9 Seepage and Groundwater Flow

N.A. de Ridder[1]† and G. Zijlstra[1]

9.1 Introduction

The underground flow of water can create significant problems for land drainage. These problems can be divided into two categories: those of seepage and those of groundwater flow. Seepage problems concern the percolation of water through dams and into excavations, and the movement of water into and through the soil from bodies of surface water such as canals, streams, or lakes. Groundwater-flow problems concern the natural processes of infiltration and the subsequent flow of water through layers of high and low permeability until the flow discharges into springs, rivers, or other natural drainage channels. A quantitative knowledge of seepage and/or groundwater flow is needed to determine the drainable surplus of a project area (Chapter 16). Seepage from open watercourses can be determined by direct measurements at various points (inflow-outflow technique), or by subjecting the flow system to a hydrodynamic analysis (analytical approach). The latter requires that the relevant hydraulic characteristics of the water-transmitting layers and the boundary conditions be known.

This chapter is mainly concerned with the analytical approach to some of the seepage and groundwater-flow problems frequently encountered in land drainage. For a more thorough treatment of the subject, we refer to textbooks: e.g. Harr (1962), Verruijt (1982), Rushton and Redshaw (1979), Muskat (1946), Bear et al. (1968), Bouwer (1978).

9.2 Seepage from a River into a Semi-Confined Aquifer

A water-bearing layer is called a semi-confined or leaky aquifer when its overlying and underlying layers are aquitards, or when one of them is an aquitard and the other an aquiclude. Aquitards are layers whose permeability is much less than that of the aquifer itself. Aquicludes are layers that are essentially impermeable. These terms were defined in Chapter 2.2.3.

Semi-confined aquifers being common in alluvial plains, we shall consider the seepage along a river that fully penetrates a semi-confined aquifer overlain by an aquitard and underlain by an aquiclude. We assume that the aquifer is homogeneous and isotropic, and that its thickness, D, is constant. As the hydraulic conductivity of the aquifer, K, is much greater than the hydraulic conductivity of the overlying confining layer, K′, we are justified in assuming that vertical velocities in the aquifer are small compared with the horizontal velocities. This implies that the hydraulic head in the aquifer can be considered practically constant over its thickness. Whereas horizontal flow predominates in the aquifer, vertical flow, either upward or downward, occurs in the confining top layer, depending on the relative position of the watertable

[1] International Institute for Land Reclamation and Improvement

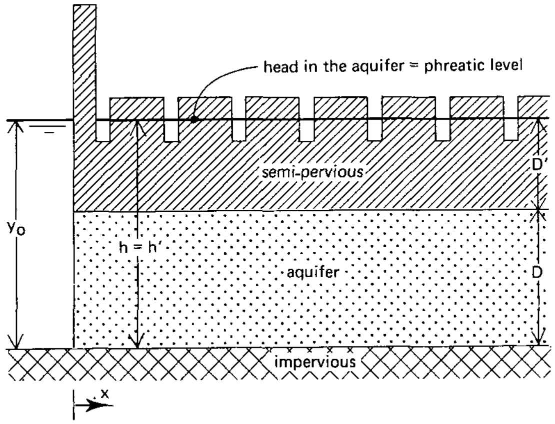

Figure 9.1 Semi-confined aquifer cut by a straight river; equilibrium conditions, groundwater at rest

in the top layer and the piezometric surface in the aquifer.

As a start, let us consider a situation where the groundwater is at rest (Figure 9.1). The watertable in the confining layer and the piezometric surface in the aquifer coincide with the water level in the river, y_o.

At high river stages the hydraulic head, h, in the aquifer increases and may rise above the phreatic level h′ in the confining layer, or even rise above the land surface. The high river stage induces a seepage flow from the river into the aquifer, and from the aquifer into the overlying confining layer (Figure 9.2). At low river stages, the head in the aquifer decreases and may fall below the watertable in the overlying confining layer. The low river stage induces a downward flow through the confining layer into the aquifer, and a horizontal flow from the aquifer towards the river channel (Figure 9.3). The upward or downward flow through the confining layer causes the watertable in that layer to rise or fall. Rainfall and evapotranspiration also affect the elevation of the watertable.

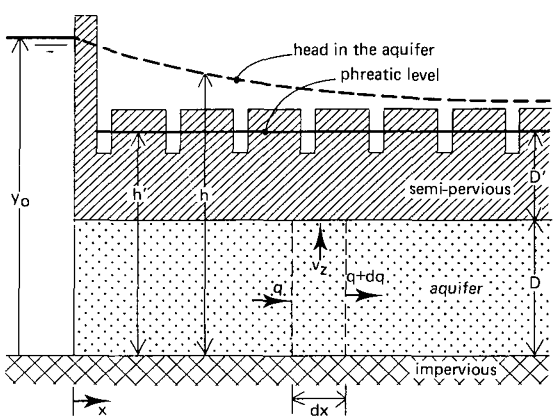

Figure 9.2 Semi-confined aquifer cut by a straight river; seepage flow

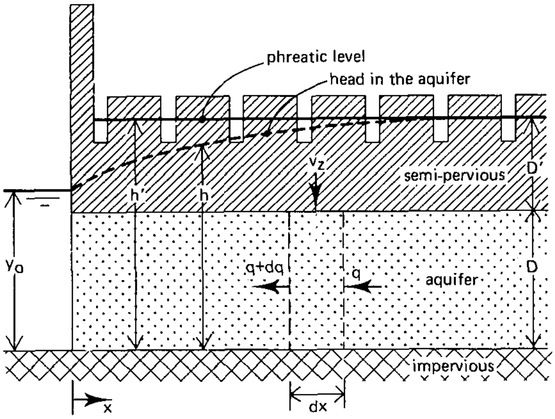

Figure 9.3 Semi-confined aquifer cut by a straight river; drainage flow

A solution to the above problem can be obtained by assuming that the watertable in the confining layer is constant and uniform at a height, h′, above the horizontal surface of the impermeable base, although a constant watertable in the confining layer, independent of changes in the hydraulic head in the aquifer, is possible only when narrowly-spaced ditches and drains are present.

