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Abstract
In problems related to agricultural land use exploration, nearly optimal solutions of linear programming models constitute alternative land use allocations that result in good, albeit not optimal, levels of satisfaction of objectives. In this paper, we develop a framework to study nearly optimal solutions. The principle is to generate a group of nearly optimal solutions, to summarize the generated solutions by low dimensional vectors called 'aspects of the solutions' and, finally, to present graphically these vectors. Different procedures are described to generate the nearly optimal solutions. The framework is applied to a linear programming model developed for land use exploration at the European level. Regional, crop, and technical aspects of a group of nearly optimal solutions are presented graphically and reveal a great diversity of land use allocations with similar performance in terms of objective. Such presentation allows a stakeholder to choose a solution according to issues that are not taken into account by the model. The results indicate a high sensitivity of the nearly optimal solutions to the procedure used to generate them.

1. Introduction

Linear programming is recognized as an important tool for agricultural land use exploration (De Wit et al., 1988 ; Van Keulen, 1990). It can be used to explore land use allocations that optimize agricultural, economic or environmental objectives at the farm level (Rossing et al., 1997) and at the regional level (WRR, 1992).

A linear programming model is defined by

$$Min\ \{z = c'x\} \qquad (1)$$

$$D\,x \leq b \qquad (2)$$

$$x \geq 0 \qquad (3)$$

Scalar z is the objective function, c and x are two n-dimensional vectors, and b is a p-dimensional vector. D is a matrix $p{\times}n$. The n elements of the vector x are the values of the decision variables. The classic outputs of such models are an optimal solution x^*

and the optimal value of the objective function z^*. As noted by Brill (1979), a linear programming model may not take into account all the objectives and all the constraints that are important for the stakeholder. Many issues cannot be quantified satisfactorily and the calculated optimal solution x^* is not necessarily the best solution in the real world. Better solutions may be found in the set of nearly optimal solutions, S_α, defined by (2), (3) and by

$$z \leq (1 + \alpha)\, z^* \qquad (4)$$

in which α is a small and positive constant which represents a tolerable deviation from z^*. The solutions included in S_α are all good in terms of objective function value but can differ considerably in terms of decision variable values.

In problems related to agricultural land use exploration, all the decision variables are somehow related to areas allocated to activities and, thus, are bounded. This implies that S_α defines a polytope in the n-space of the decision variables. The extreme points, vertices, of S_α are particularly interesting because there the maximum or the minimum of linear objective functions can be found. These solutions are useful to discriminate between the characteristics of the optimal land use allocation x^* that really make a difference for the objective value and the characteristics that do not. A study of nearly optimal solutions can thus provide information on the robustness of the optimal land use allocation to a slight deviation in the objective function value. Such a study avoids overemphasis on the optimal land use allocation and allows the stakeholder (*e.g.* policy maker or farmer) to choose a solution according to issues that are not quantified in the model.

A study of nearly optimal solutions involves two different problems: the generation of the nearly optimal solutions and the presentation of the generated solutions. The models developed for agricultural land use exploration often have several hundreds or even thousands of decision variables. This high number of decision variables complicates both the generation and the presentation of the nearly optimal solutions. A first consequence is that the extreme nearly optimal solutions are very numerous. Methods have been developed to enumerate all the extreme points of a polytope (Mattheiss and Rubin, 1980) but, when the number of decision variables is too high, the calculations are intractable. As a rule of thumb, Burton et al. (1987) fixed the upper limit on model size for a complete enumeration of the extreme points at 50 decision variables and 50 constraints. Thus, in practice, one can generate only a part of the possible extreme nearly optimal solutions. Different methods were developed in the 80's to generate some extreme nearly optimal solutions and were applied for water resource planning (Chang et al., 1982 ; Chang and Liaw, 1984 ; Harrington and Gidley, 1985), for public sector planning (Brill et al., 1982), to study the impact of free trade agreement on industry (Gibson et al., 1991) and for agricultural planning (Jeffrey et al., 1992 ; Willis and Willis, 1993 ; Abdulkadri and Ajibefun, 1998). However, the properties of the various methods developed to generate nearly optimal solutions have received little attention. As only a part of the possible nearly optimal solutions can be generated, some methods may generate solutions that are all situated in a particular region, subset of S_α. Others may provide a wider coverage of S_α.

A second consequence of the high number of decision variables is that each solution is equivalent to a high dimensional vector. Such vector cannot be presented directly in a graph or a short table. Consequently, it is necessary to summarize the nearly optimal solutions before presenting them.

