COMMISSIE VOOR HYDROLOGISCH ONDERZOEK TNO
COMMITTEE FOR HYDROLOGICAL RESEARCH TNO

VERSLAGEN EN MEDEDELINGEN No. 26
PROCEEDINGS AND INFORMATIONS No. 26

RESEARCH ON POSSIBLE CHANGES IN THE DISTRIBUTION OF SALINE SEEPAGE IN THE NETHERLANDS

RESEARCH ON POSSIBLE CHANGES IN THE DISTRIBUTION OF SALINE SEEPAGE IN THE NETHERLANDS

COPYRIGHT © BY CENTRAL ORGANIZATION FOR APPLIED SCIENTIFIC RESEARCH IN THE NETHERLANDS TNO, 1980

COMMISSIE VOOR HYDROLOGISCH ONDERZOEK TNO
COMMITTEE FOR HYDROLOGICAL RESEARCH TNO

Verslagen en Mededelingen No. 26
Proceedings and Informations No. 26

RESEARCH ON POSSIBLE CHANGES IN THE DISTRIBUTION OF SALINE SEEPAGE IN THE NETHERLANDS

PROCEEDINGS OF
TECHNICAL MEETING 36
(March 1979)

VERSL. MEDED. COMM. HYDROL. ONDERZ. TNO 26 – THE HAGUE – 1980

CONTENTS

AUTHORS

Prof. dr. ir. J.C. VAN DAM	Delft University of Technology, Delft
Dr. ir. J. WESSELING	Institute for Land and Water Management Research, Wageningen
Ir. W.H.C. TEN HOORN	Delft University of Technology, Delft
Dr. ir. P. VAN DER VEER	Rijkswaterstaat, Data Processing Division, Rijswijk
D. PEREBOOM*	Delft University of Technology, Delft
Ir. G.L. MOLENKAMP	Delft University of Technology, Delft
J. SCHONEVELD*	Delft University of Technology, Delft
Ir. R. AWATER	Rijkswaterstaat, Data Processing Division, Rijswijk
Ir. J.D. LEENEN	Agricultural University, Department of Hydraulics and Catchment Hydrology, Wageningen
Ir. C.R. MEINARDI	National Institute for Water Supply, Leidschendam
Ir. A.N.M. OBDAM	National Institute for Water Supply, Leidschendam
F.C. VAN GEER*	Delft University of Technology, Delft

* Graduate student majoring in hydrology.

INTRODUCTION AND PROBLEM FORMULATION

J.C. VAN DAM

SUMMARY

The paper gives an introduction to the problem of saline seepage in the Netherlands and puts it in a prospect for the future. A review is given of the state of the art, i.e. the knowledge at present available on the subject of groundwater flow with different densities. The ongoing activities, both national and international, are briefly mentioned. Finally the program of the meeting is given and the speakers are introduced.

1. INTRODUCTION

As the title on this Technical Meeting indicates, the subject is closely related to conditions in the Netherlands. The hydrological conditions in the Netherlands, especially in the northern and western provinces are very peculiar indeed. This is not only due to the geological history of the Netherlands, located at the downstream end of the rivers Rhine and Meuse, so that it was often subject to transgressions of the sea. Equally important for the explanation of the present situation – in terms of salinity distribution of the groundwater, groundwater flow and seepage – is the influence of men. Men's activities are diking (since the early Middle Ages), drainage, reclamation of lakes (as early as in the 17th century (Van Veen, 1962)) closure of tidal inlets, thus creating fresh water lakes, reclamation of large parts of the former Zuyderzee (in the present century) and pumping of groundwater for various purposes.

The present situation is reasonably well known from borings, geo-electrical prospecting (Van Dam and Meulenkamp, 1967), observations of groundwaterlevels, piezometric levels and physical and chemical analysis of groundwater in observation wells, electrical resistivities measured with permanent electrode systems (salinity watchers) installed in the underground, water- and salt-balance studies of the surface water in polders.

Meinardi (1973), Volker and several other authors (Volker, 1961) have tried to explain the present salinity distribution in the groundwater by means of flow processes caused by infiltration of fresh water, transgression and regression of the sea, taking into account the differences in density, diffusion, dispersion, permanent freezing of the soil during ice ages, etc.

2. PROBLEM FORMULATION

Remarkably so far little has been published about the prospects for the future. The present situation is certainly not an ultimate one. The various men's activities, undertaken at different times in the past have not yet fully worked out. There is also an interaction between the various influences. So, even in the absence an any future men's activity, there will be further changes in the distribution of the salinity of the groundwater as well as of the seepage water in the deeper polders. The present situation can become worse or better. It is also remarkable that at this point there is so much misunderstanding and unacquaintedness. This appears for instance from frequently heard statements that the seepage of saline water should be continuously increasing.

An attempt to describe, in a simplified and schematized way, the influence of the reclamation of a single polder was made by Van Dam (1976). The conclusions of that paper were that there are various aspects of the problem of seepage that were not yet known; and that the type of models used offer the possibility of forecasting of what may happen under given circumstances. Further elaboration of this type of research forms a challenge to both the author and his coworkers for further study and verification in more extended models.

It is obvious that before starting any new activity that may affect the groundwater regime one should carefully study how it interacts with the ongoing processes and what consequences it will have, especially in the long run. One might even think of adaption of plans, measures to be taken or even special works to be realized to obtain a desired situation in the near or far future. Before a solution can be given for such a problem the knowledge of the actual processes occurring has to be extended so as to be able to collect the proper data and to draw the right conclusions on the consequences of any future activity.

Examples of future developments are:

a. the future Groundwater Act, that will cover the subjects now regulated by the Provincial Groundwater Regulations. The Act will probably prescribe the provinces to prepare provincial groundwater plans, subject to the approval by the state;
b. possible future abstraction of brackish and saline groundwater for the production of fresh water by desalination for domestic and industrial use;
c. possible reclamation of the last and greatest polder in the former Zuyderzee, the Markerwaard.

So far the importance and interest of the subject in the Netherlands. The problem of salt water intrusion in groundwater has also drawn international attention. The so-called SWIM-meetings (SWIM = Salt Water Intrusion Meeting), first held in Hannover 1967 at the initiative of the late prof. dr. W. Richter and since at regular intervals in the countries surrounding the North Sea (see the proceedings of the SWIM meetings at Vogelenzang (1970), Copenhagen (1972), Ghent (1974) and Medmenham (1977)) satisfy the need for regional exchange and cooperation between some 20-40 experts from these countries. This activity takes place in the framework of the International Hydrological Programme (IHP), formerly the International Hydrological Decade, (IHD). The national IHP-committee's have, in turn, kindly provided the required facilities for these meetings and the proceedings thereof.

It is also worthwhile to note that in two out of the eight Scientific Projects of the IHP attention is paid to the subject of the present meeting, viz.:

project 5: Investigations of the hydrological and ecological effects of men's activities and their assessment.

Concerned with the influence ofland use practices, the effects of hydraulic works on channel erosion and sedimentation, changes in the hydrological regime resulting from irrigation, *estimation of changes in the salt-fresh water balance in coastal areas resulting from men's activities,* study of the hydrological consequences of the exploitation of new energy sources and of the possibility of establishing hydro-ecological indices for evaluation of water projects.

project 8: Long-term prediction of groundwater regime taking into account human activities.

Concerned with study of the *response of aquifers to heavy pumping and exploitation and the long-term prediction of reserves available, determination of the limits of rational exploitation and capabilities of models for the prediction of quantitative and qualitative changes in the groundwater regime.*

3. STATE OF THE ART

The aim of the Technical Meetings of the Committee for Hydrological Research TNO is to draw attention to topical problems, originating from the water management practice, and to new developments in practically oriented hydrological research. The meetings and their proceedings serve as tools in the transfer of research results – facts or methods – to practical implementation. During the meeting of the members of the Committee for Hydrological Research (CHO) in October 1978 the problem of salt water intrusion has been indentified as one that deserves continuous attention.

Before describing the recent and present developments in the Netherlands it is worthwhile to pay attention to the international literature on this subject. A reference list is added at the end of this paper. The papers show a great variety in scope and approach. An annotated bibliography prepared by Van Geer, a graduate student majoring in hydrology, is reproduced as Appendix A (see page 180).

The first contribution to the theory of fresh water – salt water relationships dates back to 1889 when the Dutchman Badon Ghijben formulated his wellknown principle. The paper of Badon Ghijben (1889), in Dutch, remained rather unknown. Twelve years later, the German Herzberg (1901) independantly formulated the same principle based on his observations of the fresh water lenses on the German islands off the northern coastline.

In 1940 Edelman applied this principle and made calculations of the shape of the fresh water lens in sanddunes of the Amsterdam waterworks (Report 1940). He wrote the basic equations describing the flows of fresh and saline water, separated by a sharp interface, in the semi-confined aquifer. The dimensions of his problem were such that the flowlines in the aquifer were almost horizontal. So the Dupuit-approach of one-dimensional flow was justified. The equations were solved by approximation methods.

In the sixties Venhuizen (see proceedings SWIM-meeting Ghent, 1974) made analytical calculations of the shape of the fresh water lens in the same sanddunes. As in Edelman's calculations the fresh water lens in the semiconfined aquifer was fed by infiltration and below the fresh water lens there was an inland flow of saline water towards the deep polders behind the sanddunes. He also assumed a sharp interface and described the flow in the aquifer as horizontal flow.

Another Dutch contribution were De Josselin de Jong's papers (1960 and 1977) on the vortex theory. He described two-dimensional flow patterns in water of varying densities. His theory is now applied by Haitjema (1977), (and see proceedings SWIM-meeting at Medmenham, 1977) and extended for three-dimensional cases.

Strack (1972 and 1973) described two-dimensional flow systems with particular emphasis on abstraction of groundwater by drains and the effect of upconing in two-phase systems.

Bruggeman developed an analytical solution for moving fronts in two- and three-dimensional groundwater flow (see proceedings SWIM-meetings Copenhagen, 1972 and Ghent,1974), when ignoring the differences in density of water of different chemical composition.

Van der Veer (1977[a,b,c]) (and see proceedings SWIM-meeting Medmenham, 1977), gave analytical solutions for some special interface problems. In his Ph.D.-thesis Van der Veer (1978) developed a boundary element method, the Analytical Function Method (AFM), to describe transient two-dimensional flow patterns, comprising also water of different densities, separated by sharp interfaces.

In this context mention should also be made of various applications of the powerful finite differences methods and the finite element methods.

The recent report of the Ad-hoc group on groundwater models and numerical computer software, installed by the CHO, gives an inventory of groundwater models nowadays available in the Netherlands (Commissie voor Hydrologisch Onderzoek TNO, 1978). One of the conclusions is: "The number of models with which multi-phase problems (e.g. fresh water – salt water) can be calculated is small, whereas mostly a simple methodology is applied'.

This conclusion led to the following recommendation: 'Since fresh water – salt water problems occur in large parts of the Netherlands further development of relevant models is important".

The large scale problems in the lower parts of the Netherlands are of great practical importance. Therefore they get serious and continuous attention of water management authorities and services.

The hydrology and water management group of the Civil Engineering Department of Delft University of Technology recognized that these problems form an attractive object for research and training (see Appendix B, page 203).

Graduate students are taking part in the research programme. The purpose of it is to develop suitable models to solve practical problems.

The real problem is very complicated; it is three-dimensional and transient. The aquifer systems, characterized by their geohydrological constants, are complicated. Even so the boundary conditions (as e.g. the polder levels). The water is of gradually varying density. Therefore we are still far from realizing the dream of a universal and comprehensive model, if ever realisable.

Nevertheless the Small Committee of the CHO judged that a glance in the kitchen would be instructive for the users and stimulating for the researchers.

4. PROGRAMME OF THE MEETING

After this introduction Wesseling describes and quantifies the present situation in terms of salinity distribution and seepage of saline groundwater in the western part of the Netherlands. Next Ten Hoorn briefly presents the physical fundamentals. Then Van der Veer reviews possible calculation methods nowadays in use. In the next four papers Pereboom, Molenkamp, Schoneveld and Awater successivily present results obtained with different methods and for different problems. Pereboom deals with the results obtained for steady one-dimensional flow in two semi-confined aquifers by application of the finite differences method. Molenkamp deals with the results obtained for steady, radial flow in one semi-confined aquifer by application of an analytical method. Schoneveld deals with the results obtained for transient one-dimensional flow and for steady flow in two horizontal dimensions in one semi-confined aquifer by application of the finite elements method. Awater deals with transient two-dimensional flow in a vertical profile in one semi-confined aquifer by application of a boundary element method. Because both the problems dealt with and the methods applied differ, the results obtained are not directly comparable.

These four papers serve merely as an illustration of the present state of the research on large scale two-phase flows in semi-confined aquifers for the prediction of possible changes in the distribution of saline seepage in the Netherlands.

For actual, practical applications the results must be verified experimentally. This goes with great problems because:

- the geographical reality is often much more complicated than assumed in the highly schematized calculations;
- various causes dating back to different instants in the past are still active and mutually influence their effects;
- future and ultimate situations cannot be measured at present and situations of the past are often insufficiently known by lack of data;

Leenen presents the results of a geo-electrical survey in the deep polder Groot Mijdrecht, which is subject to seepage of groundwater that is at present brackish.

Next Meinardi presents his views on what data are required for proper verification.

The meeting closes with conclusions and recommendations based on these conclusions. The recommendations are intended for authorities responsible for (ground)water management, services responsible for data collection as well as for research institutes.

REFERENCES

A. References on salt water intrusion and relevant calculation methods

BADON GHIJBEN, W. and J. DRABBE, (1889) – Nota in verband met de voorgenomen putboring nabij Amsterdam. (Notes on the probable results of the proposed well drilling near Amsterdam.) Tijdschr. Kon. Inst. v. Ingenieurs, pp. 8-22.

BEAR, J. (1960) – The transition zone between fresh and salt waters in coastal aquifers. Ph.D.-thesis University of California, Berkeley (California).

BEAR, J. and G. DAGAN (1964) – Moving interface in coastal aquifers. Proceedings ASCE, Hydraulics Division, No. HY 4, Vol. 90: 193-215.

BEAR, J. (1970) – Two-liquid flows in porous media. Advances in Hydroscience, Vol. 6: 142-252. Academic Press, New York.

BEAR, J. (1972) – On the flow of two immiscible fluids in fractured porous media. Fundamentals of transport phenomena in porous media, Elsevier, Amsterdam, pp. 177-202.

BEAR, J. and Y. MUALEM (1974) – The shape of the interface in steady flow in a stratified aquifer. Water Resources Research, Vol. 10: 1207-1215.

BEAR, J. and G. DAGAN (1964) – Some exact solutions of interface problems by means of the hodograph method. Journ. of Geophysical Research, Vol. 69, No. 8: 1563-1572.

BONNET, M. and J.P. SAUTY (1975) – Un modèle simplifié pour la simulation des nappes avec intrusion saline. Proceedings of the Bratislava Symposium, September 1975, IAHS Publication, No. 115.

COLLINS, M.A. (1976) – The extended Boussinesq Problem. Water Resources Research, Vol. 12, No. 1: 54-56.

COOPER, H.H. Jr., F.A. KOHOUT, H.R. HENRY and R.E. GLOVER (1964) – Sea water in coastal aquifers. U.S. Geological Survey, Water Supply Paper 1613C, Washington DC, pp. 84.

DAM, J.C. VAN, (1976) – Partial depletion of saline groundwater by seepage. Journ. of Hydrology, Vol. 29, No. 3/4: 315-339.

DAM, J.C. VAN, (1977) – Determination of horizontal and vertical groundwater flow from piezometric levels observed in groundwater of varied densities. Delft Progress Report, Vol. 3: 19-34.

DAM, J.C. VAN and W.H.C. TEN HOORN, (1977) – Effects of urbanization on the distribution of fresh and saline groundwater in the Netherlands. Proceedings of the Amsterdam Symposium, October 1977, IAHS Publication, No. 123: 488-499.

DAM, J.C. VAN and J.J. MEULENKAMP (1967) – Some results of the geo-electrical resistivity method in groundwater investigations in the Netherlands. Geophysical Prospecting, Vol. XV, No. 1.

GLOVER, R.E. (1959) – The pattern of fresh-water flow in a coastal aquifer. Journ. of Geophysical Research, Vol. 64, No. 4: 457-459.

HAITJEMA, H.M. (1977) – Numerical application of vortices to multiple fluid flow in porous media. Delft Progress Report, Vol. 2: 237-248.

HANTUSH, M.S. (1968) – Unsteady movement of fresh water in thick unconfined saline aquifers. Bulletin of the International Association of Scientific Hydrology, XIII, No. 1: 40-60.

HENRY, H.R. (1959) – Salt intrusion into fresh-water aquifers. Journ. of Geophysical Research, Vol. 64, No. 11: 1911-1919.

HERZBERG, (1901) – Die Wasserversorgung einiger Nordseebäder. Journal für Gasbeleuchtung und Wasserversorgung, XLIV Jahrgang, No. 44: 815-819 and No. 45: 842-844.

JOSSELIN DE JONG, G. DE (1960) – Singularity distributions for the analysis of multiple-fluid flow through porous media. Journ. of Geophysical Research, Vol. 65, No. 11: 3739-3758.

JOSSELIN DE JONG, G. DE (1965) – A many-valued hodograph in an interface problem. Water Resources Research, Vol. 1, No. 4: 543-555.

JOSSELIN DE JONG, G. DE, (1977) – Review of vortex theory for multiple fluid flow. Delft Progress Report, Vol. 2: 225-236.

KISHI, Y. and Y. FUKUO (1977) – Studies on salinization of groundwater: I. Theoretical consideration of the three-dimensional movement of the salt water interface caused by the pumpage of confined groundwater in fan-shaped alluvium. Journ. of Hydrology, Vol. 35: 1-30.

KOHOUT, F.A. and H. KLEIN (1967) – Effect of pulse recharge on the zone of diffusion in the Biscayne aquifer. Extract of publication No. 72, Symposium of Haifa, 1967. IASH.

KONIKOW, L.F. (1977) – Modeling Chloride Movement in the Alluvial Aquifer at the Rocky Mountain Arsenal Colorado. U.S. Geological Survey, Water Supply Paper 2044, Washington DC.

LEE, CHUNG-HIAN and, RALPH TA-SHUN (1974) – On seawater encroachment in coastal aquifers. Water Resources Research, Vol. 10, No. 5: 1039-1043.

MEINARDI, C.R. (1973) – Het zoutvoorkomen in de ondergrond van de lage gedeelten van Nederland, H_2O 6/18:454-460.

PINDER. G.F. and H.H. COOPER, jr. (1970) – A numerical technique for calculating the transient position of the saltwater front. Water Resources Research, Vol. 6, No. 3: 875-882.

PINDER, G.F. and W.G. GRAY (1977) – Finite element simultation in surface and subsurface hydrology. (in particular pp. 169-183). Academic Press, New York.

PROCEEDINGS OF THE SALT WATER INTRUSION MEETINGS (SWIM) – a. Vogelenzang 1970 (Dutch National I.H.P. Committee) – b. Copenhagen 1972 (Danish National I.H.P. Committee) – c. Ghent 1974 (Belgian National I.H.P. Committee) – d. Medmenham 1977 (H.M.S.O.)

RAPPORT 1940, (EDELMAN et al.) – De watervoorziening van Amsterdam. Stadsdrukkerij van Amsterdam.

SEGOL, G. and G.F. PINDER (1976) – Transient simultation of saltwater intrusion in southeastern Florida. Water Resources Research, Vol. 12, No. 1: 65-70.

STICHTING POSTAKADEMIALE VORMING GEZONDHEIDSTECHNIEK (1970/1971) – Cursus zout grondwater in Nederland. Deel I. Vol. A en B.

STRACK, O.D.L. (1972) – Some Cases of Interface Flow towards Drains. Journ. of Engineering Mathematics, Vol. 6: 175-191.

STRACK, O.D.L. (1973) – Many-valuedness encountered in groundwater flow. Ph.D. thesis, Delft University of Technology.

STRACK, O.D.L. (1976) – A single-potential solution for regional interface problems in coastal aquifers. Water Resources Research, Vol. 12, No. 6: 1165-1174.

TODD, D.K. (1974) – Saltwater intrusion and its control. Journal A.W.W.A., Vol. 66, No. 3: 180-187.

VAPPICHA, V.N. and S.H. NAGARAJA (1976) – An appropriate solution for the transient interface in a coastal aquifer. Journ. of Hydrology, 31: 161-173.

VEER, P. VAN DER (1977[a]) – The pattern of fresh and salt water flow in a coastal aquifer. Delft Progress Report, Vol. 2: 137-142.

VEER, P. VAN DER (1977[b]) – Analytical solution for steady interface flow in coastal aquifer involving a phreatic surface with precipitation. Journ. of Hydrology, Vol. 34, No. 1/2: 1-11.

VEER, P. VAN DER (1977[c]) – Analytical solution for a two-fluid flow in a coastal aquifer involving a phreatic surface with precipitation. Journ. of Hydrology, Vol. 35: 271-278.

VEER, P. VAN DER (1978) – Calculation methods for two-dimensional groundwater flow. Ph.D. thesis, Delft University of Technology; also as: Rijkswaterstaat Communications No. 28, The Hague.

VERRUYT, A. (1968) – A note on the Ghijben-Herzberg formula. Bulletin of the IASH, XIII, 4-12/1968 pp. 43-46.

VERRUYT, A. (1969) – An interface problem with a source and a sink in the heavy fluid. Journ. of Hydrology, Vol. 8: 197-206.

VOLKER, A. (1961) – Source of brackish groundwater in pleistocene formations beneath the Dutch polderland. Economic Geology, Vol. 56: 1045-1057.

B. Other references

COMMISSIE VOOR HYDROLOGISCH ONDERZOEK TNO (CHO-TNO) (1978) – Verslag van de TNO-ad hoc groep Grondwatermodellen en computerprogrammatuur, Rapporten en Nota's, No. 2 (English version No. 2b).

VEEN, J. VAN (1962) – Dredge, drain, reclaim; the art of a nation. Martinus Nijhof, The Hague.

17

SALINE SEEPAGE IN THE NETHERLANDS, OCCURRENCE AND MAGNITUDE

J. WESSELING

SUMMARY

In order to explain the origin of saline seepage in the lower parts of the Netherlands an overall history of the human occupation of the country has been given. Next the geohydrological conditions have been reviewed shortly.

On the basis of the present hydrological situation the occurrence of saline seepage has been explained. Next some water balances of larger drainage districts have been considered in order to arrive at the magnitude of the seepage. It appears that the seepage in the mid-western part of the Netherlands is responsible for a salt load of about 200 000 tons of Cl^- per annum.

In order to remove this salt, together with other polluting agents from the surface water a total amount of some 450 million m^3 of river water is used annually to maintain the quality of the surface waters in this region.

1. INTRODUCTION

Topographically the Netherlands can be divided into two parts, namely the western and north-western relatively flat half with an elevation equal or below sealevel and an eastern and south-eastern more undulating and higher part (fig. 1).

With an average annual rainfall of about 750 mm and a yearly evapotranspiration of say 450 mm there is an excess of water during the winter periods.

Part of the drainage water from the higher regions is removed by gravity through a system of small natural rivers and brooks supplemented by a narrower system of man-made drainage ditches, furrows and pipes. The remainder part of the water flows as groundwater towards the larger rivers, the lower regions and towards the sea.

The lower half of the country is divided into numerous separate polders, each of them having its own embankment and its own drainage outlet.

The drainage water from the separate polders is sometimes transferred towards the North Sea through sluices that can let out water at low sea tides. From the major part of the polders the water is pumped into a system of natural rivers and lakes and manmade canals, known as the "boezem". Through this boezem the water is transported towards the larger rivers or the sea where it discharges its water by means of sluices or pumps. The polder and boezem system originated from the human occupation of the area. The history of the area is schematically given in figure 2 taken from Hellinga (1952).

Before occupation by man the area behind the dunes were drained by a system of broad slowly moving streams. Flood water from high river stages and from the sea could easily enter the area. The first activity of man to protect the area against flooding started about 1100 A.D. and consisted of throwing up embankments along the main rivers and

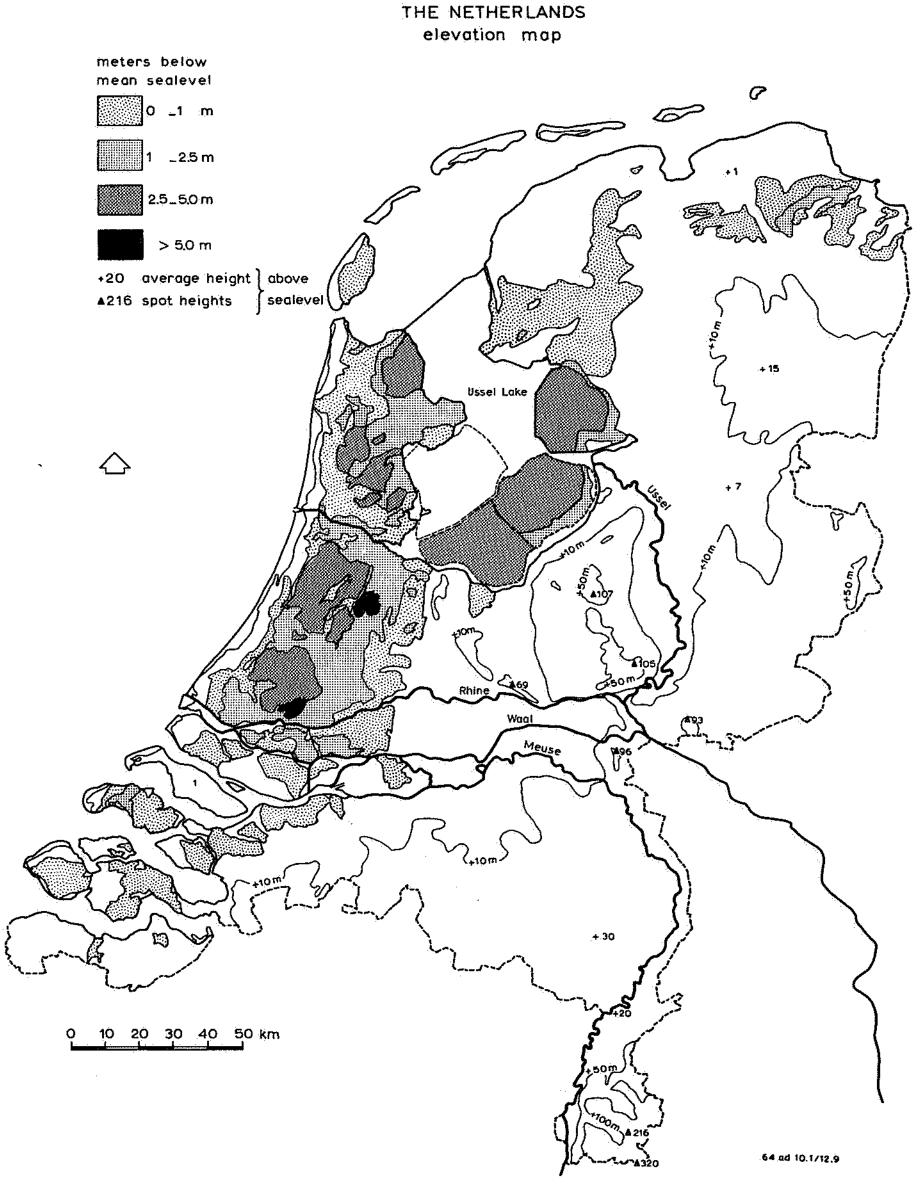
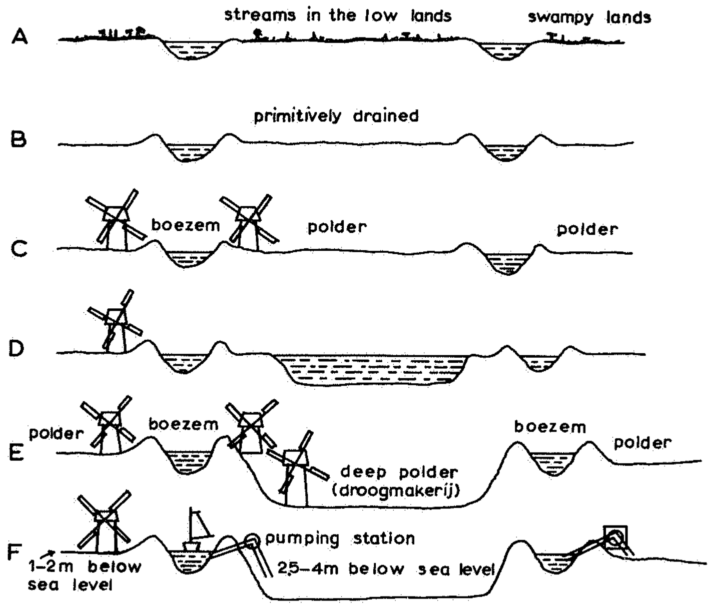


Fig. 1 Elevation map of the Netherlands.



A. Before occupation by man; B. After damming of the streams at their mouths and their embankment; separation of 'boezem' and 'polder' by small dikes; C. Subsidence of the peaty polder soils and pumping by windmilss; D. Digging out of some polders for peat making; E. After draining of the lake originating from peat making; F. Present situation

Fig. 2 Development of the Dutch Polder Region (after Hellinga, 1952).

damming up and building sluices at the outlets of smaller streams. Gradually smaller inland streams were embanked resulting into a pattern of small polders surrounded by dikes. With the invention of the windmill and the paddle wheel not only the water from the polders could be removed easier, but it became possible to reclaim larger and deeper lakes (so-called droogmakerijen) that partly originated from digging and drying peat for heating purposes. The use of steam followed by electrical and diesel engines finally lead to the reclamation of the deepest lakes (e.g. Haarlemmermeer) and the new IJssellake polders. What has been left from a swampy flat area with some natural streams and lakes is a pattern of smaller and larger polders, each having its own embankment and a con-

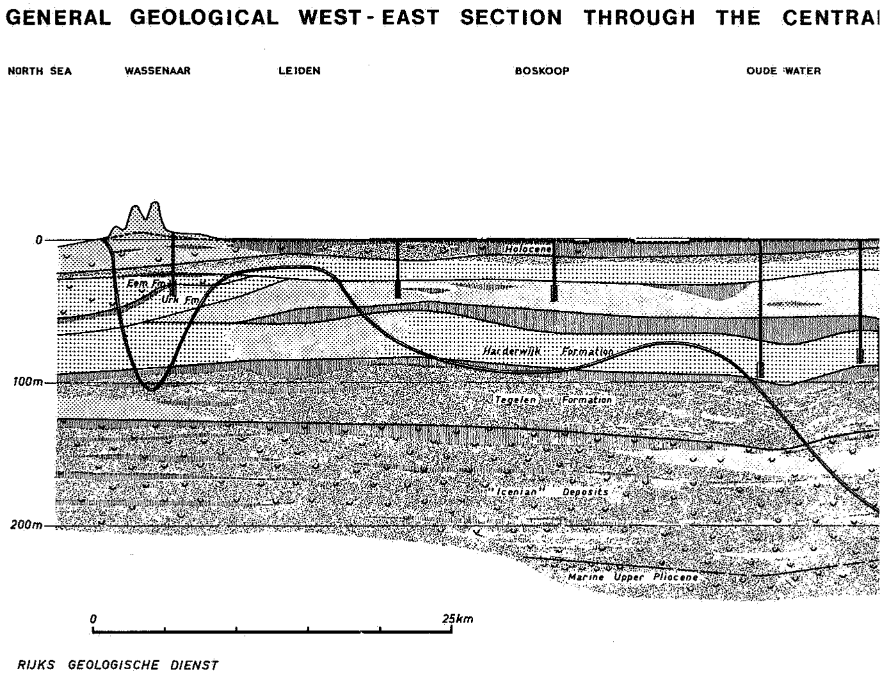
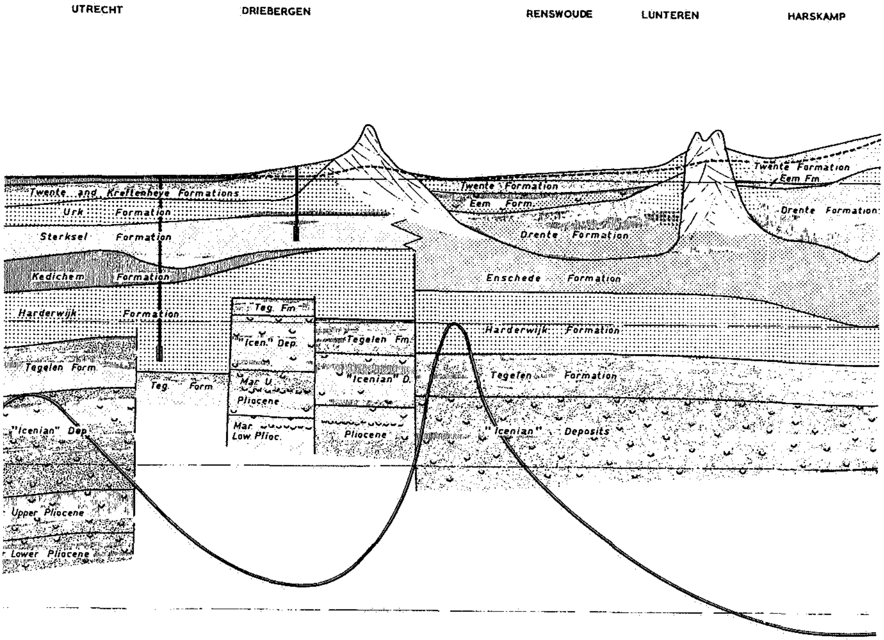


Fig. 3 Geological cross-section (West-East) through the Netherlands (Breeuwer and Jelgersma, 1973)

trolled waterlevel at a different elevation depending on the natural elevation of the surface soil type and land use.

Due to the control of the watertable in the polder area and its low elevation there is a more or less continuous flow of groundwater from the higher regions to the lower ones, the water appearing in the polder area as the so-called seepage. The intensity of this groundwater stream is highly dependent on the geohydrological conditions in the underlying soil profile, the thickness and permeability of the confining layers overlying the aquifer and last but not least on the water levels surrounding the area. Since the groundwater in the Dutch polder region generally is brackish or salt, the seepage is accompanied by a load of salt that appears in the open drainage system. Due to the excess of rainfall available in winter periods, salts accumulated in the soil during the drier summer period will be leached out easily, so that problems of salinization of the soil, common in drier irrigated areas, generally do not occur in the Dutch polders. Pumping of the brackish seepage water into the "boezem" system, however, causes a continuation of this system so that water present in it cannot be used for several purposes, like

OF THE NETHERLANDS



(Western part).

sprinkling in horticultural glasshouses, unless huge amounts of water are used to flush the system in order to keep the salinity at a reasonably low level.

The occurrence of saline seepage causes problems which are peculiar for alluvial valleys close to the sea under humid climatological conditions.

2. GEOHYDROLOGICAL CONDITIONS

Geologically the Netherlands is situated at the edge of the North Sea basin. Due to tectonic subsidence of this basin during the Tertiary and Quaternary huge amounts of sediments have been deposited. The base of the Quaternary lies in the north-western and western part of the country between 300 and 500 meters below the present surface. In the eastern and southern part the Tertiary sediments subcrop near the surface. In the utmost southern part also older Mesozoic and Paleozolic strata are found near or at the surface.

Figure 3 taken from Breeuwer and Jelgersma (1973) gives a geohydrological West-East cross-section through the central part of the Netherlands. The figure shows that the

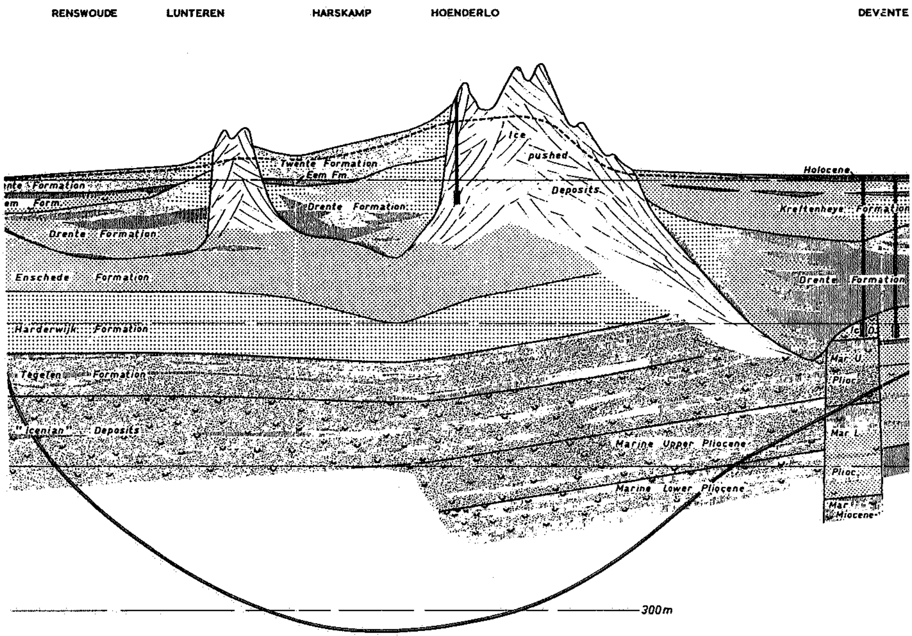
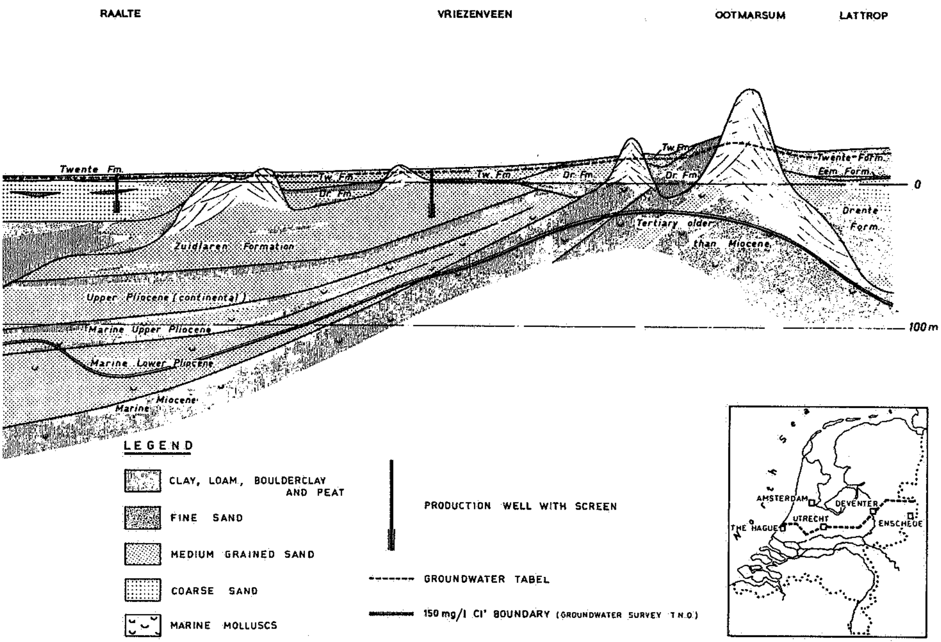


Fig. 3 Geological cross-section (West-East) through the Netherlands (Breeuwer and Jelgersma, 1973)

marine Miocene and Pliocene deposits are generally fine textured. Apart from irregularities at places where tectonic faults occur, these strata are rather continuous and have a nearly constant thickness. This does not hold for the younger strata. Due to a sequence of glacial, interglacial and interstadial periods, the sedimentation varied considerably both in place and in nature. In large parts of the country fluviatile sands were deposited by the rivers Rhine, Meuse, Ems and Weser. As a result of the advancing ice tongues during the Saalien Glaciation deep valleys and high ice-pushed ridges were developed. Depending on the degree of filling up of these valleys, they were flooded due to transgression of the sea during the Eemien Interglacial that followed the Saalien.

The same occured again during the Holocene transgression (Calais and Duinkerken), leaving strata of widely varying deposits. Figure 3 also gives the fresh-salt boundary of the groundwater. This line shows a very irregular pattern with a depth varying between some 10 meters in the western to a few hundred meters in the middle part of the country.

It is obvious that during periods of marine sedimentation the groundwater was saline. This explains why a fresh salt boundary should be in the Miocene and Pliocene strata.



(Eastern part).

During the Glacials the salt groundwater can have been pushed down considerably. Due to excess rainfall joining the groundwater the saline water under the ice-pushed ridges moved further downwards. The latter phenomenon can also explain the fresh water pockets below the Holocene sand dune ridges along the coast.

A hydrological map produced by Jelgersma and Visser (1972) clearly shows that fresh water pockets down to 100 meters depth are present below part of the dune area along the coast. In a 30 to 50 km wide strip along the coast the groundwater is saline. The pattern is, however, very irregular. Sometimes small areas with fresh groundwater are found while in other places saline water is overlying fresh water layers. The inverse, namely fresh water layers above saline ones, also occur. Geirnaert (1973) is of the opinion that the saline groundwater in the western part of the country originates from seawater that infiltrated in the fluviatile Pleistocene strata during transgressions. Part of the saline water should then have been replaced by fresh groundwater flowing down from the higher parts of the country (the ice-pushed ridges). Mazure (1940) and Volker (1961) could prove that the diffusion theory could explain the salt profile below the bottom of the former

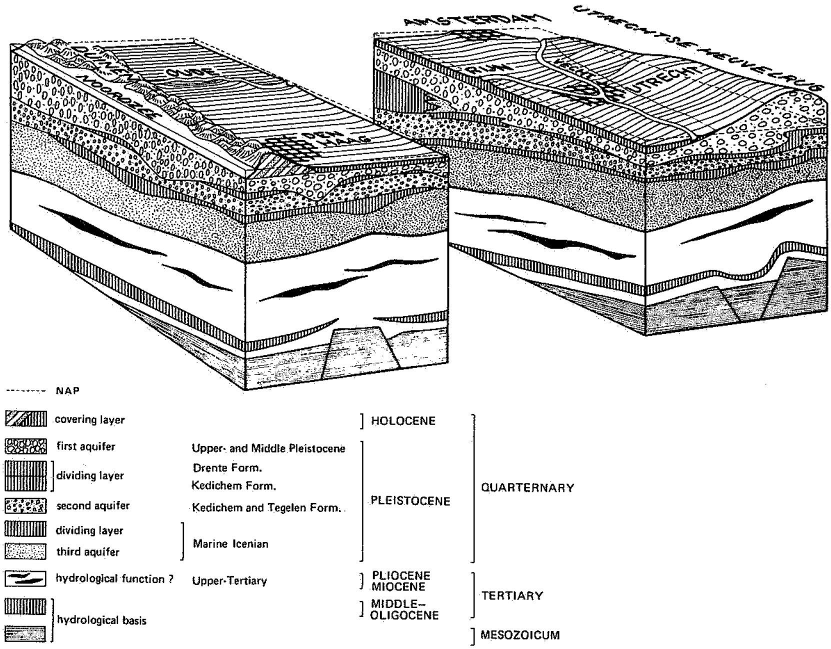


Fig. 4 Schematical diagnosis of the subsoil in the mid-western part of the Netherlands (ICW, 1976).

Zuyderzee. Meinardi (1973), however, rejects the displacement of salt due to flow of fresh groundwater because this process is too slow. For the same reason he rejects the opinion that part of the saline groundwater is originating from inflow of seawater into the aquifer in the western part. Both he and Wit (ICW, 1976) show that due to the reclamation of for instance the Haarlemmermeer, now about 120 years ago, the seawater could hardly have been moved more than 2 km inland despite a large gradient in the groundwater caused by a polder level of 5 m -NAP. Apart from diffusion Meinardi mentions the dispersion at the boundary between moving fresh and salt groundwater bodies as an important reason for the present situation. The moving of groundwater then should originate from the natural flow from the higher middle part to the lower parts of the country and from the artificial gradients in the lower part caused by reclaiming the polders and maintaining different polder levels in that area. In order to describe the possibilities of groundwater flow in the low western part of the country a picture taken from Pomper (ICW, 1976) can be used (see fig. 4). This picture describes schematically the profile in this region. Although the Mesozoic rock and the Oligocene heavy clay strata form in fact the hydrological base of the aquifer, the Miocene and Pliocene sediments can be considered as relatively impermeable, so that the real aquifer base is formed by the

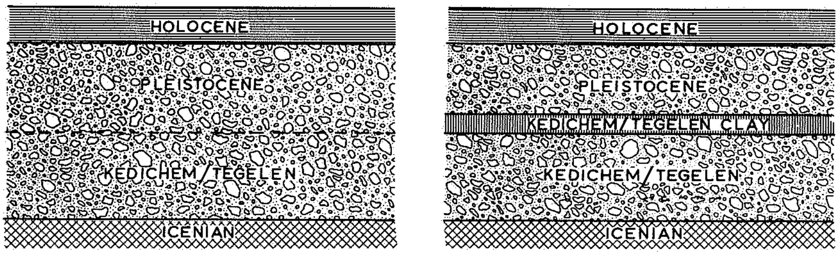


Fig. 5 Hydrogeological schematization of the main groundwater aquifers.

top of these strata. The lowest aquifer then is formed by the Icenian deposits from which the upper part consists of layers of fine textured sands and clay. The medium aquifer is formed by medium coarse sands of the Formation of Kedichem/Tegelen. The upper part of these strata often consists of heavy clay or glacial loams of the Drente deposits. Above these strata there is an upper aquifer formed by coarse sands of Pleistocene origin which in turn is covered by a 0 to 20 meters thick Holocene layer of peat, clay and sands. From the Icenian deposits in the area not much is known, but in any case the lowest aquifer may be considered having little or no influence on the transfer of water in the aquifers above. So in fact, there are only two different aquifers. Since, however, the upper parts of the Kedichem/Tegelen Formation often have been removed by erosion, there may be a direct connection between the middle and upper aquifer. So in fact in certain parts of the area there is only one aquifer of importance. Both situations are schematically given in figure 5.

As compared to the other strata, the permeability of the Holocene covering strata is very small so the aquifers are semi-confined. The left hand situation in figure 4 is known in groundwater hydrology as the "Holland profile" since it is representative for large areas in the western part of the country.

3. THE SEEPAGE PROBLEM

As indicated in the introduction, the seepage problem originates from the low elevation and the fact that water levels in the polder area are controlled. In hydrological terms this means that the phreatic surface in the confining upper layer is maintained within certain boundaries. If the piezometric level of the groundwater in the aquifer is higher than the phreatic polder level upward seepage flow will occur. The intensity of this flow will depend on the differences in piezometric level and the resistance the flow meets in the confining layer.

Saline seepage in the polder region presents itself in the presence of high salt contents in the open water. The results of a survey of Snijders (1960) in the dry summer of 1959 is given in figure 6.

Fig. 6 Chloride contents of the open water in the summer of 1959 (Snijders, 1960).

Along the northern part of the coast the picture is similar, namely a mozaic pattern of strongly varying chloride contents. Concerning the northern area a survey of Bots et al (1978) gives similar results. The picture clearly shows the influence of the dune ridge, of the North Sea canal and of the close vicinity of the sea around the isles in the Delta area in the south-west.

In fact the difference between the areas south of Rotterdam and north of that city is

Table 1 Average yearly amounts of water transported through the boezem system of Rijnland and Delfland over the years 1960 to 1970.

		Rijnland	Delfland
Area		98 000 ha	29 900 ha
Content 'boezem'		80 $10^6 m^3$	5.5 $10^6 m^3$
Total inlet	summer	266 $10^6 m^3$	60 $10^6 m^3$
Total inlet	winter	105 $10^6 m^3$	107 $10^6 m^3$
Total outlet	summer	238 $10^6 m^3$	127 $10^6 m^3$
Total outlet	winter	462 $10^6 m^3$	170 $10^6 m^3$
Turnover	summer	3.0 times	23.1 times
Turnover	winter	5.8 times	30.9 times

somewhat false because in the latter area large amounts of fresh water from the Rhine and the IJssellake have been used not only to meet the evaporative demand but also to flush the boezem and polder system. In order to demonstrate this, data of Toussaint (ICW, 1976) can be used. This author set up water balances of the main drainage districts Rijnland and Delfland for the period 1960/70 and arrived at the data given in table 1.

A small boezem system as that of Delfland is turned over in summer 23 times in winter even 31 times. Such a system is impossible in the south-western Delta area where no fresh water is available at all and in the northern part of the country where not enough fresh water can be transported.

A far better picture of the distribution of the seepage is obtained from water balance studies. Figure 7 taken from ICW (1976) shows the rates of seepage and infiltration (negative seepage) in the central-western part of the Netherlands. Here we see that polders with a high seepage intensity are bordered by regions with infiltration. Especially the deeper polders cause a flow of water from the surrounding, so that there a loss of groundwater occurs. Seepage intensities in some of these polders may go up to 6 mm/day, so about 2 000 mm per year.

