Time-dependent reliability analysis of flood defences using gamma processes

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ABSTRACT: Inclusion of time-dependent effects in flood risk analysis is useful to inform inspection and maintenance strategies. Several deterioration processes are incorporated in a reliability analysis of an earth embankment along the Thames Estuary based on multiple failure modes. Also, a biavariate gamma process model is developed to take correlated deterioration processes into account. That model is applied to motor crossing events that damage both the crest level and the vegetation on the rearslope. As neither process-based models nor statistical data on deterioration processes due to motor crossing are available, an expert judgment-based model is developed. The model is valid under restrictive assumptions about stationarity, independency and linear proportionalities. The results highlight that the sensitivity of the reliability to a change in different random variables can significantly vary among the variables. That sensitivity has repercussions for the influence of the correlation model in this application. It is concluded that assumptions regarding the gamma process model can have a large impact on the time-dependent reliability, and should be made with care. When developing a time-dependent reliability model, the variables in the 'snap shot' reliability model for which the reliability is highly sensitive require most attention.

1 INTRODUCTION

Flood risk analysis methods widely receive attention as a valuable means to obtain insight in the most vulnerable areas in a floodplain, see HR Wallingford (2004), Vrijling (2001). Within flood risk assessment, methods to determine the reliability of a single flood defence and of flood defence systems as a 'snapshot in time' are well established, e.g. Voortman et al. (2002).

Additional to 'snap shot' reliability, insight in the time-dependent development of flood defence reliability and hence flood risk is desired. In the UK, more than 80% of the flood defence budget is spent on maintenance and repair of flood defences. Inclusion of time-dependencies in flood risk analyses is then useful to inform inspection and maintenance strategies.

This paper presents a time-dependent reliability analysis of earth embankments situated along the Thames Estuary near Dartford Creek (see figure 1 for site location). The probabilistic model of a *singular* deterioration process is modelled with a gamma process as suggested in Van Noortwijk et al. (1997). In this paper, in addition, that gamma process deterioration model is used to represent singular deterioration processes in multiple correlated failure modes. Additionally, this paper explores the possibilities to extend the gamma process model to a situation where two different condition parameters deteriorate sharing a common deterioration source.

The rest of this paper is built up as follows:

- Section 2: description of failure modes and deterioration processes included in the reliability analysis.
- Section 3: explanation of the gamma process deterioration model for a singular deterioration process and for correlated processes due to a common deterioration source.
- Section 4: discussion of the results
- Section 5: conclusions

2 'SNAP SHOT' RELIABILITY MODEL AND APPROACH OF DETERIORATION PROCESSES THEREIN

2.1 *'Snap shot' reliability model*

The failure modes taken into account in this study are: uplifting, piping and overtopping/overflow leading to erosion of the rearslope and breach. In table 1 a definition of the failure modes and associated limit

