NOTA 1421 April 1983
Instituut voor Cultuurtechniek en Waterhuishouding
Wageningen

CALCULATION OF ACTUAL EVAPOTRANSPIRATION AND CROP PRODUCTION FOR THE SOUTHERN PEEL AREA

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CONTENTS

1. CALCULATION OF ACTUAL EVAPOTRANSPIRATION

In the model approach for the Southern Peel Area actual evapotranspiration is needed for two main reasons:

- to serve as an input of the water balance of the unsaturated zone in the calculation of saturated groundwater flow (see report of Van Bakel, Smidt and Wit);
- to serve as an input for the crop production model.

From steady state solutions of moisture flow in soils it can be derived that their exist a relationship between height of capillary rise above the groundwater table and soil moisture pressure head at different fluxes of upward flow. Assuming a more or less constant pressure head at the underside of the root zone, one can derive a single relation between the average upward flux $\bar{v}_z$ and the groundwater table depth, h^* (Fig. 1) for different soil types.

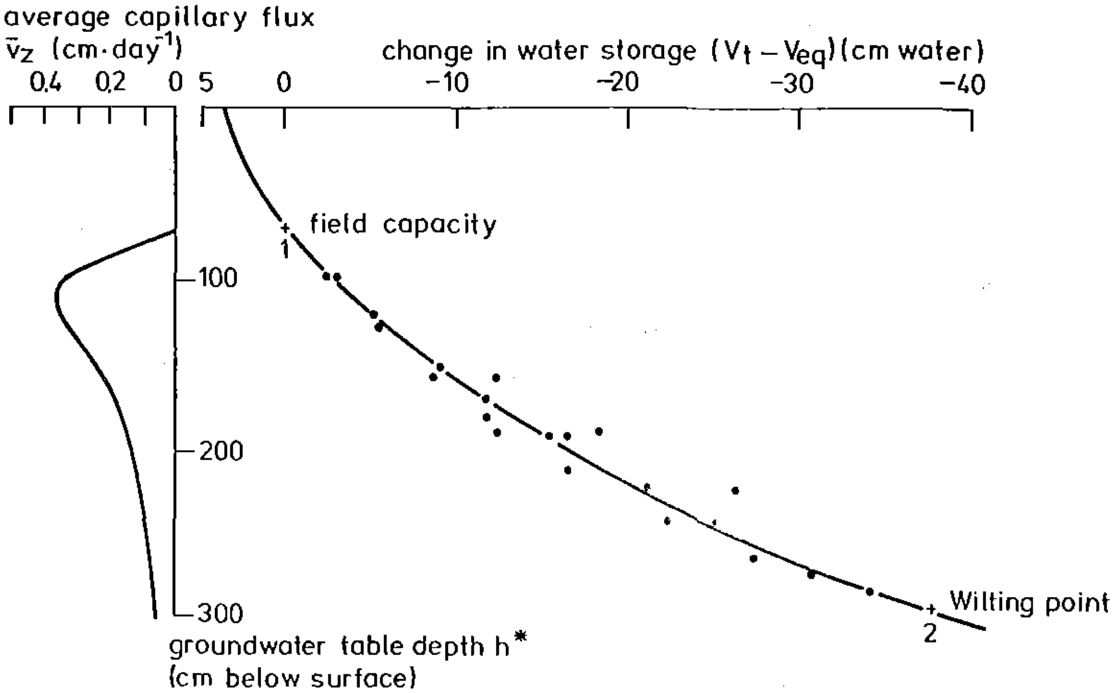

Fig. 1. Fixed relationship between average capillary flux, v_z, change in water storage, $(V_t - V_{eq})$, and groundwater table depth, h^*_o, for a sandy loam soil with an initial groundwater table depth of 70 cm, an initial equilibrium pressure head profile, a fixed rooting depth of 30 cm

It appears that there exist also a relationship between the change in water storage of the profile ($V_t - V_{eq}$) and the average capillary flux $\bar{v}_z$ (Fig. 1). These type of relationships will have to be tested later by the SWATRE program for a number of crops and soil profiles for a few selected actual years that represent both 'dry' and 'wet' conditions.