This distribution picture agrees far better with the pattern of the piezometric pressure of the groundwater. Figure 8 representing the piezometric pressure of the groundwater at a depth of about 25 m –NAP clearly shows the influence of the deeper polders with a low level of the phreatic water. Also the huge extraction of groundwater used for industrial cooling in the neighbourhood of Delft turns out to have a big influence. Part of the above mentioned effects are not found in the piezometric levels at larger depths. The smoothening effect of the Kedichem/Tegelen clay layer indicated in figure 6 is one of the reasons for this.

One should take into account, that apart from the differences in piezometric pressure between the freatic and deep groundwater, the seepage rate appearing at the surface of the soil also depends on the resistance of the covering Holocene layer.

Saline seepage occurs in a considerable part of the lower half of the Netherlands as may be clear from the preceding descriptions. Its magnitude depends highly on local circumstances from which apart from the geohydrological conditions in the subsoil, the

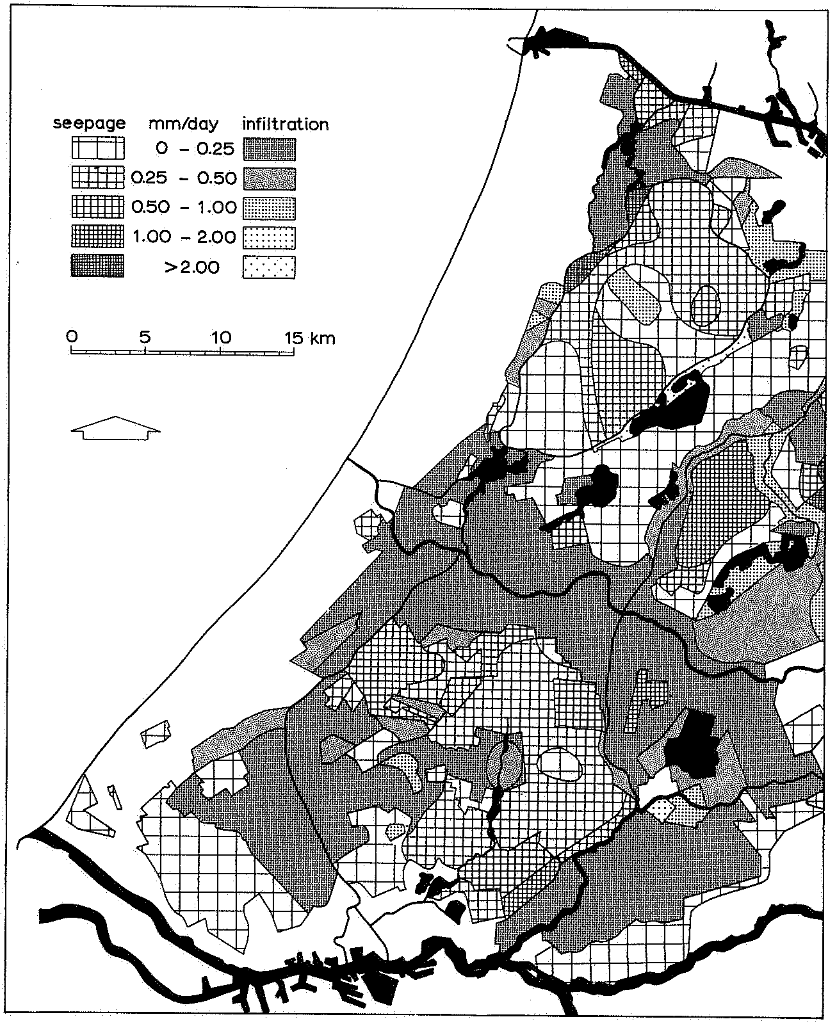


Fig. 7 Seepage and infiltration (ICW, 1976).

elevation of the area itself and that relative to the surrounding determines to a large extent the magnitude of the seepage.

Seepage itself increases the costs of drainage, because more water has to be removed from the area and a more intensive drainage system is required to obtain favourable con-

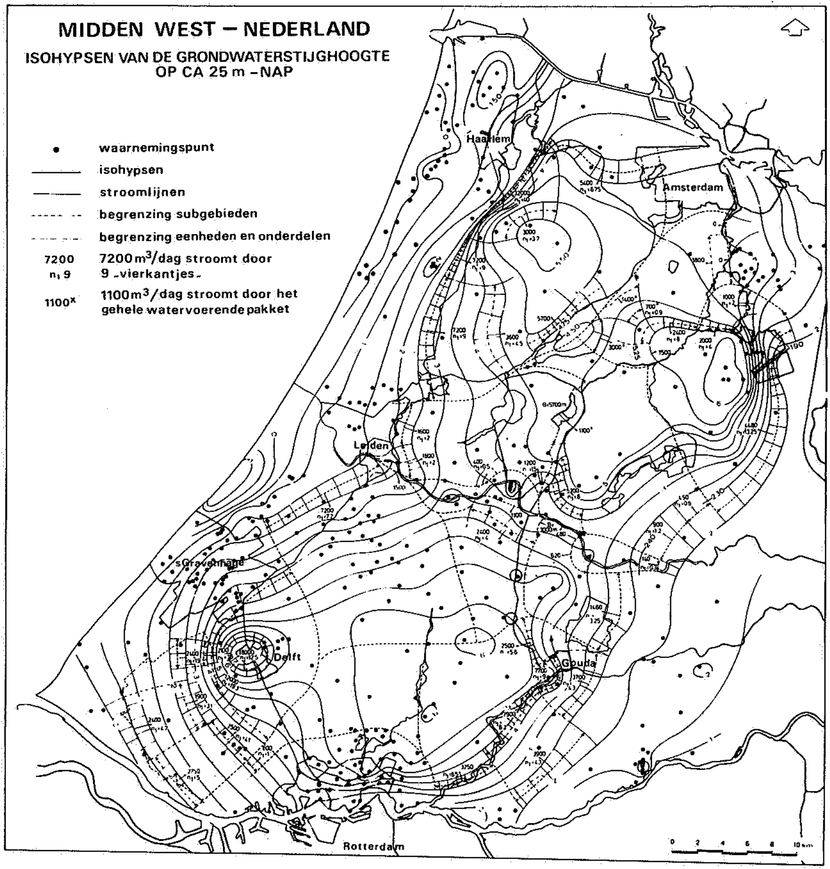


Fig. 8 Piezometric levels of the groundwater at a depth of about 25 m below surface (ICW, 1976).

ditions for agriculture. As long as the seepage rate is not too high, so that drainage is still possible without excessive costs, it forms no serious problem for agriculture. The reason for this is, that under the humid climatic conditions prevailing in the Netherlands there is enough rainfall to prevent salinization of the topsoil. Salts, accumulated in the upper part of the soil during the dry season (summer) are washed out by the excess winter rainfall (during the winter season). Problems arise, when surface water is used for irrigation for instance in glasshouses, for horticulture in the open air or for agriculture. Without going into details it must be said that for glasshouse use the chloride content of the irrigation water must be below a level of about 200 ppm in order to prevent damage to production and quality. Without flushing the open water courses, the chloride content in various regions will rise higher than that level due to the occurrence of saline seepage. Therefore huge quantities of water are required to lower the Cl^--content of the water, the amount required being higher the more seepage water is intercepted by the drainage system.

4. THE MAGNITUDE OF THE SALINE SEEPAGE

From the preceding section it will be clear that saline seepage forms a problem in certain parts of the country. In this respect, two main questions arise. They are:

a. What is the magnitude of the saline seepage?
b. What factors are governing the seepage and what can be expected in the future on the changes in seepage rates with and without measures to reduce it?

Determining the seepage rate in a certain area requires a lot of information. Both geological and hydrological data must be available, not only from the area itself but also from the surrounding. It is therefore obvious, that the exact rate of saline seepage for all parts of the Netherlands is not yet known.

One region where a thorough investigation has been carried out is the mid-western part of the country (ICW, 1976). Combining all kinds of geological and hydrological data yearly water balances could be set up. Combining the results with data on water quality the average yearly chloride load in the area, due to different sources could be established. The result is given in table 2.

The load due to industrial activities pertains to extraction of groundwater that is used for cooling purposes and after use delivered to the surface water.

The gaswells were originally constructed for domestic use. For this purpose a pipe was driven or bored through the covering Holecene layer. Due to the overpressure of the groundwater in the aquifer water containing methane gas flowed out of the pipe. By putting a clock-type cover over the top of the pipe the gas was collected, the water flowing off into the open water courses. The gas was mainly used for cooking purposes in

Table 2 Chloride load in tons Cl^-pa and sources for the central- western part of the Netherlands based on the 1972 situation (ICW, 1976)

Drainage district	Seepage	Industry	Gaswells	Total
Rijnland	75 800	3 000	7 200	86 000
Delfland	5 800	18 600	–	24 400
Schieland	14 000	–	230	14 230
Amstelland	52 600	1 450	1 760	55 810
Total	148 200	23 050	9 190	180 440
Percentage of total	82	13	5	

farmhouses. Nowadays many of the gaswells are no more used for this purpose mainly because they do not deliver any gas. Some of them are still used for cooking purposes, but the major part is abandoned nowadays. The flow of saline groundwater, however, continues. A recent survey of Toussaint and Boogaard (1978) showed that in the mainland north of the North Sea canal there are still 880 active gaswells. The total chloride load of them amounts to roughly 9 000 tons per year.

Although the gaswells find their existence in the same conditions as the saline seepage, their water and chloride production is generally not considered as seepage. The drainage boards try to clay these wells up and to neutralize their effect as much as possible.

Not taking the gaswells as seepage sources, still more than 80% of the total chloride load in the central-western part of the Netherlands finds its origin in saline seepage.

A rough estimate of the total amount of river water used for flushing in the area shows that about 450 million m^3 is used annually. Taking an average Cl^--concentration of 150 ppm, one arrives at a Cl^--load of 700 000 tons per year.

As pointed out before, the Dutch polder system has been developed during the past say 800 years. The last big reclamation of lakes in this area was the Haarlemmermeer, some 120 years ago. Further reclamation on a large scale may not be expected any more with the exception of parts of the IJssellake.

In answering the questions, whether the present situation is in a steady state or not, one has to take into account that due to improvement of the drainage systems in the existing polders, there occur still minor changes in the present hydrological situation.

It must be expected that many of the human activities that affect larger regions, like the converting of lakes into polders still influence the present salt and fresh water distribution, because not enough time has been elapsed to reach the equilibrium state. This is mainly due to the fact that several of these activities gave rise to the movement of considerably large amounts of subsoil water. Moreover, changes in the magnitude and pattern of extraction of groundwater, either temporarily or permanently, can change the conditions also.

On the basis of existing gradients in the groundwater and the present Cl^--content of that water along the boundaries of the area one can set up a computation of the total underground inflow of salt. Figure 9 gives the principal data and information used for such a calculation for the mid-western part of the Netherlands. It appears that on this basis there is an underground flow of salt from the North Sea that amounts 170 000 to 200 000 tons of Cl^- per year. Through the eastern border the inflow is estimated to be about 2 000 and from the southern border about 4 000 tons giving a total of 175 000 to 206 000 tons per year. Comparing this value with the load of same 180 000 tons following from table 2 and taking into account the accuracy of the data used one arrives at the conclusion that as far as it concerns the whole area there is an equilibrium or nearly equilibrium situation. This, however, does not mean that the same holds for the redistribution of the saline water within the area itself. As mentioned above several human activities caused, changes in the hydrological situation. This effect will not yet

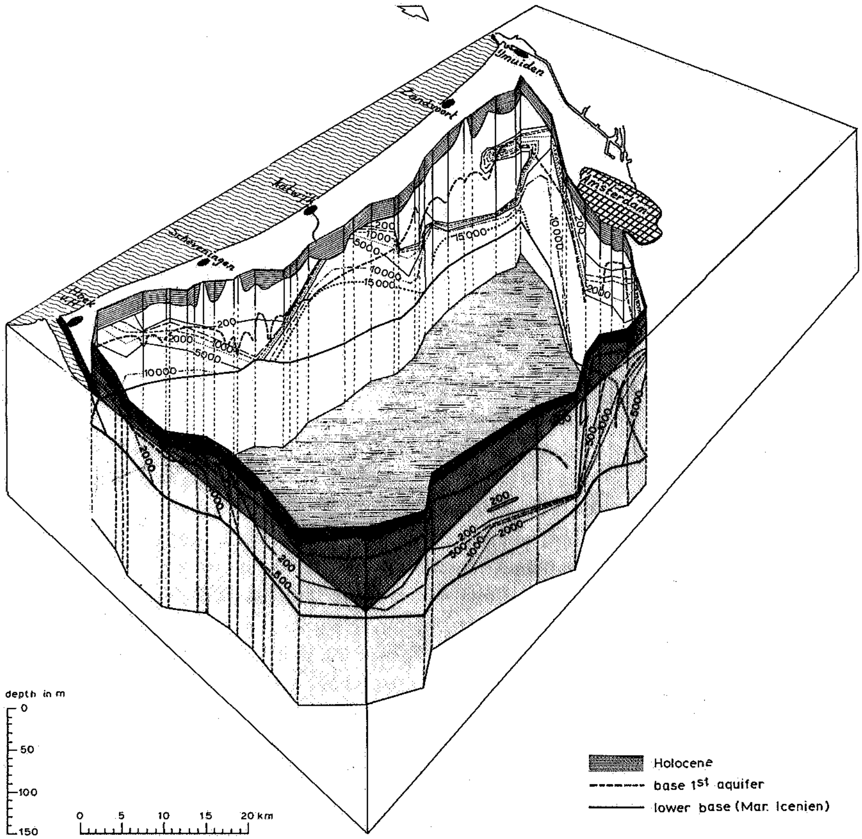


Fig. 9 Schematical representation of the flow of saline groundwater to the mid-western part of the Netherlands (personal contribution Van Rees Vellinga).

have been worked out because they caused the movement of large amounts of groundwater. Moreover, human activities are going on in the form of improving the drainage situation, extraction of groundwater, feeding of the aquifer with fresh water, especially in the dune area etc.

REFERENCES

BOTS, W.C.P.M., P.C. JANSEN and G.J. NOORDEWIER (1978) – Fysisch-chemische samenstelling van oppervlakte – en grondwater in het Noorden des Lands. Regionale Studies ICW, nr. 13, 109 pp.

BREEUWER, J.B. and S. JELGERSMA (1973) – An East-West geohydrological section across the Netherlands. Verhandelingen K.N.G.M.G. 29:105-106.

GEIRNAERT, W. (1973) – The hydrology and hydrochemistry of the lower Rhine fluvial plain. Leidse Geologische Mededelingen.

HELLINGA, F. (1952) – Water control. Soil Sci. 74: 21-32.

INSTITUUT VOOR CULTUURTECHNIEK EN WATERHUISHOUDING (ICW) (1976) – Hydrologie en waterkwaliteit van Midden West-Nederland. Regionale Studies ICW, nr. 9, 100 pp.

JELGERSMA, S. and W.A. VISSER (1972) – Hydrogeological Maps of the Netherlands. Scale 1 :1.500.000. Geologie en Mijnbouw 51:7-10.

MAZURE, J. (1940) – Rapport van de Commissie Drinkwatervoorziening Westen des Lands. Bijlage IX.

MEINARDI, C.R. (1973) – Het zoutwatervoorkomen in de ondergrond van de lage gedeelten van Nederland. H_2O 6: 454-461.

SNIJDERS, J.H. (1960) – Het Nederlandse verziltingsvraagstuk. Mededelingen ICW, nr. 20, 19 pp.

TOUSSAINT, C.G.T. and T. BOOGAARD (1978) – Chloride-, stikstof- en fosfaatbelasting van polderwater door gas- en koelbronnen. Nota ICW, nr. 1061, 18 pp.

VOLKER, A. (1961) – Source of brackish groundwater in Pleistocene formations beneath the Dutch polderland. Economic Geology 56:1045-1057.

PHYSICAL FUNDAMENTALS OF THE FLOW PROBLEM

W.H.C. TEN HOORN

SUMMARY

The flow of a fluid in a porous medium is considered for the case that the density varies with place. The relation between density of the fluid and the hydraulic conductivity of the soil is discussed. Ways are given to calculate groundwaterflow, both in the case of gradually varying density as in the case of a sharp interface between fluids of different densities.

Finally, the required schematization is discussed which is needed when considering a problem in practice.

1. INTRODUCTION

The flow of a fluid in a porous medium is a complicated phenomenon, which depends on the properties both of the porous medium and of the fluid. In order to be able to find answers to groundwater problems encountered in practice, it is necessary to schematize the natural situation to a model which we can handle.

As a rule this schematization only involves the porous medium, dividing it in aquifers, aquitards and aquicludes with appropriate geohydrological values, the groundwater being considered to be a homogeneous liquid. When there are rather considerable differences in density of the groundwater, as is the case in the western and northern part of the Netherlands, it is necessary to take these differences into account.

In the following it will be tried to give an outline of the relevant properties and mechanisms and of the manners in which the problem can be simplified, in order to be able to tackle it.

2. FLOW THROUGH POROUS MEDIA

Like all motion, the flow of a fluid through a porous medium is the result of the forces, acting on that fluid.

In the case of saturated flow, to which case we shall confine ourselves in the following, these forces are due to:

- differences in the pressure p
- gravity
- resistance to the flow

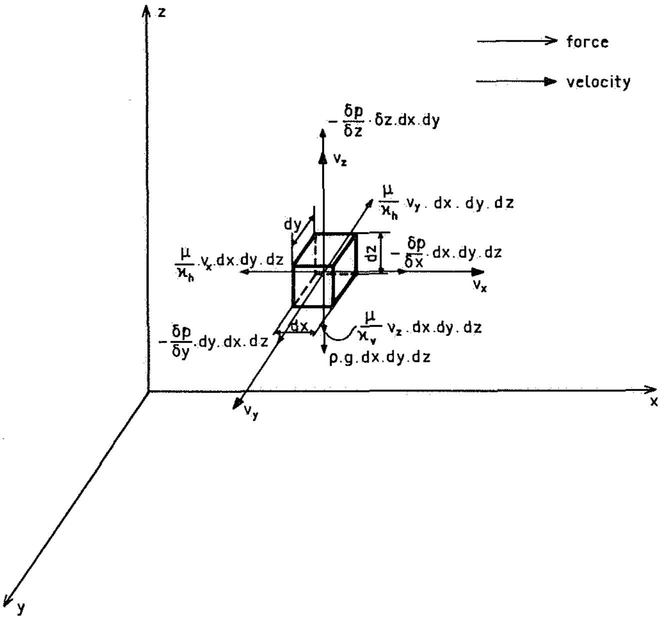


Fig. 1 Elementary volume of groundwater with forces and velocities.

If we choose a rectangular coordinate system x, y, z, with a vertical z-axis, being positive in upward direction, we find the following forces, acting on an elementary volume of water dx dy dz (fig. 1):

1. due to differences in the pressure p:

in x-direction: $-\frac{\partial p}{\partial x} \cdot dx \cdot dy \cdot dz$ $\quad (1^a)$

in y-direction: $-\frac{\partial p}{\partial y} \cdot dy \cdot dx \cdot dz$ $\quad (1^b)$

in z-direction: $-\frac{\partial p}{\partial z} \cdot dz \cdot dx \cdot dy$ $\quad (1^c)$

2. due to gravity:
in z-direction: $-\rho \cdot g \cdot dx \cdot dy \cdot dz$ $\quad (2)$

3. due to the resistance to the flow:

$$\text{in x-direction: } -\frac{\mu}{\kappa_h} v_x \cdot dx \cdot dy \cdot dz \tag{3^a}$$

$$\text{in y-direction: } -\frac{\mu}{\kappa_h} v_y \cdot dx \cdot dy \cdot dz \tag{3^b}$$

$$\text{in z-direction: } -\frac{\mu}{\kappa_v} v_z \cdot dx \cdot dy \cdot dz \tag{3^c}$$

where:

ρ	–	density of the fluid	$[L^{-3}M]$
g	–	acceleration of gravity	$[LT^{-2}]$
κ	–	intrinsic permeability	$[L^2]$
μ	–	dynamic viscosity	$[L^{-1}MT^{-1}]$

v_x, v_y, v_z-specific discharges in respectively x-, y- and z-direction $[LT^{-1}]$

The intrinsic permeability accounts for the properties of the porous medium, that are relevant for the flow of fluids through this medium. Important properties in this respect are grain-size distribution, shape of grains, degree of compaction. Owing to the way in which the soillayers were deposited the soil is as a rule not isotropic but anisotropic, resulting in a difference between permeability for flow in horizontal direction and for flow in vertical direction. In the formulae (3) this difference is accounted for by the subscripts h and v.

As in groundwaterflow acceleration is negligible, the force are in equilibrium, from which is found:

$$\text{in x-direction: } -\frac{\partial p}{\partial x} \cdot dx \cdot dy \cdot dz - \frac{\mu}{\kappa_h} \cdot v_x \cdot dx \cdot dy \cdot dz = 0 \tag{4^a}$$

$$\text{in y-direction: } -\frac{\partial p}{\partial y} \cdot dy \cdot dx \cdot dz - \frac{\mu}{\kappa_h} \cdot v_y \cdot dx \cdot dy \cdot dz = 0 \tag{4^b}$$

$$\text{in z-direction: } -\frac{\partial p}{\partial z} \cdot dz \cdot dx \cdot dy - \rho \cdot g \cdot dx \cdot dy \cdot dz - \frac{\mu}{\kappa_v} \cdot v_z \cdot dx \cdot dy \cdot dz = 0 \tag{4^c}$$

The specific discharges, found from this formulae, are:

$$v_x = -\frac{\kappa_h}{\mu} \frac{\partial p}{\partial x} \tag{5^a}$$

$$v_y = -\frac{\kappa_h}{\mu} \frac{\partial p}{\partial y} \tag{5^b}$$

$$v_z = -\frac{\kappa_v}{\mu}(\rho g + \frac{\partial p}{\partial z}) \tag{5^c}$$

In this formulae p will and ρ may vary with place, so $p = p(x, y, z)$ and $\rho = \rho(x, y, z)$. If no variations in density occur, the pressure in the water will be:

$$p = (\phi - z)\rho g, \tag{6}$$

in which ϕ is the piezometric head and z is the height above the reference level. (6) in (5) yields:

$$v_x = -\frac{\kappa_h \rho g}{\mu}\frac{\partial \phi}{\partial x} \tag{7^a}$$

$$v_y = -\frac{\kappa_h \rho g}{\mu}\frac{\partial \phi}{\partial z} \tag{7^b}$$

$$v_z = -\frac{\kappa_v \rho g}{\mu}(1 + \frac{\partial \phi}{\partial z}) \tag{7^c}$$

As a rule we combine $\frac{\kappa \rho g}{\mu}$ in a new constant k, the hydraulic conductivity $[LT^{-1}]$. So, depending on whether we consider horizontal or vertical flow:

$$k_h = \frac{\kappa_h \rho g}{\mu} \tag{8^a}$$

$$k_v = \frac{\kappa_v \rho g}{\mu} \tag{8^b}$$

In the hydraulic conductivity the properties of the soil as well as of the fluid are combined. The fluid affects the hydraulic conductivity both by its density and by its dynamic viscosity. Strictly speaking the density of a fluid is influenced among other things by the pressure, but normally this is disregarded, the fluid being considered incompressible. Density is also influenced by temperature, but this influence is likewise neglected, being of little importance.

However, change in temperature has a considerable influence on the dynamic viscosity, the latter varying from 1,79 $\times 10^{-3}$ kg/ms at 0°C to 1,01 $\times 10^{-3}$ kg/ms at 20°C and 0,41 $\times 10^{-3}$ kg/ms at 70°C.

In spite of the great influence which temperature has on the dynamic viscosity and hence on the hydraulic conductivity, still the hydraulic conductivity is always given as a geohydrological constant without reference to temperature. The explanation for this lays in the constant temperature of the groundwater. However, in cases where this temperature varies considerably during the year, as in the case when surface water is infiltrated for e.g. drinking water purposes, it may be compelling to take this variance into consideration.

3. FLOW OF GROUNDWATER WITH GRADUALLY VARYING DENSITY

Taking into account the difference in density between fresh water and saline water it can be expected that in the subsoil fresh water is present at the top of saline water.

However, the occurrence of fresh and saline water in the subsoil is the outcome of a geological history and in the course of that history there were also periods in which saline water infiltrated in a subsoil saturated with fresh water (e.g. during transgressions). In that case one can expect a descent of the saline water combined with both a flushing of the fresh water and a mixing with the fresh water. Also can fresh water pockets stay untouched under impermeable layers. In this way a more complicated system is created than the simple: fresh over saline. Moreover, fresh water and saline water are miscible fluids, i.e. there is no interfacial tension between the two fluids and the two fluids dissolve in each other owing to diffusion. This fading of the interface can be accelerated when one or both fluids are flowing, as in that case the fluids while flowing through the pores are mixed, the total process of mixing and diffusion being known as dispersion.

In reality all above mentioned processes took and take place, resulting in a complex present situation.

In this case a calculation of the groundwater flow that takes into consideration density differences is only possible if sufficient reliable data on piezometric head and density are at hand.

If such data are available indeed one can try to draw conclusions from the differences in pressure, observed and/or calculated in horizontal planes; or, if at different places groundwater of the same density occurs the piezometric levels in this groundwater can be compared (Van Dam, 1977).

4. FLOW OF FRESH AND SALINE GROUNDWATER, IN CASE OF A SHARP INTERFACE

Owing to the fact that fresh and saline water are miscible fluids, a relatively sharp interface between the two can only be expected in some special cases. A rather sharp interface is found under the dunes in the western part of the Netherlands where fresh water flows away on the top of saline water. As the fresh water is continuously renewed by precipitation, the flow velocities are rather high. This combined with the flow length which depends on the width of the dunes and is only a few kilometers, results in a relatively short time during which dispersion can take place. In this way the mixing zone between fresh and saline water is restricted to a thickness of only a few meters. Also in other areas where there is a definite flow of fresh water over saline water there may be a rather sharp interface.

From the condition that there has to be an equilibrium between the pressure in the fresh water just above the interface and the pressure in the saline water just under the interface (the interface considered to be a sharp one, i.e. there is a sudden change from fresh water to saline water), it follows (fig. 2):

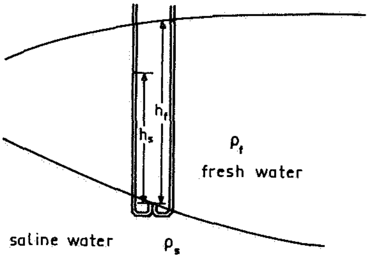


Fig. 2 Equilibrium at the interface between fresh and saline water.

$$h_f \rho_f = h_s \rho_s \tag{9}$$

where:

h_f = pressure head of the fresh water
h_s = pressure head of the saline water
ρ_f = density of the fresh water
ρ_s = density of the saline water

In the case of phreatic water and horizontal flow (the vertical component of the flow is as a rule so small that it may be disregarded), we find h_f to be the thickness of the fresh water layer.

From (9) we can find:

$$h_f = \frac{\rho_s}{\rho_f} h_s \tag{10}$$

The difference in hydraulic head is

$$h_f - h_s = \frac{\rho_s - \rho_f}{\rho_f} h_s, \text{ or}$$

$$h_f - h_s = \alpha \cdot h_s \tag{11}$$

with $\alpha = \dfrac{\rho_s - \rho_f}{\rho_f}$, the relative difference in density.

So, the depth of the interface with respect to the piezometric level of the saline water turns out to be:

$$h_s = \frac{1}{\alpha}(h_f - h_s) \tag{12}$$

This relation between the depth of the interface and the difference between the pressure heads (or, which is the same, the difference between the piezometric levels) is the well known Badon Ghijben-Herzberg relation, first published by Badon Ghijben (1889) and Herzberg (1901).

From (8): $k = \frac{\kappa \rho g}{\mu}$ we see that the hydraulic conductivity depends among other things on the density and the dynamic viscosity. The dynamic viscosity can be considered to be independent of the salt content of the fluid, but, in order to derive the proper formulae when dealing with both flowing fresh and saline groundwater, we find from:

$$k_f = \frac{\kappa \rho_f g}{\mu} \text{ for fresh water and}$$

$$k_s = \frac{\kappa \rho_s g}{\mu} \text{ for saline water, that}$$

$$k_s = \frac{\rho_s}{\rho_f} k_f = (\alpha + 1) k_f$$

5. SCHEMATIZATION

The solution of a geohydrological problem is always based on schematization. Object of schematization are:

- the system,
- the boundary conditions,
- the state of the system.

The choice of the system involves the division in aquifers and aquitards, together with the geohydrological constants (transmissivity, hydraulic resistivity, hydraulic conductivity) and eventually the distribution of different types of water with respect to density. It is based on information gathered from different sources: borings, samples (of ground and water), geophysical prospecting, geology, groundwater survey, pumping test data etc. A great part of this information is local and in between there may be considerable deviations. The choice of the system is also influenced by the way in which it is intended to solve the problem: the availability of a certain computer program may limit the number of layers or the choice for an analytical solution may restrict the variation in geohydrological constants. The extent of the system is coupled with the information about conditions on the boundary. If it is possible to find a nearby boundary on which the situation is well known then the system can end at this boundary. If not, then it may be necessary to consider an area which is so large that faulty conditions on the boundaries are not of great influence at the point where we are interested in the results. The mathematical solution that is chosen may impose certain requirements on the boundary conditions and this again may influence the extent of the system. If a calculation is made, starting from the present situation it may be necessary to smooth the latter, as it is

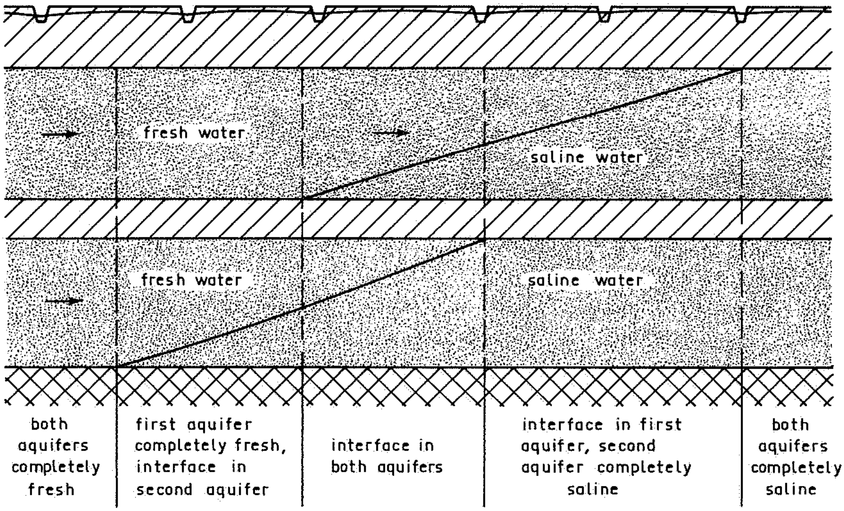


Fig. 3 Some possible positions of the interface between fresh water and saline water.

influenced by local irregularities that are not accounted for in the system and by deviations in the available data.

As a rule the geohydrological system is considered to consist of one or more aquifers, separated by impermeable or semi-impermeable layers. In large scale problems there will be little objection against neglecting vertical flow in the aquifers and horizontal flow in the semi-permeable layers. In this way we can reduce two-dimensional flow in the vertical plane to one-dimensional flow, i.e. horizontal flow in the aquifers and vertical flow in the semi-permeable layers. This schematization at the same time solves the problem of anisotropy, for accounting for vertical flow in the aquifer, would also force us to account for another hydraulic conductivity in vertical direction. In the case dealing with fresh and saline water and a sharp interface it may be necessary to account for several possibilities, depending on the situation of the interface (fig. 3). Here the transmissivity of a layer is partly available for the flow of fresh water and partly for the flow of saline water.

Often it is not known whether the flow in the subsoil has reached the steady state. As groundwater movement is very slow and a modest change in polder level can introduce a far greater change in the interface between fresh and saline water, it can hardly be expected that anywhere in the subsoil of the western part of the Netherlands a steady state flow has been reached.

Although from this a non steady state calculation seems to lay at hand, this offers on the other hand many problems. For besides problems in the computational field as more complex and more expensive programs, we don't know where to start, as we don't know the boundary conditions. Moreover, the situation as it is at the moment is caused also under influence of dispersion.

When accomplishing a steady state calculation we are wrong in so far that we don't account for changes in hydraulic head and in the situation of the interface. In a polder area however, the phreatic level is kept essentially constant (polder level), which means that during a certain period also the hydraulic head of the fresh water will but gradually change. As the difference of the hydraulic heads of fresh and saline water determine the place of the interface (eq. 12), the principal inaccuracy of applying a steady state calculation lies in the assumption of a fixed interface. For this reason we find that the law of continuity does not hold, that is, if we consider a certain period we have to account for some storage. This storage can be obtained by a change in the interface and the next period can be calculated with the adjusted interface. As the change in interface takes place very gradually the periods need not to be taken very short, in practice a period of a year will often do.

Quite another schematization follows when it is reflected that often there is a very gradual change in density. In this case we may make the calculation as if we have to deal with a homogeneous fluid. If we calculate the stream pattern we can follow the movement of water with a certain density along the flowpath.

Up will now it is not clear whether, when schematizing the interface in an aquifer overtopped by another aquifer the fresh water tongue in the lowest aquifer can extend below saline water in the aquifer above. Although in nature this is often found, e.g. on the west side of the dunes where a fresh water tongue in the second aquifer may extend under sea, it is difficult to match this phenomenon with the supposition of a sharp interface, as the fresh water from the second aquifer will after seepage appear in a saline environment, without changing this significantly. Here we can think of secondary currents, flushing the fresh water that emerges in the upper aquifer.

Near the point where the rising interface intersects the bottom of a semipermeable layer there may be a sharp rise in the interface and it is doubtful whether it is allowed to neglect the vertical component of the flow there, whereas also in small scale problems two-dimensional calculation seems inevitable.

6. CONCLUSIONS

In order to be able to calculate the fluid flow in case the fluid doesn't have one density over the whole flow domain, it is essential to base the calculation on differences in pressure and not on differences in piezometric head.

Differences in piezometric head can only be used to calculate the flow in a plane or in a space where the fluid has a constant density, the latter being the case when a sharp interface exists between two types of water with different densities ('fresh' and 'saline').

As the hydraulic conductivity depends on properties of both the porous medium and the fluid, a change in density also effects the hydraulic conductivity. The solution of a groundwater problem in the case that water with varying density is involved requires more information than in the case that the water is homegeneous with respect to density. Not only is it necessary to know hydraulic heads and geohydrological constants, but also sufficient data are needed on the density of the groundwater and on the piezometric head at different levels.

In case a sharp interface is considered, the transmissivity of an aquifer has to be distributed between the fresh and the saline region.

The usual schematization to horizontal flow in aquifers and vertical flow in aquitards may be objectionable in certain cases, just as the simplification of a transition zone to a sharp interface.

REFERENCES

BADON GHIJBEN, W. (1889) – Nota in verband met de voorgenomen putboring nabij Amsterdam. (Notes on the probable results of the proposed well drilling near Amsterdam). Tijdschr. Kon. Instituut v. Ingenieurs, pp. 8-22.

BEAR, J. (1972) – Dynamics of fluids in porous media. Am. Elsevier Publ. Comp., Environm. Science Series, New York.

DAM, J.C. VAN (1977) – Determination of horizontal and vertical groundwater flow from piezometric levels observed in groundwater of varied densities. Delft Progress Report, Vol. 3: 19-34.

HERZBERG, (1901) – Die Wasserverzorgung einiger Nordseebäder. Journal für Gasbeleuchtung. und Wasserversorging, XLIV Jahrgang, No. 44: 815-819 und No. 45: 842-844.

VERRUYT, A. (1970) – Theory of groundwater flow. McMillan and Co. Ltd., London.

WIEST, R.J.M. DE (1965) – Geohydrology. John Willey and Sons, Inc., New York.

NOTES ON CALCULATION METHODS FOR GROUNDWATER FLOW

P. VAN DER VEER

SUMMARY

These notes give a review of the most important calculation methods for groundwater flow problems. Special attention is paid to problems of fresh and salt groundwater flow. Calculation methods are mentioned without mathematical analysis. The presentation is restricted to the main ideas behind the calculation methods, presented at the CHO-TNO meeting 1979.

1. INTRODUCTION

In reality groundwater flow problems are always three-dimensional. The soil properties are generally not constant, and in general the properties of the fluid are not constant too. The most important step in the way from physical reality to mathematical description is the schematization.

Generally in groundwater flow analysis the properties of the soil are assumed to be constant or sectionally constant. Dealing with problems of fresh and salt water flow further assumptions have to be made in order to enable a mathematical description of the phenomenon. Some authors neglect the influence of density differences in the fluid (see e.g. Bruggeman, 1975). Although it may be a rather rough approximation this approach has important practical advantages. The mathematical formulation becomes the significantly simplier, or more complicate problems can be solved using known mathematical techniques.

Many approaches involving the influence of the density difference between fresh and salt water are based on the assumption of the presence of a sharp interface between fresh and salt water and the salt water being at rest. In many practical situations the specific discharges in the salt water are indeed very small in comparison with these in the fresh water. So it may be a good approximation. Further simplification can be made if the flow is assumed to be one-dimensional. In many cases then the flow is described by a rather simple differential equation that can be solved analytically (see e.g. Van Dam).

However sometimes even this simple approach leads to a differential equation that is usually only numerically solved (see e.g. Van Dam, 1976). The one-dimensional approach is a suitable one for those problems in which vertical components of the specific discharges may be neglected and where, in addition, one profile is relevant for a large region. In many cases two- or even three-dimensional solutions are necessary or wanted to investigated the interface (e.g. near the coast or in the vicinity of wells).

For two-dimensional steady problems the hodograph method is a very elegant and powerfull method (see e.g. Verruyt, 1970). In some complicated two-dimensional problems the hodograph can be drawn but nevertheless the hodograph method fails.

Then sometimes a solution can be found by generalizing solutions of similar problems (Van der Veer, 1977[b]), but generally a numerical approach has to be applied. Numerical methods are generally also necessary for calculations of non-steady problems and for problems involving flow in anisotropic inhomogeneous soil. Some numerical methods have an analytical character (vortex theory, see De Josselin de Jong, 1960, 1969, 1977) or a semi-analytic character (boundary element methods, see e.g. Brebbia, 1978). Others are discretization methods, using a subdivision of the flow region into elements (finite difference methods, see e.g. Carnahan et. al., 1969, and finite elements methods, see e.g. Zienkiewicz, 1971).

In the following section some notes are made on the most important calculation methods.

2. ANALYTIC CALCULATION METHODS

In analytic calculation methods a closed form solution is found.

2.1. *One-dimensional*

In many cases interface problems are met in extensive aquifers where the groundwater flow is mainly directed horizontally.

Consequently a one-dimensional approximation may be used for problems where one profile is relevant for a large region. This means that the vertical components of the specific discharges are neglected. In this approach a one-dimensional differential equation is found by combining Darcy's Law with the continuity condition for a segment of the profile. This yields a relationship between the coordinate x, the groundwater head ϕ and the depth h of the interface below a reference level. This relationship has the form of a differential equation. Further the condition holds that at both sides of the interface the pressure is equal. This yields a relationship between ϕ and h. This is usually called the Badon Ghijben relationship on account of the (hydrostatic) analysis of the problem by Badon Ghijben (1889) which is equivalent to an approach of one-dimensional flow. A combination of the differential equation with the Badon Ghijben relationship yields a differential equation in ϕ or h. Several authors investigate the accuracy of one-dimensional approximations by comparing them with exact two-dimensional solutions (see e.g. Bear and Dagon, 1964, Verruijt, 1968 and Van der Veer, 1977[b]). They report that generally the one-dimensional approach is very accurate. For the problem of Badon Ghijben (coastal aquifer involving interface and phreatic surface) the formula for the interface according to the one-dimensional approach is even exactly the same as the one following from the exact two-dimensional solution, provided that the origin is suitably chosen (Van der Veer, 1977[b]).

2.2. *Two-dimensional*

a) In some cases a one-dimensional approximation can not be applied although vertical components of specific discharges may be neglected. This occurs for example in coastal aquifers with a sharply curved shore line or when there is a sink near the coast, causing pertubation of the one-dimensional flow towards the coast (see e.g. Strack, 1976).

b) In some cases, especially when one is interested in the interface near the coast or below a sink the vertical components of the specific discharge may no longer be neglected. Dealing with problems of steady flow, the salt water still assuming to be at rest, the hodograph method is a powerful and elegant method. In the hodograph method the theory of complex functions is used (conformal mapping).

For steady flow generally the representation of the flow region in the plane of the complex potential Ω is known ($\Omega = \Phi + i\psi$; $\Phi = k\phi$ where k: coefficient of permeability and ϕ: groundwater head, ψ: stream function).

Unfortunately the representation of the flow in the physical plane (z-plane, $z = x + iy$, x and y: coordinates) is not known as the position of the interface in unknown beforehand. However, the representation of the flow in the hodograph-plance (v_x, v_y): components of specific discharge) is known.

Consequently the representation of the flow in the place of $d\Omega/dz = -v_x + v_y$ is known. (From a simple derivation it follows that the representation of an interface in that plane is a circle, see e.g. Verruijt, 1970). As the flow can be represented in the Ω-plane as well as in the $d\Omega/dz$-plane one can find a relationship between Ω and $d\Omega/dz$ by application of the theory of conformal mapping. By evaluation of the resulting formula the relationship between Ω and z is found. For poblems involving sinks and sources within the region, the hodograph method is classical form cannot be applied. For such cases Strack (1972, 1973) gave an extension. Strack and Asgian (1978) presented a function that enables the use of the hodograph method for problems involving a phreatic line as well as a seepage line.

2.3. *Note on the simultaneous flow of several fluids*

As mentioned in the preceding section at the interface between two fluids the condition holds that at both sides the pressure is equal. As a consequence of the density difference between the two fluids there is a discontinuity in the groundwater head at the interface. Denoting for an interface point the groundwater head on both sides of the interface by ϕ_f and ϕ_s and its vertical coordinate by y the mentioned pressure condition is given by:

$$y = a\phi_f + b\phi_s$$

where a and b are constants depending on the fluid densities. If the salt water is assumed to be at rest the term $b\phi_s$ is a constant. If both fluids are moving, there are, in principle, two approaches for a mathematical analysis:

a) Finding one function being the solution for the whole region involving two (or several) fluids. As a consequence of the discontinuity of the groundwater head at the interface, the solution has to involve these discontinuities. This approach (singularity, distributions, see De Josselin de Jong, 1969, 1969, 1977) has the advantage of being suitable as well for problems involving fluids with gradually varying density. Although the behaviour of an interface can be given in a closed form expression the method does not generally provide a closed form solution for the steady position of an interface as the method deals with the more general case of non-steady flow (of course from that a calculation in time towards the steady position of the interface is well possible).

   The approach published by De Josselin de Jong provides a solution in the stream function or in a variable that is simular to the groundwater head.

b) Finding several functions, each of them describing the flow of one fluid. So in this approach there are two functions in fresh/salt water flow, one for the fresh water flow, the other for the salt water flow. Application of this approach is, of course, restricted to those problems in which the densities are sectionally constant. (So there are no gradual density variations). It may be advantegeous that now complex function formulations may be used. Then the potential as well as the stream function are included in the solution.

   So far this approach has not been developed to a generally applicable method. A few ad-hoc solutions using the philosophy of the approach for the simultaneous flow of two fluids were given by Van der Veer (1977[a], 1977[c]).

3. NUMERICAL METHODS

In numerical methods a discretization in space or time is used to evaluate a solution. In these methods there is not a closed form solution (in space and time variables) available.

3.1. *Methods having an analytical character*

In the preceding section the method of singularity distributions (vortex theory, De Josselin de Jong, 1960, 1969, 1977) was mentioned. In this theory those distributions are used to generate the effects of varying density. The strength of the singularity distribution is known beforehand as a consequence of density variation. The problem is then reduced to a homogeneous problem having a solution of two parts: one part accounting for the effects of density variation and another that accounts for the boundary conditions. For complicated problems the theory of singularity distributions has to be combined with other methods in order to find a solution that satisfies the boundary conditions.

Generally for the singularity distributions a discretization technique is used (see e.g. Haitjema, 1977).

This theory is used for non-steady flow problems where the position of the interface at one moment is known beforehand. The interface position is followed in time.

3.2. *Semi-analytical methods*

In these methods basic solutions are used that satisfy the differential equation. Therefore there is no necessity for subdividing the flow region into elements as used in discretization methods (finite difference method and finite element method). The basic solution contains a number of degrees of freedom that are used to meet the boundary conditions at a number of points of the boundary.

These methods are called Boundary Element Methods. Generally the theory is in real variables (see e.g. Jaswon and Symm, 1977, Brebbia, 1978). Sometimes a complex function formulation is used (Van der Veer, 1978). Boundary element methods have practical advantages because only boundary elements are used. Consequently the amount of work for preparing input data and checking them is rather small. In boundary elements methods the solutions are exact within an approximative boundary as the used basic solutions satisfy already the differential equation. The applicability to these methods is generally restricted to problems involving soils and fluids that have sectionally constant properties.

3.3. *Discretization methods*

In these methods the flow region is subdivided into a number of elements in order to find a solution that satisfies approximately the differential equation within the flow region. In the finite difference method the mathematical formulation is based on a discretization of the differential equation. Depending on the type of discretization there are several finite differences techniques.

In the finite element method there are also several mathematical formulations, e.g. variational principles (see e.g. Verruijt, 1970), Galerkin's method (see e.g. Finlayson, 1972). Generally the elements have an triangular form having the practival advantage of flexibility in approximating curved boundaries.

REFERENCES

BADON GHIJBEN, W. (1889) – Nota in verband met de voorgenomen putboring nabij Amsterdam. (Notes on the probable results of the proposed well drilling near Amsterdam). Tijdsch. Kon. Inst. Ingenieurs, pp. 8-22.

BEAR, J. and DAGAN, J. (1964) – Some exact solutions of interface problems by means of the hodograph method. J. Geophys. Res., 69 (8): 1563-1572.

BREBBIA, C.A. (1978) – The boundary element method for engineers. Pentech Press. London.

BRUGGEMAN, G.A. (1975) – Analysis of moving fronts in two- and three-dimensional groundwater flow in anisotropic aquifers. RID-Med. 75-4.

CARNAHAN, B., H.A. LUTHER and J.O. WILKES (1969) – Applied Numerical Methods. Willey, New York.

DAM, J.C. VAN (1976) – Partial depletion of saline groundwater by seepage. Journal of Hydrol., 29: 315-339.

DAM, J.C. van, – Fresh water – salt water. Stencil Delft University of Technology, Delft.

DE JOSSELIN DE JONG, G. (1960) – Singularity distributions for the analysis of multiple-fluid flow through porous media. J. Geophys. Res., 65 (11): 3739-3758.

DE JOSSELIN DE JONG, G. (1969) – Generating functions in the theory of flow through porous media. In: Flow Through Porous Media (Editor: R.J.M. de Wiest), Academic Press, New York: 377-400.

DE JOSSELIN DE JONG, G. (1977) – Review of vortex theory for multiple fluid flow. Delft Progr. Rep., 2: 255-236.

FINLAYSON, B.A. (1972) – The method of weighted residuals and variational principles. Academic Press, New York.

HAITJEMA, H.M. (1977) – Numerical application of vortices to multiple fluid foow in porous media. Delft Progr. Rep., 2: 237-248.

JASWON, M.A. and G.T. SYMM (1977) – Integral equation methods in potential theory and elastostatics. Academic Press, London.

STRACK, O.D.L. (1972) – Some cases of interface flow towards drains. J. Eng. Math., Vol. 6, no. 2.

STRACK, O.D.L. (1973) – Many valuedness encountered in groundwater flow. Ph.D. thesis, Delft University of Technology, Delft.

STRACK, O.D.L. (1976) – A single-potential solution for regional interface problems in coastal aquifers. Water Res. Research Vol. 12, no. 6: 1165-1174.

STRACK, O.D.L. and M.I. ASGIAN (1978) – A new function for use in the hodograph method Water Res. Research, Vol. 14, no. 6: 1045-1058.

VEER, P. VAN DER (1977[a]) – The pattern of fresh and salt water flow in a coastal aquifer. Delft Progress Report, Vol. 2: 137-142.

VEER, P. VAN DER (1977[b]) – Analytical solution for steady interface flow in a coastal aquifer involving a phreatic surface with precipitation. Journ. of Hydrol., 34 No1/2: 1-11.

VEER, P. VAN DER (1977[c]) – Analytical solution for a two-fluid in a coastal aquifer involving a phreatic surface with precipitation. J. Hydrol. 35: 271-278.

