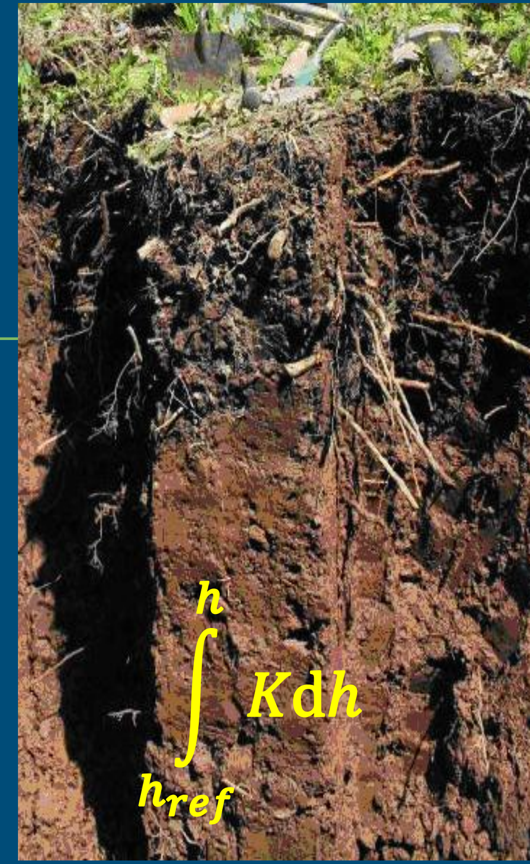
The matric flux potential: history and root water uptake



Marius Heinen, Peter de Willigen (Alterra, Wageningen-UR)
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Contents

- History
  - on the birth and name-giving “matric flux potential”
  - use, properties etc. of matric flux potential
- Root water uptake
  - steady-rate solution of RWU with matric flux potential
  - RWU and hysteresis
- Resume

Soil water movement

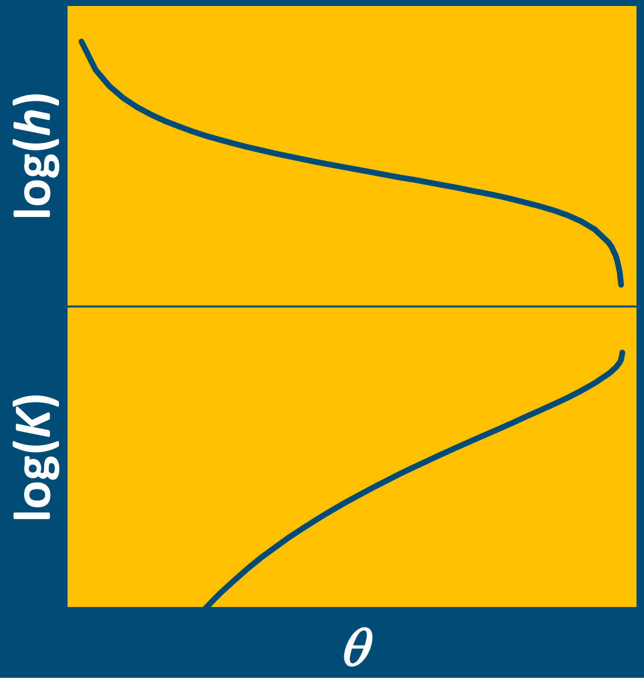
- Richards equation (1931)

$$\frac{\partial \theta}{\partial t} = -\nabla \cdot q$$

- Darcy-Buckingham (1856, 1907)

$$q = -\underbrace{K\nabla h}_{\text{matric}} - \underbrace{K\nabla z}_{\text{gravity}}$$

- Hydraulic properties
  - h, θ, K
- How to solve the non-linear PDE ?



History

- Earlier solutions making use of a transformation
  - e.g., Klute (1952), Gardner (1958), Philip (1966)
- June 1966: Symposium "Water in the Unsaturated Zone", Wageningen
- Discussion session led by G.H. Bolt
  - How to proceed with solving Richards equation

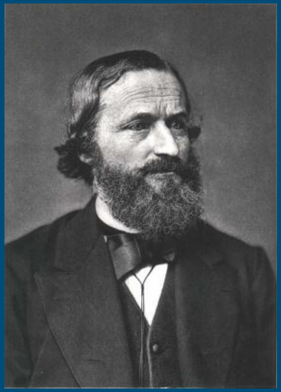
i) Introduction of

$$s = p[\int D \mathrm{d}\theta + q]$$

s: name and symbol?

The transformation

- The Kirchhoff transformation: heat transport
  - Gustav Robert Kirchhoff (1824-1887)



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VON

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VON

DR. MAX PLANCK,

PROFESSOR DER THEORETISCHEN PHYSIK AN DER UNIVERSITÄT BERLIN.

Es sei k eine bekannte Function von ϑ: man setze

$$K = \int^{\vartheta} k \, d\vartheta$$

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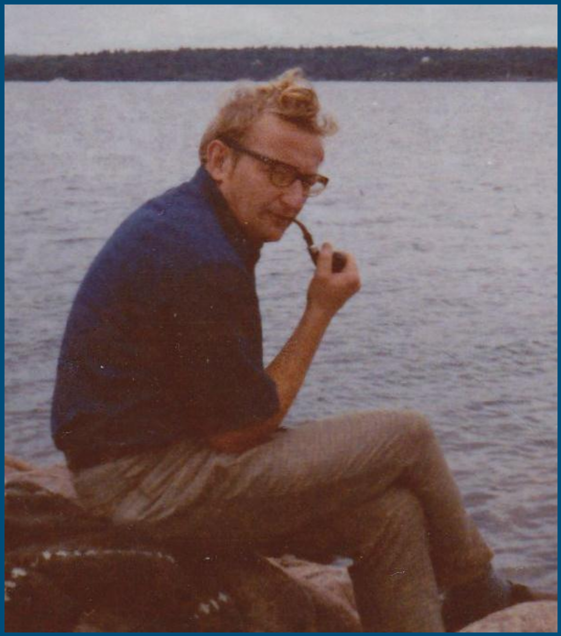
Raats (1970, SSSAP 34: 709-714)

$$\phi = \int_{h_{ref}}^{h} K(h)\,\mathrm{d}h$$

$$q = -K\nabla h \quad - K\nabla z$$

matric gravity

$$q = -\nabla\phi \quad - K(\phi)\nabla z$$



At any point the flux may be regarded as the sum of a matric component and of a gravitational component. The matric component is given by the gradient of ϕ and, therefore, it is appropriate to call ϕ the *matric flux potential.*

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Steady-state Richards equation with ϕ

- $$0 = -\nabla^2\phi - \frac{\partial K(\phi)}{\partial z}$$ non linear
- Exponential $K(h)$ relationship (Gardner, 1958)

$$K(h) = K_s \exp[\alpha h]$$

$$\phi = M = \int_{-\infty}^{h} K(h)\,\mathrm{d}h = \frac{K}{\alpha}$$

$$\nabla^2\phi = \alpha \frac{\partial \phi}{\partial z}$$

linear; Laplace (un)saturated

ϕ and the stream function

- Stream function ψ: " … *corresponding to any plane flow problem there exists a conjugate problem which is obtained by interchanging ϕ and ψ* " (same Laplace equation) (Raats, 1970)

$$\nabla^2\phi = \alpha\frac{\partial\phi}{\partial z}$$

$$\nabla^2\psi = \alpha\frac{\partial\psi}{\partial z}$$

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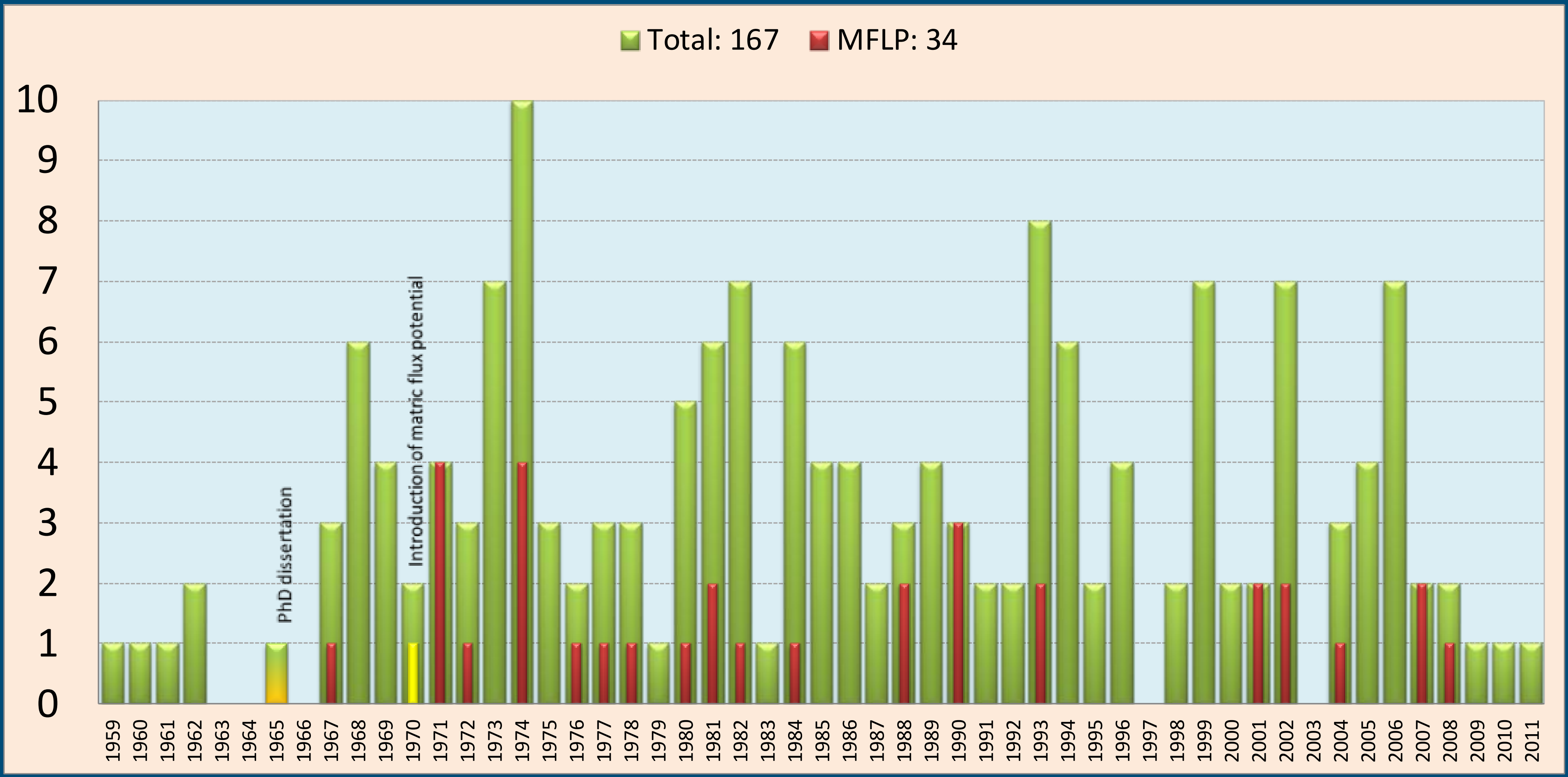
Raats publications on ϕ