Over a period of 10 days, the water balance can be written as:

$$V_t = V_{eq} + P_n + P_{irr} + \bar{v}_z \cdot \Delta t - E_a \qquad \text{(cm)} \qquad (1)$$

where:

V_t = moisture storage of the soil profile at time t

V_{eq} = moisture storage of the soil profile at equilibrium conditions, i. e. at field capacity

P_n = net natural rainfall, i. e. accounted for rainfall interception by the crop

P_{irr} = irrigated amount of rainfall by sprinkling

$\bar{v}_z \cdot \Delta t$ = average upward flow from the groundwater table

Δt = 10 days

E_a = actual evapotranspiration

For eq. (1) the condition holds that if $V_t > V_{eq}$, then

$$\bar{v}_z \cdot \; t = V_{eq} - V_t \qquad (2)$$

and in the next time step one starts again with V_{eq}.

This assumption is reasonable as most of the groundwater tables in the Southern Peel area are within 2 m of the soil surface.

In eq. (2) actual evapotranspiration, E_a is estimated as a simple function of potential evapotranspiration, E_p:

$$E_a = \alpha E_p \qquad (3)$$

The most simple thing now to do is to assume that the ability of plants to satisfy their water needs is proportional to the ratio of actual water stored in the soil, V_t, to the amount that can be stored at equilibrium conditions V_{eq}, hence that (Fig. 2):

$$\alpha = \frac{V_t}{V_{eq}} \qquad (4)$$

or that

$$E_a = \frac{V_t}{V_{eq}} \cdot E_p \qquad (5)$$

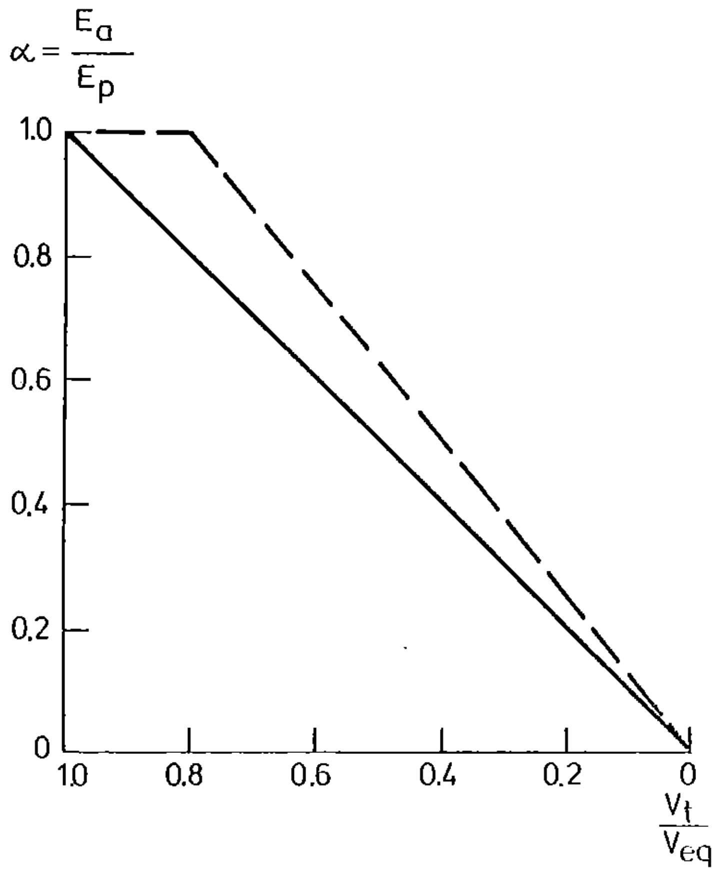

Fig. 2. Relationship between relative evapotranspiration, E_a/E_p and relative available soil moisture, V_t/V_{eq}, (continuous line). An alternative relationship could be the dotted line

An alternative expression is the dotted line, also shown in Fig. 2. Calculations with SWATRE are needed to find out which kind of relationship is the best.

Notes:

- In order to establish the time of irrigation, one can take that if $\alpha = V_t/V_{eq} \leqslant 0.7$ say (or any other limit) irrigation, P_{irr} will be applied to fill up the soil to equilibrium conditions again (or any other required limit).

- As roots act on the moisture situation inside the root zone, one should in fact take into account the soil moisture storage inside this root zone and not in the entire soil profile. It is expected that there is not much difference, however, taking either V_t or $V_{rootzone,\ t}$.

Introducing now eq. (5) into eq. (1) one can write (after some re-arrangement)

$$V_t = \frac{P_n + P_{irr} + \bar{v}_z \Delta t + V_{eq}}{1 + \frac{E_p}{V_{eq}}} \quad \text{(cm)} \qquad (1a)$$

Eq (1a) can now be solved: P_n and E_p are for each 10-day period, V_{eq} is a fixed number and $\bar{v}_z \Delta t$ is determined from the groundwater table depth at the previous 10-day period. If the latter assumption is too crude some kind of iteration procedure has to be included.

