

Figure 19.24 Compound canal sections

Calculate the discharge through areas $2A_z$ and A_b and add the results to find the discharge capacity of the canal. This is an iteration process, in which the water depths must be balanced.

19.5 Maximum Permissible Velocities

19.5.1 Introduction

The stability analysis of earthen channels and those with vegetation is important to the design of a drainage canal system. To evaluate such a system, an engineer needs to know the relationships between flowing water and the earthen materials forming the boundary of the channel. He also needs to understand the expected flow response when lining, vegetation, or other features are imposed. These relationships may be the determining factors for channel alignment, hydraulic gradient, and dimensions of the cross-section.

To analyze the stability of earthen channels, an engineer uses two fundamental approaches. In the first, he assumes that the bed and banks of the channel are mobile; in the second, that they are rigid. The conditions for these assumptions are described below.

Mobile Boundary

Stability of a channel with mobile bed and banks is attained when the rate at which sediment enters a channel reach from upstream is equal to the sediment transport capacity of that reach. Stability in such channels may be determined with the sediment transport equations in Section 19.5.2.

Rigid Boundary

Stability of a channel with rigid bed and banks is attained when the erosive forces



Figure 19.25 Procedural guide to find the maximum permissible velocity

of the flowing water are effectively resisted by the soil material forming the channel boundary. Properly designed channels of this type have a cross-section that remains essentially unchanged during all stages of flow. The stability of the channel boundaries can be evaluated with either the allowable velocity approach or the tractive stress approach.

Both approaches are appropriate for the design of drainage canals. A procedural guide is presented in Figure 19.25 to assist the designer in determining the maximum permissible velocity.

19.5.2 The Sediment Transport Approach

To consider the tractive force exerted by flowing water on a channel bed and bottom, we study a unit length of channel section like the one in Figure 19.26. For uniform flow to occur, the acceleration of flow must be zero, so that according to Newton's second law of motion, F = ma, the resultant of all forces acting on the water in the considered channel section should be zero. As the net hydraulic thrust in that section is zero, the net force in the flow direction is zero if

 $\tau = \rho g A \sin \alpha \tag{19.30}$



Figure 19.26 Definition sketch for the tractive stress equation

where

 $\tau = tractive force (N)$

 α = canal slope angle (degrees)

If we assume that the canal slope is slight, we can write

 $\sin\alpha\approx tg\,\alpha=s$

so that per unit length of canal the total tractive force, τ , may be expressed as

$$\tau = \rho g A s \tag{19.31}$$

Hence, the average tractive force per unit of the wetted perimeter P, known as tractive stress, equals

$$\tau_{o} = \rho g R s \tag{19.32}$$

This stress acts on the discrete soil grains on the channel bed together with the gravity force, G per unit area.

For the two-dimensional example in Figure 19.27, the discrete grain will be in



Figure 19.27 Forces acting on discrete channel bed material (two-dimensional)

equilibrium if

$$\frac{\tau_o}{G} = \frac{\rho_w \, gRs}{a_1(\rho_m - \rho_w) \, gd} \le tg \, \phi \tag{19.33}$$

or

$$\frac{R_s}{\rho_r d} \le tg\,\phi \tag{19.34}$$

where, in addition to the terms defined earlier

G = gravity force acting on the bed material per unit area (Pa)

 $\rho_m = mass density of the bed material (kg/m³)$

 a_1 = percentage of solid material in a layer of thickness d_{50}

 $d = d_{50} = median grain diameter (m)$

 ϕ = angle of internal repose of the bed material (degrees)

$$\rho_{\rm r} = (\rho_{\rm m} - \rho_{\rm w})/\rho_{\rm w}$$

A considerable amount of research has been done on this subject, providing that the bed material of a channel will not move if

$$Y = \frac{\mu Rs}{\rho_r d_{50}} \le 0.047$$
(19.35)

where

Y = flow parameter (-)

 μ = so-called ripple factor, which is a factor of ignorance, used to obtain agreement between calculated and measured values of bed-load transport. It varies between 0.5 for slightly rough beds, to 1.0 for smoother bed forms

