

**COMMISSIE VOOR HYDROLOGISCH ONDERZOEK TNO**  
**COMMITTEE FOR HYDROLOGICAL RESEARCH TNO**

**VERSLAGEN EN MEDEDELINGEN No. 19**  
**PROCEEDINGS AND INFORMATIONS No. 19**

**THE HYDROLOGICAL  
INVESTIGATION PROGRAMME  
IN SALLAND (THE NETHERLANDS)**



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(THE NETHERLANDS)**

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RESEARCH IN THE NETHERLANDS TNO 1974**





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PROCEEDINGS OF  
TECHNICAL MEETING 27





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## INTRODUCTION

G. SANTING

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The aim of the hydrological studies being carried out in Salland (fig. 1), an area of some 500 km<sup>2</sup> with good aquifers in the province of Overijssel, is to develop directives for the optimum management and exploitation of the groundwater resources of that area, for the benefit of all parties interested in groundwater management, viz. public and industrial water supply, agriculture and nature conservancy, the latter including the protection of the landscape and the environment as a whole. The studies are carried out by a working group consisting of representatives of various interested groups and experts of the relevant disciplines.

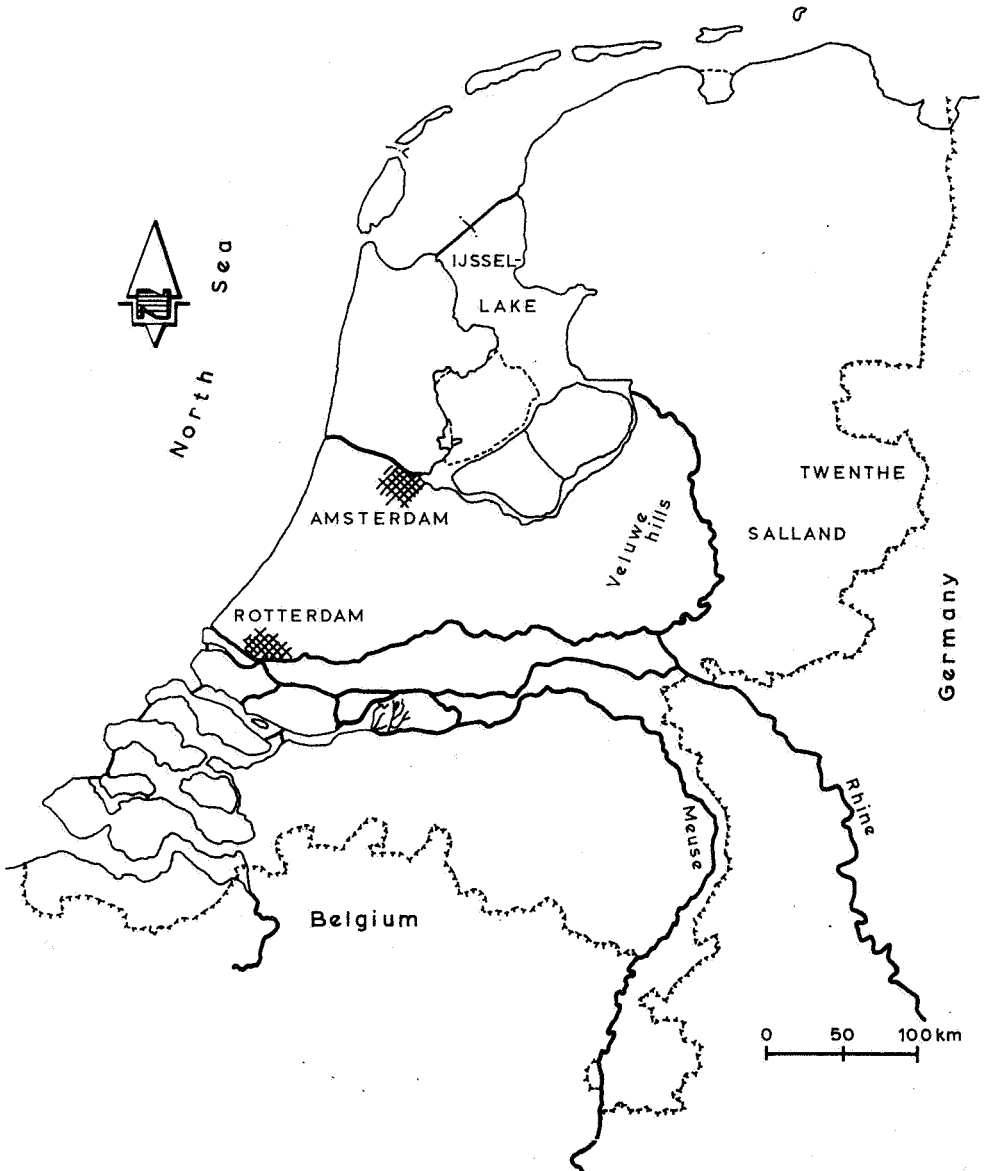
Although the studies have not yet been finished and final results cannot be shown, the working group deemed it useful to report at an early stage about the set-up of the investigations, the new methods used and the problems that still have to be solved. A second reason for this early publicity is the working group's expectation that the study may be useful also for other areas in The Netherlands.

The working group (Werkgroep Hydrologisch Onderzoek Overijssel) was established in December 1968 by a provincial committee (Technische Werkgroep van de provinciale Commissie Drinkwatervoorziening Overijssel) and consists presently of representatives of the following organisations and institutions.

1. Waterleidingmaatschappij "Overijssel" N.V.
2. Gem. Gas- en Drinkwaterleidingbedrijven Enschede.
3. Gem. Energie- en Waterleidingbedrijven Deventer.
4. Provinciale Waterstaat Overijssel.
5. Cultuurtechnische Dienst, Provinciale Directie Overijssel.
6. Agrarische Bedrijfsontwikkeling, Provinciale Directie.
7. Waterschap Salland.
8. Rijkswaterstaat, Dienst voor de Waterhuishouding.
9. Rijks Geologische Dienst.
10. Koninklijk Nederlands Meteorologisch Instituut, KNMI.
11. Instituut voor Cultuurtechniek en Waterhuishouding, ICW.
12. Rijksinstituut voor Drinkwatervoorziening, RID.
13. Landbouwhogeschool.
14. Dienst Grondwaterverkenning TNO, DGV-TNO.



Fig. 1. The Netherlands.



15. Commissie Grondwaterwet Waterleidingbedrijven.
16. N.V. Heidemaatschappij Beheer.
17. Vereniging Krachtwerktuigen, afd. industriewater.

Numbers 1, 2 and 3 are water works, 4 and 8 are provincial and state water authorities, 7 is a regional water board, 5, 6, 11, 13 and 16 are agricultural or land improvement organisations and institutions, 9 is the State Geological Survey, 10 is the Royal Meteorological Service, 12 is the Government Institute for Water Supply, 14 is the TNO Groundwater Reconnaissance Service, 15 is a state advisory committee dealing with the allocation of concessions to public water works for groundwater pumping, 17 represents the industrial interests in groundwater management.

The working group, after having defined its aims and worked out its programme, estimated to need two years for the collection of basic data, for field investigations and field reconnaissance and for partial studies, and another year for the final integrated study. The collection of some of the basic data started in the course of 1969, but only from January 1st 1970 the full measuring programme was being carried out. Measuring was stopped in the course of 1972. The elaboration of the results and the integrated studies will require some more time than it was estimated; it is expected that the final report will be ready by the end of 1973 or early in 1974.

Because of the wide scope of the study and the many disciplines involved, the working group set up four separate study groups, viz. for the hydro-meteorological aspects, for drainage and surface water discharge, for the hydrogeology, and for the agriculture studies. The latter group should also integrate the partial studies of the four groups and formulate the final conclusions. As only selected subjects and investigations of these four study groups will be dealt with in the following papers, a brief summary of the whole working programmes of the groups is given below. It should be pointed out thereby that the basic principle of the Salland investigation programme is to divide the area into a small number of more or less hydrologically uniform areas and to develop a mathematical model that can be applied to these areas and that includes the main hydrological characteristics and variables and even some economical factors.

— Study group A, hydrometeorology.

The programme of this group included first of all the collecting of basic data (rainfall and evaporation) required as input data for the water balance model.

In view of the high variability of rainfall, a dense network of 60 groundlevel rain gauges was installed (KNMI). The gauges are measured daily at the same time by volunteers. Furthermore 6 recording rain gauges (Rijkswaterstaat) and radar maps (KNMI) are used in order to obtain a better knowledge of the variations of rainfall in the course of the day.

The evaporation (of an open water surface), being less variable than rainfall, will be calculated according to Penman's method from measurements of the net radiation, relative humidity, air temperature and wind velocity. These factors are registered at a meteorological station at Schoonheten (Heidemij). In addition an evaporation pan with rain gauge and recorder was installed in the vicinity of this station (ICW).

The programme of group A further includes a statistical study of many years' series of rainfall data from 5 KNMI stations (frequency distributions of total amounts of rainfall in  $k$ -day periods ( $k = 1, 2, 3, \dots$ ), applicability of the Gumbel statistic for extreme values, significance of differences of monthly averages), a study of the optimum network density and a study of the applicability of Penman's method to short periods.

— Study group B, drainage and surface-water discharge.

The field of study of this group concerns drainage and its relations with the groundwater level, storage and rainfall.

In the Salland area a great many recorders for the measurement of the surface water discharge have been installed, complementary to the existing networks of Heidemij (Rijkswaterstaat, Prov. Waterstaat, ICW). The data are being processed by computer. Furthermore some drainage formulas are being checked by means of these data.

The groundwater level is measured weekly in 36 key wells of the national observation network of DGV-TNO; some of these wells were provided with recorders. Additional rows of shallow wells were installed across brooks and rivulets, to determine the effects of weirs on the groundwater level and on water storage.

— Study group C, hydrogeology.

This group investigates the geological and hydrological conditions of the sub-soil, extent and thickness of permeable and semi-permeable formations, the formation constants, the groundwater movement and balance. It also compiles an inventory of existing groundwater pumping stations and of hydrologically suitable locations for future pumping stations.

After a preliminary geo-electrical reconnaissance of the area by DGV/TNO a drilling programme was carried out, including 6 borings of medium depth (15 m), 10 deep borings (40 m) and one very deep boring (200 m). The purpose of the latter was to obtain supplementary information about the presence and depth of the deep brackish groundwater.

The geological data were worked out in cross-sections and maps, showing thickness and extent of aquifers and of less permeable formations. The extent of a clay layer at a depth of about 15 m over a small area near Schalkhaar was mapped in detail by means of soundings and a few borings.

Contour maps of the phreatic surface and of the head of the deeper groundwater,



and maps showing the areas of groundwater recharge and discharge were made for the purpose of studying the groundwater movement and the water balance. A study of the chemistry of the groundwater has nearly been completed. The further programme includes several additional pumping tests for the determination of the transmissivity and other geotechnical constants; a provisional transmissivity map has already been made.

— Study group D, agricultural aspects and integration of the partial studies.

The agricultural studies relate in particular to the effect of groundwater abstraction on farming. Since most of the arable lands of the Salland area are used as pastures, the studies could be confined to that type of agriculture.

The lowering of the groundwater table causes a reduction of crop yields. Moreover there are other effects on the economic profits of the farmer, on cultivation methods, on the required drainage, on the maintenance costs of the irrigation and drainage systems etc. It is, therefore, necessary to study farming and cultivation as a whole. For that purpose a survey was carried out on some 20 pasture farms during two grazing seasons (grazing, manuring, harvesting etc.).

The second task of group D is to integrate the partial studies of the other groups, with its own agricultural study, into a final study. A great deal of this work will have to be done at the end of the period of research, when the results of the field and other partial studies are available. Preliminarily group D is developing mathematical models for the integrated study; it also studies various possible groundwater development schemes, suitable principles for indemnification and compensation to the farmers, and a key for the apportionment of the costs of groundwater development among the public water works and the farmers.

Mathematical models of the water balance of a very small area were developed; by means of these models the groundwater level and the evaporation at that location can be determined day by day from the water balance equation. The groundwater abstraction by pumping stations is also accounted for in that model. The groundwater level and the evaporation will be used at their turn in another mathematical model that is being developed and that will represent the growth of the crops. Finally the crop yield determined in this way will make it possible to calculate the economic profits of the farm under various conditions of groundwater development; thus the damage caused to the farms by the groundwater development can be calculated.

Moreover it will be possible to determine by means of these models how a given yearly groundwater abstraction should be performed (spacing and location of wells and well fields, adaptation of the rates of abstraction to agricultural demands with respect to groundwater level and soil humidity in the course of the seasons) in order to obtain the smallest possible damage to agriculture. These models will have to be made available as a means for routine studies of water management questions.

From this review of subjects of study and also from the durations of the studies and the great number of participating institutes it is clear that the costs of this research programme are not small; they are estimated at approximately 1.5 million D. guilders.

The programme however is not of local importance only; if the studies are successful, their results may also be useful for other areas of similar hydrological character in the country. It would be of particular importance if policy decisions concerning management and development of the groundwater resources could be taken on the basis of reliable methods to determine the optimum use of the resources.

Apart from this aspect it should be remarked that the mere partial studies in the fields of meteorology, hydrogeology etc. already justify their being carried out, even if no integration of these studies would be made; several partial studies would still have been carried out by the institutes concerned, be it at other times or in other areas.

Finally it should be pointed out that, through the concerted studies and the joint aim of the optimum development of the groundwater resources, the previous conflict between the interests of public water supply and those of agriculture may decrease.

#### *Acknowledgement*

The readiness of the many institutes and services represented in the working group to collaborate in the research programme should be mentioned. In particular the great amount of work done by the ICW deserves special gratitude.

# I. THE AIM OF MODERN HYDROLOGY

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## INTRODUCTION

In recent years the aim of hydrologic research has changed rapidly as a result of the modern elaboration types. A few years ago investigations concerning a single effect, dealing with a single term or process in the water balance, were habitual. This kind of research might deal with formulae for the flow of water or with evaporation, with the behaviour of capillary moisture or with the reaction of the vegetation on lack or excess of water.

Gradually, however, the interest shifted more and more to the study of all the effects on the totality of the water balance caused by one single intervention. An instance is for example the many undesirable effects of the abstraction of large quantities of water out of the deep subsoil in cases where the groundwater is present at a shallow depth. The natural or the artificial agricultural vegetation may be damaged by drought, the rivers may receive less drainage water, other wells may dry out and nature may become less attractive for recreational purposes or for wild life.

These investigations of a multi-purpose type are of importance for practice as well as for scientific progress. To solve integrated problems an integrated study is required. In hydrology this integrated approach is easiest to realise by investigating the water balance in its entirety. Every term or property can be accounted for and can, if desired, be placed in the center of the elaboration.

The calculations may for example end in the assessment of the groundwater depth. The same calculating precept, however, makes it possible to assess also the discharge of a river.

The water balance model enables one to assess all the terms of the balance, which in this study also includes capillary soil moisture stress and capillary rise as they simplify the formulae to be evolved. A curve fitting procedure can then be used to calculate soil constants (see table 1, column 4) and the quantities of flow or the amount of stored water (see column 5). This opens the possibility to assess hydrologic soil constants providing data which up to recently were used to be determined in the laboratory or by means of field investigations.



In the multi-purpose solution many quantities of moisture or soil moisture constants require assessment. Also productivity estimates with respect to crop production or the richness of the growth of natural plant associations can be calculated with the model. Which parameters are solved depends on the scope of the problem to be studied. One can for example, include river flow or limit the area of study to the channel-soil interface and leave the river out. Another possibility is to deduce the reaction of agriculture on abstraction of water for civil water supply or on the other hand, leave it outside the scope of the investigation. The nucleus of the elaboration, however, is the water balance present at a certain spot. This balance can be deduced from data on groundwater depth which at several thousands of places are determined at 14-day intervals, and made generally available by the Netherlands Archive for Groundwater depths.

#### AVAILABLE TECHNIQUES

The study of processes which determine the moisture relations in an area is conducted according various techniques.

One method is based upon the moisture relations that can be evolved from the lysimeter equation

$$N + E_w + A + \Delta B + A_L = 0 \quad (1)$$

where:

$N$  = rainfall

$E_w$  = real evaporation

$A$  = discharge

$\Delta B$  = change in amount of stored water

$A_L$  = lateral deep groundwater flow

This equation can be worked out to an accurate description of the water management situation around a selected point. The description of the areal situation is less accurate as it only accounts for the discharge from the selected point to one or more drainage bases. The location of these drainage bases often will not be accurately known, although the depth of the drainage basis – but not its location – can be accurately calculated.

A second elaboration is the one of the finite element method, in which the area is divided into a not too large number of small but finite elements. This method makes use of the continuity equation

$$\frac{L}{P} \frac{d}{ds} \left( kD \frac{dh}{ds} \right) = \mu \frac{dh}{dt} + \frac{d}{dt} (N - E_w - A) \quad (2)$$

where:

$kD$ = horizontal transmissivity	$t$ = time
$h$ = moisture potential	$P$ = area of finite element
$s$ = length along flow path	$L$ = width of the area of flow

In equation (2) it is stated that the variation in the horizontal flow is compensated for by the variation in storage, in real evaporation and in discharge. The rainfall  $N$  is inserted to make the discharge agree with the other water balance terms. For the same reason the area  $P$  of the finite element is taken in order to obtain the discharge in mm per day, the dimension of  $N$  and  $E_w$ .

This formula describes the horizontal flow and it is easiest used for areas with a deep groundwater level, which excludes the influence of groundwater depth on evaporation, discharge to field drains, furrows and all other drainage flow to nearby discharge conduits. The processes, governing the vertical movement of moisture, described by  $N$ ,  $E_w$  and  $A$  are generally given by simple functions because of the application of this method to areas with deep water tables.

It is obvious that at several instances the desire has been expressed to combine the two approaches for areas with a shallow water table. To this purpose in formula (1) the lateral flow  $A_L$  has to be inserted. Here the influence of the changes in storage and groundwater depth at larger or smaller distances from the point to be studied is of importance.

This desire to combine the local study based on the lysimeter equation with the regional description of the equation of continuity is for instance apparent by the extension of the elaboration of the continuity equation, worked out with electric analogons, with the digital technique to a hybrid method. The extension of the lysimeter equation in the direction of the regional study also is a point of investigation.

## TWO-LAYER MODEL

The water balance can be illustrated by fig. 1, showing the balance for a two-layer problem.

The upper layer is described by

$$N - E_w - I + V_c + B_{z_{i+1}} - B_{z_i} - A_{\text{surf}} = 0 \quad (3)$$

where:

$N$ = rainfall	$V_c$ = capillary rise
$E_w$ = real evaporation	$B_z$ = moving storage
$I$ = infiltration	$A_{\text{surf}}$ = surface discharge

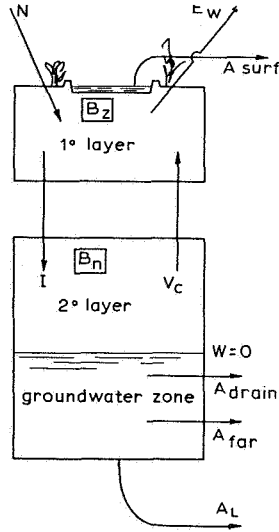


Fig. 1. The soil profile is considered to consist of two layers. The upper layer is assumed to be the rootzone as well as the zone of unsteady capillary conditions. The second zone is partly a capillary, partly a groundwater zone with steady situations.

$B_{z_i}$  represents the capillary downward flowing surplus water which is on its way to the groundwater,  $B_{z_{i+1}}$  is the amount of stored moving water on the following day. If on day  $i$  this excess of water is present, then the capillary rise  $V_{c_i}$  is zero. If, however,  $B_{z_i}$  is zero then the surface discharge  $A_{surf}$  is zero; the capillary rise  $V_{c_i}$ , however, will have a certain positive value.

The water balance equation therefore has two shapes. For the upper layer with excess water present

$$N - I - E_w - A_{surf} + 0 + B_{z_i} - B_{z_{i+1}} = 0 \quad (4a)$$

is valid and with excess water absent

$$N - I - E_w - 0 + V_c + B_{z_i} - 0 = 0 \quad (4b)$$

The discharge  $A_{surf}$  represents the water that does not penetrate into the soil due to a high groundwater table or to a restricted permeability constant.

The second layer has a moisture build-up, represented by the following equation

$$I - \Sigma A - V_c + B_{h_i} - B_{h_{i+1}} = 0 \quad (5)$$

where:

$\Sigma A$  = discharge to different drainage bases

$B_h$  = capillary fixed storage

In formula (5) the discharge  $\Sigma A$  may consist of water flowing to tile drains, but at the same time to furrows, boundary ditches and deeper rivers and channels. The deep lateral flow also belongs to this balance term. The stored water  $B_{hi} - B_{hi+1}$  in this layer largely consists of capillary fixed water that only flows off in case of lowering of the groundwater table or in case of increase in rate of capillary rise.

#### MATHEMATICAL REPRESENTATION OF THE WATER BALANCE MODEL

For every term in the water balance as given in formulae (4) and (5) a mathematical expression is available. Several terms have been given much attention in soil physics and are known in minute detail. Other terms have been studied less intensively and need further attention, as the equations with the least detail and refinement determine the precision attainable with the model. A detailed and accurate formula provides only a restricted reliability if combined in a model with terms of restricted detail and larger uncertainties.

A model which describes the functional as well as the quantitative representation of the several interrelated processes within the water balance equation is by the set of formulae given in table 1.

The model enables one in case the soil constants are known, to calculate successively all the terms of the water balance, the groundwater depth  $W$  and the soil moisture stress  $\psi$ . These unknowns are found at the stage of calculation of the model as indicated in column V. The parameters indicated in column IV are assumed to be given in advance. In reality an estimate is made and by a curve fitting process the values are gradually improved till an optimal fit is found with respect to an observed series of values of one of the terms in column V.

With the parameter values of column IV each process function can be calculated, using the formulae of column III. The calculation is carried out for the first day, using the starting values  $W_0$ ,  $V_{c0}$ ,  $B_{z0}$ ,  $B_{h0}$ , of which  $W_0$  and  $B_{h0}$  are mutually dependent according to the storage function. For the deep seepage two sets of groundwater depth values are needed in order to assess the gradients under which water flows to the spot where the groundwater depth is measured and under which gradient the water flows away. The gradients used in the model are calculated using the actual difference with the observed groundwater depth into two well situated other observation wells, divided by their distance  $L_a$  or  $L_b$  from the well under investigation (see fig. 2).

It is assumed that the gradient is less variable than the groundwater depth. If groundwater levels are known with a 14-day interval, then the mean gradient over two weeks may be used for daily values even if for the groundwater depth a daily value is to be calculated.

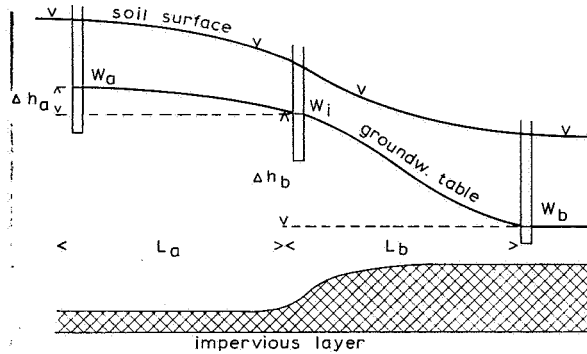


Fig. 2. The horizontal deep flow of groundwater depends on the permeability, the surface of flow and the gradient. A decrease in the area of flow at some part of the flow path requires a larger gradient and causes an increase in stored water in the upstream part of the flow path. This relation is described by the continuity equation.

The water relations in the upper layer and the drainage do not require much discussion with regard to the calculation of the balance terms. Evaporation is defined by two situations, the first where evaporation is climatically defined and described by  $gE_0$ . Here  $g$  is dependent on the height of the vegetative cover and is calculated on a monthly basis. It is, however, possible that  $gE_0$  will be too large, and therefore in table 1 the second term of formula (9a) then also will be too large, so the product of the two terms in formula (9a) will become larger than unity. This will make the log unreal, which is a sign that the evaporation should not be calculated according formula (7a) but formula (7b).

If this is the case and the magnitude of the evaporation is clearly controlled by soil moisture stress and the by this stress indicated moisture depletion conditions of the soil, the soil moisture stress  $\psi$  must be calculated by an iterative procedure using formula (7b).

Now that  $\psi$  is known the value of the capillary rise  $V_c$  can be calculated for wet and dry soils, making it possible to assess the permanent capillary storage  $B_{hi+1}$ . This leads to the calculation of  $\Delta W_i$  and the groundwater depth on the next day  $W_{i+1}$ .

The water balance terms, the groundwater depth and soil moisture stress then are calculated in this way for day  $i+1$ , using the data obtained for day  $i$ . This can be repeated for the days  $i+2$  from  $i+1$ ,  $i+3$  from  $i+2$  and so on. The water balance can be assessed for any length of time, if only the initial values  $W_0$ ,  $B_{z0}$ ,  $B_{h0}$ ,  $V_{c0}$ , the weekly or fortnightly values for the gradients  $(W_{ai} - W_i)/L_a$  and  $(W_i - W_{bi})/L_b$  and daily values for  $N$  and  $E_w$  are available.