2. CALCULATION OF ACTUAL CROP PRODUCTION

2.1. Optimal nutrient supply

For the calculation of dry matter production Q ($kg.ha^{-1}$) under optimal nutrient conditions over a period of 10 days one may use an equation simular to the one proposed by Feddes et al (1978) in the so called CROPR model (see also Fig. 3).

$$\left(1 - \frac{Q_{act}}{A_w W}\right)\left(1 - \frac{Q_{act}}{Q_{pot}}\right) = \xi \qquad (6)$$

where

Q_{act} = actual dry matter yield ($kg.ha^{-1}$)
A_w = water use efficiency factor ($kg.mbar.cm^{-1}.ha^{-1}$)
W = evapotranspiration E_a (cm)/average vapour pressure deficit $\overline{\Delta e}$ mbar
Q_{pot} = potential dry matter yield ($kg.ha^{-1}$)
ξ = mathematical constant ($\xi \approx 0.01$?)

When solving eq. (6) explicitly for the actual dry matter yield Q_{act}, one obtains:

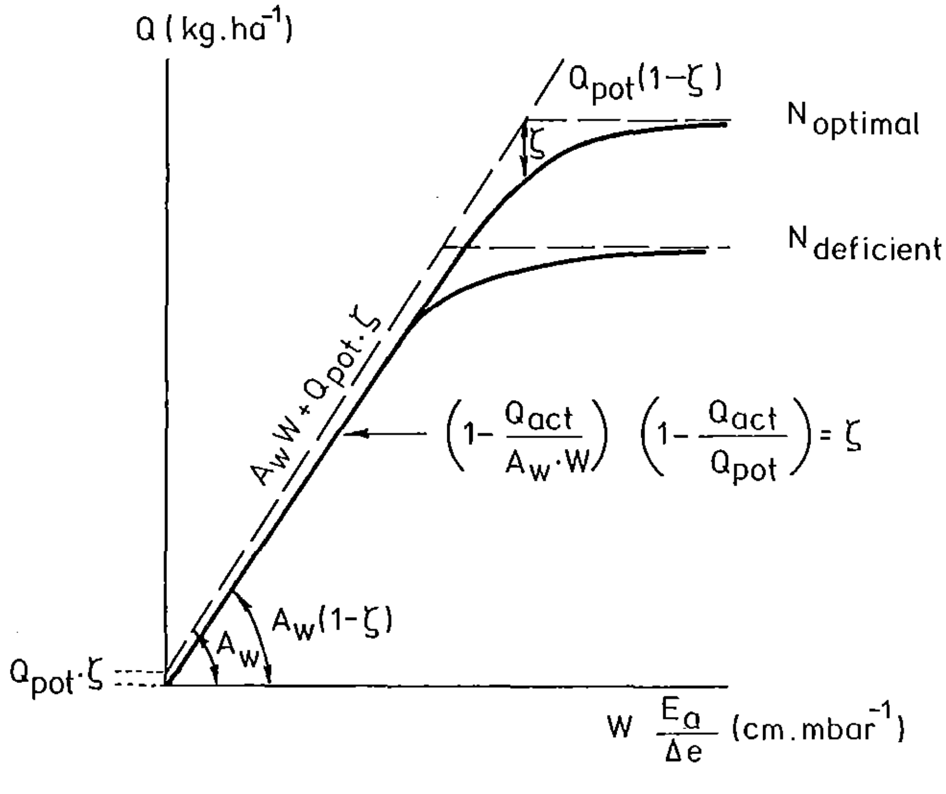

Fig. 3. Dry matter yield Q versus the growth factor water, $E_a/\Delta e$. The upper curve is for optimal nutrient conditions, the lower curve for deficiency in nitrogen, N, conditions

When solving eq. (6) explicitly for the actual dry matter yield Q_{act}, one obtains:

$$Q_{act} = \tfrac{1}{2}\left\{A_w \frac{E_a}{\overline{\Delta e}} + Q_{pot} - \left[\left(Q_{pot} + A_w \frac{E_a}{\overline{\Delta e}}\right)^2 - 4 Q_{pot} A_w \frac{E_a}{\overline{\Delta e}} (1-\zeta)\right]^{\frac{1}{2}}\right\} \quad (kg.\,ha^{-1}) \quad (7)$$

