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Space-time Kalman filtering of soil redistribution

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Objective

Soil redistribution is the net result of erosion and sedimentation. In this work we use space-time Kalman filtering to combine soil redistribution predictions from a physical-deterministic tillage erosion model with independent measurements. This is attractive because information from process knowledge and information from measurements are both utilised, while taking the respective accuracies of both sources of information into account.

Tillage erosion modelling

During each tillage event, material flows downhill whereby the amount of soil flux has a linear relationship with the slope angle. The result is illustrated in Fig. 1 for a seven-hectare section of a research site near Hepburn, Saskatchewan, where tillage erosion was simulated from 1963 to 2000.

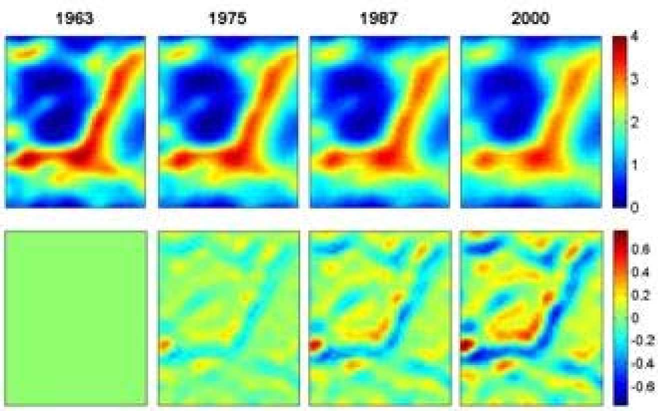

Fig. 1. Four time slices showing the evolution of elevation [m] (top row) and net soil redistribution [m] (bottom row) over time for the Hepburn research site, as computed with the LAPSUS physical-deterministic tillage erosion model.

Soil redistribution measurements

Cumulative soil redistribution from 1963 to 2000 was measured on the Hepburn site at 99 locations on a regular grid, using ^{137}Cs as a tracer. The measurement error standard deviation was estimated as 0.05 m. Fig. 3 (left panel) presents a scatter plot of measurements against model predictions.

Kalman filter equations

$Z(t+1) = A(t)\cdot Z(t) + \varepsilon(t)$ (state equation, Z=state, t=time, ε=model error)

$Y(t) = C(t)\cdot Z(t) + \eta(t)$ (measurement equation, η=measurement error)

$\hat{Z}^-(t+1) = A(t)\cdot \hat{Z}^+(t)$ (time update)

$\hat{Z}^+(t+1) = \hat{Z}^-(t+1) + K(t+1)\cdot(Y(t+1) - C(t+1)\cdot \hat{Z}^-(t+1))$ (measurement update)

$K(t+1) = \Sigma^-(t+1)\cdot C(t+1)^T \cdot (C(t+1)\cdot \Sigma^-(t+1)\cdot C(t+1)^T + \Sigma_\eta(t+1))^{-1}$ (Kalman gain)

$\Sigma^-(t+1) = A(t)^T \cdot \Sigma^+(t)\cdot A(t) + \Sigma_\varepsilon(t)$ (variance of time update)

$\Sigma^+(t+1) = \Sigma^-(t+1) - K(t+1)\cdot C(t+1)\cdot \Sigma^-(t+1)$ (variance of measurement update)

Results

Fig. 2 shows that the Kalman filter makes marked adjustments to the predicted soil redistribution, particularly along the transportation routes near measurement locations. Fig. 3 (right panel) shows that the adjusted soil redistribution at measurement locations is close to the measured value, indicating that measurement error is small compared to model error. Fig. 4 shows that the adjusted redistribution map is much more accurate than that of the physical-deterministic model.

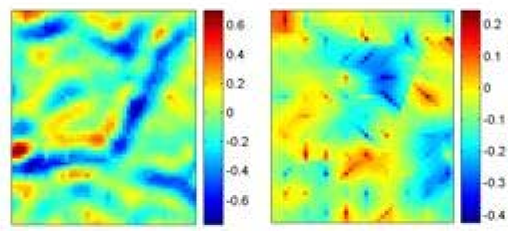

Fig. 2. Predicted cumulative soil redistribution after Kalman filter adjustment (left) and difference between predicted soil redistribution before and after adjustment (right).

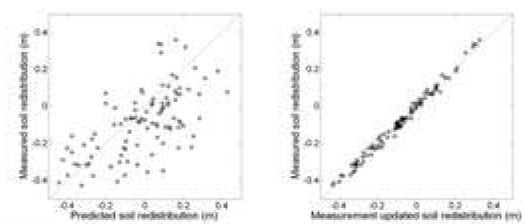

Fig. 3. Scatter plot of predicted against measured soil redistribution, before (left) and after (right) Kalman filter adjustment.

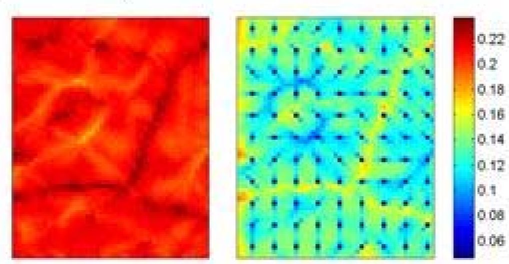

Fig. 4. Soil redistribution prediction error standard deviation map before (left) and after (right) Kalman filter adjustment.

Main conclusions

The space-time Kalman filter adjusts predictions made by the physical-deterministic model in a much more intelligent way than plain spatial interpolation of observed differences between model predictions and measurements. However, the method is computationally demanding and estimation of the stochastic model parameters deserves attention. More details and results of sensitivity analyses are presented in Heuvelink et al. (2006), *Geoderma* 113, 124–137.

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