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Raats (2001; Developments in soil-water physics since the mid 1960s; Geoderma 100:355-387)

Quasi-linear analysis of steady flows (1)

- Flows from (sub)surface drip irrigation sources
  - Raats (1970, 1971, 1976)
  - Recently elaborated upon by Communar & Friedman (SSSAJ, 2010): coupled source-sink steady water flow model
- Flows to sinks: porous cup samplers, suction lysimeter
  - Raats (1971, 1976)
  - Merrill, Raats & Dirksen (1978)

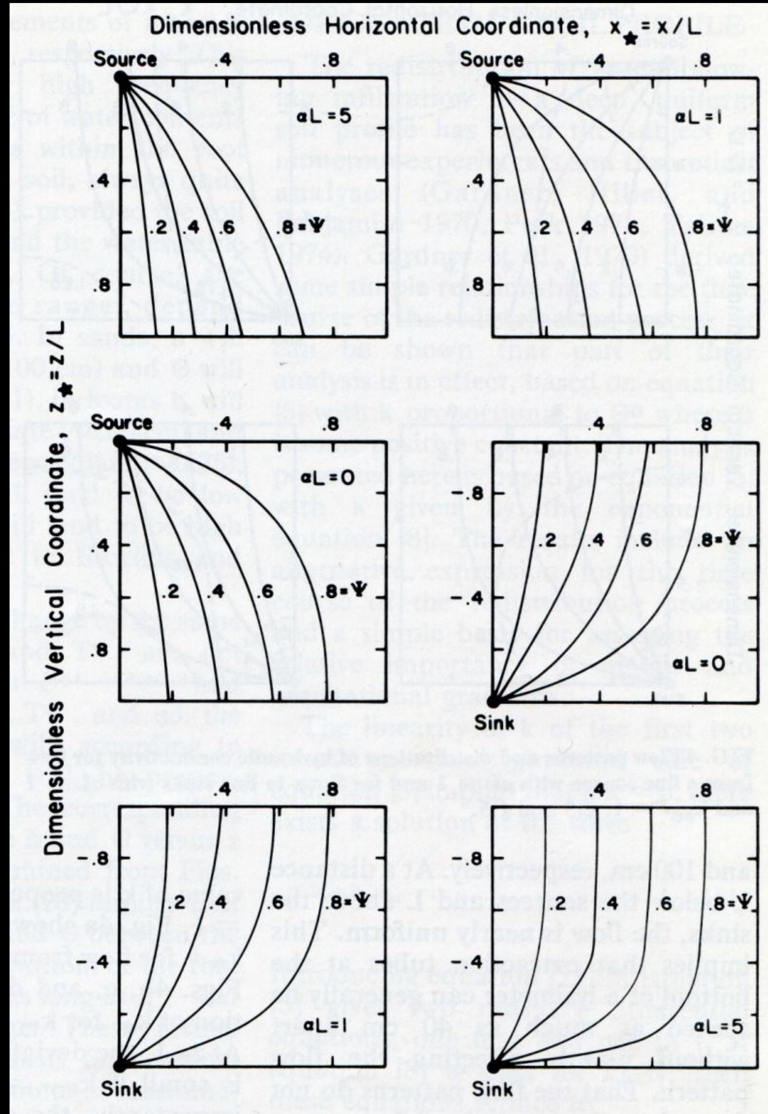

FIG. 3 Flow patterns for flows from line sources and to line sinks for αL = 5, 1, and 0.

Raats (1976; Trans ASAE 19:683-689)

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Quasi-linear analysis of steady flows (2)

$$Q = \pi R^2 K_0 + 2\pi R K_0 \frac{2}{\pi\alpha}$$

- Flows from surface disc
  - Wooding (1968)
- Flows from bore-hole permeameters
  - Reynolds *et al*. (1985); Philip (1985); see also Heinen & Raats (1990)

$$Q = AK_0 + B\phi = K_0\left(A + \frac{B^*}{\alpha}\right)$$

- Stability of steady flows
  - van Duijn, Pieters, & Raats, 2004
- And several others ...

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Disadvantages

- Does not apply in layered or heterogeneous soils, unless
  - solved by discretization (Ross,WRR, 1990)
  - by averaging ϕ (ten Berge et al., SAWAH, 1992)
- Cannot be used in combination with hysteresis (e.g. Raats, 1990)

How to obtain ϕ

- Analytical solution of the integral
- Numerical integration
- Analytical approximation
  - e.g. for Mualem-van Genuchten
    (de Jong van Lier *et al*., 2009; WRR 45 W02422)

- Measure
  - Steady-state evaporation (ten Berge et al., NJAS 1987)
  - Non steady-state absorption (Clothier et al., SSSAJ, 1983)

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Example

- Root water uptake (RWU)
  - single root uptake
  - steady-rate solution in ϕ
- Partial root-soil contact
- Up-scaling into macro-model
- Properties of RWU model
- Model by de Jong van Lier et al.: also ϕ

Root water uptake

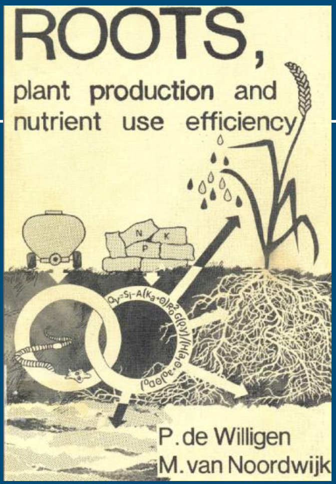
- Richards equation extended with RWU

$$\frac{\partial \theta}{\partial t} = -\nabla \cdot q - S_w$$

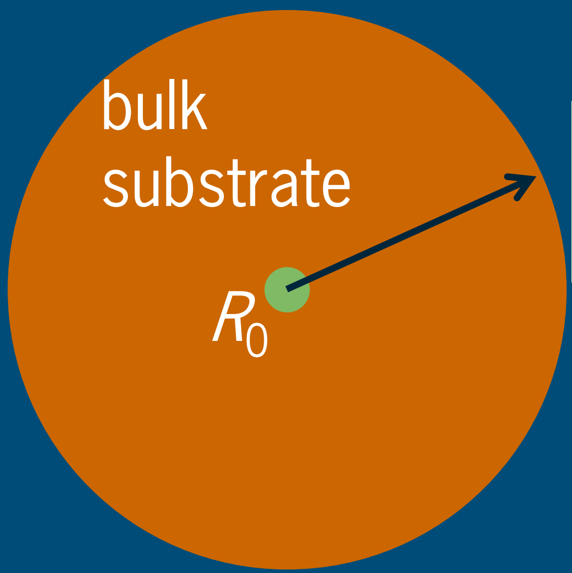
- Raats and RWU, e.g.:
  - 1974: SSSAP 38:717-722 – steady state with plant roots, ϕ
  - 1990: scaling in soil physics – depletion ahead of moving root front
  - review
    - 2004: Feddes & Raats
    - 2007: TIPM 68:5-28