In eq. (7) A_w is fixed; E_a is obtained from eq. (5), $\overline{\Delta e}$ from weather records; For the estimation of Q_{pot} one can follow the approach of Feddes at al (1978, verg. 7.4/7.5, with changes from $q \rightarrow Q$ and $P_{o,c} \rightarrow P^1_{o,c}$) :

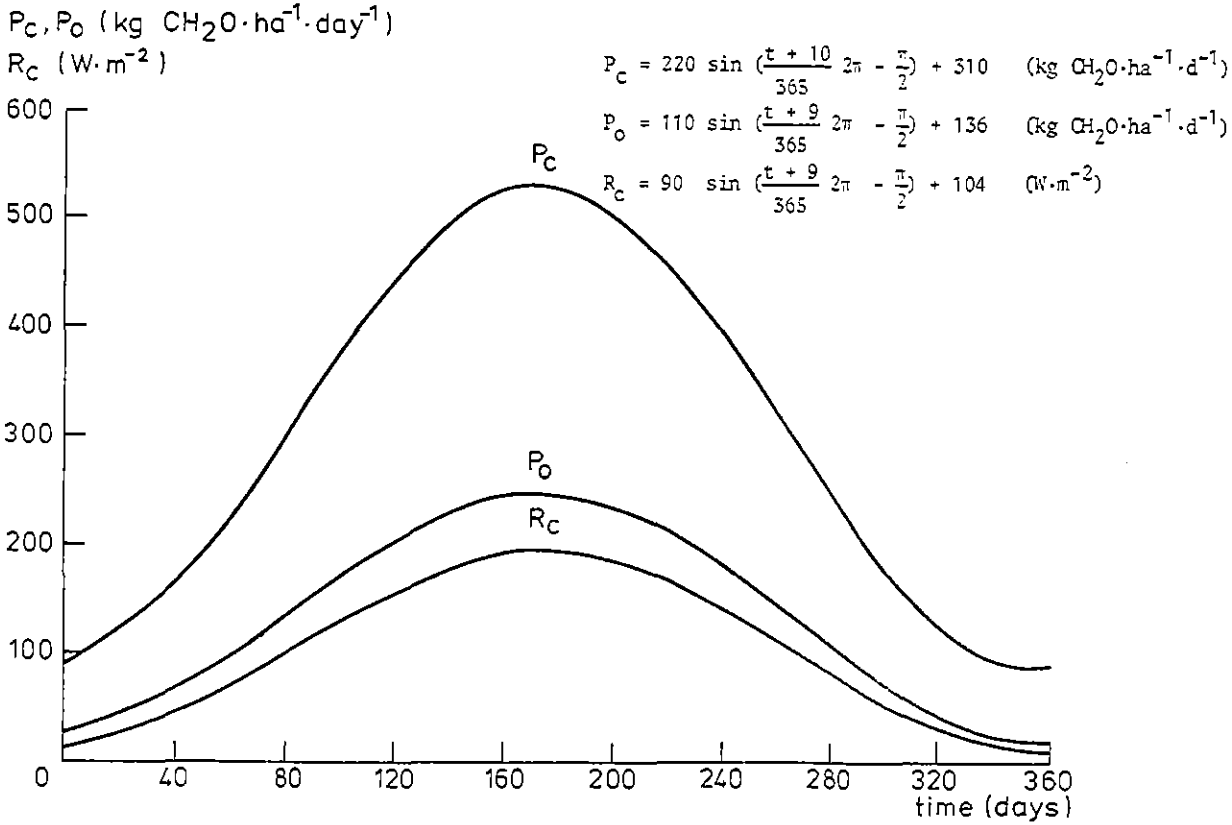

Fig. 4. Annual variation of gross photosynthesis at 52 °North latitude (Netherlands) of a 'standard canopy' on clear days (P_c) and on overcast days)P_o). Also shown is the variation of the solar radiation flux (R_c) involved in photosynthesis on clear days. Based on the model of Goudriaan and Van Laan (1978), De Wit (1965).

$$Q_{pot} = [\bar{\Lambda}\ \bar{P}_o^1 + (1-\bar{\Lambda})\ \bar{P}_c^1]\ \phi_r .\ \bar{S}_c \quad (kg.\ ha^{-1}) \tag{8}$$

where $\bar{\Lambda}$ = average fraction of time the sky under the actual conditions is overcast (-)

$\bar{P}_o^1, \bar{P}_c^1$ = average gross dry matter yield of a theoretical standard crop on overcast resp. clear days (kg. CH_2O ha^{-1}), see Fig. 4.

