7 Basics of Groundwater Flow

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7.1 Introduction

In drainage studies, we are interested not only in the depth at which the watertable is found and in its rise and fall, but also in the flow of groundwater and the rate at which it flows. The terms groundwater and watertable are defined in Section 7.2. In Section 7.3, because we are dealing with groundwater as a fluid, we present some of its physical properties and the basic laws related to its movement. This movement is governed by well-known principles of hydrodynamics which, in fact, are nothing more than a reformulation of the corresponding principles of mechanics. On the basis of these principles, we shall formulate the equation of continuity and the equations of groundwater movement. We give special attention to Darcy's equation in Section 7.4, to some of its applications in Section 7.5, and to the theory of streamlines and equipotential lines in Section 7.6.

The equations for flow and continuity are partial differential equations which can only be solved if we know the boundaries of the flow regions. These boundaries or 'boundary conditions' are discussed in Section 7.7. Further, to solve groundwater-flow patterns bounded by a free watertable (known as an unconfined aquifer, Chapter 2), we have to make additional assumptions to simplify the flow pattern. The Dupuit-Forchheimer theory, which deals with these assumptions, gives good solutions to problems of flow to parallel drains and pumped wells (Section 7.8). Finally, as an example of an approximate method to solve the partial differential equations, Section 7.9 presents the relaxation method.

It should be noted that the equations in this chapter are not intended for direct use in drainage design, but are expanded upon in subsequent chapters on Subsurface Flow to Drains (Chapter 8), Seepage and Groundwater Flow (Chapter 9), and Single-well and Aquifer Tests (Chapter 10).

7.2 Groundwater and Watertable Defined

The term 'groundwater' refers to the body of water found in soil whose pores are saturated with water. The locus of points in the groundwater where water pressure is equal to atmospheric pressure defines the 'watertable', which is also called the free water surface or the phreatic surface (Figure 7.1). The watertable can be found by measuring the water level in an open borehole that penetrates the saturated zone. Pressure is usually expressed as relative pressure, p, with reference to atmospheric pressure, p_{atm}. At the watertable, by definition, $p = p_{atm}$.

The groundwater body actually extends slightly above the watertable owing to capillary action, but the water is held there at less than atmospheric pressure. The zone where capillary water fills nearly all of the soil's pores is called the capillary

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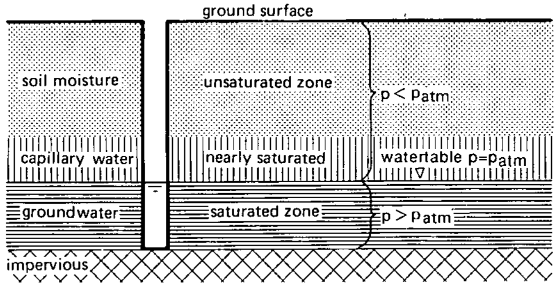

Figure 7.1 Schema of the occurrence of subsurface water

fringe. Although it occurs above the watertable, the capillary fringe is sometimes included in the groundwater body. The capillary water occurring above the capillary fringe belongs to the unsaturated zone, or zone of aeration, where the soil's pores are filled partly with water and partly with air (Chapter 11).

7.3 Physical Properties, Basic Laws

7.3.1 Mass Density of Water

The density of a material is defined as the mass per unit of volume. Mass density may vary with pressure, temperature, and the concentration of dissolved particles. Temperature, for example, causes the mass density of water to vary as follows (see also Table 7.1)

$$\rho = 1000 - \frac{(T-4)^2}{150} \tag{7.1}$$

where

ρ = density of water (kg/m^3)
T = water temperature (°C)