In the current paper, we present a framework to study nearly optimal solutions of linear programming models developed for agricultural land use exploration. The principle is to generate a group of nearly optimal solutions, to summarize it and, finally, to present it graphically. The framework is applied to a model developed for land use exploration at the European level. Three different procedures are compared to generate nearly optimal solutions of this model.

2. A framework to study nearly optimal solutions

2.1. General outline

In this framework, a group of nearly optimal solutions is summarized by projection into different low dimensional spaces. The result of a projection of a solution is a low dimensional vector called 'an aspect of the solution'. The elements of such vector are called 'attributes of the solution'. Having a low dimension, aspects of the solutions can be presented graphically. Information on the characteristics of the group of nearly optimal solutions can be provided to stakeholders by defining different aspects.

The framework consists of three steps: (1) definition of the different aspects of solutions that will be shown; (2) generation of a group of nearly optimal solutions; (3) graphical presentation of the different aspects of the generated solutions.

2.2. Definition of the aspects of solutions

In the linear programming models developed for agricultural land use exploration, the decision variables are usually areas allocated to production activities. This implies that an optimal or nearly optimal solution is a vector whose elements are the areas allocated to the different production activities taken into account by the model. A production activity is defined by Van Ittersum and Rabbinge (1997) as cultivation of a crop or crop rotation in a particular physical environment, completely specified by its input and output. Consequently, an optimal or nearly optimal solution can be summarized by the allocation of the agricultural area between different regions, by the allocation of the agricultural area between different crop rotations or by the allocation of the agricultural area between different types of production techniques. These three types of allocations define respectively the regional ('where'), crop ('what') and technical ('how') aspects of a solution. Such aspects provide relevant information to stakeholders.

A particular aspect of a solution x can be seen as an m-dimensional vector, $m < n$, denoted by $a(x)$ and defined by

$$a(x) = R_m' x$$

$$R_m = [r_1, \ldots, r_j, \ldots, r_m]$$

R_m is an $n \times m$ matrix. The n-dimensional vectors $r_1, \ldots, r_j, \ldots, r_m$ each represent a group of activities and consist of zeros and ones such that $r_{j1}' r_{j2} = 0, \forall j_1 \neq j_2, j_1, j_2 = 1, ..., m$. Each element $a_j(x)$ of $a(x)$ is a sum of decision variables and represents an attribute of the solution x. If the decision variables are areas allocated to production activities, an attribute is an area allocated to a group of production activities. The vectors r_j can be chosen to identify groups of production activities characterized by particular regions, by particular crop rotations or by particular production techniques. An attribute of x is then an area allocated to a region, to a crop rotation or to a production technique and the vector $a(x)$ represents a regional, a crop or a technical aspect of the solution x.

2.3. Generation of optimal and nearly optimal solutions

The optimal solution is generated by common linear programming. Several ways can then be considered to generate nearly optimal solutions. In all cases, one must define the tolerable deviation α allowed for the objective function. This coefficient defines the set of nearly optimal solution, S_α. In a multi-objectives model, different levels of tolerable deviation can be used.

Nearly optimal solutions can be generated by maximizing or minimizing the function

$$f(x) = u'x$$

where u is an n-dimensional vector whose elements are 0 and 1. Maximization or minimization of f under the constraints defined by the relations (2), (3) and (4) results in an extreme nearly optimal solution. Various methods can be used to define vector u, for instance:

i. In the HSJ ('Hop, Skip, Jump') method (Brill et al., 1982), a first vector u is chosen such that the function f is the sum of the decision variables that are non-zero in the optimal solution x^*. This function is then minimized to obtain a nearly optimal solution. The vector u is then updated to generate other nearly optimal solutions that minimize the decision variables which are non-zero both in the optimal and the previous nearly optimal solutions.

ii. Another method is to generate randomly different vectors u and to maximize the corresponding functions f. Chang et al. (1982) propose to select randomly s decision variables and to maximize their sum.

iii. A third possibility is to choose u such that $u = R_m v$, where v is an m-dimensional vector whose elements are 0 or 1. Then, the function f is a particular attribute or a particular sum of attributes. The minimization or the maximization of f will generate a nearly optimal solution minimizing or maximizing a particular attribute or a particular sum of attributes.

The former methods allow generation of several nearly optimal solutions. The optimal solution and the generated nearly optimal solutions constitute a group of t solutions which can be represented in a $t \times n$ matrix X_t whose rows are the t solutions and whose columns are the n decision variables. An aspect of this group of solutions is defined by

$A_t = X_t\ R_m$

A_t is a $t \times m$ matrix. Each row of A_t is the aspect of a solution: $a(x)'$. Each column of this matrix represents the value of an attribute of the t solutions: $X_t\ r_j$. The next step is to present graphically the matrix A_t corresponding to one of the aspects of the group of solutions.