Table 1. Formulae used in a water balance model describing functionally as well as quantitatively several interrelated water balance processes

I	II	III	IV	V
Formula number	Kind of flow	Process function	Parameters given or estimated	Balance term calculated
<b>Water relations upper layer</b>				
1	Infiltration	$I_i = (\Delta B_{zi-1} + N_i)e^{-\beta W_{i-1}}$	$W_0 B_{z0} B_{h0} V_{c0} W_{ai} W_{bi}$	$I$
2	Temporary storage	$B_{zi} = (B_z - E_w + V_c)_{i-1} + (N - I)_i - A_{\text{surf}}$	$\beta$	$B_{zi}$
<b>Drainage</b>				
3	Surface	$A_{\text{surf}} = 10(B_{zi} - B_0)$	$B_0$	$A_{\text{surf}}$
4	Pipe drain	$A_{\text{pipe}} = B_1(S_1 - W_{i-1}) + B_2(S_1 - W_{i-1})^2$	$B_1 B_2 S_1$	$A_{\text{pipe}}$
5	Field drain	$A_{\text{field}} = B_3(S_2 - W_{i-1}) + B_4(S_2 - W_{i-1})^2$	$B_3 B_4 S_2$	$A_{\text{field}}$
6	Deep seepage	$A_L = \frac{2kD}{La + Lb} \left( \frac{W_{ai-1} - W_{i-1}}{La} \right) - \left( \frac{W_{i-1} - W_{bi-1}}{Lb} \right)$	$(W_a W_b W')_{i-1}$ $kD La Lb$	$A_L$
<b>First alternative: evaporation determined by climate</b>				
7a	Real evaporation	$E_w = gE_0$	$g_1 - g_{12}$	$E_w$
8a	Water balance upper layer	$V_{ci} = gE_{0i} + I_i - N_i - B_{zi}$		$V_c$
9a	Soil moisture stress	$\psi_i = \frac{-1}{\alpha} \ln \left\{ 1 - (1 - e^{-\alpha W_i}) \left( 1 + \frac{gE_0 - \Delta B_{zi}}{k_0} \right) \right\}$	$\alpha k_0$	$\psi$
<b>Second alternative: evaporation determined by soil moisture content</b>				
7b	Real evaporation	$E_w = d_1 \psi^{-d_2}$	$d_1 d_2$	$E_w$
8b	Water balance upper layer	$V_c = d_1 \psi_i^{-d_2} + I_i - N_i - B_{zi}$		$V_c$
9b	Soil moisture stress	$\psi_i = \frac{-1}{\alpha} \ln \left\{ 1 - (1 - e^{-\alpha W_i}) \left( 1 + \frac{(d_1 \psi_i^{-d_2} - \Delta B_{zi})}{k_0} \right) \right\}$	$\alpha k_0$	$\psi$
<b>Water relations lower layer</b>				
10	Storage lower layer	$B_{hi+1} - B_{hi} = (I - \Sigma A - V_c)_i$		$B_{hi+1}$
11	Storage factor	$\mu = C_1 \psi_i^{C_2}$		$\Delta W$
	Change groundwater depth	$\Delta W_i = \frac{B_{hi+1} - B_{hi}}{C_1 \psi_i^{C_2}} = \frac{\Delta B_h}{\mu}$	$c_1 c_2$	
12	Groundwater depth next day	$W_{i+1} = W_i + \Delta W_i$		$W_i$

# PHYSICAL BASIS OF THE FORMULAE

In solving integrated problems the interest for each factor generally will not be of the same intensity. The advantage is that a complete model, even if some of the factors are treated lightly, eliminates for a large part the influence of varying values with respect to these uninteresting factors and increases the accuracy of the calculations for the factors of main interest. It will be of importance to have available a number of formulae of different accuracy and of different laboriousness in the application. The greater the interest in a factor is, the more complicated process function for this factor will be acceptable.

## INFILTRATION

The formula for infiltration depends on the assumption that the speed of infiltration  $V_i$ , equal to the rate of decrease of the temporary storage  $dB_z/dt$ , is represented by:

$$V_i = \frac{dB_z}{dt} = -\beta W B_z \quad (6)$$

$$\frac{dB_z}{B_z} = -\beta W dt$$

$$\ln \frac{B_z}{B_{z0}} = -\beta W (t - t_0) \quad (7)$$

$$B_z - B_{z0} = e^{-\beta W (t - t_0)} \quad (8)$$

The formula states that the logarithm of the infiltration rate decreases proportionally with the depth of the water table. This simple formula is used to make sure that soils with a dense structure will be corrected for the density-caused slow infiltration. Because the available rainfall data are on a 24-hour basis – which is too long for an accurate analysis of the infiltration in normal pervious agricultural soils – the general results are, because of an order of magnitude for  $\beta$  of 0.001, that the water can be taken to disappear totally in 24 hours in the groundwater.

If infiltration is considered to be of interest, then hourly rainfall and evaporation observations should be available. Self-registering rain and evaporation gauges have been constructed (Bloemen, 1972). But this excludes the possibility to select an interesting year from the available records, because for such a year only data with 24-hour intervals will be available. The second requirement is, that a more accurate formula should be available.

The formula might be based on:

$$V_i = k_c \left( \frac{d\Psi}{dz} + 1 \right) \quad (9)$$

$$V_i = k_0 \frac{e^{\alpha z} - e^{-\alpha \psi}}{e^{\alpha z} - 1} \quad (10)$$

where:

$$k_c = k_0 e^{-\alpha \psi}$$

$$z = W - M$$

$M$  = depth below soil surface of the point of observation in the soil profile

The first formula (8) is based on the draining away of water through non-capillary pores, the second formula (10) on the assumption that flow only occurs in the capillary pores. By combining formulae (8) and (10) with an adjustable share  $p$  in the total infiltration, the formula becomes:

$$I = p(B_{z_0} + N)e^{-\beta W} + (1 - p)k_0 \frac{e^{\alpha z} - e^{-\alpha \psi}}{e^{\alpha z} - 1} \quad (11)$$

The ratio  $p/(1 - p)$  between the capillary and non-capillary pores has to be solved by means of curve fitting techniques.

#### DRAIN DISCHARGE

The drain discharge functions in formulae (4) and (5) (table 1) are the normal ones. The only point is that often it is not realised that the water drains away to more than one drainage basis at the same time, as is described in table 1, formulae (3) to (6). By lowering of the groundwater table one drainage basis after the other stops functioning. The deepest drainage basis, however, will often still discharge water in the driest year imaginable. The surface drainage is rather crudely described in formula (3), table 1. Here also the time interval of rain gauging is too long. Ponding and surface discharge generally do not last much longer than some tens of minutes and can be studied only with expressly collected observations with short intervals for rainfall and precisely known moments of groundwater depth readings. Still this surface discharge function, that is seldomly important, is kept in the formula to be sure that a dense impervious soil is correctly described by the model.

In the drainage formulae the depth of the drainage basis  $S$  is included. If the value for  $S$  is determined separately for two months periods, the value for  $S$  appears to vary. In fig. 3 the result of such an investigation is shown.

As the drainage depth is a hydrologic constant the explanation is that more than one drainage process is present. Then, if the discharge to the highest basis stops, a drainage basis at larger depth continues the discharge. This deeper basis may in fig. 3 be situated at 1.60 m below soil surface, the higher basis at 90 cm below soil surface. The drainage expedients which are shallowest can be calculated when the



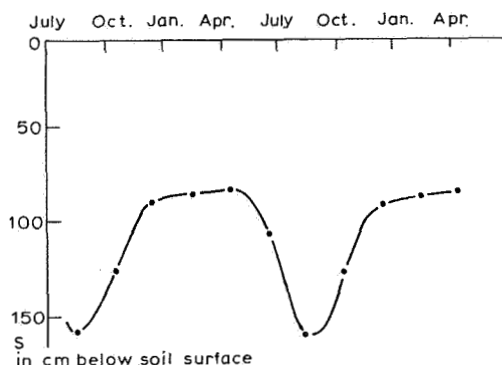


Fig. 3. Calculating the drainage basis  $S$  for separate two-month periods the depth in the month with excess rain and low evaporation is found to be shallow and in the month with excess evaporation to be deeper indicating the presence of a drainage basis at 90 and one at 160 cm below surface.

water table draws down. It may be possible that the water table found deepest is caused by an accidental result of the actual water balance and therefore depends on the equilibrium between rainfall and evaporation without an effect of drainage. In that case for each year a different drainage basis will be found.

The depth of the water table may also be determined by the water level in a river or channel. This is shown in fig. 4. Here clearly one system exists that depends on a

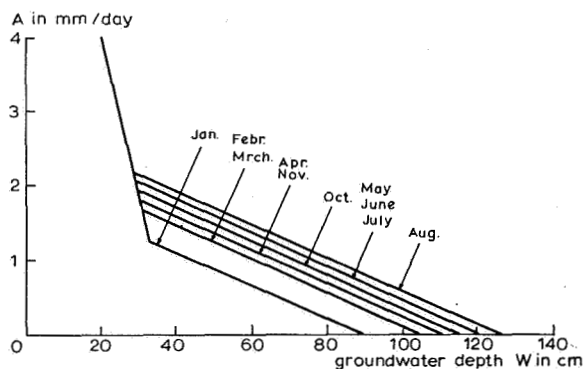


Fig. 4. The drain discharge depends on the type of drainage basis. Here between 20 and 35 cm water drains to a fixed basis, for instance a furrow. From 35 to 130 cm water drains to a river with a varying water level.

fixed drainage basis at a depth of 35 cm. A second system is present with a variable drainage basis, here determined at monthly intervals which is situated at a depth varying between 90 and 130 cm. Here the drains clearly discharge on a river. Which

kind of drainage is present can be assessed in the field or by inserting different drainage formulae in the model.

The drainage equations (3), (4), (5) and (6) in table 1 express the complexity of the drainage situation which depends on the difference in depth of the drainage basis and the distances to the draining channel. By calculating the ratio of the two drainage parameters  $B_1 = 8kD/l^2$  and  $B_2 = 4k/l^2$ , one finds:

$$\frac{B_1}{B_2} = \frac{8k_1 D l^2}{4k_2 l^2} = \frac{2k_1 D}{k_2} \quad (12)$$

If the permeability constants  $k$  are equal or are known, then the soil constant  $D$  can be calculated. At this depth of the lower boundary of the drainage layer very often no impervious layer is present. This is the reason that in practical use this  $D$ -value is often very inaccurate and it is of importance to have available a method to determine the value of  $D$ , which in a certain region may be considered to represent a fair mean value.

An interesting refinement of the model is obtained if the discharge of the draining river is first found by the adjustment procedure developed in these pages. Then on a day to day basis the water depth in the river is calculated. This water depth, substituted in the water balance equation as the drainage basis, then causes the ground-water depth and the field drainage discharge to assume a correct value. This, however, is rather time consuming and is only acceptable if the water balance as well as the river discharge are of high importance.

The aim of remarks given above is to show that a model should be reshaped for each project. The different possibilities, however, should be prepared in advance so that people with only a partial hydrologic training still can select the correct equations to construct a well fitting model.

## EVAPORATION

The weakest point in the description of evaporation is the relation between the capillary conductivity  $k_c$  and soil moisture stress  $\psi$ . Two formulae are available:

$$k_c = k_0 e^{-\alpha(\psi - \psi_0)} \quad (13)$$

$$k_c = k_0 \left( \frac{\psi_0}{\psi} \right)^n \quad (14)$$

The first formula (Rijtema, 1965) has the advantage that it can be applied to many different problems without much mathematical difficulties. The accuracy, however, is only guaranteed over a short range of  $\psi$ .

The second formula (Wesseling, 1957) is applicable over a wide range of  $\psi$ -values, but the mathematical elaboration often will involve the binominal integral, which

is only solvable with a series of formulae which is of different length and complexity for different values of  $n$ . For the determination of evaporation this laboriousness is of slight importance, but in this case the interactions with other factors is involving that much labour, that for practical reasons formula (13) must be preferred.

In equation (7a) of table 1 the common formula for evaporation is used. In formula (7b) an exponential equation of less general use is applied. This equation follows from the assumption that a plant root functions in the same way as water abstraction by a well in a capillary domain of circular shape (Visser, 1963).

The well formula is:

$$\pi (d^2 - x^2) E dx = k_c 2\pi L x d \Psi \quad (15)$$

The influence of capillarity is described by:

$$k_c = k_0 \left( \frac{\psi_0}{\psi} \right)^n \quad (16)$$

Integration leads to (Visser, 1964):

$$E = d_1 \left( \frac{1}{\psi_d^{n-1}} - \frac{1}{\psi_r^{n-1}} \right) \quad (17)$$

$$d_1 = \frac{1}{(n-1)R^2} \frac{4Lk_0\psi_e^n}{\ln \frac{R^2}{r^2} - 1 - \frac{r^2}{R^2}} \quad (18)$$

where:

$d$  = radius of the area of extraction

$L$  = thickness partly dewatered root zone

$k_0$  = capillary permeability for  $\psi = \psi_e$

$\psi_e$  = air entry value

$r$  = radius of root

$R$  = radius of root occupied domain

$\psi_r$  = soil moisture tension at root surface

$\psi_d$  = soil moisture tension at limit of root domain

The value of  $d_1$  is constant for a certain combination of soil profile and vegetation. Often  $\psi_r$  can be neglected. The equation shows that there should be a relation between  $n-1$  in formula (17) -  $n$  is generally of the order of 1.5 - and  $d_2$  in formula (7b) of table 1.

The formula for evaporation which is limited by the desiccation of the soil, is dependent on the soil moisture stress. The climate limits evaporation by the value

of  $E_0$  present in the formula  $gE_0$ . But moreover the vegetation is of importance as it determines the value of  $g$ .

If for each month the best fitting value of  $g$  is determined, then the influence of  $g$  on the real evaporation is found as depicted in fig. 5. The graph shows that in December and January the value of  $g$  is negative. This might mean that some dew formation occurs which here shows up as negative evaporation. The influence of incorrect relations in the model may also be the reason. As evaporation is low in these months, the influence of this negative  $g$  is of restricted importance.

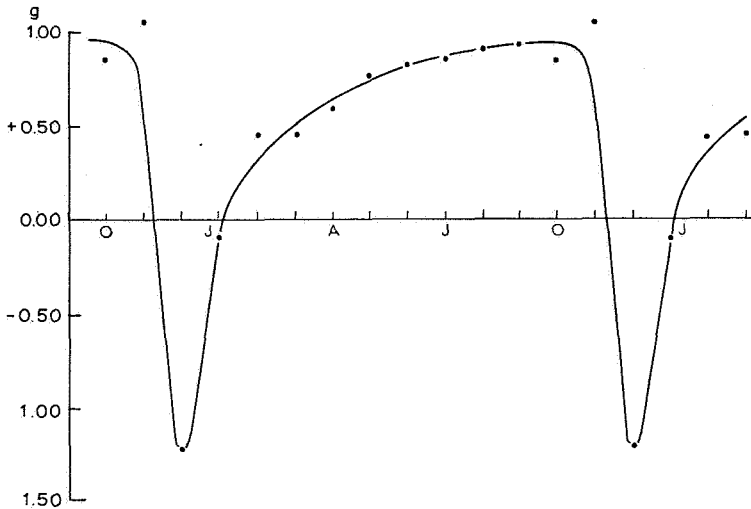


Fig. 5. The evaporation described with  $gE_0$  depends by means of  $E_0$  on the climate, but by means of  $g$  also on the vegetative cover. Calculation of  $g$  for each month shows that probably in winter dew formation causes negative evaporation. During the year the expanding vegetative cover increases the evaporation up to 95% of the climatologically possible evaporation.

As the vegetative cover grows the value of  $g$  first becomes positive and then becomes higher at a decreasing rate till in the autumn a maximum value of 0.95 is reached. The influence of  $g$  is such that in spring the not yet full-grown vegetation evaporates water in a somewhat restricted way with  $g = 0.5$  to  $0.6$ , but in the last part of the growth season a value for  $g$  of 0.95 shows how the full-grown vegetation is able to use nearly the full evaporative capacity available in the atmosphere.

In table 1 it is striking that there is a difference between the formulae for evaporation according to formula (7a) and (7b) and the question has to be answered how can be assessed which of the two formulae is valid. This is done by means of formula (9a) where, as already mentioned, too large a value of  $E_0$  may enlarge the product of the two terms between brackets to such an extent that the argument of the logarithm becomes negative and the value for  $\psi$  unreal. This is corrected by using the lower

value for the evaporation given by formula (7b), and by determining the value of  $\psi$  from this more complex formula. The solving with an iterative technique of equations, which cannot be solved with the well known techniques of algebra, will be met more often when process descriptions of physical or plant physiological origin are used. The degree of complexity of the methods of solving the unknowns, however, should not influence too much the choice of the process functions to be used. This choice should more depend on the required accuracy with the acceptance of the laboriousness of the calculation. It is for instance practiced to eliminate many difficulties by describing the processes with polynomials of the type:

$$a + bx + cx^2 + dx^3 + \dots$$

As the constants  $a, b, \dots$  have no physical meaning, the results cannot be checked visually or be transferred to other situations. Therefore this approach with polynomials should be contemplated very critically.

#### STORAGE

The balance term for storage is probably least studied. The storage as description of a steady state is dependent on the desorption curve. For this soil characteristic a formula was developed (Visser, 1966):

$$\psi = \frac{G(P + \Delta P - v)^n}{v^m} \quad (19)$$

This is a S-shaped curve (see fig. 6) which varies for the moisture content  $v$  from zero to the volume percentages of the porespace  $P$  and for  $\psi$  from the air entry point  $\psi_a$ , going with a moisture volume  $P$  to equal to infinity and  $v$  equal to zero. The  $\Delta P$  enables one to determine the air entry point, which shows that the largest soil pores are of a capillary size. A high value of  $\psi_a$  points to a dense soil. If large non-capillary cracks are present  $\Delta P$  becomes negative. This equation, however, is mathematically a laborious description and often is substituted by a formula for the wet and for the dry part of the desorption curve separately:

$$\text{For wet soils} \quad \mu = c_1 \psi^{c_2} \quad (20)$$

$$\text{For dry soils} \quad \mu = P - f_1 \psi^{-f_2} \quad (21)$$

The  $\mu$  in this concept is considered to be identical with the soil air volume in the upper layer of the soil. To simplify the determination of  $\mu$  the value of  $P + \Delta P$  often is fixed at 50%. The formulae (20) and (21), however, only describe the stationary equilibrium situation.

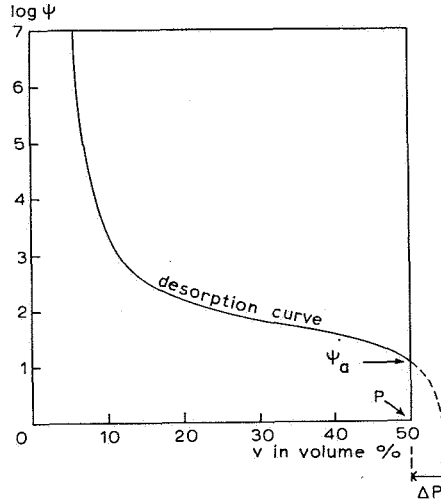


Fig. 6. The desorption curve for steady storage conditions shows that storage increases with lowering of the soil moisture stress  $\psi$ . The lowest definable moisture stress is the air entry point  $\psi_a$  for a moisture content  $v\%$  of  $v$  volume percent showing that the largest pores have a finite size. If non-capillary cracks are present  $\Delta P$  is negative.

A formula for the non-steady state has been developed on the assumption that the variation  $\Delta\psi$  of  $\psi$  will over a small range be related to the variation  $\Delta W$  of  $W$  by:

$$\Delta\psi = \gamma \Delta W \quad (22)$$

By differentiating formula (9) of table 1 to  $W$ , the following result is obtained:

$$\gamma = \frac{d\psi}{dW} = \frac{e^{-\alpha W}}{e^{-\alpha W} - \frac{V_c}{V_c + k_0}} \quad (23)$$

Formula (23) is inserted in formula (22) and, assuming that  $\gamma$  over the small interval is not dependent on  $W$ , as result formula (24) is obtained:

$$B_h = \frac{C_1}{1 + C_2} (\gamma_i^{C_2} W_i^{C_2+1} - \gamma_{i+1}^{C_2} W_{i+1}^{C_2+1}) \quad (24)$$

If this formula is used, the calculation of  $W_{i+1}$  is not carried out according formula (12) in table 1 with  $\Delta W_i$  but is done with the formula

$$\gamma_{i+1}^{C_2} W_{i+1}^{C_2+1} = \gamma_i^{C_2} W_i^{C_2+1} - (I_i - V_{c_i} - \Sigma A_i) \left( \frac{1 + C_2}{C_1} \right) \quad (25)$$

This equation makes it possible to estimate the amount of stored water under un-

steady conditions. Up to now it has not been used because the solution is rather time consuming. The more detailed process definition is given, however, to show that a model can be built from various different and separate parts which elucidate the problem that has to be investigated in the best way.

#### DETERMINATION OF THE PARAMETER VALUES

To many people it will be unexpected that a complex mathematical treatment is based on estimates instead of on careful determinations of the parameter values and that, whatever hydrologic property has to be determined, the observations required are always the same: rainfall, potential evaporation and one of the other balance terms or hydrologic properties, for which generally the groundwater depth or the discharge of a rivulet is taken. The groundwater depth or discharge is used as checking value to ensure that the model can be used to find values for each parameter and factor comparing well with the actual observed value.

This comparison with one of the balance terms would not be necessary if a certainty existed that the available formulae and parameters were accurate. Small deviations of the true value may, however, influence the water balance results in a way and to an extent which is difficult to predict. The formulae often hold for an idealized situation and not for the actual heterogeneous condition.

The curve fitting technique provides one with the magnitude of the parameters, which ensures the closest agreement between calculated and observed water balance terms. If the process function is not entirely correct – as it never will be due to the neglecting of soil heterogeneity or the complex agricultural use of the land, the variations in water management of the catchment board or the uncertainties in rain gauge data – then it has to be expected that with the actual laboratory type determinations no correct description of the hydrologic situation will be obtained. They will not fit in the inaccurate mathematical area description.

The curve fitting technique, in which the water balance must account for all balance terms and errors in one term are of necessity counterbalanced by the magnitudes calculated for the other balance terms, has the property to compensate the error for one parameter by a complementary error for the other parameters. Due to this property of compensating errors the result of the calculation of balance terms may be of a high accuracy while at the same time the agreement between the parameters obtained by the curve fitting technique may compare less well with the parameter values determined in the laboratory.

The determination of the parameters is generally carried out by estimating the parameter values and calculating the checking value, generally the groundwater depth. Then one of the parameters is slightly altered and in some way the goodness of fit of

calculated and observed checking values is assessed before and after the small correction. If the degree of fit improves, the correction was chosen in the right direction. The correction is repeated in the same direction till a minimum of the general deviation is passed. If all parameters show an optimum fit the solution is considered to be obtained.

In order to speed up this process of correcting, many different methods are available. But all have the adverse property that uncertainties arise if the number of unknowns becomes larger than 7 to 10. For a complex practical problem, however, such a number of parameters is too restricted and methods of curve fitting are developed which can handle a larger number of unknowns or instructions should be formulated whether data of wet or dry years and longer or shorter sequences of rainfall and evaporation should be taken and which kind or number of checking values should be used. Eventually the curve fitting technique might be based on more than one checking value, for instance groundwater depth, discharge, moisture content and soil moisture stress.

No results of the error calculation are discussed here because a technique accounting for all the observed difficulties is in the process of development and will in due time be available.

#### RETARDETION IN THE UNSATURATED DOWNWARD FLOW

Infiltration is a property which only in dry climates gets much attention from hydrologists. The importance for agricultural purposes of soil density is large. But for a drainage project it also is of importance as retarded infiltration and surface runoff may be expected to occur when the soil density increases.

Formulae (1) and (2) in table 1 should be included in the model as a security measure giving a warning in case the infiltration is slow. However, on the usual agricultural soils, where measures certainly would be taken if the top layer was too dense and impervious, the general result of the calculation shows that the value of  $\beta$  is of the order of 0.005 to 0.001. This means that the time interval of 24 hours between the rainfall measurements is too long. As rainfall data of the Meteorological Service are used, which makes it possible to choose as most interesting a wet or a dry year from the many years for which these data are available, the freedom of calculating the hydrologic situation for exceptional years must be considered to be more valuable than a more detailed insight into the process of infiltration to be obtained with more frequent observations in the one arbitrary year.

#### RIVER DISCHARGE

The river discharge can be calculated in two ways. The first method assesses the



drainage from field to river. The discharge according to different drainage bases is multiplied by a factor, accounting for the area with that discharge.

The sum of the products of the areal factor and the discharge for the group of pertinent catchment basins according the depth of the drainage basis gives the river discharge. The deep flow will get a small areal constant, because the larger part drains to channels outside the catchment area. The tile drainage on the other hand will require a nearly 100% areal factor. The areal values are calculated by curve fitting using the river discharge as test value.

If only the river discharge is of interest, the second method will be used. If interest is also given to the effect of field drainage on the river discharge, then the first method gives the required information in a better way (Bloemen, 1974, this volume).