Table 7.1 Variation in mass density and viscosity of water with temperature

Temperature (°C)	Mass density (kg/m^3)	Dynamic viscosity (kg/m s)	Kinematic viscosity (m^2/s)
0	999.87	1.79×10^{-3}	1.79×10^{-6}
5	999.99	1.52×10^{-3}	1.52×10^{-6}
10	999.73	1.31×10^{-3}	1.31×10^{-6}
15	999.13	1.14×10^{-3}	1.14×10^{-6}
20	998.23	1.01×10^{-3}	1.007×10^{-6}
25	997.07	0.89×10^{-3}	0.897×10^{-6}
30	995.67	0.80×10^{-3}	0.804×10^{-6}
40	992.24	0.65×10^{-3}	0.661×10^{-6}

Because mass density varies with temperature, water of 15°C will not mix spontaneously with water of 20°C, and there will be even less mixing between fresh water and sea water. Because of its salt content, the mass density of sea water is about $\rho_s = 1027$ kg/m^3. This variation in mass density must, of course, be taken into account when hydraulic heads are being measured.

7.3.2 Viscosity of Water

In a moving fluid, a fast-moving layer tends to drag a more slowly-moving layer along with it; the slower layer, however, tends to hold back the faster one. Because layers of fluid flow at different velocities, the fluid body opposed by an internal stress will be deformed. The internal stress that causes the deformation of the fluid during flow is called viscosity. Basically, viscosity is the relation between the shear stress acting along any plane between neighbouring fluid elements, and the rate of deformation of the velocity gradient perpendicular to this plane. Thus, if the fluid element A, shown in Figure 7.2, travels at an average velocity, v, in the x-direction, it will be deformed at an angular rate equal to dv/dy. According to Newton, the shear, τ, along plane a-a will then be

$$\tau = \eta \frac{dv}{dy} \tag{7.2}$$

where

τ = shear stress (Pa)
η = the dynamic viscosity of the fluid (kg/m s)

Kinematic viscosity is defined by the relation

$$\upsilon = \frac{\eta}{\rho} \tag{7.3}$$

where
υ = kinematic viscosity (m^2/s)

Table 7.1 gives the variation in viscosity with temperature.

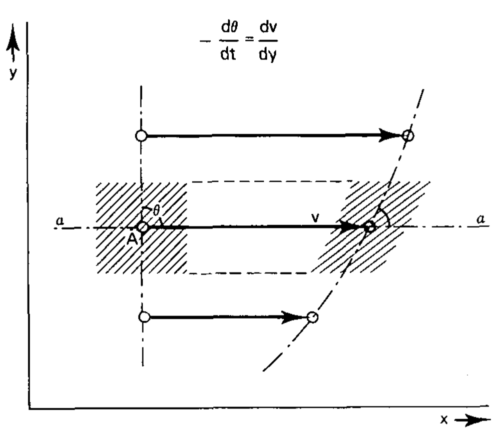

Figure 7.2 Angular deformation of a fluid element (Rouse 1964)

7.3.3 Law of Conservation of Mass

A fundamental law of hydrodynamics is the law of conservation of mass, which states that, in a closed system, fluid mass can be neither created nor destroyed. In other words, a space element, dx, dy, dz, in which the fluid and the flow medium are both incompressible, will conserve its mass over a time, dt. Therefore, the fluid must enter the space element at the same rate (volume per unit time) as it leaves. The rate at which a given volume is transferred across a section equals the product of the velocity component perpendicular to the section and the area of the section.

If the velocity distribution over the flow profile is non-linear, we may assume a linear velocity distribution over the elementary distances, dx, dy, and dz. Hence, we can write the average velocity components perpendicular to the lateral faces of the space element as indicated in Figure 7.3.