Using Darcy's law, we can express the horizontal flow in the aquifer as

$$q = -KD\frac{dh}{dx}$$

or, differentiating with respect to x,

$$\frac{dq}{dx} = -KD\frac{d^2h}{dx^2} \qquad (9.1)$$

where

q = the flow per unit width of the aquifer (m^2/d)

Small quantities of water leave the aquifer through the confining layer of low permeability. The principle of continuity requires that the change in the horizontal flow in the aquifer brought about by these water losses be taken into account.

If the vertical flow through the confining layer, v_z, is taken positive in the upward direction, then

$$v_z = -\frac{dq}{dx} \qquad (9.2)$$

Using Darcy's law, we can write the upward flow through the confining layer as

$$v_z = K'\frac{h-h'}{D'} = \frac{h-h'}{c} \qquad (9.3)$$

where

K' = hydraulic conductivity of the confining layer for vertical flow (m/d)
D' = saturated thickness of the confining layer (m)
c = D'/K' = hydraulic resistance of the confining layer (d)
h' = phreatic level in the overlying confining layer (m)

Combining Equations 9.1, 9.2 and 9.3 gives the general differential equation for steady one-dimensional seepage flow

$$KD\frac{d^2h}{dx^2} - \frac{h - h'}{c} = 0 \tag{9.4}$$

which may alternatively be written as

$$\frac{d^2h}{dx^2} - \frac{h - h'}{L^2} = 0 \tag{9.5}$$

where

$L = \sqrt{KDc}$ is the leakage factor of the aquifer (m)

Equation 9.5 can be solved by integration; the solution as given in handbooks on calculus (e.g. Dwight 1971) is

$$h - h' = C_1 e^{x/L} + C_2 e^{-x/L} \tag{9.6}$$

where C_1 and C_2 are integration constants that must be determined from the boundary conditions

for $x \rightarrow \infty$, $h = h'$
for $x = 0$, $h = h_o$
and h' = constant

Substituting the first condition into Equation 9.6 gives $C_1 = 0$, and substituting the other two conditions gives $C_2 = h_o - h'$. In this expression, h_o, is the hydraulic head in the aquifer at a distance $x = 0$ from the river, or $h_o = y_o$.

Substituting these results into Equation 9.6 gives the solution

$$h - h' = (h_o - h')\, e^{-x/L} \tag{9.7}$$

which, after being rewritten, gives the relation between the hydraulic head in the aquifer, h, and the distance from the river, x

$$h = h' + (h_o - h')\, e^{-x/L} \tag{9.8}$$

The equation for the seepage can be obtained as follows. First the flow rate, v_x, is determined by differentiating Equation 9.8

$$v_x = -K\frac{dh}{dx} = \frac{K(h_o - h')}{L} e^{-x/L} \tag{9.9}$$

The total seepage per unit width of the aquifer at distance x from the river is obtained by multiplying the flow rate by the aquifer thickness, D

$$q_x = \frac{KD(h_o - h')}{L} e^{-x/L} \tag{9.10}$$

The seepage into the aquifer at x = 0 is then found by substituting x = 0 into Equation 9.10. This gives

$$q_o = \frac{KD}{L}(h_o - h') \tag{9.11}$$

From Equations 9.10 and 9.11, it follows that

$$q_x = q_o e^{-x/L} \tag{9.12}$$

This equation shows that the spatial distribution of the seepage depends only on the leakage factor, L. For some values of x, the corresponding ratios q_x/q_o and the seepage as a percentage of the seepage entering the aquifer at the river are as follows:

Distance from the river	q_x/q_0	Seepage over distance x as percentage of q_0
x = 0.5L	0.61	39
x = 1.0L	0.37	63
x = 2.0L	0.13	87
x = 3.0L	0.05	95

These figures indicate that the seepage in a zone extending from the river over a distance x = 3L equals 95% of the water entering the aquifer (at x = 0); only 5% of the water appears beyond this zone. Both Equation 9.7 and Equation 9.12 contain a damping exponential function ($e^{-x/L}$), which means that the rate of damping is governed by the leakage factor, L. At a distance x = 4L, the watertable in the confining layer and the piezometric head in the aquifer will practically coincide and, consequently, the upward flow through the confining layer will be virtually zero. Thus, a knowledge of the value of L is of practical importance.

The question now arises: how can we determine the leakage factor? One method is to conduct one or more aquifer tests (Chapter 10). From the data of such tests, the transmissivity, KD, and the hydraulic resistance, c, can be determined, giving a value of $L = \sqrt{KDc}$.

Another method is to use water-level data collected in double piezometer wells placed in rows perpendicular to the river. Equation 9.7 gives the relation between the hydraulic head difference, h – h′, and the distance, x

$$h - h' = (h_o - h')\, e^{-x/L}$$

Taking the logarithm and rewriting gives

$$L = \frac{x}{2.30\,\{\log (h_o - h') - \log (h - h')\}} \tag{9.13}$$

Plotting the observed data of (h – h′) against the distance x on single logarithmic paper (with h − h′ on the ordinate with logarithmic scale and x on the abscissa with a linear scale) will give a straight line whose slope is −1/2.30L. Such plots thus allow the value of the leakage factor to be determined (Figure 9.4).

The figure refers to a study (Colenbrander 1962) in an area along the River Waal, a branch of the River Rhine. The coarse sandy aquifer is covered by a 12 m thick layer of clayey fine sand, clay, and peat. Three double piezometers were placed in a line perpendicular to the river at distances of 120, 430, and 850 m from the dike. The slope of the straight line equals −0.2/800.