Root water uptake: single root model

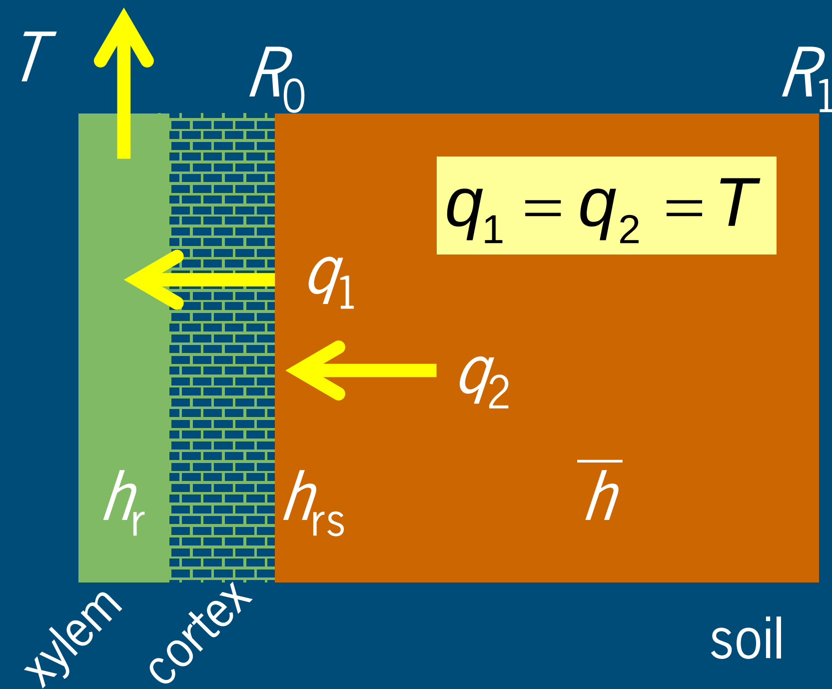
PhD Thesis 1987



- Single root with cylindrical shell of soil



$$R_1 = \frac{1}{\sqrt{\pi L_{rv}}}$$



Steady-rate RWU in ϕ

$$q_1 = \Delta z L_{rv} K_1 \left(h_{rs} - h_r\right)$$

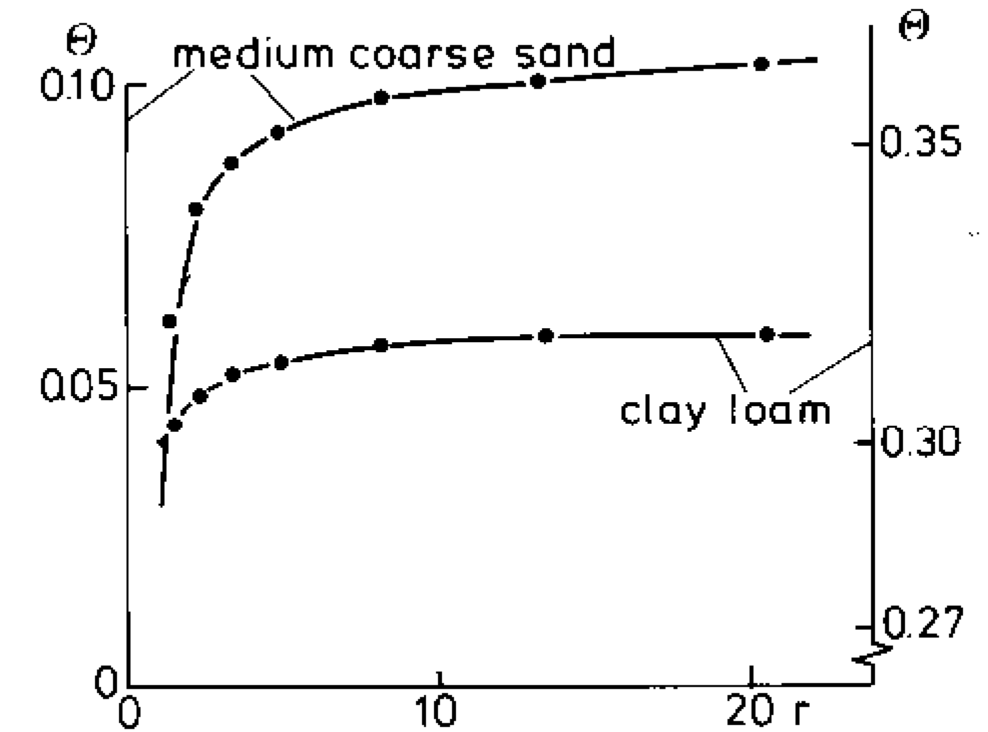
$$q_2 = \Delta z \pi L_{rv} f(\rho)\left(\bar{\phi} - \phi_{rs}\right)$$

$$T_{act} = T_{pot} \frac{1}{1 + \left(\frac{h_r}{h_{r,1/2}}\right)^a}$$

$$f\left(\rho = R_1/R_0\right) = \frac{1}{2}\left(\frac{1-3\rho^2}{4\left(\rho^2-1\right)} + \frac{\rho^4 \ln(\rho)}{\left(\rho^2-1\right)^2}\right)^{-1}$$

- Iteratively find h_r, h_{rs} and T_{act} for known $\bar{h}$

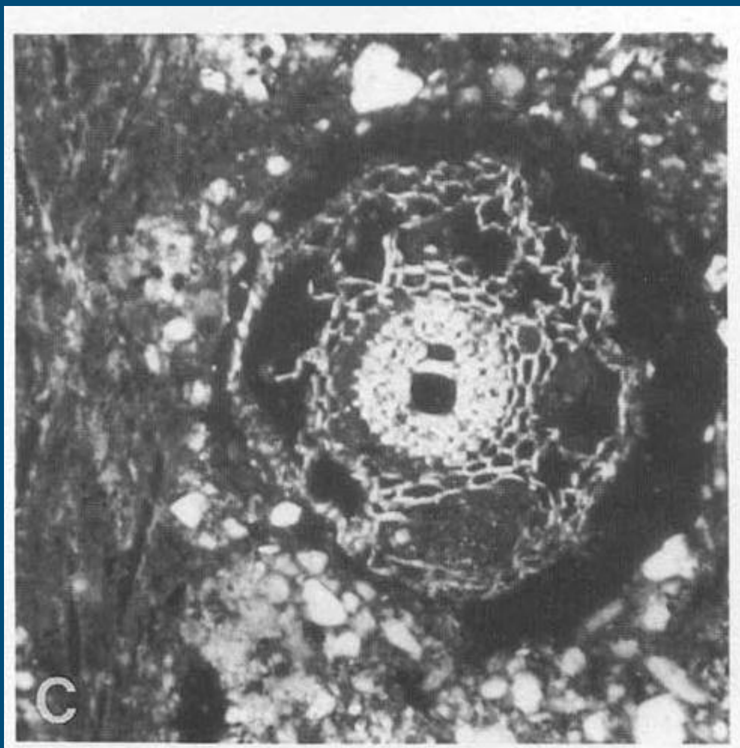
de Willigen and van Noordwijk, 1987



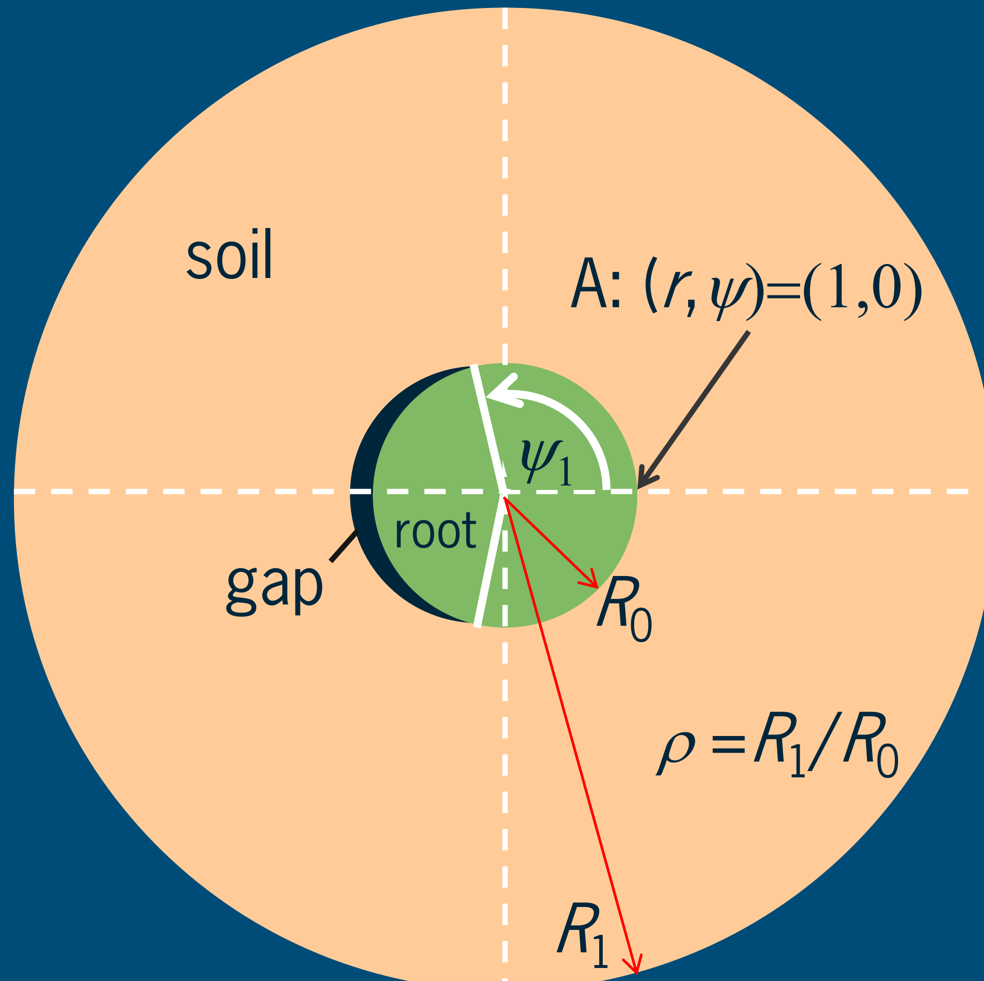
Numerical solution

Steady-rate approximation

Root-soil contact: model



van Noordwijk et al., 1993;
Geoderma, 56:277-286



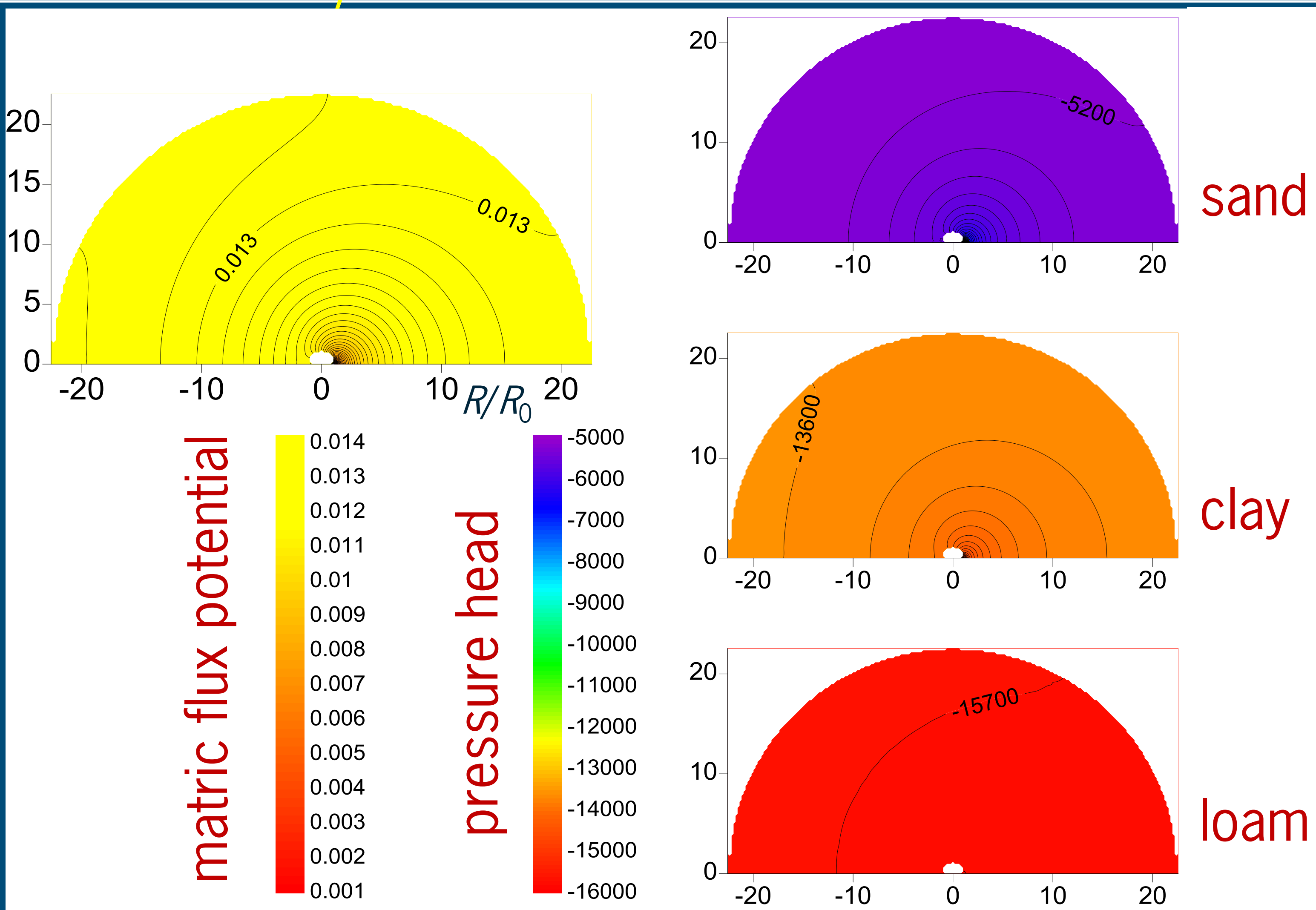
Steady-rate solution for partial contact