The difference between the volume of water leaving the space element and the volume of water entering it in the x-direction over time, dt, equals

$$(v_x + \frac{\delta v_x}{\delta x} dx)\, dy\, dz\, dt - v_x\, dy\, dz\, dt \tag{7.4}$$

or

$$\frac{\delta v_x}{\delta x} dx\, dy\, dz\, dt \tag{7.5}$$

Analogous expressions can be derived for the y- and z-directions

$$\frac{\delta v_y}{\delta y} dy\, dx\, dz\, dt \tag{7.6}$$

and

$$\frac{\delta v_z}{\delta z} dz\, dx\, dy\, dt \tag{7.7}$$

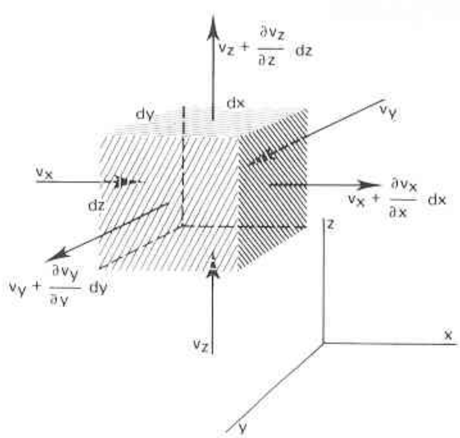

Figure 7.3 Velocity distribution in a fluid space element

According to the law of conservation of mass, the total difference between the volume transferred into the space element and that transferred out of it must equal zero. Hence

$$\frac{\delta v_x}{\delta x} dx\, dy\, dz\, dt + \frac{\delta v_y}{\delta y} dy\, dx\, dz\, dt + \frac{\delta v_z}{\delta z} dz\, dx\, dy\, dt = 0 \tag{7.8}$$

For flow independent of time, $(\partial v/\partial t = 0)$, this equation reduces to

$$\frac{\delta v_x}{\delta x} + \frac{\delta v_y}{\delta y} + \frac{\delta v_z}{\delta z} = 0 \tag{7.9}$$

which is a general form of the continuity equation.

In fluid mechanics, it is common practice to select a coordinate system whose x-direction coincides with the direction of the flow vector at a given point. In other words, the x-direction is parallel to the tangent of the path line at that point. Consequently, $v_x = v$, $v_y = 0$, and $v_z = 0$. Because, in these circumstances, there is a transfer of volume in the x-direction only, the difference between the volume of water transferred into and out of the space element, over time, dt, must equal zero. Hence

$$(v_x + \frac{dv_x}{dx} dx)\, dy\, dz\, dt - v_x\, dy\, dz\, dt = 0 \tag{7.10}$$

and because $dA = dydz$

$$v_{(x+dx)}\, dA - v_x\, dA = 0$$

or

$$(vdA)_{(x+dx)} = (vdA)_x = dQ \tag{7.11}$$

Thus, the rate of flow, dQ, is a constant through two elementary cross-sections at an infinitely short distance from each other. In fact, we considered an elementary part of a stream tube, bounded by streamlines, lying on the dx-dy and dx-dz planes.

If we now consider a finite area of flow, A, we can write the continuity equation as

$$Q = \int_A vdA = \bar{v}A \tag{7.12}$$

where $\bar{v}$ is the average velocity component perpendicular to the crosssectional area of flow.

7.3.4 The Energy of Water

Although heat and noise are types of energy that can influence the flow of water, let us assume for our purposes that they exert no energy. An elementary fluid particle then has three interchangeable types of energy per unit of volume

$\rho v^2/2$ = kinetic energy per unit of volume (Pa)
ρgz = potential energy per unit of volume (Pa)
p = pressure energy per unit of volume (Pa)

Let us assume that a fluid particle is moving in a time interval, Δt, over a short distance (from Point 1 to Point 2) along a streamline, and is not losing energy through friction or turbulence. Because, on the other hand, the fluid particle is not gaining energy either, we can write

$$\left(\frac{\rho v^2}{2} + \rho g z + p\right)_1 = \left(\frac{\rho v^2}{2} + \rho g z + p\right)_2 = \text{constant} \quad (7.13)$$

This equation is valid for points along a streamline only if the energy losses are negligible and the mass density, ρ, is a constant. According to Equation 7.13

$$\frac{\rho v^2}{2} + \rho g z + p = \text{constant} \quad (7.14)$$

or

$$\frac{v^2}{2g} + \frac{p}{\rho g} + z = H = \text{constant} \quad (7.15)$$

where, as shown in Figure 7.4

$v^2/2g$ = the velocity head (m)
$p/\rho g$ = the pressure head (m)
z = the elevation head (m)
$p/\rho g + z$ = the piezometric head (or potential or hydraulic head) or the water level in the stilling well (m)
H = the total energy head (m)

The latter three heads all refer to the same reference level. The reader should note that each streamline may have its own energy head.