The leakage factor is found from

$$2.30L = \frac{800}{0.2}$$

or

$$L = \frac{800}{2.30 \times 0.2} = 1740\,\text{m}$$

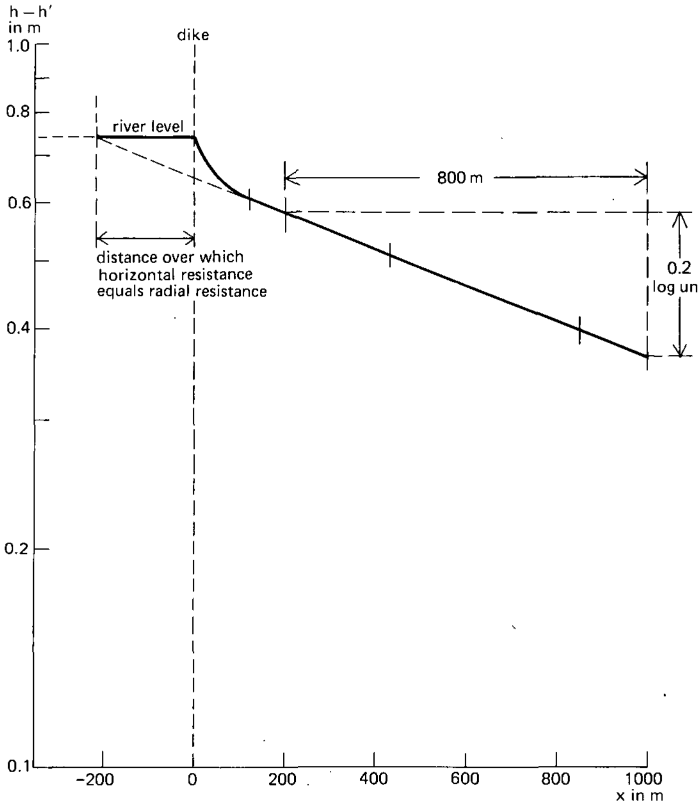

Figure 9.4 Relation between hydraulic head differences and the distance from a river in three double piezometers placed in a semi-confined aquifer

In Figure 9.4 we see that there is a deviation from the straight-line relationship near the river dike. This is because the river channel may be some distance from the dike and may not fully penetrate the aquifer. The assumption of horizontal flow in the aquifer may not hold near the river, but a certain radial flow resistance must still be taken into account. This can be done either by reducing ($h_o - h'$) to an effective value, or by expressing the effect of the radial flow in metres of horizontal flow. In Figure 9.4, the extended straight line intersects the river level at 215 m from the dike; hence the radial resistance due to the river's partial penetration of the aquifer is equal to a horizontal flow resistance over a distance of 215 m.

Example 9.1

For a situation similar to the one shown in Figure 9.2, the following data are available: transmissivity of the aquifer KD = 2000 m^2/d, hydraulic resistance of the covering confining layer c = 1000 days, the water level in the river y_o = 10 m above mean sea level, and the watertable in the confining layer h' = 8 m above mean sea level.

Calculate the upward seepage flow in a strip of land extending 1000 m along the river and 500 m inland from the river.

From the above data, we first calculate the leakage factor

$$L = \sqrt{KDc} = \sqrt{2000 \times 1000} = 1414 \text{ m}$$

The upward seepage flow per metre length of the river is found by substracting the flow through the aquifer at x = 500 m (Equation 9.12) from the flow through the aquifer below the river dike (Equation 9.11)

$$q_o - q_x = \frac{KD}{L}(h_o - h')(1 - e^{-x/L})$$

Substituting the relevant values then gives

$$q_o - q_{500} = \frac{2000}{1414}(10 - 8)(1 - e^{-500/1414}) = 0.843 \text{ m}^2/\text{d}$$

For a length of river of 1000 m, the upward seepage is

$$Q = 1000 \times 0.843 = 843 \text{ m}^3/\text{d}$$

or an average seepage rate of

$$\frac{843}{500 \times 1000} = 1.7 \times 10^{-3} \text{ m/d or } 1.7 \text{ mm/d}$$

9.3 Semi-confined Aquifer with Two Different Watertables

Figure 9.5 shows a semi-confined aquifer underlain by an aquiclude and overlain by an aquitard. In the covering confining aquitard, two different watertables occur, h_1' and h_2'; the transition between them is abrupt. In the right half, there is a vertical downward flow through the confining layer into the aquifer and a horizontal flow through the aquifer towards the left half, where there is a vertical upward flow into

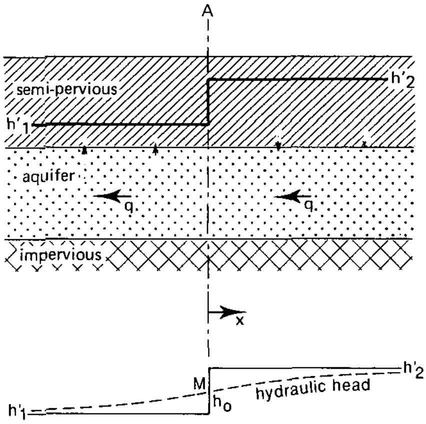

Figure 9.5 Semi-confined aquifer with two different watertables in the overlying aquitard (after Edelman 1972)

the confining layer. The lower part of Figure 9.5 shows the hydraulic head distribution in the aquifer, which is symmetrical about the point M, where

$$h = h_o = \frac{1}{2}(h'_1 + h'_2) \tag{9.14}$$

This consideration reduces the problem to the previous one. For the right half of the aquifer, we thus obtain (substituting Equation 9.14 into Equation 9.7)

$$h'_2 - h = \frac{h'_2 - h'_1}{2} e^{-x/L} \tag{9.15}$$

and

$$q_x = q_o e^{-x/L} \tag{9.16}$$

where

$$q_o = \frac{KD}{L} \frac{h'_2 - h'_1}{2} \tag{9.17}$$

9.4 Seepage through a Dam and under a Dike

9.4.1 Seepage through a Dam

A seepage problem of some practical interest is the flow through a straight dam with vertical faces (Figure 9.6). It is assumed that the dam, with a length L and a width B, rests on an impermeable base. The water levels upstream and downstream of the dam are h_1 and h_2 respectively, with $h_1 > h_2$.