VEER, P. VAN DER (1978) – Calculation methods for two-dimensional groundwater flow. Ph.D. thesis Delft 1978. Rijkswaterstaat Communications no. 28/1978, The Hague.

VERRUYT, A. (1968) – A note on the Ghijben-Herzberg formula. Bull. Int. Assoc. Sci. Hydrol., XIII: 4-12 1968, pp. 43-46.

VERRUYT, A. (1970) – Theory of groundwater flow. Mc.Millan and Co. Ltd., London.

ZIENKIEWICZ, O.C. (1971) – The finite element method in engineering sciences. McGraw-Hill, London.

SOME RESULTS OBTAINED WITH A FINITE DIFFERENCE METHOD

D. PEREBOOM

SUMMARY

This paper deals with the steady flow of fresh and salt groundwater in a system with one aquifer and in a system with two aquifers, seperated by a semi-pervious layer. Using a finite difference method, calculations are made to reflect the influence of changes in the values of the interval length Δx and some of the geohydrological constants. The results of the calculations are discussed. Finally, conclusions are drawn concerning the calculation method in use and proposals for further investigations are made.

1. INTRODUCTION

Van Dam (1976) made a start for the solution of a fresh-salt groundwater flow problem in a system with one semi-confined aquifer by using an approximate analytical solution method. Van der Knaap (1977) used a finite difference method is a similar problem as Van Dam did. Both Van Dam and Van der Knaap divided the length of their models in a number of N intervals each with length Δx (discretization). The unknown values of the variables are calculated only in the nodal points i of the developed network.

To find a solution they both used a "shooting method". After a first estimate of the values of the variables (piezometric levels and/or flows) in x = o, that were not given as boundary conditions, the values of the piezometric levels and flows are calculated by going through the model from left (x = o) to right (x = L), with the use of so-called explicit formulae.

The unknown values in any next nodal point are expressed in the values calculated in the foregoing nodal points.

Once arrived at the right hand side of the model, generally there will be a difference between the calculated values and the values given as boundary conditions. Several repetitions will be required with renewed estimates in order to arrive at the correct solution for all unknown values finally.

In contrast to Van der Knaap, who used a so-called explicit finite difference method, the author of this paper uses a implicit finite difference method (Pereboom, 1979). After discretization of the model the differential equations are changed into difference equations.

Subsequently these are worked out into a system of implicit linear equations containing all unknown values of the variables (piezometric levels and flows) in all nodal points. By using the Gauss elimination method (Zaat, 1972) and an iteration procedure

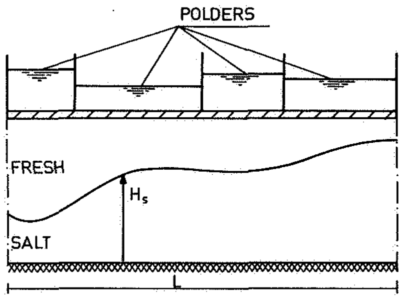


Fig. 1 Model 1.

the solution of the system is obtained. The position of the interface is calculated with the formula of Badon Ghijben-Herzberg.

In the following chapters, respectively, a review will be given on the groundwater flow models (chapter 2), the equations and boundary conditions (chapter 3), the finite difference method as it is in use here (chapter 4) and the results that are obtained (chapter 5). Finally, conclusions and proposals for further investigations will be given (chapter 6).

2. MODELS

2.1. *Model 1*

The first model, see figure 1, describes a system with one aquifer, bounded by an impermeable base and a semi-pervious toplayer. In or above the semi-pervious toplayer different polderlevels are maintained. In the aquifer there is both fresh and salt groundwater. The flow is described as one-dimensional flow; in the aquifer the flow is taken only horizontal and in the semi-pervious toplayer the flow is taken only vertical. The calculations are made for the steady state only.

The values of the geohydrological constants k, D and c and the polderlevels p are supposed to be known just like the lengths of the model (L) and of the individual polders and the densities ρ_f and ρ_s of the fresh and salt groundwater.

The boundary conditions at the left and right hand side of the model are given by the values of the piezometric levels of the fresh groundwater (h_f) and those of the salt groundwater (h_s). By application of the principle of Badon Ghijben-Herzberg the position of the fresh-salt interface H_s at the boundaries can be calculated directly from these boundary values.

It should be mentioned here that in connection with the formulae used, it is not accounted for the fresh-salt interface to touch the impermeable base or to touch or to intersect the semi-pervious toplayer.

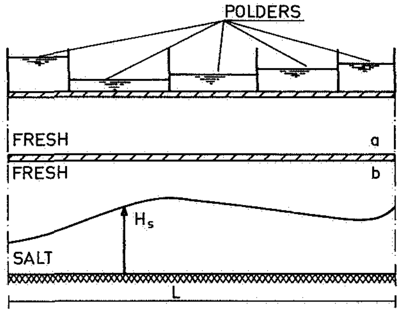


Fig. 2 Model 2.

2.2. *Model 2*

The second model, see figure 2, can be considered to be an extension of the first one. It describes a system with two aquifers, a and b, separated by a semi-pervious layer, bounded by the impermeable base and a semi-pervious toplayer like in the first model. In aquifer b there is both fresh and salt groundwater, in aquifer a there is only fresh groundwater. Again the flow is assumed to be one-dimensional; the flow in the aquifers is taken only horizontal while the flow in the semi-pervious layers is taken only vertical. The flow is in steady state.

The values of k, D, c and p, L, ρ_f and ρ_s are supposed to be known. The boundary conditions at the left and right hand side of the model are given by the piezometric levels of the fresh groundwater in aquifer a (h_{fa}), of the fresh groundwater in aquifer b (h_{fb}) and of the salt groundwater in aquifer b (h_{sb}). From these, the position of the fresh-salt interface H_s at the boundaries can be calculated directly by application of the Badon Ghijben-Herzberg principle. Like in the first model, the position of the fresh-salt interface is restricted to be between the impermeable base and the lowest semi-pervious layer.

3. EQUATIONS AND BOUNDARY VALUES

The flow of groundwater is described by the equations of continuity and motion. The solution of these equations depends on the boundary conditions and in the case of non-steady flow also on the initial conditions. Here, in this paper, the flow is restricted to the steady state.

Referring to the one-dimensional flow of liquids with different densities, which is the case here, the equations of continuity and motion together with the principle of Badon Ghijben-Herzberg and the boundary conditions govern the flow.

Both equations and the principle of Badon Ghijben-Herzberg are given in the Annexes I and II referring to Model 1 and Model 2 respectively. After reworking there remains

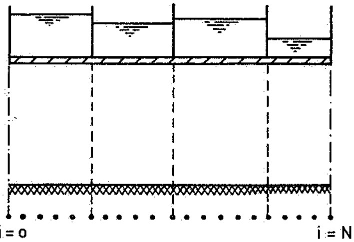


Fig. 3 Discretization.

a system of four differential equations for Model 1 and a system of six differential equations for Model 2. Some of these equations are non-linear in the unknown values of the piezometric levels of fresh and salt groundwater; this is also made clear in the annexes.

As stated above, the solutions of these systems depend also on the boundary values (Annexes I and II). Mathematically, this is a boundary value problem: a solution has to be found for a system of differential equations, governed by boundary values on both sides.

4. THE IMPLICIT FINITE DIFFERENCE METHOD

Boundary value problems can be solved in many ways. A review of the different possibilities is given by Roberts and Shipman (1972), Collatz (1960) and by Zaat (1971).

Here, the finite difference method in its implicit form will be used. A distinction can be made into three parts:

1. discretization of the length of the model;
2. transformation of the differential equations into difference equations and
3. linearization of these equations and use of an iteration procedure in finding the numerical solution.

Each of these three parts asks for a brief explanation:

1. The discretization of the length of the model is to be done in such a way that all points where a change in polderlevel occurs coincides with one of the nodal points (fig. 3).
   As the differential equations hold all over the length of the model, they hold also for any nodal point i and for the so-called "difference points" j, situated half-way between these nodal points (fig. 4).
2. In these "difference points" j the differential equations change into difference equations by using a central difference approximation that uses the values of the variables in the nodal points i and i + 1, on both sides of "difference point" j (fig. 4). So, since in calculations a network is used, it is possible to give k, D, c and p a specific value for each "difference point" j.

in "difference point" j :

$\frac{dhf}{dx}_j \simeq \frac{hf_{i+1} - hf_i}{\Delta x}$

where $i = 0, 1, \ldots, N-1$ (nodal points)

and $j = i + 1/2$ (difference points)

Fig. 4 **Central difference approximation.**

3. The final solution in this calculation method is found after linearization and use of an iteration procedure. As already indicated in chapter 3, some of the differential equations are not linear in the piezometric levels of fresh and salt groundwater. The corresponding set of non-linear difference equations can not easily be solved. A linearization of the difference equations is necessary. This is done by substituting for all nodal points a first estimate of the values of the piezometric levels of the fresh and salt groundwater into the equations.

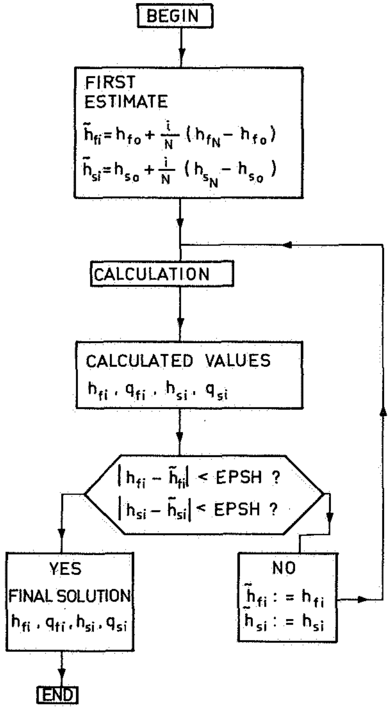


Fig. 5 **Iteration process (Model 1).**

h_{fo} = 300.00 m
h_{fN} = 303.75 m
h_{so} = 294.00 m
h_{sN} = 300.00 m
H_{so} = 54.00 m
H_{sN} = 150.00 m

c = 5 000 d
k = 20 m/d
ρ_s = 1 025 kg/m^3
ρ_f = 1 000 kg/m^3
∞ = 0.025

p_1 = 304 m
p_2 = 298 m
p_3 = 302 m
p_4 = 305 m

For this first estimate a linear interpolation between the known boundary values is used, as illustrated in the Annexes I and II. Generally, after the first calculation, there is a difference between the estimated and calculated values. The newly calculated values are then used as new and better estimates and the calculation is repeated. This iteration procedure comes to an end as soon as the difference between estimated and calculated values becomes smaller than a certain preset criterion (fig. 5).

5. RESULTS

Both for Model 1 and for Model 2 computer calculations have been carried out on the IBM 370/158 computer of the Delft University of Technology Computing Centre. Annex III gives the outlines of this; the results are discussed below.

5.1. *Model 1*

5.1.0. *Standard*

A system is considered, in which four different polderlevels are maintained above the semi-pervious toplayer. Numerical values are given in figure 6.

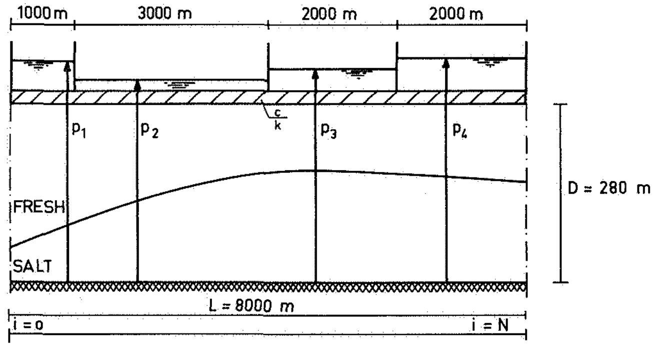


Fig. 6 Model 1. Standard (5.1.0.).

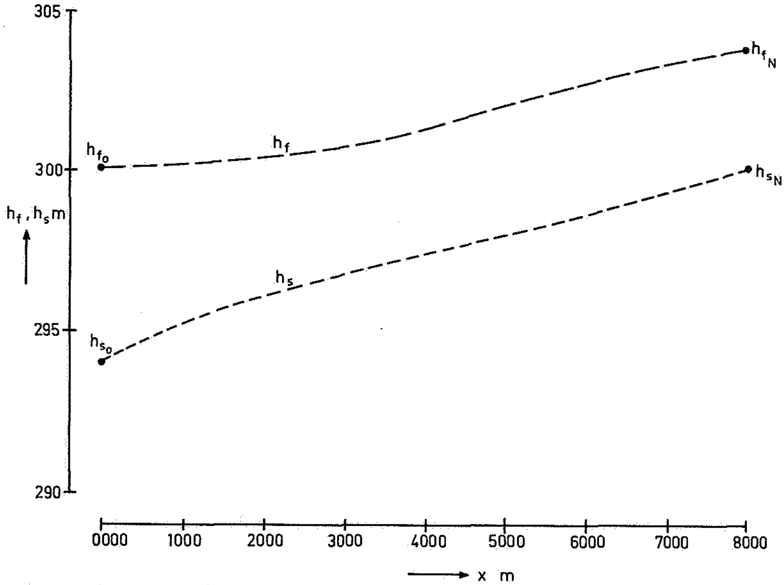


Fig. 7 Results Model 1. Standard (5.1.0.)
Piezometric levels h_f, h_s.

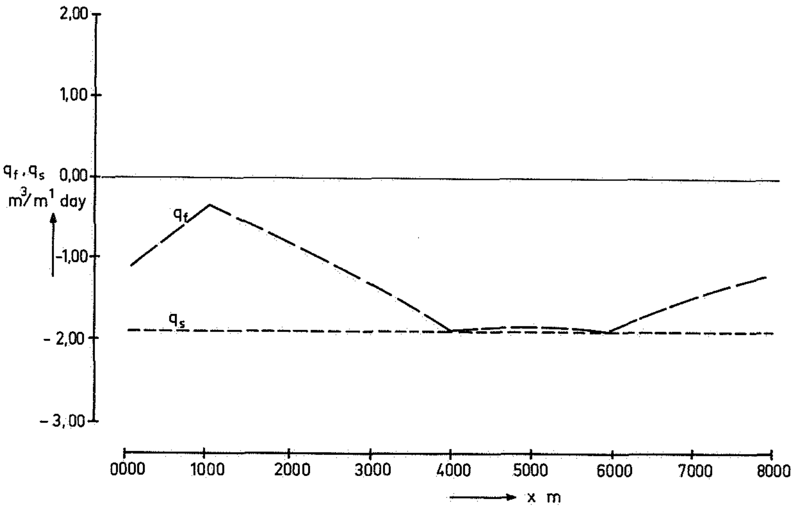


Fig. 8 Results Model 1. Standard (5.1.0.).
Flows q_f, q_s.

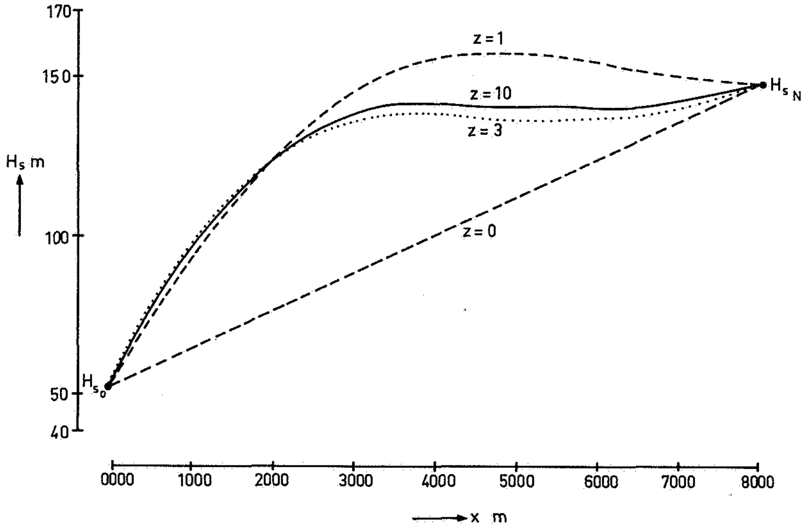


Fig. 9 **Results Model 1. Standard (5.1.0.).**
Position interface H_s for various iteration steps.

For the interval length $\Delta x = 200$ m figures 7, 8 and 9 show the results for h_f and h_s, q_f and q_s and H_s.

As already could be expected by integration of the equation of continuity (Annex I, eq. 9 and 17), the flow of the salt groundwater q_s (fig. 8) is constant over the length of the model.

Figure 9 gives an impression of the iteration process. It shows the calculated position of the fresh-salt interface for different values of steps z in the iteration process. As appears clearly, the position for $z = 3$ is already close to the final position, calculated in iteration step $z = 10$.

5.1.1. *Influence of interval length* Δx

For the total length $L = 8\,000$ m, calculations were not only made for $\Delta x = 200$ m, but also for $\Delta x = 500$ m and $\Delta x = 1\,000$ m.

Table 1 summarizes the number of iteration steps and the cpu-time used for these different interval lengths.

The results for these three calculations do not differ much. The position of the fresh-salt interface, calculated for $\Delta x = 500$ m and $\Delta x = 1\,000$ m, changes with a maximum value of 0.14 m and 0.62 m compared with that for $\Delta x = 200$ m, see figure 10. So, as could be expected, enlarging the interval length Δx causes a more rough way of

Table 1

	number of iteration steps	cpu-time
$\Delta x =$ 200 m	10	9 s
$\Delta x =$ 500 m	10	6 s
$\Delta x =$ 1 000 m	9	5 s

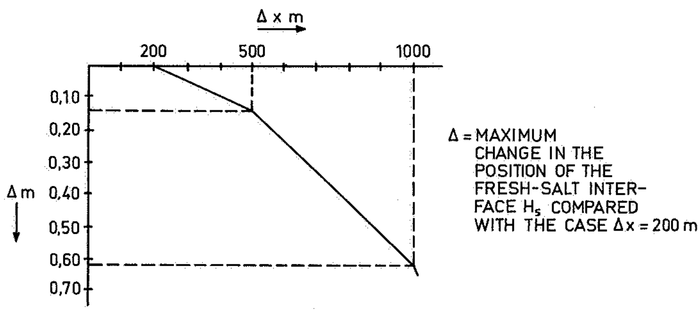


Fig. 10 **Results Model 1.**
Influence of interval length Δx (5.1.1.).
Maximum changes in the position of the fresh-salt interface H_s.

calculating the position of the fresh-salt interface, but still the results are very close to the results with a small Δx-value

5.1.2. *Influence of changes in one of the polderlevels*

The influence of changes in one of the polderlevels is illustrated for $p_2 = 293$ m, $p_2 = 298$ m and $p_2 = 302$ m.

For the interval length $\Delta x = 200$ m, figure 11 shows the position of the fresh-salt interface for these different polderlevels. As could be expected, the position of the interface is the lower the higher the polderlevel is.

Figure 12 shows the rather drastic change in the flow of the fresh groundwater q_f.

In this model a higher polderlevel p_2 causes even a reserval of the flow of fresh groundwater in the left hand part of the model and a decrease in the right hand part. As indicated, the flow of the salt groundwater q_s is also influenced.

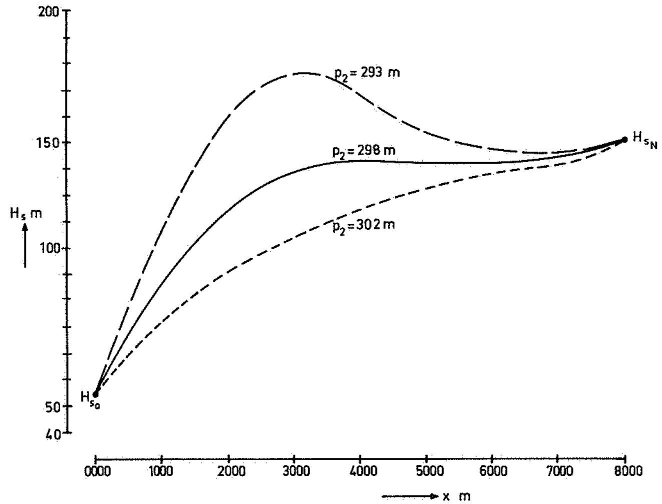


Fig. 11 Results Model 1.
Influence of changes in one of the polderlevels (5.1.2.).
Changes in the position of the fresh-salt interface H_s.

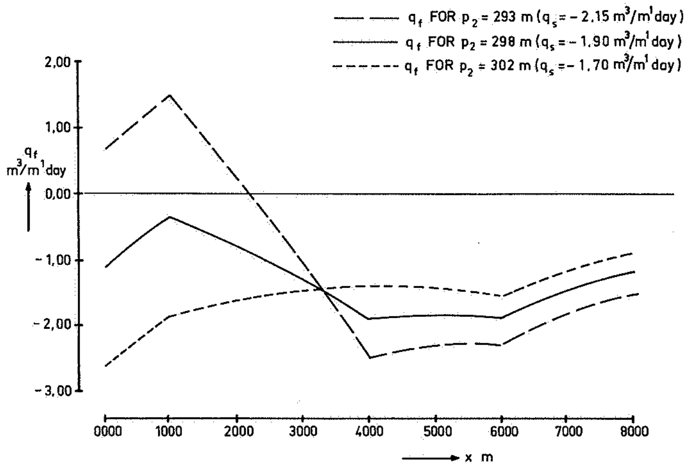


Fig. 12 Results Model 1.
Influence of changes in one of the polderlevels (5.1.2.).
Changes in the flows q_f, q_s.

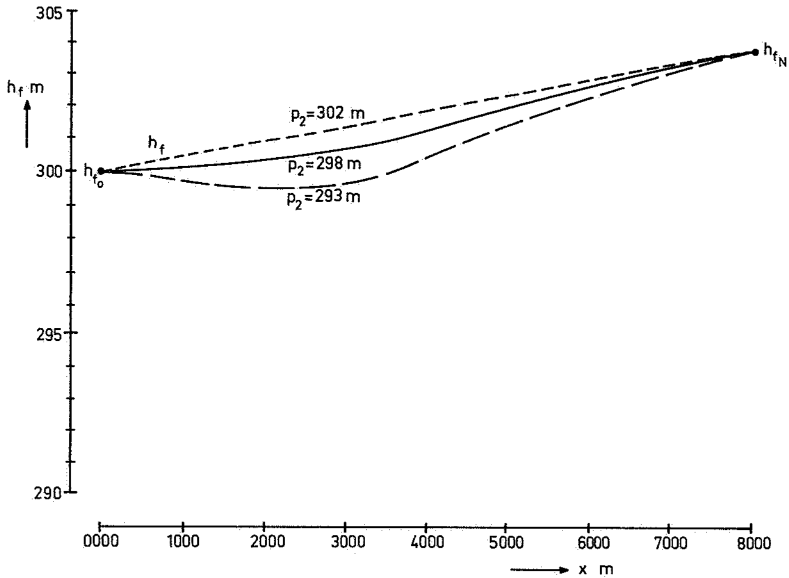


Fig. 13 Results Model 1.
Influence of changes in one of the polderlevels (5.1.2.).
Changes in the piezometric level h_f.

Figure 13, finally, shows how the piezometric level of the fresh groundwater h_f changes with the polderlevel. Changes in h_s were too small to draw.

Table 2 gives the number of iteration steps and the cpu-time that were needed.

Table 2

	number of iteration steps	cpu-time
p_2 = 293 m	11	10 s
p_2 = 298 m	10	9 s
p_2 = 302 m	9	9 s

5.2. *Model 2*

5.2.0. *Standard*

A system is considered in which five different polderlevels are maintained above the semi-pervious toplayer. Numerical values of geohydrological constants, polderlevels, lengths and boundary conditions are given in figure 14.

For the interval length Δx = 200 m figures 15, 16 and 17 shows the results for h_{fa}, h_{fb} and h_{sb}, for q_{fa}, q_{fb} and q_{sb} and for H_s.

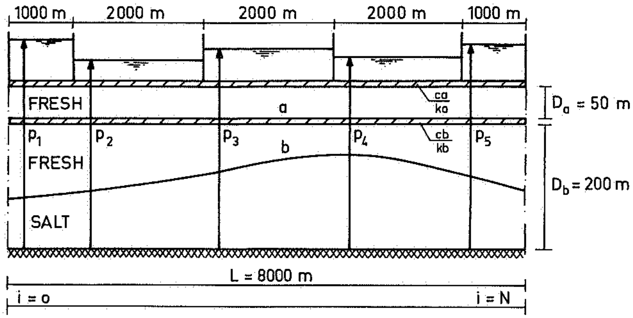


Fig. 14 Model 2. Standard (5.2.0.).

$N = 40$	$h_{fbo} = 278.50$ m	$H_{so} = 79.60$ m
$\Delta x = 200$ m	$h_{fb_N} = 276.50$ m	$H_{s_N} = 103.10$ m
$h_{fao} = 279.00$ m	$h_{sbo} = 274.60$ m	$\rho_s = 1\,020$ kg/m^3
$h_{fa_N} = 276.00$ m	$h_{sb_N} = 273.10$ m	$\rho_f = 1\,000$ kg/m^3
		$\alpha = 0.02$
$k_a = 20$ m/d	$p_1 = 280$ m	$p_4 = 270$ m
$c_a = 5\,000$ d	$p_2 = 269$ m	$p_5 = 278$ m
$k_b = 15$ m/d	$p_3 = 275$ m	
$c_b = 6\,000$ d		

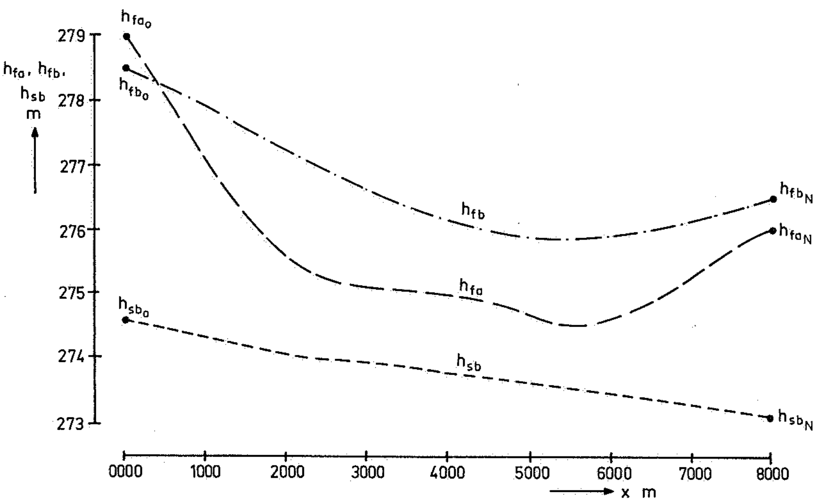


Fig. 15 Results Model 2. Standard (5.2.0.).
Piezometric levels h_{fa}, h_{fb}, h_{sb}.

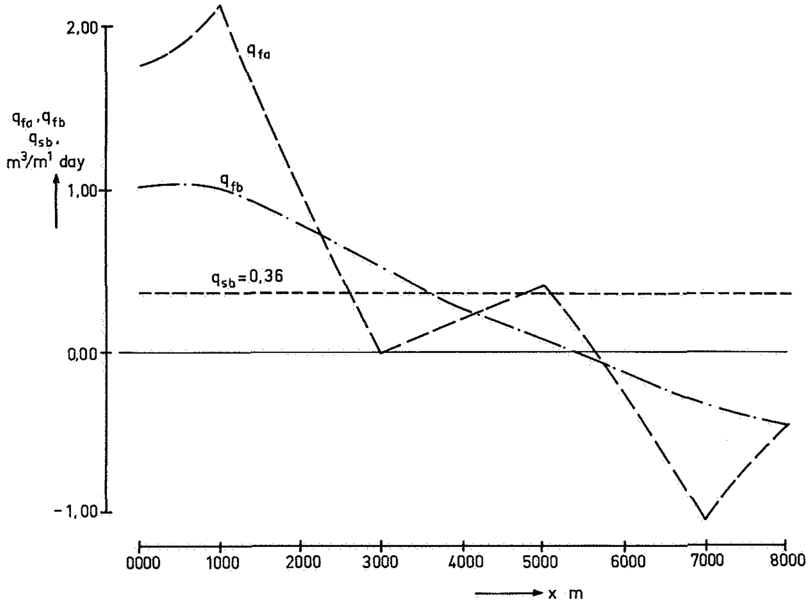


Fig. 16 Results Model 2. Standard (5.2.0.).
Flows q_{fa}, q_{fb}, q_{sb}.

As already could be expected by integration of the equation of continuity (Annex II, eq. 13 and 23), the flow of the salt groundwater q_s (fig. 16) is constant over the length of the model.

Figure 17 gives an impression of the iteration process.

It shows the calculated position of the fresh-salt interface for different values of steps z in the iteration process. As appears clearly, the position for $z = 3$ is already close to the final position, calculated in iteration step $z = 8$.

5.2.1. *Influence of changes in one of the polderlevels*

The influence of changes in one of the polderlevels is illustrated for $p_3 = 272.5$ m, $p_3 = 275$ m and $p_3 = 280$ m.

For the interval lenght Δx = 200 m, figure 18 shows the position of the fresh-salt interface H_s for these different polderlevels. The interface is the lower the higher the polderlevel is.

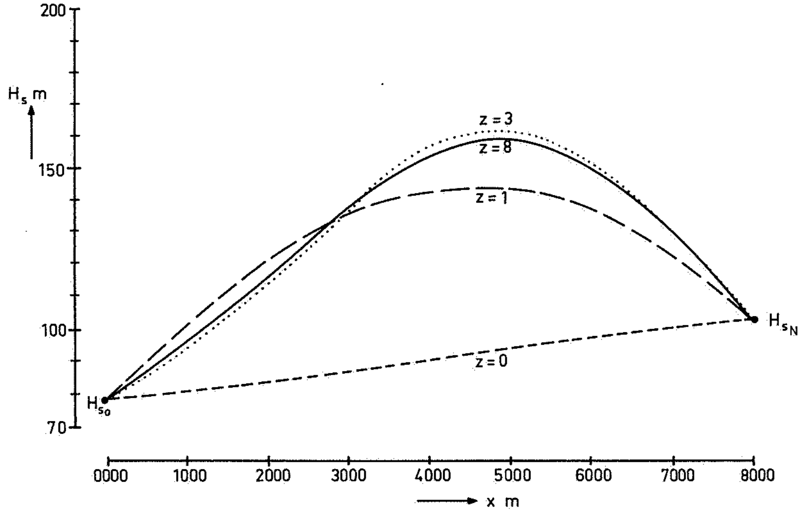


Fig. 17 Results Model 2. Standard (5.2.0.).
Position interface H_s for various iteration steps.

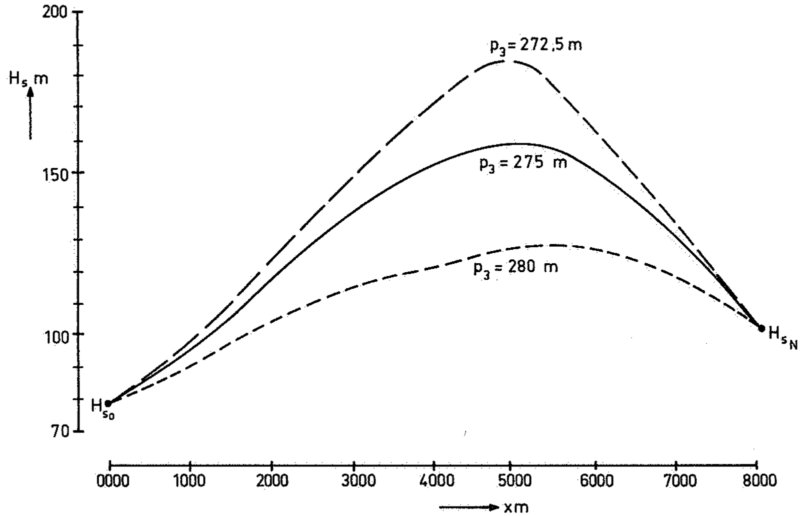


Fig. 18 Results Model 2.
Influence of changes in one of the polderlevels (5.2.1.).
Changes in the position of the fresh-salt interface H_s.

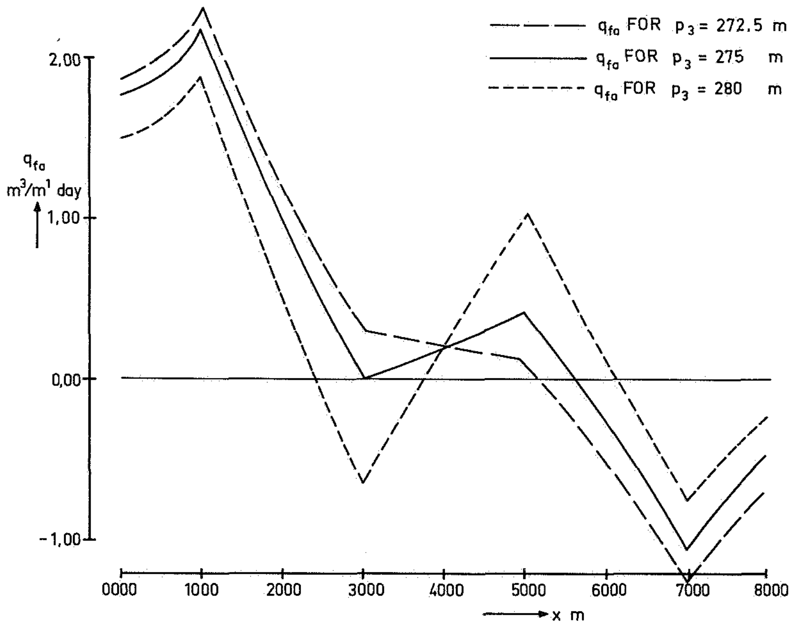


Fig. 19 Results Model 2.
Influence of changes in one of the polderlevels (5.2.1.).
Changes in the flow q_{fa}.

Figure 19 shows the flow q_{fa} and figure 20 the flow q_{fb} for these different polderlevels. As indicated in figure 20, the flow of the salt groundwater q_{sb} is also influenced.

Figures 21 and 22 finally, show the changes in the piezometric levels h_{fa} and h_{fb} respectively, due to the changes in polderlevel. As could be expected, these changes follow the change in polderlevel. This applies also to the piezometric level of the salt groundwater h_{sb}. However, changes in h_{sb} were too small to draw.

Table 3 summarizes the number of iteration steps and the cpu-time used in calculation for these different polderlevels.

Table 3

	number of iteration steps	cpu-time
p_3 = 272.5 m	12	17 s
p_3 = 275 m	8	13 s
p_3 = 280 m	6	11 s

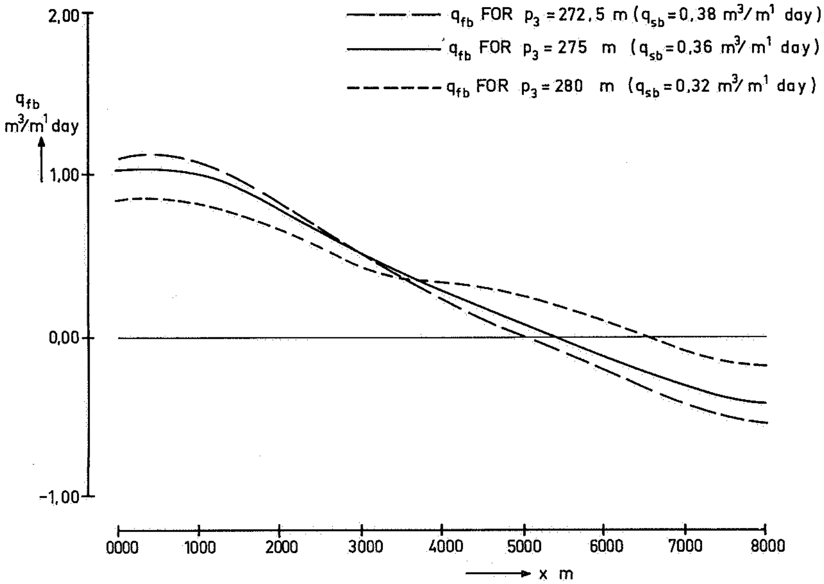


Fig. 20 Results Model 2.
Influence of changes in one of the polderlevels (5.2.1.).
Changes in the flows q_{fb}, q_{sb}.

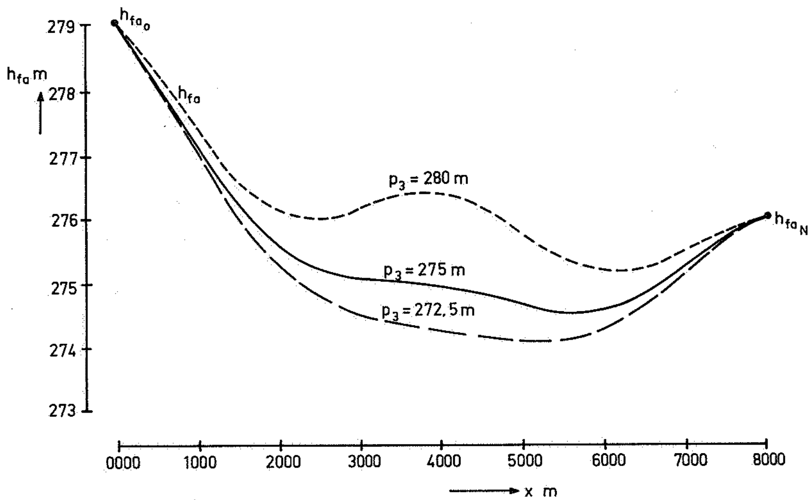


Fig. 21 Results Model 2.
Influence of changes in one of the polderlevels (5.2.1.).
Changes in the piezometric level h_{fa}.

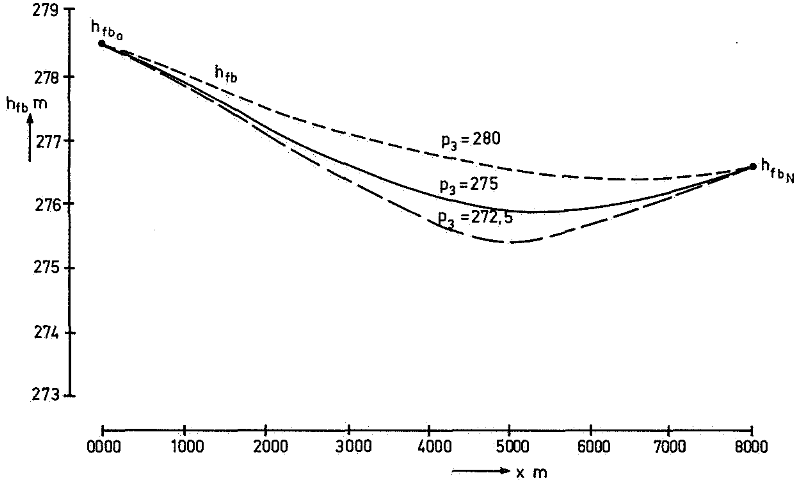


Fig. 22 **Results Model 2.**
Influence of changes in one of the polderlevels (5.2.1.).
Changes in the piezometric level h_{fb}.

5.2.2. *Influence of changes in the value of* $c_{a_{AB}}$

The groundwater system to be discussed now, is almost completely described by the numerical values given in figure 14.

However, now in the third polder the polderlevel p_3 = 280 m is maintained; changes are introduced in the value of c_a over the distance AB, x = 3 000 m to x = 5 000 m, see figure 23. As used before, c_a = 5 000 days. Over the distance AB c_a will be changed into 100 days, which is a very small resistance and into 15 000 days, which is very high.

The influence of these changes on the values of the piezometric levels, the flows and the position of the fresh-salt interface will be studied. The interval length remains Δx= 200 m.

The number of the iteration steps and the cpu-time used in calculation are given in table 4.

Table 4

	number of iteration steps	cpu-time
$c_{a_{AB}}$ = 5 000 days	6	11 s
$c_{a_{AB}}$ = 100 days	6	11 s
$c_{a_{AB}}$ = 15 000 days	7	12 s

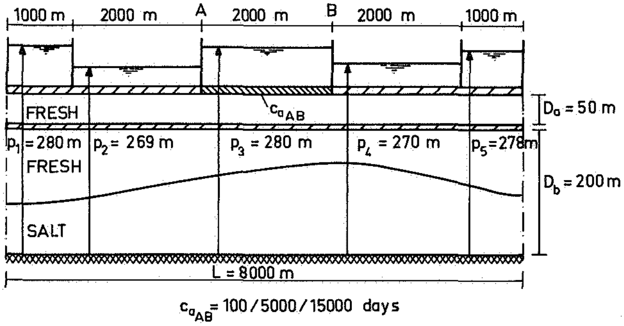


Fig. 23 Model 2.
Influence of changes in the value of $c_{a_{AB}}$ (5.2.2.).

Figure 24 gives the value of h_{fa} for these different $c_{a_{AB}}$-values, figure 25 the values of h_{fb}, figure 26 the values of q_{fa}, figure 27 the values of q_{fb} and q_{sb} and figure 28 the position of the fresh-salt interface H_s. The values of h_{sb} undergo only small changes and have not been drawn.

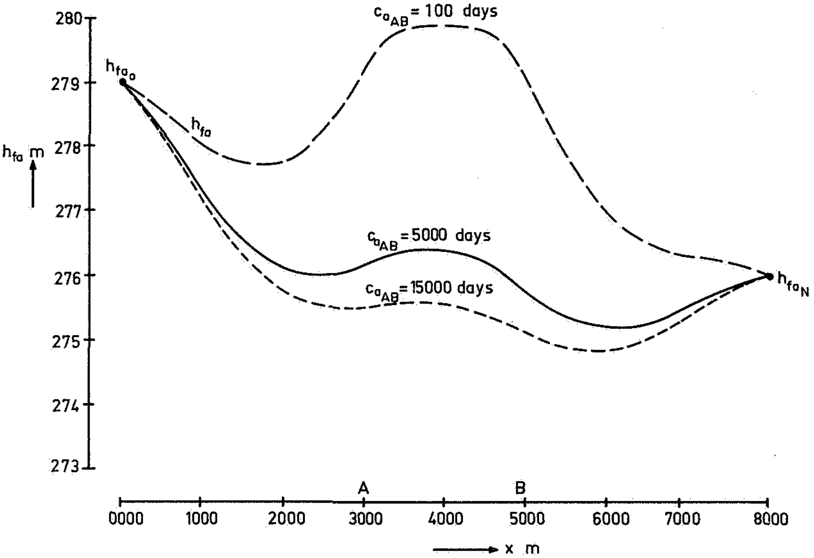


Fig. 24 Results Model 2.
Influence of changes in the value of $c_{a_{AB}}$ (5.2.2.).
Changes in the piezometric level h_{fa}.

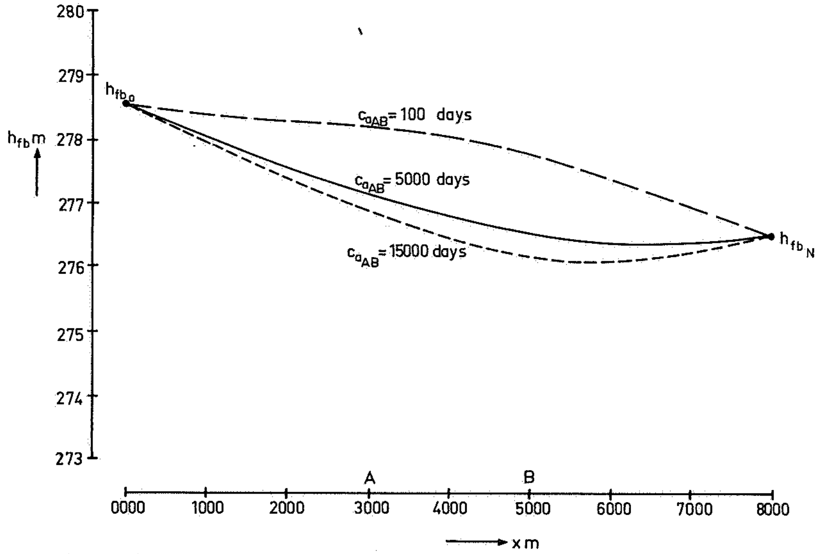


Fig. 25 Results Model 2.
Influence of changes in the value of $c_{a_{AB}}$ (5.2.2.).
Changes in the piezometric level h_{fb}.

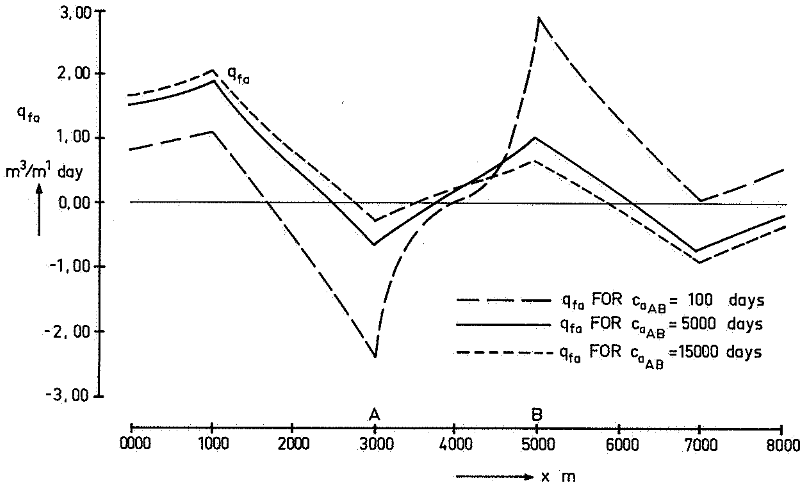


Fig. 26 Results Model 2.
Influence of changes in the value of $c_{a_{AB}}$ (5.2.2.).
Changes in the flow q_{fa}.

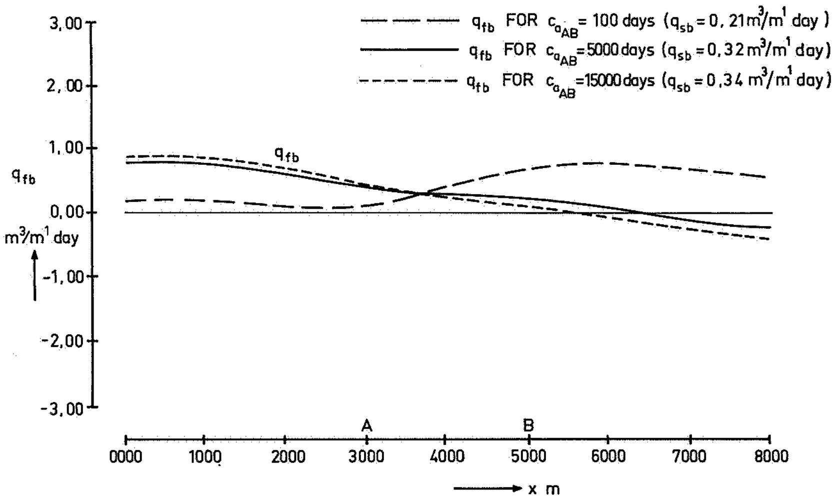


Fig. 27 Results Model 2.
Influence of changes in the value of $c_{a_{AB}}$ (5.2.2.).
Changes in the flows q_{fb}, q_{sb}.

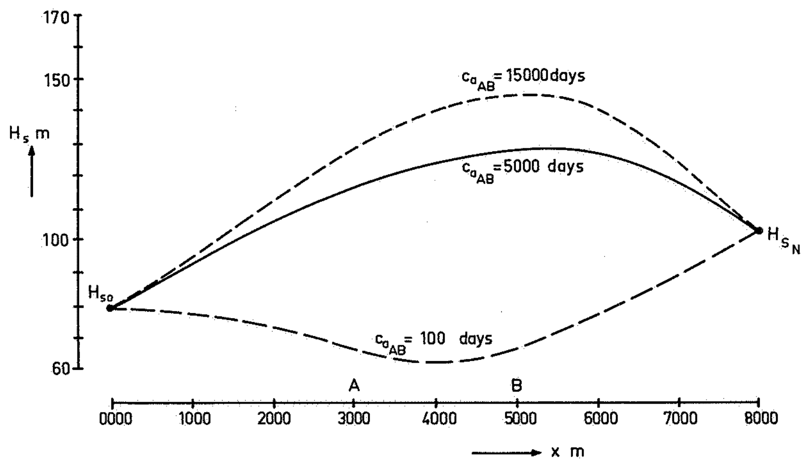


Fig. 28 Results Model 2.
Influence of changes in the value of $c_{a_{AB}}$ (5.2.2.).
Changes in the position of the fresh-salt interface H_s.

Reduction of $c_{a_{AB}}$ from 5 000 days to 100 days increases the influence of the (high) polderlevel p_3 = 280 m. This causes an increase of h_{fa} along AB, almost to the value h_{fa}

$\approx p_3 = 280$ m. The flow q_{fa} increases, as shown in figure 26. Due to the small $c_{a_{AB}}$-value there is very much infiltration along AB, which causes increasing flow in both directions, from x = 4 000 m (middle of the polder) to x = 5 000 m and from x = 4 000 m to x = 3 000 m.