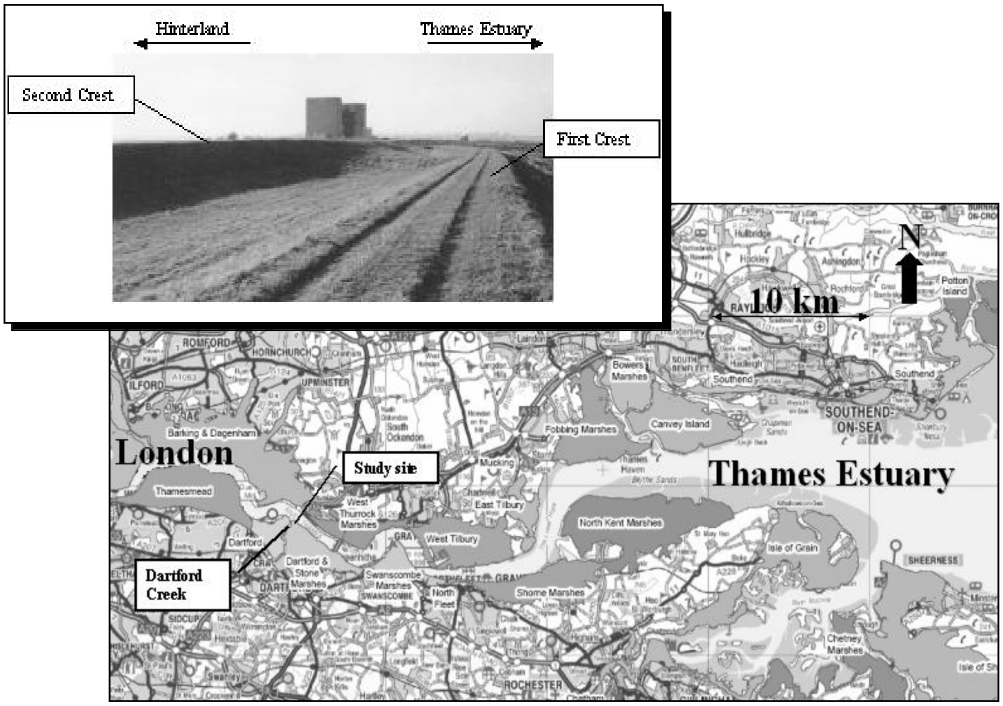


Figure 1. London, floodcells along the Thames Estuary and the location of the earth embankments near Dartford Creek

state functions are given that are used in this paper, see for details of the underlying models: Vrouwenvelder et al. 2001, Lassing et al. 2003, Buijs et al. 2003. The failure modes of the 'double crested' earth embankment are analysed for three different water level zones, see figure 2, resulting in the following selections:

- <u>*Zone 1, h ≤ hc1:*</u> uplifting (Pf_u) and piping (Pf_p), this combination is required as the hydraulic head first causes uplifting of the impermeable layers and then drives a water flow through the burst passage leading to erosion of the sand layer underneath the impermeable layers, a process called piping (see table 1). Therefore both failure modes must occur for failure of the embankment → $Pf_u \cap Pf_p$
- <u>*Zone 2, hc1 < h < hc2:*</u> uplifting and piping, or wave overtopping (Pf_o) →

  $$\left(Pf_u \cap Pf_p\right) \cup Pf_o$$
- <u>*Zone 3, h ≥ hc2:*</u> uplifting and piping, or overflow (Pf_{ov}) →

  $$\left(Pf_u \cap Pf_p\right) \cup Pf_{ov}$$

The probability of failure for each failure mode are calculated with methods based on those applied in structural reliability, see for instance: Ditlevsen et al., 1996 and Thoft-Christensen, 1982. In this approach a limit state function is used to define failure due to a failure mode:

$$Z = g\left(\overrightarrow{X}\right) \tag{1}$$

Where Z is a function of $\overrightarrow{X}$, the vector with base variables. Probability distributions are assigned to the different base variables in the models, in this study they are taken from Vrouwenvelder et al. 2001, CUR 190 and Baecher, 2003. Failure is defined as the values of Z that are smaller than zero. Below is explained how the probability of failure, $P(Z \leq 0)$, is calculated.

Table 1. Limit state functions of the flood defence failure modes in this reliability analysis

Failure mode	**Limit state func.**	**Definition parameters**
Uplifting: Bursting of the impermeable layers behind the embankment, driven by the hydraulic head due to high river water levels	$Z=\Delta h_{up}-(h-h_i)$	Δh_{up}= critical water head causing uplifting of the impermeable layers. $h-h_i$= actual occurring water head
Piping: Erosion of sand layer underneath impermeable layers, driven by the hydraulic head	$Z=\Delta h_p-(h-h_i)$	Δh_p= critical water head causing piping after uplifting has occurred. $h-h_i$= see above
Overtopping: Wave overtopping causing erosion at the rear face of the embankment, leading to breach	$Z=qc-qa$	qc= critical overtopping discharge associated with breach qa=actual occurring discharge
Overflow: River water level exceeds the crest level, the overflowing water causes erosion and breach	$Z= \Delta h_{over}-(h-h_{c2})$	Δh_{over}= actual occurring difference between river water level and crest level driving overflow $h-h_{c2}$= difference between river water level and crest level