- The steady-rate distribution of ϕ in the soil cylinder with partial radial contact is given by

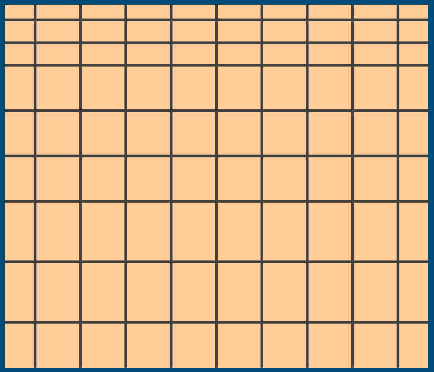
$$\phi(r,\psi)-\phi(1,0) = -\omega\left(\underbrace{\frac{r^2-1}{2(\rho^2-1)} - \frac{\rho^2\ln(r)}{\rho^2-1}}_{\text{full contact}} + \underbrace{\frac{2}{\psi_1}\sum_{k=1}^{\infty}\frac{r^{2k}+\rho^{2k}}{r^k(\rho^{2k}-1)}\frac{\sin(k\psi_1)}{k^2}\cos(k\psi) - \frac{\rho^{2k}+1}{\rho^{2k}-1}\frac{\sin(k\psi_1)}{k^2}}_{\text{adjustment when } 0 < \psi_1 < \pi}\right)$$

- At A: *h* is at wilting point (-160 m), so that $\phi(1,0) = 0$
- Iso-ϕ lines can be constructed

Partial contact: ϕ and h distributions



From a single root to a root system: up-scaling



- Numerical simulation models: finite elements, control volumes, finite difference grids
- Assume: roots are regularly distributed within each element, and they are parallel: a given L_{rv}
- For each time step use single root model for each element: distributed S_w
- Sum of all element uptakes must equal total T_{act}
- N+1 equations with N+1 unknowns
  - N values of h_{rs} (+ ϕ_{rs}) + h_r (NB: N values of $\overline{h}$ are known)

Properties of the RWU model

- Under wet conditions: RWU distribution is proportional to L_{rv} distribution
- In case of local dry conditions: compensation through uptake by roots under more favorable conditions
- Hydraulic lift
- Root resistance is dominating
- The model is extended with osmotic hindrance
  (cf. Dalton, Raats & Gardner, 1975; Agr. J. 67:334-339)

$$q_1 = \Delta z L_{\mathrm{rv}} K_1 \left(h_{\mathrm{rs}} - h_{\mathrm{r}} + \sigma \left(h_{\mathrm{o,rs}} - h_{\mathrm{o,r}} \right) \right)$$

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RWU model and hysteresis (1)

- Raats: what is the effect of hysteresis on your RWU model?
- As said before: an analysis with ϕ can only be done when hysteresis is disregarded

- Numerical analysis around a single root
  - alternating 2 d drying and 2 d wetting: effect on uptake
  - extremely large difference in drying – wetting retention curves: sand

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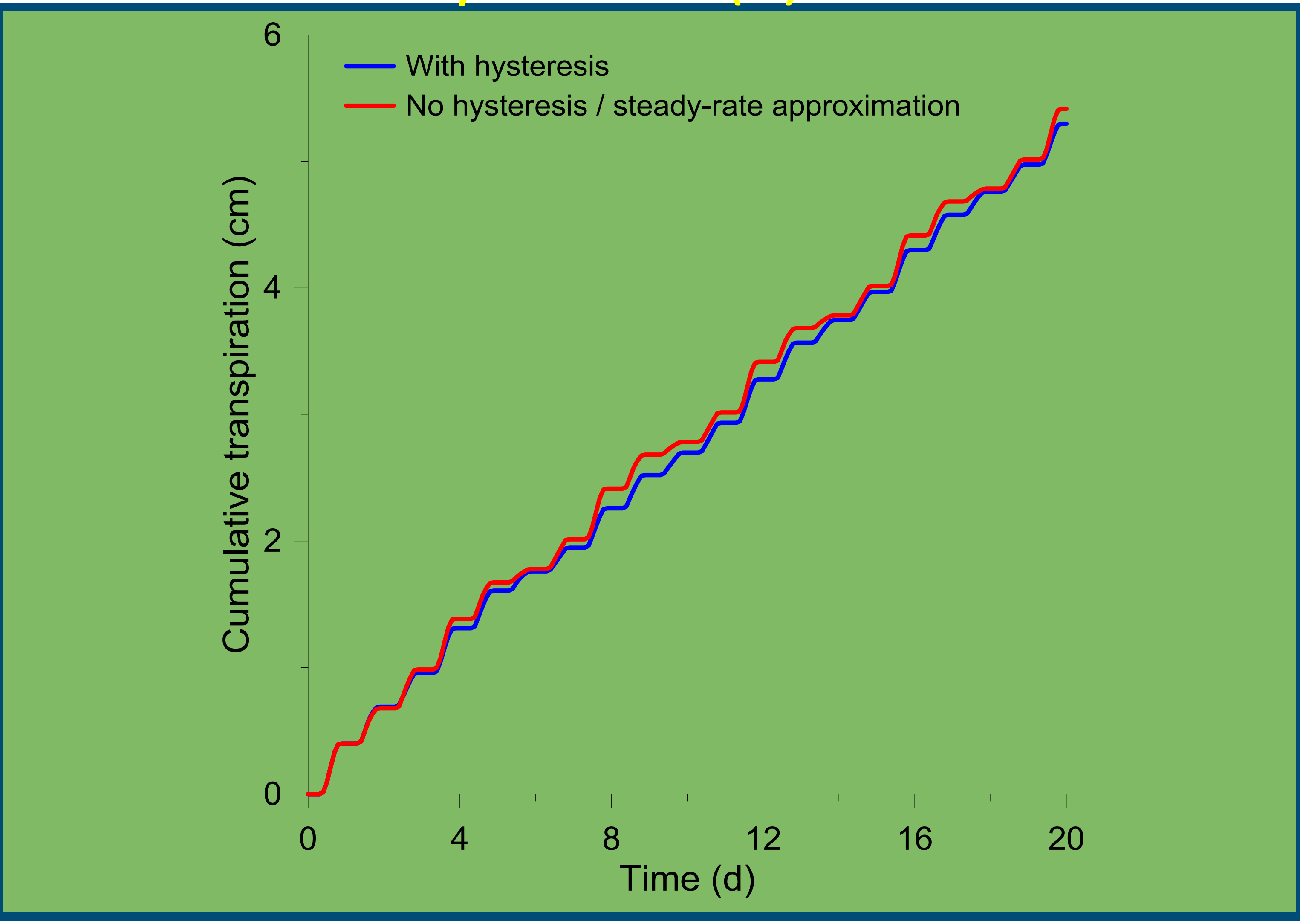
RWU model and hysteresis (2)



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de Jong van Lier, Metselaar, van Dam

- Assume $h \geq h_{\text{wilting}}$ at root – soil interface (no root!)
- Steady-rate solution for ϕ
- Confirmed steady rate assumption by numerical modeling
- Derived: $$\frac{T_a}{T_p} = \frac{\phi}{\phi_{crit}}$$ Relative transpiration scales as relative ϕ
- ϕ_{crit} : ϕ at the on-set of transpiration reduction
  - dependent on root density, and atmospheric demand

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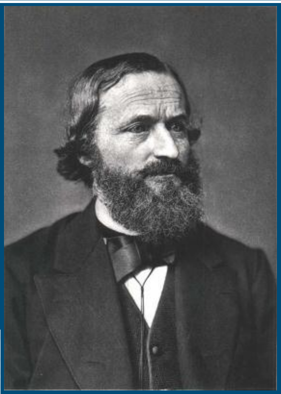
Resume

- Peter Raats introduced and promoted the term matric flux potential, and used it for numerous analytical analyses of flow problems. Others have used it as well.
- We have successfully applied the matric flux potential concept in steady-rate root water uptake models
  - up-scaling to root systems
  - compensation; hydraulic lift
  - root resistance mostly dominating
- Hysteresis is not of great importance with respect to RWU


I wonder if the matric flux potential of this soil can explain the presence of these sand mushrooms ...

The transformation

- The Kirchhoff transformation: heat transport
  - Gustav Robert Kirchhoff (1824-1887)



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Es sei k eine bekannte Function von ϑ: man setze

$$K = \int^{\vartheta} k \, d\vartheta$$

- Soil physics

$$\phi = M = \int_{h_{ref}}^{h} K(h)\,\mathrm{d}h = \int_{\theta_{ref}}^{\theta} D(\theta)\,\mathrm{d}\theta$$

Experimental evidence

Raats (1980; Report visit New Zealand)

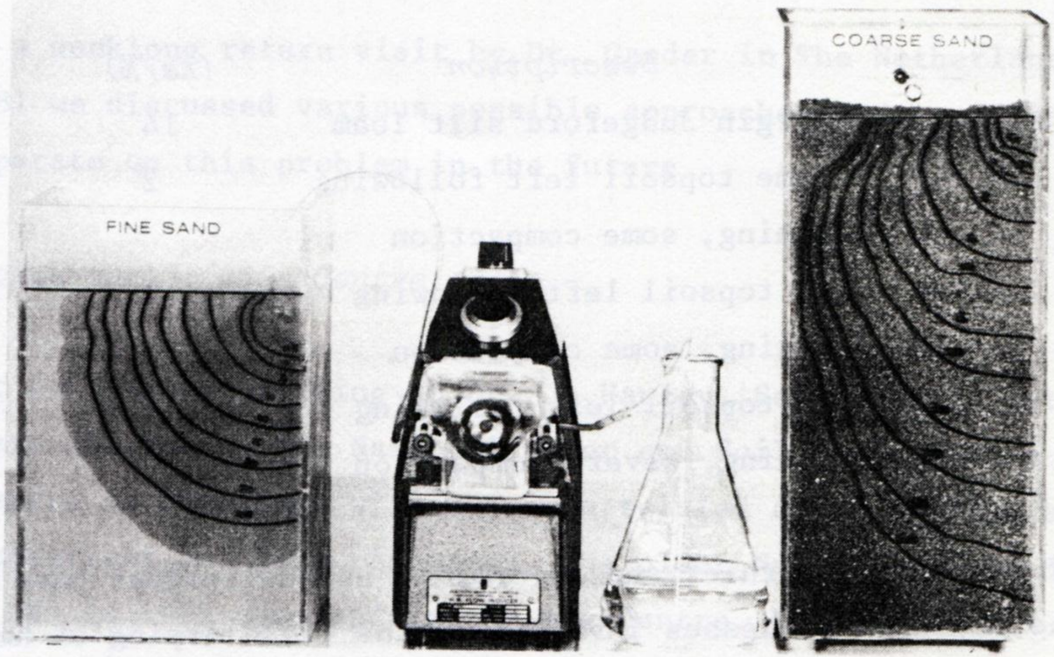

Fig. 7. This infiltration experiment shows the effect of soil texture on water distribution during trickle irrigation. In the fine sand the pattern remains symmetric whereas in the coarse sand elongation due to gravity occurs quite quickly.
From: DSIR Plant Physiology Div. 1980 Report.