2.4. Presentation of an aspect of a group of solutions

2.4.1. Plots of pairs of attributes

One possibility to graphically represent A_t is to plot pairs of attributes, that is, to plot the points whose coordinates are in $X_t\ r_{j1}$ and in $X_t\ r_{j2}$, $j_1 \neq j_2$, j_1, $j_2 = 1, \ldots m$. The number of possible plots of pairs of attributes is equal to $\binom{m}{2}$. If the considered aspect has just two attributes, it can be represented by a simple graph. If a larger number of attributes has been defined, the number of possible plots of pairs of attributes may be high. For instance, one could draw 3 different figures if 3 attributes are considered, 6 different figures if 4 attributes are considered, 10 different figures if 5 attributes are considered. Thus, if the number of attributes is high, plotting all the possible pairs of attributes will overload the decision maker with information. In this case, a more synthetic presentation is called for.

2.4.2. Plots of principal components

A principal component analysis (Hotelling, 1933 ; Krzanowski and Marriot, 1990) can be used to reduce the dimension of the dataset A_t. The principle is to determine few linear combinations of the attributes that explain as much as possible of the variability of the data. We defined $A_t{}^*$ as the matrix A_t centered about the mean so that column totals are zero. The matrix $V_t = A_t{}^{*\prime} A_t{}^*/(t-1)$ is considered as an estimate of the variance-covariance matrix of the aspect $a(x)$. Let $\lambda_1, \ldots, \lambda_j, \ldots, \lambda_m$ be the m eigenvalues of $A_t{}^{*\prime}\ A_t{}^*$ arranged in decreasing order and $d_1, \ldots, d_j, \ldots, d_m$ their corresponding eigenvectors. We know that $d_{j1}'\ d_{j2} = 0\ \ j_1 \neq j_2\ \forall j_1, j_2$ and that d_1 is the axis which displays the highest variance of the data set, d_2 is the second one, and so on. The jth principal component of A_t is defined as the vector $A_t{}^*\ d_j$, where d_j is the eigenvector corresponding to the jth largest eigenvalue λ_j, and $var(A_t{}^*\ d_j) = \lambda_j$. The total inertia of A_t is defined as $I = \lambda_1 + \ldots + \lambda_m$. Consequently, the importance of the jth component in a more parsimonious description of the system can be measured by λ_j/I. This ratio is useful to determine the number of principal components that should be plotted to represent much of the variability of A_t.

3. Application

3.1. The GOAL-QUASI model

The GOAL-QUASI model (Van Ittersum et al., 1995 ; Hijmans and Van Ittersum, 1996) is a simplified version of the GOAL model (General Optimal Allocation of Land use) (Scheele, 1992 ; Rabbinge and Van Latesteijn, 1992). GOAL is a multiple goal linear programming model which was developed for exploration of land use options in the European Community (EC 12). The target group of the model were policy makers in Europe. The GOAL-QUASI model takes into account 706 decision variables. Each decision variable corresponds to a production activity describing where to produce, which crops and how to produce it. The constraints of the model include product balances, area constraints, water use constraints and manure balances. One of these constraints ensures self-sufficiency for agricultural products within the EC. Nine different agricultural, economic or environmental objectives can be optimized by the model. In this study we consider only the objective 'minimization of nitrogen loss (N loss) for total agricultural production in the EC'.

3.2. Definition of the aspects of solutions

Regional ('where'), crop ('what') and technical ('how') aspects and their corresponding attributes are given in Table 1.

Table 1: Attributes associated to regional, crop and technical aspects

Aspect	Attribute
Regional	Area allocated to the north of the European Community (North)
	Area allocated to the south of the European Community (South)
Crop	Area allocated to roughage (ROU)
	Area allocated to short rotations (SHO)
	Area allocated to long rotations without field bean (LO1)
	Area allocated to long rotations with field bean (LO2)
Technical	Area allocated to yield oriented and irrigated activities (YOP)
	Area allocated to yield oriented and non-irrigated activities (YOW)
	Area allocated to environmental oriented and irrigated activities (EOP)
	Area allocated to environmental oriented and non-irrigated activities (EOW)
	Area allocated to extensive activities (EXT)

These attributes are only examples. Others could be defined. Here, the regional aspect of a solution is a two dimensional vector [North, South]' whose attributes are the area allocated to the north of the European Community (Ireland, United Kingdom, Belgium, The Netherlands, Germany and France north) and the area allocated to the south of the European Community (Portugal, Spain, Italy, Greece and France south). The crop aspect of a solution is a four dimensional vector [ROU, SHO, LO1, LO2]' whose attributes are the areas allocated to four kinds of crops or crop rotations. The technical aspect of a solution is a five dimensional vector [YOP, YOW, EOP, EOW, EXT]' whose attributes are the areas allocated to five production orientations (a production orientation is defined as the aims and restrictions that direct the output and input levels of a production activity, Van Ittersum and Rabbinge, 1997).