Considering h_{fb} and q_{fb}, the additional influence of the second semi-pervious layer causes a simular, but somewhat reduced effect. The fresh-salt interface is "pushed" down by the reduction of $c_{a_{AB}}$; the flow of the salt groundwater q_{sb} decreases.

The increase of $c_{a_{AB}}$ from 5 000 days to 15 000 days causes the opposite effect: decrease of h_{fa} and q_{fa}, h_{fb} and q_{fb} and a rise of the fresh-salt interface. The flow of the salt groundwater increases somewhat.

Summarizing, the possibility of changing the c_a-value over some distance in the model gives an insight in the effect of the resistance of the semi-pervious toplayer. This can also be done for other geohydrological constants of the system.

6. CONCLUSIONS AND RECOMMENDATIONS

From the text above it becomes clear that the implicit finite difference method as it has been used here gives good results when applied to the fresh-salt groundwater models described in this paper. The influence of the interval length Δx was studied and yielded acceptable results. Though not mentioned in this paper, the results achieved with this method are in good agreement with those achieved with other calculation methods (Pereboom, 1979).

Therefore the use of an implicit finite difference method for flow problems of fresh and salt groundwater, often being boundary value problems, is recommended.

However, if one wants to compute practical cases, it will be clear that the approach followed needs further development. Among others, attention can be paid to the following points:

1. Application to the non-steady state.
2. Tackling of more complex flow problems: two-dimensional (both in the horizontal and in the vertical place) or even three-dimensional problems, in stead of only one-dimensional ones.
3. Tackling of the problems that arise if the fresh-salt interface reaches the impermeable base or intersects one or more semi-pervious layers.
4. Introduction of inhomogenity and anisotropy.

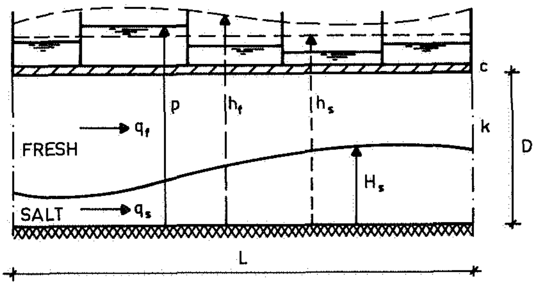
With respect to point 1, it will probably be possible to apply a finite difference method also to non-steady state flow problems. With regard to point 2, probably a finite elements method is better suitable for this kind of problems.

The problems raised in 3 and 4 should be studied further.

REFERENCES

COLLATZ, L. (1960) – The numerical treatment of differential equations. Springer Verlag, Berlin.
DAM, J.C. VAN (1976) – Partial depletion of saline groundwater by seepage. Journal of Hydrology, Vol. 29, No. 3/4: 315-339.
KNAAP, F.C.M. VAN DER (1977) – Verschillende rekenwijzen ter bepaling van de diepteligging van het grensvlak tussen zout en zoet water. Master Thesis, Department of Civil Engineering, Delft University of Technology.
PEREBOOM, D. (1979) – Onderzoek naar de ligging van het zoet-zout grensvlak in een vijftal geohydrologische profielen met behulp van de eindige differentie methode. Master Thesis, Department of Civil Engineering, Delft University of Technology.
ROBERTS, S.M. and J.S. SHIPMAN (1972) – Two point-boundary value problems: shooting methods. American Elsevier Publishing Company Inc. New York.
ZAAT, J.A. (1972) – Numerieke methoden in de lineaire algebra. Notes on Lectures A 107, Delft Inc., University of Technology.
ZAAT, J.A. (1971) – Numerieke Analyse A1. Delftsche Uitgevers Maatschappij NV, Delft.

ANNEX I



MODEL 1

Fresh groundwater:

Continuity : $$\frac{dq_f}{dx} = \frac{p - h_f}{c} \quad 1$$

Motion : $$\frac{dh_f}{dx} = \frac{-q_f}{k(D - H_s)} \quad 2$$

Salt groundwater:

Continuity : $$\frac{dq_s}{dx} = 0 \quad 3$$

Motion : $$\frac{dh_s}{dx} = \frac{-q_s}{k\frac{\rho_s}{\rho_f}H_s} \quad 4$$

Badon Ghijben-Herzberg:

$$h_f - h_s = \alpha(h_s - H_s) \qquad 5$$

where $\alpha = (\rho_s - \rho_f)/\rho_f$ 6

5 into 2 and 4 together with 1 and 3:

$$\frac{dq_f}{dx} = \frac{p - h_f}{c} \qquad 7$$

$$\frac{dh_f}{dx} = \frac{-q_f}{k\,[D - \frac{1}{\alpha}(\frac{\rho_s}{\rho_f}h_s - h_f)]} \qquad 8$$

$$\frac{dq_s}{dx} = 0 \qquad 9$$

$$\frac{dh_s}{dx} = \frac{-q_s}{k\frac{\rho_s}{\rho_f}\frac{1}{\alpha}(\frac{\rho_s}{\rho_f}h_s - h_f)} \qquad 10$$

8 and 10 are non-linear in h_f and h_s

Boundary values:

$$h_f(x = o) = h_{fo} \qquad 11$$
$$h_f(x = L) = h_{fL} \qquad 12$$
$$h_s(x = o) = h_{so} \qquad 13$$
$$h_s(x = L) = h_{sL} \qquad 14$$

Linearization, first approximation:

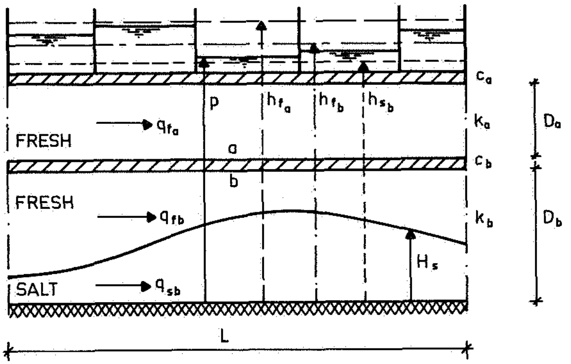
$$h_{fi} = h_{fo} + \frac{i}{N}(h_{f_N} - h_{fo}) \qquad 15$$

$$h_{si} = h_{so} + \frac{i}{N}(h_{s_N} - h_{so}) \qquad 16$$

Integration of 9:

$$q_s = \text{constant} \qquad 17$$

ANNEX II



MODEL 2

Fresh groundwater a:

Continuity : $$\frac{dq_{fa}}{dx} = \frac{p - h_{fa}}{ca} + \frac{h_{fb} - h_{fa}}{c_b} \qquad 1$$

Motion : $$\frac{dh_{fa}}{dx} = \frac{-q_{fa}}{k_a D_a} \qquad 2$$

Fresh groundwater b:

Continuity : $$\frac{dq_{fb}}{dx} = \frac{h_{fa} - h_{fb}}{c_b} \qquad 3$$

Motion : $$\frac{dh_{fb}}{dx} = \frac{-q_{fb}}{k_b (D_b - H_s)} \qquad 4$$

Salt groundwater b:

Continuity : $$\frac{dq_{sb}}{dx} = 0 \qquad 5$$

Motion : $$\frac{dh_{sb}}{dx} = \frac{-q_{sb}}{k_b \frac{\rho_s}{\rho_f} H_s} \qquad 6$$

Badon Ghijben-Herzberg:

$$h_{fb} - h_{sb} = \propto (h_{sb} - H_s) \qquad 7$$

where $$\propto = (\rho_s - \rho_f)/\rho_f \qquad 8$$

7 into 4 and 6 together with 1, 2, 3 and 5:

$$\frac{dq_{fa}}{dx} = \frac{p - h_{fa}}{c_a} + \frac{h_{fb} - h_{fa}}{c_b} \qquad 9$$

$$\frac{dh_{fa}}{dx} = \frac{-q_{fa}}{k_a D_a} \qquad 10$$

$$\frac{dq_{fb}}{dx} = \frac{h_{fa} - h_{fb}}{c_b} \qquad 11$$

$$\frac{dh_{fb}}{dx} = \frac{-q_{fb}}{k_b\,[D_b - \frac{1}{\alpha}(\frac{\rho_s}{\rho_f} h_{sb} - h_{fb})]} \qquad 12$$

$$\frac{dq_{sb}}{dx} = 0 \qquad 13$$

$$\frac{dh_{sb}}{dx} = \frac{-q_{sb}}{k_b \frac{\rho_s}{\rho_f} \frac{1}{\alpha} (\frac{\rho_s}{\rho_f} h_{sb} - h_{fb})} \qquad 14$$

12 and 14 are non-linear in h_{sb} and h_{fb}

Boundary values:

$$h_{fa}(x = o) = h_{fao} \qquad 15$$

$$h_{fa}(x = L) = h_{fa_L} \qquad 16$$

$$h_{fb}(x = o) = h_{fbo} \qquad 17$$

$$h_{fb}(x = L) = h_{fb_L} \qquad 18$$

$$h_{sb}(x = o) = h_{sbo} \qquad 19$$

$$h_{sb}(x = L) = h_{sb_L} \qquad 20$$

Linearization, first approximation:

$$\tilde{h}_{fbi} = h_{fbo} + \frac{i}{N}(h_{fb_N} - h_{fbo}) \qquad 21$$

$$\tilde{h}_{sbi} = h_{sbo} + \frac{i}{N}(h_{sb_N} - h_{sbo}) \qquad 22$$

Integration of 13:

$$q_{sb} = \text{constant} \qquad 23$$

ANNEX III

RESULTS

5.1. MODEL 1

5.1.0.	*Standard*	fig. 6
	Piezometric levels h_f, h_s	fig. 7
	Flows q_f, q_s	fig. 8
	Position interface H_s	fig. 9
5.1.1.	*Influence of interval length* Δx	
	(Δx = 200 m, Δx = 500 m, Δx = 1 000 m)	
	Table 1: number of iteration steps; cpu-time	table 1
	Maximum changes in the position of the interface H_s	fig. 10
5.1.2.	*Influence of changes in one of the polderlevels*	
	(p_2 = 293 m, p_2 = 298 m and p_2 = 302 m)	
	Table 2: number of iteration steps; cpu-time	table 2
	Changes in the position of the interface H_s	fig. 11
	Changes in the flows q_f, q_s	fig. 12
	Changes in the piezometric level h_f	fig. 13

5.2. MODEL 2

5.2.0.	*Standard*	fig. 14
	Piezometric levels h_{fa}, h_{fb}, h_{sb}	fig. 15
	Flows q_{fa}, q_{fb}, q_{sb}	fig. 16
	Position interface H_s	fig. 17
5.2.1.	*Influence of changes in one of the polderlevels*	
	(p_3 = 272.5 m, p_3 = 275 m and p_3 = 280 m)	
	Table 3: number of iteration steps; cpu-time	table 3
	Changes in the position of the interface H_s	fig. 18
	Changes in the flow q_{fa}	fig. 19
	Changes in the flows q_{fb}, q_{sb}	fig. 20
	Changes in the piezometric level h_{fa}	fig. 21
	Changes in the piezometric level h_{fb}	fig. 22

5.2.2. *Influence of changes in the value of* $c_{a_{AB}}$ fig. 23

($c_{a_{AB}}$ = 100 days, $c_{a_{AB}}$ = 5 000 days, $c_{a_{AB}}$ = 15 000 days)

Table 4: number of iteration steps; cpu-time table 4

Changes in the piezometric level h_{fa} fig. 24

Changes in the piezometric level h_{fb} fig. 25

Changes in the flow q_{fa} fig. 26

Changes in the flows q_{fb}, q_{sb} fig. 27

Changes in the position of the interface H_s fig. 28.

SOME RESULTS OBTAINED WITH AN ANALYTICAL SOLUTION IN A RADIAL SYMETRIC PROFILE

G.L. MOLENKAMP

SUMMARY

There are several analytical methods for the solution of problems with simultaneous flow of fresh and saline groundwater. For symmetric geohydrologic profiles (e.g. square, rectangular or circular) it is possible to arrive at a solution by a temporary linearization of the equations that describe the groundwater flow.

In the first chapters the mathematical basis will be presented and illustrated by an example. Then several cases of steady flow in radial symmetric geohydrologic profiles will be considered. The method used can, after expansion, also be applied to non-steady flows.

1. INTRODUCTION

The steady groundwater problem, described in the following chapters, was considered in order to find a solution for the simultaneous flow of fresh and saline groundwater in a radial symmetric profile. (Molenkamp, 1978).

There are several methods available to solve simultaneous groundwater flows. Some of them can only be used in rather simple problems (trial and error method; Van Dam, 1976), others may lead to a very slow convergence process (difference methods). Recently an analytical solution was found by Sikkema that will soon be published.

The mathematical arsenal, used in the following chapters, consists of a method to obtain the real roots of simultaneous equations and of an Approximative Solution Method, APSOM, to solve integrals.

APSOM is based on a temporary linearization (Dronkers, 1969; Annex IV) of the basic equations that describe groundwater flow:

- equation of continuity;
- equation of motion (Darcy).

This linearization makes it possible to arrive at an analytical solution.

The advantage of a temporary linearization will become clear after a comparison, outside the scope of these chapters, with methods that make use of a permanent linearization, as some difference methods do.

- A temporary linearization shows a rapid convergence of the start values, necessary to begin the computation, to the final results.
- A permanent linearization causes often a very slow, computer time consuming, convergence.

2. CALCULATION OF THE REAL ROOTS OF SIMULTANEOUS NON-LINEAR EQUATIONS

Consider two non-linear equations $f_1(x,y) = 0$ and $f_2(x,y) = 0$ with the approximate roots x_1 and y_1. Expanding the functions $f_1(x,y)$ and $f_2(x,y)$ in the near vicinity of x_1 and y_1 in a Taylor series yields:

$$f_1(x,y) = f_1 + \frac{\partial f_1}{\partial x}\cdot\Delta x + \frac{\partial f_1}{\partial y}\cdot\Delta y + \ldots\ldots = 0 \qquad (1a)$$

$$f_2(x,y) = f_2 + \frac{\partial f_2}{\partial x}\cdot\Delta x + \frac{\partial f_2}{\partial y}\cdot\Delta y + \ldots\ldots = 0 \qquad (1b)$$

where $f_1 = f_1(x_1, y_1)$
$f_2 = f_2(x_1, y_1)$
$\Delta x = x - x_1$
$\Delta y = y - y_1$
(x,y) : the exact solution, that is why: $f_1(x,y) = 0$.
$f_2(x,y) = 0$

Truncation of the Taylor series expansions after two terms, as in the equations (1a) and (1b), gives

$$\frac{\partial f_1}{\partial x}\cdot\Delta x + \frac{\partial f_1}{\partial y}\cdot\Delta y = -f_1$$

$$\frac{\partial f_2}{\partial x}\cdot\Delta x + \frac{\partial f_2}{\partial y}\cdot\Delta y = -f_2$$

or, written in matrix form:

$$\begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix} \cdot \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = - \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

These equations are solved for Δx and Δy and, with $x_2 = x_1 + \Delta x$ and $y_2 = y_1 + \Delta y$, one has obtained new and better estimates for the roots x and y. With these new estimates the calculation can be repeated, which results in still better estimates, etc. etc. An example is given in Annex I.

In the case of more than two simultaneous non-linear equations, the same procedure can be followed. Then the calculation of the corrections must be performed by solution of a system of simultaneous linear equations. By expansion of the functions in Taylor series, in the same manner as before, one obtains the following system:

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1}\cdot\Delta x_1 + \frac{\partial f_1}{\partial x_2}\cdot\Delta x_2 + \frac{\partial f_1}{\partial x_3}\cdot\Delta x_3 + & \ldots & + \frac{\partial f_1}{\partial x_n}\cdot\Delta x_n = -f_1 \\ \frac{\partial f_2}{\partial x_1}\cdot\Delta x_1 + \frac{\partial f_2}{\partial x_2}\cdot\Delta x_2 + \frac{\partial f_2}{\partial x_3}\cdot\Delta x_3 + & \ldots & + \frac{\partial f_2}{\partial x_n}\cdot\Delta x_n = -f_2 \\ \frac{\partial f_3}{\partial x_1}\cdot\Delta x_1 + \frac{\partial f_3}{\partial x_2}\cdot\Delta x_2 + \frac{\partial f_3}{\partial x_3}\cdot\Delta x_3 + & & + \frac{\partial f_3}{\partial x_n}\cdot\Delta x_n = -f_3 \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \frac{\partial f_n}{\partial x_1}\cdot\Delta x_1 + \frac{\partial f_n}{\partial x_2}\cdot\Delta x_2 + \frac{\partial f_n}{\partial x_3}\cdot\Delta x_3 + & \ldots & + \frac{\partial f_n}{\partial x_n}\cdot\Delta x_n = -f_n \end{bmatrix}$$

By means of a matrix this is written as:

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} \frac{\partial f_1}{\partial x_2} \frac{\partial f_1}{\partial x_3} & \ldots\ldots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} \frac{\partial f_2}{\partial x_2} \frac{\partial f_2}{\partial x_3} & & \frac{\partial f_2}{\partial x_n} \\ \frac{\partial f_3}{\partial x_1} \frac{\partial f_3}{\partial x_2} \frac{\partial f_3}{\partial x_3} & & \frac{\partial f_3}{\partial x_n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \frac{\partial f_n}{\partial x_1} \frac{\partial f_n}{\partial x_2} \frac{\partial f_n}{\partial x_3} & \ldots\ldots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \cdot \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \\ \\ \Delta x_n \end{bmatrix} = - \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ \\ \\ f_n \end{bmatrix}$$

For the solution of this system of equations a method will be used that is known as 'exchange process' or 'pivoting', (Zaat, 1971).

3. AN APPROXIMATIVE SOLUTION METHOD, APSOM, TO SOLVE INTEGRALS

3.1. *Modification of the equations that describe the groundwater flow*

Figure 1 shows a geohydrologic profile.

The aquifer as well as the semi-pervious layer are homogeneous. The groundwater in the semi-pervious layer and in the upper part of the aquifer is fresh. There is a sharp interface between fresh and saline groundwater. The fresh groundwater is in steady state, the saline groundwater does not flow.

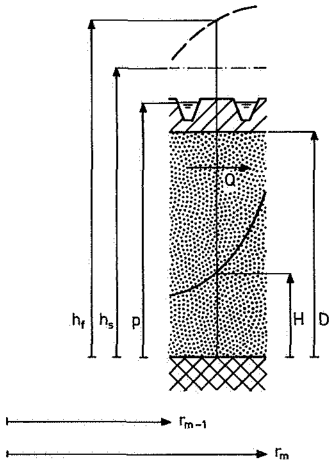


Fig. 1 Geohydrologic profile with symbols.

h_f	: fresh water potential above reference level	[L]
p	: polder water level above reference level	[L]
h_s	: saline water potential above reference level (constant)	[L]
D	: aquifer thickness	[L]
H	: height of the interface above reference level	[L]
c	: resistance of the semi-pervious layer	[T]
k	: coefficient of permeability	$[LT^{-1}]$
q_f	: fresh water discharge	$[L^3 L^{-1} T^{-1}]$
ρ_f	: density of fresh water	$[ML^{-3}]$
ρ_s	: density of saline water	$[ML^{-3}]$
α	: $\alpha = \dfrac{\rho_s - \rho_f}{\rho_f}$	[–]

The equilibrium between fresh and saline groundwater, generally referred to as the Badon Ghijben-Herzberg relation, and the equations of motion and continuity, per unit width, give:

Equation of motion (Darcy):

$$q \quad = -k(D - H)\frac{dh_f}{dx} \tag{2a}$$

Equation of continuity:

$$dq \quad = -\frac{(h_f - p)}{c}dx \tag{2b}$$

Badon Ghijben-Herzberg relation:

$$h_f - h_s = (h_s - H)\cdot\alpha \tag{2c}$$

The elimination of H, by means of equation (2c), from equations (2a) and (2b) gives:

$$q = -\frac{k}{\alpha}[\alpha D + h_f - (\alpha+1)\,h_s]\,\frac{dh_f}{dx} \tag{3a}$$

$$dq = -\frac{(h_f - p)}{c}\,dx \tag{3b}$$

Until recently it was not possible to find an exact solution to these two equations (the analytical solutions of these equations were found by Sikkema and will soon be published) but a good approximate solution was obtained by replacing the 'free' variable h_f in the equations by its average value over an interval (Annex IV). The word 'free' means here: not belonging to a differential quotient $\frac{d(...)}{dx}$.

A consequence of this approach is the replacement of the expressions $\int h_f dh_f$ and $\int h_f dx$ by $\bar{h}_f \int dh_f$ and $\bar{h}_f \int dx$ respectively. In the following chapters this procedure is called APSOM.

Although equation (3a) càn be integrated without the help of APSOM (h_s is constant in this example and the integration of $h_f dh_f$ does not give any problem), it will consequently by maintained in the next chapters that in every equation 'free' variables are to be averaged over an interval.

A very simple example of this approach follows in the next paragraph. For illustration the solution of this problem will be given analytically, with a difference method and with the help of APSOM.

3.2. APSOM *applied to a mathematical example*

Suppose that one wants to integrate the function $f(x) = h\,\frac{dh}{dx}$ (see figure 2).

Analytically:

$$F(x) = \int_{x_1}^{x_2} f(x)\,dx \triangleq \int_{h_1}^{h_2} h\,dh = [\tfrac{1}{2}h^2]_{h_1}^{h_2} = \tfrac{1}{2}\,[h_2^2 - h_1^2]$$

With a difference method:

$$f(x) = h\frac{dh}{dx} \approx h\frac{\Delta h}{\Delta x}$$

$$F(x) = \int_{x_1}^{x_2} f(x)\,dx \approx f(x)\cdot\Delta x \triangleq h\cdot\Delta h \approx \bar{h}\,[h_2 - h_1] = \tfrac{1}{2}\,[h_1 + h_2]\,[h_2 - h_1]$$
$$= \tfrac{1}{2}\,[h_2^2 - h_1^2]$$

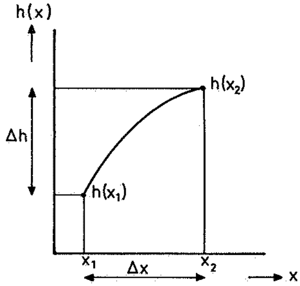


Fig. 2 The function f(x).

With APSOM:

$$F(x) = \int_{x_1}^{x_2} f(x)\,dx \triangleq \int_{h_1}^{h_2} h\,dh \approx \int_{h_1}^{h_2} \bar{h}\,dh = \bar{h}\int_{h_1}^{h_2} dh \qquad = \bar{h}\,[h]_{h_1}^{h_2}$$

$$= \tfrac{1}{2}\,[h_1 + h_2]\,[h_2 - h_1]$$

$$= \tfrac{1}{2}\,[h_2^2 - h_1^2]$$

As one sees in this, very simple, example all three methods yield the same results.

3.3. APSOM *applied to groundwater flow*

3.3.1. *Two concentric polders the underground of which consists of an aquifer and a semi-pervious toplayer above which different polder water levels are maintained.*

Figure 3 shows a cross-section through two concentric polders. The circular inner polder, with a low polder water level, is surrounded by an annular polder with a high polder water level. The geohydrologic profile consists of an aquifer bounded by an impervious base, a semi-pervious toplayer and by a circular impervious wall. Both the aquifer and the semi-pervious layer are homogeneous.

The flow of fresh groundwater, caused by the difference of the polder water levels, is in steady state. The saline groundwater under the fresh groundwater in the aquifer is stagnant. The interface between fresh and saline groundwater is sharp.

In spite of the variable height of the profile that is available for the flow of fresh groundwater, D-H, the vertical flow component in the aquifer is neglected. This does not cause great errors as long as these variations are small with respect to the radius of the polder.

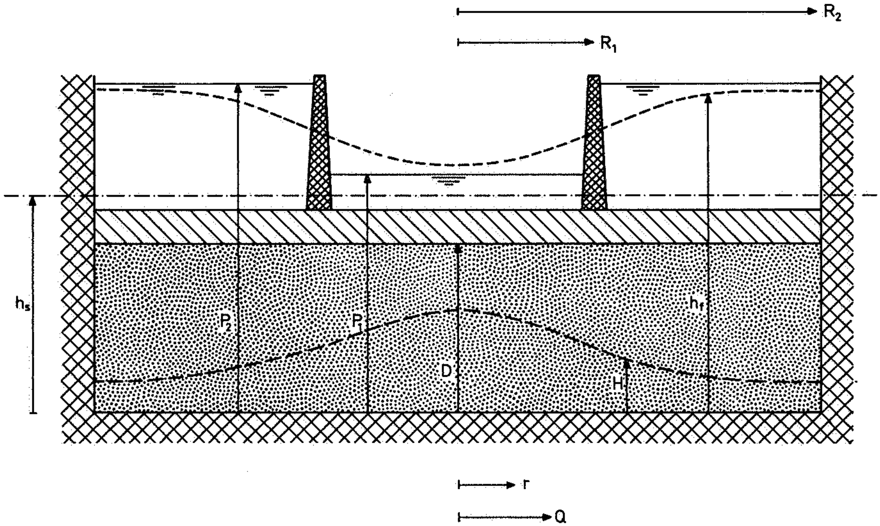


Fig. 3 Cross-section through two concentric polders.

R_1	:	radius of the inner polder	[L]
R_2	:	radius of the outer polder	[L]
k	:	coefficient of permeability	$[LT^{-1}]$
c	:	resistance of the semi-pervious toplayer	[T]
p_1	:	polder water level in the inner polder above reference level	[L]
p_2	:	polder water level in the outer polder above reference level	[L]
h_f	:	fresh water potential above reference level	[L]
h_s	:	saline water potential above reference level (constant)	[L]
Q	:	fresh water discharge	$[L^3 T^{-1}]$
D	:	aquifer thickness	[L]
H	:	height of the interface above reference level	[L]
ρ_f	:	density of fresh water	$[ML^{-3}]$
ρ_s	:	density of saline water	$[ML^{-3}]$
α	:	$\alpha = \dfrac{\rho_s - \rho_f}{\rho_f}$	[–]

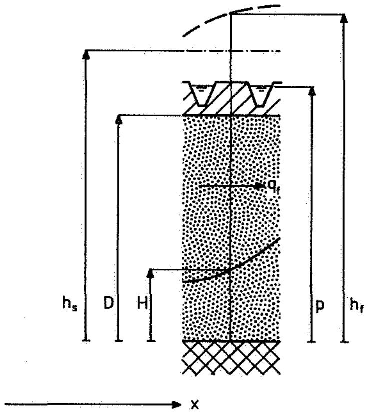


Fig. 4 **Geohydrologic profile with symbols.**

For an annular space between $r = r_{m-1}$ and $r = r_m$, as depicted in figure 4, the following equations can be written:

Equation of motion (Darcy):

$$Q \quad = -2\pi rk(D-H)\frac{dh_f}{dr} \tag{4a}$$

Equation of continuity:

$$dQ \quad = -\frac{2\pi_r}{c}(h_f - p)\,dr \tag{4b}$$

Badon Ghijben-Herzberg relation:

$$h_f - h_s = (h_s - H)\cdot\alpha \tag{4c}$$

The elimination of H, by means of equation (4c), from equations (4a) and (4b) gives:

$$Q \quad = -\frac{2\pi rk}{\alpha}[\alpha D + h_f - (\alpha+1)h_s]\,\frac{dh_f}{dr} \tag{5a}$$

$$dQ \quad = -\frac{2\pi r}{c}(h_f - p)dr \tag{5b}$$

With the help of APSOM the following two equations can be derived from the two equations above. They are comparable with $f_1(x, y)$ and $f_2(x, y)$ in chapter 2.

$$f_{2m-1}(h_{f_{m-1}}, Q_{m-1}, h_{f_m}, Q_m) = h_{f_m} - h_{f_{m-1}} + \frac{\alpha Q_{m-1} \ln\left(\frac{r_m}{r_{m-1}}\right)}{\pi k\,[2\alpha D + h_{f_{m-1}} + h_{f_m} - 2h_s\,(1+\alpha)]} - \frac{\alpha\,[h_{f_{m-1}} + h_{f_m} - 2p]\,[r_m^2 - r_{m-1}^2 - 2r_{m-1}^2 \ln\left(\frac{r_m}{r_{m-1}}\right)]}{4kc\,[2\alpha D + h_{f_{m-1}} + h_{f_m} - 2h_s\,(1+\alpha)]} \tag{6a}$$

$$f_{2m}(h_{f_{m-1}}, Q_{m-1}, h_{f_m}, Q_m) = Q_m - Q_{m-1} + \frac{\pi}{2c}\,[h_{f_{m-1}} + h_{f_m} - 2p]\,[r_m^2 - r_{m-1}^2] \tag{6b}$$

How to arrive at the equations (6a) and (6b), starting from (5a) and (5b), is shown in Annex II. This annex contains also the derivatives of (6a) and (6b) which are required to compute the roots of the equations (see chapter 2).

The equations (6a) and (6b) hold for compartment m; they can be applied to each compartment.

At every compartment boundary there are two unknowns: h_f and Q. As n compartments are bounded by n+1 boundaries, this means that there are n. 2 equations and (n + 1). 2 unknowns. So, to get as many unknowns as equations, one needs two boundary conditions. In the above example (see also figure 3) these are:

$$Q(r = 0) = Q_0 = 0 \text{ and } Q(r = R_2) = Q_n = 0.$$

Equations (6a) and (6b) will be used to compute the unknowns h_f and Q at the compartment boundaries just as in chapter 2 the functions $f_1(x, y)$ and $f_2(x, y)$ were used to compute the unknowns x and y. In figure 5 the compartment boundaries are shown and the unknowns at every boundary.

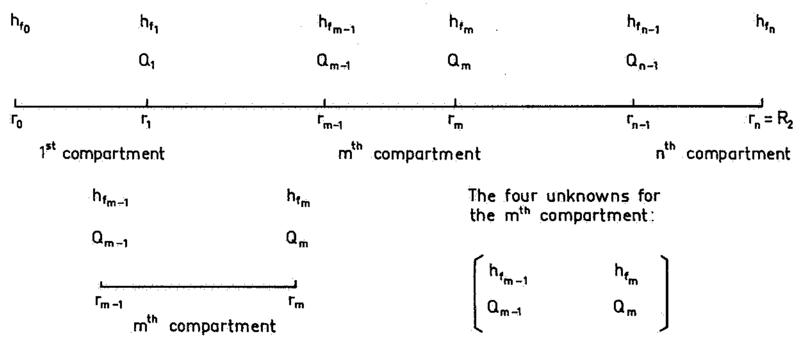


Fig. 5 **Unknowns at the compartment boundaries when the geohydrologic profile of figure 3 is divided into n compartments.**

3.3.2. *Results of APSOM when the compartments have a width of 1 000 m and 250 m respectively.*

Suppose that for the computation of Q, H and h_f the polder in figure 3 is divided into two compartments:

- the first one bounded by $r = 0$ and $r = R_1$;
- the second one bounded by $r = R_1$ and $r = R_2$.

When in the equations (6a) and (6b) and in their derivatives (Annex II) m is given the value 1 and 2 respectively, then figure 6 can be drawn.

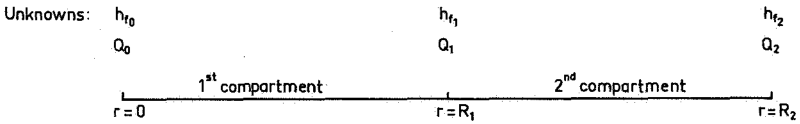


Fig. 6 **Unknowns at the compartment boundaries when the geo-hydrologic profile of figure 3 is divided into two compartments.**

Each of the two compartments has two equations and four unknowns. For the first compartment these are $f_1\,(h_{f_0}, Q_0, h_{f_1}, Q_1)$ and $f_2\,(h_{f_0}, Q_0, h_{f_1}, Q_1)$, for the second compartment $f_3\,(h_{f_1}, Q_1, h_{f_2}, Q_2)$ and $f_4\,(h_{f_1}, Q_1, h_{f_2}, Q_2)$.

According to chapter 2 it is possible to set up the following matrix for the solution of these four equations:

$$\begin{bmatrix} \frac{\partial f_1}{\partial h_{f_0}} & \frac{\partial f_1}{\partial Q_0} & \frac{\partial f_1}{\partial h_{f_1}} & \frac{\partial f_1}{\partial Q_1} & 0 & 0 \\ \frac{\partial f_2}{\partial h_{f_0}} & \frac{\partial f_2}{\partial Q_0} & \frac{\partial f_2}{\partial h_{f_1}} & \frac{\partial f_2}{\partial Q_1} & 0 & 0 \\ 0 & 0 & \frac{\partial f_3}{\partial h_{f_1}} & \frac{\partial f_3}{\partial Q_1} & \frac{\partial f_3}{\partial h_{f_2}} & \frac{\partial f_3}{\partial Q_2} \\ 0 & 0 & \frac{\partial f_4}{\partial h_{f_1}} & \frac{\partial f_4}{\partial Q_1} & \frac{\partial f_4}{\partial h_{f_2}} & \frac{\partial f_4}{\partial Q_2} \end{bmatrix} \cdot \begin{bmatrix} \Delta h_{f_0} \\ \Delta Q_0 \\ \Delta h_{f_1} \\ \Delta Q_1 \\ \Delta h_{f_2} \\ \Delta Q_2 \end{bmatrix} = - \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}$$

At first sight it appears as if one has to solve a matrix of six unknowns with the help of only four equations. The boundary conditions are: $Q(r = 0) = Q_0 = 0$ and $Q(r = R_2) = Q_2 = 0$. As one can introduce the right values for Q_0 and Q_2, this means that $\Delta Q_0 = 0$ and $\Delta Q_2 = 0$ in the above matrix. So, the right matrix notation of the set of equations becomes:

$$\begin{bmatrix} \frac{\partial f_1}{\partial h_{f_0}} & \frac{\partial f_1}{\partial h_{f_1}} & \frac{\partial f_1}{\partial Q_1} & 0 \\ \frac{\partial f_2}{\partial h_{f_0}} & \frac{\partial f_2}{\partial h_{f_1}} & \frac{\partial f_2}{\partial Q_1} & 0 \\ 0 & \frac{\partial f_3}{\partial h_{f_1}} & \frac{\partial f_3}{\partial Q_1} & \frac{\partial f_3}{\partial h_{f_2}} \\ 0 & \frac{\partial f_4}{\partial h_{f_1}} & \frac{\partial f_4}{\partial Q_1} & \frac{\partial f_4}{\partial h_{f_2}} \end{bmatrix} \cdot \begin{bmatrix} \Delta h_{f_0} \\ \Delta h_{f_1} \\ \Delta Q_1 \\ \Delta h_{f_2} \end{bmatrix} = - \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}$$

In the case of eight instead of two compartments, figure 7 gives the unknowns at the compartment boundaries:

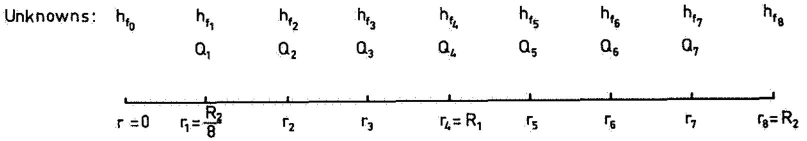


Fig. 7 **Unknowns at the compartment boundaries when the geohydrologic profile of figure 3 is divided into compartments.**

The matrix notation of the set of, now sixteen, equations belonging to figure 7 becomes:

$$\begin{bmatrix}
\frac{\partial f_1}{\partial h_{f_0}} & \frac{\partial f_1}{\partial h_{f_1}} & \frac{\partial f_1}{\partial Q_1} & 0 & & & & & 0 \\
\frac{\partial f_2}{\partial h_{f_0}} & \frac{\partial f_2}{\partial h_{f_1}} & \frac{\partial f_2}{\partial Q_1} & & & & & & \cdot \\
0 & \frac{\partial f_3}{\partial h_{f_1}} & \frac{\partial f_3}{\partial Q_1} & \frac{\partial f_3}{\partial h_{f_2}} & \frac{\partial f_3}{\partial Q_2} & & & & \cdot \\
\cdot & \frac{\partial f_4}{\partial h_{f_1}} & \frac{\partial f_4}{\partial Q_1} & \frac{\partial f_4}{\partial h_{f_2}} & \frac{\partial f_4}{\partial Q_2} & & & & \cdot \\
\cdot & \ddots & \ddots & \ddots & \ddots & & & & \cdot \\
\cdot & & & & & & & & \cdot \\
\cdot & & & & \frac{\partial f_{13}}{\partial h_{f_6}} & \frac{\partial f_{13}}{\partial Q_6} & \frac{\partial f_{13}}{\partial h_{f_7}} & \frac{\partial f_{13}}{\partial Q_7} & \cdot \\
\cdot & & & & \frac{\partial f_{14}}{\partial h_{f_6}} & \frac{\partial f_{14}}{\partial Q_6} & \frac{\partial f_{14}}{\partial h_{f_7}} & \frac{\partial f_{14}}{\partial Q_7} & 0 \\
 & & & & & & \frac{\partial f_{15}}{\partial h_{f_7}} & \frac{\partial f_{15}}{\partial Q_7} & \frac{\partial f_{15}}{\partial h_{f_8}} \\
0 & \cdot & \cdot & \cdot & \cdot & 0 & \frac{\partial f_{16}}{\partial h_{f_7}} & \frac{\partial f_{16}}{\partial Q_7} & \frac{\partial f_{16}}{\partial h_{f_8}}
\end{bmatrix}
\cdot
\begin{bmatrix}
\Delta h_{f_0} \\ \Delta h_{f_1} \\ \Delta Q_1 \\ \Delta h_{f_2} \\ \Delta Q_2 \\ \Delta h_{f_3} \\ \Delta Q_3 \\ \Delta h_{f_4} \\ \Delta Q_4 \\ \Delta h_{f_5} \\ \Delta Q_5 \\ \Delta h_{f_6} \\ \Delta Q_6 \\ \Delta h_{f_7} \\ \Delta Q_7 \\ \Delta h_{f_8}
\end{bmatrix}
= -
\begin{bmatrix}
f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \\ f_9 \\ f_{10} \\ f_{11} \\ f_{12} \\ f_{13} \\ f_{14} \\ f_{15} \\ f_{16}
\end{bmatrix}$$

An example, based on figure 6 and figure 7, is given below. It is a summary of results from Annex III.

Data:

$R_1 = 1\,000$ m	$h_s = 520$ m	$p_1 = 519$ m
$R_2 = 2\,000$ m	$c = 1\,000$ d	$p_2 = 525$ m
$D = 500$ m	$k = 25$ m/d	$\alpha = 0.02$

Boundary conditions:

$Q(r = 0) = 0 \quad Q(r = R_2) = 0$

Table 1 Results when the width of the compartments, Δr, is 1 000 m

r [m]	Q [m³/d]	H [m]	h_f [m]
0	0	362.300	523.154
1 000	−13516.268	347.450	523.451
2 000	0	335.950	523.681

Table 2 Results when the width of the compartments, Δr, is 250 m

r [m]	Q [m³/d]	H [m]	h_f [m]
0	0	364.100	523.118
250	−810.376	363.150	523.137
500	−3263.639	360.400	523.192
750	−7425.249	355.700	523.286
1 000	−13401.300	349.500	523.410
1 250	−10692.509	343.850	523.523
1 500	−7576.560	340.450	523.591
1 750	−4026.670	338.650	523.627
2 000	0	338.100	523.638

N.B. $H = \dfrac{(1+\alpha)h_s - h_f}{\alpha}$

4. CONCLUSIONS

Before ending these chapters some conclusions about APSOM, used for groundwater flow, can be drawn from the foregoing.

The analytical method in the foregoing chapters can only be applied succesfully when dealing with a strongly schematized mathematical and physical formulation of a groundwater flow problem: symmetric profiles, simple boundaries and simple boundary conditions.

In case of non-symmetric profiles and/or complicated boundary conditions, one may expect that the used method cannot be applied and one has to turn to a numerical method, e.g. a different method.

This does not mean that APSOM is of little importance compared to the nowadays rather well developed numerical techniques because next to these comprehensive numerical methods there is room for simpler ones for problems requiring rough and quick information only. In this respect APSOM may prove its value.

REFERENCES

DAM, J.C. VAN (1976) – Partial depletion of saline groundwater by seepage, Journal of Hydrology, Vol. 29 (1976), no 3/4: 315-339.
DRONKERS, J.J. (1969) – Tidal computations for rivers, coastal areas and seas, Proceedings of the American Society of Civil Engineers, Journal of the Hydraulics Division, Januari, 1969.
MOLENKAMP, G.L. (1978) – Een analytische oplossing voor stationaire zout-zout grondwater stromingsbeelden in radiaal-symmetrische, geo-hydrologische profielen. Master thesis Department of Civil Engineering, Delft University of Technology.
VERSPUY, C. (1973) – Lecture notes on fluid mechanics: Differentie-methoden, februari 1973, Department of Civil Engineering. Delft University of Technology.
ZAAT, J.A. (1971) – Numerieke Analyse A 1. Delftsche Uitgevers Maatschappij NV, Delft.

ANNEX I

EXAMPLE OF THE CALCULATION OF THE REAL ROOTS OF SIMULTANEOUS NON-LINEAR EQUATIONS

The following example is chosen:

$$f_1(x,y): x^2 + y^2 - 16 = 0 \tag{7a}$$
$$f_2(x,y): y - \tfrac{1}{2}\sqrt{3}x^2 = 0 \tag{7b}$$

The derivatives of the functions (7a) and (7b) are:

$$\frac{\partial f_1}{\partial x} = 2x \qquad \frac{\partial f_1}{\partial y} = 2y$$

$$\frac{\partial f_2}{\partial x} = \sqrt{3}x \qquad \frac{\partial f_2}{\partial y} = 1$$

From chapter 2 it is known that for the computation of the roots of the functions (7a) and (7b) the following matrix can be set up:

$$\begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix} \cdot \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = - \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

where:

$$\Delta x = x - x_1$$
$$\Delta y = y - y_1$$

The iteration process will be stopped when Δx and Δy are both smaller than 0.001

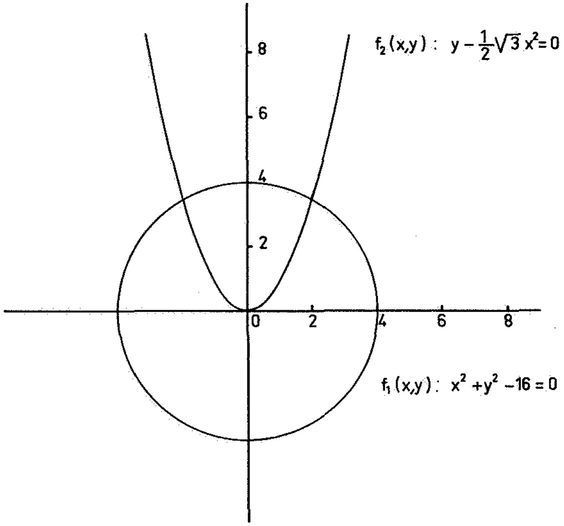


Fig. 8 The functions $f_1(x, y) = x^2 + y^2 - 16$ and $f_2(x, y) = y - \tfrac{1}{2}\sqrt{3}x^2$.

$$\left.\begin{array}{l} \text{1st estimate for } x: x_1 = 2.500 \\[2ex] \text{1st estimate for } y: y_1 = 2.500 \end{array}\right] \begin{array}{llll} f_1 = -3.500 & \dfrac{\partial f_1}{\partial x} = 5.000 & \dfrac{\partial f_1}{\partial y} = 5.000 \\[2ex] f_2 = -2.913 & \dfrac{\partial f_2}{\partial x} = -4.330 & \dfrac{\partial f_2}{\partial y} = 1.000 \end{array}$$

Substitution of these values in the matrix yields $\Delta x = -0.415$ and $\Delta y = 1.115$

2nd estimate for x : $x_2 = x_1 + \Delta x = 2.500 - 0.415 = 2.085$
2nd estimate for y : $y_2 = y_1 + \Delta y = 2.500 + 1.115 = 3.615$

Repetition of such calculations leads to the results as tabulated below:

Table 3 Computation of the roots of the simultaneous equations $f_1 = x^2 + y^2 - 16$ and $f_2 = y - \tfrac{1}{2}\sqrt{3}x^2$

x_i	y_i	$f_1(x_i, y_i)$	$\frac{\partial f_1}{\partial x}$	$\frac{\partial f_1}{\partial y}$	$f_2(x_i, y_i)$	$\frac{\partial f_2}{\partial x}$	$\frac{\partial f_2}{\partial y}$	Δx	Δy
2.500	2.500	−3.500	5.000	5.000	−2.913	−4.330	1.000	−0.415	1.115
2.085	3.615	1.415	4.170	7.230	−0.150	−3.611	1.000	−0.083	−0.148
2.002	3.467	0.028	4.004	6.934	−0.004	−3.468	1.000	−0.002	−0.003
2.000	3.464	−0.001	4.000	6.928	−0.000	−3.464	1.000	0.000*)	0.000*)

2.000 3.464 : *final results*

*) smaller than 0.001

Because of symmetry, another solution is: $x = -2.000$, $y = 3.464$. This can easily be verified analytically:

$$\left.\begin{aligned} x^2 + y^2 &= 16 \\ y &= \tfrac{1}{2}\sqrt{3x^2} \end{aligned}\right] \tfrac{3}{4}x^4 + x^2 = 16$$

The real roots are: $x = \pm\, 2.000$, $y = 2\sqrt{3}$

ANNEX II

THE TWO COMPARTMENT EQUATIONS, OBTAINED WITH THE HELP OF APSOM, AND THEIR DERIVATIVES

The equation of motion (Darcy), the equation of continuity and the Badon Ghijben-Herzberg relation are:

$$Q = -2\pi rk(D - H)\frac{dh_f}{dr} \tag{8a}$$

$$dQ = -\frac{2\pi r}{c}(h_f - p)\,dr \tag{8b}$$

$$h_f - h_s = (h_s - H)\cdot\alpha \tag{8c}$$

The elimination of H, by means of equation (8c), from equations (8a) and (8b) gives:

$$Q = -\frac{2\pi rk}{\alpha}\left[\alpha D - (\alpha + 1)\,h_s + h_f\right]\frac{dh_f}{dr} \tag{9a}$$

$$dQ = -\frac{2\pi r}{c}(h_f - p)\,dr \tag{9b}$$

The application of APSOM to equations (9a) and (9b) gives:

$$Q = -\frac{2\pi rk}{\alpha}\left[\alpha D - (\alpha + 1)\,h_s + \overline{h}_f\right]\frac{dh_f}{dr} \tag{10a}$$

$$dQ = -\frac{2\pi r}{c}(\overline{h}_f - p)\,dr \tag{10b}$$

Integrating successively the equations (10a) and (10b) yields:

$$h_f = \frac{\alpha r^2\,(\overline{h}_f - p)}{4kc\,\left[\alpha D - (\alpha + 1)\,h_s + \overline{h}_f\right]} - \frac{\alpha A_1 \ln r}{2\pi k\,\left[\alpha D - (\alpha + 1)\,h_s + \overline{h}_f\right]} + A_2 \tag{11a}$$

$$Q = -\frac{\pi r^2}{c}(\overline{h}_f - p) + A_1 \tag{11b}$$

The boundary conditions at $r = r_{m-1}$ are $h_f = h_{f_{m-1}}$ and $Q = Q_{m-1}$, so:

$$A_1 = Q_{m-1} + \frac{\pi r_{m-1}^2}{c}(\overline{h}_f - p) \tag{12a}$$

$$A_2 = h_{f_{m-1}} - \frac{\alpha\,[\overline{h}_f - p]\;[r_{m-1}^2 - 2r_{m-1}^2 \ln r_{m-1}]}{4kc\,[\alpha D - (\alpha+1)\,h_s + \overline{h}_f]}$$

$$+ \frac{\alpha Q_{m-1} \ln r_{m-1}}{2\pi k\,[\alpha D - (\alpha+1)\,h_s + \overline{h}_f]} \tag{12b}$$

The substitution of A_1 and A_2 in equations (11a) and (11b) and the replacement of $\overline{h}_f$ by $(h_{f_{m-1}} + h_{f_m})/2$ in these equations give:

$$h_{f_m} = h_{f_{m-1}} + \frac{\alpha\,[h_{f_{m-1}} + h_{f_m} - 2p]\;[r_m^2 - r_{m-1}^2 - 2r_{m-1}^2 \ln(\frac{r_m}{r_{m-1}})]}{4kc\,[2\alpha D + h_{f_{m-1}} + h_{f_m} - 2h_s\,(1+\alpha)]} -$$

$$- \frac{\alpha Q_{m-1} \ln(\frac{r_m}{r_{m-1}})}{\pi k\,[2\alpha D + h_{f_{m-1}} + h_{f_m} - 2h_s\,(1+\alpha)]} \tag{13a}$$

$$Q_m = Q_{m-1} - \frac{\pi}{2c}\,[h_{f_{m-1}} + h_{f_m} - 2p]\;[r_m^2 - r_{m-1}^2] \tag{13b}$$

The four unknowns in these equations are $h_{f_{m-1}}$, Q_{m-1}, h_{f_m} and Q_m. Equations (13a) and (13b) are reduced to zero and called $f_{2m-1}\,(h_{f_{m-1}}, Q_{m-1}, h_{f_m}, Q_m)$ and $f_{2m}\,(h_{f_{m-1}}, Q_{m-1}, h_{f_m}, Q_m)$ respectively. They are comparable with $f_1\,(x, y)$ and $f_2\,(x, y)$ in chapter 2. According to chapter 2 one needs for the calculation of the roots simultaneous equations, apart from these equations, also their derivatives:

$$\frac{\partial f_{2m-1}}{\partial Q_{m-1}},\ \frac{\partial f_{2m-1}}{\partial h_{f_{m-1}}},\ \frac{\partial f_{2m-1}}{\partial Q_m},\ \frac{\partial f_{2m-1}}{\partial h_{f_m}},\ \frac{\partial f_{2m}}{\partial Q_{m-1}},\ \frac{\partial f_{2m}}{\partial h_{f_{m-1}}},\ \frac{\partial f_{2m}}{\partial Q_m},\ \frac{\partial f_{2m}}{\partial h_{f_m}}.$$

The functions f_{2m-1} and f_{2m} are equations (6a) and (6b) in paragraph 3.3.1.; with little effort their derivatives can be found by the reader himself.