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Some aspects concerning ϕ

- Analytical expressions for ϕ: Raats & Gardner (1971): 6 $K(h)$ relationships
  - constant, exponential, power
- Critical pressure head (Bouwer, 1964):
  - K-weighted mean pressure head between $h = 0$ and $h = -\infty$

$$h_{\mathrm{crit}} = \frac{\int_{-\infty}^{0} K(h)\,\mathrm{d}h}{K_{\mathrm{s}}} = \frac{1}{\alpha}$$

- Stream function ψ: " … *corresponding to any plane flow problem there exists a conjugate problem which is obtained by interchanging ϕ and ψ* " (same Laplace equation) (Raats, 1970)

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Quasi-linear analysis of steady flows (2)

- Flows involving specified extraction patterns by roots
  - Raats (1981)
- Flows around obstructions: solid object or air-filled cavities
  - Philip

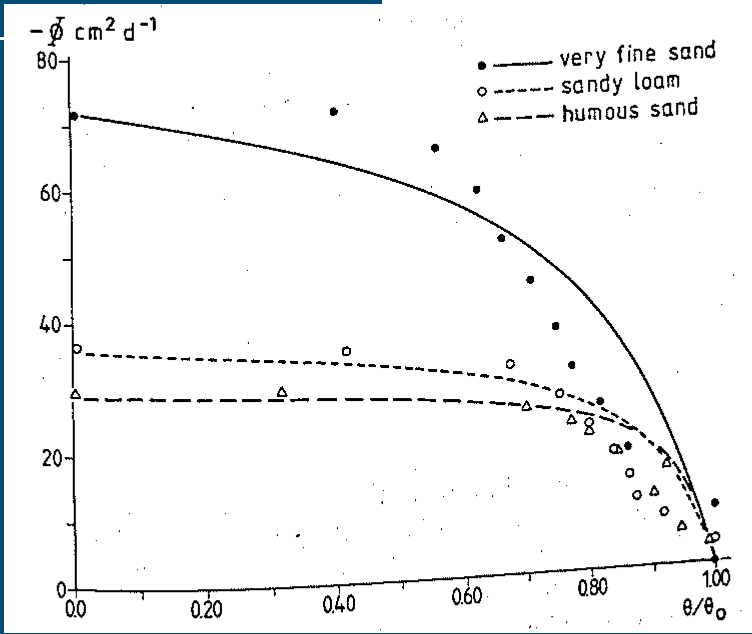
Advantages

- Do revisit analytical solutions (Laplace equation)
- Robust and stable numerical solutions - ϕ known as integrated average
  (e.g. Shaykewich and Stroosnijder; NJAS, 1977)

Disadvantages

- Does not apply in layered or heterogeneous soils, unless
  - solved by discretization
    Ross (WRR, 1990);
  - by averaging ϕ
    ten Berge et al. (SAWAH, 1992)
- Do exclude hysteresis,
  e.g. Raats (1990)

Measurement of ϕ



ten Berge et al., NJAS, 1987

- Steady-state evaporation (ten Berge et al., NJAS 1987)
  - measure flux and weight

$$\Phi(\theta) = -\frac{A(1-\theta^*)}{B-\theta^*}$$

- Non steady-state (Clothier et al., SSSAJ, 1983)
  - measure water content and Boltzmann transform λ

$$\frac{d\phi}{d\theta} = D(\theta) = -\frac{1}{2}\frac{d\lambda}{d\theta}\int \lambda d\theta$$

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Root water uptake

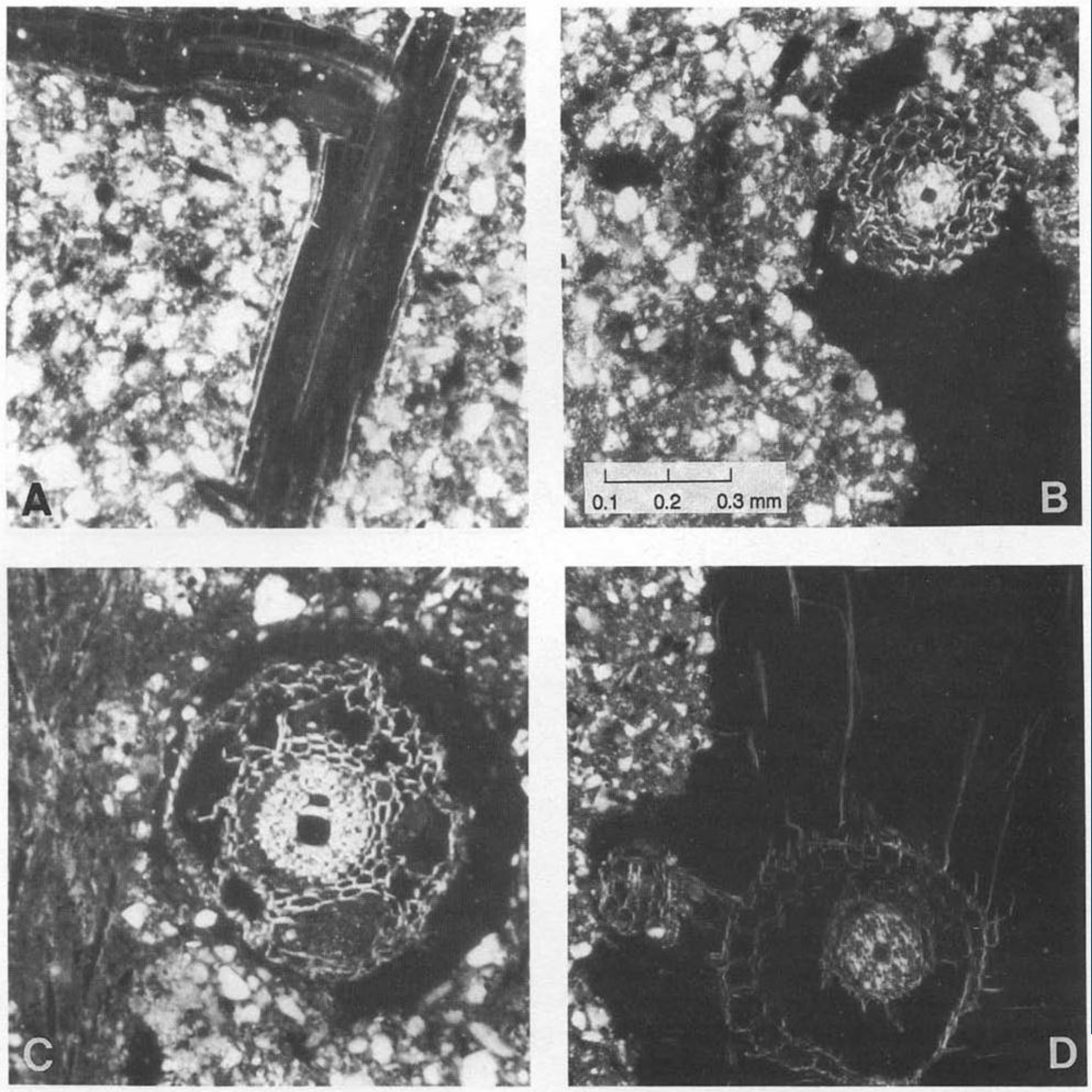
- Richards equation extended with RWU

$$\frac{\partial \theta}{\partial t} = -\nabla \cdot q - S_{\mathrm{w}}$$

- Raats and RWU, e.g.:
  - 1974: SSSAP 38:717-722 – steady state, ϕ
  - 1990: scaling in soil physics – depletion ahead of moving root front
  - review
    - 2004: Feddes & Raats
    - 2007: TIPM 68:5-28