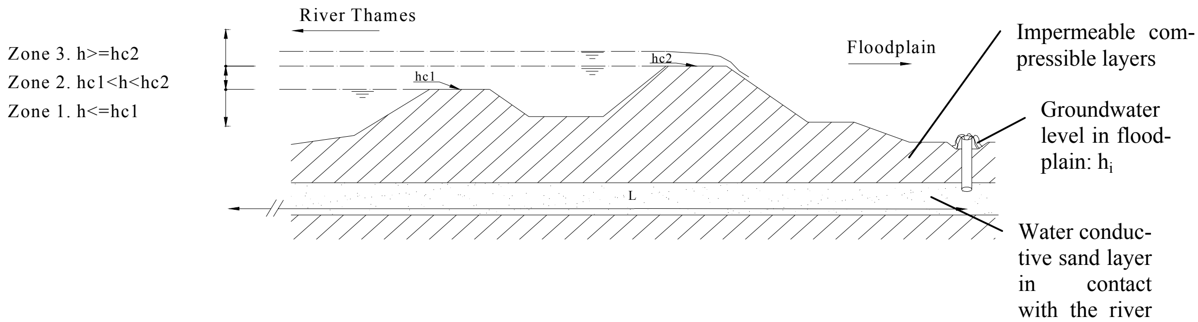


Figure 2. 'Double crested' earth embankments and the definition of three zones of water levels

Calculation of the probability of failure based on a limit state function requires integration of the integral:

$$\int_{z\leq 0}\cdots\int f\left(\overline{X}\right)dX_1\cdots dX_n \tag{2}$$

As an analytical solution of this integral is often not available, numerical methods are used to approximate the probability of failure. Several calculation methods are available to do this: e.g. FORM, SORM, Monte Carlo, Directional Sampling. In this study, the Monte Carlo simulation method is chosen.

When failure of a flood defence section can occur due to failure modes Z_1 or Z_2, the total probability of failure of the section is determined taking correlations into account. For more details about the modelling approach of combinations of failure modes see e.g. Vrouwenvelder et al. (2001), Steenbergen et al. (2004).

Multiplying the probability of failure given a scenario of water level and wave conditions with the consequences of flooding gives the risk for that scenario. Repeating that for a large number of flooding scenarios entails the integration of flood risk. Expressing the conditional probabilities of failure given different hydraulic boundary conditions is also called fragility, see for use in flood risk, Dawson and Hall, 2002, and more generally Casciati, 1991.

2.2 *Approach to deterioration processes in 'snap shot' reliability model*

The following deterioration processes are considered:

- Settlements of the crest level as a result of the compressible foundational soils and overall settlements taking place in the region. The behaviour of settlements due to compressible foundational soils is the result of the increased levels of the earth embankments in the 1970s. This type of settlement is known to have a primary component, behaving logarithmically in time, and secondary component, with a non-linear development in time, Van Tol et al, 1999. The main portion of the settlements therefore appears in the years shortly after the increase in loading of the soil. At present, after thirty years, the effect of these settlements is negligible.
- Car/motor 'crossing' activities damage for instance the crest and the vegetation of the earth embankment. The crest level and quality of the vegetation play a role in the limit state functions of the overtopping and overflow failure modes. The crest level is hereby represented by h_{c2} and the vegetation with an erosion strength coefficient c_g which varies between 300,000 [m·s] being very poor quality grass and 1000,000 [m·s] being excellent quality grass.
- Deteriorating filter material that prevents the sand material from eroding through the pipes relieving the water conductive sand layer. Partly deteriorated filter material in the pipe results in a slowly reducing seepage length L, as during each high water some sand washes out. A fully deteriorated filter leads to a large increase in the probability of failure due to uplifting. In this case

Table 2. The parameters in the failure modes that are affected by the deterioration processes

Deterioration process	Parameter definition
$h_{c2;t} = h_{c2} - \sum \Delta h_{s;dt} - \sum \Delta h_{m;dt}$	$h_{c2;t}$ = the crest level at time t h_{c2} = the crest level at time t=0 $\Delta h_{s;dt}$ = the deterioration increment occurring in a time interval dt due to settlements $\Delta h_{m;dt}$ = the deterioration increment occurring in dt due to car/motor crossing
$c_{g;t} = c_g - \sum \Delta c_{g;m;dt}$	$c_{g;t}$ = the erosion endurance coefficient representing the strength of the vegetation at time t c_g = the coefficient at t=0 $\Delta c_{g;m;dt}$ = the deterioration increment of c_g occurring in dt due to car/motor crossing
$A_{f;t} = A_f - \sum \Delta A_{f;dt}$	$A_{f;t}$ = the area of the filter material at time t A_f = the area of the filter material at time t = 0 $\Delta A_{f;dt}$ = the deterioration increment of the area of filter material occurring in dt
$L_t = L - \sum \Delta L_{dt}$	L_t = the seepage length at time t L = the seepage length at t=0 ΔL_{dt} = the increment of the seepage length occurring in dt as a result of the deteriorating filter material

Note: Summation over the increments gives the accumulated deterioration at time t.

this increase is conservatively modelled with a probability of uplifting equal to 1. The deteriorating filter material is modelled with a decreasing filter area, bounded by the total area of the pipe.