3.3. Generation of nearly optimal solutions

We follow the steps presented in section 2.3. The optimal solution is calculated for the objective 'Minimization of nitrogen loss'. The optimal value of N loss in the EC is 2234.1 10^6 kg. We assume a deviation tolerance α of 5% from the optimal value for the level of N loss. This deviation tolerance defines a set of nearly optimal solutions. Three different methods are used to generate nearly optimal solutions:

Method i. A group of 15 nearly optimal solutions, called 'NOS1', is generated with the HSJ method. A first nearly optimal solution is generated by minimizing the sum of the decision variables that are non-zero in the optimal solution. A second nearly optimal solution is generated by minimizing the sum of the decision variables that are non-zero both in the optimal solution and in the first nearly optimal solution, and so on.

Method ii. A second group of 15 nearly optimal solutions, called 'NOS2', is generated by selecting randomly 15 different sets of 30 decision variables and by maximizing in successive runs the sums of values of these sets.

Method iii. A third group of 22 nearly optimal solutions, called 'NOS3', is generated by both maximizing and minimizing each one of the 11 attributes defined in Table 1.

All calculations are performed with OMP software (OMP manual, 1993).

3.4. Presentation of the aspects of the solutions

3.4.1. Regional aspect

The regional aspect of the group of solutions is presented graphically in a two dimensional space defined by the attributes North and South (Fig. 1). The range of variation of the two attributes is quite large. For North, the minimal and maximal values are respectively 10.9 10^6 ha (11% of the territory of the north of EC) and 50.2 10^6 ha (51% of the territory of the north of EC). For South, the minimal and maximal values are respectively 1.9 10^6 ha (1.4% of the territory of the south of EC) and 36.6 10^6 ha (28% of the territory of the south of EC). The values obtained for the optimal solution are intermediate : 26.5 10^6 ha for North and 26.0 10^6 ha for South.

Thus, the regional aspect of the nearly optimal solutions reveals that it is possible to modify greatly the optimal distribution of agricultural area between the north and the south of EC without increasing the level of nitrogen loss by more than 5%. Although the agricultural area is equally distributed between the north and the south in the optimal solution, nearly optimal solutions exist with both a high proportion of area allocated to the north and a high proportion of the area allocated to the south. The two attributes are not independent: when allocation to the north is high, allocation to the south was found to be rather small, and vice versa. This relation between attributes is due to the constraint which puts an upper bound on the total agricultural area and to the constraint which ensures self-sufficiency on the agricultural products. This later constraint implies a lower bound on the total agricultural area in EC. It should be noted that this apparent global substitutability observed between area allocated to the north and area allocated to the south of EC may not be possible for less aggregated levels, *i.e.* the level of provinces or municipalities.

Figure 1 shows that the solutions NOS3 provide a wider coverage of the space of the attributes than the solutions NOS1 and NOS2. The solutions NOS1 and NOS2 are all located in the center of the whole group of solutions and are rather close to the optimal solution. Moreover, it should be noted that the solutions NOS1 and NOS2 form disjunct clusters. For a given value of North, the solutions NOS2 have higher values of South than the solutions NOS1.

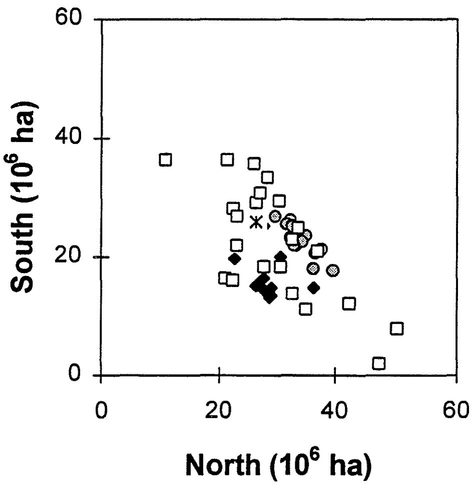

Figure 1: The regional aspect of optimal and nearly optimal solutions of GOAL-QUASI, represented by attributes North and South. The optimal solution (*) was calculated by minimizing the nitrogen loss in the EC. Fifty two nearly optimal solutions were generated using methods *i* (◆), *ii* (◎) and *iii* (□) with a tolerable deviation of 5% from the minimal value of nitrogen loss.