$$W = \frac{\phi_{\mathrm{i}} - \phi_0}{v}$$

Root-soil contact



van Noordwijk et al., 1993;
Geoderma, 56:277-286

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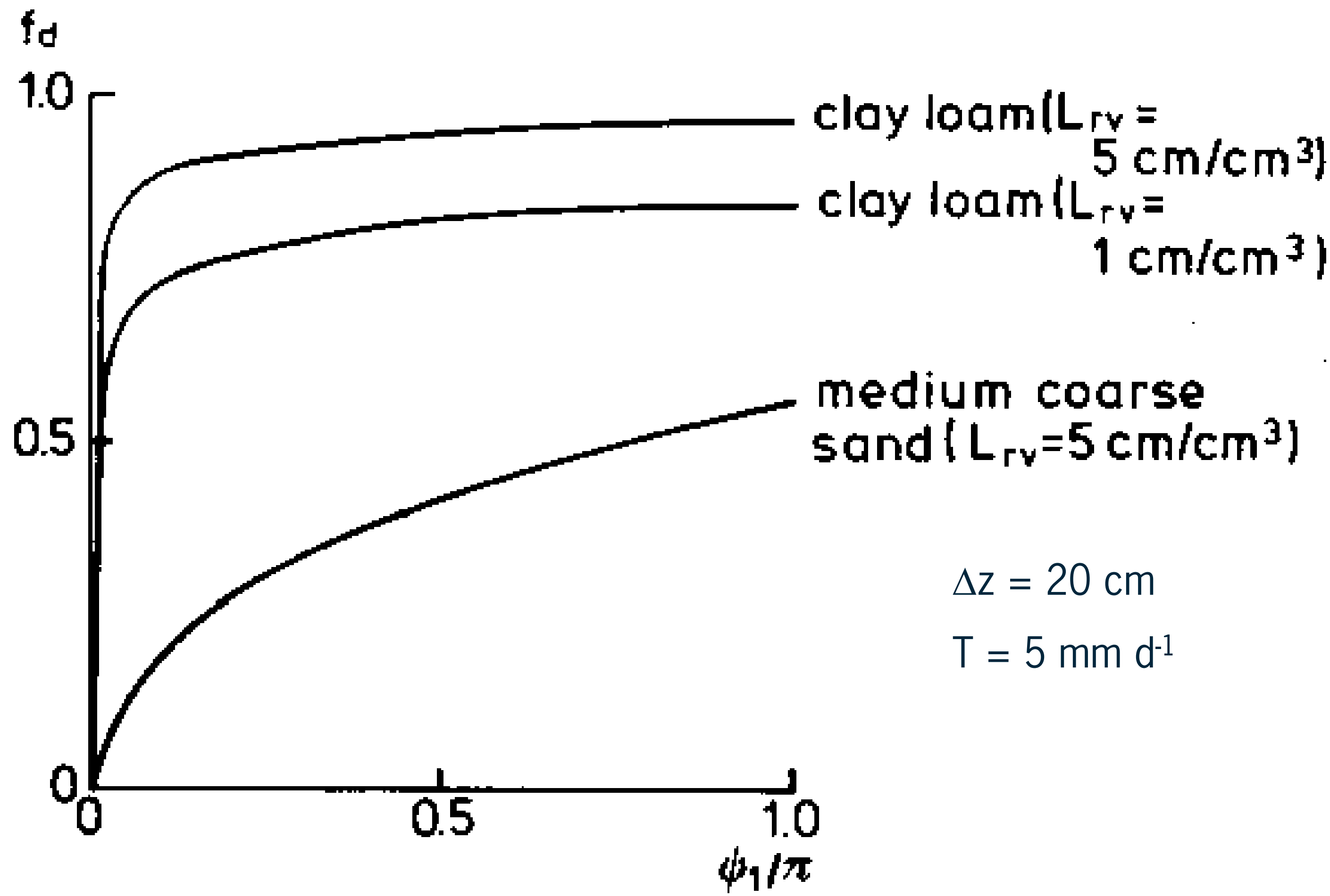
Fractional depletion



de Willigen and van Noordwijk, 1987

Some observations

- Steady-rate solution in ϕ confirmed by numerical validation
- Highest resistance to flow mostly across the root wall
  - the soil is (mostly) not limiting
- Except for coarse sands, only at small root-soil contact problems with uptake occur

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ϕ for Mualem – van Genuchten (1)

$$S(h) = \frac{\theta - \theta_{\mathrm{r}}}{\theta_{\mathrm{s}} - \theta_{\mathrm{r}}} = \frac{1}{\left(1 + |\alpha h|^{n}\right)^{m}}$$

$$K(S) = K_{\mathrm{s}} S^{\lambda}\left(1 - \left(1 - S^{1/m}\right)^{m}\right)^{2}$$

$$m = 1 - \frac{1}{n}; \quad n > 1$$

$$C(S) = \left(\theta_{\mathrm{s}} - \theta_{\mathrm{r}}\right)\frac{\mathrm{d}S}{\mathrm{d}h}$$

$$D(S) = \frac{K(S)}{C(S)}$$

$$\phi(S) = \left(\theta_{\mathrm{s}} - \theta_{\mathrm{r}}\right)\int_{\mathrm{S_{ref}}}^{\mathrm{S_a}} D(S)\,\mathrm{d}S$$

ϕ for Mualem – van Genuchten (2)

$$\phi(S_{\mathrm{a}}) = \frac{K_{\mathrm{s}}(1-m)}{\alpha(\nu-1)}\left[S_{\mathrm{a}}^{\frac{\nu-1}{m}}\left(f_1\left(S_{\mathrm{a}}^{1/\mathrm{m}}\right) + f_2\left(S_{\mathrm{a}}^{1/\mathrm{m}}\right) - 2\right) - S_{\mathrm{ref}}^{\frac{\nu-1}{m}}\left(f_1\left(S_{\mathrm{ref}}^{1/\mathrm{m}}\right) + f_2\left(S_{\mathrm{ref}}^{1/\mathrm{m}}\right) - 2\right)\right]$$

$$f_1(x) = {}_2\mathrm{F}_1(-m, \nu-1, \nu, x)$$
$$f_2(x) = {}_2\mathrm{F}_1(m, \nu-1, \nu, x)$$
$$\nu = m(\lambda+1)$$

- ${}_2\mathrm{F}_1$ (Michel & Stoitsov, 2008; Computer Physics Communications 178: 535–551)
  - power series expansion around $x = 0$
  - series converges always for $|x| < 1$
  - singularities for $x = 0$, $x = 1$: approximations available
  - Fortran and C++ routines available

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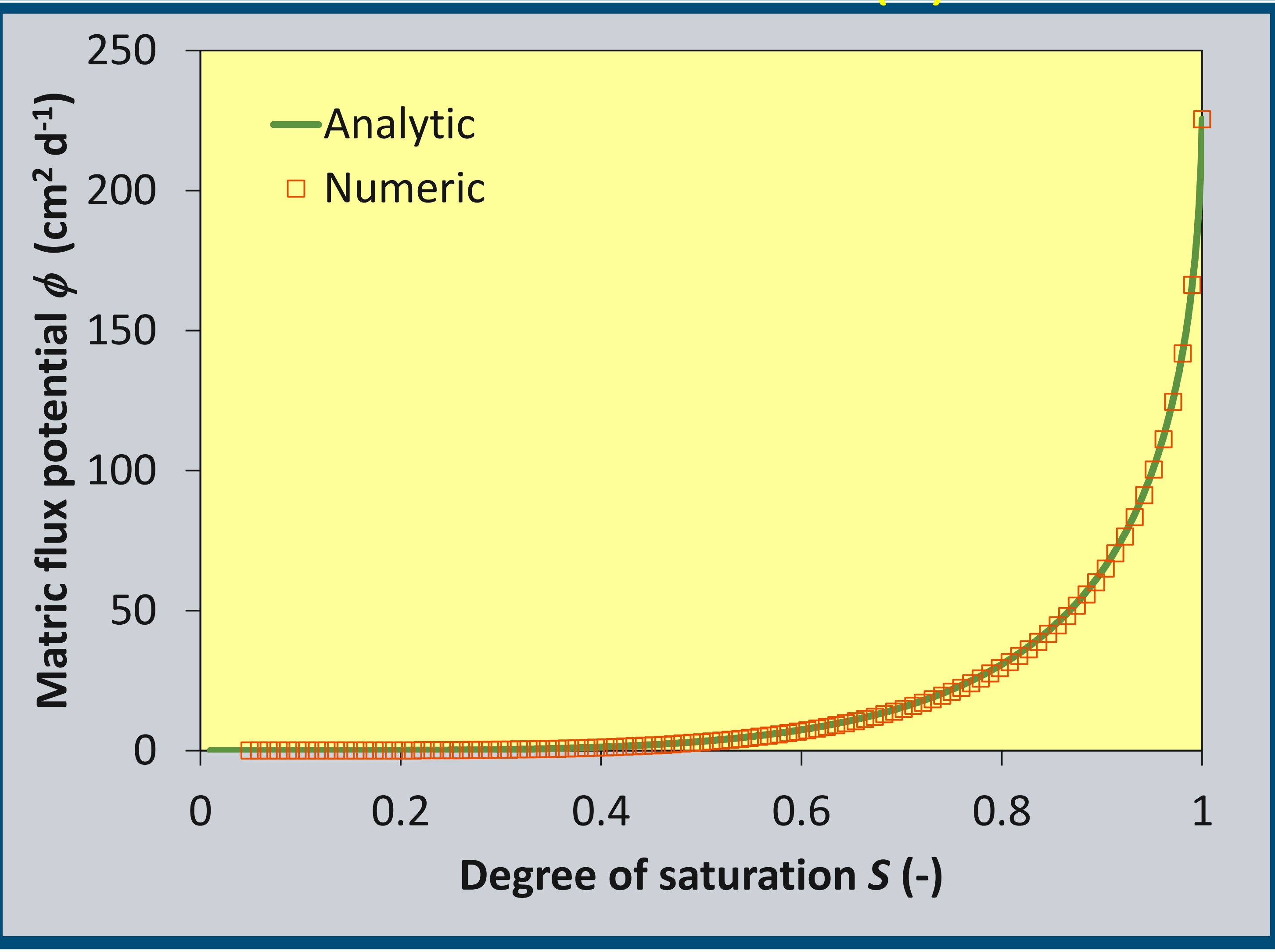
After: de Jong van Lier *et al.* (2009; WRR 45 W02422)
Note: their Eq. [A-10] contains an error

ϕ for Mualem – van Genuchten (3)