If the flow parameter, Y, exceeds 0.047, the sediment particles on the channel bed start to slide, roll or jump over and near the bed, generally in the form of moving bed shapes such as dunes and ripples. This is called bed-load transport, and it can be calculated with the equation of Meyer-Peter and Müller (1948), which reads

$$\mathbf{X} = \mathbf{A}_1 (\mathbf{Y} - 0.047)^{3/2} \tag{19.36}$$

where

 $A_1 = a$ factor with an average value of 8

X = the transport parameter (dimensionless), which is

$$X = \frac{T}{\sqrt{\rho_r g d_{50}^3}}$$
(19.37)

where

T = transport of soil material expressed in solid volume per second for one unit width of channel

The bed material load over the full width of the channel, expressed in m³/s soils, equals

$$Q_{\rm m} = Tb \tag{19.38}$$

Equation 19.38 does not hold for the suspended load, i.e. the bed material being transported, because the gravity force is counterbalanced by the upward forces from turbulence. This means that while the grains make large or small jumps, they return eventually to the channel bed. A strict division between bed load and suspended load is not possible; in fact, the mechanics behind the two are related. It is therefore not surprising that the equation for the bed material load (bed load plus suspended load) is based on the above flow and transport parameters (Engelund and Hansen 1967)

$$\mathbf{X} = 0.05 \, \mathbf{Y}^{5/2} \tag{19.39}$$

Sediment transport also can be expressed in parts per million (ppm) by mass

$$ppm = \frac{Q_m \rho_m}{Q \rho_w} \times 10^6$$
(19.40)

If the suspended sediment concentration equals or exceeds 20 000 ppm by mass, the flow is termed 'sediment-laden'. If the concentration is 1000 ppm or less by mass the flow is 'sediment-free'.

The calculation of flow rate and sediment transport through a channel with an essentially mobile bed and banks, is complex. Technical assistance from a qualified hydraulics laboratory is recommended for the design of important channels.

19.5.3 The Allowable Velocity Approach

This method of evaluating the erosion resistance of the rigid bed and banks of an earthen channel is based on data collected by Fortier and Scobey (1926), Lane (1952), by investigators in the U.S. Soil Conservation Service (1977), Eastgate (1969), Ree (1977), and others.

The maximum allowable velocity is determined in two steps; (i) find the basic allowable velocity for a straight channel with a water depth of 1 m, and (ii) determine some correction factors. Basic velocities are presented below for channels with bottom and sides of discrete earth materials, of coherent earth materials, and for grassed channels. Subsequently, we will treat five correction factors: (A) for the void ratio, (B) for the frequency of design flow, (C) for the design water depth, (D) for channel curvature, and (E) for the side slope of the channel bank.

Basic Allowable Velocity, v_b

The allowable velocity is strongly influenced by the concentration of fine material carried in suspension. There are two distinct types of flow with respect to sediment concentration:

- 1) Sediment-free flow; which is when material is carried in suspension at concentrations of 1000 ppm or less by mass. If sediment-free water flows with increasing velocity, its sediment transport capacity can only be reached if the water erodes material from the channel bed and banks;
- 2) Sediment-laden flow; which is when the water carries 20 000 ppm or more by mass of soil material. This high concentration will increase boundary stability, either through replacement of eroded material, or through formation of a protective cover because of settling. As a result, sediment-laden water has a significantly higher allowable flow velocity than sediment-free water.

Discrete Earthen Materials

Figure 19.28 gives the basic allowable velocities for channels with a boundary of discrete earthen materials. We can make a linear interpolation between the two curves for water with suspended sediment concentrations between 1000 ppm and 20 000 ppm. We can best estimate the sediment concentration by sampling channels in the area to be drained. If we cannot measure the concentration, we can calculate it with Equations 19.38 to 19.40 of Section 19.5.2. The basic velocity for discrete earthen materials should be corrected with the factors C, D and E.

Coherent Earthen Materials

In coherent earthen materials, the individual grains are cemented together so that the allowable velocity is greater than when the soil grain diameter alone is the determining factor. It is advisable to use the 'Unified Soil Classification System' as given in Section 19.3.4 to name the soil and determine the basic allowable velocity. Figure 19.29 gives the basic allowable velocity as a function of the plasticity index PI for each classification name (code). The basic velocity for coherent earthen materials should be corrected with factors A through D.