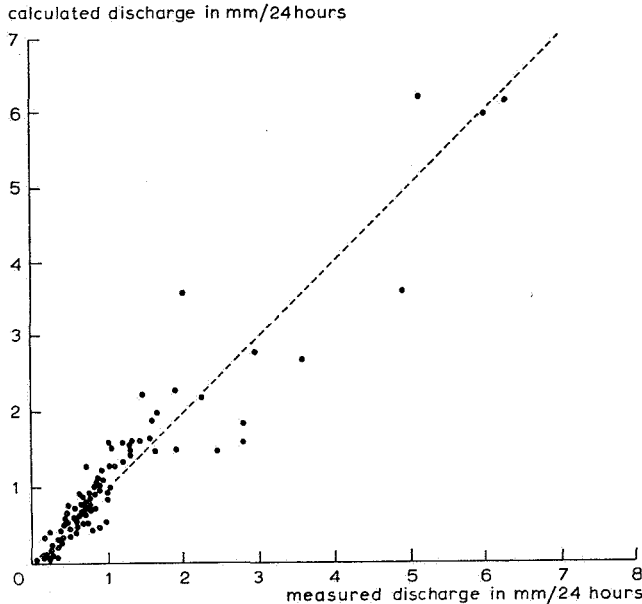
Another point that needs attention is the question, which dimension of discharge should be taken. This dimension may be the discharge over another time interval than 24 hours. It also may be over an at random chosen time interval or it may be the highest or lowest discharge over some length of time. In those cases the discharge differs from the discharge which fits in into the water balance. The time interval is not in agreement with the time interval for the rainfall. For a length of several days this can be remedied by using rainfall and evaporation data over the same length of time. If the time interval is restricted to several hours, then the elaboration is only possible if rainfall and evaporation data over the same short time interval are as well available. This, however, makes the use of interesting climatic circumstances from earlier periods impossible.

It is however, possible to make a list of the discharges over the specific period – for instance the maximum 3-hour rainfall for each day, indicated as  $A_{3/24 \text{ max}}$  – as well as the list of 24-hour discharges ( $A_{24}$ ) with which the water balance is calculated. It is possible to calculate the difference between the average for  $A_{24}$  and  $A_{3/24 \text{ max}}$ , but also the ratio between the differences  $\Delta A_{24}$  and  $\Delta A_{3/24 \text{ max}}$ . These two values make it possible to calculate the variance for both values,  $A_{24}$  for the discharge as term of the water balance,  $A_{3/24 \text{ max}}$  as a value of importance for the assessment of the design discharge. The more stochastically based methods will still have to be tried out.

In fig. 7 the discharge calculated with the water balance model is plotted against the measured discharge. In this calculation it could be shown that the river discharge  $A$  can be calculated with an accuracy of 0.3 to 0.5 mm/day. Here the measurements of the rain intensity on a 24-hour basis does not allow to calculate the peak discharge because the time interval is too large. If a time interval of a few hours is taken in the calculation the summer period had better been neglected and only the wettest few months should be used in order to prevent the calculation to become too laborious.

The discharge of the river is calculated for hydrological conditions as prevailed

during the period of gauging of the river discharge. Often the knowledge of discharge situations which only seldom occur are desired, not only with respect to large discharges, but also concerning the duration of restricted discharge.



*Fig. 7.* The curve fitting technique can use as testing value the river discharge as well as the ground-water depth using the rainfall and the evaporation. This technique enables one to calculate the water balance parameters over a few years and determine the river discharge over as many years as rainfall and potential evaporation data are available.

The discharge over a longer period can be calculated if over the full length of time rainfall and evaporation data are available. The discharge is then again obtained for the conditions of the hydrologic situation during the period of river gauging. If the rivulet has been improved repeatedly and the discharge data are only available over the last few years, then the historical discharges will not be found by the calculation. For projects this is a desirable result however, as what is required is the discharge under the present-day conditions, but under extreme rainfall or drought conditions which often have not occurred during the relatively short period of river gauging.

In fig. 8 the probability relations between discharge and the frequency of exceedance are given. The parameters were calculated from discharge measurements during one and a half year. The discharges were calculated over 17 years. Contrary to expectation this extrapolation of the discharges appeared to be acceptable.

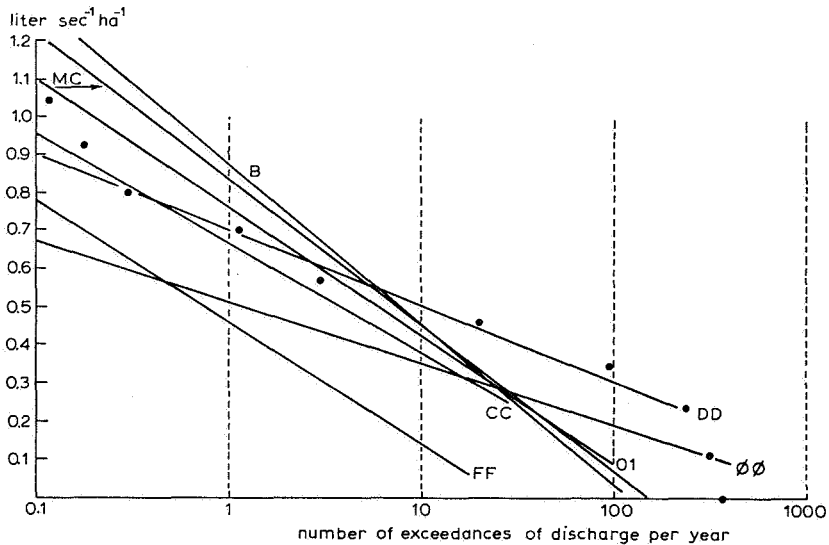


Fig. 8. The calculated discharges allow to calculate how many times a certain discharge is exceeded. The results for 7 rivulets show what design discharge should be taken, if the acceptable number of exceedance is known.

#### INFLUENCE OF WELL PUMPING

The influence of a new well often will not be calculated with the aid of parameters determined with a curve fitting procedure. The hydrologic constants generally will be derived from geologic observations and pumping trials. At the place where the well will have to be built no pumping station will have previously been in exploitation.

The mathematical treatment generally will be some kind of simulation technique. The aim of the calculation will be, to assess the degree of drawdown of the groundwater table as a function of the intensity of abstraction and the distance to the well head. In shallowly drained land the increase in depth of the groundwater table is a measure for the damage done to agricultural crop production, for the impoverishment of the natural vegetation and the diminishing of the river discharge with its decrease in dilution of polluted water that village sewers may discharge on the river. Also of other wells in the neighbourhood the screens may come above the water table and here also the drawdown of the water table as function of abstraction intensity is of importance. Further the lowering of the water table changes the division of the water over evaporation, drainage, storage and well discharge.

In fig. 9 the daily values of the groundwater depth at 6 different distances from the well head are given for an abstraction of 0.5 mm per day or 2.3 million m<sup>3</sup> of water a

year. The data hold for the normal year 1958, the excessively dry year 1959 and the year 1960 with a dry first half year and a very wet second half. It is clear that in a dry year with zero abstraction the water table is already deeper than is desirable for agriculture. But a water table of more than 1.50 to 1.75 m deep in a sandy area will

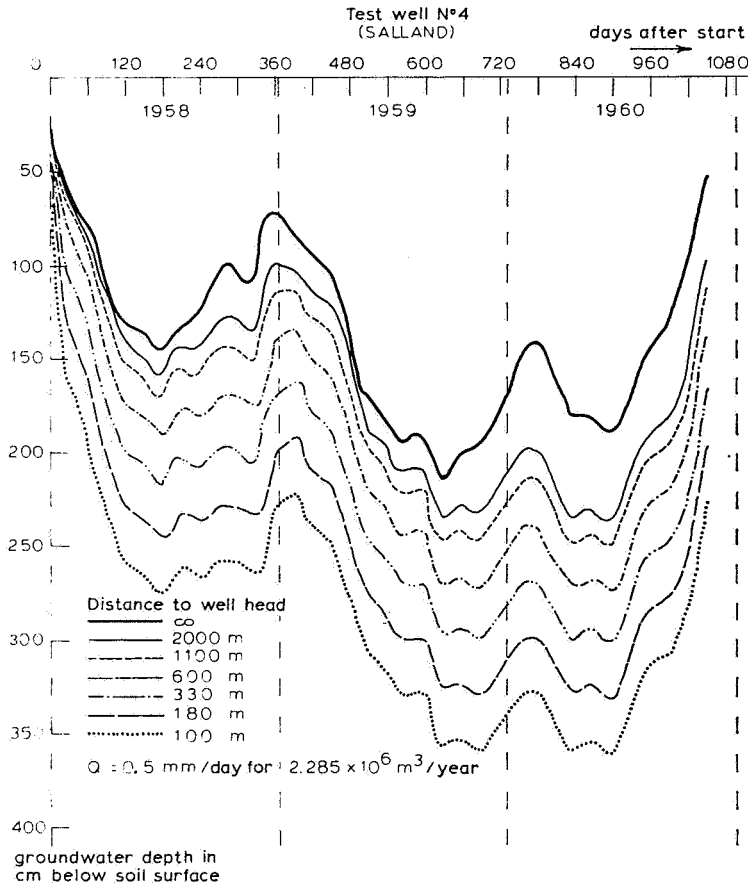


Fig. 9. If the formula for water abstraction by a well is inserted in the water balance formula, the depth of the water table for an abstraction of 0.5 mm/day can be calculated for any desired distance from the well head.

already let the soil desiccate to such an extent that an increase in water depth is no longer of importance to agricultural land use. Abstraction of water at a groundwater depth of less than 60 cm is not harmful to crop production neither is abstraction at a groundwater depth deeper than 1.70 m.

In fig. 10 the influence of variation in the abstraction rate is shown for a distance to the well head of 1100 m, assuming at the start a constant water depth over the entire pumped area. The curves for different abstraction rates show that the lowering of the groundwater table at rates in excess of 0.5 mm becomes excessive. The rate of

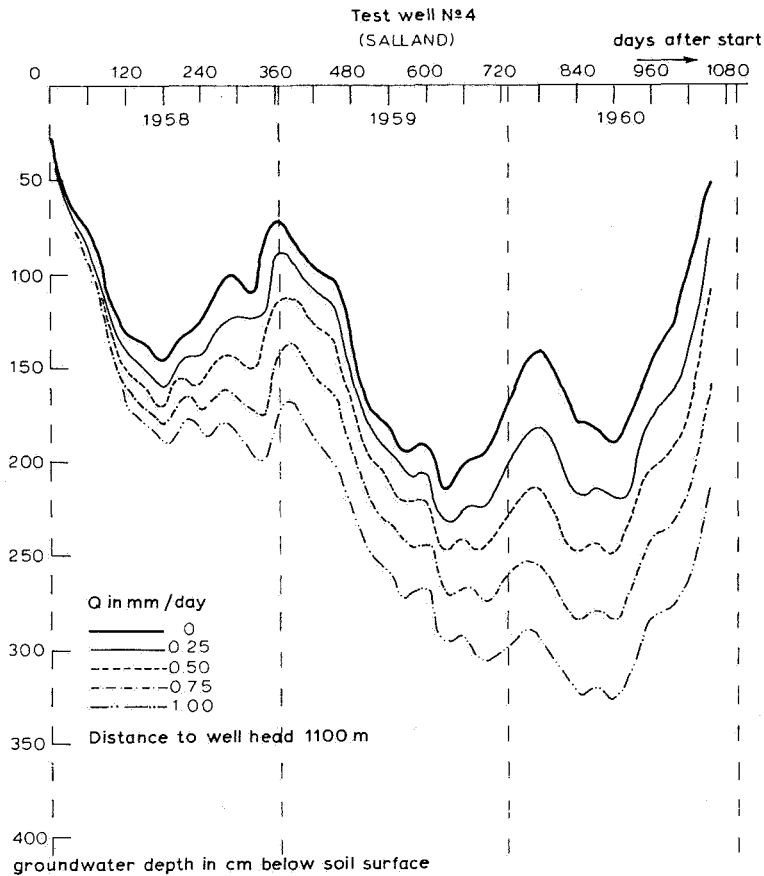


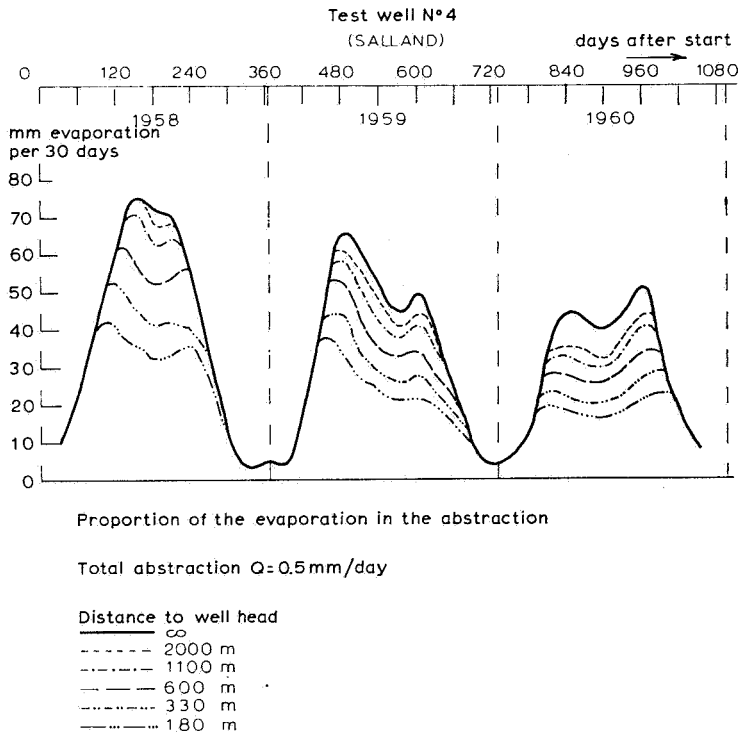
Fig. 10. For a distance of 1100 m from the well head the effect of the rate of abstraction  $Q$  was calculated. The result shows how much deeper the water table is in a dry (1959) year then when adjusted in a wet (1958) and, having an adverse effect on vegetation growth.

pumping then begins to surpass the quantity of water which may be harmlessly available from discharge and from evaporation. The water in excess of these two balance quantities will have to be abstracted from the storage and this shows up in the lowering of the water table.

It is usually considered that a third or a fourth part of the open channel discharge

might be pumped without much damage to agriculture. This is a rule of thumb, which does, however, not account for the large number of soil properties which are determining to what extent water may be abstracted.

A rather important factor is the diminishing of the evaporation by decreasing water table height, depicted in fig. 11. The curves for the magnitude of the evaporation are given here for an abstraction of 0.55 mm/day and 6 different distances from the well head. The evaporation is limited by the magnitude of the capillary rise which brings



*Fig. 11.* The increase in water depth as a consequence of water abstraction decreases the evaporation. The magnitude of this decrease is depicted by the various curves showing that in a dry year considerable less water evaporates and is available for the water production of the well.

water from the more or less deep groundwater table to the root zone. It is also limited by the rate of infiltration which conveys rain water from the soil surface to the groundwater but leaves a greater part of the rain water within reach of the plant roots if infiltration is less rapid.

The deep water table in the spring of 1960, together with the lack of rainfall caused the evaporation to remain rather low and the increased rainfall in autumn came too late

to increase evaporation and plant response. It will be clear that all kinds of complications but also all kinds of simplifications can be quantitatively expressed in the model.

#### DEEP LATERAL LOSS OF WATER

The water which infiltrates into the soil partly flows through the deeper subsoil to a main river or to the sea. This part of the water balance is difficult to measure at the lower end of the flow path and has to be determined by the use of gradients and transmissivity data. The data can be treated with the aid of formula (2) for the continuity equation.

In this formula are to be inserted the values for evaporation, field drainage and storage as obtained by means of the water balance type of description of the hydrologic system. A preliminary result is that given in table 2 for an area of 40.000 ha and a quadrangular shape of which the boundaries are indicated with north border, south border and so on.

Table 2. Deep moisture flow in  $\text{m}^3/\text{year}$  at the four borders of the quadrangle of 40.000 ha.

Year	West	South	East	North	Area total
1958	— 800 000	— 870 000	12 200 000	— 1 540 000	+ 9 000 000
1959	— 4 380 000	— 390 000	11 440 000	— 1 730 000	+ 4 940 000
1960	— 2 500 000	— 815 000	10 310 000	— 1 900 000	+ 5 100 000
average					
yearly total	— 2 560 000	— 690 000	+ 11 320 000	— 1 720 000	

The amount of water entering the area and an increase in quantity of moisture within its boundaries has been given a positive, water flowing out of the area and a decrease in moisture storage a negative sign.

Considerable amounts of water are lost to the large river at the west border and much water enters the area from the east, draining out of a glacial ridge. The north and south border are to a certain extent parallel to the flow lines and are of restricted importance for the water balance of the area.

The effect of the dry year 1959 and the first half of 1960 follows from the lower values in the last column for total areal discharge for these years. It is obvious that the effect on discharge in a dry year is a complicated process. The discharge at the west border to the main river is largest in the driest year 1959 and smallest in the wettest year 1958.

Apparently the low river level in 1959 caused a considerable quantity of water to drain out of the area (Povel, 1973). A comparable result can be obtained when operating with smaller distance units along the boundaries and with shorter time intervals. The parameters for  $kD$ ,  $\mu$  and discharge  $A$  on open channels can be corrected by means of curve fitting techniques, so that the combination of the approach according the water balance equation promises an improved description of the hydrological situation and of the effect of deep discharge on the available quantity of groundwater for drinking water abstraction.

#### AGRICULTURAL RESPONSE

The agricultural response is described by an agrohydrologic model which takes into account the time of the year, the evaporation and the aeration (Kouwe, 1974). With the water balance model the number of terms is limited and an account of all the factors involved in the hydrology of an area is not entirely out of the question. In the case of agricultural response the number of growth factors is, however, that large that accounting for all factors is not a practical aim.

The set up therefore has to be simplified and one of the simplifications is, that evaporation is assumed to be equal to capillary rise so that infiltration minus rainfall minus temporary storage  $I - N - B_z$  is zero, see formula (8a) in table 1.

A further simplification is that the yield response is assumed to be linearly related to evaporation. If evaporation decreases with  $n$  percent, then if evaporation is the limiting factor the rate of dry matter production also will decrease with  $n$  percent.

The simplified graph, which is deduced from the more complicated calculation, is shown in fig. 12.

The model states that

$$V_c = E_w + I - N - B_{zi} = k_0 \frac{e^{-\alpha W} - e^{-\alpha \psi}}{1 - e^{-\alpha W}} \quad (26)$$

The simplified formula is condensed to

$$E_w = \frac{k_0}{e^{\alpha W} - 1} \quad \text{for the dry branch and}$$

$$E_w = gE_0 \quad \text{for the wet branch}$$

which formulae are depicted in fig. 12.

If the daily yield increase in the most productive time interval is suboptimally assumed to be of the order of 125 kg dry matter a day and is obtained in June with an evaporation of 2.7 mm/day, then with an evaporation of 1.7 mm/day in September the yield can be calculated to be  $125 \times 1.7/2.7 = 97$  kg/day.



This calculation assumes, however, that in June and September evaporation is the limiting factor and that the crop would by a larger evaporation show a higher growth rate. The percentual relation holds more general for the skew part of the lines in fig. 12, but to be more sure a curve for the yielding capacity of the soil should be

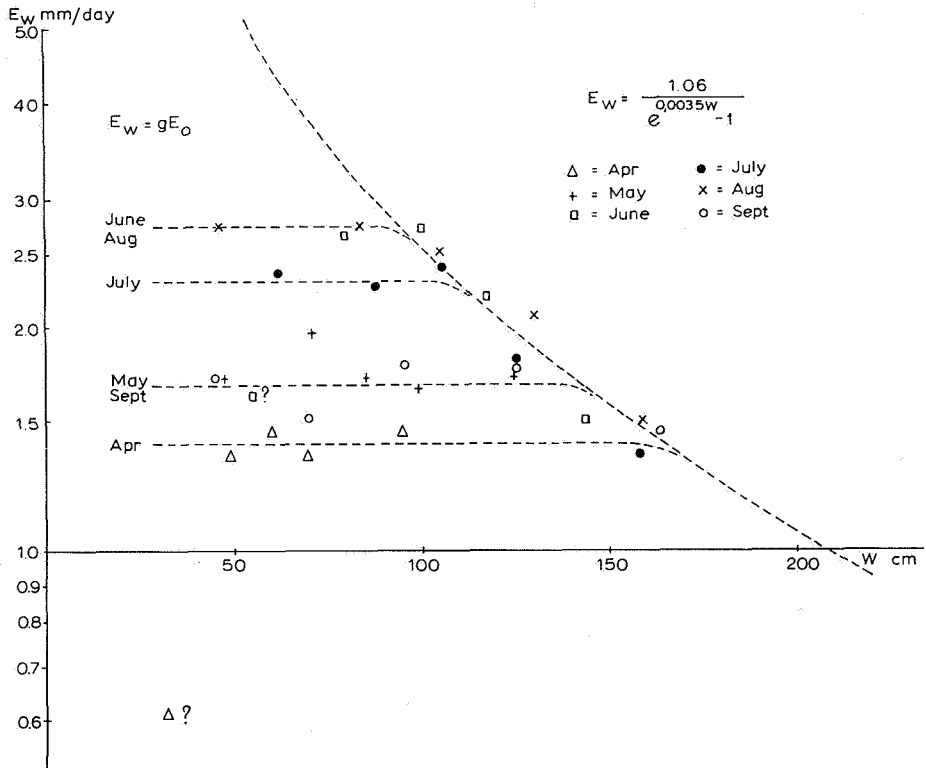


Fig. 12. The evaporation is depicted by a horizontal line in case the evaporation  $E_w = gE_0$  is governed by climate alone and by the skew line for  $E_w = k_0/(e^{0.0035W} - 1)$  in case the evaporation depends on the moisture content of the soil. The graph gives a simplified picture of the more complex relationship obtained by calculation according the model.

available also. This subject, however, will not be discussed here for brevities sake. With the water balance model and the plant production model it is possible, however, to calculate these relations into rather some detail.

## SUMMARY

Modern hydrology describes water relations with mathematical models. This use of computer models enables one to study practical problems which are defined by

many, often interdisciplinary factors and with effects in many different directions. The model approach is necessary because human interventions nearly always have next to the desired effect a number of by-effects which may be of decisive importance for the success of the project.

The hydrologic models are based on two concepts, the first derived from water balance studies of lysimeters and depicted in formulae (1), (3) and (4). The other model is based on the continuity equation, described in formula (2). The water balance model stresses the different factors as evaporation, discharge, capillary rise or infiltration, surface runoff and storage, but deals only in a restricted way with the areal distribution of the water balance terms.

The continuity model stresses the regional interdependence of the moisture relations and evaluates only in a restricted way the water balance conditions on each spot.

The water balance model is preferable under circumstances prevailing in the Netherlands because of the presence of shallow water tables and a direct influence of water table depth on open water discharge, evaporation, capillary rise and storage coefficient. The lateral discharge is in a flat country of restricted importance. This discharge will not only be compensated by changes in storage alone, as formula (2) assumes, but also by changes in open water discharge and evaporation.

The continuity model is more adapted to conditions with a deep water table and no discharging conduits or influences of the water table depth on evaporation or storage. The Netherlands, situated around the estuaries of Rhine and Meuse, had to evolve the water balance model for shallow drainage situations, while many countries used the continuity model because of the preponderant presence of deeply drained undulating terrain. The water balance model is given in table 1, the used formulae are explained and some possible alternative solutions are discussed.

So in a structured soil the infiltration is complex. The rain water penetrates quickly to the water table through the wider cracks and then moistens the higher parts of the soil profile by capillary rise. In investigations on a dense soil on the other hand, the water penetrates slowly into the soil through the fine pores. In general the flow of rain water through the upper soil could be described by  $p$  times the flow through the cracks and  $1 - p$  times the downward capillary flow, but such a detailed solution is too time-consuming.

A further point to be improved can be found in the description of the process of storage. In formulae (19) and (24) the equations for the desorption curve and the influence of capillary flow on the storage capacity are given. Storage is generally described as a function of soil moisture tension. It is, however, also influenced by vertical capillary flow. Due to this second influence, the storage capacity may differ considerably at the same groundwater depth because of drying out of the soil due to evaporation or because of moistening of a dry upper layer after some rain. These changes in storage capacity may occur without change in the groundwater depth even

if the depth of water is shallow. But the calculation of a model improved in this way is also rather time-consuming and has up to now not been practiced.

These examples may show that a considerable number of alternative models is possible, all, however, based on the water balance and the continuity equation.

The parameters in the model can be calculated by a curve fitting process. It is noteworthy that this curve fitting procedure carefully worked out by mathematicians so often is not practiced or is substituted for by some visual subjective method. This enlarges the need for simplified models in which, however, the model parameters often have no physical meaning. But it is highly desirable that the parameters which are used should in principle have a physical meaning and that comparison with direct laboratory determinations will make sense. By comparing the calculated parameter values with laboratory determination makes it possible to show that a correct model has been used, in which case the parameter value also can be used in other formulae. If comparison with laboratory determinations does not give a favorable result, then, due to the curve fitting, the quantitative result nevertheless will be good, and better than when the laboratory determination had been used.

It is highly advisable that the use of models is backed up by curve fitting and error calculation. It should be understood that in a shallowly drained, densely inhabited land as the Netherlands, multiple parameter solutions should be aimed at and that these multiple parameter solutions require the use of the available theory on error calculation.