This is a problem of one-dimensional flow (in the x-direction only); its basic differential equation reads (Chapter 7.8.2)

$$\frac{d^2h^2}{dx^2} = 0 \tag{9.18}$$

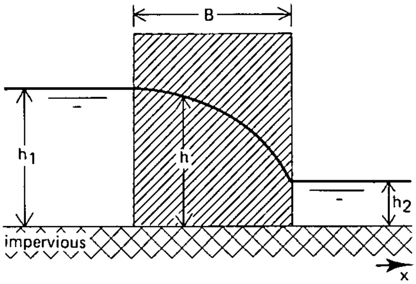

Figure 9.6 Seepage through a straight dam with vertical faces

The general solution to this equation is

$$h^2 = C_1x + C_2 \tag{9.19}$$

where C_1 and C_2 are constants to be determined from the boundary conditions, which are

$$\text{for} \quad x = 0, \quad h = h_1$$
$$x = B, \quad h = h_2$$

Substitution into Equation 9.19 gives: $C_2 = h_1^2$ and $C_1 = (h_2^2 - h_1^2)/B$. Substituting these expressions into Equation 9.19 yields

$$h^2 = h_1^2 - (h_1^2 - h_2^2)\,\frac{x}{B} \tag{9.20}$$

This equation indicates that the watertable in the dam is a parabola. Using Darcy's law, we can express the seepage through the dam per unit length as

$$q = hv_x = -\,Kh\frac{dh}{dx} = -\,\frac{K}{2}\frac{dh^2}{dx}$$

Combined with Equation 9.20, this results in

$$q = \frac{K\,(h_1^2 - h_2^2)}{2B} \tag{9.21}$$

For a given length L of the dam, the total seepage is

$$Q = \frac{KL\,(h_1^2 - h_2^2)}{2B} \tag{9.22}$$

This equation is known as the Dupuit formula (as was already derived in Chapter 7.8.2); it gives good results even when the width of the dam B is small and $(h_1 - h_2)$ is large (Verruijt 1982).

9.4.2 Seepage under a Dike

Another seepage problem is the flow from a lake into a reclaimed area under a straight, impermeable dike that separates the reclaimed area from the lake. The dike rests on an aquitard which, in turn, rests on a permeable aquifer (Figure 9.7).

On the left side of the dike, lake water percolates vertically downward through the aquitard and into the aquifer. The flow through the aquifer is horizontal in the x-direction only (one-dimensional flow). On the right side of the dike, water from the aquifer flows vertically upward through the aquitard into the reclaimed area. Thus, the problem to be solved is: what is the total seepage flow into the reclaimed area?

According to Verruijt (1982), the problem can be solved by dividing the aquifer into three regions:

Region 1: $-\infty < x < -B$
Region 2: $-B < x < +B$
Region 3: $+B < x < +\infty$

To obtain a solution for the flow in these three regions, it is necessary to introduce the values h_2 and h_3, which represent the hydraulic head in the aquifer at $x = -B$ and $x = +B$ respectively. These heads are still unknown, but can be determined later from continuity conditions along the common boundaries of the three regions.

Region 1

The flow in Region 1 is similar to that in Figure 9.3. This means that, except for the distance x, which must be replaced by $-(x + B)$, Equation 9.8 applies. Hence

$$h = h'_1 - (h'_1 - h_2)\, e^{\frac{x+B}{L}} \qquad (9.23)$$

The groundwater flow per metre length of the dike at $x = -B$ is

$$q = KD\frac{h'_1 - h_2}{L} \qquad (9.24)$$

Region 2

In Region 2, the flow rate is constant, so that according to Darcy the groundwater

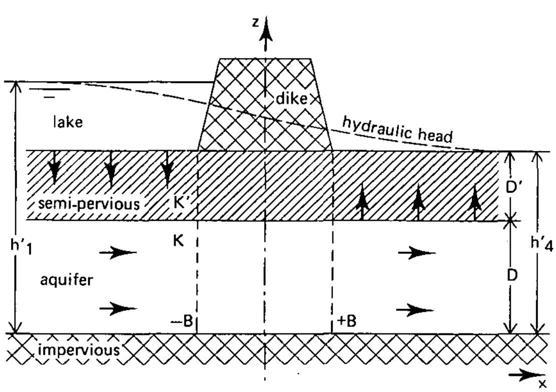

Figure 9.7 Seepage underneath a straight impermeable dike

flow per metre length of the dike in this region is

$$q = KD \frac{h_2 - h_3}{2B} \tag{9.25}$$

Region 3

In Region 3, upward vertical flow occurs from the aquifer through the aquitard. Equation 9.8 can be used if x is replaced by x – B, and h′ and h_o by h_4' and h_3. This gives

$$h = h_4' + (h_3 - h_4')\, e^{-\frac{x - B}{L}} \tag{9.26}$$

The groundwater flow at x = B is

$$q = KD \frac{h_3 - h_4'}{L} \tag{9.27}$$

The principle of continuity requires that the flows according to Equations 9.24, 9.25, and 9.27 be the same. Thus, the three unknown quantities in these equations (q, h_2, and h_3) can be solved. The solutions are

$$h_2 = h_1' - \frac{(h_1' - h_4')L}{2B + 2L} \tag{9.28}$$

$$h_3 = h_4' + \frac{(h_1' - h_4')L}{2B + 2L} \tag{9.29}$$

$$q = KD \frac{h_1' - h_4'}{2B + 2L} \tag{9.30}$$

Equation 9.30 gives the seepage into the reclaimed area per metre length of the dike. With the heads h_2 and h_3 known, the hydraulic head in the aquifer at any point can now be calculated with Equations 9.23 and 9.26.