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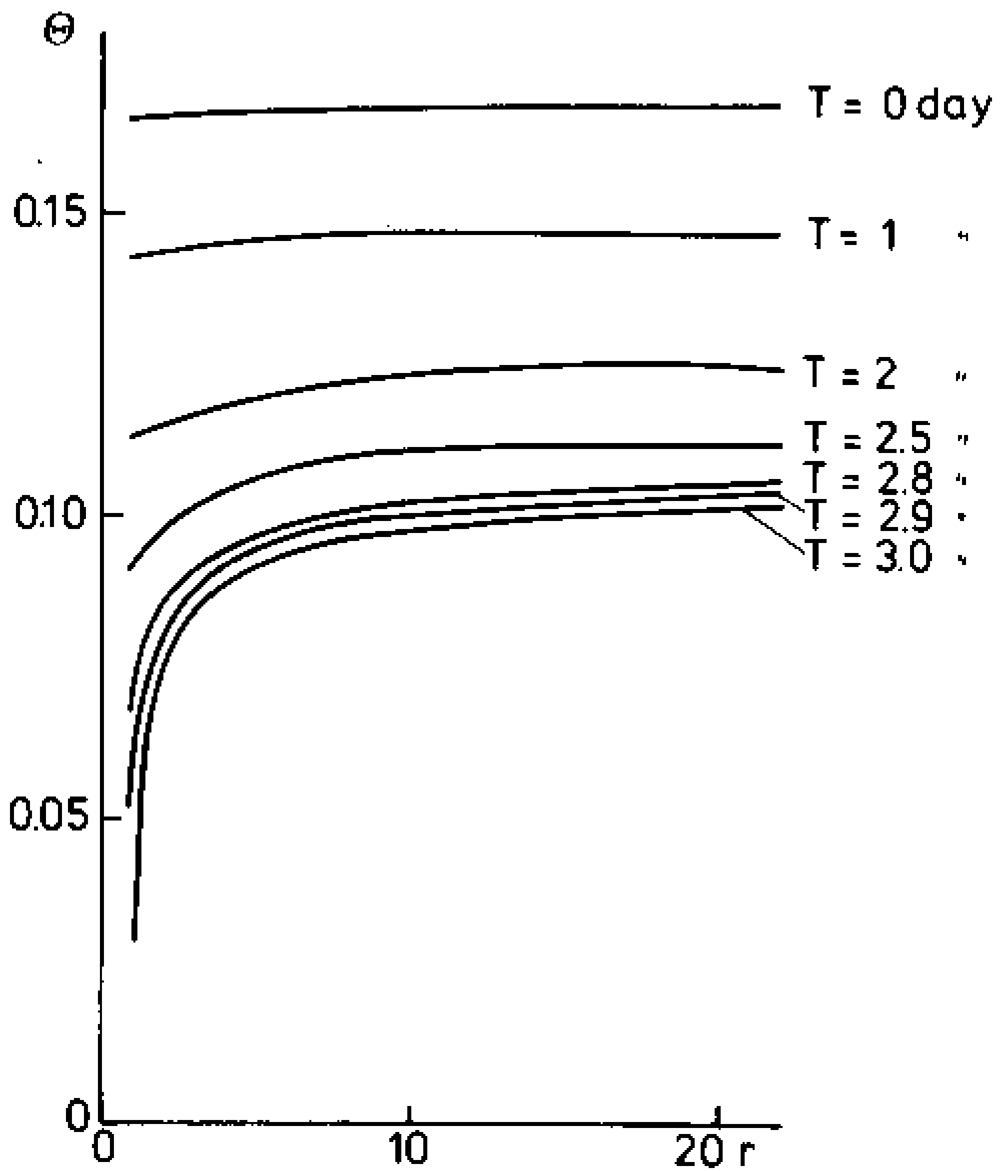

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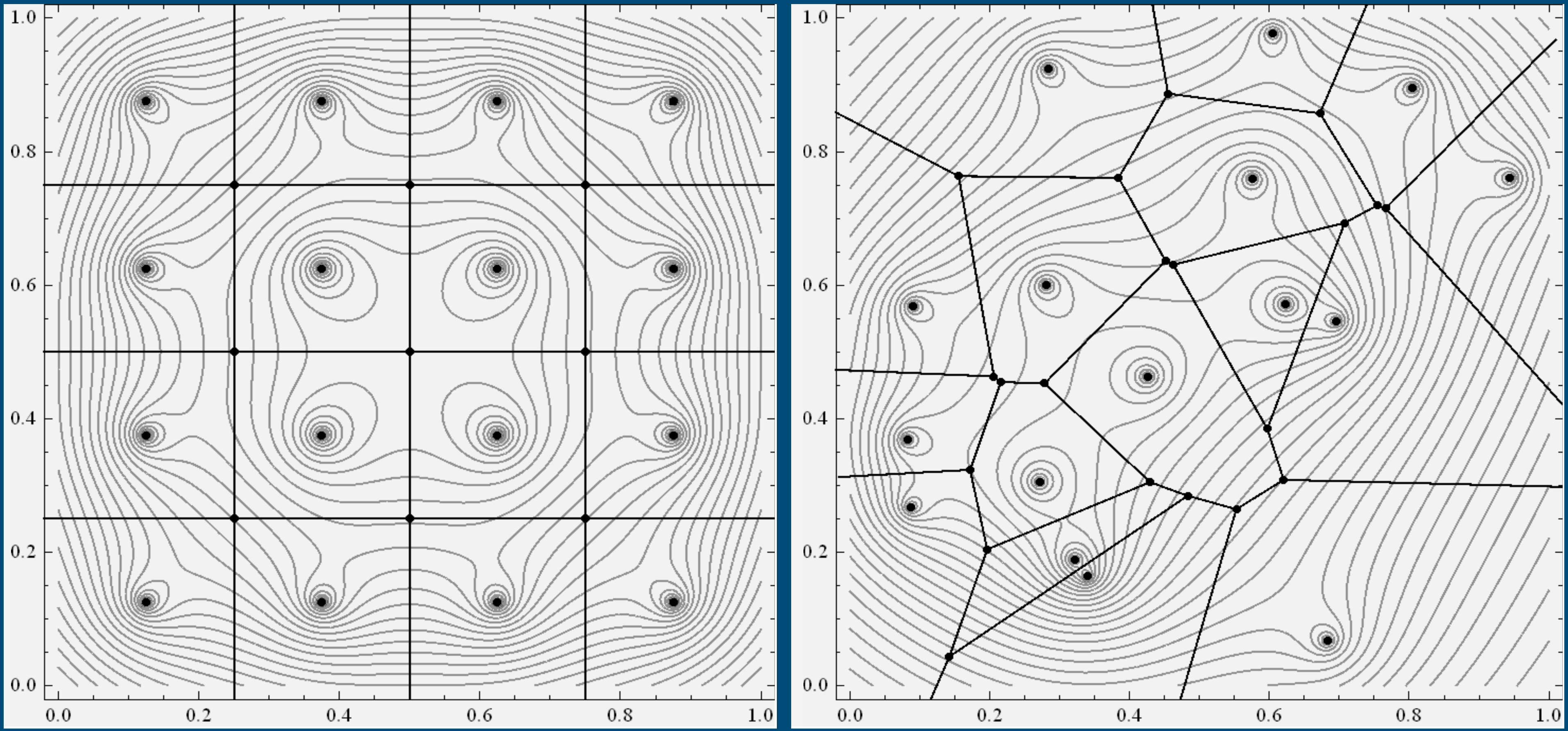
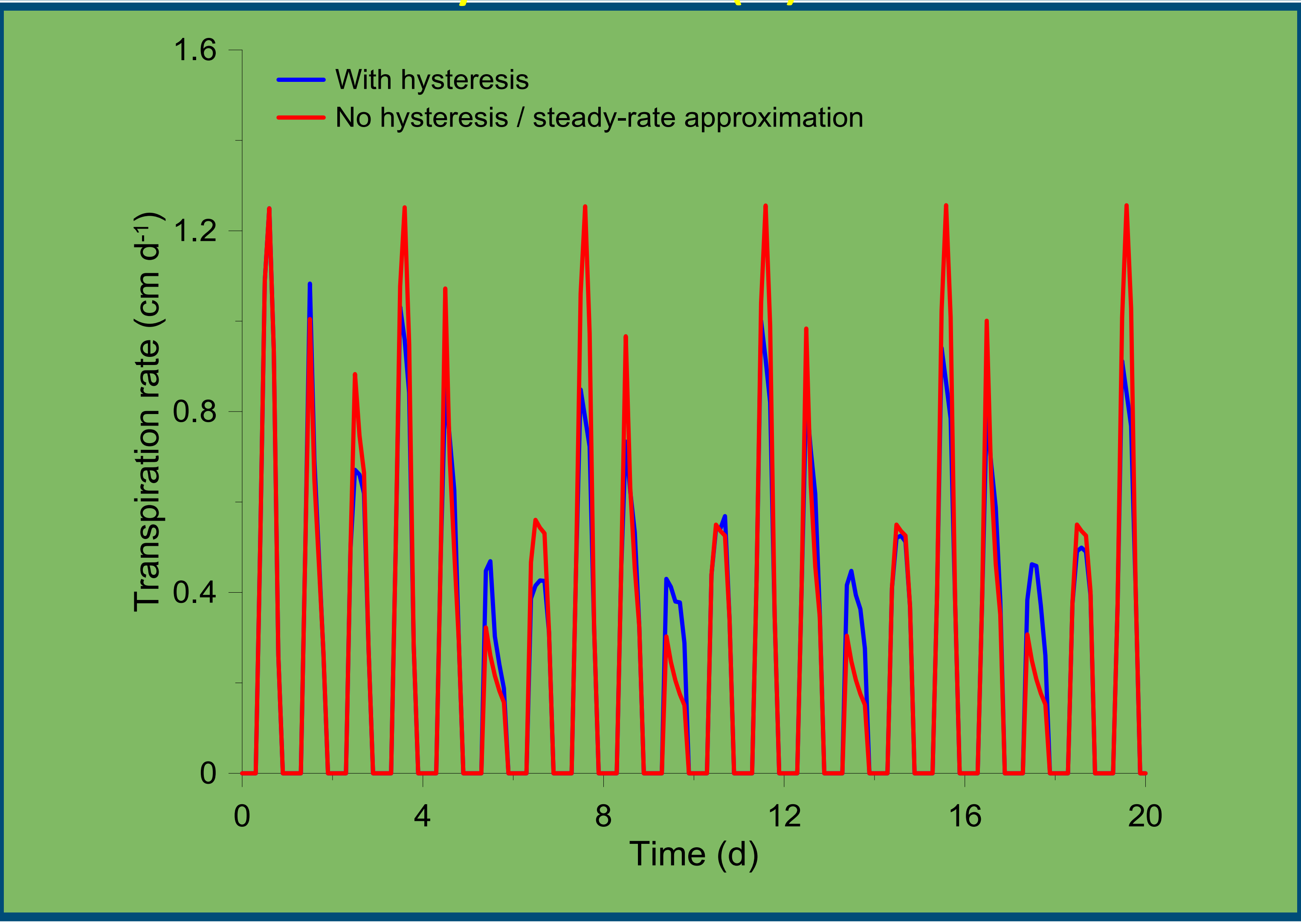
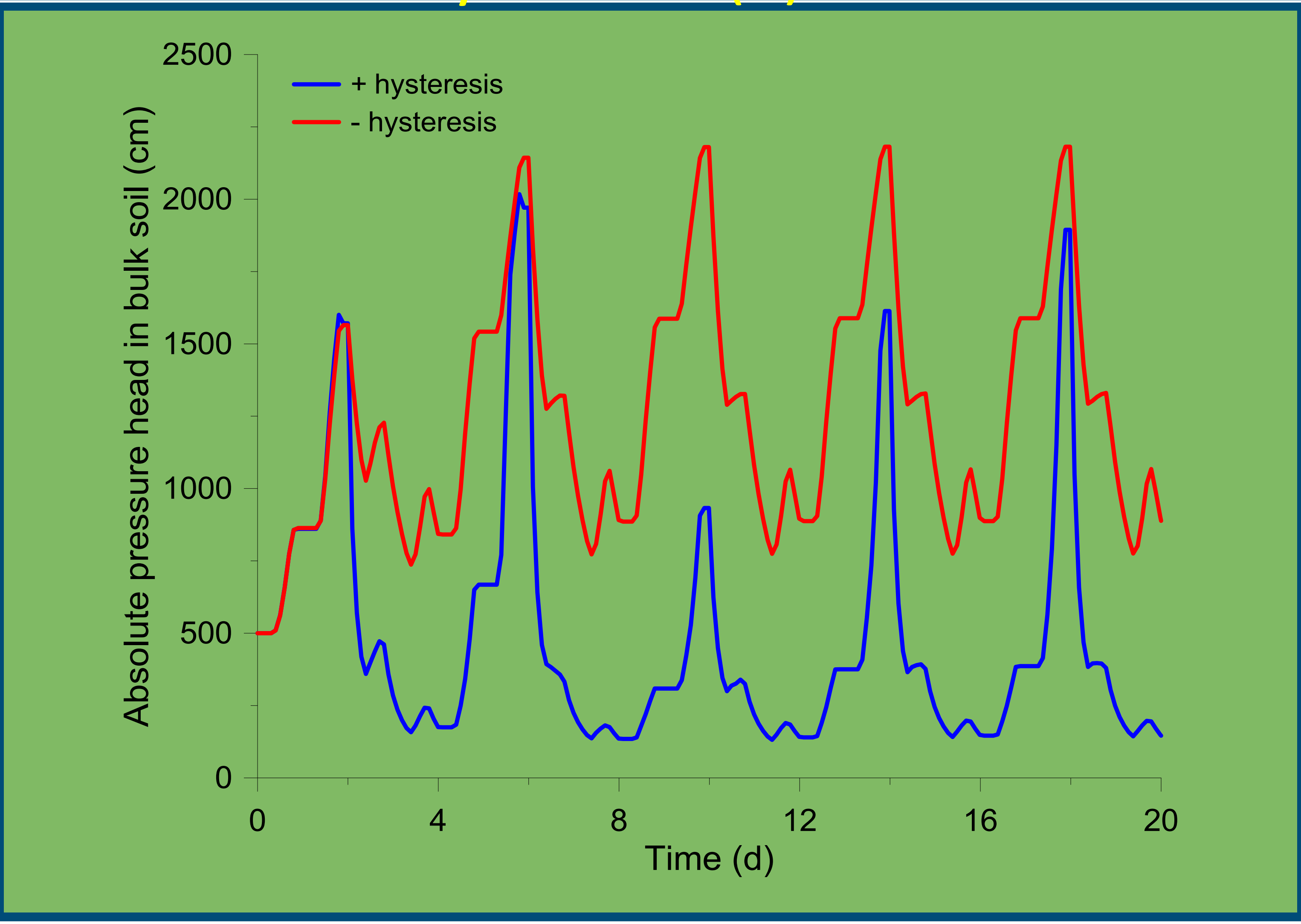