The use of subjective visual methods should be discontinued for shallowly drained areas, irrespective of the techniques which have been proved to be applicable in less densely populated deeply drained countries, as there simpler techniques may be preferred because no need for high accuracy exists.

In this paper, for brevities sake not much explanation about curve fitting is given although nearly all the results are based on it. A paper on curve fitting technique is in preparation, however.

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## II. THE INTEREST OF PUBLIC WATER SUPPLY IN THE SALLAND INVESTIGATION PROGRAMME

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### 1. INTRODUCTION

The steadily increasing water demands for domestic and industrial supplies in the Netherlands compel the water works to develop the country's groundwater resources further and further, groundwater representing the safest and cheapest resource of water of good quality. Surface water is abundant but generally of inferior quality and too vulnerable to accidental pollutions.

Up to some 10 years ago the increase of the groundwater withdrawal and the construction of new pumping stations hardly encountered any resistance from agricultural or other interested population groups. The amounts of water pumped, and the effects thereof on the groundwater regime, were still small, and damage to agriculture, landscape or nature reserves was considered to be within acceptable limits. Hence the hydrogeological investigations, required to select a suitable location for the new well field and to estimate its capacity and the expected draw-downs of the water table, could remain rather simple and inextensive. The governmental license for groundwater pumping was readily obtained.

This peaceful situation no longer exists. The increase in groundwater development has been so large in the course of the last decade, that the effects on the water regime and the water balance might no longer be negligible. Farmers and organisations for the protection of nature now strongly object to the extension of well fields and to the construction of new groundwater pumping stations. They fear harmful effects of the draw-downs of the shallow groundwater table on the vegetation and the landscape. A governmental license is, nowadays, not readily issued.

### 2. PRESENT SITUATION IN SALLAND

A few figures may illustrate the present situation in Salland.

The average annual rainfall (800 mm) over the area (500 km<sup>2</sup>) yields 400 million m<sup>3</sup> per annum. 250 million m<sup>3</sup> is withdrawn yearly from this quantity by evaporation (500 mm/annum), leaving 150 million m<sup>3</sup> per year as a potential resource for other purposes. (Practically all that water is discharged into the brooks and rivulets through the underground; there is hardly any surface run-off).

The groundwater abstraction for domestic and industrial supplies amounts to about 14 million m<sup>3</sup>/year (1970), i.e. less than 10% of the above mentioned figure. One would be inclined to conclude from this that further development of the groundwater resources would not present any problem. However, the present opposition against the extension of well fields and the construction of new pumping stations make it very unlikely that the water companies in this area would easily obtain new licenses for pumping stations of great capacity.

Now, the presence of evidently large potential groundwater resources on one hand and the growing opposition from agricultural sides on the other, a situation that threatens to leave the precious groundwater resources insufficiently exploited and to force the water works to shift too soon to surface water resources, make it necessary to find a solution. The aim should be to use the water in such a way that all parties involved will be served in the best possible way. This explains why the water works are interested in the Salland investigation programme and take an active part in it.

### 3. ECONOMICAL ASPECTS

Groundwater development in such a way that the optimum benefit is obtained for all parties involved, raises the question which rules or system should be used for the allocation of the available water to the various interested groups (water works and farmers in the first place). It is clear that this is a question with economical aspects. Optimum use of water implicates the highest social benefit against the smallest sacrifices. As a rule the term sacrifice is understood to mean financial sacrifice. Because of this economical aspect it is necessary to give some information on the costs of the development of different types of water resources for public and industrial supply and the financial aspects of damage to agriculture. The following table shows some figures of costs of the production of water from 4 different sources. The figures have been taken from studies carried out by the RID.

The figures show that groundwater represents the most economic resource. The table, however, is not complete: costs of transport, indemnification to farmers, distribution and administration costs should also be taken into account. The latter two are the same both for groundwater and for surface water and do not alter the picture. Transportation costs, however, are generally much higher for surface water than for groundwater; a well field is generally situated near the area to be supplied, whereas surface water storage reservoirs may be situated at great distances. Transport of e.g. 10 million m<sup>3</sup> water per year over a distance of say 25 km may be put at an average cost of 5 ct per m<sup>3</sup>.

On the other hand groundwater development may cause damage to agriculture (lower crop yields) and lead to payment of indemnifications to the farmers. The ICW, in recent years, carried out a number of studies on the decrement of crop yields

Source and pumped quantity	Costs per m <sup>3</sup> (1971 level)		
	Recovery	Treatment	Together
Groundwater from the Veluwe area (10 million m <sup>3</sup> per year)	5 ct	nil	5 ct
Groundwater from Salland (10 million m <sup>3</sup> per year)	5 ct	5 ct	10 ct
Surface water from Rhine, Meuse or IJssel Lake (80 million m <sup>3</sup> per year)	5...10 ct *)	15...20 ct	20...30 ct
Surface water from storage reservoirs in Twenthe (80 million m <sup>3</sup> per year)	5 ct *)	15...20 ct	20...25 ct

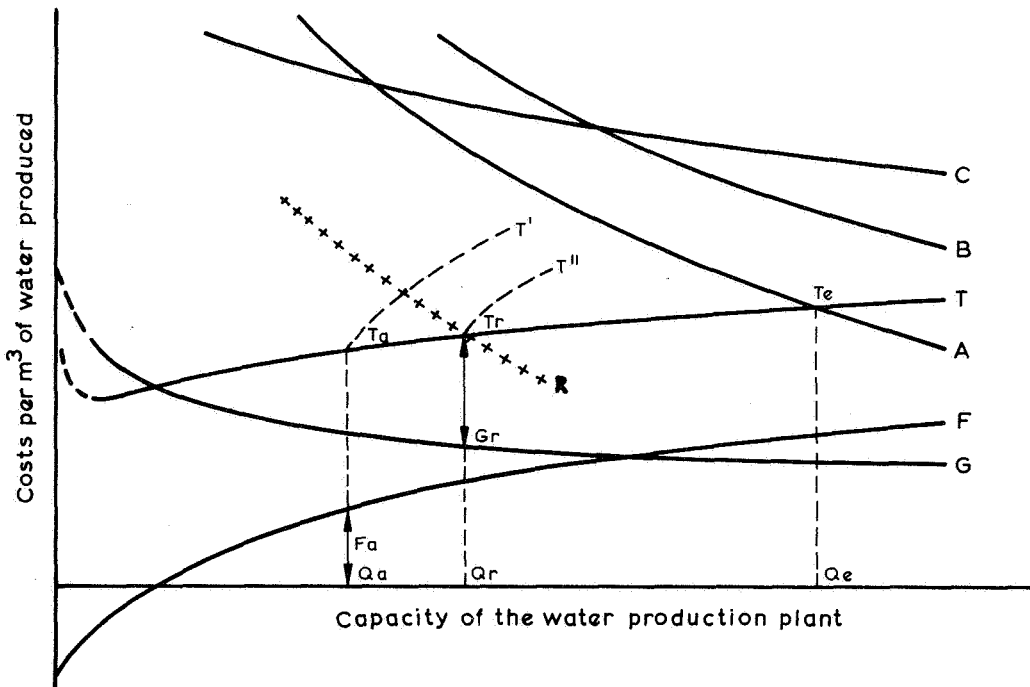
\*) Including costs of storage and of transport from the source to the storage reservoirs.

due to a lowering of the groundwater table and the economic loss to the farmer thereby. These studies show that the withdrawal of 1 m<sup>3</sup> of water from the amounts of water normally consumed by the vegetation would cause a decrease in profit of 5 ct to the farmer. Now only part of the groundwater pumped by a water works may be considered as having been withheld from the vegetation. The replenishment of the groundwater by the rainfall excess (rainfall minus evapotranspiration) in autumn and winter may partially be pumped by the water works without causing any damage to the vegetation. Moreover, in some areas the groundwater table is too deep below ground surface to have any effect on crop yields. Finally, not all areas are cultivated farmland. This means that an agricultural profit loss of 5 ct per m<sup>3</sup> groundwater pumped should be considered as the upper limit; generally the indemnification to be paid by the water works will be much less.

From this review it is evident that groundwater is a much cheaper (and furthermore a much safer) resource than surface water. It need not be explained any further that the water works industry will, therefore, base their plans for future supply on the optimum development of the country's groundwater resources.

Now, if only economic factors would be involved, the groundwater development by the water works would go on up to the technical limits, i.e. up to the moment that the use of another resource or another method of water production would become more advantageous. The diagram (fig. 1) illustrates this tendency.

As line *G* shows, the cost of production of water from a groundwater pumping station decreases slightly with increasing yields. However the growing draw-down



- $G$  = Net costs of production of water at a groundwater pumping station  
 $A, B, C,$  = Costs of production of water from other sources or by other methods  
 $F$  = Costs of indemnification to the farmers  
 $T$  = Total costs of production of water at a groundwater pumping station;  $T = G + F$   
 $R$  = Costs of water produced by the reference method

Fig. 1. Costs of rooler produced from different sources.

of the groundwater table in the surrounding area will cause increasing damage to agriculture, involving steadily rising indemnifications to be paid by the water works to the farmers or other affected interests (line  $F$ ). (It should be noted that at the initial stages the abstraction of groundwater may have a favourable effect on farming in areas suffering from excess of water. Furthermore peak discharges of surface water will be reduced by groundwater development, decreasing the costs of drainage). The total costs of production of water from the groundwater pumping station is indicated by line  $T$  ( $T = G + F$ ).

The lines  $A, B, C$  show the costs of production from other sources or by other methods (surface water, storage reservoirs, artificial recharge, desalinisation etc.). These costs are much higher than those of pumping groundwater (see table in this §).

Hence the water works people are inclined to continue the groundwater development up to a production  $Q_e$ , at a cost  $T_e$ ; from that stage on it will be more economical to maintain that production and to start using another resource to cover further increasing water demands.



#### 4. SOCIAL ASPECTS

Now the course of matters described at the end of the preceding § may entail such big draw-downs and such drought effects that farming in the conventional way (without irrigation, sprinkling or other expensive methods) would no longer be technically possible. Although the farmers would receive full indemnification in money, they might not be able to cultivate their fields in the conventional way any more, they would have to use other crops and quite different cultivation methods. This result is not acceptable from a social point of view. It is the policy of the government, i.e. the Minister of Public Health and Environmental Hygiene (who decides on the licenses to the water works), to allow groundwater development to continue up to the point that only "acceptably" slight damage is caused to agriculture. Here acceptable damage is understood to mean so small a damage that farmers are still able to continue using their present cultivation system and methods. In the diagram the groundwater abstraction causing this "acceptable" damage is indicated by  $Q_a$ , produced at a cost  $T_a$ . It is obvious that  $Q_a$  is a rather arbitrarily chosen quantity.

After having arrived at a capacity  $Q_a$  of the groundwater pumping station, no further extension would be allowed, and the water works would have to start using another resource to cover the yearly access in the water demands. The total costs of the raw water produced from the two sources would then be indicated by the line  $T^1$ .

It is clear that for the water works this solution is less economical than a continuation of the groundwater development; the costs  $T^1$  are higher than  $T$ .

For the agricultural party the solution is "acceptable", but not ideal. It is true that farming remains technically possible, but under less favourable conditions. An additional problem is the calculation of the indemnification to be paid to the farmers. The amount is not the same every year; in some years there will be damage, in other (wet) years there will be none. Should there be a yearly taxation of the crop yield in the field? Or should the indemnification be based on the magnitude of the draw-down of the groundwater table?

Another disadvantage of the adoption of the method of "acceptably" slight damage is the fact that it is a solution only for the benefit of the water works; water supply and agriculture remain antagonists with respect to the use of the groundwater resources. This will constitute an everlasting source of controversies.

It would therefore be wiser to look for a solution offering a broader utility, namely the solution of a concerted water management for the benefit of both water supply and agriculture. This management should be based on the principle of the optimum allocation of water to the interested parties, resulting in the highest social profits against the least sacrifices. It is clear that this will involve, amongst other things, the conservation of water in periods of excess, in order to bridge periods of shortage.

The advantages of this solution for the water works are evident: because of the

conservation and other management measures, the joint development of the groundwater resources by the water works and the agricultural organisations may result in a larger allocation of groundwater for public water supply purposes than when the method of "acceptably slight damage" were employed; the costs of production of that extra water will most likely be less than the costs of water produced by the methods A, B and C in fig. 1.

Also the farmers will benefit from the joint water management; a better equipped and more elaborate system of works for water management and conservation can be built if the two interested parties together provide the financial means.

## 5. THE REFERENCE-METHOD OF WATER PRODUCTION

The problem now is how to determine a just and justified allocation of water to the interested parties and to determine the financial consequences. A difficulty in the latter respect is the fact that water for public supplies is not a commercial product; the economical law of supply and demand is not fully applicable. Water supply in the Netherlands is a monopoly business and the water price is not the result of commercial competition; the price is connected with the water quality standards adopted and the exploitation and treatment methods chosen by the water works in question.

Farming products on the other hand are market products, although governmental subventions influence the farmer's revenues. This means that a loss of farming revenues due to groundwater development is not economically comparable with an economic gain for the water works by their continuing groundwater development instead of switching to another water source with higher production costs. Nor are the individual farmers themselves comparable with a public utility service as a water works, from the point of view of prosperity and technical equipment.

To solve this problem of comparing two incomparable items, the ICW developed a means that might be called the reference-method of water production. It might be defined as a (hypothetical) method of water production that does not cause any damage to agriculture and that furthermore is of the same prosperity level as the conventional farming methods. It is a method that should be technically feasible and that is less costly than the high-standard methods A, B and C actually used by the water works (fig. 1). The costs of water production by that method might be presented by the line R.

According to the principle of the reference-method, the water works would be allowed to abstract an amount of groundwater  $Q_r$ . The difference between the net production costs  $G_r$  and the production costs by the hypothetical reference-method,  $T_r$ , would be the water-works's contribution to the construction of the water management and conservation works for the benefit of all parties concerned.

After having arrived at a capacity  $Q_r$ , the water works will have to start using an additional source of water (e.g. method A, B or C). The production costs will then follow the line  $T''$ , which lies below the line  $T'$ .

An additional advantage of the reference-method is that the "acceptably" slight damage need not be determined. On the other hand it is necessary now to work out a suitable (hypothetical) waterproduction technique that might be used as reference technique. The working group is still studying this subject.

### III. PLANT PRODUCTION AND HYDROLOGICAL CONDITIONS

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#### INTRODUCTION

Agricultural production on the sandy soils of the Netherlands is being influenced to a large extent by the soil moisture and soil aeration conditions in the root zone. The holding in these areas for centuries has been of the mixed farming type, which on the one hand consists of arable crop production and on the other of dairy farming as well as on other activities which are not bound to the soil, such as raising chickens and pigs. As these latter activities are tied up neither to soil moisture conditions nor to soil type, they are neglected in this paper.

In the mixed farming type holding the arable crop production sector is to a large extent being used for the nutrient supply to the dairy sector. So products of the arable crops are transformed on the same holding into meat and milk and give an important addition to the cattle fodder produced by pasture. Aside from supply fodder for the cattle, the marketable products of the arable crops contribute directly to the farmers' income.

This traditional way of farming may be considered as having attained some kind of equilibrium between a number of factors as climatic conditions, soil type, drainage and soil aeration and social and economic position of agriculture within the larger community of the nation. This original equilibrium between traditional farming practices and physical and economical possibilities started to become disrupted in an ever increasing measure about three quarters of a century ago. First by the introduction of chemical fertilizers, which made possible the reclamation of large areas of waste lands, and subsequently by quick developments in social and economic conditions within the country itself and the world as a whole. By means of large drainage and land improvement projects, physical farming conditions have been changed considerably for the better, thus opening a wide field of new agricultural practices.

However, in an ever increasing measure these developments have put traditional farming under great stresses. A few of these are: the increasing population and as a consequence the rapid urbanisation and industrialisation, the unbalanced economic expansion of industry and agriculture causing an ever widening gap between the

economic position of the farmers and the rest of the population, the development of an industrialized type of farming, commonly called "bio-industry", and last but not least the ever growing claim which civil water supply exerts on groundwater resources. It may be expected that between the years 1980 and 2000 for large areas in the eastern part of the Netherlands the demands for drinking water of reasonable quality may not be met by pumping groundwater, as for other reasons is already the case for the western provinces.

In the near future pumping groundwater for civil water supply will locally constitute a serious threat to agriculture, which by its nature is an activity tied to a certain location and depends on soil and water management conditions on the spot. Now, groundwater extraction in certain regions starts to interfere with the latter by a drawdown of the groundwater table. The tragedy is that agriculture cannot migrate to a more favourable location and that it is not able to compete with industry for the monetary value of water.

It is clear that if, for reasons of national or world policy, a sound and strong agricultural activity is regarded as being of importance for the Netherlands, a sufficient water supply to agriculture will have to be safeguarded by the government. Groundwater extraction then should be limited to such an extent as determined by yield depressions regarded by mutual agreement as being acceptable both to agriculture and to the rest of the society. As a consequence of such a decision civil water supply, at least partly, will have to resort to more expensive methods of production.

The problem of domestic and industrial groundwater supply in relation to the interests of other parties involved (agriculture, nature and wildlife, river boards, pollution of open water, etc.) is a very complicated multi-dimensional and multi-disciplinary one. It can only be solved in a satisfactory manner by simultaneously taking into account all interests.

Farming practice is in fact a curious going together of pedological, hydrological, ecological, technical and economic features, which all are interrelated. In the mentioned context it is impossible to regard one of these aspects without being confronted with the others. An integrated study of groundwater supply problems in agriculture should account as well as possible for all these aspects. The possibility exists to do this by means of computerized mathematical models describing the water balance and plant growth, into which the effects of the most important factors influencing agriculture find their expression. Such models should make it possible to calculate the effects and consequences of interventions in the existing situation of water management.

In the following pages the effect of water management conditions on farming results will be treated in short. Then some remarks will be made on the effects of groundwater deficiency. Finally attention will be given to the development of a

mathematical model to calculate plant growth as a means to determine the yield depression caused by groundwater extraction for non-agricultural purposes.

## FARM REVENUE BUDGETING

In the mixed farming type revenues accrue by way of two kinds of production. On the one side there are the arable crops giving marketable products. The gross revenue of each individual crop can be calculated by taking: area in ha  $\times$  yield units per ha  $\times$  market price per yield unit — total direct costs (labour and non-factor costs). Applying water-yield functions, the influence of water management on gross revenue of arable crops can be calculated.

Then there is the animal husbandry sector. Grass and other fodder crops from the arable sector are transformed on the farm into milk and meat. The revenue of this sector depends on the current prices of the latter.

The labour income of the farm holding is found by subtracting from the gross revenue of the arable and dairy sectors the amount of fixed costs, such as interest on investments and depreciation of buildings and inventory. Here also the influence of the moisture situation in the field can be assessed.

Livestock farming of the traditional type is the result of a carefully planned fodder balance scheme, which is based on the need of food of one livestock unit (equivalent to a milk producing cow), expressed in number of units of digestible protein and starch equivalents. The traditional farmer will, to an extent as high as possible, meet the demand of food of the cattle by growing enough grass and forage crops in order to reduce to a minimum the buying of forage or concentrated fodder.

Modern developments in livestock farming tend to the development of highly specialised farms with very large numbers of head depending for fodder on other, fodder producing, farms. In this paper, however, only the traditional mixed farming type will be considered.

## SOIL AND WATER MANAGEMENT

Plant production occurs in an environment which is mainly determined by climate and soil. Climate determines rainfall, evaporation, temperature and radiation. The function of the soil is threefold: as site, as supplier of foodstuffs and as stratum of the solid, liquid and gaseous phase in which the root system of the plant grows and functions. Man tries to improve these conditions by supplying foodstuffs in the form of fertilizers and by regulating moisture and air content of the soil by drainage, irrigation, tillage and soil improvement measures. There remains the influence of the site, the kind is expressed by the ecology and topography of the area.

This totality of climate, soil and site determines the water management situation, as described by the water balance equation:

$$\text{precipitation} = \text{discharge} + \text{evapotranspiration} + \text{storage} + \text{water extraction}$$

This water balance can be drawn up for large areas as river basins of tens to thousands of hectares, as well as for very small areas.

An adequate soil moisture and soil air supply of the crop is of the utmost importance to enable the plant to transpire in an undisturbed manner. The plant then can extract water and mineral foodstuffs from the soil, transport it to the leaves and transpire water through the stomata. The plant will take up carbon dioxide from the air, produce carbohydrates by means of photosynthesis and convert these through assimilation into organic matter.

With regard to plant production the soil has two important faculties. First the ability to retain water and air and secondly to conduct it. The soil is the medium in which water movement from a locality with a high to one with a low potential takes place under influence of gravity and of capillary forces and in which oxygen is conducted to the roots by means of diffusion. The water and air retaining capacity of the soil and its conductivity for water and air are properties which are depending on the pore space size and distribution in the soil. As water and air management are in close negative correlation in the unsaturated zone of the soil and water management measures can more easily be taken, in the following pages attention will be given to water management only, always bearing in mind that excess moisture can give rise to air deficiency.

The capacity of the soil to retain a certain amount of moisture in the unsaturated zone above the water table and gradually to yield it under the influence of differences in potential is a very important hydrological property. The rainfall distribution mostly is irregular, so periods during which the soil moisture is being recharged, alternate with periods during which the plant has to draw on the amount of water stored in the root zone. The larger this moisture storage, the longer the plant can cope with rainless periods.

Now, the variations in rainfall and an insufficient moisture retaining capacity of the soil to a certain extent can be compensated by the depth at which the groundwater table is located. Soil moisture in the unsaturated zone has a negative tension as a result of the capillary forces acting between soil particles and water molecules. In a soil which after complete saturation was permitted to drain freely to the subsoil, the suction of the water at a height of 50 cm above the water table is equal to that exerted by a 50 cm column of water. There exists an equilibrium of gravitational and capillary forces and no water transport will take place. The relation between soil moisture content and tension is depicted by the desorption curve for the soil under consideration (fig. 1). If, however, this equilibrium of forces acting on the water

particles in the soil has been disrupted, e.g. because water has been abstracted at a certain locality, a potential is called into existence and consequently a capillary flow of water to the point of lowest potential takes place. The intensity of this rate of flow of water is proportional to the gradient of the potential and the capillary conductivity of the soil for water. However, the latter is not constant but depends on the moisture content of the soil and consequently also on the moisture tension (fig. 2). The more the soil moisture tension increases the more the capillary conductivity decreases. In terms of plant growth the point of extraction of water is the root and by the uptake

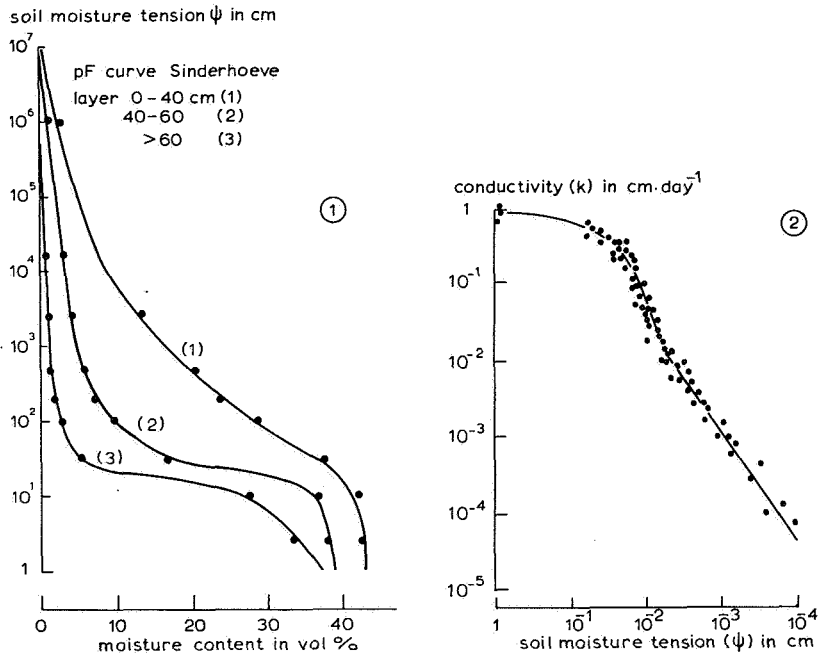


Fig. 1. Desorption curves for three layers of a sandy soil (after Endrödi and Rijtema, 1969).

Fig. 2. Relation between capillary conductivity (K) of humous loamy sand and soil moisture tension ( $\psi$ ) (after Rijtema, 1965).

of water the root zone of the soil is becoming desiccated. Because of the initiated moisture tension gradient moisture begins to flow in the direction of the plant roots. If the groundwater table is located at a favourable depth below the roots, at a constant rate of uptake of soil moisture from the root zone a constant vertical soil moisture flow will rise from the water table to the root zone; an equilibrium flow will exist as long as the conditions remain the same (fig. 3). In order that such a constant flow of moisture from the water table to the root zone is effectuated, a certain amount



of stored soil moisture has to be extracted from the soil layers between the water table and the root zone in order to generate the soil moisture gradient required. To

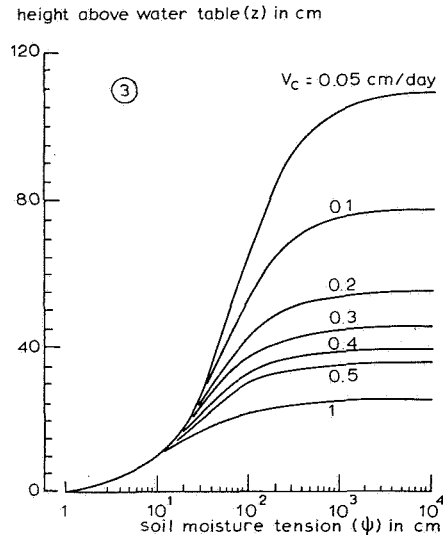


Fig. 3. Relation between height above the water table ( $z$ ), soil moisture suction ( $\psi$ ) and capillary rise ( $v_c$ ) (after Wesseling, 1957).

the plant it makes no difference if in a rainless period it receives the total amount of moisture needed, say 100 mm, by way of capillary rise or 50 mm in this manner and the other 50 mm from the soil moisture storage.