Example 9.2

Calculate the seepage and hydraulic heads at x = –B and x = + B for a situation as shown in Figure 9.7, using the following data: h_1' = 22 m, h_4' = 18 m, aquifer thickness D = 15 m, hydraulic conductivity of the aquifer K = 15 m/d, thickness of the confining layer D′ = 3 m, hydraulic conductivity of the confining layer K′ = 0.005 m/d, and width of the dike 2B = 30 m.

The hydraulic resistance of the confining layer c = D′/K′, or 3/0.005 = 600 d. The leakage factor $L = \sqrt{KDc}$, or $\sqrt{15 \times 15 \times 600} = 367$ m.

Substituting the appropriate values into Equation 9.30 gives the seepage rate per metre length of the dike

$$q = \frac{15 \times 15\,(22 - 18)}{30 + (2 \times 367)} = 1.18\ \mathrm{m^2/d}$$

The hydraulic heads at x = –B and x = + B are found from Equations 9.28 and 9.29 respectively

$$h_2 = 22 - \frac{(22 - 18)\,367}{30 + 734} = 20.08\ \mathrm{m}$$

and

$$h_3 = 18 + \frac{(22 - 18)\,367}{30 + 734} = 19.92\,\text{m}$$

9.5 Unsteady Seepage in an Unconfined Aquifer

Some one-dimensional, unsteady flows of practical importance to the drainage engineer are: the interchange of water between a stream or canal and an aquifer in response to a change in water level in the stream or canal, seepage from canals, and drainage flow towards a stream or ditch in response to recharge in the area adjacent to the stream or ditch.

Figure 9.8 shows a semi-infinite unconfined aquifer bounded on the left by a straight, fully-penetrating stream or canal, and bounded below by an impermeable layer.

Under equilibrium conditions, the watertable in the aquifer and the water level in the canal coincide, and there is no flow out of or into the aquifer. A sudden drop in the water level of the canal induces a flow from the aquifer towards the canal. As a result, the watertable in the aquifer starts falling until it reaches the same level as that in the canal. Until this new state of equilibrium has been reached, there is an unsteady, one-dimensional flow from the aquifer into the canal. For the Dupuit assumption to be valid (Chapter 7), we assume that the drop in the watertable is small compared with the saturated thickness of the aquifer. Hence we can assume horizontal flow through the aquifer, and constant aquifer characteristics. This flow problem can be described by the following equations

- Darcy's equation for the flow through the aquifer

$$q = + KD \frac{\partial s}{\partial x} \tag{9.31}$$

which, after differentiation, gives

$$\frac{\partial q}{\partial x} = + KD \frac{\partial^2 s}{\partial x^2} \tag{9.32}$$

- The continuity equation

$$\frac{\partial q}{\partial x} = + \mu \frac{\partial s}{\partial t} \tag{9.33}$$

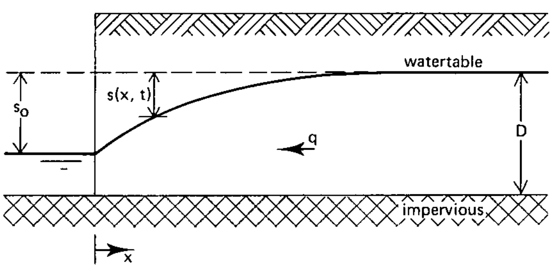

Figure 9.8 Unsteady, one-dimensional flow in a semi-infinite unconfined aquifer

Eliminating $\partial q/\partial x$ from these two equations gives the general differential equation

$$\frac{\partial^2 s}{\partial x^2} = \frac{\mu}{KD}\frac{\partial s}{\partial t} \qquad (9.34)$$

where

s = drawdown in the aquifer (m) positive downwards
x = distance from the canal (m)
μ = specific yield of the aquifer (–)
KD = transmissivity of the aquifer (m^2/d)
t = time after the change in the water level of the canal (d)

A general solution to this differential equation does not exist and integration is possible only for specific boundary conditions. Edelman (1947, 1972) derived solutions for four different situations:
- A sudden drop in the water level of the canal;
- The canal is discharging at a constant rate;
- The water level in the canal is lowered at a constant rate;
- The canal is discharging at an increasing rate, proportional to time.

Here we shall consider only the first and the third situations.

9.5.1 After a Sudden Change in Canal Stage

In the case of a sudden drop in the canal stage, the initial and boundary conditions for which the partial differential equation, Equation 9.34, must be solved are

for $t = 0$ and $x > 0$: $s = 0$
for $t > 0$ and $x = 0$: $s = s_o$
for $t > 0$ and $x \to \infty$: $s = 0$

Edelman (1947) solved this problem by introducing a dimensionless auxiliary variable, u, incorporating x and t as follows

$$u = \frac{1}{2}\sqrt{\frac{\mu}{KD}}\frac{x}{\sqrt{t}} \qquad (9.35)$$

The partial differential Equation 9.34 can then be written as the ordinary differential equation

$$\frac{d^2 s}{du^2} + 2u\frac{ds}{du} = 0 \qquad (9.36)$$

and for the boundary conditions

$s = s_o$ $\quad$ $u = 0$
$s = 0$ $\quad$ $u = \infty$

the solution is

$$s_{x,t} = s_o\left(1 - \frac{2}{\sqrt{\pi}}\int_o^u e^{-u^2}du\right) = s_o\,E_1\,(u) \qquad (9.37)$$

where $\frac{2}{\sqrt{\pi}}\int_o^u e^{-u^2}du = \text{erf}\,(u)$ is called the error function.