Grassed Channels

In Table 19.8, we can find the basic allowable velocities for channels where a grass lining was established on the bottom and side slopes upon construction, to act as a stabilizer. These velocities apply to average, uniform stands of each type of vegetal cover. Basic allowable velocities must be less than 1.5 m/s if there is no proper maintenance. The basic allowable velocities require correction with factors C, D, and E.





Figure 19.28 Basic allowable velocity, v_b , for discrete earthen materials and related correction factors C through E (adapted from USDA 1977)



Figure 19.29 Basic allowable velocity, v_b , for coherent earthen materials, and related correction factors A through D (adapted from USDA 1977)

The 'erosion resistant soils' are probably clay loams or better, while the 'easily eroded soils' are probably as erosion resistant as a sandy loam soil.

The following conditions must be met before we can rely on grass lining as a stabilizer (Theuer 1979):

- The climate must be conducive to establishing and supporting a grass cover that will provide year-round protection;
- The soils in the channel boundary must be capable of supporting permanent vegetation;

	Basic allowable velocity (m/s)		
Cover	Slope Range (s in %)	Erosion resistant soils	Easily eroded soils
Kikuyu (Pennisetum clandestinum), Bermuda grass or African star grass (Cynodon dactylon)	0 - 5 5 - 10 > 10	2.40 2.10 1.80	1.80 1.50 1.20
Buffalo grass (Buchloe dactyloides), Kentucky bluegrass (Poa pratensis) Smooth brome (Bromus inermis) Blue grama (Bouteloua gracilis) Rhodes grass (Chloris gayana)	0 - 5 5 - 10 > 10	2.10 1.80 1.50	1.50 1.20 0.90
Grass mixture	0 - 5 1.50 1.20 5 - 10 1.20 0.90 Do not use on slopes steeper than 10%		
Lespedeza sericea, Weeping love grass (Eragrostis curvula), Kudzu (Pueraria thunbergiana), Queensland Bluegrass (Dichanthium sericeum), Alfalfa (Medicago sativa), Crabgrass (Digitaria sanguinalis)	0 - 5 1.00 0.75 Do not use on slopes steeper than 5%, except for side-slopes in a combination channel		
Annuals - used on mild slopes or as temporary protection until permanent covers are established: Common Lespedeza (Lespedeza striata), Sudan grass (Sorghum sudanense)	0 - 5 Use on slope not recomme	1.00 es steeper than ended	0.75 5% is

Table 19.8 Basic allowable velocities for grass-lined channels (adapted from U.S. Dept. of Agriculture 1977, and Eastgate 1969)

- The bed of the channel and that portion of the side-slope under base flow must be stable. Armouring or rip-rap may be needed to stabilize these parts of the channel boundary; or flow must be intermittent enough to allow vegetation to be established;
- Design flows in the channel must not cause scouring. Vegetation should never be intended as a stabilizer for sloughing banks or banks undermined by seepage.

Adjustments in the basic allowable velocity, v_b , to reflect the modifying effects of the void ratio, frequency of design flow, design water depth, curvature in alignment, and channel bank slope, should be made with the correction factors A through E of Figures 19.28 and 19.29. They are:

- A) The void ratio correction factor, which applies to coherent earthen materials only, and corrects for the compactness of the soil. The void ratio (e) in Figure 19.29 is the ratio of (i) the volume of void space to (ii) the volume of solid particles in a given soil mass. This factor decreases with the increase of void space;
- B) The frequency correction factor, which applies to coherent earthen materials only,

and is based on the assumption that an infrequent discharge causes little erosion damage in channels with coherent boundaries. Channels designed for flood discharges of less than 10% frequency, however, should be checked for stability at the 10% chance frequency discharge and related water depth;

- C) The water depth correction factor, which is needed because the initial basic velocities are valid only for channels with a depth of 1 m (y = 1.0 m). A greater water depth (y > 1 m) causes lower velocities along the channel boundary than shallow depth (y < 1 m) if the average velocity, v = Q/A, is the same. This factor applies to channels in all soils;
- D) The channel curvature correction factor, which applies to all channels in all soils. It is necessary because the spiral flow in curves tends to erode the outer channel embankment. For sharp curves, it can be a good solution to armour the outer embankment instead of lowering the average velocity (see Photo 19.4);
- E) The bank slope correction factor, which applies to channels whose banks are constructed in earth with discrete particles. Because of the lack of cohesion, these particles tend to roll down the bank slope.