#### HYDROLOGICAL PROPERTIES OF THE SOIL

In farming practice the farmer always has taken into account the properties of the soils. The different crops which are included in the cropping pattern make different demands on water supply. Some crops will manage with less water than others. So one usually sees a crop like rye grown on the drier soils and sugar beets or pastures on the soils with the best moisture supply. This does not mean that rye would not do better on these soils with better hydrological properties. Experience has taught the farmer, however, to reserve the higher and drier soils, which often also make part of his holding, for growing rye. This for the simple reason that this crop as an element of the total cropping pattern will do better on these soils than will other crops and in this way will contribute to achieve the best farming results attainable.

On the sandy soils in the eastern and southern parts of the Netherlands a mixed type of farming has developed of which livestock forms the core. The soils with the best

moisture holding properties are reserved for pastures, while the drier soils are used for arable crops. Part of the products of the arable sector is used for dairy farming and part is being marketed directly. Shortages in fodder are met by buying concentrated foods.

Snijders (1968, 1969), Visser (1968) and Snijders and Visser (1968) have investigated in what manner the farmer uses the parcels with different hydrological properties of his holding to arrive at the best possible farming results. These investigations pertained to:

- cropping pattern
- fertilizer practices,
- moisture availability or excess,
- economic farming results.

The hydrological qualities of the soil have been indicated on maps in different ways, such as classifications by:

- drought sensitiveness,
- depth of groundwater table in summer and winter season,
- groundwater table fluctuation (Van Heesen, 1970).

Whatever indication technique is being followed, it invariably amounts to an attempt to characterize the effects of the climatic conditions (amount of rain, distribution of rainfall and evaporation) and the hydrological properties of the soil (runoff, moisture holding capacity, capillary rise, depth of groundwater table and its fluctuation during the seasons) by one or more index figures. During a nation wide investigation initiated by the Netherlands Committee on Agro-hydrological Research (COLN) this was effectuated by classifying the soils into a number of soil humidity or so-called "desiccation" classes, such as: frequently desiccating (FD), drought sensitive (DS), good water retaining capacity (GC), water logged (OF) and soils alternately waterlogged during winter and spring time but desiccating during summer. Snijders (1968) supplemented this classification by adding a class of soils which on the whole are rather wet although actually not of the water logged type (TW).

In a subsequent stage the Netherlands Soil Survey Institute introduced a scheme of "water table classes". These typify the depth of the water table and its fluctuation by its "mean highest" level in the winter season and its "mean lowest" level during the summer.

#### YIELD AND DEPTH OF GROUNDWATER TABLE

The connection between soil water management and crop yield can be depicted by the yield versus depth of water table curve. This curve shows the manner in which

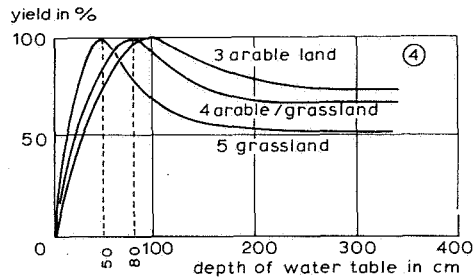


Fig. 4. Yield curves for arable and grassland in relation to the depth of the water table for three hydrological soil groups (after Kouwe, 1968).

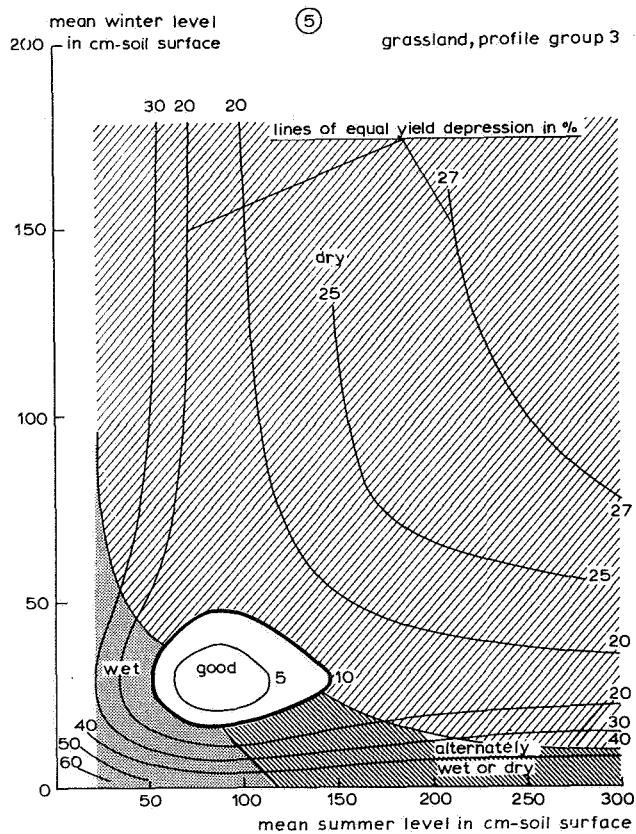


Fig. 5. Yield depression for grassland in relation to the mean depth of the water table during winter and summer for soil group 3 (after Bon, 1968).

the crop responds to differences in moisture and air supply of the soil. By the COLN-research group mentioned above such yield curves were introduced for a total of seven soil groups for grassland respectively the pertinent arable cropping scheme (fig. 4). In addition these curves were differentiated to the mean winter and summer water table, resulting in isocarp diagrams (fig. 5). These show lines of equal yield, or equal yield depression as may be the case, in relation to the water table in winter and summer. So, by means of these isocarp diagrams for each soil group the yield (depression) can be read for each combination of winter and summer depth of the water table. Until recent these yield curves and isocarp diagrams constituted the only material for characterizing the productivity of soils in large scale surveys of soil moisture.

However, for civil water supply purposes the Salland Research Group is developing new methods by which plant growth and moisture movement in the soil are incorporated into growth models (Visser, 1972).

The COLN-yield curves and the isocarp diagrams are rather coarse ways of determining yield, expressed as an annual mean for a long term period of say 10 years. Distinction is only being made between grassland and arable land. Besides, in case of the latter the diagrams apply for the integral cropping scheme, i.e. for all the composing crops as a whole. It will be clear that a coarse approximating method, as the COLN-yield curves, is not a very suitable instrument for a detailed research of plant growth under the climatic conditions of a specific year nor for the financial consequences of groundwater abstraction according to different strategies, if compensation of damages has to be given. For instance, the stage of maturity of the crop is not being considered, nor are evapotranspiration and precipitation and their distribution over the period of growth. Such refinements can be incorporated in a growth model, however.

Another approach was made by Rijtema (1971) by using the real evapotranspiration during the season as a basis for the availability of soil moisture to the plant, the depletion of the soil moisture storage and the capillary rise from the groundwater table. This calculation programme is operational and has been applied to solve indemnification cases in the area surrounding the Losser pumping plant in the eastern Netherlands.

#### CROPPING SCHEME AND WATER MANAGEMENT

The farmer takes into consideration the moisture retaining capacity of his soils when planning his cropping scheme. In what way he does this is being investigated by means of a cropping pattern research. This is being executed by registering for each parcel of a large number of farm holdings data on for example: type of crop, depth of water table, desiccation class, i.e. groundwater class, fertilization practices,

etc. All the parcels are being grouped according to these classifications after which the frequency distribution of each crop is calculated.

For each crop individually these frequencies of occurrence are being plotted on a logarithmic scale against the desiccation class (or any other classification preferred). This approach is based on the assumption that incorporating a crop into a cropping scheme is in fact a compromise between ecological and economic reasoning. Average curves can then be drawn and when a sufficient number of data is available it is to be expected that they constitute sections of a normal probability distribution. By using a log scale when plotting the frequencies of occurrence, the frequency curve will be transformed into a parabola, which has the advantage that it can be analysed more easily. The curve sections for the different crops can be integrated to form a complete curve of the frequency parabola by shifting them in a horizontal and vertical direction (fig. 6). The horizontal shift is an indication of the demand the crop exerts on the

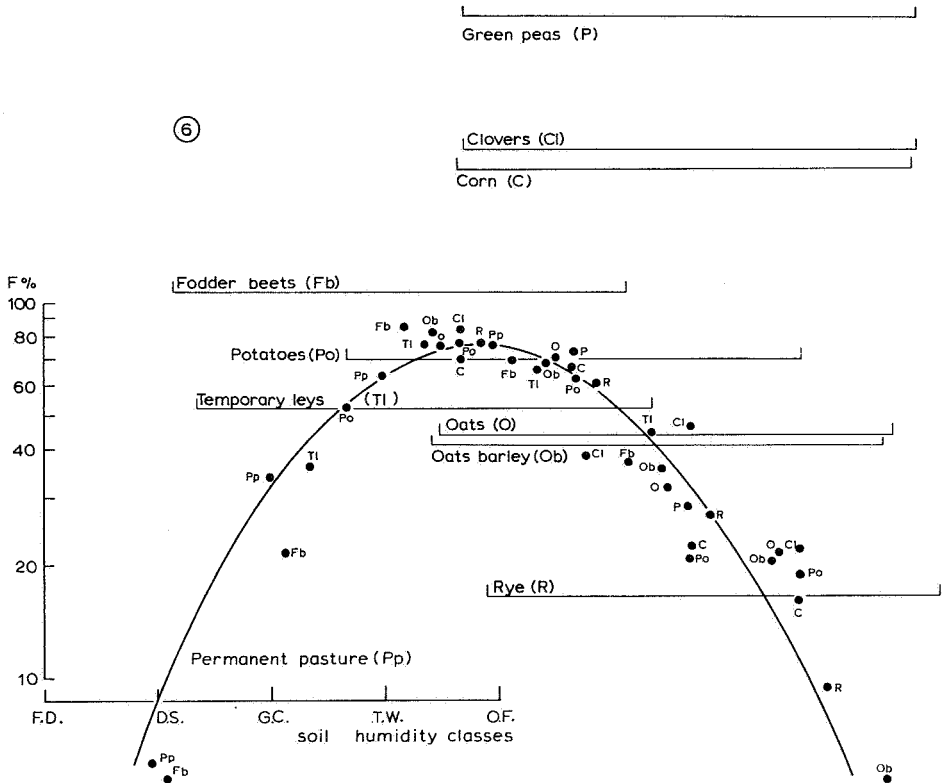


Fig. 6. Cropping pattern parabola for sandy soils with different hydrological properties, integrated from the separate probability curves for the individual crops of the cropping pattern by shifting the indicated abscissae to coincide with the one for permanent pasture (after Snijders & Visser, 1968).

environmental conditions; the vertical shift indicates the preference the crop receives in accordance with its economic significance for the farming results. The grouping together of the curve sections into one curve has the additional advantage that a certain adjustment to the normal frequency distribution curve can be performed. In this way for each crop the mean frequency distribution over each soil group can be determined, from which the average cropping pattern for each soil group can be deduced (fig. 7). When a relation between the depth of groundwater level and the classification used is established, the cropping patterns for certain depths of water table can be determined (fig. 8). An elaboration as explained above can also be performed for second crops as turnips, clover, etc.

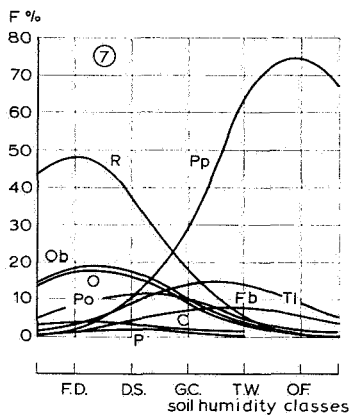


Fig. 7. Average frequency distribution curves of arable crops and grassland for the hydrological soil groups as derived from the cropping pattern parabola given in fig. 6 (after Snijders & Visser, 1968).

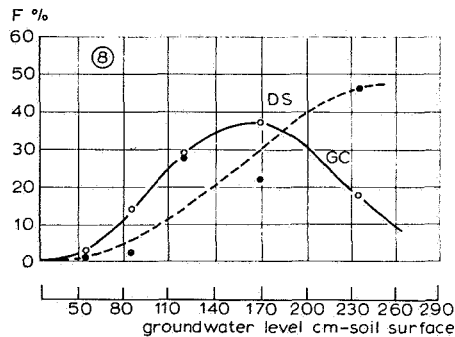


Fig. 8. Relation between frequency of depths of water table for drought sensitive (DS) soils and soils of good water retaining capacities (GC) (after Snijders & Visser, 1968).

## FERTILIZING AND WATER MANAGEMENT

Only a few remarks will be made on this part of agricultural water management research. The influence of the hydrological properties of the soil on fertilizing practices realizes itself mainly by way of the cropping pattern. There is a tendency for heavier gifts on dry soils than on those with better water management conditions, but the effect of soil hydrology on the total need of fertilizers of the holding is outweighed by the fertilizer needs of the individual crop, particularly because the main crops like rye and grass, which do not differ much in their need of fertilizers, are gradually exchanged for each other when soil moisture conditions change. On the other hand

there is a very distinct relation between the drought sensitivity of the soil and its acidity, on which the need for lime dressing depends (fig. 9).

The manure position of the farm is closely related to the number of cattle (number of livestock units) and the area of pasture land (fig. 10). The latter in its turn depends on the desiccation class of the soil, and consequently on the cropping pattern of this soil group. Furthermore the number of livestock units per unit area of pasture land

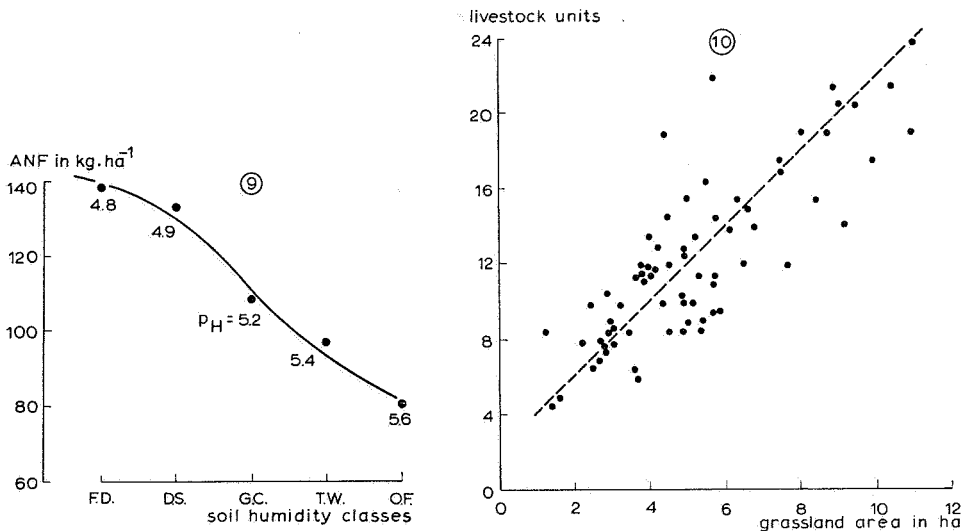


Fig. 9. Relation between the amount of acid neutralizing fertilizer compound (ANF) dressings, the hydrological properties of the soil and soil acidity (pH) (after Snijders, 1968).

Fig. 10. Relation between the number of livestock units and grassland area (after Snijders, 1968).

depends on the productivity of it. In the mixed farming type of holding the grassland area determines the number of livestock units and this the availability of manure. Although the variability may be large, there nevertheless is a distinct relation between the area of pasture land and the number of livestock units. From the available data lines could be drawn denoting equal numbers of livestock units for different areas of pasture land and fodder crops (fig. 11).

## FINANCIAL RESULTS

The financial results of farming are not only determined by the crop yields but also by the costs, for instance for sowing seeds, seed potatoes, fertilizers, purchases of concentrated foods and forage, soil tillage, agricultural piece work, interest and

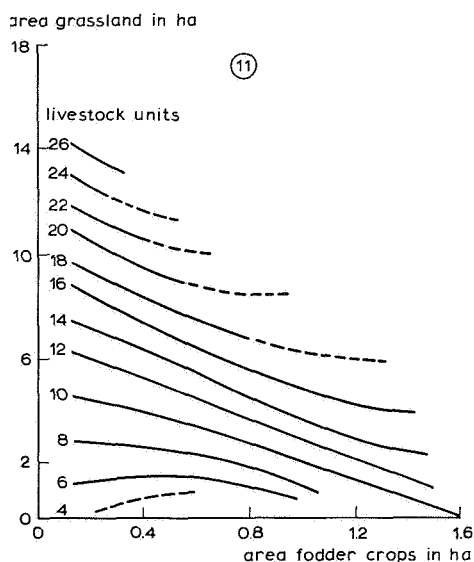


Fig. 11. Relation between area of grassland and fodder crops and number of livestock units (after Snijders, 1968).

depreciation on buildings and inventory. Some of these costs are indirectly related with the water management conditions of the holding; others are independent of it.

The crop yields for the different hydrological soil groups can be calculated by using the cropping pattern for the soil group with good water retaining capacity. The crop yields for each of the other desiccation classes can be deduced from it by applying their respective reduction factors. These reduction factors, representing the relative yields for the cropping patterns of the desiccation classes involved as compared to the optimum hydrological conditions, can be taken from the COLN-investigations. If, however, the mean depth of the groundwater table during the growing season, or differentiated for winter and summer period, is being used, the depression percentages have to be read from the yield curves or the isocarp diagrams respectively.

The physical crop yields of the cropping patterns of each of the hydrological soil groups (or groundwater table classes) can be converted into monetary values either directly by applying market prices to the marketable products or indirectly by taking the prices of milk and meat produced by the dairy farming sections. The labour income of the farmer is found by subtracting the costs from the revenue. It appears, as might be expected, that the results are best on the soils of good water retaining capacity (fig. 12).



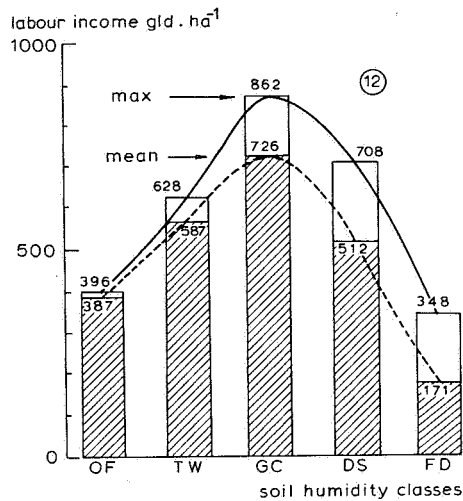


Fig. 12. Labour income (maximum and average) of farmers on sandy soils with different hydrological properties (after data from Visser & Snijders, 1968).

#### GROUNDWATER ABSTRACTION, YIELD AND INCOME

The technique of calculating farming results by way of a classification of soils into pedo-hydrological groups, a cropping pattern analysis and farm revenue budgeting, as described in the preceding paragraphs, may also be used to investigate the consequences of groundwater extraction for civil water supply. This has been done by Snijders (1969) by determining the sphere of influence of the depression cone of the groundwater table, subdividing it into a number of zones concentric to the pumping centre and calculating the farming results for each zone separately. The calculation was carried out for an area with a radius of 2000 m from the pumping plant for civil water supply near the village of Eibergen in the eastern part of the Netherlands. In the first place, for comparison reasons, the cropping pattern on the soil groups under non-influenced conditions has to be established. This was done in the vicinity outside the influenced zone. Subsequently for each point of a lattice of 800 points, and each concentric zone the relative frequency of the pedo-hydrological soil groups were determined, both for the non-influenced and the influenced situation.

From the investigation within the outer perimeter of the influenced area it appeared that the cropping patterns of the respective soil groups did not deviate appreciably from those on the same groups outside the zone of influence. This partly can be attributed to the continuation of the familiar traditional type of mixed farming, which, as has been explained before, is to a large extent based on cattle farming. It was seen, however, that, notwithstanding the deteriorated hydrological conditions, pasture land

is being maintained on soils which have become unsuitable for the purpose. To some extent this is caused by the relatively high prices of dairy products.

This outcome gives rise to a number of questions. Are the farmers' decisions purely based on conservatism or do they think to be better off by keeping to their traditional farming? In case only conservatism is the motive for the decision to stick to the old pattern, is it then reasonable to consider indemnification? The answer to this last question presumably would be negative. However, the possibility also exists that the lowering of the groundwater table constitutes a drastic intervention in an actual situation to which, furthermore, the farmer is being exposed unasked. What an indemnification contract should look like, along what process the size of the damage incurred should be determined, especially when long periods are involved during which only with increasing difficulty a basis of comparison may be determined, is a problem to which the answer has yet to be found.

The mean crop yield in per cent of the yield without groundwater extraction in the Eibergen case amounted, respective the distance from the pumping centre, from 0 to 13% (fig. 13, curve a). The calculation of the earned income, however, resulted in a

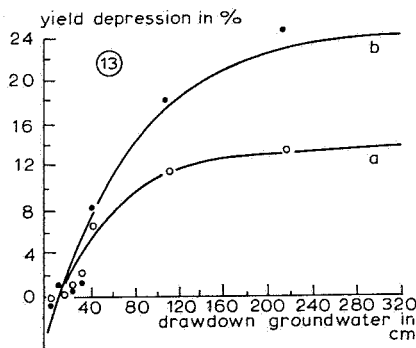


Fig. 13. Influence of a lowering of the water table due to groundwater extraction on crop yield (a) and income (b) (after Snijders, 1969).

depression of 0 to 20% (curve b). The difference between the two results is to be attributed to the need of forage purchases and of concentrated food in combination with the continuation of practically the same cropping scheme as was applicable under better hydrological conditions. The decreased productivity of grassland on the now drier soils proved to prohibit a close fodder balance in at about 75% of the area under grass and 25% as arable land.

In cases as that of the Eibergen pumping plant one may observe the fact that although the soils have become more liable to desiccation because a groundwater table drawdown, farmers nevertheless do not shift from mixed farming to the more dry farming type with mainly arable crops in the cropping scheme. Obviously the

farmer only partly adapts to the new situation and is unknowing of or accepts less favourable results instead of exploring new ways of production which might prove to be more remunerative.

#### A MODEL TO CALCULATE PLANT GROWTH

The method described to calculate the revenues of farming is not very suitable for operating in indemnification cases relating to the effects on agriculture by groundwater abstraction for civil water supply purposes. Such cases may lead to court actions where the simplified solutions and approximations may often be insufficiently convincing.

To investigate the effects on farming results of groundwater abstraction at different locations and a variety of abstraction rates, also to decide on the most desirable strategy of abstracting, the availability of a mathematical model on plant growth will have many advantages. With such a model a large variety of situations during the period of plant production can be taken into account, particularly when using a computer program able to handle these complex and intricate problems. The basic data pertaining to the problem have to be registered in such a way that they may readily be introduced into a computer, as also the necessary initial elaboration results which constitute part of the input.

Plant growth may be regarded as an interplay of hereditary factors, environmental conditions and energy transformation. The result is an amount of organic material which can be expressed in terms of dry matter stored in the body of the plant (roots, stems, leaves, fruits). This growth shows a certain sequence: germination, vegetative growth, maturation, flowering, fruiting and finally dying. This growth rhythm is to a large extent independent of the dry matter production.

Plant growth is governed to a large extent by the availability of sufficient water and air in the soil. Conditions of growth corresponding with the "wet" branch of the yield versus depth of water table curve are mainly determined by aeration of the soil, those corresponding with the "dry" branch of the curve by soil moisture content. Near the optimum of the curve both branches meet.

The demands of the plant during the different stages of development are not the same. The need of water in the juvenile stage is small, but progresses gradually as the crop grows and attains a peak as the plant reaches maturity. Then the need of water diminishes again when the crop starts ripening and is reduced to zero at the end of this stage. This, at any rate, holds for crops like cereals, potatoes and such. The moisture demand of grassland differs from this pattern, because it is either grazed by cattle or mowed for forage.

In the process of growing the transpiration capacity of the crop plays a predominant role. Dry matter production is almost directly proportionate to the amount of

water evaporated. This evapotranspiration is being regulated on the one hand by the possibility to withdraw water from the soil and on the other by the evaporating capacity of the atmosphere. The first limitation in its turn is being determined by the moisture potential of the soil water and the capillary conductivity of the unsaturated zone of the soil depending on it. As long as the plant is able to evaporate to its maximum capacity the evapotranspiration is equal to its potential value. As soon as evapotranspiration starts to slack off because the plants' need of water is not met, stomata start to close and actual transpiration is less than potential.

Simply presented it can be stated that dry matter production is delimited by two asymptotes: one which shows a proportional increase of growth with increasing availability of one growth factor (for instance possible evapotranspiration) and a second one imposed by the availability of another growth factor (for instance soil aeration) which limits production to a certain maximum or ceiling level (fig. 14).