Tables with values of this function for different values of u are available in mathematical handbooks: e.g. Abramowitz and Stegun (1965), and Jahnke and Emde (1945). Values of the function $E_1(u)$ are given in Table 9.1. A more elaborate table is given by Huisman (1972).

The flow in the aquifer per unit length of canal at any distance x is found by differentiating Equation 9.37 with respect to x, and substituting the result in Darcy's equation according to Equation 9.31. Disregarding the sign for flow direction, we get

$$q_{x,t} = \frac{s_o}{\sqrt{\pi}}\frac{1}{\sqrt{t}}\sqrt{KD\mu}\ e^{-u^2} \qquad (9.38)$$

The discharge from the aquifer into the canal per unit length of canal is found by substituting x = 0; thus u = 0

$$q_{o,t} = \frac{s_o}{\sqrt{\pi}}\frac{1}{\sqrt{t}}\sqrt{KD\mu} \qquad (9.39)$$

so that Equation 9.38 reduces to

$$q_{x,t} = q_{o,t}\,e^{-u^2} = q_{o,t}\,E_2\,(u) \qquad (9.40)$$

Values of the function $E_2(u)$ are also given in Table 9.1.
Equation 9.39 gives the discharge from one side of the canal. If the drop in the water level of the canal induces groundwater flow from two sides, the discharge given by Equation 9.39 must be multiplied by two.

Note: The above equations can also be used if the water level in the canal suddenly rises, inducing a flow from the canal into the aquifer, and resulting in a rise in the watertable in the aquifer.
The equations can also be used to calculate either the change in watertable in the aquifer if the hydraulic characteristics are known, or to calculate the hydraulic characteristics if the watertable changes have been measured in a number of observation wells placed in a row perpendicular to the canal.

Example 9.3
Using the following data, calculate the rise in the watertable at 10, 20, 40, 60, 80, and 100 m from the canal 25 days after the water level in the canal has risen suddenly by 1 m: saturated thickness of the aquifer D = 10 m, hydraulic conductivity K = 1 m/d, and specific yield μ = 0.10.

Table 9.1 Values of the functions $E_1(u)$, $E_2(u)$, $E_3(u)$, and $E_4(u)$

u	$E_1(u)$	$E_2(u)$	$E_3(u)$	$E_4(u)$
0.00	1.0000	1.0000	1.0000	1.0000
0.01	0.9887	0.9999	0.9824	0.9776
0.02	0.9774	0.9996	0.9650	0.9556
0.03	0.9662	0.9991	0.9477	0.9341
0.04	0.9549	0.9984	0.9307	0.9129
0.05	0.9436	0.9975	0.9139	0.8920
0.06	0.9324	0.9964	0.8973	0.8717
0.07	0.9211	0.9951	0.8808	0.8515
0.08	0.9099	0.9936	0.8646	0.8319
0.09	0.8987	0.9919	0.8486	0.8125
0.10	0.8875	0.9900	0.8327	0.7935
0.12	0.8652	0.9857	0.8017	0.7566
0.14	0.8431	0.9806	0.7714	0.7212
0.16	0.8210	0.9747	0.7419	0.6871
0.18	0.7991	0.9681	0.7132	0.6542
0.20	0.7773	0.9608	0.6852	0.6227
0.22	0.7557	0.9528	0.6581	0.5924
0.24	0.7343	0.9440	0.6317	0.5633
0.26	0.7131	0.9346	0.6060	0.5353
0.28	0.6921	0.9246	0.5811	0.5085
0.30	0.6714	0.9139	0.5569	0.4829
0.32	0.6509	0.9027	0.5335	0.4583
0.34	0.6306	0.8908	0.5108	0.4346
0.36	0.6107	0.8784	0.4888	0.4121
0.38	0.5910	0.8655	0.4675	0.3906
0.40	0.5716	0.8521	0.4469	0.3699
0.42	0.5525	0.8383	0.4270	0.3501
0.44	0.5338	0.8240	0.4077	0.3314
0.46	0.5153	0.8093	0.3891	0.3133
0.48	0.4973	0.7942	0.3712	0.2963
0.50	0.4795	0.7788	0.3539	0.2799
0.52	0.4621	0.7631	0.3372	0.2643
0.54	0.4451	0.7471	0.3211	0.2495
0.56	0.4284	0.7308	0.3056	0.2353
0.58	0.4121	0.7143	0.2907	0.2219

Table 9.1 (cont.)