The three deterioration processes mentioned at the three bullets above are considered to be independent. The deterioration processes are considered as sequences of deterioration increments, the total deterioration at time t is the sum of increments over the interval from time zero until time t. In table 2 the parameters in the failure modes that are affected by the deterioration processes mentioned above are listed.

The deterioration increments $\Delta h_{m;dt}$ and $\Delta c_{g;m;dt}$ are the result of the same source sequence of loading events caused by the car/motor crossing. The cumulative deterioration processes of these two parameters are therefore correlated. The deterioration increments of the filter material $\Delta A_{f;dt}$ and the seepage length ΔL_{dt} are also related as the deterioration of the seepage length is caused by the filter material deterioration. Besides for the settlement process caused by compressible soils, no physical process-based models are currently available to describe the deterioration due to filter material or car/motor crossing.

3 THE PROBABILISTIC MODEL OF THE DETERIORATION PROCESSES

3.1 *Gamma process for a singular deterioration process*

3.1.1 *Formulation of the model*

In Van Noortwijk et al., 1997, a generalised gamma process model is recommended to represent deterioration processes such as erosion and the occurrence of scour holes. Two assumptions underlie the gamma process model. The first is that the deterioration increments are non-negative and exchangeable. The second is that the amounts of deterioration are l_1-isotropic, see Van Noortwijk et al. 1995. The latter property entails that the decision-maker is indifferent in the way in which the average rate of deterioration is achieved; all combinations of deterioration increments leading to the same average receive the same belief. The mathematical consequence is that the vector of deterioration increments is invariant under all n! permutations of the increments:

$$\left(\delta_1(\tau),...,\delta n(\tau)\right)=\left(\delta_{\pi(1)}(\tau),...,\delta_{\pi(n)}(\tau)\right) \qquad (3)$$

Where δ_i is the i^{th} deterioration increment, τ is the length of the time interval and the subscript π stands for permutation. Based on the assumptions described above, the joint probability density function for an infinite sequence of deterioration increments given the average rate of deterioration θ turns out to be:

$$p(\delta_1,...,\delta_n)=\int_0^\infty \prod_{i=1}^n p(\delta_i)dP(\theta)$$
$$\theta=\lim_{N\to\infty}\frac{1}{n}\sum_{i=1}^n \delta_i \qquad (4)$$

Which shows that the deterioration increments are conditionally independent given the average rate of deterioration. $dP(\theta)$ represents the uncertainty in θ which are not addressed in this case study. The model corresponds with the De Finetti general representation theorem, see Bernardo & Smith, 2000.

The temporal variability of the deterioration has been modelled as a so-called gamma process (van Noortwijk et al., 1997). A gamma process is a non-decreasing stochastic process for which the increments are statistically independent, gamma-distributed random quantities with identical scale parameter. The mathematical definition of the gamma process is given as follows. Recall that a random quantity X has a gamma distribution with shape parameter $v > 0$ and scale parameter $u > 0$ if its probability density function is given by:

$$\text{Ga}(x\,|\,v,u)=\frac{u^v}{\Gamma(v)}x^{v-1}\exp\{-ux\}, \quad x>0,$$

$$\Gamma(a)=\int_{t=0}^{\infty}t^{a-1}e^{-t}dt \qquad (5)$$

The latter is the gamma function for $a > 0$. It is assumed that the probability density function of the cumulative deterioration $X(t)$ can be written as

$$p_{X(t)}=Ga\left(x\middle|a,b\right)=Ga\left(x\middle|\left[\mu^2 t\right]/\sigma^2,\mu/\sigma^2\right)$$
$$E(X(t))=\mu t \qquad (6)$$
$$Var\left(X(t)\right)=\sigma^2 t$$

In which μ is the expected deterioration per unit time and σ is the standard deviation of the deterioration per unit time. A consequence of the application of the model as described above is that the development of the deterioration in time is a linear model. Estimates of μ and σ can be made in the following ways:

1. expert judgement
2. in-situ data about the deterioration process
3. physical process-based models of the deterioration process

In some situations a combination of the three might be desired.