3.4.2. The crop aspect

The crop aspect of the group of solutions was studied first by plotting the different pairs of attributes. Two of these plots are shown in Figure 2: the plot of ROU and LO2 (Fig.2a) and the plot of SHO and LO1 (Fig.2b).

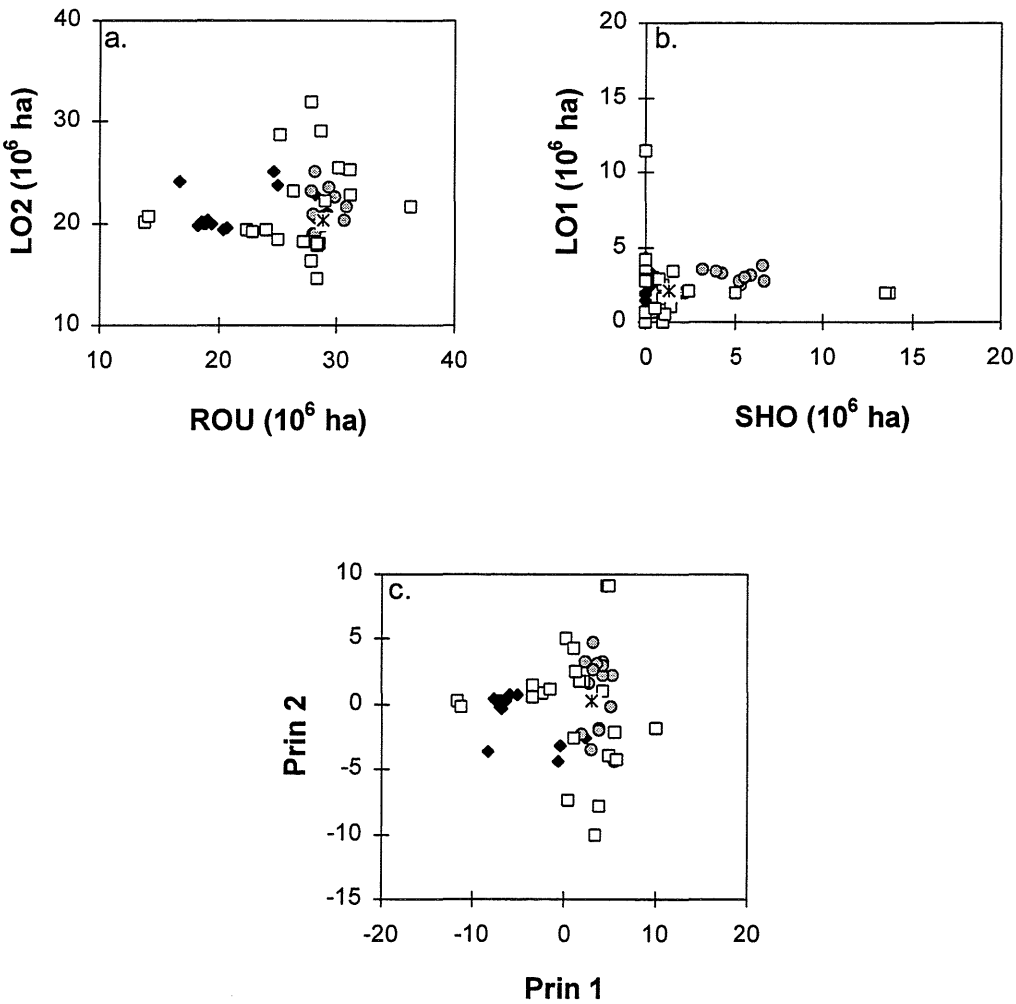

Figure 2: The crop aspect of optimal and nearly optimal solutions of GOAL-QUASI, represented by (a) attributes ROU and LO2, (b) attributes SHO and LO1, and (c) the first two principal components (Prin1, Prin2). The optimal solution (*) was calculated by minimizing the nitrogen loss in the EC. Fifty two nearly optimal solutions were generated using methods *i* (◆), *ii* (◎) and *iii* (□) with a tolerable deviation of 5% from the minimal value of nitrogen loss.

Prin1 = 0.96 ROU + 0.24 SHO - 0.08 LO1 + 0.15 LO2 (centered values)

Prin2 = 0.0007 ROU + 0.57 SHO + 0.24 LO1 - 0.78 LO2 (centered values)

The values of ROU and LO2 are higher than the values of SHO and LO1. Moreover, ROU and LO2 have a wider range of variation among the solutions than SHO and LO1. No particular relation between attributes has been noticed.