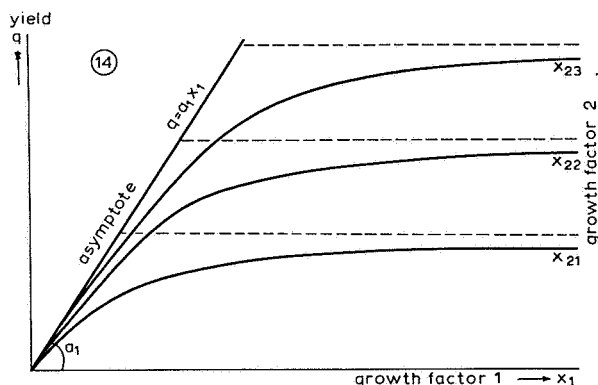


Fig. 14. Relation between crop yield and two growth factors according the Blackmann principle (after Visser, 1968).

In reality the process of growth naturally is more complicate than the above presentation suggests. This because not only evapotranspiration, soil aeration and the maturity stage of the plant determine plant growth, but also a number of other factors (for instance soil nutrients), some of which will have a large influence while others are of less significance. All these growth conditions impose their respective limitations on dry matter production depending on their relative availability. Therefore the presentation of the growth relation should be a multidimensional one (see for example fig. 15).

The influence of each growth factor is supposed to proceed according a simple linear transport function, such as is valid for mass flux and diffusion. However, on this linear function of factor 1 a limitation is imposed by growth factor 2, because the

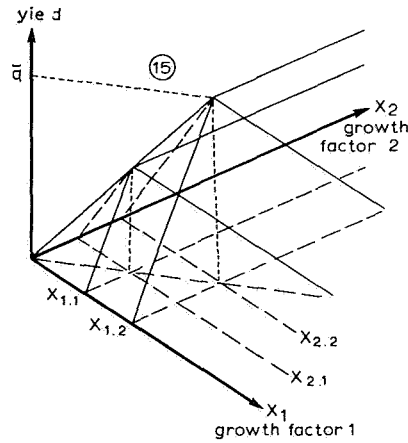


Fig. 15. Spatial presentation of the production function for two growth factors according eq. (1).

latters availability increasingly turns towards a minimum and ultimately constitutes a ceiling to plant production. Vice versa, in a different situation factor 1 may impose a similar production ceiling on the linear function of factor 2.

This principle of limiting growth factors for some time is a familiar one in biology and has been described by Blackman (1905). According the law of limiting growth factors can be written:

$$\text{asymptote 1: } q = ax_1 \text{ or } 1 - \frac{q}{ax_1} = 0 \text{ for } ax_1 \neq 0$$

$$\text{asymptote 2: } q = bx_2 \text{ or } 1 - \frac{q}{bx_2} = 0 \text{ for } bx_2 \neq 0$$

where:

$q$  = dry matter production

$x_1$  and  $x_2$  = growth factors

$a$  and  $b$  = growth parameters

As the growth curve is deflected from the asymptotes, it is possible to write:

$$\left(1 - \frac{q}{ax_1}\right) \left(1 - \frac{q}{bx_2}\right) = F \quad (1)$$

where  $F$  expresses the degree of adaptation to the asymptotes  $ax_1$  and  $bx_2$ , and therefore not necessarily needs to be equal to zero.

When dry matter production of a plant or crop is plotted against time the result

is a typical S-shaped curve (fig. 16). This starts at time equal to zero ( $t_0$ ) at the moment of sowing with a yield  $q_0$ . As time elapses the growth rate  $dq/dt$  gradually in-

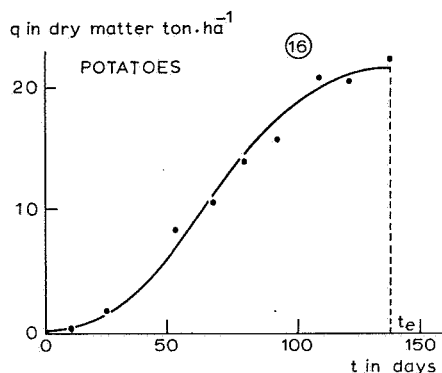


Fig. 16. Relation between dry matter production ( $q$ ) and time ( $t$ ) for a potato variety in a particular year (after data from Sibma, 1968).

creases to a maximum and, at attaining maturity, decreases again to become zero when the ripening process has been completed ( $t_e$ ). The ultimate yield  $Q$  is reached and

$$Q = q_0 + \sum_{t_0}^{t_e} \Delta q(t) \quad (2)$$

If no limiting factors would be present the process of growth would be unlimited. For higher plants, even under optimum conditions a natural limit will be set by the plants' biological and hereditary properties. Mostly, however, growth will also be limited by restrictions imposed by certain sub-optimal conditions.

Visser (1973) describes the relation between optimum growth and time according the following equation:

$$\left\{ \frac{a}{q_t - q_0} + \frac{1}{Q - q_t} \right\} dq = c \left\{ \frac{b}{t - t_0} + \frac{1}{t_e - t} \right\} dt \quad (3)$$

In this formula ( $q_t - q_0$ ) is proportional to the quantity of plant cells engaged in production of dry matter at time  $t$ , while ( $Q - q_t$ ) is the amount of matter which according to the natural capabilities will still be produced. In the same manner ( $t - t_0$ ) stands for the time elapsed since growth started and ( $t_e - t$ ) for the amount of time until the natural end of production when the maximum possible yield  $Q$  has been attained. The constant  $a$  indicates that production in the juvenile stage proceeds at a somewhat different rate as in later phases of development. Constant  $b$  describes the fact that time in the early phases might have a different significance

than in a later one. Both constants together cause the upper bend of the S-shaped growth curve to be non-similar to the lower bend. Integrating equation (3) gives:

$$\frac{(q_t - q_0)^a}{Q - q_t} = d \left\{ \frac{(t - t_0)^b}{t_e - t} \right\}^c \quad (4)$$

in which  $d$  is the integration constant. The constants  $a$ ,  $b$ ,  $c$  and  $d$  can be determined from data of growth experiments such as given in fig. 16. By means of subsequent elaborations the other constants  $Q$ ,  $q_0$ ,  $t_0$  and  $t_e$  ought to be determined.

In equation (3) two interdependent variables are present:  $dq/dt$  and  $q$ . In order to arrive at an expression in which only  $dq/dt$  figures as an unknown  $q$  must be eliminated. This can be done by solving equation (4) for  $q$ , which, however is an intricate algebraic problem. For simplification purposes it may be assumed that  $a = b = 1$ . In this case the solution of eq. (4) is:

$$q_t = \frac{q_0 + PQ}{1 + P} \quad \text{with} \quad P = d \left( \frac{t - t_0}{t_e - t} \right)^c \quad (5)$$

With the aid of eq. (5),  $(q_t - q_0)$  and  $(Q - q_t)$  can be determined and the results substituted in eq. (3) which after some regrouping of the terms yields:

$$\frac{dq}{dt} = \left\{ c \frac{P}{(1 + P)^2} \cdot \frac{(t_e - t_0)}{(t - t_0)(t_e - t)} \right\} (Q - q_0) = D(t) \cdot (Q - q_0) \quad (6)$$

in which  $D(t)$  stands for the expression between braces, which is only time dependent. For  $a$  and  $b$  unequal to 1,  $dq/dt$  as function of  $t$  is to be solved numerically.

This expression for the rate of growth can be combined with the influence of limiting factors according to the Blackman principle, described previously.

However, when introducing limiting growth factors another feature needs to be considered. Optimum, natural, growth ( $n$ ) means that ultimately a maximum yield  $Q$  is attained. Sub-optimal conditions during a certain period cause a limited rate of growth ( $l$ ) which results in an ultimate yield:

$$Q_l < Q$$

According to eq. (2) the yield depression is:

$$\Delta Q = Q - Q_l(t_e) = \sum_{t_0}^{t_e} \Delta q_n(t) - \sum_{t_0}^{t_e} \Delta q_l(t)$$

It is assumed that the plant is not able to undo a once incurred reduction in the rate of growth, even if in a later phase conditions have become optimum again

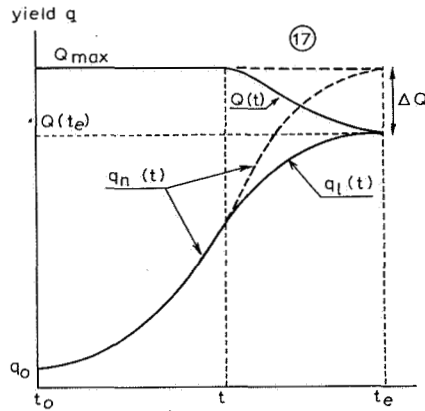


Fig. 17. Optimum growth conditions produce a maximum yield ( $Q_{\max}$ ); sub-optimum conditions result in a yield depression ( $\Delta Q$ ) decreasing  $Q_{\max}$  to  $Q(t_e)$ .

(fig. 17). The consequence of this assumption is that from the moment natural growth is limited,  $Q$  is no longer a constant, but is time dependent according to:

$$Q_l(t) = Q - \left\{ \sum_{t_0}^t \Delta q_n(t) - \sum_{t_0}^t \Delta q_l(t) \right\} \quad (7)$$

So for limited conditions in expression (6)  $Q$  has to be replaced by  $Q_l(t)$ . Growth limited by a reduction of transpiration ( $E_r < E_{\text{pot}}$ ) can be described by:

$$\left[ 1 - \frac{\frac{dq_l}{dt}}{D_r(Q(t) - q_0)} \right] \left[ 1 - \frac{\frac{dq_l}{dt}}{f(E_r(t))} \right] = F \quad (8)$$

in which  $E_r(t)$  is the actual transpiration as a function of time determined either by the evaporating capacity of the atmosphere or by the rate of capillary rise of soil moisture to the root zone;  $F$  is a constant determining the degree of adaptation of the growth curve to its asymptotes (see fig. 14). Of course it is possible to insert more limiting growth factors as terms in eq. (8), for instance the air content of the soil in the root zone, which up to a certain level may limit growth because of oxygen shortage.

It may be remarked that neither temperature nor radiation have been mentioned, although these factors are of great importance for plant production. As a gross approximation both could be accounted for by using the temperature sum

$$T_t = \int_{i=0}^t (t_i - t_{i-1})(\tau_i - \tau_0) \quad \text{and} \quad T_0 = \int_{i=0}^{t_0} (t_i - t_{i-1})(\tau_i - \tau_0)$$

instead of the time  $t$  and  $t_0$  proper. In these expressions  $t_i$  stands for the time elapsed



since sowing,  $\tau_i$  the mean temperature at day  $t_i$  and  $\tau_0$  the temperature below which no growth takes place. This supposition accounts to a large extent, because of the relationship between radiation and temperature, for the radiation as well.

#### USE OF THE GROWTH MODEL

The model described will be used on a day-to-day basis. From models for the water balance of the soil under varying climatic conditions, the actual evapotranspiration together with other quantities such as depth of water table, air content, discharge and seepage will be calculated. Either for an undisturbed hydrological situation or for situations in which water is being pumped for civil water supply purposes at different extraction rates. These series of evapotranspiration data will be substituted into the model in order to determine possible yield depressions and to establish a relation between extraction rate and yield depression per  $\text{m}^3$  of groundwater extracted, in terms of labour income of the farmer; In the process of the day-to-day calculation of growth equation (7) and (8) will have to be used alternately, both for the undisturbed and changed hydrological circumstances. The purpose of the investigation being to find a strategy of groundwater extraction which produces the maximum amount of water at acceptable losses to agriculture. These losses then have to be compensated for by some arrangement of indemnification.

#### FINAL REMARKS

At the time this article was written the water balance model was available and the one for plant growth was nearing the end phase of development. However, anticipating the result of combining the two models, it might be of interest to give an idea of the losses the farmer is likely to suffer as a result of pumping groundwater for civil water supply purposes. Such calculation has been made for the surroundings of the pumping plant Eibergen according to the method described in the first part of this article. At an extraction rate of  $1.5 \times 10^6 \text{ m}^3$  per year the average agricultural losses on the area influenced amounted to a total of 55,200 Dutch guilders. This comes to a mean of 3.7 cents per  $\text{m}^3$  of water extracted. Close to the pumping station the damage will be much higher while at the perimeter of the area no influence will be felt. It was found that the results for different years with varying climatic conditions did show a variance in the yield depressions caused by pumping the same amount of water. In years with a favourable precipitation distribution hardly any yield depressions were incurred. Under less favourable conditions of plant growth they are much more dependent on soil moisture storage and capillary rise from the water table. Then losses may be heavy, because of the severe drawdown of the water table.

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## IV. ON THE EVALUATION OF PARAMETER VALUES IN WATER BALANCE MODELS

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### INTRODUCTION

Data of groundwater levels and run-off values of a watershed may occasionally both be available in sufficiently large numbers that it is worthwhile to try to get more information on the hydrology of the watershed from these data. Such a study can be carried out with models based on the water balance equation, because run-off is one item on the water balance and groundwater level fluctuations are the result of the interplay of all items. In this paper results are discussed of the application of a water balance model to data from watersheds in the district Salland in the province of Overijssel, collected in the years 1970, 1971 and 1972.

### MODELLING THE WATER BALANCE

#### *A very simple model*

With a water balance model daily values of run-off, actual evapotranspiration and storage coefficient are calculated. For day  $k$  run-off  $R_k$  and evapotranspiration  $E_k$  are subtracted from precipitation  $P_k$ , giving a change in storage. Dividing by the storage coefficient  $\mu_k$  this is converted into a change in groundwater level  $\Delta G_k$ . This added to the groundwater level  $G_k$  will give a groundwater level for the next day. So the principle is

$$\frac{P_k - R_k - E_k}{\mu_k} = \Delta G_k \quad (1)$$

and

$$G_{k+1} = G_k + \Delta G_k \quad (2)$$

The items present in the water balance model are described as functions in which the parameters are combined with variables expressed in terms of open water evaporation, precipitation and depth or head of the phreatic water. The simplest example of such a deterministic model has 6 parameters and rests on a study to determine storage, discharge and evapotranspiration from groundwater level data (Bloemen, 1970).

The drainage process is described with the equation

$$R_k = a(S - G_k)^2 \quad (3)$$

in which  $S$  represents an unknown drainage level and  $a$  incorporates all resistances to groundwater flow.  $G$  is the depth of the phreatic level below soil surface.

Actual evapotranspiration  $E$  is calculated as the lowest value obtained from two equations. One is

$$E_k = gE_{0k} \quad (4)$$

in which  $E_0$  is the measured or calculated open water evaporation and  $g$  is a reduction factor according to Penman's theory. The other is

$$E_k = d_1 G_k^{-d_2} \quad (5)$$

in which  $d_1$  and  $d_2$  are parameters expressing how  $E$  is limited by the depth  $G$  of the groundwater table.

Finally the storage coefficient is calculated as a function of the groundwater depth with parameters  $f$  and  $m$  as

$$\mu_k = fG_k^m \quad (6)$$

In this model the amounts of water  $B$  involved in the changes of the groundwater level from  $G_1$  to  $G_2$  must be found by integration because (6) is non-linear. The solution is

$$\int B = \int_{G_1}^{G_2} fG^m dG = \left[ f \frac{G^{m+1}}{m+1} \right]_{G_1}^{G_2} \quad (7)$$

$$B = \frac{f}{m+1} [G_2^{m+1} - G_1^{m+1}] \quad (8)$$

This implies that the groundwater level  $G_k$  after a 24-hour change in water storage  $B$ , calculated as  $B = P - R - E$  can be calculated from the groundwater level  $G_{k-1}$  at zero hours as

$$G_k = \sqrt[m+1]{\frac{-(m+1)(P - R - E)}{f}} k + G_{k-1}^{m+1} \quad (9)$$

Equation (9) is the simplest model for calculating successive groundwater levels. Sometimes it is necessary to allow for positive or negative infiltration ( $I$ ). This item is proportional to the difference between phreatic and piezometric heads and inversely proportional to the hydraulic resistance in vertical direction, or

$$I_k = \frac{h_{phrk} - h_{piezk}}{c} \quad (10)$$

When only phreatic heads are measured, infiltration can be calculated from the differences  $\Delta h$  between the phreatic head within and outside the watershed describing the grade of the phreatic level toward it over a length  $L$ . It is assessed as

$$I_k = \sum_{i=1}^n \Delta h_{ik} \frac{W_i k D_i}{A} \quad (11)$$

in which  $W_i$  is the width of stream flow in the direction of  $\Delta h_i$  and  $A$  is the area of the catchment.

In fig. 1 a flow chart of the simple model of eq. (9) is given. There are 6 parameters.

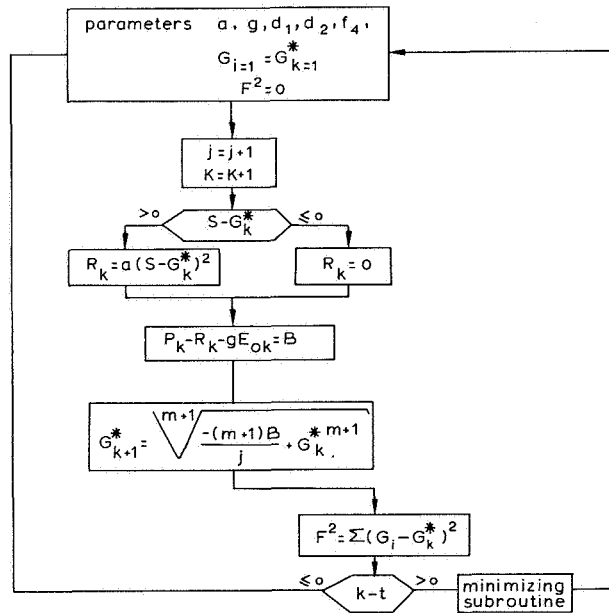


Fig. 1. Flow chart of a simple water balance model with 6 parameters and an error function of groundwater levels.

### *A better representation of physical reality*

Of course a better representation of the physical aspects of run-off, evapotranspiration, etc. can be given. There are three main points:

1. It is obvious that the calculation of evapotranspiration and the storage coefficient is dependent on the assumption that there is a continuous equilibrium between suction and the distance to the phreatic level. Of course, a better conception is that the moisture tension  $\psi$  at a fixed depth  $z$  under soil surface is a function of ground-

water depth  $G$  and capillary conductivity  $K_c$ . The equations can be derived from the function (Rijtema, 1965)

$$K_c = K_0 e^{-\alpha\psi} \quad (12)$$

where  $K_0$  is capillary conductivity at zero suction,  $\psi$  is suction and  $\alpha$  is a soil constant. The derived equations read (Visser, 1969)

$$\psi_k = \frac{1}{-\alpha} \ln T_k \quad (13)$$

and

$$T_k = 1 - (1 - e^{-\alpha z}) \left( 1 + \frac{V_{ck}}{K_0} \right) \quad (14)$$

$V_c$  is the upward capillary flow and as a simplification it is assumed to equal actual evaporation. A test on the value of  $T_k$  decides whether eq. (4) is valid or the equation

$$E_k = d_1 \psi_k^{-d_2} \quad (15)$$

If eq. (4) does not hold,  $\psi$  will be imaginary ( $T_k < 0$ ) and eq. (15) is substituted for  $V_c$ . Then  $E$  and  $\psi$  are solved by iteration. If, however, on the particular day precipitation exceeded  $E$  according eq. (4) then  $E$  following from this equation is applied in the water balance instead of  $E$  according eq. (15), but  $\psi$  is calculated as if eq. (15) was valid.

Consequently the storage coefficient  $\mu$  is calculated as

$$\mu_k = c_1 \psi_k^{c_2} \quad (16)$$

2. It is clear that a part of the precipitation measured with 24-hour intervals may have an effect on the groundwater level in the next interval. So it is necessary to calculate the amount  $F$  of rainfall  $P$  which percolates to the groundwater in the same interval in dependence of the depth of the phreatic level. The equation is

$$F_k = (P_{k-1} - F_{k-1} + P_k) e^{-q G_{k-1}} \quad (17)$$

The parameter is  $q$  and its physical meaning is somewhat obscure because as a rule there are time lags between the 24-hour intervals of measuring precipitation and measuring groundwater level. Besides, precipitation is a 24-hour total and groundwater level is a 24-hour average at best. Details like these are not accounted for in the model.

3. The drainage process will have to be described in a more complicated form. In medium high grounds to which the model of the water balance is applied a rising phreatic level means an increase of drainage flow, contributing to the depletion of

groundwater storage, because the outflow occurs via an increasing number of channels of increasingly lower order. Integration of groundwater flow obtained at an observation point with a high phreatic level will show a smooth curve as shown in fig. 2, where run-off of a watershed and the phreatic head at a site in it are plotted.

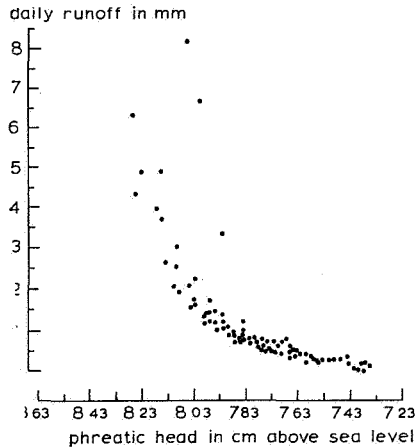


Fig. 2. Runoff of a watershed plotted against phreatic head at a particular site inside the catchment area.

Such a relationship can only be acceptably described on a physical basis by schematizing arbitrarily to for instance three drainage levels, as was done to obtain fig. 2. A further simplification was carried out in connection with deeper channels, which generally have larger mutual distances than smaller ones have. As impermeable layers appear in this area only at a considerable depth it was assumed that when applying the Hooghoudt formula for steady flow the quadratic term in it can be neglected as far as flow to channels at large mutual distances is concerned.

So the drainage of these medium high grounds is described in the water balance model as

$$R_k = b_0(S_1 - G_k) + b_1(S_1 - G_k)^2 + b_2(S_2 - G_k) + b_3(S_3 - G_k) \quad (18)$$

The drainage levels  $S_i$  are solved as parameters, the parameters  $b_i$  stand for the resistance of groundwater flow to the water levels  $S_i$  in the channels.

In fig. 3 a flow chart is shown of the water balance model after the preceding remarks have been applied in the formula. There are 16 parameters.

#### *Accounting for non-steady outflow*

A further development of the model is attractive because in eq. (18) only steady groundwater flow is accounted for. In 24-hour intervals there will be a fair chance

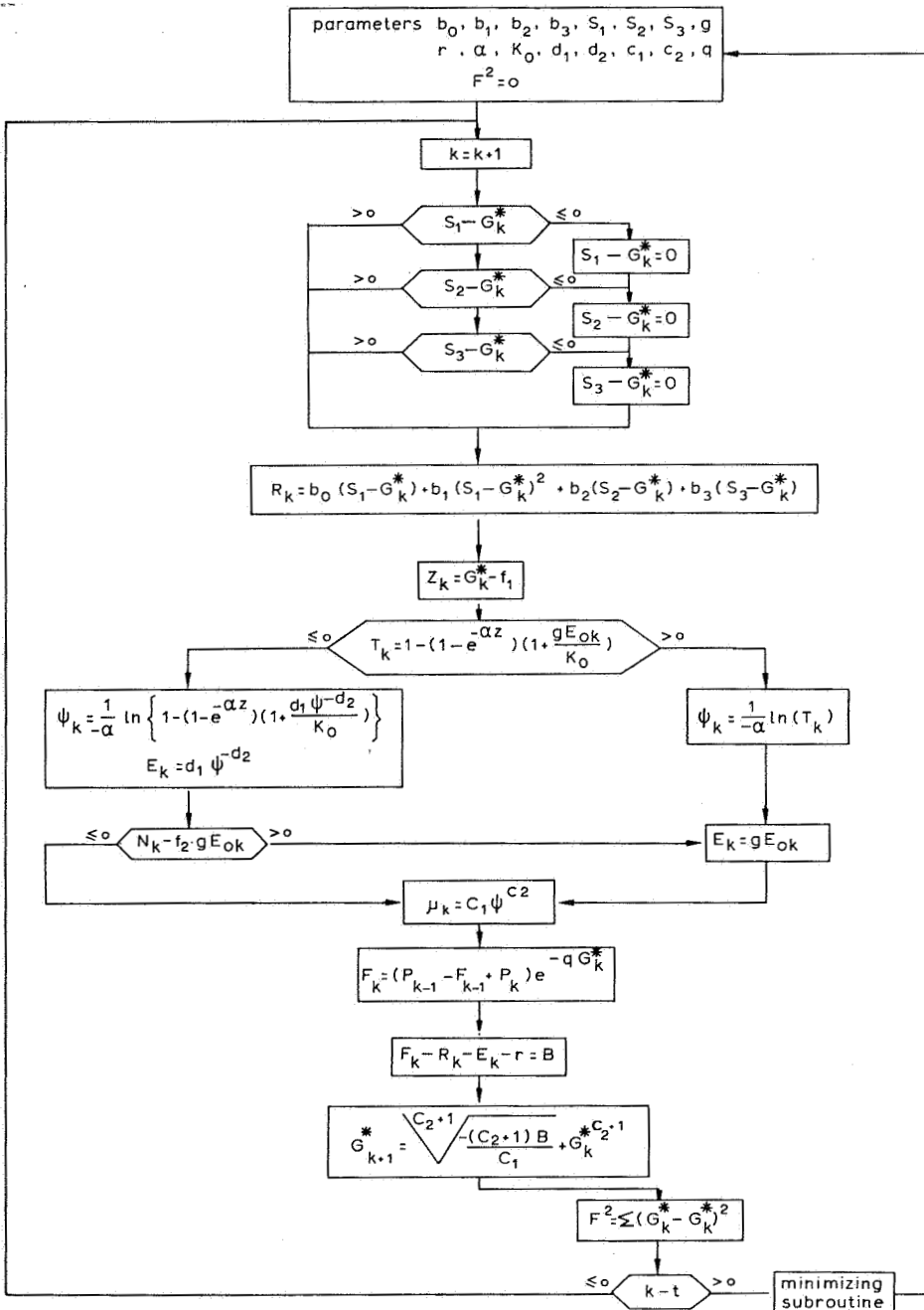


Fig. 3. Flow chart of a water balance model with 16 parameters and an error function of groundwater levels.



that non-steady flow will be of considerable importance. As a matter of fact the scatter in fig. 2 emphasizes that non-steady flow probably occurs when groundwater levels are high. It would complicate things too much to describe both steady and non-steady flow to different levels. A simplification is justified by the triviality of the quantities of water which is drained to channels at large mutual distances. When these amounts are ignored run-off can best be represented by a description of steady and non-steady flow to the group of most frequent channels.