3.1.2 *Practical issues*

The practical use of the gamma process model in reliability theory was first pointed out by Abdel-Hameed, 1975. This approach has several advantages. In the absence of proper models to describe the deterioration in time, expert judgement can be used to estimate the average rate of deterioration. Secondly, the gamma process model takes the temporal variability of the deterioration process into account, see Pandey and Van Noortwijk, 2004. Another advantage is that the gamma process model only takes on non-negative values which makes spontaneous improvement of the structure impossible. A comprehensive review of gamma processes and other probabilistic methods that have been developed to model the deterioration of civil structures is given in Frangopol et al. (2004).

The settlement process of the crest level in this case study is modelled with the gamma process explained above. The non-linear contribution of the settlement of foundational compressible soils can be neglected but the overall settlement process in this area is taken into account. In this paper average deterioration rates are estimated based on expert judgement, or type 1. The deterioration processes in the different failure modes are not interacting and can therefore be independently modeled.

3.2 *Model for correlated deterioration increments with common deterioration source*

3.2.1 *Development of the model*

The deterioration of the vegetation, c_g, and the crest level, h_c, is caused by the same sequence of motor crossing events and is therefore correlated, see section 2.2. The following notes are made:

- An expert judgment-based gamma process solution is attractive there is no process-based model describing how c_g and h_c exactly deteriorate as a function of the motor crossing events. Neither are there sufficient data to base a statistical analysis on.
- Preferably, the average deterioration rates of c_g and h_c and their mutual correlation are separately estimated, as the lack of information about the physical processes also undermines real estimates of mutual dependencies between c_g and h_c.
- Concluding: the aim is to develop a model allowing expert judgment, separately, on average deterioration rates for marginal gamma processes c_g and h_c, and on a correlation defining their mutual correlation.

First the correlated deterioration processes due to a common source are considered in general, see figure 3. An exchangeable sequence of loading events involves cumulated increments of different characteristics of loading: Y_1, Y_2 and Y_3. The distribution functions of these loading increments can then separately be represented by gamma processes. The independent gamma distributed loading increments (Y_1, Y_2, Y_3) are assumed to induce deterioration increments (X_1, X_2) for two condition variables 1 and 2. The deterioration increments are distributed according to marginal gamma processes if they are a linear function of Y_1, Y_2, Y_3. The dependency is introduced by the fact that both deterioration processes are caused by two independent loading increments, of which one is shared: Y_2.

Note that it can be easily verified that the correlation coefficient of Y_1+Y_3 and Y_2+Y_3 equals ρ. In the context of a Monte Carlo calculation, inversion-free bivariate gamma distributed random couples can be generated with trivariate reduction methods, see Devroye, 1986. The procedure requires the scale and shape parameters of the marginal distributions of X_1, $Ga(x_1|a_1,b_1)$, and X_2, $Ga(x_2|a_2,b_2)$, and the mutual correlation ρ, which must lie between $0 \leq \rho \leq \min(a_1,a_2)/\sqrt{(a_1 a_2)}$. The random couple is generated as follows:

- Generate $Y_1 \sim Ga(a_1 - \rho \sqrt{(a_1 a_2)},1)$
- Generate $Y_2 \sim Ga(a_2 - \rho \sqrt{(a_1 a_2)},1)$
- Generate $Y_3 \sim Ga(\rho \sqrt{(a_1 a_2)},1)$
- Return $(X_1,X_2)=(b_1(Y_1+Y_2),b_2(Y_2+Y_3))$

The so-obtained bivariate gamma density function is called the double-gamma distribution and is treated in Kotz et al. (2000). This model allows expert judgment-based estimates of the average deterioration per unit time for both marginal gamma processes and their mutual dependency. Restrictions are discussed in section 3.2.2.

In this case study, the exchangeable sequence of loading is represented by the car/motor crossing events. The condition variables subject to damage are respectively: the crest level and the vegetation (resp. X1 and X_2). It is assumed that the damage due to the car/motor crossing is (linearly) proportional to three sets of independent characteristics, indicated with Y_1, Y_2 and Y_3, representing characteristics of