u	$E_1(u)$	$E_2(u)$	$E_3(u)$	$E_4(u)$
0.60	0.3961	0.6977	0.2764	0.2089
0.62	0.3806	0.6809	0.2626	0.1969
0.64	0.3654	0.6639	0.2494	0.1853
0.66	0.3506	0.6469	0.2367	0.1743
0.68	0.3362	0.6298	0.2245	0.1639
0.70	0.3222	0.6126	0.2129	0.1541
0.72	0.3086	0.5955	0.2017	0.1448
0.74	0.2953	0.5783	0.1910	0.1358
0.76	0.2825	0.5612	0.1807	0.1275
0.78	0.2700	0.5442	0.1710	0.1195
0.80	0.2579	0.5273	0.1616	0.1120
0.82	0.2462	0.5105	0.1527	0.1049
0.84	0.2349	0.4938	0.1441	0.0982
0.86	0.2239	0.4773	0.1360	0.0919
0.88	0.2133	0.4610	0.1283	0.0860
0.90	0.2031	0.4449	0.1209	0.0803
0.92	0.1932	0.4290	0.1139	0.0750
0.94	0.1837	0.4133	0.1072	0.0700
0.96	0.1746	0.3979	0.1008	0.0654
0.98	0.1658	0.3827	0.0948	0.0609
1.00	0.1573	0.3679	0.0891	0.0568
1.02	0.1492	0.3533	0.0836	0.0529
1.04	0.1414	0.3391	0.0785	0.0492
1.06	0.1339	0.3251	0.0736	0.0458
1.08	0.1267	0.3115	0.0690	0.0426
1.10	0.1198	0.2982	0.0646	0.0396
1.14	0.1069	0.2726	0.0566	0.0341
1.18	0.0952	0.2485	0.0494	0.0293
1.22	0.0845	0.2257	0.0431	0.0252
1.26	0.0748	0.2044	0.0374	0.0215
1.30	0.0660	0.1845	0.0325	0.0184
1.34	0.0581	0.1660	0.0281	0.0156
1.38	0.0510	0.1489	0.0242	0.0133
1.42	0.0446	0.1331	0.0208	0.0113
1.46	0.0389	0.1186	0.0179	0.0095

Table 9.1 (cont.)

u	$E_1(u)$	$E_2(u)$	$E_3(u)$	$E_4(u)$
1.50	0.0339	0.1054	0.0153	0.0080
1.60	0.0237	0.0773	0.0102	0.0052
1.70	0.0162	0.0556	0.0067	0.0033
1.80	0.0109	0.0392	0.0044	0.0021
1.92	0.0066	0.0251	0.0025	0.0011
2.00	0.0047	0.0183	0.0017	0.0007
2.10	0.0030	0.0122	0.0011	0.0005
2.20	0.0019	0.0079	0.0006	0.0003
2.30	0.0012	0.0050	0.0004	0.0002
2.40	0.0007	0.0032	0.0002	0.0001
2.50	0.0004	0.0019	0.0001	0.0000

The transmissivity of the aquifer KD = 1 × 10 = 10 m^2/d is assumed to be constant, although, with the rise of the watertable, the saturated thickness D, and hence KD, increases slightly to, say, 10.5 m^2/d. Substituting into Equation 9.35 gives

$$u = \frac{x}{2}\sqrt{\frac{0.10}{10}}\frac{1}{\sqrt{25}} = 0.01\ x$$

For the given distances of x, the value of u is calculated and the corresponding values of $E_1(u)$ are read from Table 9.1. Substitution of these values and s_o = 1 m into Equation 9.37 yields the rise in the watertable after 25 days at the given distances from the canal (Table 9.2).

Example 9.4

Analyzing the change in the watertable caused by a sudden rise or fall of the water level in a canal makes it possible to determine the aquifer characteristics. For this purpose, the change in watertable is measured in a few observation wells placed in a line perpendicular to the canal. Suppose three observation wells are placed at distances of 10, 20, and 40 m from the canal. At $t < 0$, the watertable in the aquifer has the same elevation as the water level in the canal. At $t = 0$, the water level in

Table 9.2 The rise in the watertable after 25 days

Distance x (m)	u (-)	$E_1(u)$ (Table 9.1) (-)	Watertable rise (m)
10	0.1	0.8875	0.89
20	0.2	0.7773	0.78
40	0.4	0.5716	0.57
60	0.6	0.3961	0.40
80	0.8	0.2579	0.26
100	1.0	0.1573	0.16

Table 9.3 Observed rise in the watertable (m) in three wells

Distance of observation well (m)	Time since rise in canal stage (d)				
	t = 0.5	t = 1	t = 2	t = 3	t = 4
10	0.25	0.29	0.32	0.34	0.35
20	0.13	0.19	0.25	0.26	0.27
40	0.02	0.065	0.125	0.165	0.19

the canal suddenly rises by an amount $s_o = 0.50$ m. The watertable measurements made in the three observation wells are given in Table 9.3.

Calculate the transmissivity of the aquifer, assuming that its specific yield $\mu = 0.10$. Analyze the flow in the vicinity of the canal. Calculate the seepage from the canal at t = 1 d and t = 4 d.

Equations 9.35 and 9.37 indicate that $\log(s/s_o)$ varies with $\log(x/\sqrt{t})$ in the same manner as $\log E_l(u)$ varies with log u. Solving Equation 9.35 for μ/KD therefore requires matching a logarithmic data plot of s/s_o ratios against their corresponding values of $x/\sqrt{t}$ to a logarithmic type curve drawn by plotting values of $E_l(u)$ against corresponding values of u. The type curve is drawn with the aid of Table 9.1. To prepare the logarithmic data plot of s/s_o versus $x/\sqrt{t}$, we use the data from Table 9.3:

	Time since rise in canal stage (d)				
	t = 0.5	t = 1	t = 2	t = 3	t = 4
For x = 10 m					
$x/\sqrt{t}$	14.1	10.0	7.1	5.8	5.0
s/s_o	0.50	0.58	0.64	0.68	0.70
For x = 20 m					
$x/\sqrt{t}$	28.3	20.0	14.1	11.5	10.0
s/s_o	0.26	0.38	0.50	0.52	0.54
For x = 40 m					
$x/\sqrt{t}$	56.6	40.0	28.3	23.1	20.0
s/s_o	0.04	0.13	0.25	0.33	0.38

We now plot these data on another sheet of double logarithmic paper with the same scale as that used to prepare the type curve of $E_l(u)$. We then superimpose the two sheets and, keeping the coordinate axes parallel, we find a position in which all (or most) of the field-data points fall on a segment of the type curve (Figure 9.9). As match point, we select the point z with logarithmic type curve coordinates u = 0.1, $E_l(u) = 1.0$. On the field-data plot, this point has the coordinates $x/\sqrt{t} = 4$ and $s/s_o = 0.8$.