Steady flow is calculated with the Hooghoudt formula as a function of the mean phreatic head  $h_k$ , which is equal to  $S - G_k$ . This mean is of course the average of two observations with a 24-hour interval. To prevent that the calculation results in too large a run-off when major changes in phreatic head occur, it is necessary to integrate between  $h_{k-1}$  and  $h_k$ . The equation is

$$R_k = \frac{\int_{h_{k-1}}^{h_k} (ah + bh^2) dh}{h_k - h_{k-1}} \quad (19)$$

$a$  and  $b$  are the well known constants in the Hooghoudt formula.

Non-steady flow can be described as a function of the change in groundwater level during the 24-hour intervals. This conception is based on a well known fact that after rainfall groundwater outflow increases more than agrees with the relatively slow rise of the groundwater table. Groundwater levels are only constant when outflow and percolation are in equilibrium. In that case an equation for steady flow like eq. (19) is valid. Also when no percolation occurs there is a fixed relation between outflow and phreatic head. This tail recession outflow is in the ratio of  $1 : \frac{12}{\pi^2} = 0.82 : 1$  to steady outflow (Kraijenhoff van de Leur, 1958).

In fig. 4 it is schematically shown how deviations from steady outflow are related to changes in groundwater level. As an explanation it may be accepted that only when outflow is steady, the actual phreatic head and the distance to the channels

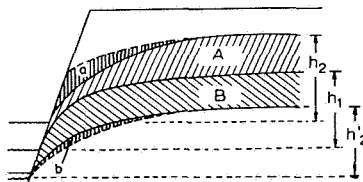


Fig. 4. A rise of the groundwater level from  $h_1$  to  $h_2$  gives an increased water storage A and an extra runoff a. A fall from  $h_1$  to  $h'_2$  gives a decrease B in water storage and a reduction b of runoff.

are balanced at every point between two channels. It is assumed that the ground-water table is elliptical in cross section, which is not the case when outflow is non-steady.

When the phreatic head changes from  $h_{k-1}$  to  $h_k$ , the area  $A$  of the vertical cross section of the groundwater reservoir, perpendicular to and halfway across the channel distance and above the open water level has changed with

$$\Delta A_k = \frac{\pi \cdot l(h_{k-1} - h_k)}{4} \quad (20)$$

with  $l$  half the distance between two channels.

On the assumptions that the phreatic level would rise in a parallel manner if no discharge occurred, that precipitation amounts remained equal and that the storage coefficient is homogeneous, the mentioned cross sectional area of the groundwater reservoir above the water level in the channel would change with

$$\Delta A'_k = l(h_{k-1} - h_k) \quad (21)$$

The deviations from steady outflow then will be proportional to

$$\Delta R_k :: l(h_{k-1} - h_k) - \frac{\pi \cdot l(h_{k-1} - h_k)}{4} \quad (22)$$

or

$$\Delta R_k :: \left(1 - \frac{\pi}{4}\right) \cdot l(h_{k-1} - h_k) \quad (23)$$

Introducing the storage coefficient gives the identity

$$\Delta R_k = \left(1 - \frac{\pi}{4}\right) \cdot \mu \cdot l(h_{k-1} - h_k) \quad (24)$$

which is expressed in millimeters of water

$$\Delta R_k = \left(1 - \frac{\pi}{4}\right) \cdot \mu(h_{k-1} - h_k) \quad (25)$$

Finally it is assumed that fluctuations of the water level in the channels may be neglected, so  $h_{k-1} - h_k = G_k - G_{k-1}$  (where  $G$  is groundwater depth), and that  $\Delta R$  is non-linearly dependent on groundwater depth. Now eq. (23) can be rewritten as

$$\Delta R_k = \left(1 - \frac{\pi}{4}\right) \cdot e^{-\beta(G-x)} \cdot \mu(G_k - G_{k-1}) \quad (26)$$

where  $\beta$  and  $x$  are parameters and  $\bar{G} = \frac{1}{2}(G_k + G_{k-1})$ . The term  $\left(1 - \frac{\pi}{4}\right)$  is denoted with the symbol  $p$  and it may have other values, for instance  $p = \left(1 - \frac{2}{3}\right)$  or  $p = \left(1 - \frac{2}{\pi}\right)$  when the phreatic level is parabolic or sinusoidal in cross section.

Now a substitute for eq. (18) is

$$R_k = \frac{\int_{h_{k-1}}^{h_k} (ah + bh^2) dh}{h_k - h_{k-1}} + p \cdot e^{-\beta(\bar{G}-x)} \mu(G_{k-1} - G_k) \quad (27)$$

As  $G$  is groundwater depth below soil surface, a rise of the groundwater will make the second term in eq. (27) positive. A fall will make it negative, however, with no percolation occurring (tail recession outflow) to the limit of 18% of the first term, as stated before.

#### *Accounting for channel characteristics*

In the first term of eq. (27) stationary flow is calculated as a function of the head of phreatic water over the water level  $S$  in the channel. Often somewhere downstreams in the channel there is a water level gauge. The data obtained from this gauge can be used as given  $S$ , but then there has to be allowed for the slope in water level between gauge and point of measuring the outflow including weirs or a drop

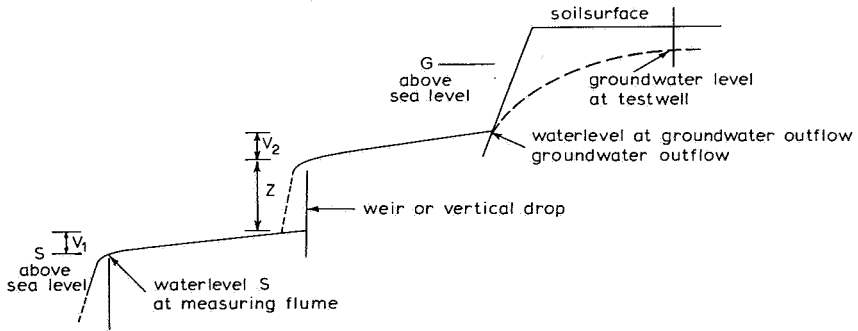


Fig. 5. Determination of the head responsible for runoff; see eq. (28).

of the water level at weirs or overflows (fig. 5). It is clear that the head  $h$  which is responsible for run-off is written as

$$h_k = S_k - G_k - V_k - Z \quad (28)$$

where  $S$  and  $G$  are elevations above sea level,  $V$  is a fall in water surface and  $Z$  a vertical drop. Both depend on the amount of run-off but for  $Z$  this is ignored for convenience sake.

The slope  $V$  is calculated with Manning's formula, reduced to (Visser, 1971)

$$Q = K_m(0.49 + 0.8W/D)D^{2.67}I^{0.5} \quad (29)$$

$Q$  is run-off in  $\text{m}^3 \cdot \text{s}^{-1}$ , computed from run-off  $R_k$  in  $\text{mm} \cdot \text{day}^{-1}$  as  $0.000116A \cdot R_k$ , in which  $A$  is the area of the watershed,  $K_m$  the factor of Manning for bed roughness,  $W$  the width of the channel bed,  $D$  the water depth in the channel at the point where  $S$  is measured and  $I$  the hydraulic gradient. Now when  $H$  is introduced as the elevation of the channel bed,  $D$  is written as  $S - H$  and it follows that

$$I^{0.5} = \frac{Q}{K_m\{0.49 + 0.8W/(S - H)\}(S - H)^{2.67}} \quad (30)$$

and

$$V_k = L \left[ \frac{Q_k}{K_m\{0.49 + 0.8W/(S - H)\}(S - H)^{2.67}} \right]^2 \quad (31)$$

where  $L$  is the estimated distance along the channel between water level gauge and testwell,  $K_m$  and  $Z$  are parameters.

In fig. 6 a flow chart is given of a model in which eq. (27) for run-off and eq. (31) for channel flow are incorporated. It must be noted that the calculation of the change in groundwater level  $\Delta G_k$  only takes into account the first term of the equation for run-off because the second term actually is a change in storage. It is assumed that groundwater levels are not measured so near water courses that this second term would effect groundwater level. Tests are provided to decide as explained before if the second term of eq. (27) should be set at its limit of 18% of the first term, because no percolation occurs. The model has 15 parameters.

## APPLYING THE MODELS TO WATERSHED DATA

### Parameter evaluation

The parameters in the models are adjusted to their optimum position by an iterative procedure. This requires a criterion of fit. Though in principle every variable magnitude, which is measured and also calculated with the model can be used, there generally are only data available on run-off of watersheds and on groundwater levels in the watersheds. For the models in fig. 1 and 3 the criterion of fit is the sum of squares of differences between observed and computed groundwater levels at corresponding days. In the model of fig. 6 the criterion of fit is an error function with which the run-off measured at the point where the water level  $S$  was gauged is com-

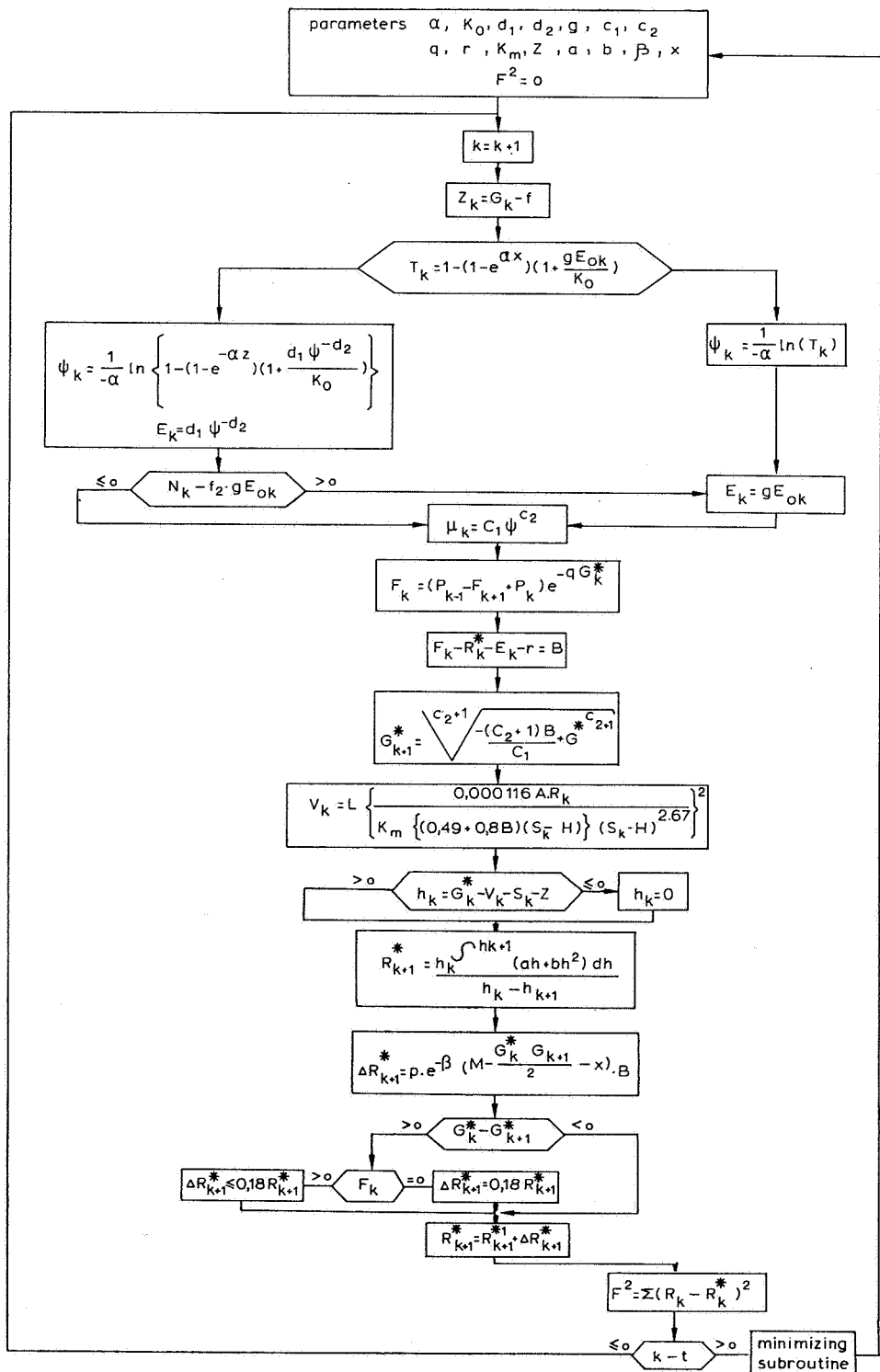


Fig. 6. Flow chart of a water balance model with 15 parameters and an error function of runoff.

pared to run-off computed with the model. In both cases the error function can be written as

$$F^2 = \sum_1^n (\text{obs}_i - \text{comp}_i)^2 \quad (32)$$

Although there is a diversity of techniques to minimize the error function a simple univariate technique was used. It proved its usefulness in 10 comparative tests against the Simplex method for function minimization (Nelder and Mead, 1965) by scoring with less trials a higher proportion of the initial variance of the observed ground-water levels that is included in the calculated groundwater levels.

Univariate methods have in common that for only one coordinate at a time a minimum is sought (Spang, 1962). As a rule the rotation of coordinates is determined objectively, for instance by means of the first derivatives. Minima are found via second derivatives or polynomials. Iterations are continued until no further decrease in  $F^2$  is obtained.

The technique that was used for the water balance models started from the idea that in the beginning of the process of adjustment minima for separate parameters do not have much significance for the location of the space minimum. The technique

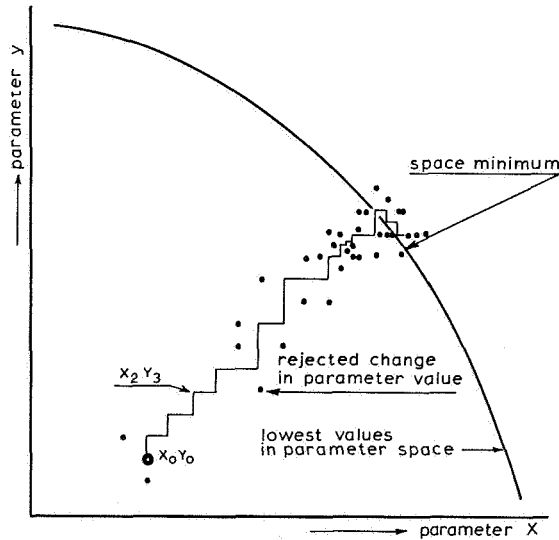


Fig. 7. Example with 2 parameters of the univariate minimizing routine used in this study.

can be demonstrated with a problem containing only two parameters  $x$  and  $y$ . The error function  $f(x, y)$  is the third dimension. In the coordinates  $x$  and  $y$  in fig. 7 estimates of the parameter values give the point  $x_0 y_0$ . Now the value of one parameter,

for instance  $x$ , is reduced with a fixed amount  $\Delta x$ . If  $f(x_0 - \Delta x, y_0) > f(x_0, y_0)$  this change is rejected and the value of  $x$  is increased with the same amount. If  $f(x_0 + \Delta x, y_0) < f(x_0, y_0)$  this change is accepted and  $x_1 = x_0 + \Delta x$ . In that case as also when the change had been rejected the other parameters' value is reduced with  $\Delta y$ . If  $f(x_1, y_0 - \Delta y) > f(x_1, y_0)$  the change is rejected and  $y_0 + \Delta y$  is tried. After that the first parameter is tried again and in rotation the parameters are adjusted to values which give the minimum value of  $F^2$  at for instance  $F^2 = f(x_{16}, y_{13})$ . The rotation of the parameters, the size and direction of the changes are arbitrary but fixed at their initiation so the minimization of  $F^2$  can be an automatic procedure. The number of parameters is not of importance. Some further details are:

- Changes in parameter values in a consecutive step can be programmed to have the same direction as in the preceding one if that was successful. If accepted, it is the next parameters' turn again, if not the opposite direction is tried. This can save steps.
- When with initial step sizes no further progress is made, a second stage begins with step sizes which are reduced in a fixed ratio. A third and fourth stage can be programmed.
- Iterations can be stopped when a specific criterion is met.

Fig. 7 shows how the adjustment of two parameters may work out.

#### *Performance of the models*

The performance of the models can be studied by defining their efficiency with the coefficient  $R^2$  as the proportion of the initial variance accounted for by the model (Murray, 1970).

$$R^2 = \frac{F_0^2 - F^2}{F_0^2} \quad (33)$$

in which  $F_0^2$  is the initial variance defined by the sum of the squared deviations of observed magnitudes from their mean, as

$$F_0^2 = \Sigma(\overline{\text{obs}} - \text{obs})^2 \quad (34)$$

The coefficient  $R^2$  of the efficiency of the model has its principal significance as a relative measure when comparing results of different models with the same data or of the same model with different data. For a more definite appreciation the standard error between computed and observed quantities should also be known. It can be computed by

$$S_a = \sqrt{\frac{\Sigma(\text{obs} - \text{comp})^2}{n - p}} \quad (35)$$

where  $n$  is the number of observations and  $p$  the number of parameters in the model which has been solved.

In table 1 the coefficient  $R^2$  and the standard error  $S_a$  are given for the output of the models of figures 1 and 3. The average number of groundwater levels used for parameter evaluation was 62. The number of trials is listed. From 57 to 96 per cent of the initial variance is accounted for. The model with 15 parameters averaged 91 per cent, the model with 6 parameters 81 per cent. So the model with more parameters gave a better output.

Table 1. Performance of the water balance models with 6 and 16 parameters when minimizing an error function of groundwater depth.  $R^2$  = coefficient of efficiency of the model;  $S_a$  = standard error between computed and observed groundwater levels in cm;  $n$  = number of trials

Watershed	Number of parameters					
	6			16		
	$R^2$	$S_a$	$n$	$R^2$	$S_a$	$n$
Mb	0.947	8.9	183	0.965	8.3	655
B	0.917	9.2	673	0.934	8.8	510
F	0.727	24.6	370	0.910	15.5	1220
II	0.608	18.5	475	0.786	14.0	1265
JJ	0.828	16.5	182	0.911	14.7	1494
Ha	0.572	16.3	283	0.878	14.3	820
Ib	0.719	13.2	862	0.864	12.7	1056
01	0.908	11.4	253	0.933	10.8	996
00	0.947	11.5	456	0.942	11.4	958
04	0.933	10.5	126	0.954	10.2	719

The standard error  $S_a$  ranged from 8.3 cm to 24.6 cm.  $S_a$  can be high when  $R^2$  is low and vice versa. The standard errors are large because of the large initial variances in groundwater levels in the type of soils under consideration.

In table 2,  $R^2$  and  $S_a$  are listed for the output of the model given in fig. 6.  $R^2$  is between 47 and 89 % and  $S_a$  is between 0.21 and 0.66 mm.day<sup>-1</sup>. There is no correlation between  $R^2$  and  $S_a$ . The result of the operations on some of the watersheds is definitely insufficient but as a whole they are satisfactory. The low number of trials needed with this model is striking.



Table 2. Performance on the water balance model with 15 parameters when minimizing an error function of runoff.  $R^2$  = coefficient of efficiency of the model = proportion of variation in observed runoff that is included in calculated runoff;  $S_a$  = standard error between calculated and observed runoff in mm.day<sup>-1</sup>;  $n$  = number of trials

Watershed	$R^2$	$S_a$	$n$	Watershed	$R^2$	$S_a$	$n$
01	0.83	0.43	163	Ha	0.61	0.21	285
3	0.70	0.45	202	II	0.69	0.34	242
4	0.72	0.41	198	LL	0.68	0.46	198
5	0.78	0.24	260	Mc	0.85	0.40	172
6	0.88	0.43	173	OO	0.80	0.34	115
7	0.89	0.26	103	B	0.76	0.50	273
CC	0.80	0.39	175	F	0.47	0.59	366
FF	0.71	0.33	170	G	0.68	0.66	200
GG	0.62	0.60	267				

Tables 1 and 2 show that the effect of minimization can be very different. It is possible that this is caused by a restricted appropriateness of the models, but the inaccuracy of input data will have had a major effect. Water level recording, calibration of weirs, assessment of areal precipitation and open water evaporation are sources of error which will have their influence in the output. A special instance is the factor for bed roughness  $K_m$  in the model of fig. 6. This coefficient will not be constant as it is supposed to be in the model, because variations in the density of aquatic vegetation will normally occur.

#### *Parameter values*

The optimum values of the parameters in the models of fig. 3 and 6 are listed in tables 3 and 4. As parameter  $q$  in eq. (17) was found to have exceedingly small values it was assumed that  $q = 0$ ,  $p$  in eq. (27) was fixed at a value of 0.3. As no data were available to evaluate parameter  $c$  in eq. (10) or  $r$  in eq. (11) they were evaluated as free constants and so lost their physical significance.

The difference between table 3 and 4 is that in table 3 parameter values are listed which were obtained by minimizing the sum of squares of the differences between observed and computed groundwater levels in a watershed. The parameter values in table 4 were obtained by striving to an agreement between observed and computed run-off of the same watersheds as a criterion of fit.

There is hardly any conformity in the comparable values of the same parameter.

Table 3. Parameter values for 16 watersheds after minimization of an error function of groundwater depth. The parameters appear in the following equations: moisture tension (14), storage coefficient (16), evapotranspiration (4) or (15), infiltration (11) and runoff (18).