Equations 7.13, 7.14, and 7.15 are alternative forms of the well-known Bernoulli equation.

It is stressed again that these equations are only valid:
- When a fluid particle is moving along a streamline under steady-flow conditions;

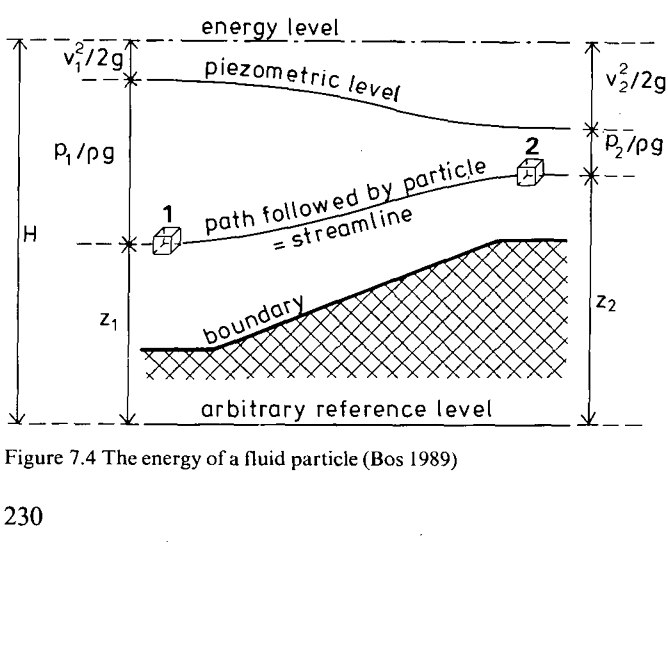

Figure 7.4 The energy of a fluid particle (Bos 1989)

– When the energy losses are negligible;
– When the mass density of the fluid, ρ, is a constant.

Because, in nature, velocities of groundwater flow, v, are usually low, the kinetic energy in Equation 7.15 can be disregarded without any appreciable error. Hence, Equation 7.15 reduces to

$$\frac{p}{\rho g} + z = h \qquad (7.16)$$

The physical meaning of Equation 7.16 is illustrated in Figure 7.5. Using Equation 7.16, we can measure the piezometric head at various points in the groundwater body. Subsequently, we can determine the hydraulic gradient and the direction of groundwater flow.

Pressure is usually expressed as relative pressure, p, with reference to atmospheric pressure, p_{atm}. Thus, in this context, p_{atm} equals zero pressure. Mean sea level is sometimes used as a reference level in measuring elevation.

7.3.5 Fresh-Water Head of Saline Groundwater

If heads are measured in piezometers installed in a deep layer containing groundwater of different salt concentrations, these heads should, as a rule, be converted into fresh-water heads so that the true hydraulic gradient can be determined. Expressing the fresh-water head, h_f, as (see Figure 7.6)

$$h_f = z + \frac{p}{\rho_f g}$$

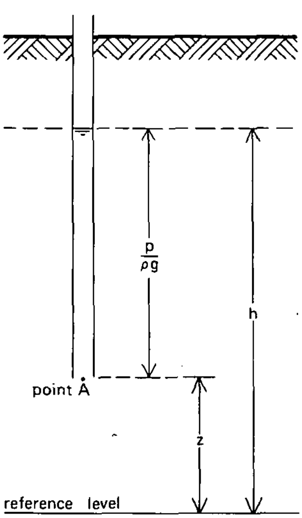

Figure 7.5 Piezometric or hydraulic head, h, at a point A, located at a height z above a reference level