The maximum allowable velocity, v_{max} , is found by multiplying the basic allowable velocity, v_{b} , by the relevant correction factors.

For discrete earthen materials and grassed channels (Figure 19.29 and Table 19.8)

$$\mathbf{v}_{\max,\text{discr.}} = \mathbf{v}_{b} \times \text{CDE} \tag{19.41}$$

For coherent earthen materials (Figure 19.29)

$$v_{\text{max. coh.}} = v_{\text{b}} \times \text{ABCD}$$
(19.42)

19.6 Protection Against Scouring

19.6.1 Field of Application

Equation 19.35 illustrates that the discrete earthen material on the bed of a channel starts to move if the flow parameter, Y, exceeds 0.047. The designer can reduce the numerical value of $Y = \mu Rs/\rho_r d_{50}$ in three ways:

- By selecting a high b/y ratio for the channel so that the hydraulic radius, R, is relatively low (see Section 19.3.5);
- By selecting a flatter hydraulic gradient, s, for the channel and using a drop structure to dissipate the remainder of the hydraulic energy (see Section 19.7.1);
- By armouring the channel bed and banks with a material having a discrete particle diameter large enough to remain stable.

The remainder of Section 19.6 will deal with the latter possibility.

19.6.2 Determining Stone Size of Protective Lining

Channel with Uniform Flow

We can determine the size of the discrete particles (called rip-rap) of the protective

lining of a channel with uniform flow by using the v_b versus d_{75} relationship of Figure 19.28 and the correction factors C, D, and E.

For example: a trapezoidal channel with a design water depth, y = 1.50 m, a curve radius/water surface width ratio of 12, and a cotangent of the side slope angle, z = 2.0, transports sediment-free water at an average velocity, v = 2.00 m/s. What is the diameter, d_{75} , required for the rip-rap stones of the protective lining?

Step 1

Find the correction factors C, D, and E of Figure 19.28:

For $y = 1.50 m$,	C = 1.08
If radius/width is 12,	D = 0.96
For $z = 2.0$,	E = 0.71

Step 2

Working in the reverse order of Section 19.5.3, divide the actual average velocity by the correction factors to find the basic allowable velocity v_b

$$v_{b} = \frac{v}{CDE}$$
(19.43)

Hence, the basic allowable velocity is

 $v_{\rm h} = 2.00/(1.08 \times 0.96 \times 0.71) = 2.72 \, {\rm m/s}$

Step 3

Enter Figure 19.28 with the calculated value of v_b and find $d_{75} = 0.14$ m.

Downstream of a Structure

A protective lining may also be needed for the channel bed and banks immediately downstream of a structure. The erosive currents leaving the downstream end of a weir, flume, culvert, and so on, often damage the earthen channel. This can be prevented by increasing the diameter of the rip-rap stones (discrete particle size) over a short channel reach. The length of this reach is affected by several factors. As a rule of thumb, do not choose a length of rip-rap that is (i) less than 4 times the (maximum) anticipated water depth in the channel downstream of the structure, (ii) less than the length of the earthen transition between structure and channel, (iii) or less than 1.50 m.

Flow leaving a structure is characterized by a variable combination of factors such as local velocity, flow direction, turbulence, and waves. Because of the unpredictable combination of these factors, the velocity at which water will strike the rip-rap is difficult to predict, but it will certainly be higher than that along the boundary of a channel with uniform flow. As a result, the size of the rip-rap stones downstream of a structure is significantly larger than that in a channel with uniform flow.

For practical purposes, we can find the grain/stone diameter needed downstream of a structure from Figure 19.30. Calculate the average velocity above the end sill of the structure by dividing the discharge by the cross-sectional area of flow above this





sill. Figure 19.30 gives the d_{40} of the rip-rap mixture, which means that more than 60% should consist of stones that are as nearly alike as practicable in length, width, and thickness, and of curve size or larger; or they should be of curve weight or heavier, and not flat slabs.