Watershed	Parameters														
	$b_0$ $10^{-1}$ , $\text{day}^{-1}$	$b_1$ $\text{cm}^{-1}$ , $10^{-1}$ , $\text{day}^{-1}$	$b_2$ $10^{-1}$ , $\text{day}^{-1}$	$b_3$ $10^{-1}$ , $\text{day}^{-1}$	$S_1$ cm	$S_2$ cm	$S_3$ cm	$\alpha$ $\text{cm}^{-1}$	$g$ —	$r$ mm, $\text{day}^{-1}$	$c_1$ $\text{cm}^{-c_2}$	$c_2$ $\text{cm}^{(1+\alpha_2)}$ , $10^{-1}$ , $\text{day}^{-1}$	$d_1$	$d_2$ —	$K_0$ mm, $\text{day}^{-1}$
1	.0100	.00058	.0029	.03010	74.	110.	157.	.0088	.685	— .255	.798	.127	39.5	.49	3.255
3	.0656	.00090	.0180	.00160	46.	97.	102.	.0041	.580	— .113	.869	.156	50.	.50	4.680
4	.0535	.00138	.0323	.00005	76	118.	214	.0002	.580	— .083	.869	.152	50.	.50	4.575
5	.0540	.00130	.0214	.00800	73	113	193	.0062	.630	— .360	.580	.168	48.	.54	3.730
6	.0588	.00232	.0182	.00882	84	72	164	.0017	.580	— .120	.520	.225	38.	.62	5.100
7	.0453	.00080	.0179	.00530	38	86	193	.0058	.670	— .075	.550	.206	23.	.82	5.100
CC	.0150	.00000	.0250	.00270	69	109	237	.0025	.555	— .165	.805	.173	45.	.66	7.780
FF	.0318	.00097	.0033	.00440	47	96	170	.0054	.640	— .135	.795	.110	14.	1.10	3.383
GG	.0003	.00236	.0143	.00140	80	91	256.	.0074	.580	— .113	.800	.118	80.	.32	4.200
Ha	.0003	.00011	.0203	.00470	0.5	131.	226.	.0049	.625	— .015	1.023	.222	71.	.60	3.128
II	.2320	.0030	.0135	.00150	76.	91.	205.	.0071	.858	— .370	.895	.172	81.	.58	3.353
LL	.0450	.00044	.0129	.0055	51.	94.	207.	.0056	.548	— .128	.580	.150	49.	.92	3.801
Mc	.1140	.00197	.0044	.0036	25.	58.	261	.0101	.570	— .120	.750	.108	167.	.42	4.930
OO	.0656	.00090	.0180	.0016	46	97	102	.0041	.580	— .113	.869	.156	50.	.50	4.680
F	.010	.00070	.0045	.0050	38	57	237	.0001	.700	— .120	1.031	.234	50.	.50	4.178
G	.010	.00044	.0495	.0059	26	100	206	.0049	.730	.090	.874	.149	114.	.35	3.705

Table 4. Parameter values for 17 watersheds after minimization of an error function of runoff. The parameters appear in the following equations: moisture tension (14), storage coefficient (16), evapotranspiration (4) or (15), infiltration (11), steady state runoff (19) and deviation from steady state runoff (27)

Water-shed	Parameters													
	$A_1$ $10^{-1}$ , day $^{-1}$	$A_2$ $\text{cm}^{-1}$ , $10^{-1}$ , day $^{-1}$	B $\text{cm}^{-1}$	x cm	$\alpha$ $\text{cm}^{-1}$	g —	r mm, day $^{-1}$	$c_1$ $\text{cm}^{-c_3}$	$c_2$ —	$d_1$ $10 \text{ cm}^{(1+\epsilon_2)}$ , day $^{-1}$	$d_2$ —	$K_0$ mm, day $^{-1}$	$K_m$ $\text{m}^{1/3}\text{sec}^{-1}$	Z cm
1	.00157	.000401	.068	8.75	.00724	.678	.3937	.281	.128	71.88	.272	1.894	144.37	105.5
3	.00534	.000230	.016	0.00	.00004	.713	.0038	.156	.001	6.25	.550	4.594	10.00	0.0
4	.00587	.000375	.020	50.94	.01072	.910	.1650	.400	.069	60.00	.500	1.200	50.00	0.0
5	.00434	.000166	.026	32.50	.00690	.507	.5137	.487	.104	40.00	.425	3.450	27.50	240.0
6	.00422	.000265	.025	56.41	.00630	.525	.2962	.606	.070	38.75	.600	2.400	110.94	273.5
7	.00080	.000171	.049	0.0	.00795	.844	.1350	.375	.105	13.44	.866	2.494	76.56	121.0
CC	.00370	.000137	.025	0.0	.01237	.831	.5400	.500	.094	11.88	.612	5.137	104.37	16.0
FF	.00060	.000175	.037	0.0	.00499	1.081	.4387	.406	.153	50.0	.500	7.031	13.44	70.0
GG	.00365	.000490	.195	0.03	.00553	.610	.4029	.187	.098	40.37	.536	2.178	37.31	241.0
Ha	.00202	.000100	.065	2.06	.01716	.920	.4310	.330	.148	50.99	.549	2.234	27.00	4.0
II	.00302	.000235	.074	18.28	.00720	.604	.6750	.150	.107	48.44	.350	1.481	12.50	2.5
LL	.00411	.000340	.076	25.94	.00960	.542	.6750	.294	.074	13.13	.853	2.025	7.50	28.0
Mc	.00434	.000095	.032	0.00	.00952	.616	.1762	.256	.093	28.75	.706	5.174	59.99	0.5
Oo	.00450	.000407	.111	0.00	.00675	1.006	.0975	.350	.104	36.25	.669	1.912	7.50	0.0
F	.00145	.000136	.039	13.62	.01953	1.0	.8997	.268	.120	17.87	.573	1.446	49.81	9.0
G	.00014	.000085	.029	40.31	.00101	.604	.1050	.175	.001	60.00	.500	4.669	47.81	83.0

This was to be expected for the following reasons:

- Although the physical interpretation of the parameter may be the same in both models, when groundwater levels serve as a criterion of fit the parameter values will relate to a local situation, i.e. the near surroundings of the test well. In case run-off serve as a criterion of fit the parameter values will relate to the watershed as a whole.
- It is questionable whether parametric values obtained by minimization still have a physical meaning. The adjustment of the parameter to its optimum value by fitting the model to some criterion forces the parameters to assume values which are statistically the best. These need not be physically correct. This explains why a small model with six parameters can give such a good fit.
- If there is only one criterion of fit a statistical interdependence between parameters in a water balance model is inevitable. Run-off, storage coefficient and evapotranspiration are partly computed as functions of groundwater levels which in their turn are calculated from a water balance in which computed run-off, evapotranspiration, etc. appear. Such a calculation is inductive and different combinations of values of the parameters in such a model can have the same error function. To what extent a minimum error function will possibly go together with different combinations of parametric values is later discussed in the paragraph on tests of the significance of parameter values.
- Parameter evaluation has its own inaccuracies as a means of determination.

Parametric values in the same model but for different watersheds show a considerable variation, but also here the points mentioned above play a role. Especially the second and third mentioned points demonstrate their influence in some spurious parameter values which are very evidently physically not correct. That nevertheless a rather good fit is obtained is due to less notable deviations in other parameters which give a balancing correction.

For parameter  $g$  in eq. (4) and  $K_m$  in eq. (3) it can easily be judged if a parameter value is acceptable from a physical point of view. It is also obvious according to eq. (12) that a very low value of parameter  $\alpha$  gives a very low gradient of capillary conductivity as well as a very high capillary rise, so such low  $\alpha$ -values must consequently be mistrusted. The combination of a very high value of  $d_1$  and a very low one of  $d_2$ , as in table 3, must be regarded as a result of parameter interdependence.

## TESTING OF THE MODELS

### *Synthetic test data*

To what extent parameter interdependence and the capacity of the univariate method, earlier described in the paragraph on parameter evaluation, to find the

minimum error function are decisive for the stability of the optimum values of parameters, was tested with synthetic data, as recommended for the testing of different optimizing methods (O'Donnell, 1966). These synthetic data are obtained by feeding the model with a set of input data and letting it generate an output record with parametric values which are obtained via the minimizing routine from the same input data. The groundwater level or run-off record that forms the output is error free and it is compatible with a set of known parameter values and the initial sets of input data on precipitation and open water evaporation. Therefore the error function  $F^2$  has the minimum value zero for coincidental and systematic errors in the input data and a lack of adaptability of the model cannot be restrictive anymore.

The test on the stability of parameter values was carried out for the model given in fig. 3 with records of groundwater levels in a test well in watershed 01 and for the model of fig. 6 with records of run-off in watershed 6. These examples were chosen because of their very good fit according table 1 and 2.

The test consisted of repeating the evaluation of parameter values with six different combinations of six different initial parameter values with synthetic groundwater levels or run-off as a criterion of fit. When generating these sets of initial values the view was adopted that such values will always be best estimates and will be randomly scattered around the optimum values (Ibbitt, 1970). Therefore six possible initial values were given a difference with the exact values by fixed percentages, three being smaller and three larger than the correct value. With a die six combinations of these initial values were arranged by chance. These sets were starting points in the parameter space for the minimizing routine. So six values for each parameter were evaluated. These values deviate from the known correct value. To know if these deviations are statistically significant they are submitted to the test of Student. Calculated were:

— the mean deviation from the correct value  $p'$ . This is  $\Delta p$ , computed as

$$\Delta p = \frac{\sum (p_i - p')}{n} \quad (36)$$

— the standard deviation  $S_d$  of the six values from their mean  $\bar{p}$

$$S_d = \sqrt{\frac{\sum (p_i - \bar{p})^2}{n - 1}} \quad (37)$$

— the test criterion according to Student

$$t_0 = \sqrt{\frac{\Delta p \sqrt{6}}{S_d}} \quad (38)$$

### *Tests on significance of parameter values*

In table 5 the results of six parameter evaluations, the correct parameter values and  $\Delta p$ ,  $S_d$  and  $t_0$  according eqs. (30), (37) and (38) are listed for the two models with different criteria of fit. When testing two-sided with 5 degrees of freedom and 90 per cent confidence limits it holds that  $t_{0.90} = 1.476$ . So for half of the parameter values in both models the zero hypothesis exceeds the test value and the values are statistically not reliable. This must be regarded as a consequence of the models' mechanism. In the paragraph on parameter values it was argued in the third point raised that the calculations are inductive and will inevitably lead to dependences between parameters. For instance parameter evaluation on an error function of run-off may stop with parameter values that calculate groundwater potentials too high, which deviation from physical reality is balanced by drainage constants being underestimated. Repeating the minimization will perhaps give groundwater potentials too low and an overestimation of drainage constants. The fit between observed and computed run-off may be just as good in either case. Still the minimizing routine may be regarded as being efficient in view of the very high values of  $R^2$  obtained with synthetic data. It is obvious that combinations of very different parameter values can result in practically identical run-off records.

The difference between inductive and non-inductive model mechanism can be shown with a test on the significance of the parameter values in eq. (27) when run-off is calculated as a function of measured instead of computed groundwater levels. The storage coefficient now has to be computed directly as a function of groundwater depth and eq. (8) has to be incorporated in eq. (27).

$$R_k = \frac{\int_{h_{k-1}}^{h_k} (ah - bh^2) dh}{h_k - h_{k-1}} \cdot p \cdot e^{-\beta(\bar{G}-x)} \cdot \frac{f}{m+1} (G_k^{m+1} - G_{k-1}^{m+1}) \quad (39)$$

where  $h_k = S_k - W_k - Z$  and  $a$ ,  $b$ ,  $\beta$ ,  $x$ ,  $f$  and  $m$  are parameters.

Now records of daily run-off, groundwater level, precipitation and open water evaporation are input data. The parameters are evaluated on the error function of run-off. Then the record of calculated run-off was again used as synthetic test data and a test on the significance of the parameter values was conducted as before. All information is listed in table 6. The parameter values are reliable.

The conclusion at this stage is that evaluation of parameters in models with inductive mechanism on only one criterion of fit will not give reliable values for all parameters. A study of literature makes it seem not to be likely that available more sophisticated minimizing routines than the univariate method would give better results with one criterion of fit. This has nothing to do with the number of parameters. The model in fig. 1 is also inductive. It will only be possible to improve the accuracy of determination of parameters when different criteria of fit for separate parts of the

Table 5. Results of tests on significance of parameter values in the models with as a criterion of fit the groundwater level (upper part) and the runoff (lower part). For  $p'$ ,  $\Delta p$ ,  $S_a$  and  $t_0$  see eq. (36), (37) and (38), for  $R^2$  eq. (33)

Evaluation	Parameters													
	$b_0$	$b_1$	$b_2$	$b_3$	$b_4$	$S_1$	$S_2$	$S_3$	$C_1$	$C_2$	$d_1$	$d_2$	$g$	$K_0$
1	.03175	.00031	.00395	.00592	-.2775	67.	80	90	.7558	.145	35.00	.485	.7775	.00898
2	.01100	.00045	.00142	.00316	-.2775	81.	117	133	.7855	.137	36.00	.485	.6850	.00700
3	.02712	.00044	.00331	.00454	-.1487	65.	118	123	.8967	.102	183.75	.002	.6250	.00667
4	.03100	.00026	.00620	.00300	-.2400	62.	140	146	.8290	.115	29.00	.500	.5800	.00740
5	.03325	.00049	.02591	.00395	-.2400	41.	78	148	.8028	.126	35.75	.455	.6550	.00770
6	.03250	.00031	.00841	.00289	-.2212	60.	105	135	.7923	.131	38.75	.500	.6699	.00834
$p'$	.01003	.00058	.00290	.00300	-.2550	74	110.	157.	.7980	.127	39.50	.485	.6850	.00875
$\Delta \bar{p}$	.0178	.00019	.00503	.00091	.0209	11.0	4.0	28.6	.0134	.0008	20.2	.080	.0196	.00114
$\Delta p\sqrt{6}$	.0436	.00047	.01230	.00223	.0510	27.0	9.8	68.1	.0328	.0019	49.5	.196	.0480	.00279
$p$	.02777	.00038	.00820	.00391	-.2341	63.	106.	129.	.8104	.126	59.7	.405	.6654	.00768
$S_a$	.0085	.00010	.00901	.00117	.0475	12.9	24.0	21.0	.0508	.016	60.8	.198	.0653	.00085
$t_0$	8.13	4.65	1.37	1.01	1.08	2.09	4.08	3.13	0.65	0.12	0.81	0.99	0.74	3.29
														0.53
Evaluation	Parameters													
	$A_1$	$A_2$	$\beta$	$x$	$\alpha$	$g$	$R$	$C_1$	$C_2$	$D_1$	$D_2$	$K_0$	$K_m$	$Z$
1	.00496	.000253	.025	55.35	.00636	.546	.2770	.647	.063	.59.69	.717	2.640	164.38	274.0
2	.00506	.000268	.028	65.02	.00687	.541	.3037	.486	.103	23.32	.420	2.160	125.95	274.0
3	.00545	.000261	.030	66.75	.00724	.647	.1570	.666	.046	47.93	.527	2.520	168.74	274.0
4	.00461	.000264	.027	59.83	.00610	.520	.3554	.666	.053	50.70	.660	2.160	147.49	274.0
5	.00495	.000338	.026	49.21	.00790	.559	.1762	.786	.049	48.94	.660	3.195	143.90	290.0
6	.00486	.000365	.032	61.44	.00778	.541	.3003	.666	.070	48.63	.526	2.179	115.78	290.0
$p'$	.00422	.000265	.025	56.41	.00630	.525	.2962	.606	.070	38.75	.600	2.400	110.94	273.5
$\Delta \bar{p}$	.00076	.000026	.003	3.19	.00074	.046	.0346	.047	.006	7.79	.015	0.076	33.43	5.5
$\Delta p\sqrt{6}$	.00186	.000064	.0073	7.80	.00181	.113	.0850	.115	.0147	19.10	.0183	0.086	82.00	13.5
$p$	.00498	.000291	.028	59.60	.00704	.571	.2616	.653	.064	46.54	.585	2.496	144.37	279.00
$S_a$	.00027	.000048	.0026	6.48	.000735	.046	.0809	.067	.021	12.2	.114	.408	20.00	8.25
$t_0$	6.9	1.3	2.8	1.2	2.4	2.4	1.05	1.71	0.7	1.56	0.16	0.456	4.0	1.6

Table 6. Result of the test on significance of parameter values on the basis of eq. (39)

Evaluation	Parameters							
	$A_1$ $10^{-1}, \text{day}^{-1}$	$A_2$ $\text{cm}^{-1}, 10^{-1}, \text{day}^{-1}$	$\alpha$ cm	$\beta$ $\text{cm}^{-1}$	$K_m$ $\text{m}^1/3, \text{sec}^{-1}$	$Z$ cm	$f$ $\text{cm}^{-m}$	$m$ —
1	.00260	.000275	31.52	.066	50.	7.50	.2280	.097
2	.00227	.000330	36.25	.065	50.	11.56	.1500	.008
3	.00450	.000285	31.25	.066	50.	11.56	.2150	.043
4	.00477	.000345	26.25	.059	50.	16.50	.2490	.010
5	.00322	.000377	35.62	.063	50.	13.44	.1530	.012
6	.00490	.000302	31.25	.060	50.	13.44	.1900	.003
p'	.00304	.000315	31.25	.062	50	11.56	.1940	.012
p	.00371	.000319	31.98	.063	50.	12.33	.1975	.029
$\Delta P$	.00067	.000004	.73	.001	.	.77	.0035	.017
$\Delta P/\sqrt{6}$	.00164	.0000098	1.79	.0024	.	1.89	.0086	.042
Sd	.00116	.000039	3.63	.0031	.	2.97	.0404	.036
$t_0$	1.41	0.25	0.49	0.78	.	0.64	0.2	1.165



model could be used. The models' construction as well as the univariate minimizing routine would have to be adapted to this purpose. Perhaps the introduction of some new principle in the technique of minimization would make possible a more efficient evaluation of parameter values.

It has to be recognized that lack of reliability of parameter values in principle means that they lose applicability when a single parameter is torn from its context in the model. So for instance simulating a runoff record with a lower value of some drainage constant than was evaluated cannot confidently be done, nor can a new model part be added when previously optimized parameter values are maintained. Of course these restrictions are relative and depend on the confidence limits used, a matter that can be decided by personal preference and by experience in a particular line of work.

#### *Checking the predicting ability of a model*

A most interesting aspect of hydrological models is the predicting ability of the model as a whole. The most severe test on this would be for instance to check a sequence of computed daily runoff values, generated with the model, with the same sequence of measured runoff. If the sequence is very long a comparison between distributions of measured and computed runoff in the same period would give a convenient conspectus. The test should not include the data records used for parameter evaluation, unless they constitute only a small part of the test data.

The opportunity for a test presented itself in the area where all data used in this paper were collected. The Service for Water Management of the Ministry of Rijkswaterstaat started runoff measurements in the area in 1951 and presented frequency distributions of runoff for 5 watersheds, based on observations in seven winter seasons from 1 November till 1 April over the years 1951 up to and including 1958 (Tromp, 1958).

For three of these watersheds these frequency distributions were derived from runoff records generated with the model of fig. 6. To that purpose parameter values in the model were evaluated from data records between September 25, 1952 and June 30, 1953. In this period, of about the same length as that used for the evaluation of the parameter values in table 2, runoff did not show the peaks that occurred in the years 1951 up to and including 1958.

Now the parameter values were applied to records of precipitation and open water evaporation of these years. For the first day of the sequence best estimates of groundwater level, runoff and water level in the channel had to be made. Effects of errors in these estimates will disappear after a few days and do not have a perceptible influence on the frequency distribution of computed runoff.

To let the calculation continue from day to day the missing record of the water level  $S$  in the channel had to be generated.  $S$  is calculated as

$$\log S_i = a + b \log R_i \quad (40)$$

where  $a$  and  $b$  are derived from the data records of  $S$  and  $R$  used for parameter evaluation. When  $S$  and  $R$  are plotted on logarithmic scales,  $a$  is the intercept on the  $S$ -scale for  $R = 1 \text{ mm} \cdot \text{day}^{-1}$  and  $b$  is the difference between  $a$  and the intercept for  $R = 10 \text{ mm} \cdot \text{day}^{-1}$ .

In fig. 8 the three exceedance frequencies of calculated and measured runoff are compared. There is fair agreement which also shows when calculated and measured

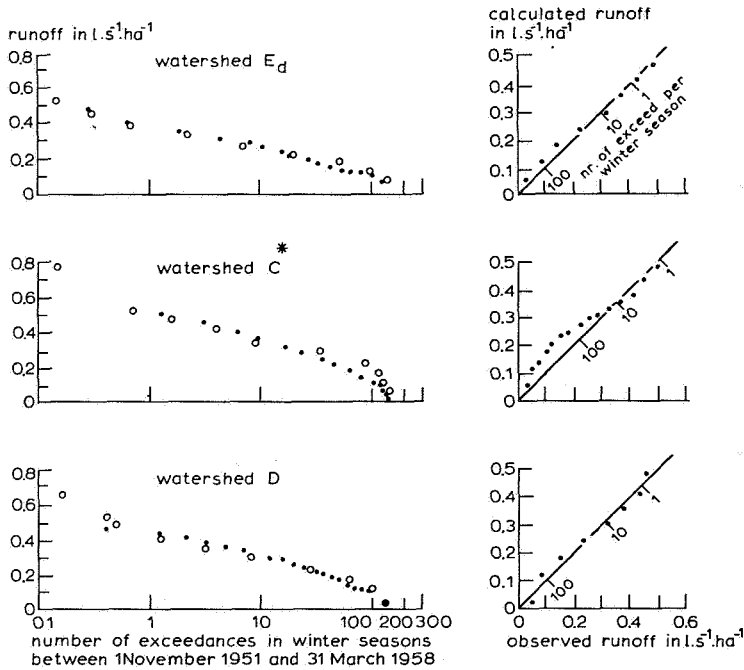


Fig. 8. Comparison of the exceedance frequencies of calculated and measured runoff of three watersheds.

runoff, exceeded with the same frequency, are plotted against each other. There is clearly some systematic deviation between measured and calculated runoff which in watershed C\* becomes rather large.

When the standard deviations between calculated and measured runoff with the same exceedance frequencies are computed ( $S^*$  in table 7), they show a trend with the efficiency coefficient  $R^2$  according eq. (33), giving an indication of the fit to the data records used for parameter evaluation.

Table 7. The better the model fits the data records used for evaluation of parameter values, the better the predicting ability of the model

Watershed	$S^*$ in $\text{l.s}^{-1} \cdot \text{ha}^{-1}$	$R^2$
$E_d$	0.025	0.95
$C^*$	0.055	0.75
$D$	0.027	0.98

## CONCLUSIONS

The gauging of runoff will generally have the object to deduce from data records some characteristics about runoff distribution. This may require an appreciable amount of time because a certain runoff occurring with a time interval of 10 years may very well show up in a gauging record of one year but just as well only once in a gauging record of 15 years. Therefore gauging has to be kept up during a considerably longer time than the time interval one is interested in. This easily will give organizational and financial problems, so the evolving methods to generate synthetic sequences of daily runoff values which can be used to estimate the  $T$ -year runoff from a short recording period must be favoured. It has been shown that a water balance model provides the possibilities for predicting exceedance frequencies of daily runoff with reasonable accuracy, the latter probably depending on data quality. In fig. 9 examples are given of frequency distributions of calculated daily runoff values for the 12 watersheds involved in this study. In the same way it would be possible to construct a water balance model with which to obtain frequency distributions of runoff peaks, which are perhaps of greater practical interest than daily values.

As there must be made reservations with respect to parameter values obtained by the minimization of one error function, it should be considered whether the best policy in water balance modelling would not be a separation of purposes, for instance:

- the construction of the smallest possible models for certain purposes as for instance for the prediction of runoff, without much concern about interpretation of parameter values;
- the construction of models which reflect the physical reality as closely as possible with adequate criteria of fit and minimizing routines that match the demand for accurate values of parameters with physical significance.

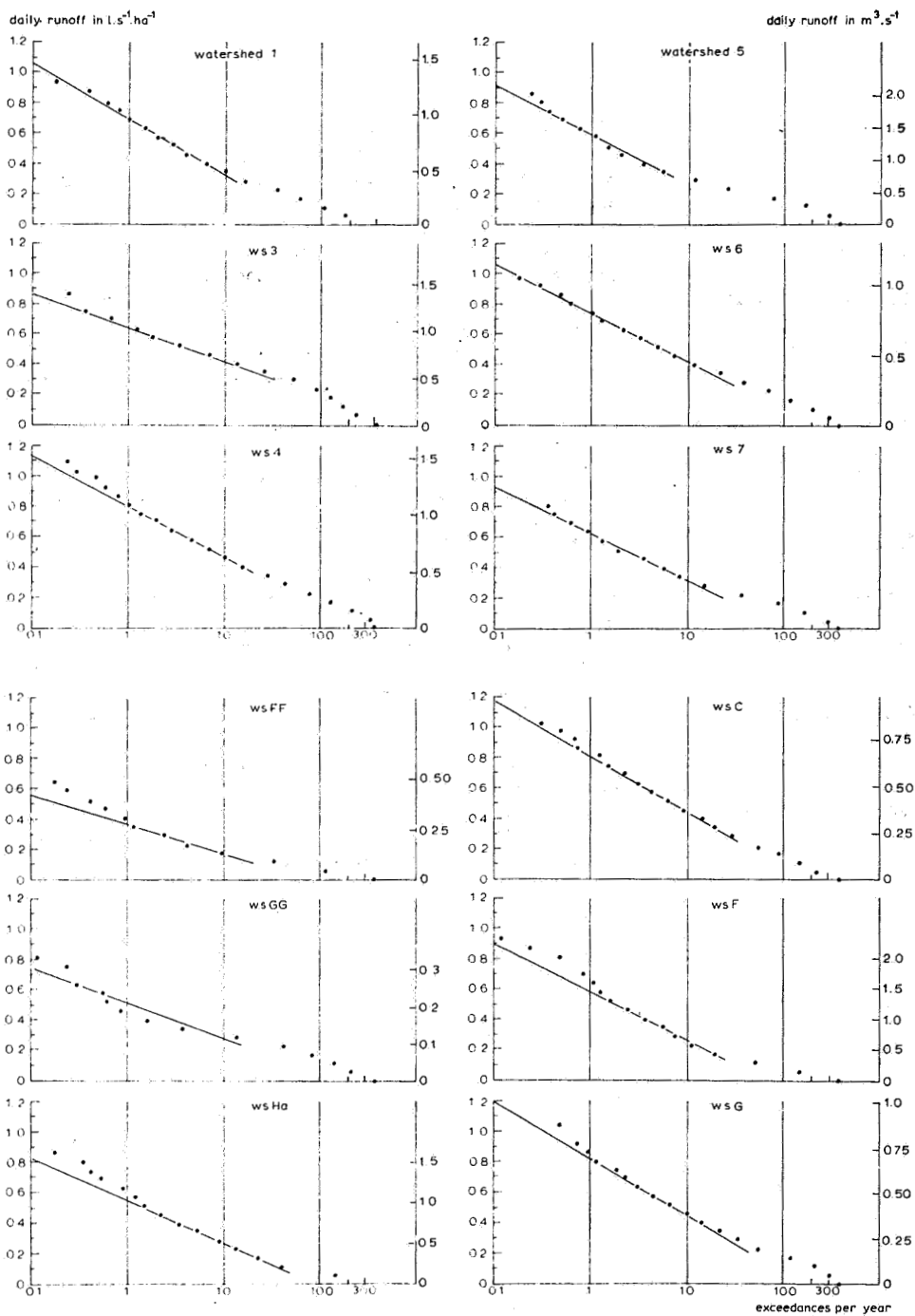


Fig. 9. Examples of frequency distributions of calculated daily runoff values for 12 watersheds.

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