Another way to study the crop aspect is to use principal component analysis. A principal component analysis was performed on the 53 optimal and nearly optimal solutions using the procedure *princomp* of the SAS software (SAS/STAT User's Guide, 1990). The four attributes of the crop aspect were considered as variables. The eigenvalues of the variance-covariance matrix are shown in Table 2.

Table 2: Eigenvalues of the variance-covariance matrix obtained for the crop aspect

Axis	Eigenvalue	Proportion[a]	Cumulative
1	26.76	0.57	0.57
2	12.88	0.27	0.84
3	6.24	0.13	0.97
4	1.42	0.03	1

[a] Proportion = $\lambda_i \Big/ \sum_{i=1}^{4} \lambda_i$

These eigenvalues can be used to calculate the proportion of variance explained by each axis. We see that the first two axes explain 84% of the total variance of the group of solutions. Thus, the plot of the first two principal components (scores of the solutions on the first two axes) seems sufficient to represent the crop aspect of the group of solutions. This plot is shown in Figure 2c. The attribute ROU receives a high positive loading on the first component. Thus, solutions with high values of ROU are on the right side of the graph and solutions with low values of ROU are on the left side of the graph. The attribute SHO and LO2 have, respectively, a high positive and a high negative loading on the second component. Thus, the second component measures the preponderance of the attribute SHO over the attribute LO2. Solutions with high values of SHO and low values of LO2 are on the upper side of the graph and solutions with low values of SHO and high values of LO2 are on the lower side of the graph. One should notice that the highest coefficients of the principal components are the coefficients of the attributes which have the widest range of variation.

The crop aspect of the nearly optimal solutions reveals that one can modify greatly the optimal distribution of agricultural area among rotations without increasing the level of nitrogen loss by more than 5%. For instance, it is possible to find nearly optimal solutions that have, compared to the optimal solution, a lower area allocated to roughage, a higher area allocated to long rotations with field bean or a higher area allocated to short rotations. On the other hand, one can not find nearly optimal solutions with very low area allocated to roughage or to long rotations with field bean. The fact that the area allocated to roughage remains relatively high among solutions may be due to the constraint in the model which ensures self-sufficiency on agricultural products. Agricultural products include animal products and the level of animal products depends on the level of roughage.

As noted also for the regional aspect, the solutions NOS3 provide a wider coverage of the space of the attributes than the solutions NOS1 and NOS2. Moreover, the solutions NOS1 and NOS2 form disjunct clusters: the solution NOS2 have higher values of SHO and of ROU than the solutions NOS1 (Fig. 2).