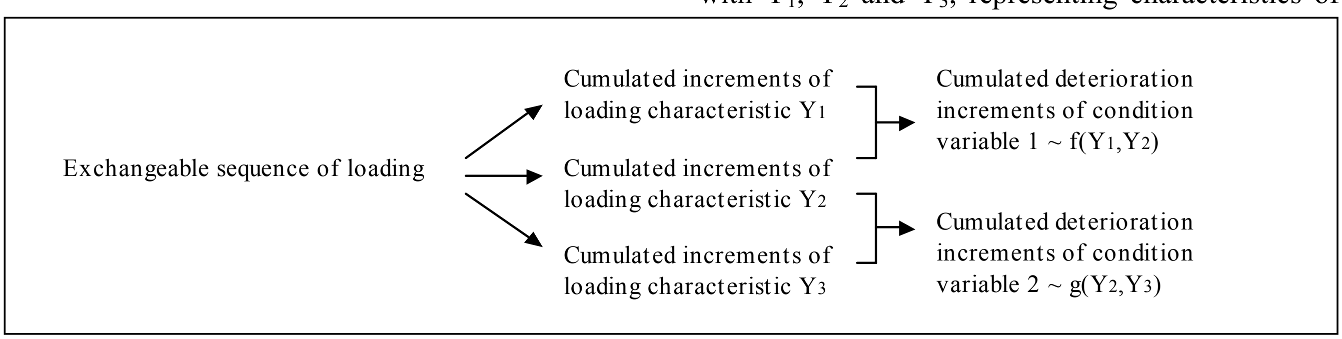
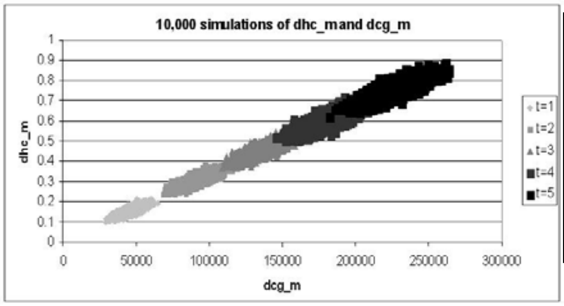


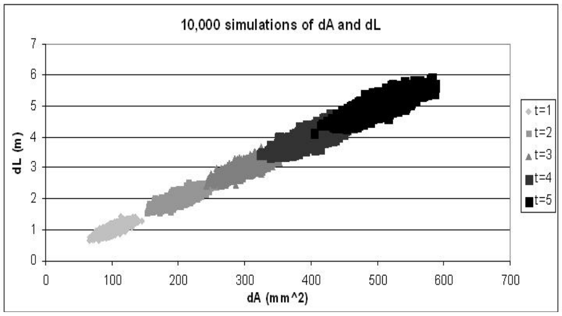
Figure 3. General model involving two deterioration processes due to a common loading source. Y_1, Y_2 and Y_3 are independent and gamma distributed. The functions f and g must be linear to support the assumption to represent deterioration of condition variable 1 and 2 with marginal gamma processes.



$\mu_{\Delta hc;m}$ = 0.15 m/year = mean value of deterioration rate 1/n*ΣΔhc;m
$\sigma_{\Delta hc;m}$ = 0.015 = stdv. of deterioration rate 1/n*Σ Δhc;m

$\mu_{\Delta cg;m}$ = 45000 /year = mean value of deterioration rate 1/n*ΣΔcg;m
$\sigma_{\Delta cg;m}$ = 4500 = stdv. of deterioration rate 1/n*ΣΔcg;m

ρ = 0.8



$\mu_{\Delta A}$ = 100 mm^2/year = mean value of deterioration rate 1/n*ΣΔA
$\sigma_{\Delta A}$= 10 = stdv. of deterioration rate 1/n*Σ ΔA

$\mu_{\Delta L}$ = 1 m /year = mean value of deterioration rate 1/n*ΣΔL
$\sigma_{\Delta L}$ = 0.1 = stdv. of deterioration rate 1/n*ΣΔL
ρ = 0.8

Figure 4. Plot of 10,000 simulations for 5 time steps of the bivariate gamma distributed deterioration processes and the applied parameters

the vehicles (e.g.: weight of the vehicles, tyre width and number of vehicles). These assumptions enable the application of trivariate reduction methods as described above.

The average rates of deterioration for the marginal gamma processes of the crest level and of the vegetation are separately estimated based on expert judgment. The mutual correlation between the two deterioration processes given the same underlying sequence of loading events can be estimated similarly.

It is noted that this paper aims to set up a theoretical model, the expert judgment entails simply a reasonable estimate. For future applications an analysis of the influence of the subjectivity of the expert judgment, and the involved uncertainties, is desired.