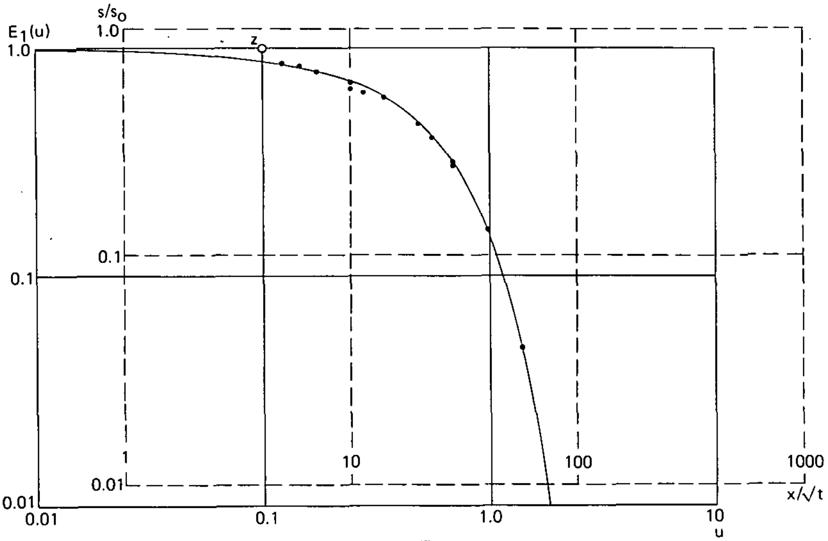

Figure 9.9 Observed data plot s/s_o versus $x/\sqrt{t}$ (points and dotted lines) superimposed on logarithmic type curve $E_1(u)$-versus-u (curve and solid lines)

Substituting these values into Equation 9.35 yields

$$\sqrt{\frac{KD}{\mu}} = \frac{x}{2\sqrt{t}}\,\frac{1}{u} = \frac{4}{2} \times \frac{1}{0.1} = 20$$

For $\mu = 0.10$, it follows that $KD = 400 \times 0.1 = 40\ m^2/d$.

According to Equation 9.37, the ratio $s/s_o = E_1(u)$. If $E_1(u) = 1$, it follows that $s = s_o$. Only at the edge of the canal (at $x = 0$) is $s = s_o$. From the coordinates of the match point z, however, it follows that, for $E_1(u) = 1$, the ratio $s/s_o = 0.8$. This means that $s = 0.8\ s_o$, or $s = 0.8 \times 0.5 = 0.4$ m. At the edge of the canal, the watertable is therefore $0.5 - 0.4 = 0.1$ m less than expected. The value of 0.1 m is the head loss due to radial flow in the vicinity of the canal, because the canal does not fully penetrate the aquifer.

The seepage from the canal after 1 and 4 days is found from Equation 9.39. Substituting the appropriate values into this equation gives

for t = 1 day

$$q_{o,1} = \frac{s_o}{\sqrt{\pi t}}\sqrt{KD\mu} = \frac{0.4}{\sqrt{3.14 \times 1}}\sqrt{40 \times 0.1} = 0.45\ m^2/d$$

for t = 4 days

$$q_{o,4} = \frac{0.4}{\sqrt{3.14 \times 4}}\sqrt{40 \times 0.1} = 0.23\ m^2/d$$

Remarks
If a canal penetrates an aquifer only partially, as is usually the case, watertable readings from observation wells placed too close to the canal may give erroneous results. The smaller the distance between the observation well and the canal, the greater the error in the calculated value of μ/KD. As is obvious, an instantaneous rise or fall in the canal level can hardly occur, which makes it difficult to determine a reference or zero time (Ferris et al. 1962). This means that observations made shortly after the change in canal stage may be unreliable. With partially penetrating canals, therefore, more weight should be given to data from wells at relatively great distances from the canal and to large values of time. However, observation wells at great distances react slowly to a relatively small change in canal stage, and it may take several days before noticeable watertable changes occur. This may be another source of error, especially when the aquifer is recharged by rain or is losing water through evapotranspiration. The solution is based on the assumption that water losses or gains do not occur. A field experiment should therefore not last longer than, say, two or three days to avoid errors caused by such water losses or gains.

9.5.2 After a Linear Change in Canal Stage

The condition of an abrupt change in the water level of a canal or stream is rather unrealistic, except perhaps in an irrigation area where some of the canals are alternately dry and filled relatively quickly when irrigation is due. A more realistic situation is a canal stage that is a function of time. In this section, a solution will be given for the situation where the change in water level of a canal is proportional to time; in other words, the water level changes at a linear rate, denoted by α.
Hence

$$s_o = \alpha t \qquad (9.41)$$

so that the initial and boundary conditions for which Equation 9.34 must be solved are

$$\begin{aligned}
&\text{for } t = 0 \text{ and } x > 0 \;:\; s = 0\\
&\text{for } t > 0 \text{ and } x = 0 \;:\; s = s_o = \alpha t\\
&\text{for } t > 0 \text{ and } x \to \infty : s = 0
\end{aligned}$$

The solution then becomes

$$s_{x,t} = s_{o,t}\, E_4(u) \qquad (9.42)$$

where

$$E_4(u) = -\frac{2u}{\sqrt{\pi}} E_2(u) + (2u^2 + 1)\, E_1(u)$$

and

$$q_{o,t} = \frac{2s_{o,t}}{\sqrt{\pi}} \frac{1}{\sqrt{t}} \sqrt{KD\mu} \qquad (9.43)$$

$$q_{x,t} = q_{o,t}\, E_3(u) \qquad (9.44)$$