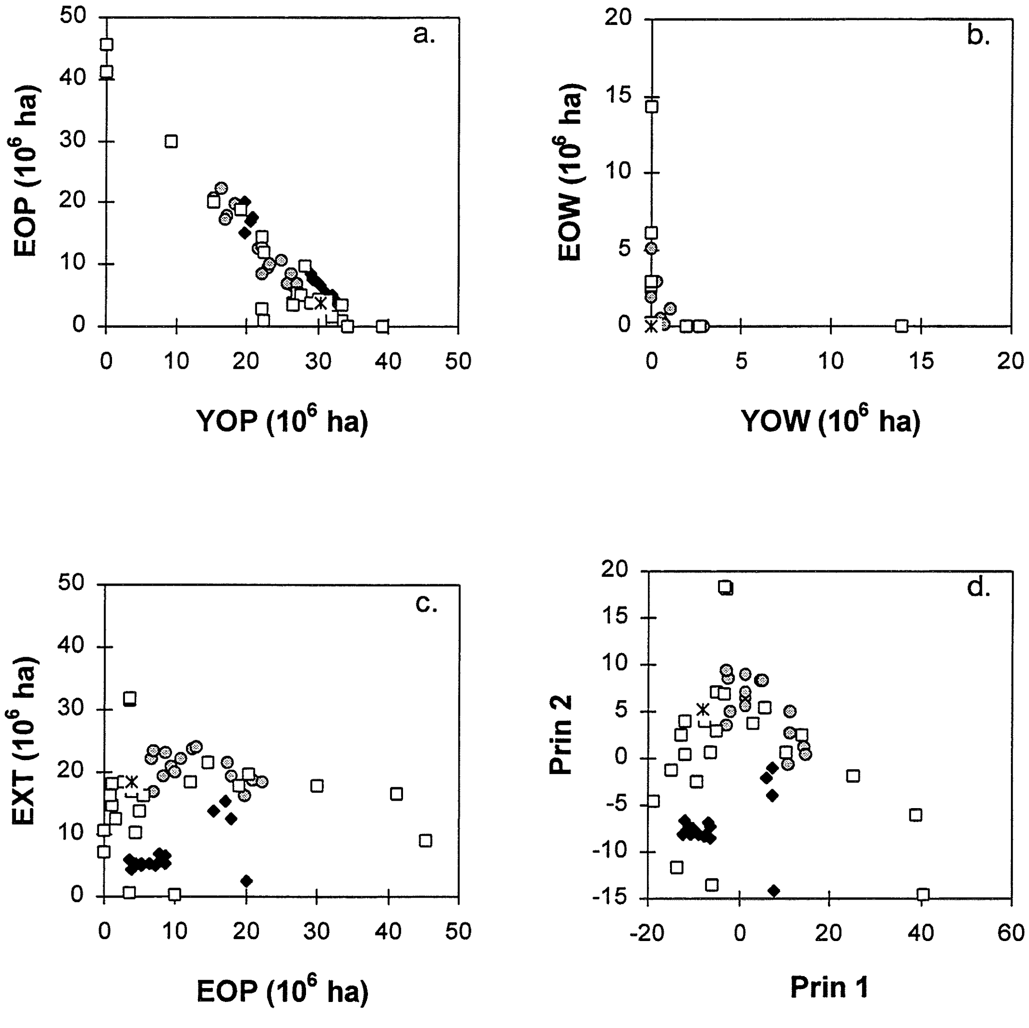

Figure 3: The technical aspect of optimal and nearly optimal solutions of GOAL-QUASI, represented by (a) attributes YOP and EOP, (b) attributes YOW and EOW, (c) attributes EOP and EXT, and (d) the first two principal components (Prin1, Prin2). The optimal solution (*) was calculated by minimizing the nitrogen loss in the EC. Fifty two nearly optimal solutions were generated using methods *i* (◆), *ii* (◎) and *iii* (□) with a tolerable deviation of 5% from the minimal value of nitrogen loss.

Prin1 = -0.63 YOP - 0.006 YOW + 0.75 EOP + 0.02 EOW + 0.2 EXT (centered values)

Prin2 = -0.05 YOP + 0.05 YOW - 0.29 EOP + 0.06 EOW + 0.95 EXT (centered values)

3.4.3. The technical aspect

The technical aspect of the group of solutions was studied first by plotting the different pairs of attributes. Three of these plots are shown in Figure 3: the plot of YOP and EOP (Fig.3a), the plot of YOW and EOW (Fig.3b) and the plot of EOP and EXT (Fig.3c). The attributes YOP, EOP and EXT have a wider range of variation than the attributes YOW and EOW. The latter two attributes remain always small. Figure 3a shows a strong substitutability between EOP and YOP: if YOP is high EOP is small and if YOP is small EOP is high. No other particular relation between attributes has been noticed.

A principal component analysis was performed on the optimal and nearly optimal solutions by considering as variables the five attributes of the technical aspect. The eigenvalues of the variance-covariance matrix are shown in Table 3.

Table 3: Eigenvalues of the variance-covariance matrix obtained for the crop aspect

Axis	Eigenvalue	Proportion[a]	Cumulative
1	148.79	0.69	0.69
2	56.5	0.26	0.95
3	5.99	0.03	0.98
4	4.99	0.02	0.998
5	0.39	0.002	1

[a] Proportion = $\lambda_i \Big/ \sum_{i=1}^{5} \lambda_i$

We see that the first two axes explain 95% of the total variance of the group of solutions. Thus, the plot of the first two principal components seems sufficient to represent the technical aspect of the group of solutions. This plot is shown in Figure 3d. The attributes YOP and EOP have, respectively, a high negative and a high positive loading on the first component. Thus, the substitution between YOP and EOP noticed in Fig.3a appears in the definition of the first principal component. This component measures the preponderance of EOP over YOP. Consequently, solutions with high values of YOP and low values of EOP are on the left side of the graph and solutions with low values of YOP and high values of EOP are on the right side of the graph. The attribute EXT has a high positive loading on the second component. Thus, solutions with high values of EXT are on the upper side of the graph and solutions with low values of EXT are on the lower side of the graph.

The technical aspect of the nearly optimal solutions shows that the optimal distribution of the area between production orientations can be modified greatly without increasing the level of nitrogen loss by more than 5%. Although the optimal solution is characterized by a high value of YOP, by a low value of EOP, and by an intermediate value of EXT, one can find nearly optimal solutions with a low value of YOP and a high value of EOP, or with quite different value of EXT. On the contrary, the values of YOW and EOW can not be greatly modified and always remain relatively low among the nearly optimal solutions.