ANNEX III

EXAMPLE OF APSOM, APPLIED TO GROUNDWATER FLOW, THAT INCLUDES THE CONVERGENCE OF START VALUES TO FINAL RESULTS WHEN THE POLDER COMPARTMENTS OF FIGURE 3 HAVE A WIDTH OF 1000 M AND 250 M RESPECTIVELY

Data:

$R_1 = 1\,000$ m	$h_s = 520$ m	$p_1 = 519$ m
$R_2 = 2\,000$ m	$c = 1\,000$ m	$p_2 = 525$ m
$D = 500$ m	$k = 25$ m/d	$\alpha = 0.02$

Boundary conditions:

$$Q(r = 0) = Q_0 = 0 \qquad Q(r = R_2) = Q_2 = 0$$

Width of the compartments: The computation was done for two compartments. The boundary is the dike between the inner polder and the outer polder. So, each of the compartments has a width of 1 000 m.

Start values: To begin the computation, with the help of the matrix in paragraph 3.3.2., one, needs start values for the four unknowns:

$$\begin{aligned} h_f(r = 0) &= h_{f_0} = 522.000 \text{ m} \\ h_f(r = 1\,000) &= h_{f_1} = 523.500 \text{ m} \\ Q(r = 1\,000) &= Q_1 = 0.000 \text{ m}^3/\text{d} \\ h_f(r = 2\,000) &= h_{f_2} = 525.000 \text{ m} \end{aligned}$$

The computation for eight instead of two compartments runs as follows:

Boundary conditions:

$$Q(r = 0) = Q_0 = 0 \qquad Q(r = R_2) = Q_8 = 0$$

Table 4 Convergence of start values to final results when there are two polder compartments 1 000 m each

	start values	1st correction	2nd correction	3rd correction	4th correction	final results
h_{f_0}	522.000	1.179	−0.025	0.000	0.000	523.154
h_{f_1}	523.500	−0.031	−0.018	0.000	0.000	523.451
Q_1	0.000	−13584.288	68.009	0.011	0.000	−13616.268
h_{f_2}	525.000	−1.352	0.033	0.000	0.000*)	523.681

*) smaller than 0.001
stop iteration process.

Width of the compartments: Each of the compartments has a width of 250 m.

Start values: The values, resulting from a linear interpolation between h_{f_0} and h_{f_8} and between Q_0 and Q_8 respectively, are chosen as start values for h_{f_i} and Q_i.

$$h_f(r = 0) \quad = h_{f_0} = 522.000 \text{ m}$$
$$h_f(r = R_2) = h_{f_8} = 525.000 \text{ m}$$

Table 5 Convergence of start values to final results when there are eight polder compartments of 250 m each

	start values	1st correction	2nd correction	3rd correction	4th correction	final results
h_{f_0}	522.000	1.120	−0.002	0.000	0.000	523.118
h_{f_1}	522.375	0.762	0.000	0.000	0.000	523.137
Q_1	0.000	−810.574	0.198	0.000	0.000	−810.376
h_{f_2}	522.750	0.449	−0.007	0.000	0.000	523.192
Q_2	0.000	−3265.795	2.154	0.002	0.000	−3263.639
h_{f_3}	523.125	0.174	0.013	0.000	0.000	523.286
Q_3	0.000	−7437.193	11.940	0.004	0.000	−7425.249
h_{f_4}	523.500	−0.075	−0.015	0.000	0.000	523.410
Q_4	0.000	−13432.522	31.218	0.004	0.000	−13401.300
h_{f_5}	523.875	−0.345	−0.007	0.000	0.000	523.523
Q_5	0.000	−10742.714	50.201	0.004	0.000	−10692.509
h_{f_6}	524.250	−0.662	0.003	0.000	0.000	523.591
Q_6	0.000	−7630.910	54.346	0.004	0.000	−7576.560
h_{f_7}	524.625	−1.009	0.011	0.000	0.000	523.627
Q_7	0.000	−4063.315	36.642	0.003	0.000	−4026.670
h_{f_8}	525.000	−1.376	0.014	0.000	0.000	523.638

ANNEX IV

TEMPORARY AND PERMANENT LINEARIZATION OF AN EQUATION

As an example the groundwater flow in a strip of land bounded by two canals (fig. 9) is taken.

With q for the fresh water discharge and h for the fresh water potential, the equation of motion (Darcy) reads:

$$q = -kh\frac{dh}{dx} \tag{14a}$$

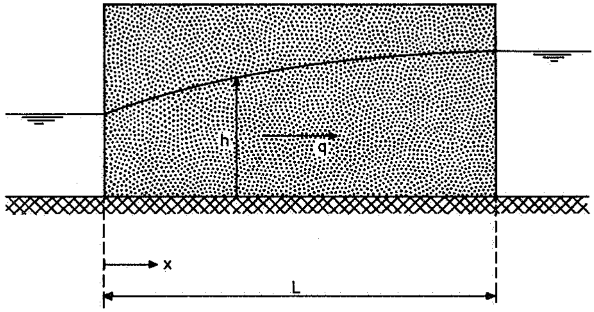


Fig. 9 **Groundwater flow in a strip of land bounded by two canals.**

For the numerical solution of an equation like (14a) it is often important that this equation is transformed into a linear equation:

$$q = -k\bar{h}\frac{dh}{dx} \tag{14b}$$

When APSOM is used, while integrating from $x = x_{m-1}$ to $x = x_m$, for the value $\bar{h}$ is chose $(h_{m-1} + h_m)/2$. In the previous paragraphs this was called a temporary linearization. For some difference methods, especially when calculating from the left boundary towards the right boundary in time-dependent problems (see Dronkers, 1969 and Verspuy, 1973); it is common use to choose for the interval (x_{m-1}, x_m) the value h_{m-1} or the average value from the previous interval: $(h_{m-2} + h_{m-1})/2$ i.e. a value already known by the intergration from x_{m-2} to x_{m-1} and so not dependent on the integration from x_{m-1} to x_m.
This is called a permanent linearization.

ANNEX V

SUPPLEMENTARY REMARKS

Figure 10 is based on the data in paragraph 3.3.2. and on figure 3. Different piezometric levels are presented when the inner polder level is successively lowered from 516.500 m via 514.500 m and 514.150 m to 514.098 m.

In the example with the level of 514.098 m the fresh-saline water interface intersects the semi-pervious layer. The consequence:

– a no-horizontal course of the interface at r = 0 m (see also Van Dam, 1976).

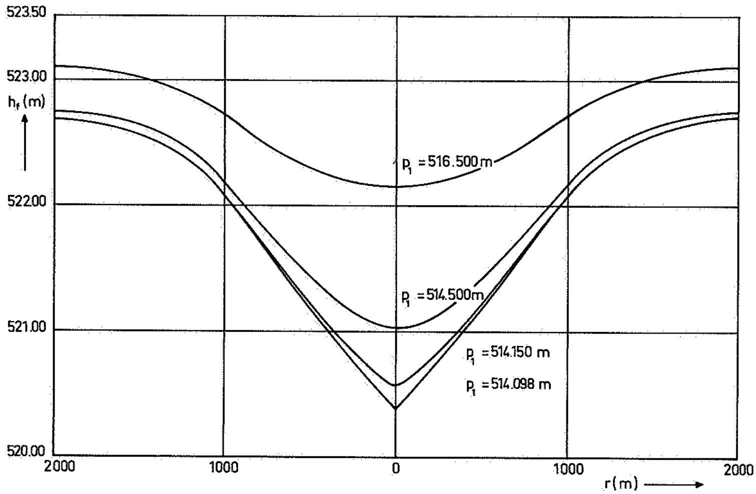


Fig. 10 Different piezometric levels, caused by different inner polder levels as in figure 3.

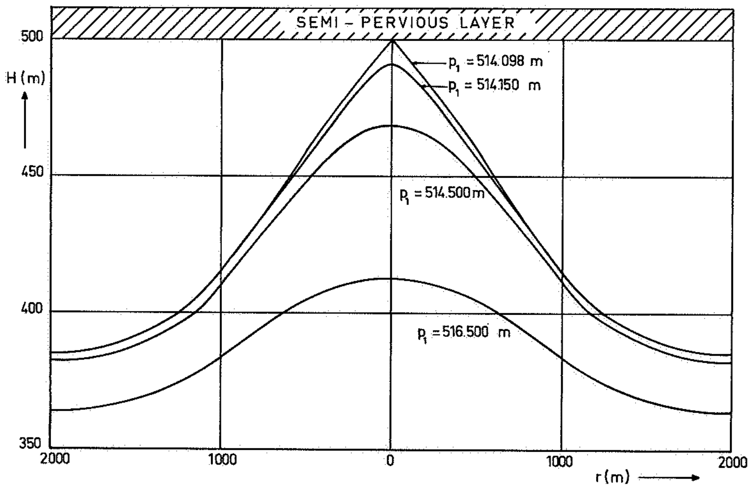


Fig. 11 Effect of different inner polder levels on the height of the interface.

Figures 11 and 12 are also based on the data in paragraph 3.3.2. and on figure 3. They show the influence of the lowering of the inner polder level on the height of the interface and the effect of different saline water potentials on the piezometric level respectively.

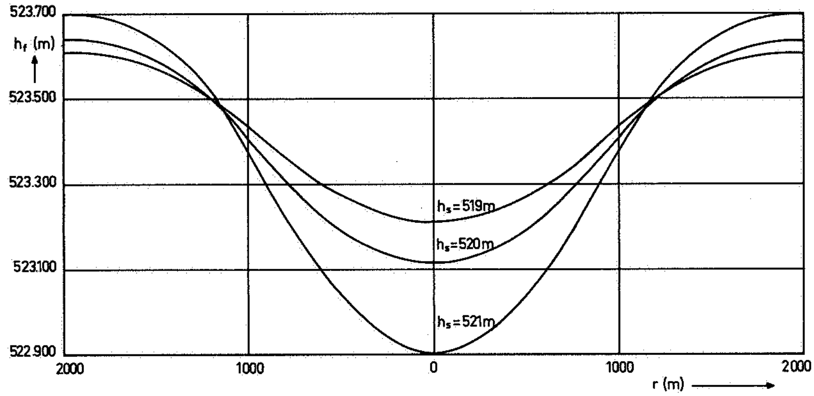


Fig. 12 Piezometric levels for different values of the saline water potential h_s.

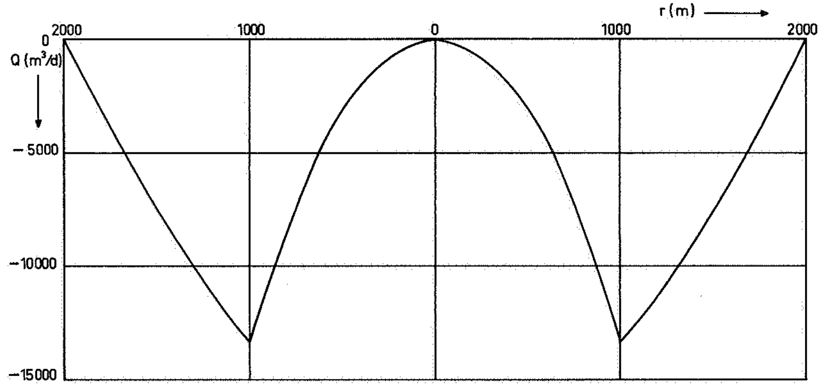


Fig. 13 Fresh water discharge.

Figure 14 shows a circular island surrounded by saline water. The geohydrologic profile is a homogeneous aquifer on an impervious base.

The flow of fresh groundwater, caused by the effective rainfall, is steady as is the flow of saline groundwater, caused by an abstraction in the centre of the island. Vertical flow components are neglected.

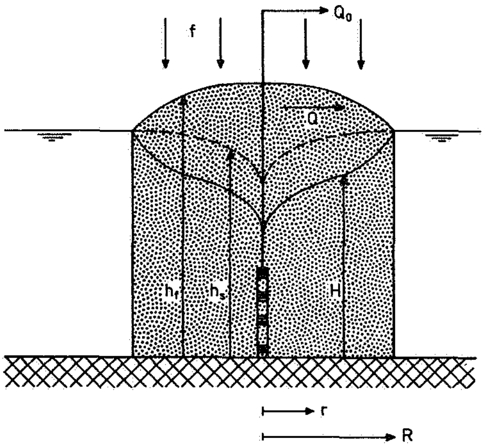


Fig. 14 Cross-section through a circular island.

R	:	radius of the island	[L]
k	:	coefficient of permeability	$[LT^{-1}]$
h_f	:	fresh water potential above reference level	[L]
h_s	:	saline water potential above reference level	[L]
f	:	effective rainfall	$[LT^{-1}]$
Q	:	fresh water discharge	$[L^3T^{-1}]$
Q_0	:	amount of saline water abstraction	$[L^3T^{-1}]$
r_0	;	radius of the well	[L]
H	:	height of the interface above reference level	[L]
α	:	$\alpha = \dfrac{\rho_s - \rho_f}{\rho_f}$	[–]

The data, with the help of which figure 14 is drawn, are:

$h_f(r = R)$	= 350 m	f	= 0.002 m/d
R	= 1 000 m	k	= 25 m/d
r_0	= 0.25 m	α	= 0.02

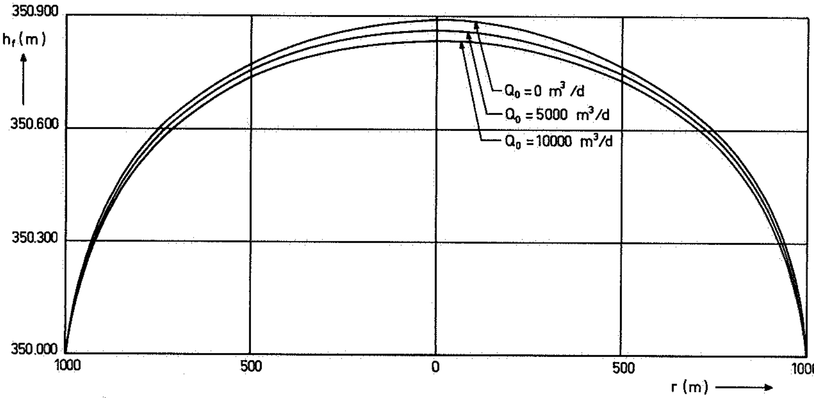


Fig. 15 **The effect of saline water abstraction on the piezometric level.**

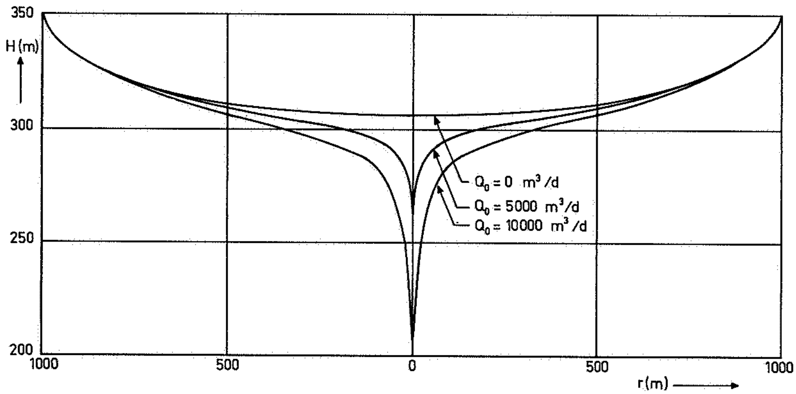


Fig. 16 **The effect of saline water abstraction on the saline water potential.**

Figures 15, 16 and 17 give the consequences of the increase of saline water abstraction, from 0 m^3/d via 5 000 m^3/d to 10 000 m^3/d, on the piezometric level, on the saline water potential and on the height and shape of the interface.

For the data, on which these figures are based: see figure 14 and its explanation.

When looking at these figures, one should always remember the Badon Ghijben-Herzberg relation: $(h_f - h_s) = (h_s - H) . \alpha$

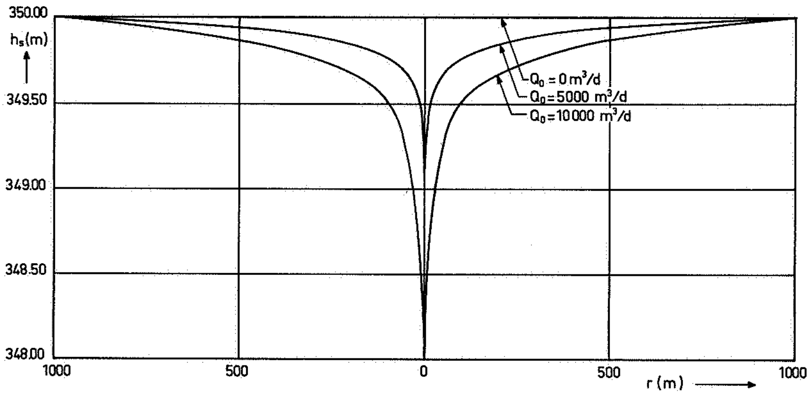


Fig. 17 The effect of saline water abstraction on the height and the shape of the interface.

THE FINITE ELEMENT METHOD IN FRESH/SALINE GROUNDWATER

J. SCHONEVELD

SUMMARY

In this contribution the basic concepts of the method will be shown with the aid of a few examples in a one-aquifer system, one- and two-dimensional, for steady and transitional stages. The latter stages will be simulated with the help of a weighted finite difference scheme. The conclusions deal shortly with the limitations of this method with respect to the practical use of it.

1. INTRODUCTION

Although originally applied in structural engineering, it has been proved in the past years that the finite element technique can be a good help in solving groundwaterflow problems. When applied to the problem of calculating the place of the interface between fresh and saline groundwater in socalled "Dutch profiles" and studying its transitional stages the method finds its base in a transformation of the governing equations into functionals.

The use of the functionals in the various elements will lead to a system of linear equations from which an approximate solution can be obtained.

The fact that applying this method needs a subdivision of the physical region into elements has the advantage that complicated geometries and varying geohydrologic conditions can be handled in a rather simple way.

2. EQUATIONS

In order to be able to describe the problem use is made of the following assumptions:

- The fresh/saline interface is a sharp one, phenomena like dispersion and diffusion are neglected.
- The saline water is in rest and has a constant piëzometric level. An exclusion of this assumption will be made in the transitional model; here the occuring flow of the saline water is not taken into account, only a constant piëzometric level on the interface will occur as a boundary condition.
- The semi-pervious top-layer is horizontal.
- The flow in the aquifer is assumed to be horizontal while in the semi-pervious layer only a vertical seepage is taken into account.

Dealing with the geohydrologic profile as presented in figure 1, in which h_f and h_s

are the piëzometric levels of the fresh and saline water and P represents the polder level above the semi-pervious layer, the governing equations are:

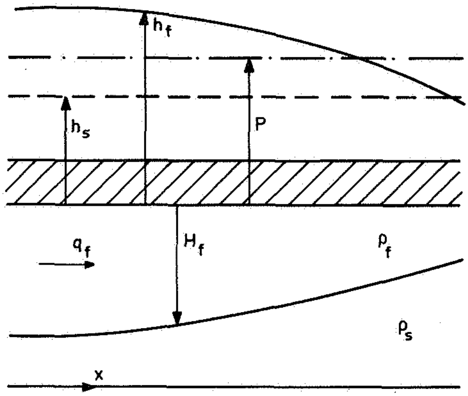


Fig. 1 Dutch profile.

Darcy : $q_x = -k_x H \dfrac{\partial h}{\partial x}$ (a)

: $q_y = -k_y H \dfrac{\partial h}{\partial y}$ (b)

Continuity : $\dfrac{dq_x}{dx} + \dfrac{dq_y}{dy} = -\dfrac{h-P}{c}$ (c)

Badon Ghijben-Herzberg: $h = H + (1 + \alpha) A$

$$\alpha = \frac{\rho_s - \rho_f}{\rho_f} \qquad \text{(d)}$$

Combination of equations (a), (b), (c) and (d) gives:

$$\frac{\partial}{\partial x}(k_x H \frac{\partial H}{\partial x}) + \frac{\partial}{\partial y}(k_y H \frac{\partial H}{\partial y}) - \frac{H}{c} - \frac{(1+\alpha)A - P}{c\alpha} = 0 \qquad \text{(e)}$$

representing a non linear second order differential equation in which the H represents the depth of the interface. So far, it was not possible to find a general solution for this differential equation.

Of the various numerical procedures for solving (e), those in which H is approximated by a trial function of the form

$$H = \sum_{k=1}^{n} a_k \phi_k(x, y) \qquad \text{(f)}$$

where a_k are undetermined parameters and where the ϕ_k are linearly independent functions, is chosen.

It is stated the describing equations of the problem can be transformed by means of a variational principle into a functional, having a minimum for the solution to be sought.

Here the functional to be used is:

$$F = \frac{1}{2} \int\int [(k_x H)_{av} \left(\frac{\partial H}{\partial x}\right)^2 + (k_y H)_{av} \left(\frac{\partial H}{\partial y}\right)^2 + \frac{H^2}{c^2} + 2 \frac{(1+\alpha)A - P}{c\alpha}] \, dxdy \quad (g)$$

having the single restriction of $q = o$ on the side boundaries. The substitution of eq. (f) into the functional and the requirement the functional satisfies the minimum condition leads to a set of equations from which an approximate solution can be obtained.

3. INTRODUCTION OF ELEMENTS

The $x - y$ area of the problem is divided into a set of subareas interconnected at the nodal points: the finite element grid in which the solution is approximated by localised functions. Having the functional for each of these elements, substitution of the approximation into the functional followed by minimization for each of these elements yields a set of n-equations with n-unknowns.

It shows out that the first two integrals of the functional will give quadratic terms which will make the equations of the non linear type. In order to obtain linearity, a preferable form to facilitate computer programming, one H is estimated and subsequently constant.

4. A ONE-DIMENSIONAL EXAMPLE

The type of examples discussed are based on the hypothetical problem Van Dam dealt with in one of his papers.

The one-dimensional case is a one-aquifer system covered by a semi-pervious toplayer above with there are two polder compartments with a constant groundwater table each.

Fig. 2 shows the profile of the two polders of length $L = 1\ 000$ m each with the, different polder levels: $P_1 = 19$ m and $P_2 = 25$ m.

The aquifer is considered to be homogeneous while the left hand and the right hand sides are supposed to be impermeable.

The total length of the polders is subdivided into elements, each having a length s, in which the unknown H is approximated by a linear variation between the two nodes of each element:

$$H = H_{i+1}\, \xi + (1 - \xi)\, H_i$$

with ξ referring to a local coordinate system. (see fig. 3).

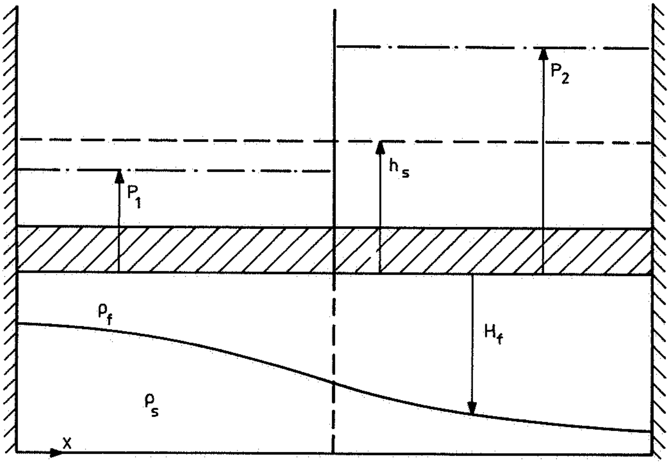


Fig. 2 One-dimensional model of two polders.

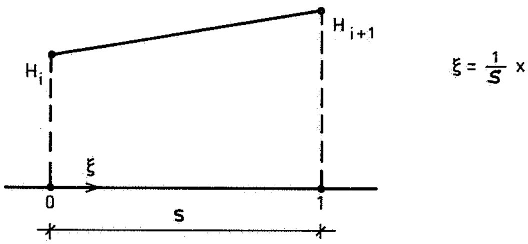


Fig. 3 Linear variation of H in an elèment.

The functional will give:

$$F = \frac{H_{av}}{2s}(H_{i+1} - H_i)^2 + \frac{s}{6kc}H_{i+1}^2 + \frac{s}{6kc}H_i^2 + \frac{s}{6kc}H_{i+1}H_i +$$

$$+ \frac{(1+\alpha)A - P}{kc\alpha}\left(\frac{s}{2}H_{i+1} + \frac{s}{2}H_i\right)$$

The minimum is found by operation on the functional:

$$\frac{\partial F}{\partial H_i} = 0 \text{ and } \frac{\partial F}{\partial H_{i+1}} = 0$$

in matrix form:

$$\begin{bmatrix} \dfrac{H_{av}}{s^2}+\dfrac{1}{3kc} & -\dfrac{H_{av}}{s^2}+\dfrac{1}{6kc} \\ \\ -\dfrac{H_{av}}{s^2}+\dfrac{1}{6kc} & \dfrac{H_{av}}{s^2}+\dfrac{1}{3kc} \end{bmatrix} \begin{bmatrix} H_i \\ \\ H_{i+1} \end{bmatrix} = \begin{bmatrix} \dfrac{P-(1+\alpha)A}{2kc\alpha} \\ \\ \dfrac{P-(1+\alpha)A}{2kc\alpha} \end{bmatrix}$$

Repeating this procedure for each element leads to a system of linear equations in H_i. This system can be solved with the help of e.g. a Gauss-procedure.

The starting values of H_{av} have to be estimated the first time, after the first iteration the correctness of the approximated H_{av}'s can be tested by

$$H_{av} = \frac{H_i + H_{i+1}}{2}$$

These new values can be used for a new iteration step.

In fig. 4 the results of a 40 element scheme are presented next to the values Van Dam has obtained.

Also the first approximations of the different H_i's are given.

5. A TWO-DIMENSIONAL EXAMPLE

A wider application can be found in the two-dimensional case. Hence the former example is extended to a four polder system in which we can distinguish four different polder levels. Also in this example the aquifer is supposed to be homogeneous and covered by a semi-pervious layer while the four sides are impervious.

In this case the area is subdivided into triangular elements. In each of them the interface is supposed to be a flat plane represented by a linear interpolation function of the type

$$H = a_1 + a_2x + a_3y$$

which will give three unknowns per element.

If we call the corners of an element nodes it is possible to relate the unknowns to the nodal values of H at nodes 1, 2 and 3 (fig. 5a).

Also quadratic functions can be applied in which we can relate the unknowns to six nodal values of H (fig. 5b).

The latter type of functions can have the advantage of giving more accurate results in comparison with the linear type if the same number of elements is used.

A disadvantage arising from the latter method is the resulting of bigger element matrices c.q. more equations, that will require more computational effort.

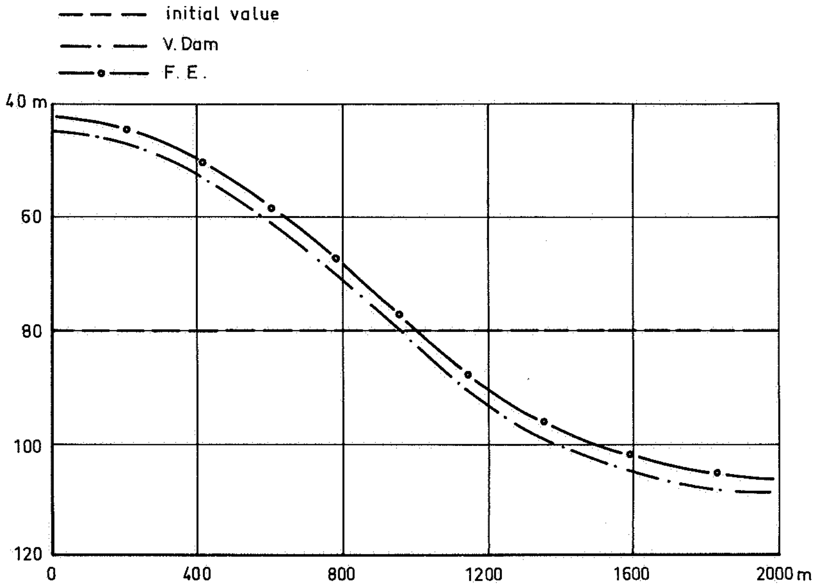
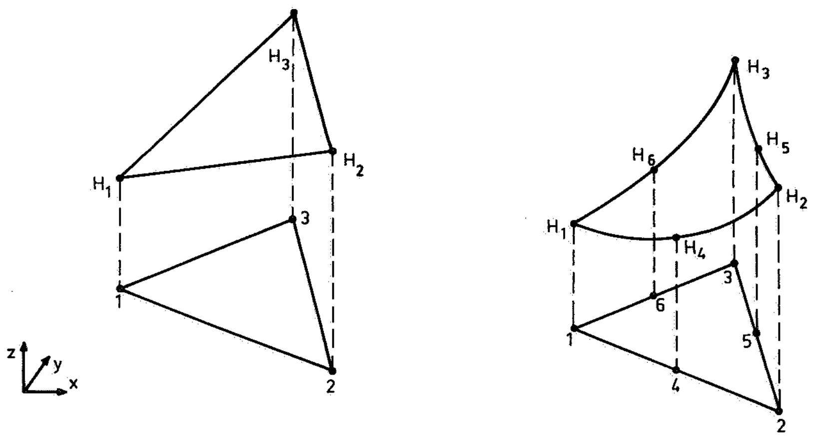


Fig. 4 Results in a two polder model.



$H = a_1 + a_2x + a_3y$

$H = a_1 + a_2x + a_3y + a_4x^2 + a_5xy + a_6y^2$

Fig. 5a and 5b Variations of H in two dimensions.

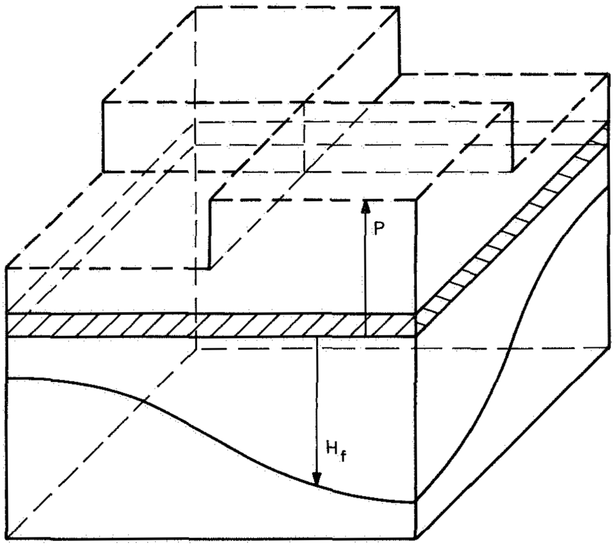


Fig. 6 Symmetric two-dimensional model.

For reasons of stability, solution of integrals and computational effort it becomes preferable to change to a local co-ordinate system.

Introduction of this functions into the functional and deriving the minimum of it will again lead to the system of n-equations with n-unknowns.

If we keep the two-polder levels symmetrically with regard to the middle of the x-y plane (fig. 6) it is stated that the solution of H along the middle axis must represent a straight line.

In figure 7 the form of the fresh water body is represented, while in figure 8 the results along the middle axis are given. These results were obtained from a 32 element scheme using linear interpolation functions.

6. TRANSIENT STAGES

To be able to simulate the development in time of the interface as a result of changing polder levels, e.g. in the case of new built up areas, the previous one-dimensional example will be studied with the help of a weighted finite difference scheme to be able to handle the time derivates.

The one-dimensional model discussed before will represent the starting point of the calculations.

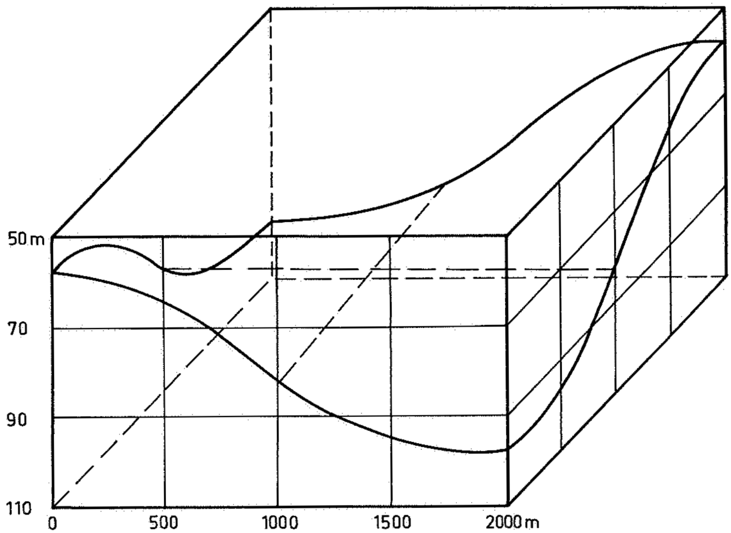


Fig. 7 Position of the interface in a two-dimensional model.

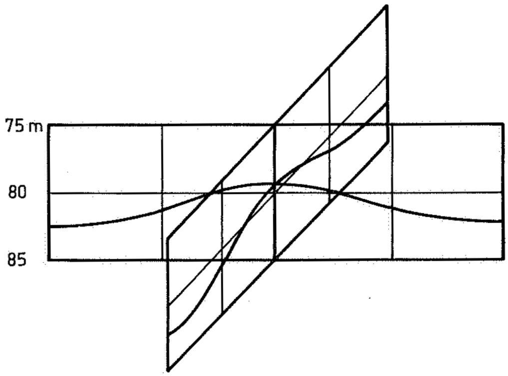


Fig. 8 Results along the middle aris.

At $t = 0$ there will be a reversal of the two polder levels. The resulting set of equations will be of the form:

$$[A]\ \{H\} + [B]\ \{\frac{\partial H}{\partial t}\} = \{F\} \qquad \text{(h)}$$

in which the matrices [A] and {F} are equal to the ones used before.

The matrix [B] is a result of the extension of the continuity equation (c):

$$\frac{dq}{dx} = -\frac{h - P}{c} - \mu \frac{\partial H}{\partial t} \qquad \text{(i)}$$

This equation is formed with the assumption of incompressible water and soil components.

Having the variational principle on the extra term in equation (i) gives the matrix:

$$[B] = \begin{bmatrix} \frac{\mu}{3k\alpha} & \frac{\mu}{6k\alpha} \\ \\ \frac{\mu}{6k\alpha} & \frac{\mu}{3k\alpha} \end{bmatrix}$$

Replacing the time-derivate in eq. (h) by a finite difference approximation and using the implicit method for reason of the stability the matrix-equation becomes:

$$([A] + (\frac{1}{\Delta t})\,[B])\ \{H\}_{t+\Delta t} - (\frac{1}{\Delta t})\,[B]\ \{H\}_t = \{F\}_{t+\Delta t}$$

With the help of this formula different time steps are simulated with at $t = 0$ the position of the interface known. The example gives the effects of the reversal of the polder levels during a period of 2 800 days (fig. 9).

7. CONCLUSIONS

Next to the features of being able to handle irregular geometries and changing geohydrologic conditions there are some restrictions to the practical use of this method.

Apart from the different hyphothesis (e.g. immiscible fluids) the main limits are the formulations of the boundary conditions. In the previous examples the side boundaries were taken to be impervious which means $q = 0$, an unrealistic assumption in practise. For the method it is necessary to know either the different H's on the boundary nodes or the flow along the boundary itself.

In practical schemes the only possible way to make this obligation less heavy seems to extend the region of study so far that one can assume that the conditions on the new boundaries are of negligeable importance for the situation inside the initial region of study.

Obiviously this will result in a considerable increase in the number of elements and subsequently in the computational efforts to solve the system of equations.

It will be clear that irrespective of the economical point of view the available computer memory will dictate the limits of the number of elements.

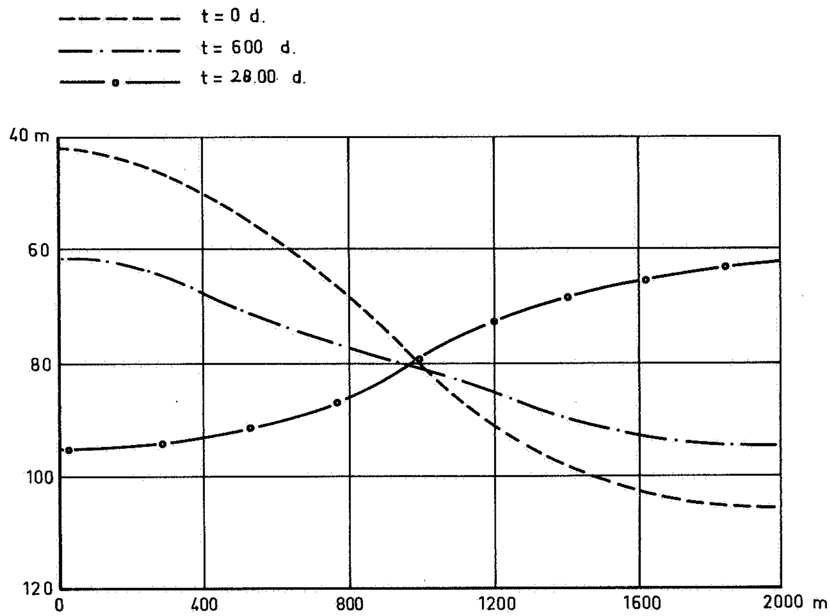


Fig. 9 Positions of the interface at different moments.

REFERENCES

CONNOR, J.J. and C.A. BREBBIA (1976) – Finite element techniques for fluid flow. Newness – Butterworths.

DAM, J.C. VAN (1976) – Partial depletion of saline groundwater by seepage. Journal of Hydrology, Vol. 29, No. 3/4: 315-339.

DAM, J.C. VAN and W.H.C. TEN HOORN (1977) – Effect of urbanization on the distribution of fresh and saline groundwater in the Netherlands. Proceedings of the Amsterdam Symposium, October 1977, IAHS Publications No. 123: 488-499.

HUISMAN, L. (1972) – Groundwater recovery, MacMillan and Co. Ltd., Londen.

I.C.W. (1976) – Hydrologie en waterkwaliteit van Midden West-Nederland. Regionale studie ICW nr. 9. 100 pp.

NORRIE, D.H. and G. DE VRIES (1972) – The finite element method. Academic Press, London.

NORRIE, D.H. and G. DE VRIES (1978) – An introduction to Finite Element analysis. Academic Press, London.

PINDER, G.F. and W.G. GRAY (1977) – Finite element simulation in surface and subsurface hydrology. Academic Press, New York.

VERRUYT, A. (1970) – Theory of groundwater flow. MacMillan and Co. Ltd, London.

VERRUYT, A. – A Finite element approach to groundwater flow problems with storage, infiltration and leakage, University of Technology, Delft.

ZAAT, J.A. (1971) – Numerieke Analyse A1. Delftsche Uitgevers Maatschappij NV, Delft.

SOME RESULTS OBTAINED WITH A BOUNDARY ELEMENT METHOD

R. AWATER

SUMMARY

A Boundary Element Method, in which a complex function formulation is used (Analytical Function Method), is applied to study the transient behaviour of the interface between fresh and salt groundwater due to a sudden change in the boundary conditions.

1. INTRODUCTION

The goal of the exercise presented in this paper is to illustrate the capabilities of a boundary element method in which a complex function formulation is used (Analytical Function Method) in solving two-dimensional groundwater flow problems. As an example the method is used to study a transient groundwater flow problem related to the reclamation of a part of a lake. The reclamation of lakes (to create polders) disturbs the original pattern of groundwater flow. In areas where underneath the fresh groundwater also salt groundwater is present, reclamation may strongly affect the position of the interface between the two fluids, sometimes even causing saline water to seep into the polders' open water courses.

As saline water makes the area less suited for agricultural use it is important to know whether saline seepage will occur and, if so, which quantities of salt water are to be expected as a function of time. Before describing the problem and its solution, the essence of the Analytical Function Method will be outlined briefly.

2. ANALYTICAL FUNCTION METHOD (AFM)

The method uses a solution that satisfies the governing equation in the domain but which has a number of unknown coefficients. The coefficients are determined by enforcing the solution to satisfy the boundary conditions. In general these techniques are called Boundary Methods or, in case the boundary of the domain is divided into a series of elements, Boundary Element Methods. In the Analytical Function Method in particular, each boundary element is covered with a continuous series (distribution) of sinks and a distribution of vortices. The distributions at all elements together constitute a complex analytical function $\Omega(z)$, in which Ω is the complex potential and $z = x + iy$, where x and y are the local coordinates. The complex potential is defined by $\Omega = \phi + i\psi$, where ϕ is the potential and ψ is the stream function. Both the real and imaginary

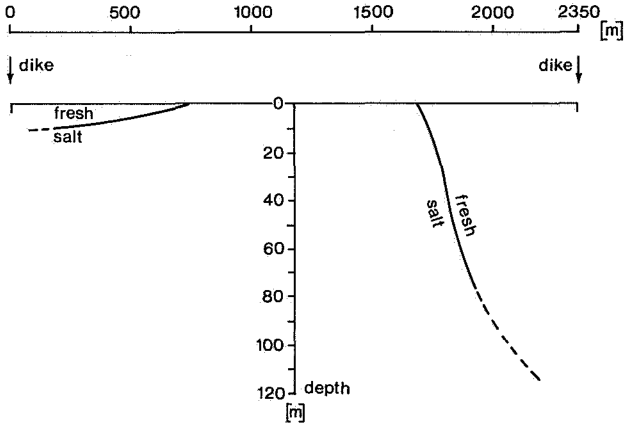


Fig. 1 Observed position of the interface below the polder "Groot-Mijdrecht".

part of an analytical function satisfy the Laplace equation. The strengths of the sinks and the strengths of the vortices at each boundary element are the unknown coefficients in the function $\Omega(z)$. The influence of sinks and sources located in the region is included in the function $\Omega(z)$ by means of superposition. Anisotropy is treated by transforming the region and the introduction of a fictive coefficient of permeability. Inhomogeneities within a flow region are handled by a division into subregions, each having constant properties of both soil and fluid. Subregions are interconnected by conditions at their separation lines. Relatively thin layers of low permeability need no distinction as a seperate subregion. The influence of these layers may be incorporated as a flow resistance at the boundary of an adjacent subregion. The unknown coefficients in the function $\Omega(z)$ are determined by enforcing the function to satisfy 2 conditions imposed at the centre point of each boundary element. The solution found is exact within the approximated boundary and comprises both the potential and the stream function. Next to the flow pattern it is possible to calculate the specific discharge in any arbitrary point within the flow region. The specific discharge values at moving boundaries are used to estimate the position of these boundaries after a step in time. The flow pattern in agreement with the new position of the boundaries may now be computed. The repeated application of this procedure allows the computation of transient flow problems. Both method and computer program have been presented by Van der Veer (1978).

3. ORIGINAL INTENTION

An evaluation of the results of geo-electrical investigations by Leenen (1978) was the original intention of the study presented in this paper. Leenen determined the position

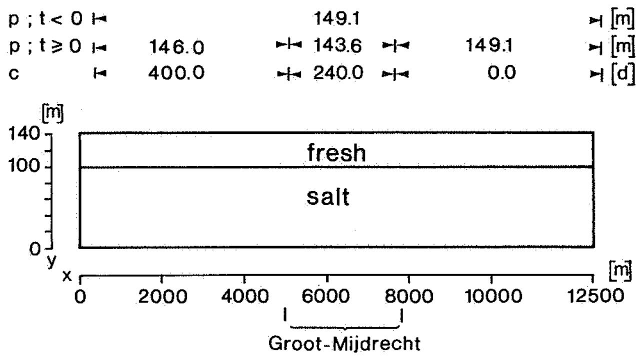


Fig. 2 Schematization of the problem "Groot-Mijdrecht".

of the interface between fresh and salt groundwater below the polder "Groot-Mijdrecht" (fig. 1), located about 20 km south of Amsterdam in the Netherlands. It remains to be seen whether the observed position is steady or not. A steady position is characterized by a zero specific discharge perpendicular to the interface. As with the computer model based on the AFM specific discharges are calculated, a judgement concerning the stability of the interface could have been given at once, provided that boundary conditions would have been available at the vertical boundaries of the polder. Unfortunately these boundary conditions are not available. This makes it necessary to enlarge the model area. Therefore vertical impermeable boundaries are chosen at a large distance from the boundaries of the polder under consideration. As the position of the interface below adjacent polders is not known this schematization does not permit a direct inquiry as indicated above. However, it allows, starting with a steady position of the interface in agreement with the open water level before reclamation, a study of the transient behaviour of the interface due to a change in the open water level in part of the area. As before reclamation the open water level is constant over the whole area, the steady position of the interface is horizontal and may be defined at an arbitrary level. As time goes on the interface may reach a position comparable to the position observed by Leenen. From this it might be possible to judge whether this position is steady and, if not, in which direction the interface tends to move and, going on, where and when it will reach its final (i.e. steady) position. Without going into details the schematization of the problem is shown in figure 2. When running the model for some time steps a rather peculiar behaviour of the interface was observed. In contradiction to an expected gradual upward bulging with a maximum value somewhere below the centre of the deepest polder, two bulges were observed with maximum values close to the boundaries of the deepest polder as illustrated in figure 3. This phenomenon seemed interesting enough to pay attention to. It was therefore decided to simplify the problem for a more elementary study of the wave-

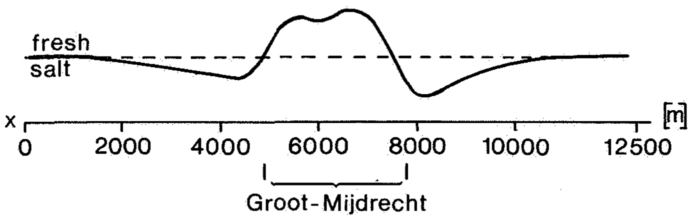


Fig. 3 Response of the interface. Problem "Groot-Mijdrecht" (not to scale).

like response of the interface. It is this elementary problem which is presented in the subsequent chapters.

4. PROBLEM DEFINITION

To study the transient behaviour of the interface between fresh an salt groundwater particularly in surroundings where sharp differences in the open water level occur, the following problem is defined: An aquifer is bounded at the upper side by a layer which has a relatively low permeability as compared with the aquifer itself. Above the layer of low permeability the hydraulic head equals the artificially maintained so called polder level or the open water level. In geohydrology this profile is known as "Holland profile". Before reclamation a lake with a constant water level is found at the top. Within the aquifer salt groundwater is present underneath the fresh groundwater. The interface between the two fluids is taken to be sharp. The groundwater is assumed to be completely at rest, no flow whatsoever occurs, so the interface is in a steady horizontal position. A reclamation of a part of the lake is assumed to happen in no time. At $t = 0$ all of a sudden the open water level is dropped by 5 meters. The difference in open water level induces the groundwater to flow which causes the interface to move.

The transient behaviour of the system described above has to be studied for $t > 0$ until at $t = \infty$ a new steady situation is attained. To be sure that a new steady situation is possible as well as to limit the horizontal dimension of the model area, it is assumed that the image of the flow region can be reflected to the left and right against vertical lines at certain distances from the discontinuity to appear in the open water level after reclamation. This allows the replacement of the "mirrors" by impermeable boundaries. It is mentioned here that for a similar problem a calculation technique yielding the steady position of the interface, assuming one-dimensional flow, has been presented by Van Dam (1976).

