.3 Research lines and/or research approaches which proved unsuccessful

None.

3.4 Potential applications of the results

Analytical solutions to sampling distributions will allow hydrologists to provide all kinds of parameters derived from surface rainfall measurements with *confidence limits* representing the expected magnitude of the fluctuations resulting from instrumental considerations. These confidence intervals will be very useful in separating sampling fluctuations from natural rainfall variability. Knowledge of the relative contributions to these fluctuations may lead to improved Z-R and k-R relationships and ultimately to improved radar rainfall estimates. Moreover, a stochastic model such as the one developed will provide a more realistic framework for simulation studies dealing with problems in radar hydrology.

3.5 Interaction with industry

None.

3.6 Benefit to the Host Institution

The grant holder brought a considerable experience on the subject of rain microphysics and its hydrological applications. The fellowship allowed the laboratories in France and Holland to deepen existing collaboration.

3.7 Benefit to the Community

Regarding the relevance of this project to Community policies, this project has had a direct relevance to Theme A of the Environment and Climate Programme: Research into the natural environment, environmental quality and global change. This is because rainfall with its large spatial and temporal variability is one of the most important links between the processes in the atmosphere and those on the land surface. Since its area of application is radar hydrology, this project has had ties with Theme B of the Programme (Environmental technologies) as well.

Moreover, one of the main objectives of the Environment and Climate individual research training grants is 'to provide opportunities for the advanced training, exchange and mobility of researchers at the postdoctoral level in science and technology relating to environment and climate'. It is clear that the postdoctoral position of a Dutch researcher at a French Laboratory with a research objective which is directly complementary to another Community-financed project under the Environment and Climate Programme has contributed significantly to this objective.

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