19.6.3 Filter Material Placed Beneath Rip-Rap

If the rip-rap stones of a protective lining are laid immediately on top of the fine material (subgrade) in which the canal is excavated, grains of this subgrade will wash through the openings between the stones. This process is due partly to the turbulent flow of canal water in and out of the voids between the stones, and partly to the inflow of water that leaks around the structures or flows into the drain.

To avoid damage to the rip-rap from washing of the subgrade, there must be a filter between the two (Figure 19.31). The protective construction as a whole and each separate layer must be sufficiently permeable to water entering the canal through its bed or banks. Further, fine material from an underlying filter layer or the subgrade must not be washed into the voids of a covering layer.

Permeability to Water

To maintain sufficient permeability of the protective construction shown in Figure 19.31, the following d_{15}/d_{15} ratios should have a value of between 5 and 40 (U.S. Bureau of Reclamation 1973, Bertram 1940)

$$\frac{d_{15} \text{ layer } 3}{d_{15} \text{ layer } 2} \text{ and } \frac{d_{15} \text{ layer } 2}{d_{15} \text{ layer } 1} \text{ and } \frac{d_{15} \text{ layer } 1}{d_{15} \text{ subgrade}} = 5 \text{ to } 40$$
(19.44)

(19.45)

where d_{15} equals the diameter of the sieve opening, through which 15% of the total sample weight passes. Depending on the shape and gradation of the grains in each layer, we can narrow the range of the ratios as follows:

 Homogeneous round grains (gravel) 	5 - 10
- Homogeneous angular grains (broken gravel, rubble)	6 - 20
- Well-graded grains	12 - 40

To prevent the filter from clogging, it is also advisable that in each layer

$$d_5 \ge 0.75 \text{ mm}$$





Stability of each Layer

To prevent the loss of fine material from an underlying filter layer or the subgrade through the openings in a covering layer, two requirements must be met:

1) The following d_{15}/d_{85} ratios should not exceed 5 (Bertram 1940)

$$\frac{d_{15} \text{ layer } 3}{d_{85} \text{ layer } 2} \text{ and } \frac{d_{15} \text{ layer } 2}{d_{85} \text{ layer } 1} \text{ and } \frac{d_{15} \text{ layer } 1}{d_{85} \text{ subgrade}} \le 5$$
(19.46)

 And the d₅₀/d₅₀ ratios should range between 5 and 60 (U.S. Army Corps of Engineers 1955)

$$\frac{d_{50} \text{ layer 3}}{d_{50} \text{ layer 2}} \text{ and } \frac{d_{50} \text{ layer 2}}{d_{50} \text{ layer 1}} \text{ and } \frac{d_{50} \text{ layer 1}}{d_{50} \text{ subgrade}} = 5 \text{ to } 60$$
(19.47)

As before, the ratio in Equation 19.47 depends on the shape and gradation of the grains:

 Homogeneous round grains (gravel) 	5 – 10
- Homogeneous angular grains (broken gravel, rubble)	10 - 30
- Well-graded grains	12 - 60

The above requirements describe the sieve curves of the successive filter layers. If we know the sieve curve of the rip-rap and the subgrade, we can plot other layers. Figure 19.32 shows the sieve curves of a construction consisting of rip-rap and two underlying filter layers.

19.6.4 Fitting of Sieve Curves

The procedure for dimensioning a protective construction in a channel with uniform flow is similar to that for dimensioning such a construction immediately downstream of a structure. The only difference is that, for a channel with uniform flow, we must



Figure 19.32 Sieve curves of a protective construction

use Figures 19.28 and 19.29 to find the size of the rip-rap stones, while for downstream of a structure, we must use Figure 19.30. In the following steps, we shall assume that the protective construction is downstream of a structure.

1) Determine and plot the sieve curve of the subgrade. For the example of Figure 19.32 we read that:

 $d_{15} = 0.05 \text{ mm}$ $d_{50} = 0.09 \text{ mm}$

 $d_{85} = 0.15 \,\mathrm{mm}$

Note that the subgrade is well-graded.

2) Use Figure 19.30 to determine the minimum d_{40} of the rip-rap layer.