5. GEOHYDROLOGICAL SCHEMATIZATION

The aquifer concerned (fig. 4) is thought to be homogeneous and isotropic. The coefficient of permeability k equals $20\ m.d^{-1}$ while the effective porosity μ is assumed

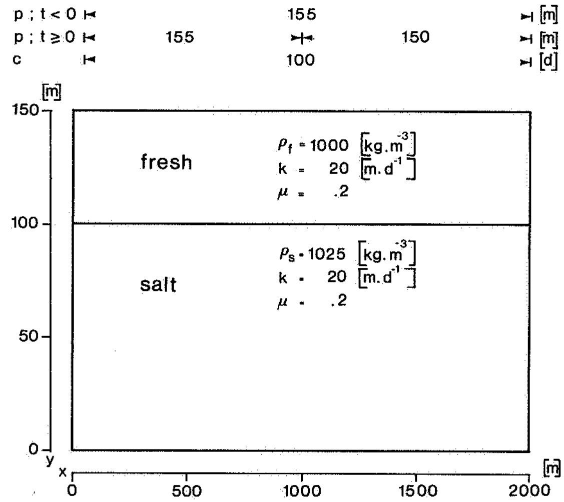


Fig. 4 Geohydrological schematization.

to be .2. The aquifer has a constant thickness D of 150 m. The semi permeable layer on top of the aquifer is characterized by a hydraulic resistance c of 100 d. The density of the lower (salt) fluid is taken equal to 1 025 kg.m^{-3} while the upper (fresh) fluid has a density of 1 000 kg.m^{-3}. The water level of the lake p is situated at 155 m above datum level; the last level being chosen at the lower boundary of the aquifer. The concept as a whole allows the interface to be chosen initially at an arbitrary but horizontal level. As a first approach the interface is taken at 100 m above datum level.

6. MODEL SCHEMATIZATION

Because of the inhomogeneity in the density of the fluid the flow region under consideration has to be divided into 2 subregions: A lower region containing salt water and an upper region containing fresh water. The boundaries of each subregion are divided into boundary elements. The boundary elements as well as the type of elements used, are indicated in figure 5. It is mentioned that the boundary element configuration as presented refers to the initial situation. In case after a time step the position of the interface appears to have changed the boundary element configuration has to be adapted. It even may be necessary to adjust the number of subregions distinguished in the schematization as time goes on.

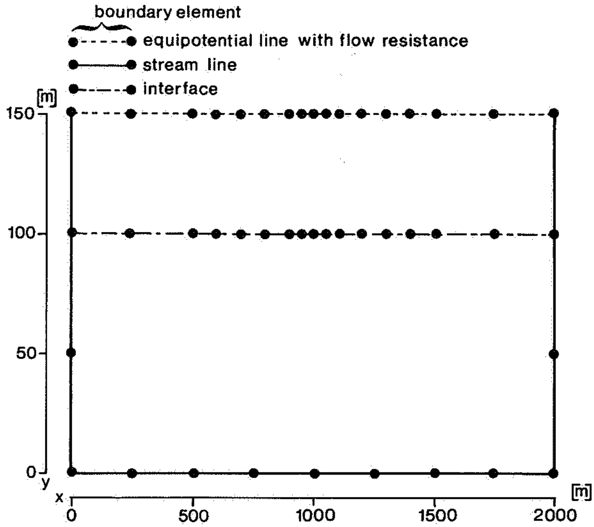


Fig. 5 Model schematization.

7. RESULTS

The computer model calculates the flow pattern in conformity with the boundary conditions imposed. The output consists of groundwater heads, stream function values and the x- and y-components of the specific discharge in any point desired. From the specific discharge perpendicular to the interface the new position after a step in time is calculated, which is then taken to be the position of the boundary for the next calculation of the flow pattern and so on and so forth. This makes it possible to present the position of the interface at several points of time, which will be done. Above this, for the same points of time, the flow pattern in terms of groundwater heads and stream function values is known. However, because of space limitations, the flow pattern will be presented for some selected points of time only and for only the most interesting part of the flow region. The points of time (expressed in days since the moment of reclamation) upon which the flow pattern has been calculated are listed in the first column of table 1. The second column gives the number of the figure in which the position of the interface is shown. The third column gives the number of the figure in which a part of the flow pattern is presented.

Table 1 Points of time and numbers of figures

Point of time since reclamation (days)	Figure showing the position of the interface (number)	Figure showing the flow pattern (number)
0	6	11
3	–	–
10	–	–
30	–	–
65	–	–
125	6	–
185	–	–
305	6	12
425	–	–
625	6	–
825	–	–
1 025	6	–
1 225	–	–
1 425	6	–
1 625	–	–
1 825	–	–
1 875 *	7	–
1 975	8	13
2 105 **	9	–
2 405 ***	10	–
∞ ****	10	14

* At t = 1 875 d the interface touches the upper boundary of the aquifer. If the thickness of the confining semi permeable layer is neglected, from now on salt water is seeping into the polder. It is noted that saline seepage first appears somewhere between the centre and the boundary of the polder. The seepage in the centre of the polder remains fresh because of the presence of a fresh water body.

** The fresh water body in the centre of the polder disappears at t = 2 105 d. The area in which saline seepage occurs is closed. However saline seepage rates remain highest near the boundary of the saline seepage area. As time goes on the interface moves towards its steady position, while the magnitude of the salt water body decreases. This process is very time consuming as stricktly speaking it takes an infinite time for the interface to reach a steady position.

*** Because of the time consuming calculation process it was decided to stop the step by step calculations at t = 2 405 d and to search for the steady position at once, which is presented in figure 10. This approach has the disadvantage that the point of time at which an "almost" steady position of the interface will be reached remains unknown. However, an indication may be obtained in the following way:

**** From the computations at t = 2 405 d, the specific discharge values within the saline seepage area are known. The magnitude of the area is also known. This makes it possible to calculate the amount of saline water q seeping into the polder. ($q = 1\,925\ m^3.m^{-1}.d^{-1}$). As times goes on the saline discharge q decreases and reaches the value q = 0 as $t \rightarrow \infty$. Now it is assumed that the discharge as a function of time may be written as

$$q = a.\exp(b.t)$$

where t is the time and a and b are constants. As the effective porosity is known, the total volume of salt water V leaving the aquifer between t = 2405 d. and $t = \infty$ can be measured from figure 10 ($V = 14\,660\ m^3.m^{-1}$). It now follows that

a = 2.6399
b = –.0001313

The function q is plotted in figure 15. It can be seen that it takes more than half a century until the value of the discharge is reduced below 10% of its value at t = 2 405 d.

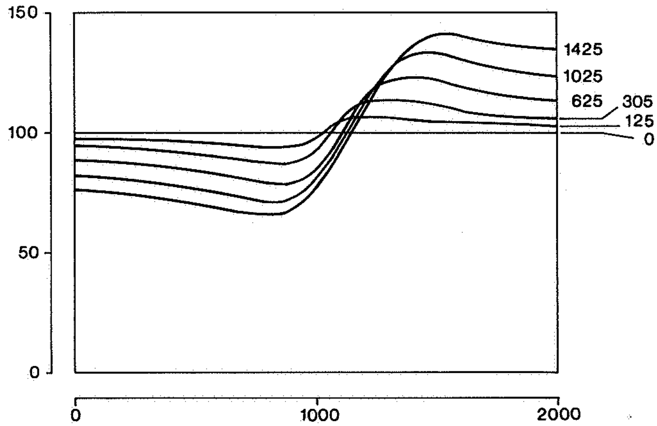


Fig. 6 The position of the interface at t = 0, 125, 305, 625, 1 025 and 1 425 d.

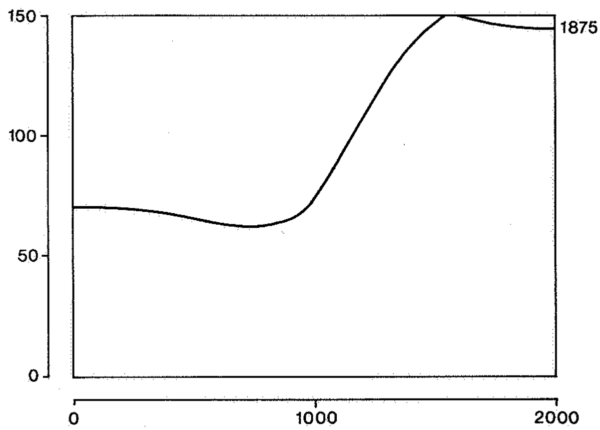


Fig. 7 The position of the interface at t = 1 875 d.

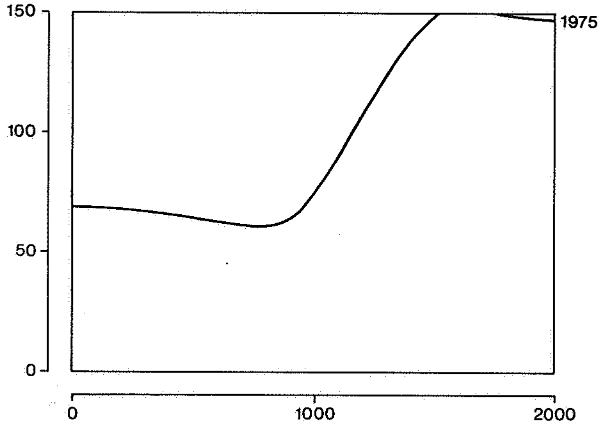


Fig. 8 The position of the interface at t = 1 975 d.

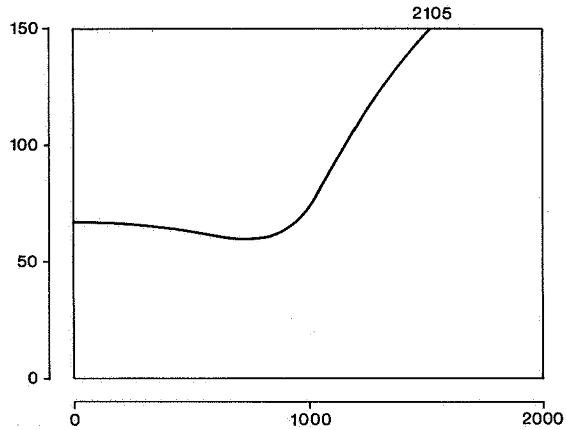


Fig. 9 The position of the interface at t = 2 105 d.

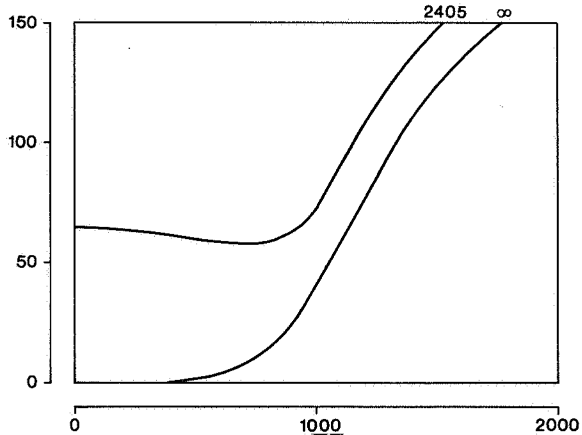


Fig. 10 The position of the interface at t = 2 405 d and the steady position.

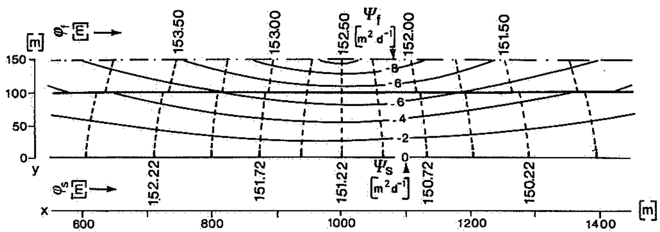


Fig. 11 The flow pattern at t = 0 d.

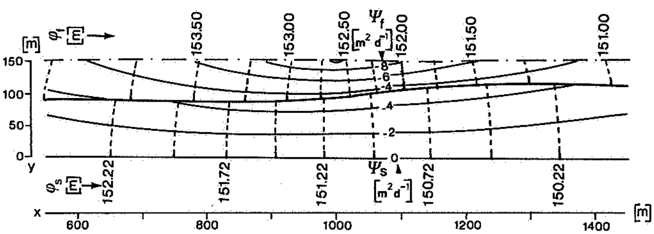


Fig. 12 The flow pattern at t = 305 d.

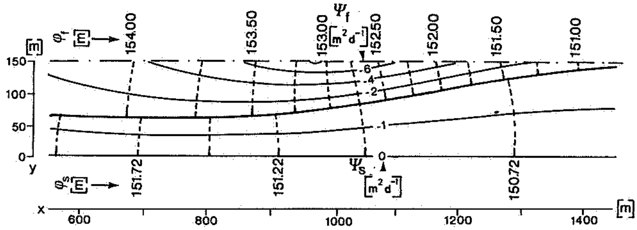


Fig. 13 The flow pattern at t = 1 975 d.

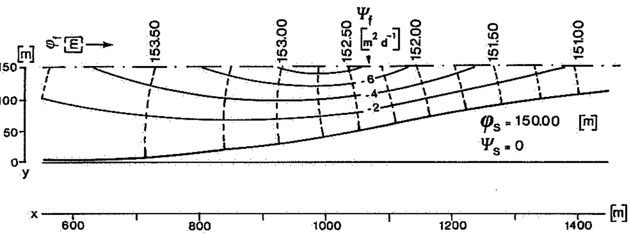


Fig. 14 The steady flow pattern.

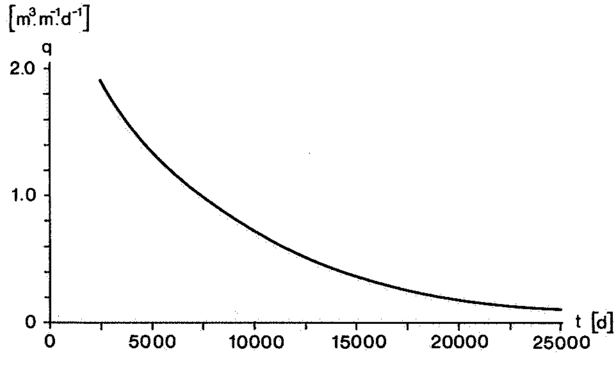


Fig. 15 The saline discharge q as a function of the time t.

8. CONCLUSIONS

- The response of an interface between fresh and salt groundwater to a sudden difference in the hydraulic head at the upper boundary of the aquifer has a wave-like character.
- If the reclamation of a lake causes seepage of saline water into the polder, the saline seepage occurs first somewhere between the centre and the boundary of the polder area.
- When comparing the position of the interface before and after reclamation it is seen that the steady position is extremely sensitive to changes in the open water or polder level. However, it takes a relatively long time to reach a new steady position when starting from an earlier steady position.
- In the schematized problem presented in this paper a steady state is reached with a stagnant salt water body remaining in the aquifer. In part of the polder area the salt water body reaches to the upper boundary of the aquifer. Nevertheless saline seepage does not occur.
- There is a conformity in the shape of the interface observed by Leenen below the eastern part of the polder "Groot-Mijdrecht" and the steady shape computed for the schematized problem. As a similar shape is found at points of time upon which the steady position is far from being reached a judgement whether the observed position of the interface is steady can not be given.

9. RECOMMENDATIONS

- As the transient behaviour of the interface depends on
  - the magnitude of the difference between the polder level and the open water level,
  - the assumed initial position of the interface and
  - the dimensions and geohydrological properties of both the aquifer and the semi permeable layer,

  the dependency on these factors has to be studied.
- As with the computer program used a time dependent calculation process has to be done by hand, the development of a program which allows an automatic time dependent calculation is recommendable.

REFERENCES

DAM, J.C. VAN (1976) – Partial depletion of saline groundwater by seepage, Journal of Hydrology, Vol. 29, No. 3/4: 315-339.

LEENEN, J.D. (1978) – Geoelektrisch onderzoek naar de verzilting van de polder "Groot-Mijdrecht" (in Dutch). Master Thesis, Department of Civil Engineering Delft University of Technology.

VEER, P. VAN DER (1978) – Calculation methods for two-dimensional groundwater flow. Rijkswaterstaat Communications, No. 28, The Hague.

RESULTS OF A GEO-ELECTRICAL SURVEY TO THE DEPTH OF THE FRESH WATER – SALT WATER INTERFACE IN THE POLDER GROOT MIJDRECHT

J.D. LEENEN

SUMMARY

The polder Groot-Mijdrecht has a relatively low phreatic water-level. According to the Badon Ghijben-Herzberg relation, this causes a rise of the fresh water – salt water interface underneath the polder. In the case of Groot-Mijdrecht the interface has reached the land surface and therefore saline seepage occurs within a certain region.

A geo-electrical survey in a cross-section of the polder Groot-Mijdrecht has been carried out to find the depth of the interface and the approximate region in which the saline seepage occurs (Leenen, 1978). Results of this survey are given in this article.

1. INTRODUCTION

The polder Groot-Mijdrecht is located approximately 20 km south of Amsterdam (fig. 1) and is one of the lowest of all Dutch polders. Its polder-level of 6.40 m below N.A.P. is 4.50 m lower than the adjacent eastern polder, which mainly consists of open water, the so-called "Vinkeveense plassen". The difference in water-level with the adjacent western polder is only 0.75 m, much less, but still respectable (fig. 2).

These differences in polder-level cause a large drop in the piezometric head of the fresh groundwater in the semi-confined aquifer underneath the polder Groot-Mijdrecht. According to the Badon Ghijben-Herzberg relation, this drop of piezometric head will cause the deep saline groundwater to rise until an equilibrium situation has been reached. In the polder Groot-Mijdrecht the drop of the piezometric groundwater level is so large that the saline groundwater in fact has risen so far that it has reached the level of the open water in the polder. This is the reason that saline seepage occurs in the middle of the polder.

The present polder levels in and around the polder Groot-Mijdrecht exist from about 1880 and it is to be expected that an equilibrium situation of the fresh water – salt water interface has not yet been reached.

Through geo-electrical surveying in an east-west cross-section through the polder (fig. 2 and fig. 3), the author has attempted to get a picture of the depth of the interface and to obtain data on the chloride concentration of the groundwater.

It should be emphasized that this survey does not give a complete picture of the salt situation in the polder, since its shape is not symmetric and the outside polder-levels have a considerable difference (fig. 2). Moreover, the geohydrological constants may show large local variations. However, tendencies can be shown and relevant information on this type of interfaces can be derived from the measurements in one cross-section.

2. GEOLOGICAL SITUATION AND ITS IMPLICATIONS

The few borings in the area, performed by the "Rijksinstituut voor Drinkwatervoor-

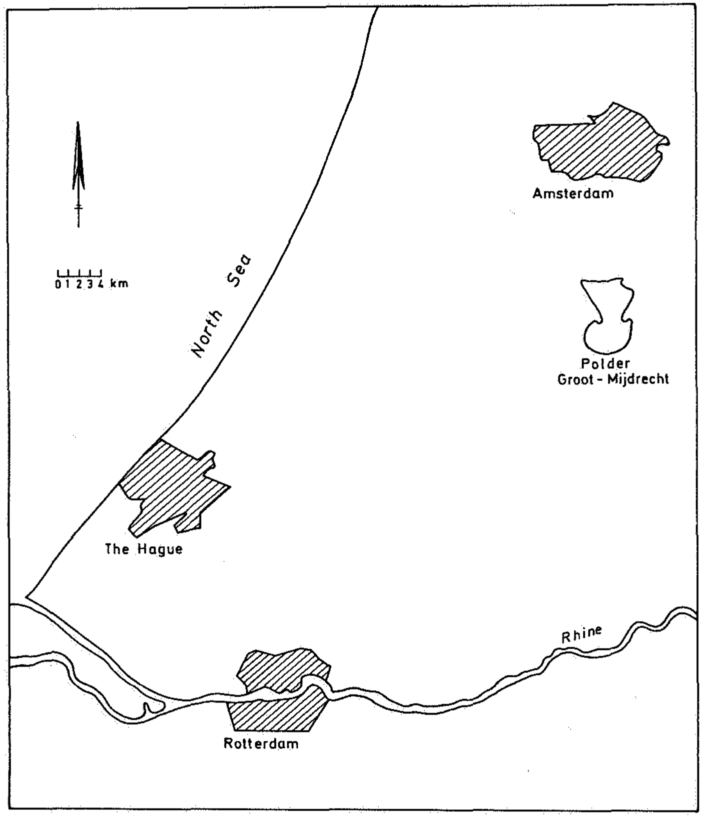


Fig. 1 Location of the polder Groot-Mijdrecht.

ziening' (National Institute for Water Supply), and the "Provinciale Waterstaat van Utrecht" (Provincial Waterboard of Utrecht), show that the subsoil is built up of sand with slight variations in coarseness. A top-layer of approximately 7 m consists

The presence of this top-layer makes it very hard to interpret the resistivity curves obtained from geo-electrical measurements, because in the region where the brackish groundwater hits the surface, it is not possible to distinguish a difference in chloride concentration from a difference in soil-layers. However, as far as the measurements pertain to the sandy layers, it is obvious that a variation in resistivity is due to a variation in salt concentration. This is the reason why, starting at the eastern edge of the polder – where the interface is still very deep – , the rising of this interface can clearly be followed as the measuring proceeds inwards into the polder.

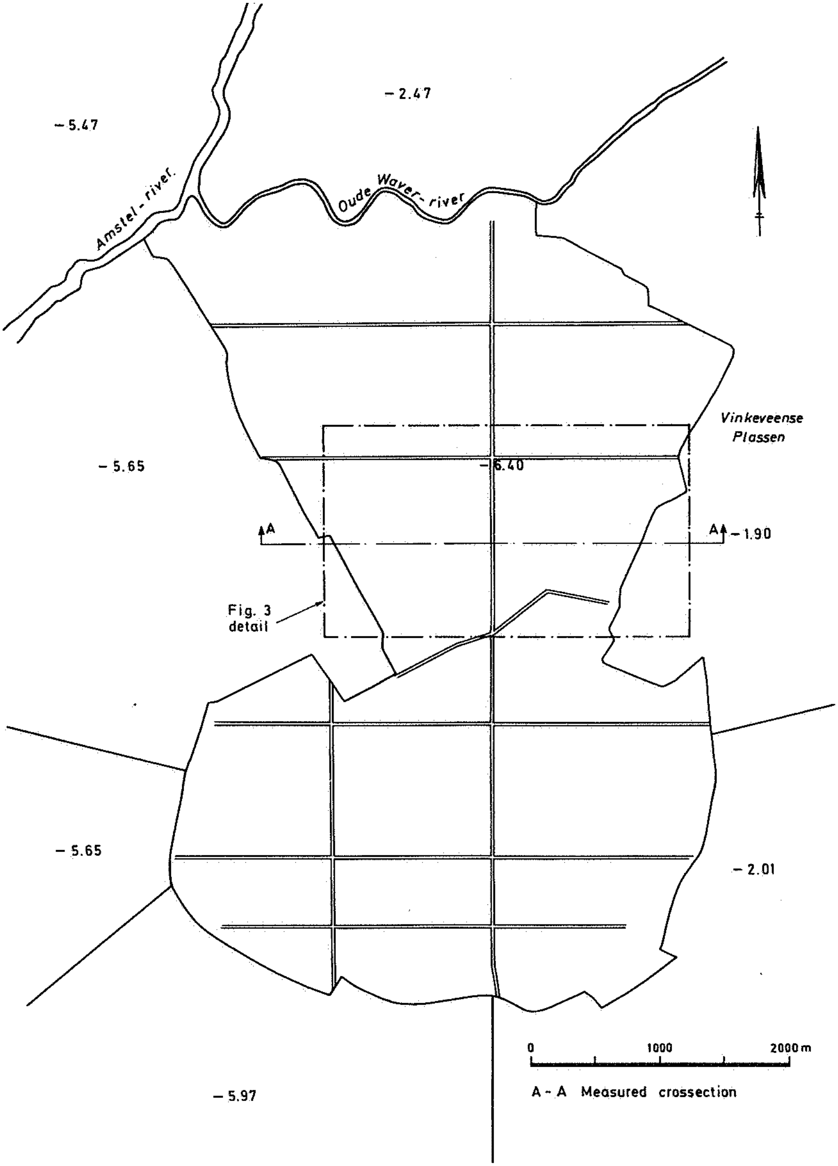


Fig. 2 Polder Groot-Mijdrecht with adjacent polder-levels all in [m] referred to N.A.P. (ordnance level).

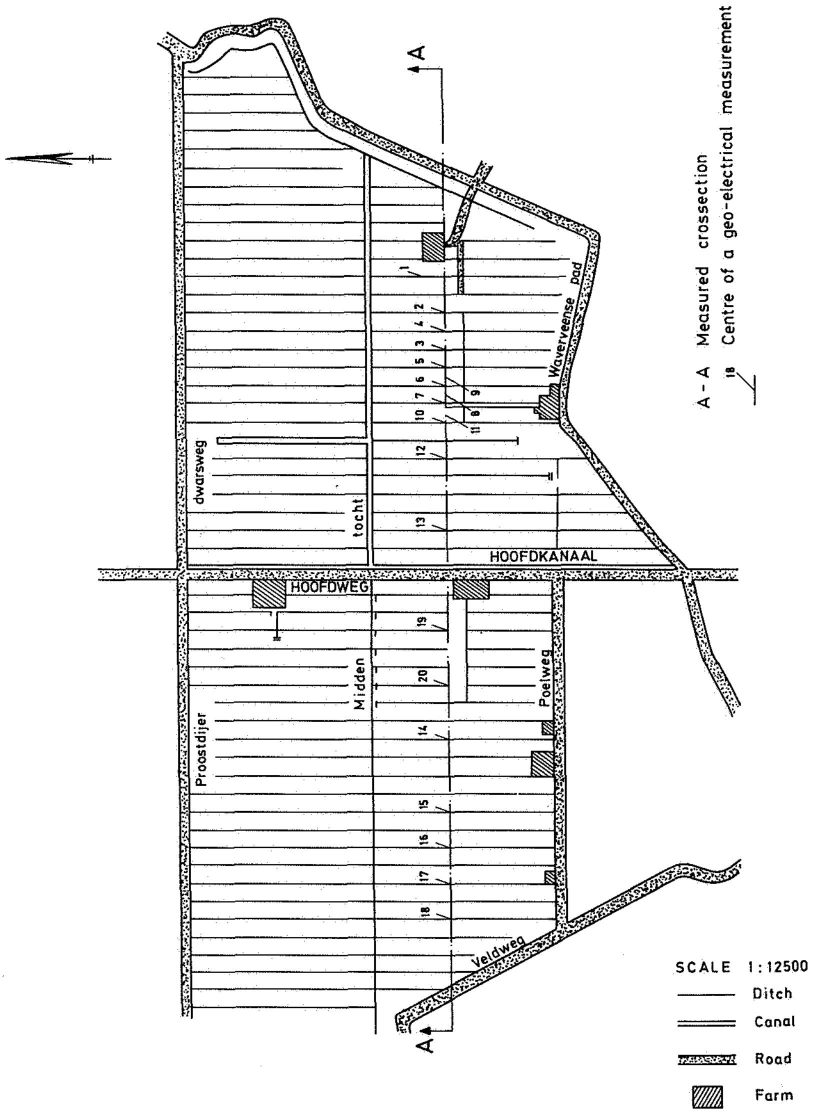
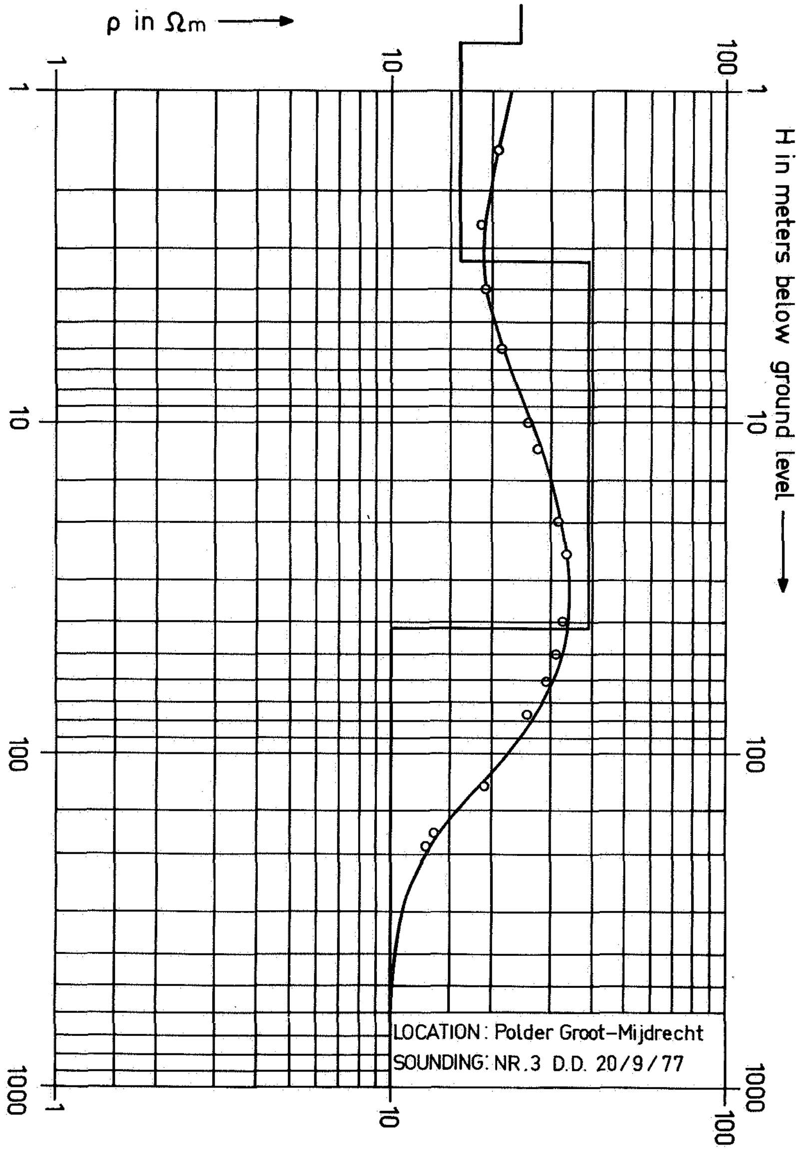
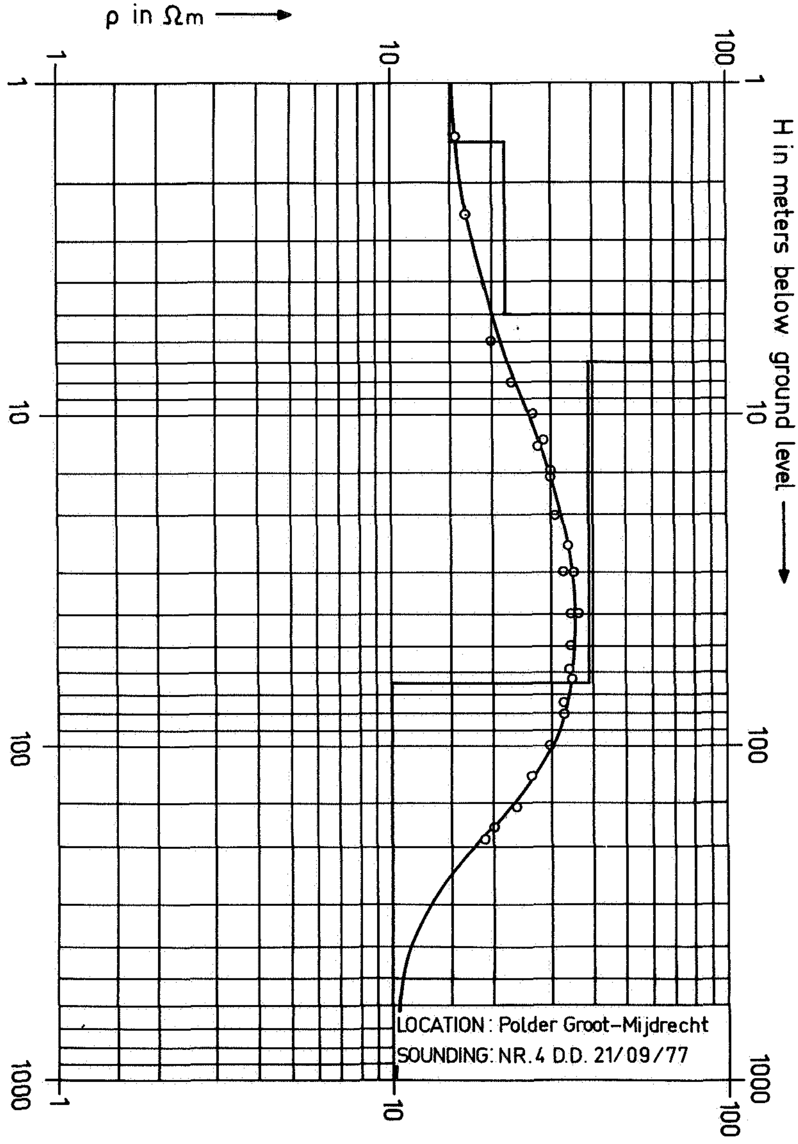


Fig. 3 Detail from fig. 2

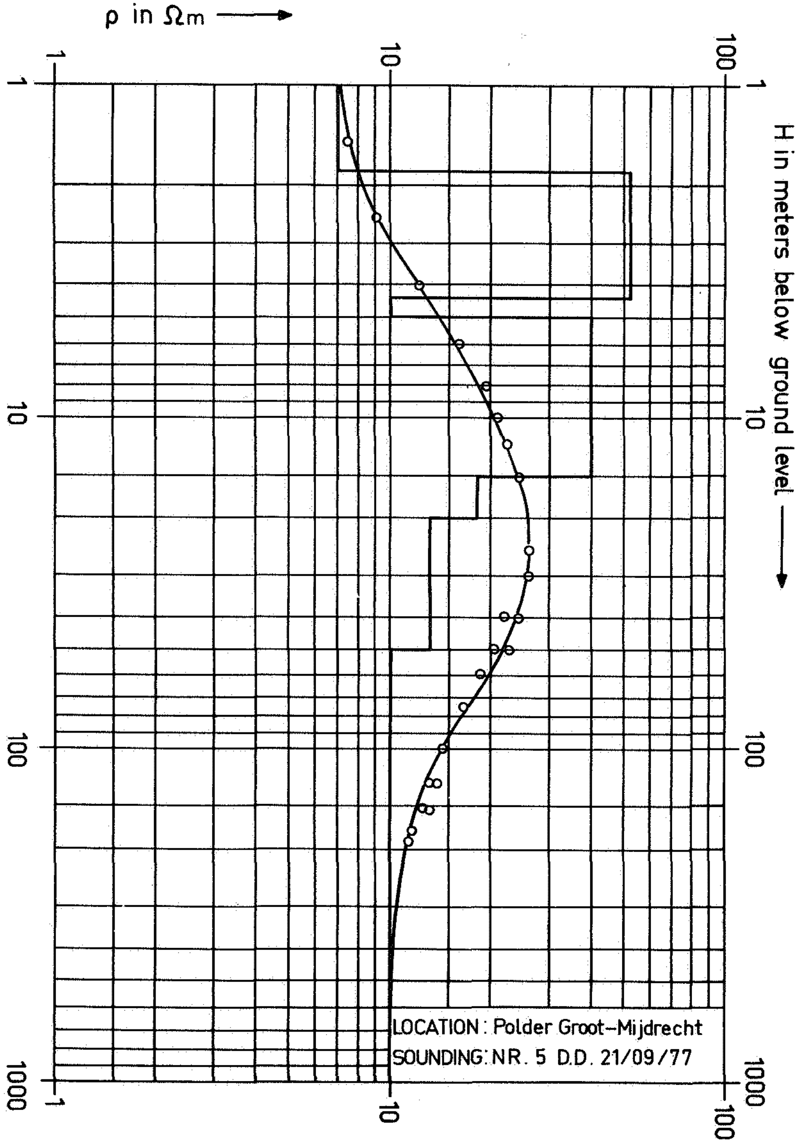
MEASUREMENT 3 IN POLDER "GROOT MIJDRECHT"



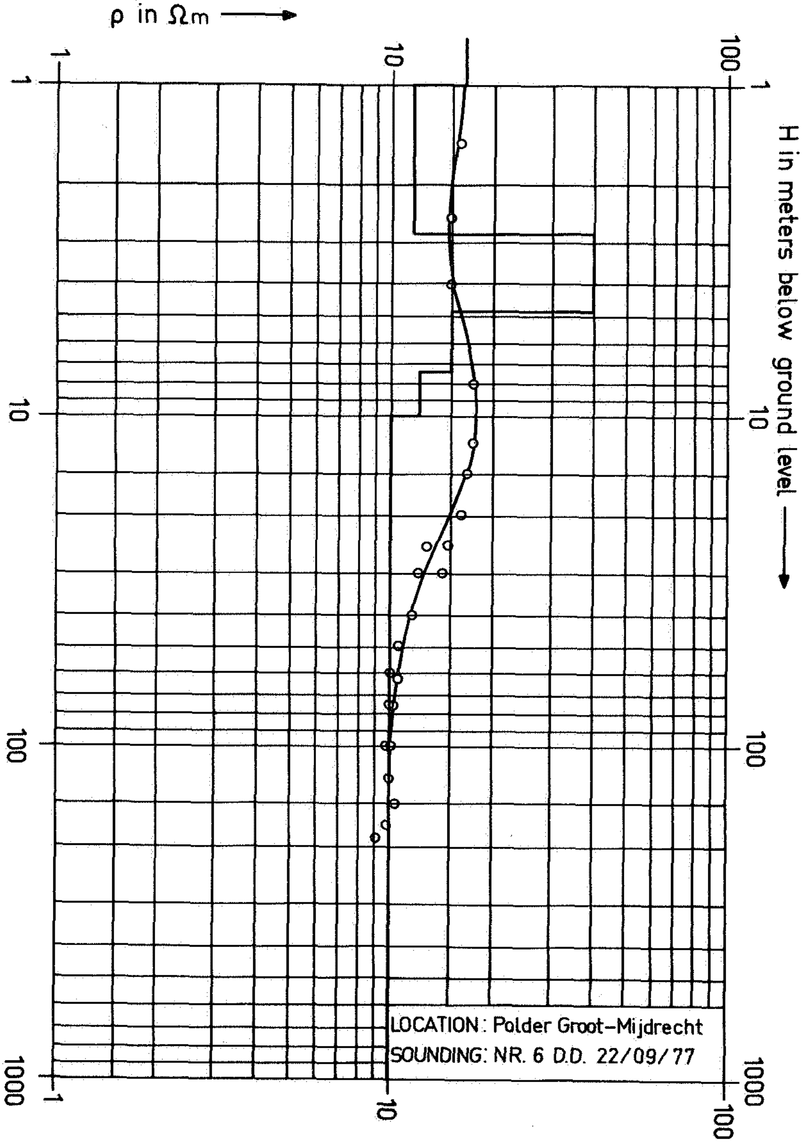
MEASUREMENT 4 IN POLDER "GROOT MIJDRECHT"



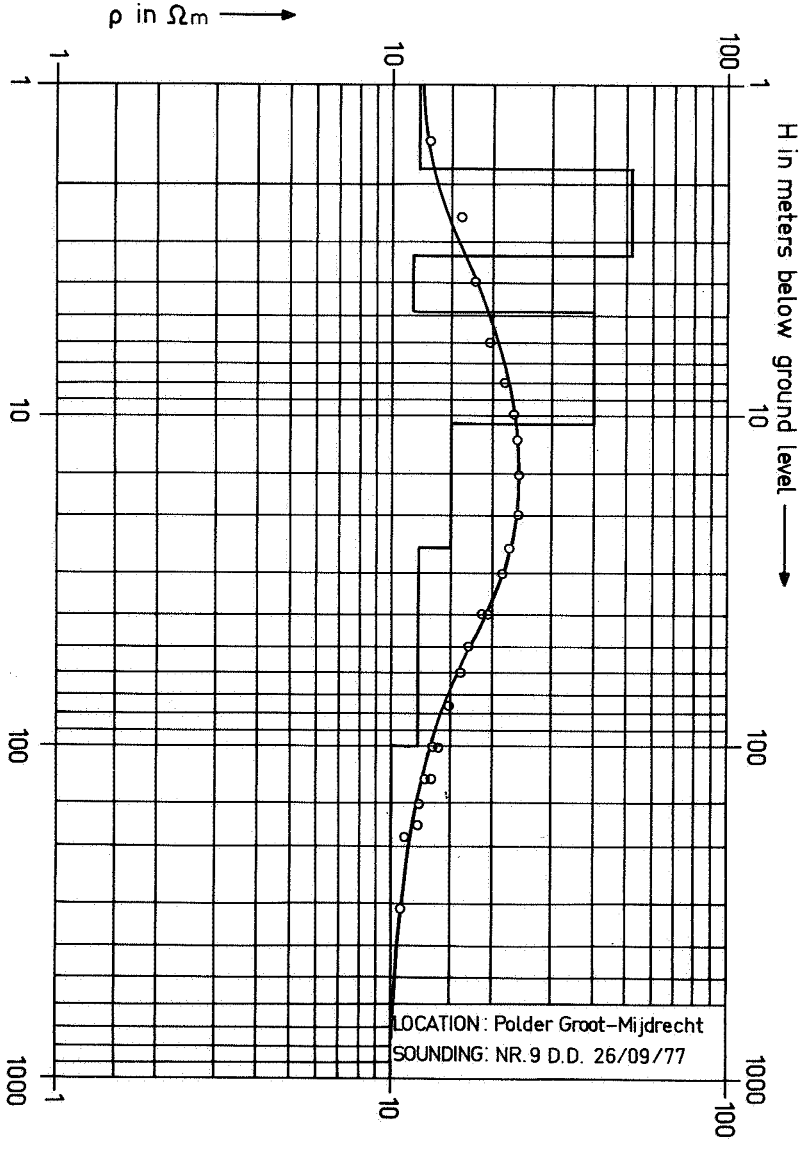
MEASUREMENT 5 IN POLDER “GROOT MIJDRECHT”



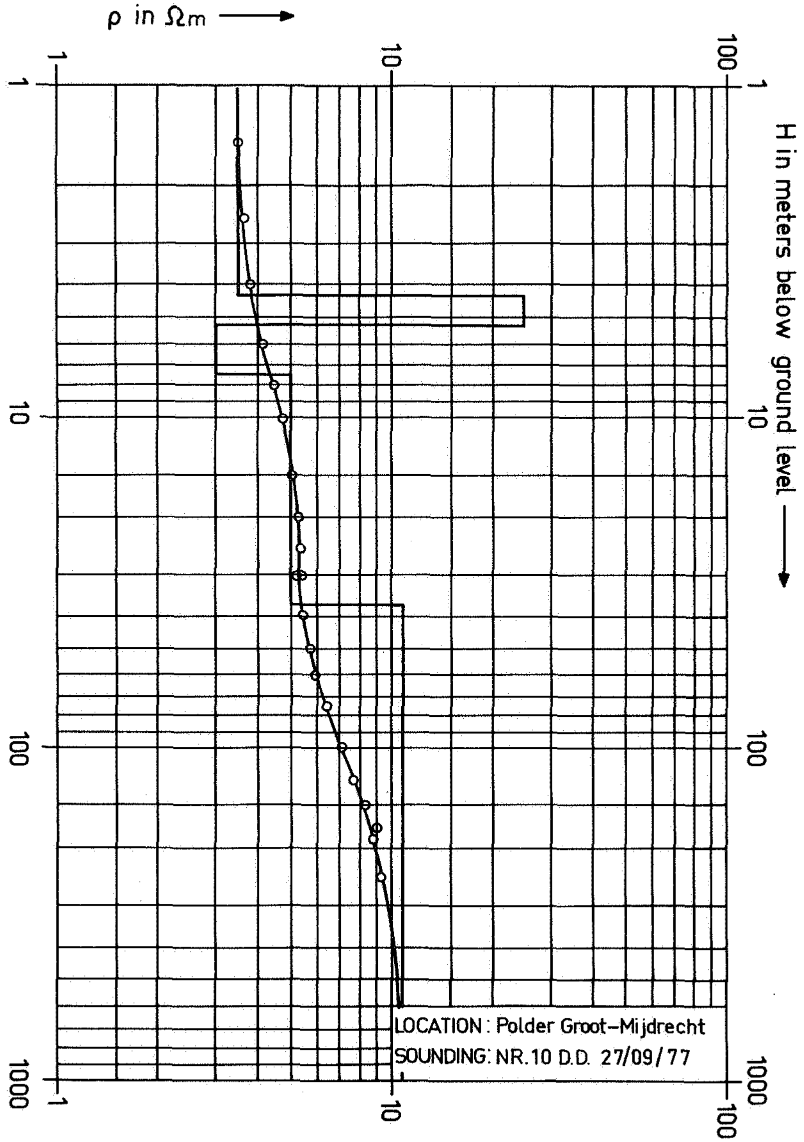
MEASUREMENT 6 IN POLDER "GROOT MIJDRECHT"



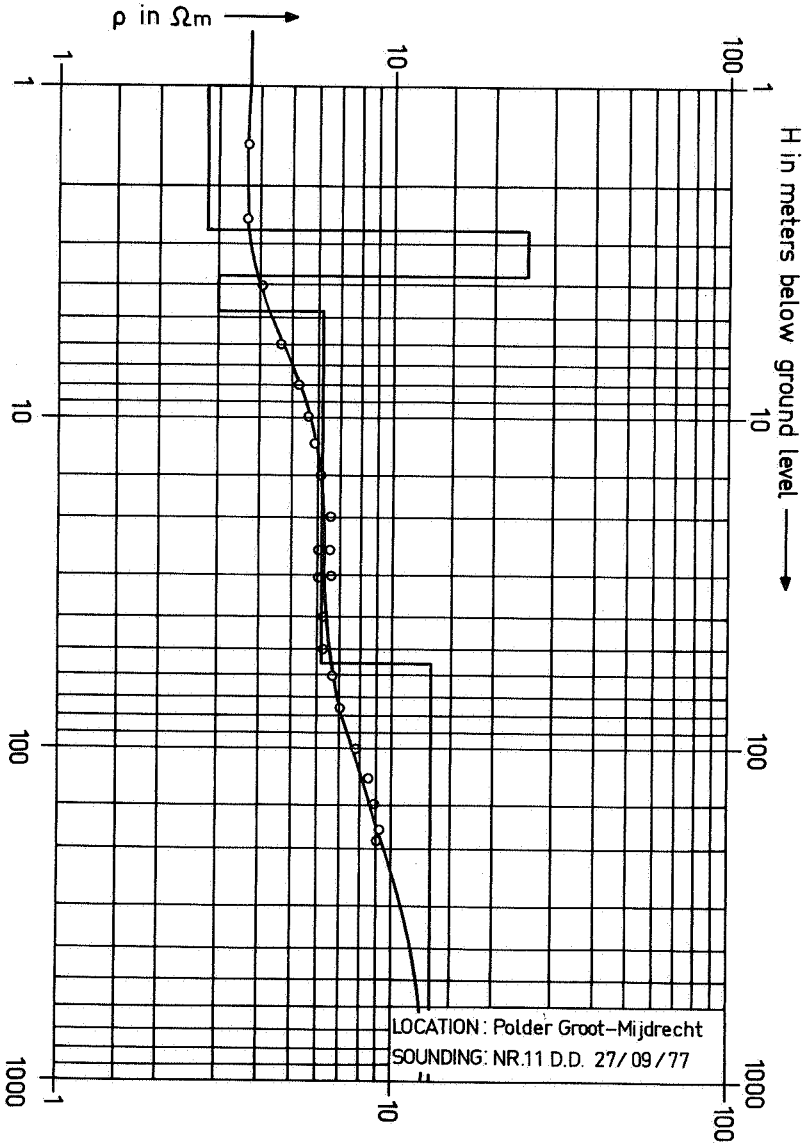
MEASUREMENT 9 IN POLDER "GROOT MIJDRECHT"



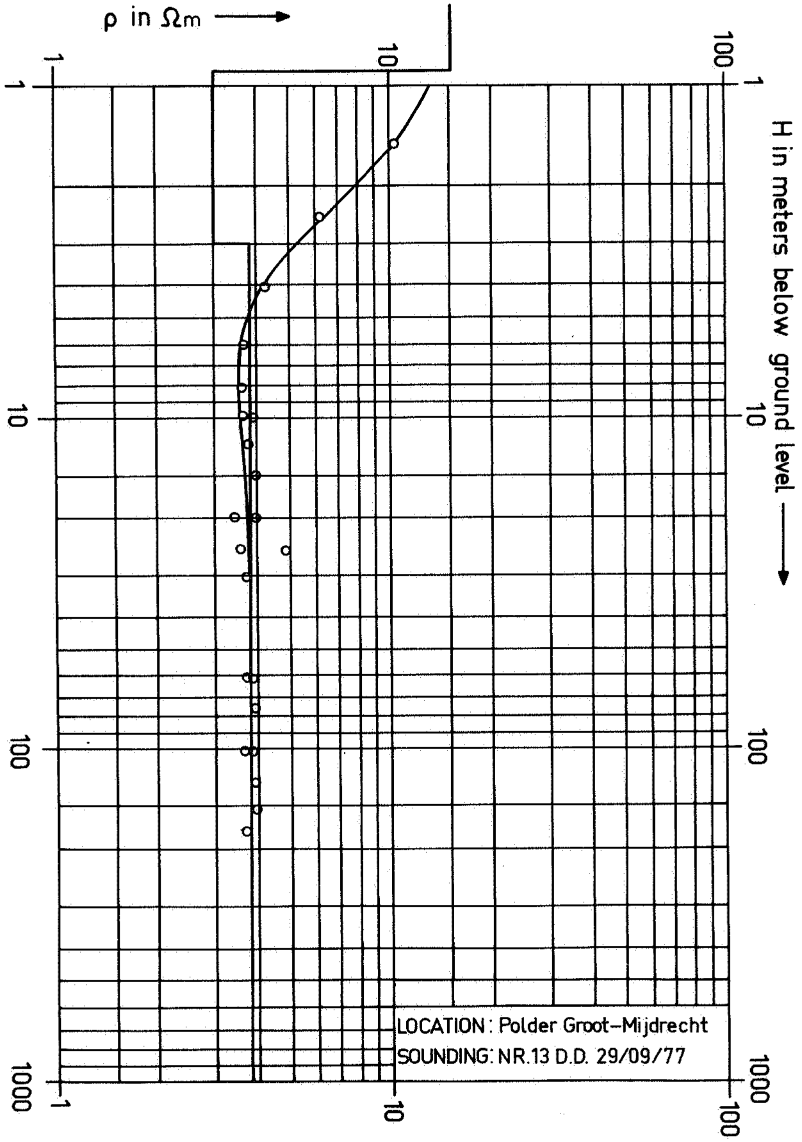
MEASUREMENT 10 IN POLDER "GROOT MIJDRECHT"



MEASUREMENT 11 IN POLDER "GROOT MIJDRECHT"



MEASUREMENT 13 IN POLDER "GROOT MIJDRECHT"



3. RESULTS

A total of 20 measurements with the so-called Schlumberger-method has been carried out in the cross-section, which has a length of about 2 200 m. Only those measurements which are important for the purpose of this article have been given here.