ϕ_r = reduction factor for respiration ($\phi_r \approx 0.7$ kg DM. kg^{-1} CH_2O)

S_c = average soil cover during the period considered (-)

The value of $\bar{\Lambda}$ can be found from the expression

$$\bar{\Lambda} = \frac{\bar{R}_c - 0.5\ \bar{R}_s}{0.8\ \bar{R}_c} \quad (-) \tag{9}$$

where: $\bar{R}_c$ = part of averaged solar radiation flux that is involved in photosynthesis (0. 4-0. 7 µm) in clear days (W. m^{-2}), see Fig. 4

$\bar{R}_s$ = averaged short wave (global) radiation flux (W. m^{-2})

Note that Q_{pot} is the theoretically maximum possible total dry matter yield that only can be reached asymptotically when nutrients are supplied optimally. To arrive at the part that is actually harvested (tubers, grain, etc.) some reduction factor will be used (see 2. 3).

2. 2. Restricted/ abundant nitrogen supply

Once the dry matter yield Q under optimal nutrient supply has been obtained, one has to correct for the actual nutrient supply, i. e. to nitrogen. Then a yield curve $N_{deficient}$ as depicted in Fig. 4 is valid.

Now we are not extending eq. (6) with the introduction of a nitrogen term, but are applying another procedure. This procedure relies entirely on data of Lammers of the 'Consulentschap voor Bodemaangelegenheden'. Relative yield data, Q_{act}/Q_{pot} have been plotted against relative nitrogen supply data, N_{app}/N_{opt} for sugar beet, potatoes, winter wheat, maize, horticulture and grass (fig. 5). Data refer to sandy soils.

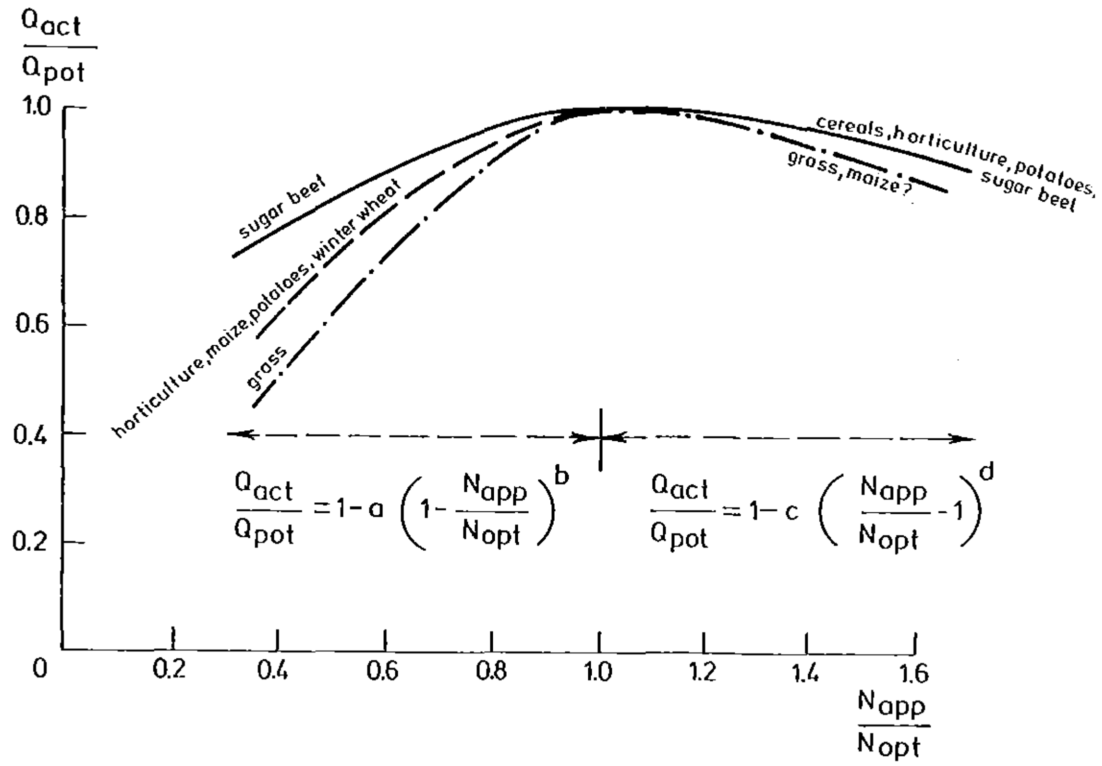

Fig. 5. The influence of relative nitrogen supply, N_{app}/N_{opt} on relative dry matter yield, Q_{act}/Q_{pot}

The assumptions underlying these data are summarized in Table 1.