The same procedure is chosen to generate bivariate gamma distributed variables for the deteriorating filter material and the seepage length. In figure 3, the values of the parameters associated with the marginal gamma processes are given. In the same figure, 10,000 simulations of the bivariate couples of crest level reduction and vegetation damage is given for t=1,..,5 years. The figure shows clearly the linear behavior in time and the wider spread in the simulations illustrates the increasing variance in the deterioration over time.

3.2.2 *Restrictions*

Some restrictions are addressed below:

- the source of loading cannot change in time: the same group of vehicles must induce the exchangeable sequence of loading, e.g. the number/type of vehicles is not subject to large changes. However, it is hard to predict whether such a group will remain more or less constant. Moreover, there is no proof that the behaviour of that group corresponds with an exchangeable sequence of events.
- It is not easy to find three clearly defined independent sets of which one set only influences the crest level, one set only influences the vegetation and one set that influences both. Features that seem obvious are e.g.: weight of the vehicles, tyre width and number of vehicles. Vehicle weight and tyre width are related to some extent which undermines the requirement of independent sets. Weight influences the crest level more than the vegetation, tyre width influences both and the number of vehicles also influences both.
- realistically it is expected that the crest level and vegetation have a non-linear relation with the loading by car/motor crossing due to compaction of the soil, moisture content, etc. Even if the loading events can be modeled with a gamma process model, the non-linear relations with the loading of the crest level and vegetation deterioration might require a different distribution function.

However, although the assumptions above restrict the deterioration model considerably it is re-iterated that this paper deals with a situation in which:

- There is no process-based model available to represent the damage by the car/motor cycles.
- There is insufficient data availability to statistically analyse the damage to the crest and vegetation

Therefore, although the deterioration model is subject to very restrictive assumptions, it does provide a theoretical structure for analysing how processes are mutually influential. A theoretical structure which could be suitable to expand to a multivariate case.

Also, the application of the deterioration model points out how the reliability model of the earth embankment responds to changes in the variables.

4 RESULTS OF THE TIME-DEPENDENT RELIABILITY ANALYSIS

The probability of failure given a significant wave height of 1.0 meter and a peak wave period of 8 seconds calculated for different water levels are given in figure 5 and 6. It is pointed out that the effect on the wave conditions with increasing water levels should be addressed for an appropriate description of the structural behavior.

The total combined probability of failure for five different time steps (time in year) is given in figure 5. Figure 6 shows which failure modes underpin the overall curve for the different water levels at t = 1 and provides understanding about the shape of the curves in figure 5. The probability of failure due to piping is high, but its influence on the total curve is overruled by the combination with the probability of failure due to uplifting. In addition, the choice for a low deterioration rate of the filter material leaves the effect of the change of seepage length unnoticed in the overall curve.

When the water level reaches the first crest level, between h=5 and h=6, depending on the time step and the deterioration in the crest level, failure due to overtopping starts to play a role. The curves show a step change when the water level exceeds the crest

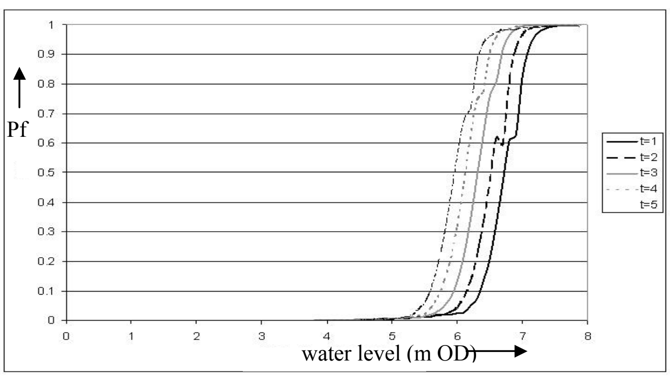


Figure 5. Time dependent probabilities of failure given different water levels, and given a significant wave height H_s = 1.0m, and a peak wave period, T_p =8. The time t is in years.