1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

RWU model and hysteresis (3)



RWU model and hysteresis (4)



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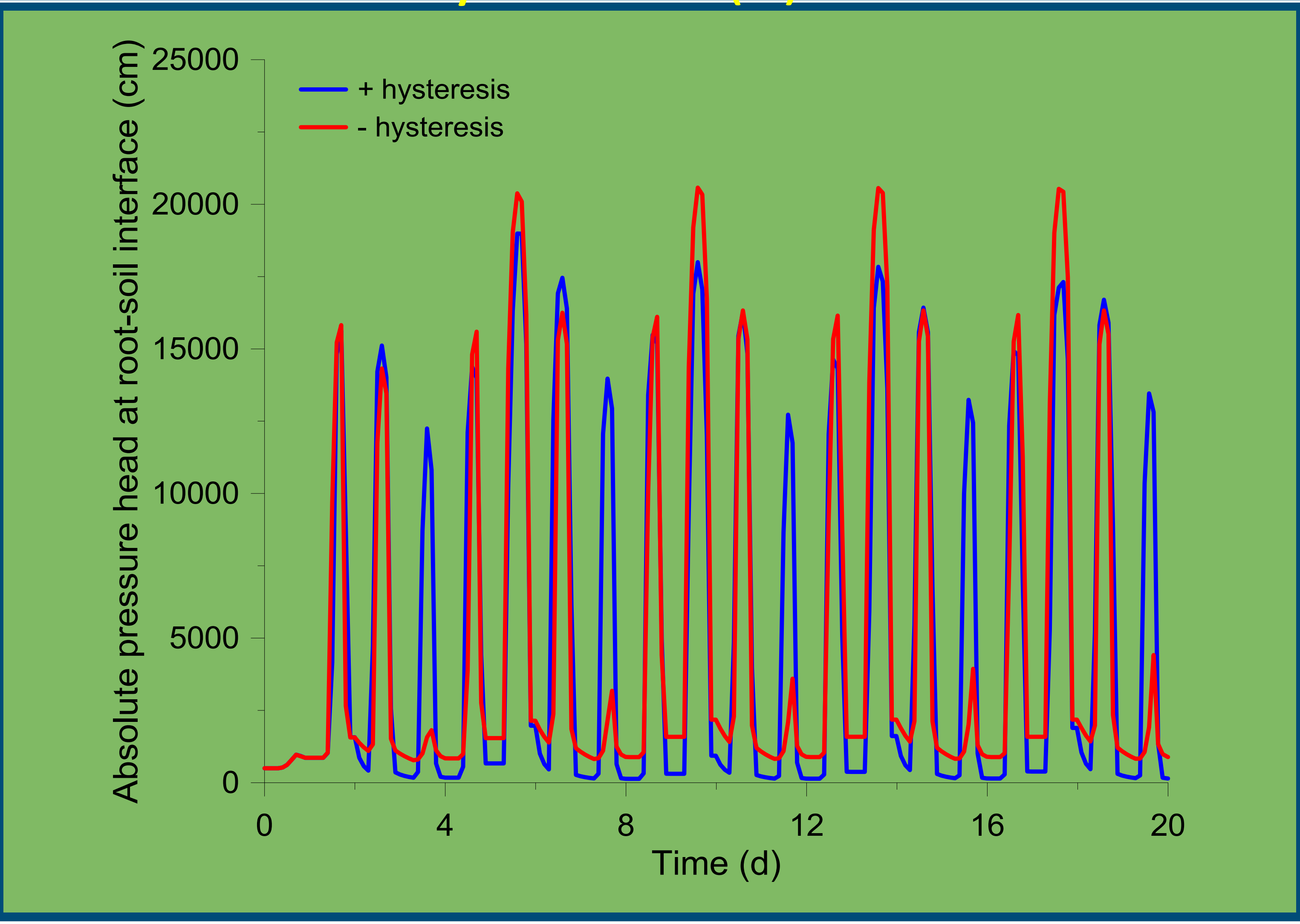
RWU model and hysteresis (5)



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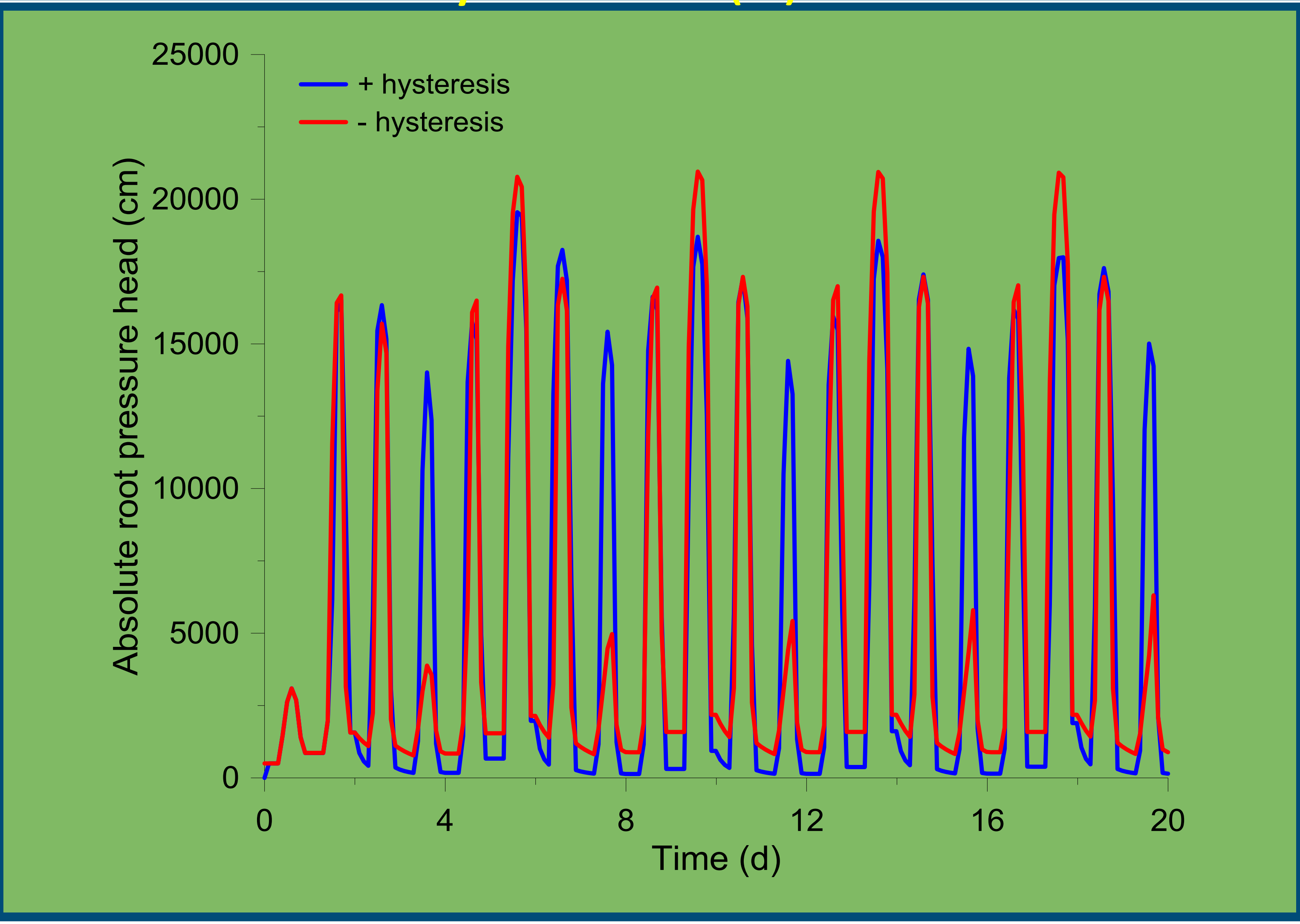
RWU model and hysteresis (6)



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