Table 4: Average 'N loss / Yield' ratios obtained for the five production orientations

Attribute	N loss ($kg.ha^{-1}$) / Yield ($t.ha^{-1}$)
YOP	2.42
YOW	5.45
EOP	2.1
EOW	4.05
EXT	3

The difference of ranges of variation between attributes in the nearly optimal solutions may be explained by the values of technical coefficients associated to the activities. Table 4 shows the average values of the ratio 'N loss / Yield' calculated, for each production orientation, from the technical coefficients of the activities. The average ratios associated to YOP and EOP activities are the lowest and are very similar. This may explain that the areas allocated to YOP and EOP activities can reach high values in the nearly optimal solutions and that the two types of activities can be mutually substituted. The high ratios associated to YOW and EOW production orientations may explain that the areas allocated to YOW and EOW activities remain low among the solutions.

As noted for the previous aspects, the solutions NOS3 provide a wider coverage of the space of the attributes than the solutions NOS1 and NOS2. Moreover, the solutions NOS1 and NOS2 form disjunct clusters: the solutions NOS2 have higher values of EXT than the solutions NOS1 (Fig. 3).

4. Conclusion

We have described a framework to visually represent images of a group of nearly optimal solutions by means of aspects. An aspect of a solution has been defined as a low dimensional vector whose elements are particular attributes. This framework is put forward as a solution to practical problems associated with using optimization techniques for decision support. In our experience linear programming is useful for structuring information in complex decision problems. At the same time, a mathematical model is necessarily a simplification of reality, and mere presentation of 'the optimal solution' is of little relevance to stakeholders. The framework was developed from the premise that not all of the stakeholder's objectives are represented in the model and that sacrificing some of the objective function optimal value may be useful to create a solution space for other, hidden, objectives. This approach thus emphasizes incomplete model specification. The description of the system is not altered except in one way, by controlled relaxation of the optimality criterion.

The first step of the framework - definition of aspects of solutions that are relevant to a stakeholder - requires close interaction between modeler and model-user. Analysis of a particular set of aspects may call for definition of new aspects. Such iterative interrogation of the model helps to learn about reality and contributes to informed decision making (Leeuwis, 1993).

In the second step a tolerance deviation is allowed for the objective function(s) and a group of nearly optimal solutions is generated. In our application we have only considered a deviation for the value of total N loss in the EC. In multi-objective models, which are common in agricultural land use exploration, a deviation can be allowed for several objective functions.

The results (Fig.1-3) reveal the sensitivity of nearly optimal solutions to the method used to generate them. The methods *i* and *ii* were found to generate solutions that have highly similar attribute values. Thus, compared to the solutions generated with the method *iii*, the solutions generated with the methods *i* and *ii* do not provide a wide coverage of the spaces of attributes. The range of variation of the attribute values within the whole set of nearly optimal solutions can be strongly underestimated with the methods *i* and *ii*. For instance, the maximum value of the area allocated to the north of the EC is equal to 36.1 10^6 ha for the solutions calculated with the method *i*, is equal to 37.5 10^6 ha for the solutions calculated with the method *ii* but is equal to 50.2 10^6 ha for the solutions calculated with method *iii*. The results obtained with the methods *i* and *ii* could be improved by increasing the number of generated solutions but it is not possible to know whether the maximal and minimal values of the different attributes are well approximated or not. The advantage of method *iii* is that it gives, by definition, exactly the minimal and maximal values of each attribute in the whole set of nearly optimal solutions.

In the third step of the method the aspects of the generated nearly optimal solutions are presented. We have proposed a graphic presentation either by pairwise plotting of attributes or by principal component analysis. The latter type of presentation is useful when the number of attributes exceeds four or five.

A study of nearly optimal solutions discriminates between the characteristics of the optimal land use allocation that can be changed without unduly depreciating the objective function value and those that can not. It has been noted in our application that some attributes vary widely among nearly optimal solutions, while others behave more conservatively. For instance, the areas allocated to yield oriented and irrigated activities (YOP) in the nearly optimal solutions varied between 0 and 39.3 10^6 ha, whereas the areas allocated to yield oriented and non-irrigated activities (YOW) did not exceed the range of 0 to 13.9 10^6 ha. This type of information on the range of solutions all resulting in an acceptable level of the objective function is expected to provide a stakeholder with 'manoevering space' necessary to resolve complex decision problems.

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