Table 1. Example of the harvested (eq.12) productions for six main crops on a sandy soil and the amounts of nitrogen applied related to the available nitrogen in that soil

Crop type	production (kg. ha-1)	N demand (kg. ha^{-1})	N available in soil (kg. ha^{-1})	N applied (kg. ha^{-1})
sugar beet	7200 kg sugar	300	68	232
winter wheat	6000 kg grain	200	40	160
potatoes	45000 kg potatoes	310	53	257
grass	15000 kg dry matter	872	297	575
maize	?	?	?	?
horticulture	?	?	?	?

The curves of Fig. 5 can be described by the following functions:

$$\frac{Q_{act}}{Q_{pot}} = 1 - a\left(1 - \frac{N_{app}}{N_{opt}}\right)^{b} \quad \text{for} \quad 0 \leqslant \frac{N_{app}}{N_{opt}} \leqslant 1 \qquad (10)$$

$$\frac{Q_{act}}{Q_{pot}} = 1 - c\left(\frac{N_{app}}{N_{opt}} - 1\right)^{d} \quad \text{for} \quad \frac{N_{app}}{N_{opt}} > 1 \qquad (11)$$

In Table 2 the values of a, b, c and d are listed for 6 categories of crops.

Table 2. Values of the coefficients a, b, c and d as used in eqs(10) and (11) for four groups of crops

Crop	a	b	c	d
grass	1.147	1.618	0.168	1.576
cereals,potatoes,sugarbeet	0.859	1.683	0.302	1.688
maize	0.859	1.683	0.168(0?)	1.576(0?)
horticulture	0.859	1.683	0.302	1.688

2.3. Crop production on farms

In practice crop productions are usually smaller than the theoretical productions as calculated for water and/or nitrogen shortages. This is mainly due to factors not included in the models, such as differences in management, problems with diseases, etc. Hence the actual production Q_{act} has to be reduced by a proportionality factor γ to reach the production found in practice, $Q_{pract.}$

$$Q^{harv.\ prod.}_{pract.} = \gamma\ Q_{act} \qquad (12)$$

The value of γ has to be derived from comparison of harvested production data in the field with theoretically, computed total dry matter yield data.

3. LITERATURE

FEDDES, R.A., P.J. Kowalik and H. Zaraday, 1978. Simulation of field water use and crop yield. Pudoc, Wageningen.

List of symbols

Symbol	Unit	Explanation
V_t	cm	moisture storage of the soil profile at time t
V_{eq}	cm	moisture storage of the soil profile at equilibrium
P_n	cm	net precipitation
P_{irr}	cm	artificial amount of rainfall
$\bar{v}_z$	$cm.d^{-1}$	upward flux from the groundwater table
E_a	cm	actual evapotranspiration
E_p	cm	potential evapotranspiration
Q_{act}	$kg.ha^{-1}$	actual dry matter yield
Q_{pot}	$kg.DM.ha^{-1}$	potential dry matter yield
$Q^{harv.}_{pract.}$	$kg.DM.ha^{-1}$	actual harvested dry matter yield by farmers
Δe	mbar	vapour pressure deficit of the air
A_w	$kg.mbar.cm^{-1}.ha^{-1}$	maximum water use efficiency
ξ	-	mathematical constant
$\bar{\Lambda}$	-	average fraction of time the sky is overcast
P_o^1, P_c^1	$kg.CH_2O.ha^{-1}$	average gross dry matter yield of the theoretical standard crop (leaf area index of 5) on overcast resp. clear days
ϕ_r	$kg.DM.kg^{-1}CH_2O$	reduction factor for respiration
$\bar{S}_c$	-	average fraction of soil cover
$\bar{R}_c$	$W.m^{-2}$	average solar radiation flux involved in photosynthesis (0.4 - 0.7 µm)
$\bar{R}_s$	$W.m^{-2}$	average shortwave (global) radiation flux
N_{app}, $N_{opt.}$	$kg.ha^{-1}$	applied resp. optimal nitrogen application
γ	-	factor to convert theoretical computed total dry matter yields into actual harvested dry matter product