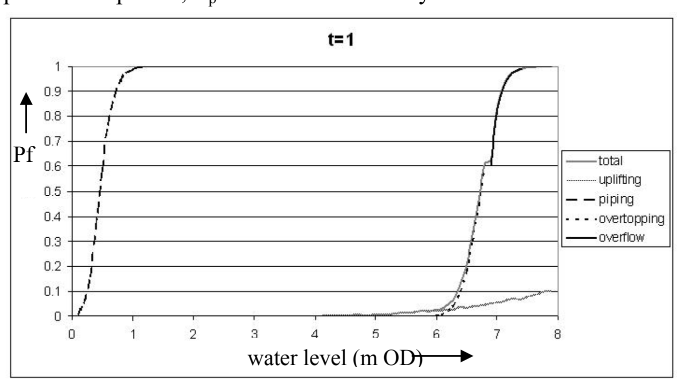


Figure 6. Contribution of the different failure modes in the different zones. Failure modes considered to be independent

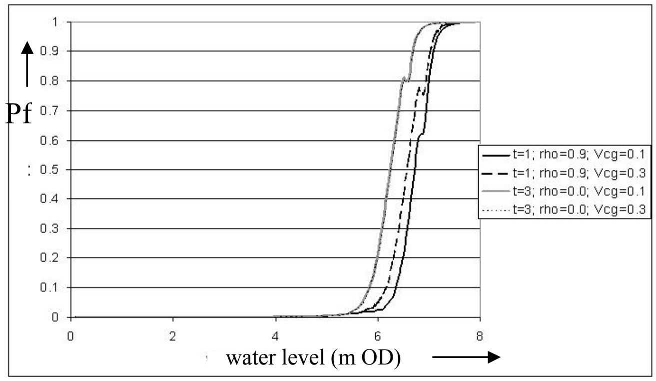


Figure 7. The change in overall probability of failure with a changing variation coefficient of the vegetation deterioration for ρ = 0.9 and ρ = 0, between the crest level and vegetation deterioration processes. Different times are chosen to enhance the clarity of the figure.

level. The wave overtopping discharge is then replaced by an overflow discharge. The discharge in the latter case is determined by the difference between the water level and the crest level, which is first not so large. Also, the waves contribute to a lesser extent to the discharge in the overflow case.

Runs for the independent case of the deterioration processes of the crest level and vegetation have been made. The influence of the deterioration due to vegetation turns out to be relatively limited in this case study. The changing curves in figure 6 therefore mainly reflect the crest level changes. Consequently, changes from a strong to very small correlation between the vegetation and crest level deterioration processes do not affect the overall curve very much.

Although correlated or uncorrelated processes do not have a direct effect on the overall curves in this case study, there is an indirect effect. A higher coefficient of variation of the vegetation deterioration affects the variation of the crest level deterioration process. The probability of failure increases with an increasing variation coefficient when there is a strong correlation. Figure 7 demonstrates that the results are unchanged when the processes are taken as uncorrelated instead.

5 CONCLUSIONS AND RECOMMENDATIONS

This paper developed an expert judgment-based bivariate gamma process deterioration model to derive time-dependent reliability of a double crested earth embankment:

- The quality of the vegetation hardly influenced the failure results. However, the correlation between the vegetation and crest level deterioration processes indirectly influenced the probability of failure: an increase in variation coefficient of the vegetation deterioration process shows an increase in probability of failure with correlation and no effect in the uncorrelated case. Therefore, the expert judgment with

respect to the correlation and the variation of the vegetation indirectly influences the behaviour of the results.
- Other (non-linear) deterioration models to describe the vegetation deterioration process may be more appropriate.
- The large influence of the crest level on the probability of failure points out that it should be verified that the overtopping model is providing realistic results in this application. Secondly, the choice to represent the crest level deterioration process with a linear model should be revisited.
- The use of the bivariate gamma distribution to model correlated deterioration processes due to a common deterioration source is developed in this paper for cases where the single deterioration processes are linear models of the source variables. This paper presented a theoretical case, but realistically it is expected that the crest level and vegetation have a non-linear relation with the loading by car/motor crossing due to compaction of the soil, moisture content, etc.
- Gamma process applications for non-linear deterioration processes have also received attention, see for example Speijker et al., 2000 and Van Noortwijk et al., 1999. In these applications it was suggested to represent the E(X(t)) in the gamma distribution by a relation proportional to the non-linear behaviour.

When developing a time-dependent reliability model, the variables in the 'snap shot' reliability model for which the probability of failure is highly sensitive require most attention. The choice for a constant average deterioration rate in time may then have large repercussions for the behaviour of the probability of failure in time – and care should be taken that it is representative of the real deterioration process, see Buijs et al. (2005). If there is clear evidence that the deterioration rate is non-linear in time, then the deterioration increments are non-exchangeable and other probabilistic distribution functions might be equally suitable to model the uncertainty in the deterioration process.

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