Interpretation from the apparent specific resistivities to real specific resistivities has been done through an indirect method by means of a Hewlett-Packard computer HP9100B with plotter. The Hydrology Group of Delft University of Technology has a program available which needs layers with a certain thickness and real specific resistivity as input. The apparent resistivity curve belonging to those layers is then plotted. This curve is compared with the measured values and the layers and their resistivities are adjusted until the best fit is reached. The program has a reach of 8 different layers which in general is more than sufficient for a good fit. Measured specific resistivities and plotted curves are given on a double-logarithmic scale. The horizontal axis is used for resistivity, the vertical one for depth.

The measurings 4, 3, 5, 9 and 6 show a steady rise of the interface, as the transition from low resistivities to high resistivities rises.

Measurement 13 – in the middle of the polder – shows about the same chloride-concentration over the whole depth.

In some measurements like 5, 6 and 9 it was necessary to use a layer-system with step-wise varying resistivities to fit the data. This clearly shows that there is no sharp interface between fresh and saline water. Instead there has to be a diffuse zone in which the chloride-concentration gradually decreases with decreasing depth. Since with the used computerprogram it is not possible to introduce a layer with a linear decreasing specific resistivity, this decrease must be simulated by a step-wise decrease. This phenomenon should be considered as the most important fact derived from the measurements. In the measurements 3 and 4 this phenomenon does not show, because of the contracted logarithmic scale. Looking at the measurements 4, 3 and 13, it is obvious that the salt concentration at great depth in the middle of the polder is higher than that at the eastern edge. This may be explained as follows:

a. There exists a large infiltration of fresh water from the "Vinkeveense plassen" into the sand-layers, which mixes with the salt water.
b. From 1956 to 1976 the "Vinkeveense plassen" have artificially been deepened through sandwinning for building purposes. As a result of this action the "Vinkeveense plassen" have in some places depths up to 35 m, which makes it easier for the fresh infiltration water to mix with the deep saline water.

Because the difference in phreatic level with the adjacent polder on the western side is not so large, there is much less infiltration in this adjacent polder and as a result the chloride-concentration at great depth at the western edge of the polder Groot-Mijdrecht does not differ very much from the concentration in the middle of the polder.

As shown in the general picture (fig. 4), the measurements at the western side indicate that the slope of the interface is not as steep as on the eastern side. This is, of course, due to the quoted differences in phreatic level.

From graphs of the "Dienst Grondwaterverkenning TNO" (Groundwater Survey, TNO), the specific electrical resistivity has been related to chloride-contents of the groundwater. The results thereof have also been given in figure 4. One rather mysterious thing may be noticed in the measurings 10 and 11. According to these, one might assume that there is a zone of groundwater with a relatively high salt-concentration above a zone with a relatively low concentration. This may indicate a tendency that the salt groundwater is gradually be replaced by fresh groundwater, flowing from the sides. However, at this point of investigation it can be no more than a theory. Other explanations for the above mentioned resistivity curves are possible.

4. CONCLUSIONS AND RECOMMENDATIONS

- Geo-electrical prospecting can be a most useful tool in determining the distribution of saline groundwater. Furthermore it is a relatively cheap method.
- In the polder Groot Mijdrecht it has been verified that the difference in freatic levels of some adjacent polders can be so large that seepage of saline groundwater occurs.
- The assumption of a sharp interface, which was necessary in many theoretical models, does not hold in polder-areas. Diffuse zones up to 20 meters thickness may be present.
- In the upper Holocene layers it is in general very hard to distinguish different layers from a change in the electrical conductivity of the groundwater. This is because of a great variance in top-layers.
- In the case-study of the polder Groot Mijdrecht the influence of a deep fresh groundwater flow from the "Vinkeveense plassen" into the polder Groot Mijdrecht appears clearly in the chloride contents of the groundwater in the eastern part of this polder.

Further investigations in the polder Groot Mijdrecht to complete the picture should be worthwile. At the moment further geo-electrical surveying is performed by the Hydrology Group of Delft University of Technology. To get more detailed information about the diffuse interface-zone borehole-logging is recommended. With the present information suitable spots can be chosen.

As mentioned earlier equilibrium conditions have not yet been reached. Therefore it deserves attention to carry out the same measurements with intervals of say 5-10 years, to be able to observe the changes in the diffuse interface and chloride contents.

Fig. 4 Cross-section A-A. Interface and chloride contents.

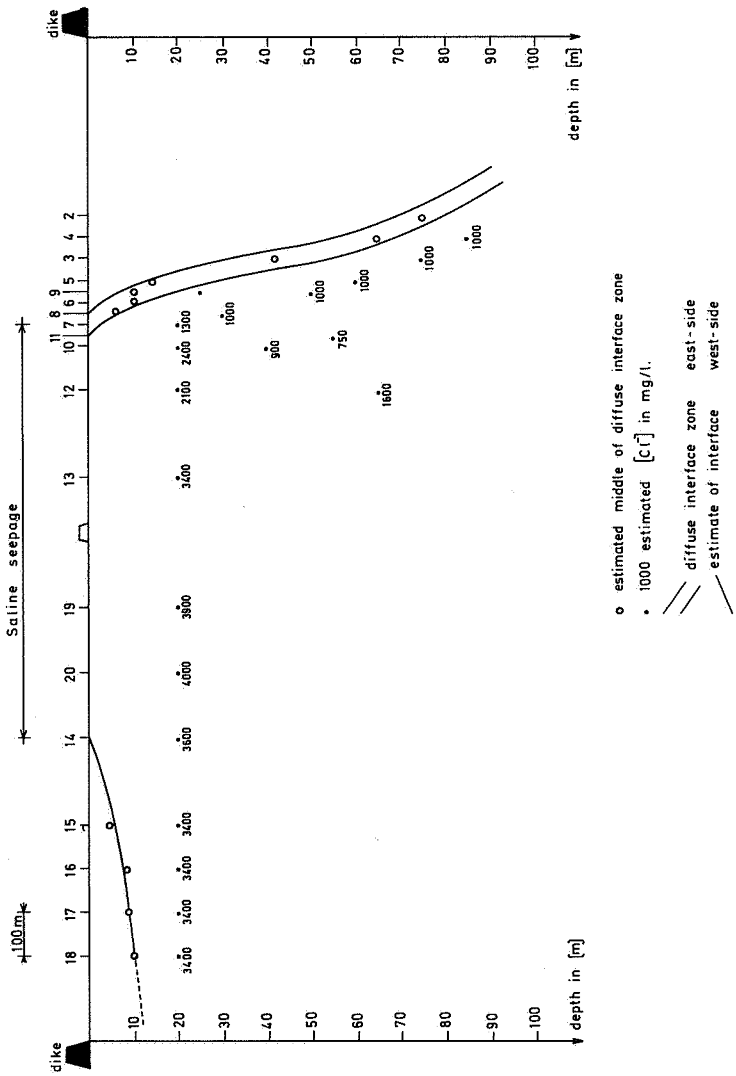


Fig. 4 Cross-section A-A. Interface and chloride contents.

REFERENCES

DAM, J.C. VAN (1964) – Geo-electrisch onderzoek in het Randgebied van Zuidelijk Flevoland. Dienstnota Rijkswaterstaat, Directie Waterhuishouding en Waterbeweging.

DAM, J.C. VAN (1967) – De theoretische grondslagen voor het geo-electrisch onderzoek. In: Water, nr. 51: 5-12.

DAM, J.C. VAN (1976) – Partial depletion of saline groundwater by seepage. Journ. of. Hydrol. Vol. 29, Nr. 3/4: 315-339.

INSTITUUT VOOR CULTUURTECHNIEK EN WATERHUISHOUDING (1973) – De chemische samenstelling van het diepe grondwater in Midden-West Nederland en de invloed hiervan op de gebruiksmogelijkheden. ICW-Nota 769.

KRUIJK, K.H. DE (1963) – Hydrologisch onderzoek Groot Mijdrecht. Interne nota, Prov. Waterstaat van Utrecht.

LEENEN, J.D. (1978) – Geo-elektrisch onderzoek naar de verzilting van de polder Groot Mijdrecht. Master thesis, Department of Civil Engineering Delft University of Technology.

ACCURACY AND VERIFICATION OF GROUNDWATER FLOW MODELS

C.R. MEINARDI and A.N.M. OBDAM

SUMMARY

No agreement may exist between results of a groundwater flow model and field observation because of either wrong model schematization, or inaccurate values of the model parameters, or incorrect boundary and initial conditions, or errors in the field observations. Consequently for model verification a determination of confidence limits of model parameters and of boundary and initial conditions is needed. Possible errors in geometry of the underground, in hydrological parameters, in water levels and in salt distribution are estimated.

Models transform the errors in input data to errors in results; two models concerning the flow of fresh and brackish groundwater have been investigated. One analytical model is based on the Badon Ghijben-Herzberg principles and the other numerical model concerns flow to a partially penetrating well, neglecting density differences. A sensitivity analysis forms part of the investigations. With the present state of data accuracy, in many cases no model verification will be possible as the confidence limits of results are too large.

1. THE RELATION BETWEEN VERIFICATION AND ACCURACY OF MODEL RESULTS

Groundwater flow obeys physical laws and therefore can be represented by physical or mathematical models of a real situation. Mostly deterministic models are used for groundwater flow simulation; the model acts as a physically determined operator between input data and results. A fixed set of input data will yield another fixed set of results. We use these models to determine the consequence of (changes in) groundwater flow, such as level drawdowns, solute transport etc. Generally the model results will deviate from observed values in the real situation. This may be due to a number of reasons (fig. 1):

- wrong schematization of the real groundwater flow system;
- inaccurate values of the model parameters;
- incorrect representation of boundary and initial conditions;
- measuring errors in observations.

If we want to verify whether we choose the right assumptions to schematize a complex reality into a simple model then we need to know the inaccuracies due to the other three sources of error. Disagreement between model results and observations does not in itself mean that we have chosen a wrong schematization and contrarily agreement between results and a limited amount of observations does not necessarily mean that our model is correct. For other (longer) periods again large deviations may occur; furthermore it will not be sure that other non-observed parameters are also correctly determined.

Thus model verification will only be possible if we have an indication of the

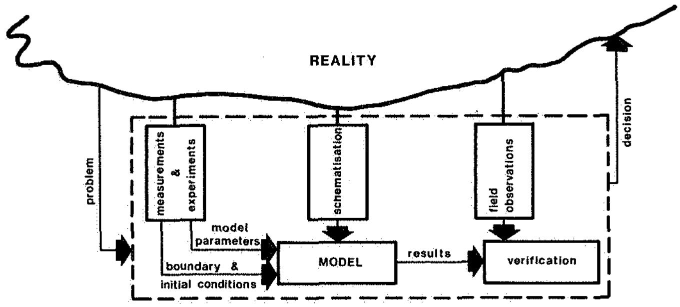


Fig. 1 The relation between modelling and verification.

inaccuracy of the model results, caused by an erroneous representation of model parameters and boundary and initial conditions. If the confidence limits of the model results are small and a statistical test on the two populations (model results and observations) gives a positive result, then we may conclude that the model gives a good representation of reality at least as to the observations done. In the following chapters some remarks will be made about inaccuracies of model parameters and initial and boundary conditions; their effect upon inaccuracy of model results will be illustrated by a few examples.

2. ACCURACY OF MODEL PARAMETERS

In the normal type of groundwater flow model the model parameters consist of the geometry of the underground and the permeability of the various layers.

For the sake of simplicity this paper will only deal with cases of saturated groundwater flow having simple boundary conditions at the upper side (polder levels, lakes etc.). Hence the model parameters to be discussed are:

- geometry
- horizontal permeability
- vertical permeability
- transmissivity
- hydraulic resistance

2.1. *Geometry*

The geometry of the underground depends on geological factors. The sedimentary deposits in the Netherlands often lead to a geohydrological schematization in approximately horizontal layers. Aquifers mostly contain sand and confining layers consist of clay-, loam- or peat layers. Sometimes a sharp boundary exists between aquifer and confining layers, but also the situation occurs that the sand layers of an aquifer gradually become finer and more silty; the boundary is not sharp but has the form of a transition zone.

Geometrical properties of the underground can be derived from bore-logs or from geophysical prospecting. The most accurate values result from an interpretation of bore-logs. However, borings only represent point observations, so the problem becomes that the position of underground surfaces has to be estimated from a relatively small number of points. Interpolation techniques and/or statistical methods have to be applied.

Apart from the above mentioned errors of a random character, the position of an underground surface may also be liable to measuring errors which are more or less of a systematic nature. These measuring errors partly depend on the boring system applied; as these errors mostly will be relatively small, they will be neglected in the following.

2.1.1. *Thickness of aquifers*

The lower boundary of an aquifer is the top of a confining layer. In case of a sharp boundary it will be the surface of a clay- or a loam layer. Such a surface will be neither fully flat nor horizontal. Deviations of the lower aquifer boundary from a horizontal plane may be divided in regional slopes (tectonic movement, general inclination of a sea floor, or a flood plane) and in local irregularities (river banks/basins at deposition, subsequent erosion, compaction). Generally the surface might be represented by a regularly constituted plane together with randomly scattered deviations. The regular plane (regional tendencies) should be determined by interpolation techniques, while the local irregularities are assumed to follow a normal distribution.

Principally the factors influencing the upper boundary of an aquifer are the same as for the lower boundary. Differences are e.g. that at deposition other irregularities were formed and that probably unequal compaction will be less important.

The thickness of an aquifer is the height H between the depth of upper and lower boundary:

$$H = d_{l(ower)} - d_{u(pper)}$$

As d_l en d_u are independent variables it follows from statistical theory that:

$$(\sigma(H))^2 = (\sigma(d_l))^2 + (\sigma(d_u))^2$$

(in which σ is the symbol for the standard deviation).

Two extreme situations can be distinguished at the determination of the thickness H of an aquifer.

a. *Local investigation with much bore-hole information*

A situation can be imagined where in a limited area a large number of borings are made for a local geohydrological investigation. All of these borings will indicate the position of the upper and the lower boundary on the aquifer concerned. As in this case regional tendencies may be neglected it may be assumed that the depths of the aquifer boundaries are normally distributed. Now first the mathematical expectation and the standard deviation of both depths should be determined and thereafter with the above formula the mean value for H and the standard deviation in H.

b. *Extrapolation of bore-hole data*

Sometimes a (rough) geohydrological investigation is needed at places where no bore-hole information is available; in other cases a regional groundwater flow model asks for geometrical information at places without borings. Now the following procedure should be applied:

- Regional tendencies in both the surface of the upper and the lower boundary are approached by interpolation techniques. The result can be expressed as a most probable value together with a standard deviation of the estimate.
- Again for both surfaces an estimate is made of a standard deviation representing the local irregularities.
- The aquifer thickness is the difference between the estimates of the positions of upper and lower boundary. The standard deviation with the estimate of aquifer thickness equals the square root of the sum of the squares of all the respective standard deviations derived from extrapolation and irregularities.

Estimatedly the error made with extrapolation may be in the order of magnitude of some metres, to be determined for each special case. The local irregularities in a river landscape also will be in the same magnitude (standard deviations between 1 and 5 metres), whereas for a sea – floor they may be either somewhat smaller (shallow sea) or much larger (estuaries). Not much research has been done as yet in this field for the Netherlands situation.

2.1.2. *Thickness and horizontal extent of confining layers*

Aquifers and confining layers are complementary; the lower boundary of an aquifer is the top of a confining layer and vice versa. For upper and lower boundaries of confining layers the same arguments hold as for those of aquifers and the way of determining mean values and standard deviations will also be the same. The standard deviation of the determination of the thickness will be in the same order of magnitude, i.e. at least some meters but often more.

In the Netherlands situation one important difference exists: Generally aquifers are much thicker than confining layers (aquifer thickness being in the order of magnitude of some tens of meters and confining layers of some meters). This means that the relative error in the determination of the thickness of confining layers often will equal the thickness itself, especially at places without borings. As the thickness of confining layers is strongly related to the hydraulic resistance of the layer, the inaccuracy in thickness determination will strongly influence estimates of hydraulic resistance based on thickness.

The above has also consequences for the estimate of the horizontal extent of confining layers. As confining layers generally become thinner at their outer boundaries it follows that also the horizontal extent of such layers can mostly only be roughly determined. Especially river clay layers have very irregular forms.

2.2. *Transmissivity and horizontal permeability*

To determine horizontal permeability in the field several methods can be applied such as:

- permeameter tests on undisturbed samples;
- granular analysis of disturbed samples;
- interpretation of geophysical bore-logs, etc..

Indirectly horizontal permeability follows from methods to determine transmissivity:

- pumping tests;
- water balance calculations;
- single well pumping tests;
- analysis of natural groundwater flow (e.g. tidal fluctuations).

We may safely assume that, of all these methods, pumping tests will yield the most reliable results. It follows that in practice the most accurate results will be reached if determination of transmissivity and horizontal permeability are fully based on a number of pumping tests.

2.2.1. *Transmissivity from pumping tests*

The interpretation of pumping test results is based on measurement of:

a. pump discharge Q $[L^3T^{-1}]$
b. distance of observation wells r $[L]$
c. time t $[T]$
d. level drawdowns resulting from pumping s $[L]$

All these measurements are liable to systematic and to random errors.

Important systematic errors may be caused by:

a. *Systematic deviation of water meter observations*

The water meter used should have been gauged against another method of water metering before the test. A systematic error of maximum 2% of water meter readings may occur in that case.

b. *Systematic errors in the distance of observation wells*

A possible error in distance results from the fact that bore-holes always deviate somewhat from the vertical. Hence the real distance between pumpscreen and observation screen will differ from the measured distance between the rims of the rising pipes. Little is known about this phenomenon.

c. *Time measurements*

The moment of full pump discharge will always lag behind the time of starting of the pump. This systematic error will mostly not exceed a few minutes and hence becomes negligibly small after some time.

d. *Level drawdowns by pumping*

An important source of errors derives from the fact that for interpretation exclusively the drawdowns caused by pumping have to be used. This means that all other (natural or artificial) fluctuations of groundwater levels have to be eliminated from the time-drawdown curve. Smooth and abrupt level fluctuations may be distinguished.

Somewhat arbitrarily it can be stated that the sum of systematic errors may lead to a difference of about 5% in the value of transmissivity.

The most important random errors are made with the measurements of pump discharge and level drawdowns. They mainly result from wrong readings and reading inaccuracy. These random errors are overcome in the classical analysis by the curve-fitting method. Smooth curves of the exact solution should as well as possible cover the randomly scattered observations. Fitting is done by eye. The best fit yields the best estimate of transmissivity at the pumping test location. No standard deviations can be obtained. Standard deviations can yet be obtained by applying a numerical method (labelled NILIRE) where fitting is done by non-linear regression (Leijnse, 1980).
The results for a given set of pumping tests are given in table 1.

These pumping tests belong to one particular hydrogeological investigation aiming to determine an areal picture of transmissivity values in the Guelders Valley area (Meinardi, 1978).

Table 1 Transmissivity from pumping tests data of Guelders Valley, analysed with various methods (PT = pumping test, RT = recovery test)

Pumping test location		(1) Curve fitting (classical analysis) transmissivity in m^2/day	(2) Non linear regression (NILIRE) without correction for extern effects, T in m^2/day ($\pm 2\sigma$)	(3) Non linear regression (NILIRE) with a correction for extern effects, T in m^2/day ($\pm 2\sigma$)	(4) Possible systematic error	(5)* Final value of transmissivity T in m^2/day + possible error (NILIRE)	(6) Difference between NILIRE and curve-fitting method
Huinerbroek	PT	7 100	4 693 ± 10%	7 358 ± 4%	5%	7 300 ± 9%	3%
Zwartebroek	PT	4 700	5 980 ± 10%	5 377 ± 8%	5%	5 400 + 13%	7%
	RT	5 000					
Amersfoort	PT	4 800		5 321 ± 11%	5%	5 300 + 16%	10%
Barneveld	PT	5 100	5 853 ± 4%	4 944 ± 8%	5%	5 000 ± 13%	3%
Achterveld	PT	6 000	5 800 ± 4%	5 700 ± 12%	5%	5 700 ± 17%	5%
Glindhorst (nearby obs. wells)	PT		4 542 ± 0.5%	4 542 ± 0.5%			
Glindhorst (all obs. well) except the well at 16 m)	PT	5 700	4 632 ± 0.5%	4 632 ± 0.5% / 5 320 ± 2%	5%	5 300 ± 7%	7%
Woudenberg	PT	3 400	3 513 ± 10%	3 491 ± 4%	5%	3 600 ± 13%	0.5%
	RT	3 800	3 745 ± 8%	3 745 ± 8%			
Ederveen	PT	3 500	2 946 ± 6%	2 946 ± 6%	5%	3 000 ± 11%	19%

* The values of kH presented in (5) are rounded values from (3).

The figures of table 1 give lead to the following remarks:

1. The standard deviation of transmissivity due to difference between measured drawdowns and expected drawdowns varies between 0.3% and 5.5% (column 3). A value of two times this standard deviation indicates the 95% confidence limits of transmissivity resulting from random errors. To these values the possible systematic error should be added to obtain the possible error in transmissivity according to the analysis with NILIRE. Variations are between 7% and 17% (column 5).
2. Large errors may occur if no correction at all is applied for external effects on groundwater levels (columns 2 and 3). Errors of 18% (Barneveld) and even 36% (Huinerbroek) result.
3. The differences between the values obtained by curve-fitting and by NILIRE may be considerable. It is reasonable to assume that they largely are caused by a different interpretation of smooth, external influences on groundwater levels.
4. The Glindhorst pumping test illustrates the danger of using observations from wells very near to the pumped well (distance errors). If the observation well at a distance of 16 m is taken into account then a 20% lower value is found compared to the case it is left out. Nevertheless the accuracy is high in both cases.

2.2.2. *Horizontal permeability derived from pumping tests*

As a good approximation it may be taken that transmissivity represents the product of horizontal permeability k_h and thickness H of the aquifer.

Hence: $T = k_h \cdot H$ $[L^2T^{-1}]$

In such a case the square of the relative error in k_h may be approximated by the sum of the squares of the relative errors in T and in H:

$$\left(\frac{\Delta k_h}{k_h}\right)^2 = \left(\frac{\Delta T}{T}\right)^2 + \left(\frac{\Delta H}{H}\right)^2.$$

For any particular pumping test the relative error in transmissivity can be derived as indicated in the fore-going. The determination of the relative error in H should depend on geological data from the pumping site (especially bore-logs). No fixed general rules can be given.

The method can be illustrated with the examples of pumping-test Huinerbroek (table 1) where a rather common situation occurs that the upper side of the aquifer has been reached by a number of borings and the lower side only by the reconnaissance boring.

Five borings indicate the upper side of the aquifer to lie at land surface minus 84.25 m; –89.0 m; –83.25 m; –81.5 and –85.25 m. From these values it can be concluded that the mean value of the depth is 84.65 m. with a standard deviation of 2.50 m. The possible absolute error in the depth determination is taken here to be two times the standard deviation 5.00 m). The lower side of the aquifer is only known from one boring to lie at landsurface

Table 2 Horizontal permeability derived from transmissivity and thickness determined at the pumping tests of table 1.

kH (m^2/day)	σ_{kH}	σ_{kH} (m^2/day)	$\sigma_{kH/s}$	$\sigma_{kH/s}$ (m^2/day)	H (m)	σ_H *	σ_H* (m)	k (m/day)	σ_k **	σ_k** (m/day)	$\sigma_{k/s}$	$\sigma_{k/s}$ (m/day)	$\sigma_k + \sigma_{k/s}$ (m/day)	
7 300	2%	146	2.5%	182	110	3.2%	3.5	66	3.8%	2.7	2.5%	1.7	6.3%	4.4
5 400	4%	216	2.5%	135	105	3.3%	3.5	51	5.2%	2.6	2.5%	1.3	7.7%	3.9
5 300	5.5%	292	2.5%	133	95	3.7%	3.5	56	6.6%	3.7	2.5%	1.4	9.1%	5.1
5 000	4%	200	2.5%	125	105	3.3%	3.5	48	5.2%	2.5	2.5%	1.2	7.7%	3.7
5 700	6%	342	2.5%	142	120	2.9%	3.5	48	6.7%	3.2	2.5%	1.2	9.2%	4.4
5 300	1%	53	2.5%	133	75	4.7%	3.5	71	4.8%	3.4	2.5%	1.8	7.3%	5.2
3 600	4%	144	2.5%	90	65	5.4%	3.5	55	6.7%	3.7	2.5%	1.4	9.2%	5.1
3 000	3%	90	2.5%	75	70	5%	3.5	43	5.8%	2.5	2.5%	1.1	8.3%	3.6

* $\sigma_H = \sqrt{2.5^2 + 2.5^2} = 3.54$

** $\sigma_k = k\sqrt{(\frac{\sigma_{kH}}{kH})^2 + (\frac{\sigma_H}{H})^2}$

$k/s = \sigma_k$ (systematic)

– 195.5 m. The boundary between aquifer and base consists of a transition from clayey sediments to coarse sand layers and probably will have a form comparable with a river landscape. It is therefore reasonable to suppose that the measured depth may be liable to an absolute error in the order of magnitude of 5 m. Hence the error in aquifer height may amount to 3.5 m. This means that the relative error in this case is about 3%.

The conclusion for the Huinerbroek pumping test is that the relative error in the value of k_h derived from the transmissivity is 6.3% (table 2).

The Huinerbroek pumping test is in the particular position that the aquifer is rather thick. For the other pumping tests in the Guelders Valley higher relative errors in aquifer height may be encountered as is shown in table 2, where k_h values are derived from the pumping test results. In table 2 the errors are given in the form of standard deviations.

2.2.3. *Transmissivity and horizontal permeability derived from pumping tests at other locations*

Very often in our models we have to substitute values for horizontal permeability or transmissivity, determined at test locations at some distance. For regional groundwater flow problems the number of pumping tests will be much smaller than the number of model nodes where a value of transmissivity is asked. For local investigations time or money may be lacking to execute a pumping test; then a value of transmissivity or horizontal permeability has to be derived from pumping tests executed formerly in the surroundings (or estimated otherwise). The interpolation or extrapolation of pumping test data to other places will introduce some error.

Pumping tests at other locations can only be used when they clearly represent the same aquifer, composed of the same geological formations. If so, then the following cases can be distinguished:

a. Both thickness of the aquifer H and horizontal permeability k_h may be regarded as variables which are normally distributed. Now the mathematical expectation of H and k_h and the respective standard deviations of the two distributions can be determined. In this case it assumingly is allowed to regard transmissivity in itself as a normally distributed variable and hence the mathematical expectation of T and the standard deviation of the distribution can be determined.
b. One of the composing parameters k_h and H is normally distributed, whereas the other one varies regularly over the aquifer. Again the mathematical expectation and the standard deviation of the normally distributed variable can be determined. The values of the varying parameter over the aquifer have to be determined with interpolation techniques.
c. Both components of transmissivity vary regularly over the aquifer. This means that for both variables interpolation techniques have to be chosen and errors have to be estimated.

The problem that either transmissivity or one of the composing factors (k_h, H) is normally distributed within the area concerned can be formulated as follows: The characteristics of the distribution have to be derived from a certain number of samples, each of which is represented by a mean value, a standard deviation and a possible systematic error. A standard technique is available to solve this problem:

- The standard deviations of the samples are cleared from a possible systematic error. The remaining standard deviation is assumed to be caused by measuring errors.
- Iteratively the mean value of the variable concerned is estimated over the area, together with values of σ and σ_m.
  The deviation σ is the true standard deviation of the distribution, due to scattering of values over the area. The additional standard deviation σ_m is the mean standard deviation, due to errors. Hence, the distribution of the variable is given by mathematical expectation and true standard deviation; however, if we have to predict the result of sampling again, then the error of prediction can be estimated to be the square root of $\sigma^2 + \sigma_m^2$.
- The mathematical expectation found in the fore-going step should be corrected with the value of the systematic error, cleared out in the first step.

If again we use the values obtained in the Guelders Valley to illustrate the above, then two approaches can be followed:

Assumption 1: Transmissivity as such is a normally distributed variable over the area of the Guelders Valley. In that case we have to regard the values of table 1, column 5. Computation as to the above maximum likelihood method results in an estimate of the mean value of transmissivity of T = 5 070 m^2/day and in an estimate of the standard deviation of 1 310 m^2/day (26%). The square root of the sum $\sigma^2 + \sigma_m^2$ equals 1 330 m^2/day.

Taking into account a possible systematic error of 5% (or of 2.5% in terms of "standard deviation") in test results, then we should conclude that transmissivity in the Guelders Valley area can be estimated to be T = 5 070 m^2/day ± 30% (standard deviation) according to assumption 1.

Assumption 2: Only horizontal permeability is a normally distributed variable, whereas thickness H of the aquifer has to be found by interpolation techniques over the area. To estimate a mean value of horizontal permeability the figures of table 2 have to be used. Applying the same method we find that an estimate of the mean value of k_h equals 54 m/day with an estimated true standard deviation of 8.2m/day (15%); the square root of $\sigma^2 + \sigma_m^2$ = 9.3 m/day.

To make an estimate of transmissivity $k_h H$ at every point it is necessary to estimate H by interpolation techniques. Without further elaborating it will assumed here that the maximum error of H will be everywhere within limits of 30%. (Comparable with two times the standard deviation).

Consequently it is possible to estimate transmissivity $k_h \circ H$ with a standard deviation of $\sqrt{15^2 + 15^2} = 21\%$.

Other assumptions (e.g. that both k_h and H vary regularly over the area) are less valid, so they will not be tried.

2.3. *Vertical permeability*

Historically pumping tests have been developed to determine transmissivity (mostly together with hydraulic resistance, storage coefficient etc.) of aquifers; this means that only a value of horizontal permeability can be derived. Only recently formulas became available and pumping tests were arranged to determine vertical permeability. These latter tests are based on partial penetration of the pumping well and on observation screens at various depths in the aquifer (Bruggeman). Again, pumping tests will constitute by far the most reliable field determination of vertical permeability.

2.3.1. *Vertical permeability from pumping tests*

As pumping tests arranged to determine vertical permeability are largely similar to classical pumping tests, it may be assumed that systematical errors as discussed before will also be about the same. One point deserves attention, viz. the distances between pumping screens and observation screens. Measurement of the effect of partial penetration implies that these distances have to be small. It is therefore likely that the systematical relative errors in distances caused by deviation of bore-holes will exert a greater influence on pumping tests to determine vertical permeability.

Random errors due to scattering in measurements of groundwater level and pumping discharge can again be estimated with a non-linear regression method (principally the same as was used for the normal pumping tests). As an example one of the pumping tests in the Guelders Valley has been taken. Although these pumping tests were only meant to determine transmissivity, at least the Achterveld test data could be used for determination of vertical permeability. For all data and the thickness H of this aquifer fixed at a value obtained from bore-logs, $k_h = 44$ m/day and $k_v = 14$ m/day. The standard deviation in the determination of k_v amounts to 7% (Table 3).

Some remarks should be made:

1. The analysis of pumping tests aiming to determine k_v is based on the assumption of anisotropy in a further homogeneous aquifer. In reality, however, the presence of claybands, thin fine-grained layers etc. will also cause (large) differences in vertical permeability.
2. The thickness will be liable to error; however, it is also possible to analyse such pumping tests with thickness (or rather the depth of the base) being one of the variables to be determined. Such analysis yields the results as presented in table 3 too.

Table 3 Analysis of the Achterveld pumping test according to various methods and different sets of observations

	Horizontal permeability (m/day) + standard deviation	Vertical permeability (m/day) + standard deviation	Thickness H of aquifer (m) + standard deviation	Hydraulic resistance (day) + standard deviation
162 observations H fixed (133 m)	41.6 ± 2%	20.0 ± 9%	133	942 ± 16%
All data H fixed (133 m)	43.6 ± 2%	14.1 ± 7%	133	773 ± 16%
All data except 2 observation screens H fixed (133 m)	41.3 ± 2%	15.3 ± 5%	133	590 ± 11%
All data except 2 observation screens H *not* fixed	38.7 ± 3%	17.2 ± 7%	147 ± 3%	675 ± 13%

The results of table 3 underline the inaccuracy of values of k_v determined with pumping tests. Generally it may be taken that the possible error in k_v determination will exceed that for k_h determination, hence the conclusion is that the standard deviation of k_v will at least be in the range of 10 to 20% (at places where a pumping test has been held).

2.3.2. *Use of* k_v *values at places without pumping tests*

Up to present relatively few pumping tests were aimed to determine vertical permeability of aquifers. No archive data will be available for extrapolation to model areas. Local geohydrological investigations, where the value of k_v plays an important role, should therefore always be preceded by a pumping test. If not, k_v has to be estimated from the value of k_h (forming in most cases an upper limit) and this procedure may entail large errors. Models, simulating regional groundwater flow, need for the same reason k_v values determined at a number of pumping tests. In any case interpolation of data will present difficulties as it may be expected that even for one and the same geological formation the values of vertical permeability may vary between wide ranges over short distances.

2.4. *Hydraulic resistance*

In the geohydrological situation of The Netherlands reliable values of the hydraulic resistance of semi-confining layers are of much importance. The groundwater part of the hydrologic cycle makes use of a sometimes complex multi-aquifer system in which practically none of the confining layers is completely impervious. In the low-lying part of the Netherlands the whole aquifer system is covered by Holocene clay and peat layers, the hydraulic resistance of which determines vertical seepage into and thereby also horizontal groundwater flow to the various polder areas.

De Glee was the first to try and determine values of the hydraulic resistance from pumping test data. He gave for a semi-confined aquifer the relation between pump discharge, distance, steady drawdowns and the aquifer constants, viz. transmissivity T and leakage factor L. These aquifer constants are related to the hydraulic resistance of the upper and lower confining layers according to:

$$L = \sqrt{T.c}\text{; and}$$

$$c = \frac{c_u \cdot c_l}{c_u + c_l}\text{, wherein:}$$

L = leakage factor [L]
T = transmissivity $[L^2T^{-1}]$
c_u = hydraulic resistance of the upper confining layer [T]
c_l = hydraulic resistance of the lower confining layer [T]

Lateron Hantush developed a method to determine the same constants from the non-steady drawdowns during pumping tests. This method has been amply used in the Netherlands at the analysis by curve fitting of pumping tests. Hantush's formula can be analysed by a numerical non-linear regression programme NILIRE (discussed before), with this respect that not L, but c is computed directly.

The determination of values of the hydraulic resistance of confining layers from pumping test data not always yields satisfactory results. Main reasons are:

1. Generally long lasting (expensive) pumping tests are needed.
2. In case that c_l and c_u are in the same order of magnitude they cannot be determined separately.
3. At thick confining layers the effect of storage should not be neglected, however, hereby the analysis becomes complicated.

Therefore other methods have been researched, one of which is the elaboration of the water balance for natural groundwater flow through aquifers. Principally c values of semi-confining layers can be determined if vertical fluxes discharged from a part of the relevant aquifer are known and groundwater levels in adjoining aquifers and in the aquifer itself have been measured.

In the following only pumping tests will be discussed.

2.4.1. *Hydraulic resistance determined by pumping tests*

The same systematic deviations influencing transmissivity values will also affect the values of hydraulic resistance derived from pumping tests. One aspect needs further attention: Computed transmissivity values are largely dependent on drawdowns in the first stage of a pumping test, whereas the hydraulic resistance is strongly related to the last stage. This means that extern influences on groundwater levels in the last week(s) of the pumping test (which may be in the order of quite a number of centimeters) have to be eliminated carefully for a correct interpretation of hydraulic resistance. A reference well not influenced by the pumping is needed for that purpose. However, inevitably no perfect elimination will be reached.

Apart from systematic errors also an error due to random scattering of observations may be expected. An indication of this error is obtained in applying the non-linear regression method NILIRE. Interpretation of the pumping tests in the Guelders Valley according to this method, yields the results of table 4.

The only definite conclusion to be drawn from the figures of table 4 is that all pumping tests, except the Glindhorst test, are less fit to determine a value of the hydraulic resistance. The deviations adjoining the determination of the hydraulic resistance by applying NILIRE are thus large that it is not justified to give confidence limits. In table 4 a mean value and estimated ranges of possible values are given. Additionally it

Table 4 Values of hydraulic resistance derived from pumping tests in the Guelders Valley according to various methods

Pumping test	C-value obtained by curve fitting (days)	C-value (days) by NILIRE without correction for extern fluctuations + "standard deviation"	C-value (days) by NILIRE at corrected observations + "standard deviation"	Systematic error (order of magnitude)	Mean value and estimated ranges of possible values for C (days)
Huinerbroek	3 500	543 ± 28%	627 ± 66%	10%	600 (100- 3 000)
Zwartebroek	450	510 ± 31%	643 ± 25%	10%	600 (300- 1 500)
Barneveld	2 000	> 50 000	2 004 ± 42%	10%	2 000 (500- 5 000)
Achterveld	1 000	773 ± 16%	675 ± 13%	10%	700 (400- 1 200)
Glindhorst (nearby observations)		1 384 ± 3%			
Glindhorst (all observations)		1 743 ± 3%			
Glindhorst (all observations except nearby well)	2 400	2 600 ± 3%	2 600 ± 3%	10%	2 600 (2 000- 3 000)
Amersfoort	2 500		3 672 ± 35%	10%	3 500 (1 000- 6 000)
Woudenberg	2 700	*No determination possible*		10%	2 500 (500-10 000)
Ederveen	1 650	418 ± 12%	418 ± 12%	10%	500 (200- 1 000)

should be remarked that the agreement, which in some cases exists between values obtained with the curve fitting method and with NILIRE, is more or less forced as observed groundwater levels in the last stage of the pumping test had to be corrected rather severely for external influences.

2.4.2. *Extrapolation or interpolation of values of hydraulic resistance*

In its most simple form the relation between hydraulic resistance c and thickness d of resisting layers is expressed by $c = \frac{d}{k}$ (k = vertical permeability of the resisting layer).

However, for soil mechanical reasons it may be assumed that k is also related to the situation of the resisting layer in the underground. Elaboration (not to be discussed here, see G.J. Heij in Meinardi et al, 1978) yields:

c = $G \cdot d(h + \frac{1}{2} d)$, with
G = a constant, pertaining to geological properties of the layer
d = thickness
h = depth of top of the layer below land surface.

For interpolation of values of hydraulic resistance several methods can be followed:

a. Hydraulic resistance as such is regarded as a normally distributed variable over the area concerned. By statistical methods (as described before) the mathematical expectation and the standard deviation of the distribution of c-values should be determined. Clearly this method will only give satisfactory results if no large variations in c-values exist; this means that thickness, depth and vertical permeability of the resisting layer should not vary too much.

b. and c. Hydraulic resistance is considered to be a function of thickness, or of thickness and depth of the resisting layer. In the first case it is assumed that vertical permeability and in the second case that the factor G are normally distributed variables. Using the known values from pumping tests the normal distribution of either k_v or G can be described. Values of the geometry parameters of the resisting layer should be derived from interpolation techniques. Combination of geometry and either k_v or G will yield a value of hydraulic resistance everywhere in the area concerned.

For the Netherlands no data are available to illustrate the above. The pumping tests in the Guelders Valley did not yield suitable data, whereas in other parts of the country not enough pumping tests have been worked out in a way, so as to give an indication of the accuracy of c-values. The Guelders Valley data do not justify a further statistical elaboration as the estimated ranges of possible values (table 4) are thus large that they are useless for computations.

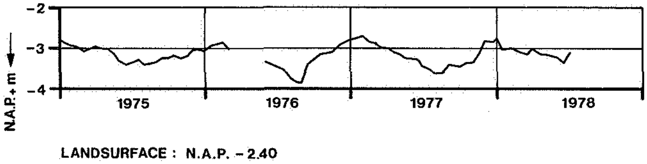


Fig. 2 Groundwater head in a polder area, observation well 37E L-11 at Pijnacker (source: CHO-TNO).

3. BOUNDARY AND INITITAL CONDITIONS

For groundwater flow models the boundary conditions may consist of given water levels or of given flows of water over the model boundaries; mostly water levels (groundwater heads or surface water levels) but exceptionally also (zero) flow conditions are used. Some common sources of errors in boundary conditions will be indicated.

Initial conditions may also take the form of water levels (groundwater heads). For problems concerning fresh and brackish groundwater an important initial condition is the original distribution of salt in the underground. In reality complex vertical salt profiles will be present; however, in our models we have to schematize, sometimes even to only two layers, one containing fresh groundwater and the other brackish groundwater of a constant salt content and density. Hence our schematization may substantially deviate from the real situation, which already in itself is not exactly known. In the following an estimate will be given of the errors made.

3.1. *Groundwater heads and surface water levels*

Values for groundwater heads or surface water levels to be substituted as boundary or initial conditions in our models, may be liable to error for a number of reasons:

a. *Difference between transient and steady state of groundwater flow*

Many groundwater flow models describe steady state groundwater flow, however, in nature the flow of water will always be transient. Water levels will show flunctuations in time, which e.g. may be diurnal (tidal effects), annual (seasonal effects), or even long-yearly (climate). Application of a steady state model implies the assumption of an average groundwater flow situation following from average boundary and initial conditions. Theoretically this may only be true in very special cases. If yet this model schematization is made then a second difficulty is the determination of average values for surface water levels and groundwater heads. Such a determination requires a long period of observations which generally is not available. Inevitably an error may be made if average values of groundwater heads, or surface water levels are estimated from a short period of observation. This error may be in the order of magnitude of decimeters (fig. 2).

b. *Difference between polder levels, surface water levels and phreatic heads*

A polder level is not a fixed water level in the ditches and water courses of a polder, but it is a desired level to be maintained by artificial means. In periods of abundant rainfall the real surface water level may be above polder level and in dry periods below. It is likely that throughout the years the average surface water level in a polder will differ from polder level. How much depends on each specific polder.

An assumption sometimes made is that phreatic groundwater levels equal polder levels (or rather surface water levels). In fact this will only hold at the locations of the ditches and even there not always. In periods of drainage (winter) the phreatic groundwater heads will be higher than ditch level and in periods of evapotranspiration excess they may sometimes be lower. The difference may be in the order of magnitude of a few decimeters, again depending on each polder.

Conclusion: if phreatic groundwater level is put equal to polder level then an error in the order of magnitude of some decimeters will be made.

c. *Errors in observations on groundwater heads*

Observations on groundwater heads are done by level measurements in observation wells. Apart from measurement errors (inaccurate instrument) a systematic error may occur, which is caused by an erroneous levelling of ground surface at the site of the well. If this last error is absent, then generally the measured levels will be accurate within a few centimeters.

d. *Extrapolation of data on groundwater heads*

Observed groundwater heads are only available at a limited number of observation wells. Generally for model calculations they have to be available as boundary or initial conditions at many more places. This means that the given data have to be inter- or extrapolated, introducing some error. Additionally an error may follow from small unknown differences in hydrologic behaviour within the model area. This type of error can be illustrated by the fact that correlation between observed groundwater heads never will be 100%.

3.2. *Salt and density distribution in the underground*

It is difficult to simultaneously solve the flow problem for fresh and brackish groundwater even under the assumption that both fluids are immiscible. Two approaches are widely used, the first assuming that the brackish groundwater is at rest and that a constant difference in density exists and the second taking that flow is not influenced by density differences. The first schematization fits rather well to a situation where a sharp transition exists between fresh and saline groundwater like under the Netherlands dunes. The second approach is more apt to the rest of the Dutch polderland where fresh

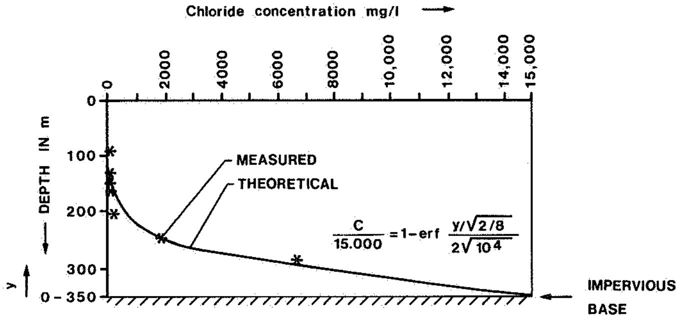


Fig. 3 Vertical distribution of chloride in boring 32 G 137.

groundwater gradually changes into brackish groundwater of low salinity (fig. 3). For models based on the assumption of brackish groundwater at rest the initial conditions consist of the position of the interface between fresh and brackish groundwater and of the constant density difference. For models of the other type the salt distribution in the underground forms an initial condition.

3.2.1. *The position of the fresh/brackish interface and the density difference*

Basically two methods exist to investigate the position of the interface between fresh and brackish groundwater, viz. geo-electrical exploration and the installation of observation wells with bore-hole logging and/or sampling.

A geo-electrical sounding can give a value for the interface which is representative for a certain area. Furthermore these soundings are relatively cheap, so they can be executed in relatively large numbers. Hence interpolation for a given model area will not pose difficulties. Unfortunately the interpretation of results always contains uncertainties, which partly are of systematic nature (wrong schematization). As a rule of thumb it sometimes is stated that the accuracy (possible error) of determination of a surface by geo-electrical prospecting is about 10% of the depth below ground level.

Observation wells can give exact values of the position of the interface, however, only just at the site of the well and at the time of observation. Observation wells are expensive, so generally their number will be limited. For an areal determination of the interface inter- or extrapolation over large distances will be necessary. In practical cases it is not to be expected that with observation wells a more accurate determination of the position of the interface will be reached than with geo-electrical soundings.

The density difference between fresh and brackish groundwater cannot be determined sufficiently with geo-electrical surface-soundings. If no observation wells are present, a first estimate is sometimes that the brackish groundwater has the salinity of sea water

($Cl^- = 19\,000$ mg/l). This estimate is unrealistic for the Netherlands, except may be for some small areas near the coast. From theoretical arguments and from observations it follows that the chloride content of most of the brackish groundwater concerned never exceeds a value of 16 000 mg/l. However, also the substitution of a density derived from 16 000 mg/l chloride will in many cases disagree with (much) lower real values.