For example, if the velocity over the end sill is 1.8 m/s, then $d_{40} > 0.17$ m or 170 mm. Check which material is – or can be made available and plot its sieve curve in Figure 19.32 (rip-rap layer 100 to 300 mm). The curve of the selected material shows that the diameter of the rounded rip-rap mixture is rather homogeneous:

- $d_{15} = 150 \text{ mm}$
- $d_{40} = 180 \text{ mm}$
- $d_{50} = 200 \text{ mm}$
- $d_{85} = 270 \text{ mm}$
- 3) Use the first part of Equation 19.44 to determine a range for the d₁₅ of layer 2 by

 $\frac{d_{15} \text{ layer } 3}{d_{15} \text{ layer } 2} = \frac{150 \text{ mm}}{d_{15} \text{ layer } 2} = 5 \text{ to } 10 \text{ (homogeneous rounded grains)}$

or

 $15 \text{ mm} < d_{15} \text{ layer } 2 < 30 \text{ mm}$

Plot this range into Figure 19.32. (///).
4) Use the last part of Equation 19.46 to determine a range for the d₁₅ of layer 1 by

 $\frac{d_{15} \text{ layer } 1}{d_{15} \text{ subgrade}} = \frac{d_{15} \text{ later } 1}{0.05 \text{ mm}} = 12 \text{ to } 40 \text{ (well-graded)}$

or

 $0.6 \,\mathrm{mm} < \mathrm{d_{15}} \,\mathrm{layer} \,1 < 2.0 \,\mathrm{mm}$

Plot this range into Figure 19.32. (////).5) Use the first part of Equation 19.46 to find and plot

 d_{85} layer 2 > 30 mm ($\leftarrow x$)

6) Use the last part of Equation 19.46 to find and plot

 d_{15} layer $1 \le 0.75 \text{ mm} (x \rightarrow)$

When we have plotted this, we shall see that the range has narrowed to 0.6 to 0.75 mm. From Equation 19.45 we find

 $d_5 \text{ layer } 1 \ge 0.75 \text{ mm} (\leftarrow 0)$

These two criteria are difficult (if not impossible) to attain and must be relaxed slightly.

7) Following a procedure similar to Steps 3 and 4, use Equation 19.47 to find and plot

 $20 \text{ mm} \le d50 \text{ layer } 2 \le 40 \text{ mm}$

and

 $1.1 \text{ mm} \le d_{50} \text{ layer } 1 \le 5.4 \text{ mm}$

- 8) Find locally available materials that have a grain size distribution within the ranges summarized in Figure 19.32. To provide a stable and effective filter, the sieve curves of the subgrade and filter layers should run about parallel for the small-diameter grains.
- 9) Determine the d_{15} , d_{50} , and d_{85} of the tentatively selected mixtures in filter layers 1 and 2. Repeat Steps 3 through 7 to check if these limitations hold for the selected mixtures. If not, shift the sieve curves slightly or add an additional filter layer.

19.6.5 Filter Construction

To obtain a fair grain size distribution throughout a filter layer, each layer should be sufficiently thick. The following thicknesses must be regarded as a minimum for filters constructed under dry conditions:

- Sand, fine gravel 0.05 to 0.10 m;
- Gravel 0.10 to 0.20 m;
- Stones 1.5 to 2 times the largest stone diameter.

For filters constructed under water, these thicknesses have to be increased considerably to compensate for irregularities in the subgrade and because it is more difficult to apply an even layer.

There are many variations on the basic filter construction. One or more of the layers often are replaced with other materials. With some protective linings, only the rip-rap layer is maintained, while the underlying filter layers are replaced by one single layer, e.g.:

- Concrete blocks on a nylon filter;
- Stones on braided hardwood strips on a plastic filter;
- Gabions on fine gravel;

- Nylon-sand mattresses.

The usual difficulty with these variants is their perviousness to the underlying material. As a rule, the openings in such a layer should not be greater than $0.5 \times d_{85}$ of the underlying material. If the openings are greater, one should not replace all the underlying layers, but maintain as many (usually one) as are needed to prevent the subgrade from being washed through the combined layer. Many manufacturers produce so called geo-textiles, which are very suitable as filter layer. The related documentation should give the above product properties. Such documentation commonly gives various design examples.

The protective construction is most subject to damage at structure-to-filter and filter-to-unprotected-channel 'joints'. This is because the filter layer is likely to subside even though the structure itself is well founded. Underlying material (subgrade) may be washed out at these joints if no special measures are taken. It is recommended