Density differences preferably should be derived from observations done on samples from observation screens in the fresh and brackish groundwater. In nearly all cases it will turn out that salinity and consequently density in the brackish water will vary (increase) with depth. Out of these varying values a constant value has to be chosen (the average, the upper values), which means that an error is introduced (fig. 3). Apart from this main error, smaller errors will follow from the fact that density is mostly not measured directly, but derived from other observations, such as conductivity or chloride content. Subsequently the values estimated from observation wells have to be extrapolated over large distances.

From the few observed data available it is rather arbitrarily estimated that the possible error made in the estimate of the density of brackish groundwater may be in the order of magnitude of 2.5 kg/m^3 (equivalent to about 1 500 mg/l chloride).

3.2.2. *Chloride distribution in the underground*

For models also considering the flow of brackish groundwater the salt distribution, or preferably the chloride contents in the underground should be known. Predictions on movement of chloride will be more reliable as chloride is a rather conservative parameter hardly or not to be changed by (bio)chemical processes during transport.

The most accurate way to determine chloride distribution is by sampling observation screens. This implies that generally only at a few discrete points exact data are available. The full distribution of chloride has to be found by inter- or extrapolation, which will be difficult as the factors governing the distribution are sometimes complex. Under special conditions (no strong vertical groundwater flow) use can be made of a theory explaining the chloride concentration in the underground (Meinardi, 1975) for a first estimate. Even then, large deviations may occur between estimate and reality.

No general rule can be given as to the errors made by estimating the chloride distribution in the underground of a given area.

4. ACCURACY OF RESULTS OF TWO MODELS CONCERNING FRESH AND BRACKISH GROUNDWATER

Each type of groundwater flow model makes use of model parameters and boundary and initial conditions in a specific way. Hence the accuracy of results will also depend on the structure of the model. The model transforms the combined inaccuracies in geohydro-

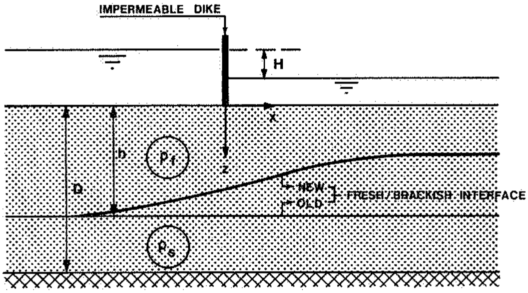


Fig. 4 The position of the interface for two polders with different level.

logical constants, groundwater heads etc. into an inaccuracy of model results, which we have to determine for each specific model. As an example two types of models both concerning fresh and brackish groundwater will be treated. The first model has been based on the assumption of brackish groundwater of constant density at rest and flowing fresh groundwater. An analytical formula is used to describe the flow problem concerned. This model gives the position of the interface, resulting from a change in the hydrological situation. In the second model density differences are neglected. The flow problem in three dimensions is solved by a numerical method and thereafter the transport of chloride can be described. The results consist of the time-dependent chloride content of the groundwater pumped by a well of given capacity.

4.1. *An analytical model for the position of a sharp interface*

The problem concerns two large polders separated by a straight dike. Originally in both polders the same level is maintained and the freshbrackish interface is a horizontal plane at depth h. In one of the polders the level is lowered a constant value H (fig. 4). What is the new equilibrium position of the interface?

The parameters playing a role in the problem are:

$$\alpha = \frac{\rho_s - \rho_f}{\rho_f} \qquad [-]$$

ρ_s =	density of the brackish groundwater	$[ML^{-3}]$
ρ_f =	density of the fresh groundwater	$[ML^{-3}]$
h =	initial depth of the interface under land surface	[L]
H =	difference in polder level	[L]
k =	permeability	$[LT^{-1}]$
D =	depth to the impermeable base	[L]

Underlying assumptions are that both fluids are immiscible and that the brackish water is at rest, except for a small flow to fill up extra storage created by changes in the position of the interface.

An additional condition is that $H/\alpha < D$. These assumptions imply that the interface is a streamline.

Bruggeman (pers. comm.) has solved the problem. He was able to define a potential function and a stream function for the flow of the fresh groundwater. For the case that $h > H/\alpha$, the interface will remain within the aquifer and its coordinates (x_i, z_i) are given by:

$$x_i = \frac{2h}{\pi} \ln \mathrm{tg}\left(\frac{\pi \psi_i}{2kH}\right) + \frac{2H}{\pi\alpha} \ln\left(2 \cos\left(\frac{\pi \psi_i}{2kH}\right)\right)$$

$$z_i = h - \frac{\psi_i}{\alpha k}$$

ψ_i = the potential function along the interface, where the stream function = 0; the potential function varies between 0 (for $x \rightarrow -\infty$) and kH (for $x \rightarrow \infty$).

At elaboration the variable k vanishes from the equation for the interface. Hence, the position of the interface is independent of k and also of the variation in k. The position of the interface is influenced by the variables h; H and ρ_s, the values of which are supposed to be independent and to have a normal distribution.

The problem has been worked out for a practical case, where the mathematical expectations of the parameters have been chosen according to a representative situation in the Netherlands and the standard deviations according to values indicated in the preceding paragraphs:

- The original depth h of the interface is taken to be 100 m with a standard deviation of $\sigma = \sqrt{5^2 + (2.5)^2} = 5.6$ m (assuming that the standard deviation of land surface equals 2.5 m and of the original position of the interface is 5 m).
- The difference in polder level H is taken to be 1 m with a standard deviation of 0.1 m.
- The density of the fresh groundwater ρ_f is assumed to have a constant value of 1 000 $kg.m^{-3}$; the density of the brackish water ρ_s is taken to be 1 025 $kg.m^{-3}$ with a standard deviation of 1.25 $kg.m^{-3}$.
- Variations of h, H and ρ_s are supposed to be independent.

The most interesting part of the problem is the outcome at $x \rightarrow \infty$, where the new position of the interface is given by:

$$z_i = h - H/\alpha, \text{ or}$$

$$z_i = h - H/\frac{\rho_s - \rho_f}{\rho_f}$$

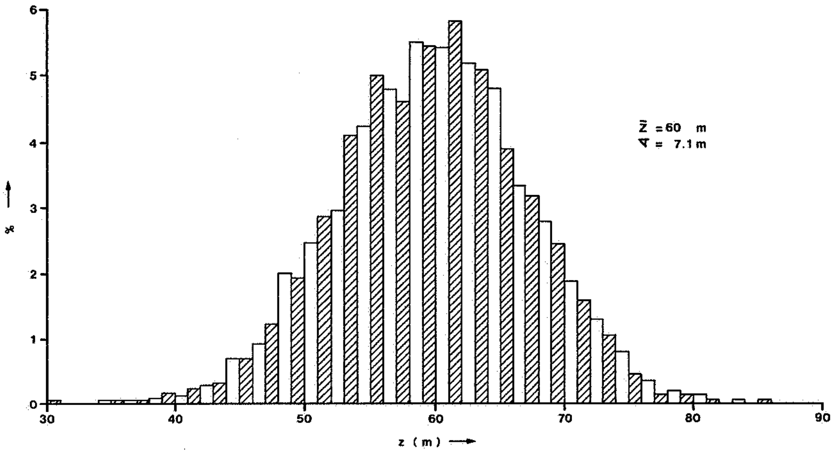


Fig. 5 Probability distribution of model results (fig. 4) for 7 000 random generated parameter combinations.

To estimate the probability distribution of values of z_i a large number (7 000) of calculations has been executed to determine z_i, using random values of the parameters concerned. These random values have been generated by computer according to the above defined normal distributions (Kovar, pers. comm.). The outcome is depicted in fig. 5. The distribution of values (for $x \to \infty$) can be characterized by a mathematical expectation of $z = 60$ m and by a standard deviation of $\sigma = 7.1$ m. With a probability of 95% the position of the interface will lie between $z_i = 46$ m and $z_i = 74$ m. Results for the full position of the interface ($-\infty < x < \infty$) are given in figure 6.

Strictly speaking the variations in the model results are large. Verification of the model results has to start now with observations on the real position of the interface for large values of x. The number of observations should be large enough to calculate a good estimate of the mean and the standard deviation and they should be cleared from possible systematic errors. With the appropriate statistical techniques it must thereafter be investigated whether and to what confidence level the calculated and the measured distributions of z-values represent the same situation. Given the relatively low accuracy of model results it may be expected that the reliability of a verification will be rather low.

The reliability of verification will improve if the accuracy of model results and hence the accuracy of the input data should be improved. The efficiency of improvements follows from a sensitivity analysis for the variables concerned. The ratio between small changes in input variables and consequent changes in the model results has to be determined. The sensivity of the model results for each parameter equals the derivative of the model results (the position of the interface) to the parameter concerned. In the

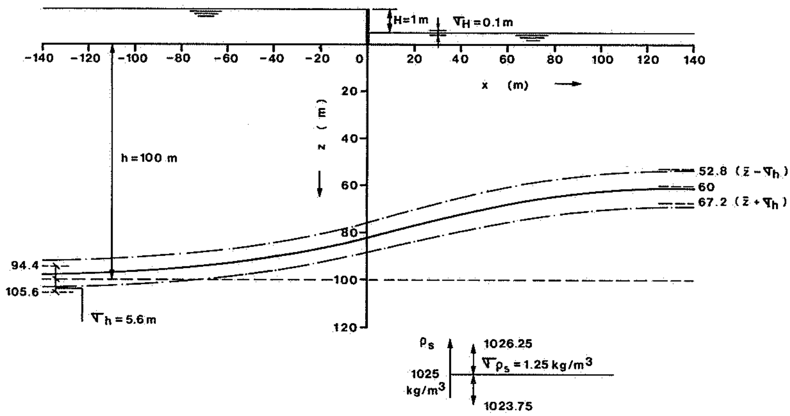


Fig. 6 Variation in model results for given variations in input data.

present case an analytical solution for the position of the interface is available and the respective derivatives can also analytically be determined. Elaboration for a value of the potential function $\psi = kH$ (at $x \rightarrow \infty$) yields the following values for the derivatives of z_i (the depth of the interface).

$$\left(\frac{\partial z_i}{\partial h}\right)_{x=\infty} = 1 \qquad [-]$$

$$\left(\frac{\partial z_i}{\partial H}\right)_{x=\infty} = \frac{-1}{\alpha} \qquad [-]$$

$$\left(\frac{\partial z_i}{\partial \rho_s}\right)_{x=\infty} = \frac{H}{\rho_f\, \alpha^2} \qquad [L^4M^{-1}]$$

For the ranges of variables, used before, it can be concluded that:

- an error of 1 m in the original position of the interface leads to an equal error of 1 m in the new position of the interface;
- an error of 0.1 m in the difference in polder levels leads to an error of -4 m in the new position of the interface;
- an error of 1 kg/m^3 ($\approx$ 600 ppm Cl^-) in the determination of the density of the brackish water results in an error of 1.6 m in the new position of the interface.

As ρ_s and α are related, the sensitivity for ρ_s will vary with

$$\frac{H \cdot \rho_f}{(\rho_s - \rho_f)^2}$$

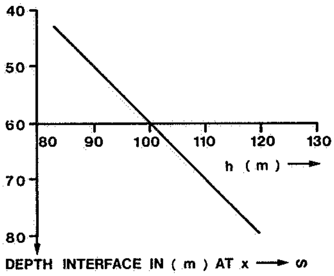


Fig. 7a Sensitivity of the interface for h.

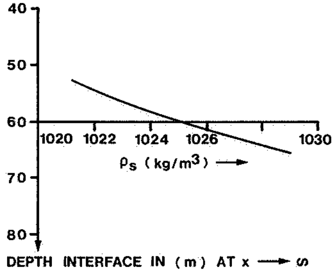


Fig. 7b Sensitivity of the interface for ρ_s.

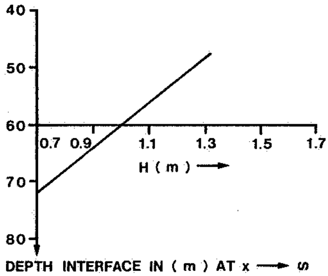


Fig. 7c Sensitivity of the interface for H.

The sensitivities of the model results for the parameters concerned are graphically represented in figure 7.

The above sensitivities can also be used to estimate the standard deviation of the depth of the interface (which was already found by random generation of input data). From statistical theory (Baird, 1962 and Squires, 1968) it follows that:

$$\mathrm{var}\,(z) = (\frac{\partial z}{\partial h})^2 \cdot \mathrm{var}\,(h) + (\frac{\partial z}{\partial H})^2 \cdot \mathrm{var}\,(H) + (\frac{\partial z}{\partial \rho_s})^2 \cdot \mathrm{var}\,(\rho_s)$$

(var (z) = the variance of z, the square of the standard deviation).

It is assumed that the respective functions can be approximated for values of the parameter around its mean value by a linear relation. Substitution of the known values in the righthand side of the equation yields for the position of the interface at $x \to \infty$:

$$\mathrm{var}\,(z) = 1.(5.6)^2 + 40^2.(0.1)^2 + \frac{10^6}{25^4}.(1.25)^2$$

$$\mathrm{var}\,(z) = 31.36 + 16 + 4 = 51.36$$

$$\sigma\,(z) = 7.2 \text{ m.}$$

A standard deviation of 7.2 m corresponds with a value of 7.1 m found before. An interesting point in this solution is that the contribution to var (z) of each inaccurate input variable can be determined. In the above example the contribution of inaccurate determination:

of h is $\frac{31.36}{51.36} = 61\%$;

of H is $\frac{16}{51.36} = 31\%$;

of ρ_s is $\frac{4}{51.36} = 8\%$.

It follows that the largest improvement in accuracy of model results is reached if the accuracy of determination of h improves. The error in determination of ρ_s only slightly affects the model results. However, this picture changes considerably if we take other mean values of ρ_s (see also fig. 8). Elaboration for a practical range of values of ρ_s, with an assumed standard deviation of 1.25 kg.m^{-3} yields the results of table 5. (Mean values and standard deviations of H and h remain unchanged).

The first thing to remark from the figures of table 5 is that the inaccuracy of H and ρ_s becomes increasingly important for lower values of ρ_s. A second point is the low accuracy of model results for values of $\rho_s = 1015$ kg.m^{-3} and lower. A density of 1 015 kg.m^{-3} corresponds with a chloride concentration of about 10 000 mg/1. Given this low accuracy it is rather doubtful whether modelling under the assumptions of a sharp interface and brackish groundwater being at rest, still makes any sense. An unrealistic accuracy of input data is needed for groundwater flow models dealing with brackish groundwater of low salinity. In such a situation the Badon Ghijben-Herzberg principles should not be used. Unfortunately this situation generally occurs underneath the Dutch polderland. In the case of brackish groundwater moving under the Dutch polders also potential flow of the brackish groundwater should be taken into account.

Table 5 The effect of a change in ρ_s on (the contribution to) the variance of z_i

mean value of ρ_s (kg.m^{-3})	variance of z (m^2)	standard deviation of z (m)	% contribution of h	% contribution of H	% contribution of ρ_s
1 030	44.40	6.66	70.1	25.0	4.4
1 025	51.36	7.17	61.1	31.1	7.8
1 020	66.13	8.13	47.4	37.8	14.8
1 015	106.67	10.33	29.4	41.7	28.9
1 010	287.61	16.96	10.9	34.8	54.3

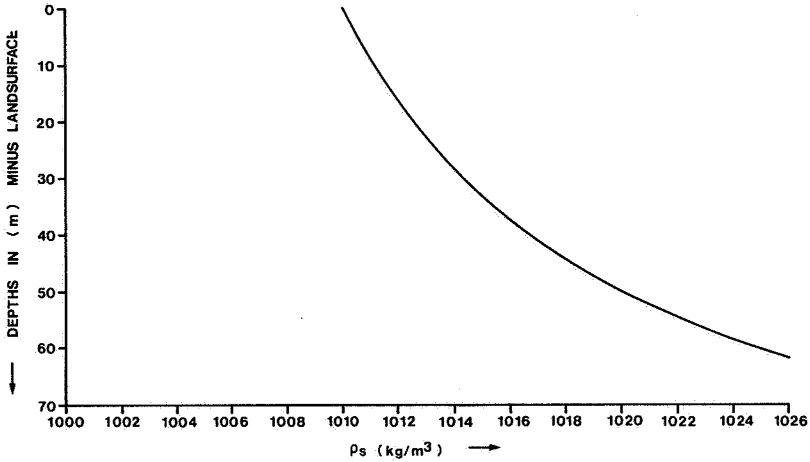


Fig. 8 Sensitivity of model results for lower values of ρ_s

4.2. *A numerical model neglecting density differences*

For drinking water purposes among others, often the following problem has to be solved: Groundwater is extracted by a partially penetrating well from an aquifer containing slightly brackish groundwater at its base (fig. 9). Chloride content of the water pumped should not exceed the limits posed by drinkability. To solve this problem the flow pattern should be known and thereafter the chloride content of the mixture pumped by the well in the course of time should be determined. Bruggeman and Sondorp (pers. comm.) have obtained a solution by neglecting density differences. In that case the basic equations describing the flow pattern can be formulated and thereafter solved numerically by a computer program labelled FLOP-3.

To investigate a practical case the following values of the parameters have been used (fig. 9):

- A pump discharge of 150 m^3/h, assumingly not liable to error.
- A mean aquifer thickness of 150 m. It is taken that the upper and lower side of the aquifer are normally distributed with standard deviations of respectively 2.5 m and 5 m. It follows that the standard deviation of D is $\sqrt{2.5^2 + 5^2} = 5.6$ m.
- A mean thickness of the zone of fresh groundwater $d_1 = 100$ m. The upper and lower side of this zone are normally distributed and independent with respective standard deviations of 2.5 m and 5 m. Again the standard deviation of d_1 can be taken to be 5.6 m. It is further taken that mean values of the brackish water zones are $d_2 = 15$ m; $d_3 = 15$ m and $d_4 = 20$ m. Changes in d_1 or in D result in a change of the thickness of the lower zone d_4, the thickness of the zones d_2 and d_3 remaining constant.

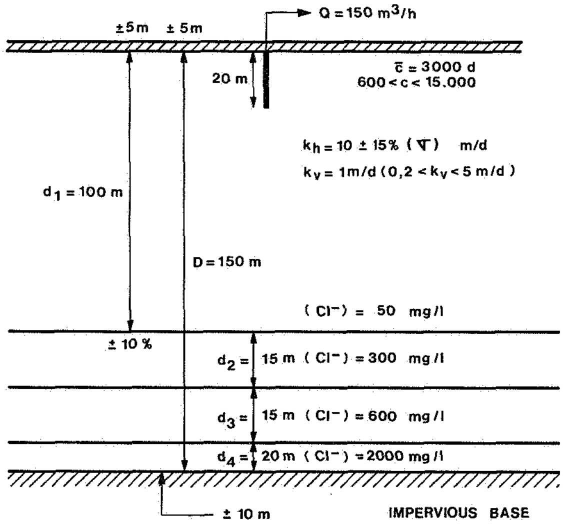


Fig. 9 Chloride content of a well in an aquifer partially filled with brackish water.

- A horizontal permeability $k_h = 10$ m/day, normally distributed with a standard deviation of $0.15 \times 10 = 1.5$ m/day.
- A vertical permeability $k_v = 1$ m/day, which firstly is taken to be normally distributed with a standard deviation of $0.15 \times 1 = 0.15$ m/day. In the second place it is taken that k_v varies between much wider margins (no adequate pumping test being done).
- A hydraulic resistance c = 3 000 days, for his parameter only margins of occurrence can be indicated, 1 800 days $< c <$ 9 000 days. It is taken that these confidence limits belong to a probability of 0.68.
- For the chloride content in the various zones it is taken that in the fresh water (Cl^-) = 50 mg/l, in the brackish groundwater zones the chloride contents are from above to below respectively 300, 600 and 2 000 mg/l. Although most certainly these values in reality will be liable to variations, they are taken here to be constant. Variations in the chloride distribution will yet occur by means of the variation in thickness of the lower zones of brackish groundwater.

Taking mean values for all input variables, the computation yields that after 20 000 days (55 years) the chloride content of the pumped water will have increased to 335 mg/l. Intermediate values are indicated by the drawn line in figure 10. In this case it is not practical to find the probability distribution of chloride contents of the pumped water by random generation of input variables. Hence the second approach, based on sensitivity

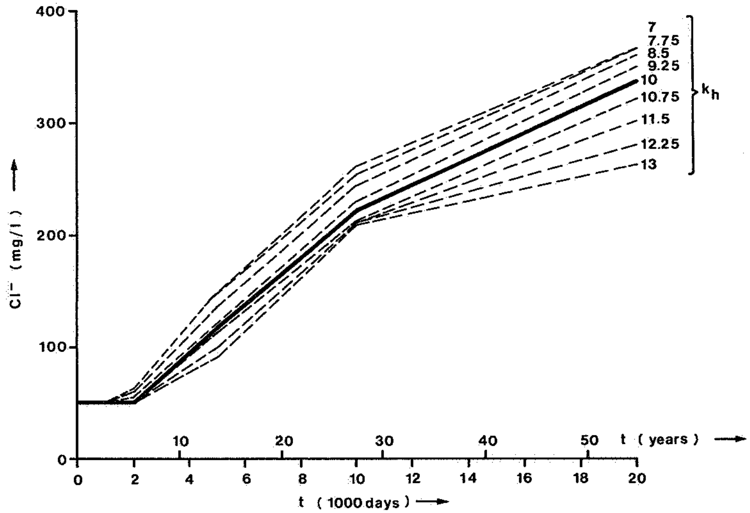


Fig. 10a Cl^- content in the course of time for varying k_h (m/day) (other parameters constant).

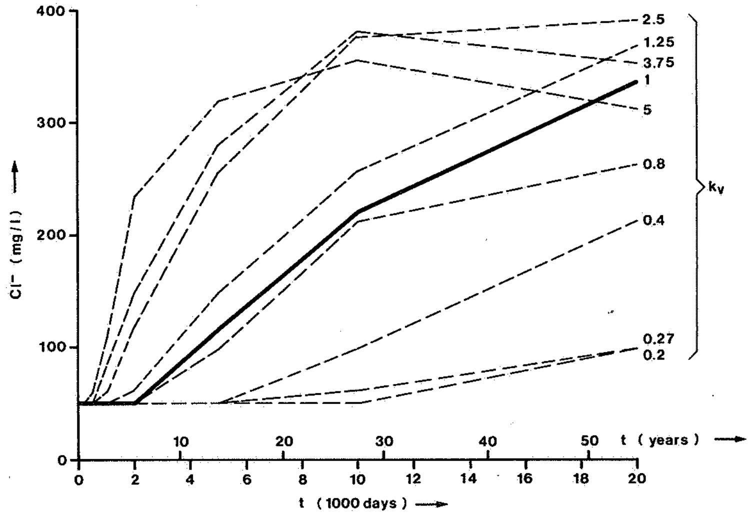
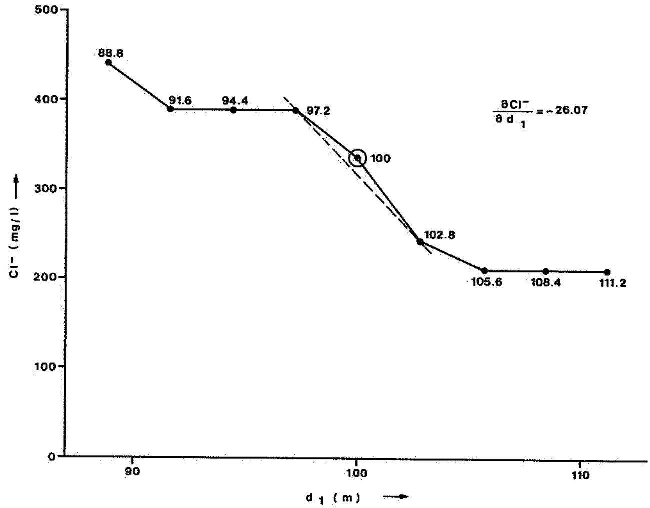
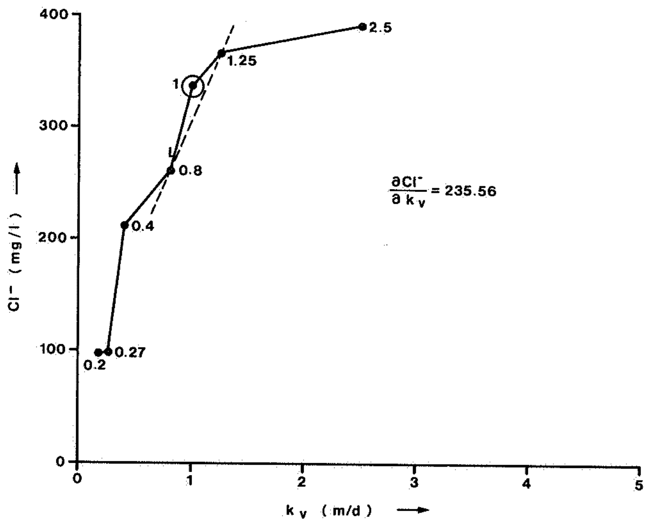


Fig. 10b Cl^- content in the course of time for varying k_v (m/day) (other parameters constant).

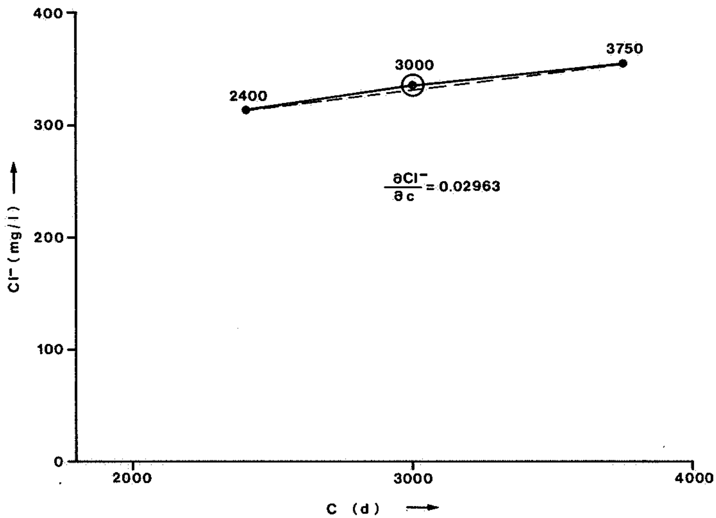
Fig. 11 Sensitivity of results of the FLOP-3 model for variations in input data.



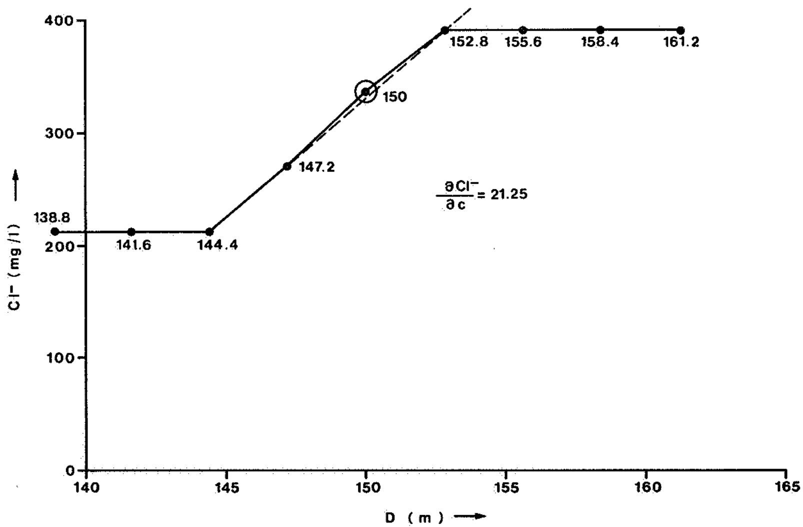
11 a Sensitivity for variation in d_1



11b Sensitivity for variation in k_v



11 c Sensitivity for variation in c



11 d Sensitivity for variation in D

determination, has to be followed. The sensitivity of the model results to variations in the input variables cannot be determined analytically, no analytical solution being available; it has to be determined by variation of the input variables and jointly numerical computations. The results of such computations are shown in figures 10a and 10b, where k_h and k_v have been varied, the other parameters having their mean values. Similar computations, to a total of 41, have been executed to determine the sensitivity of the model results (chloride content of the pumped water) for variations in the other parameters. Results for t = 55 years are shown in figure 11.

From the computations the following sensitivities for t = 55 years, holding near the mean values, can be derived:

$$\frac{\partial(Cl^-)}{\partial k_h} = -\ 19.33 \quad \left(\frac{mg/l}{m/day}\right)$$

$$\frac{\partial(Cl^-)}{\partial k_v} = 235.56 \quad \left(\frac{mg/l}{m/day}\right)$$

$$\frac{\partial(Cl^-)}{\partial c} = 0.02963 \left(\frac{mg/l}{day}\right)$$

$$\frac{\partial(Cl^-)}{\partial D} = 21.25 \quad \left(\frac{mg/l}{m}\right)$$

$$\frac{\partial(Cl^-)}{\partial d_1} = -\ 26.07 \quad \left(\frac{mg/l}{m}\right)$$

Again the rule can be applied, that:

$$\text{var}\,(Cl^-) = \left(\frac{\partial(Cl^-)}{\partial k_h}\right)^2 \text{var}\,(k_h) \quad +$$

$$+ \quad \left(\frac{\partial(Cl^-)}{\partial k_v}\right)^2 \text{var}\,(k_v) + \left(\frac{\partial(Cl^-)}{\partial c}\right)^2 \text{var}\,(c) \quad +$$

$$+ \quad \left(\frac{\partial(Cl^-)}{\partial D}\right)^2 \text{var}\,(D) + \left(\frac{\partial(Cl^-)}{\partial d_1}\right)^2 \text{var}\,(d_2 = 1)$$

The respective variances follow from the considerations given before, and hence:

$$\text{var}\,(Cl^-) = 39054\ (mg/l)^2$$
$$\sigma\,(Cl^-) = 198\ mg/l$$

The contributions of the inaccuracies in k_h, k_v, c, D and d_1 to the variance of the pumped chloride content (Cl^-) are respectively 2%; 3%; 3%; 37% and 55%. It follows

that (at the chosen inaccuracies in input variables) the inaccuracy of the model results is practically fully caused by the inaccuracies in the geometry of the system. The variations in the geohydrological constants only cause a minor part of the resulting variance. If we assume, however, a larger deviation in k_v of e.g. $\sigma(k_v) = 0.4$ – which in many cases will be more close to reality – then the following results are found:
$\sigma\ (Cl^-) = 216$ mg/l; and the contributions of k_h, k_v, c, D and d_1 to var (Cl^-) are respectively 2, 19, 3, 30 and 46%.

Although the influence of geometry remains considerable, in this case also the contribution of the vertical permeability cannot be neglected.

Before the above results can be analysed further it should be remarked that the chloride content of the pumped water is bound to physical constraints in the given example. Clearly the lower limit lies at the value of 50 mg/l. However, also an upper limit exists. This upper limit can be found by giving all parameters extreme values in this respect: D should be maximum, d_1 should be minimum and $c \rightarrow \infty$. For infinite time it is then found that the chloride content of the pumped water cannot exceed 637 mg/l. Hence the model results are not normally distributed, especially not in the lower and higher ranges of values. It may be assumed, however, that near the mathematical expectation the distribution will approximate a normal distribution. The results can be described as follows:

The 70% confidence limits are 135 mg/l and 535 mg/l, whereas the probability is more than 95% that the model results will be in between 50 and 637 mg/l. The conclusion should be that no proper verification as to model schematization is possible due to the large inaccuracy of the results. The accuracy can be improved with better knowledge of especially the geometry of the system and partly also of the vertical permeability.

The sensitivity analysis yields in this example a few other interesting points.

1. From figure 10b showing the chloride content of the pumped water for various k_v, it can be concluded that for high k_v the content will reach a maximum after a certain number of years and will thereafter decrease again. This phenomenon is caused by the fact that, seen from the pumping screen, the lowest flow lines first go downward to the base of the aquifer and subsequently will rise again to ultimately reach the upper resisting layer. In the model no dispersion is taken into account.
2. It follows from the figures showing the sensitivity of model results that for certain variables, ranges of variations exist where the sensitivity approaches zero:
   a. For values of k_v larger than 2.5 m/day the chloride content will not increase further.
   b. Values of c smaller than 800 days will not affect the chloride content anymore and similarly values of c larger than 7500 days will not cause an increase of chloride content.
   c. At the given example (the time increment of 50 years is important here) also values for D and d_1 exist where the sensitivity of the model results for further changes becomes negligible.

5. CONCLUSIONS

1. Verification of groundwater flow models as to the suitability of model schematization is only possible if we know the inaccuracy of model results due to inaccurate input data (geometry, geohydrological constants, boundary and initial conditions).
2. Confining planes in hydrogeology (e.g. top or base of an aquifer) will nearly always slope and additionally local irregularities will occur. Standard deviations of measured depths will be in between 1 and 5 meters or more. Extrapolation of bore-hole data will add an additional uncertainty. Notably for thin resisting layers the consequences of uncertainty may be large.
3. Pumping test results, being mostly used to determine geohydrological constants, are liable to systematic and to measuring errors. The confidence limits of the resulting geohydrological parameters can be estimated with a non-linear regression method.
4. Elaboration of 8 available pumping tests in a given area yielded the following results (holding at the test location):
   a. The standard deviation of transmissivity determination varies between 3.5% and 8.5%.
   b. The standard deviation of horizontal permeability varies between 6.3% and 9.2%.
   c. Only one test could be used to compute vertical permeability with a standard deviation of 7%, larger values not being excluded.
   d. Although mean values of hydraulic resistance can be determined, no standard deviation can be indicated. Possible values may vary between wide margins.
   e. Extreme care should be taken to clear pumping test results from extern influences on groundwater heads.
5. To get values of geohydrological parameters at places without tests, interpolation techniques and/or statistical methods should be used, whereby additional errors are introduced. In the given example horizontal permeability could be estimated with a standard deviation of 15%. It may be assumed that the standard deviation for the estimates of the other parameters generally will be (much) larger.
6. Boundary and initial conditions will often take the form of water levels. Each value of groundwater head not obtained from direct observation will be liable to a standard deviation in the order of magnitude of 1 decimetre or more.
7. For groundwater problems dealing with fresh and brackish groundwater two additional initial conditions are needed, viz. the geometry of and the salt (density) distribution in the brackish groundwater body. Estimates of both variables are liable to important errors.
8. Accuracy of model results depends also on the type of model. In this study two models have been investigated, an analytical model based on the Badon Ghijben-Herzberg principles and a more complicated model solving the flow problem in an aquifer with a partially penetrating well.

9. The analytical model yielded the following results:
   a. For brackish groundwater of high density the model can in principle be verified although the reliability of the model results is low due to errors in input data as given by the analyses of model parameters and boundary conditions.
   b. For brackish groundwater with a chloride content lower than 10 000 mg/l no models should be used based on the Badon Ghijben-Herzberg principles.
10. The results of the more complicated model cannot be verified with the present accuracy of input data in the Netherlands. The accuracy of model results can be improved by a better knowledge of geometry and of vertical permeability of the aquifer material.

REFERENCES

BAIRD, D.C. (1962) – Experimentation, an introduction to measurement theory and experiment design.

BRUGGEMAN, G.A. (1978) – Grondwaterstroming. In: Cursus Grondwaterkwaliteit van de Stichting Postakademiale vorming Gezondheidstechniek.

KREYSZIG, E. (1970) – Introductory Mathematical Statistics. John Wiley, New York.

LEYNSE, A. (1980 to be published). Berekening van bodemconstanten uit pompproefgegevens met behulp van niet-lineaire regressie-analyse.

MEINARDI, C.R. (1975) – Brackish groundwater bodies as a result of geological history and hydrological conditions, Symposium on brackish water as a factor in development, Beer Sheva, Israel.

MEINARDI, C.R. et al (1978) – Geohydrologische gegevens van Zuidelijk Flevoland en de Gelderse Vallei, RID-Mededeling 78-4.

SQUIRES, G.L. (1968) – Practical Physics, McGraw-Hill, London.

CONCLUSIONS

J.C. VAN DAM

1. The present distribution of fresh and saline groundwater in the lower part of the Netherlands is most complicated. This holds as much for the occurrence of saline and brackish seepage. The present, transient, situation is still far from the ultimate situation that, even without any further human interference, would only be reached after centuries. Future activities as reclamation of new polders, lowering of polder water levels, abstraction of fresh or saline groundwater will likewise influence the future and ultimate situation in terms of salinity distribution and the occurrence of seepage of saline and brackish water.

2. Several methods are available and applicable for the calculation of fresh water – salt water problems, both in steady state and transient, viz.:
   - analytical methods;
   - finite differences methods;
   - finite element methods;
   - boundary element methods.

3. The models developed so far are highly schematized compared to the complex reality.
   - all models presented dealt with a sharp interface between fresh and saline water;
   - dispersion and diffusion were not taken into account;
   - only relatively simple geometries (mostly profiles and radial symmetric situations) have been dealt with.

   The development of these simple models required a lot of effort. There is need for expansion of the models to more complex geometries and configurations.

4. The optimum way of exploitation of groundwater in areas with fresh and saline groundwater can be calculated tentatively with the very models available at present.

5. Directions for the collection of sufficient and reliable data must be based on an analysis of the sensitivity of the phenomena studied in the models.

6. For the solution of large scale problems it may be necessary to develop models which require less detailed input of data.

7. After further expansion of the models for application to larger areas it might be possible to conclude that in the ultimate situation in the lower part of the Netherlands seepage of saline groundwater will occur only in the deep polders in the first line (e.g. Haarlemmermeerpolder, Schermer, Wieringermeerpolder). The saline seepage that at

present occurs in the deep polders located more inland will turn to fresh seepage fed by infiltration of fresh surface water in the surrounding polders and lakes. In this area (e.g. IJsselmeerpolders, polder Groot Mijdrecht) stagnant saline water will permanently remain under the flowing fresh groundwater.

8. The process of partial depletion of saline groundwater, as was tentatively concluded in conclusion 7, can be accelerated by abstraction of saline groundwater in suitable locations. This artificial acceleration does not pay unless the saline water can be utilized e.g. for cooling or fresh water production through desalination. The soil-mechanic aspects (land subsidence) of such abstractions must also be taken into consideration.

9. From the viewpoint of safety against upconing of saline groundwater caused by temporary overdraft of fresh groundwater it is desirable to aim at a great thickness of the fresh water layer which can be achieved by proper location of the abstraction wells.

RECOMMENDATIONS

J.C. VAN DAM

1. (based on the conclusions 1, 2, 3, 4, 7, 8 and 9 and destined for the authorities responsible for ground-water management)
   Even the models, so far available, for fresh and saline groundwater in relatively simple geometries can be of good use for the authorities responsible for groundwater management in structuring their measures and drawing up their groundwater plans. These highly schematized models can deepen the insight into the effects of measures and works.

2. (based on the conclusions 2, 7, 8 and 9 and destined for the services in charge of data collection)
   For proper verification observations are required over long periods. Special attention, more than at present, should be given to the collection of data pertaining to the saline groundwater.

3. (based on conclusion 3 and destined for the research institutes)
   The models developed so far should be expanded for:
   - more complicated geometries and configurations, including the variations in geo-hydrological constants;
   - gradual transitions in the salinity distribution or several interfaces between groundwater of stepwise different densities;
   - inclusion of the effects of dispersion and diffusion.

4. (based on the conclusions 5 and 6 and destined for the research institutes)
   An analysis should be made of the sensitivity of the phenomena studied in the models (distribution of salinity in the underground and seepage of fresh and saline water) for changes in boundary conditions and geohydrological constants. Such an analysis should result in:
   - recommendations on the nature and density (in space and time) of data to be collected;
   - the development of simplified but still reliable large scale models, requiring less detailed data.

APPENDIX A

REVIEW OF LITERATURE OF FRESH AND SALINE GROUNDWATER

F.C. VAN GEER

1. INTRODUCTION

Badon Ghijben (1889) was the first to give an explanation for the fact that fresh groundwater was found at great depth near the sea. He explained this phenomenon by assuming that the lighter fresh groundwater floats on the heavier saline groundwater and the pressure on both sides of the interface between fresh and saline groundwater must be equal. He assumed a hydrostatic pressure distribution for both liquids and concluded that the depth of the interface (conceived as a definite boundary) below the phreatic table is proportional to the elevation of the phreatic table of the fresh water above sea level. The ratio was put at 42 (fig. 1).

He deals exclusively with phreatic water; nothing is said about aquicludes. Herzberg (1901), independently of Badon Ghijben, gave the same explanation for the fact that he found both saline and fresh groundwater at the same depth in the same aquifer on Norderney, one of the German Frisian Islands. He also stated that the cause of this phenomenon is the difference in density between fresh and saline groundwater. His theory is supported by observations during a very dry winter and summer, in which the freshwater lens was alsmost reduced to zero. After Badon Ghijben and Herzberg many investigators occupied themselves with groundwater flow in areas with both fresh and saline groundwater. They all based their calculations also on the above-mentioned principle.

2. PRINCIPLES AND BASIC EQUATIONS

Bear (1972) discusses the principles of geohydrology in a manual, in which much attention is devoted to calculations on flow of homogeneous groundwater and to processes in the unsaturated zone. In addition, two chapters deal with flow of groundwater with differences in density. No new developments are presented, but many methods and theories are described in a conveniently presented manner. One chapter deals with flow of groundwater with differences in density, in which the dispersion and diffusion are allowed to be neglected. In the next chapter, groundwater flow problems are discussed in which the dispersion and the diffusion are not allowed to be neglected. The basic equations for calculating the groundwater flow in the case of ground-

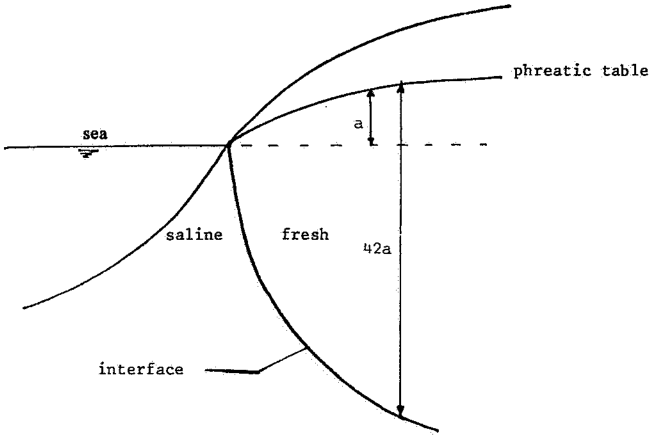


Fig. 1 Cross-section of the coastal area perpendicular to the coastline.

water with differences in density are given by Van Dam (1977). The formulae for correctly converting the pressure and potential of groundwater with any density at any point to the pressure and potential of groundwater with another density are clearly derived. In illustration of this an example is worked out for horizontal and for vertical flow.

3. THE OCCURENCE OF BRACKISH AND SALINE GROUNDWATER

Volker (1961) gives a possible cause for the occurence of brackish and saline groundwater in the Dutch polderland. His theory is based on observations to depths extending down to 100 meters below the bed of the IJssel Lake. In consequence of the enclosure of the former Zuyder Zee, the groundwater in the upper few meters of the bed of the IJssel Lake is fresh. Underneath this layer one finds groundwater of a higher chlorinity. From about 20 meters to about 80 meters depth the chlorinity gradually decreases. Under this layer the groundwater again is of a higher chlorinity (fig. 2). Volker (1961) concludes that the distribution, indicated in figure 2, can be explained on the assumption that the distribution process of the chloride ions is exclusively due to diffusion. Volker explains the groundwater chlorinity distribution at greater depth in the same way. He does not deny that there are other distribution mechanisms, but states that, in contrast with diffusion, they are not always necessarily present. Meinardi (1973) mentions three causes of the present chlorinity distribution underneath the Dutch polderland. They are dispersion, diffusion and reclamation. He developes a model for the western part of the Netherlands, with which the chlorinity distribution is explained by dispersion (fig. 3). The calculations start from a location in the middle of the Netherlands (Utrecht), where infiltration

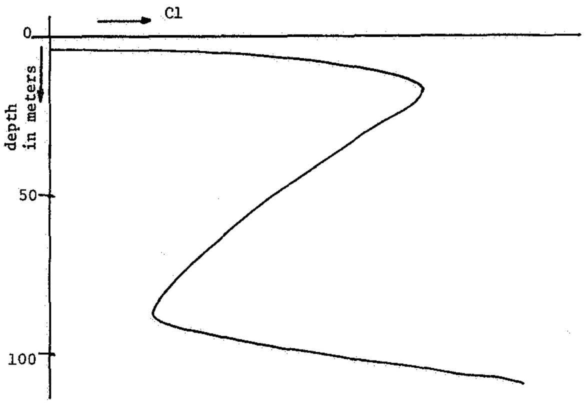


Fig. 2 Distribution of chlorinity underneath the IJssel Lake.

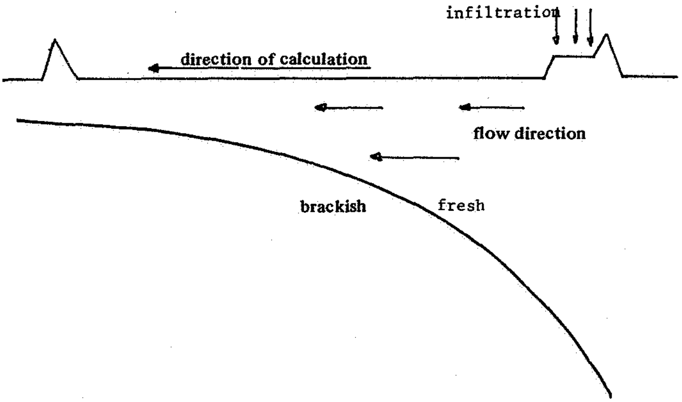


Fig. 3 Depth of the interface between fresh and brackish groundwater in consequence of dispersion.

occurs. Meinardi computes from that location to the North Sea. He makes a number of assumptions for the geohydrological constants. He concludes that if the assumption of the constants is correct, the chlorinity distribution curve can be satisfactorily reproduced. The reclamation has the effect of local upconing of the interface as a result of the fall of the groundwater table. The occurence of saline groundwater is extensively described during a course of the Stichting Postakademiale Vorming Gezondheidstechniek (1970/71). Part I of this course mainly deals with the local circumstances and contains a lot of geological information. Among other things, pillars of salt and tectonics are described. In

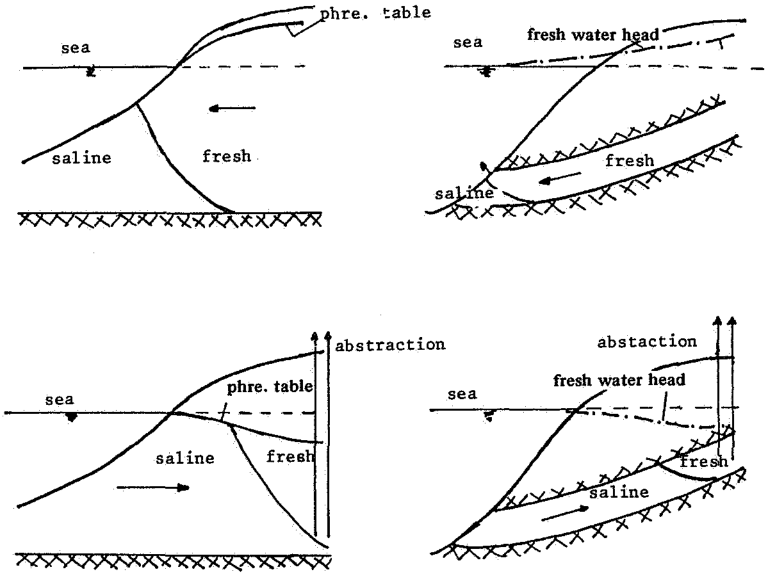


Fig. 4 Influence of freshwater abstraction on the location of the interface between fresh and saline groundwater in the case of unconfined (a and c) and confined water (b en d).

part 2 of the course methods of investigation and mappings are discussed. Todd (1974) deals qualitatively with saltwater intrusion in coastal areas. He distinguished unconfined and confined water. In both cases the influence of a freshwater abstraction on the saltwater intrusion is described (fig. 4). Todd also assumes definite interface between the fresh and the saline groundwater. This assumption is based on the circular flow of the saline water (fig. 5).

In this case he states that the thickness of the mixing range is of little importance. Six measures are mentioned which can force back the saltwater intrusion:

1. Reduction of the abstraction.
2. Artificial recharge of the fresh groundwater by means of infiltration.
3. Injection in the aquifer at a point between the sea and the location of the freshwater abstraction.
4. Saltwater abstraction between the sea and the location of the freshwater abstraction.
5. A combination of the last two measures.
6. An impermeable barrier between the location of the freshwater abstraction and the sea. Finally Todd (1974) mentions a number of areas in which one of the above-mentioned measures have been applied; a number of results are given. The circulation of the saline

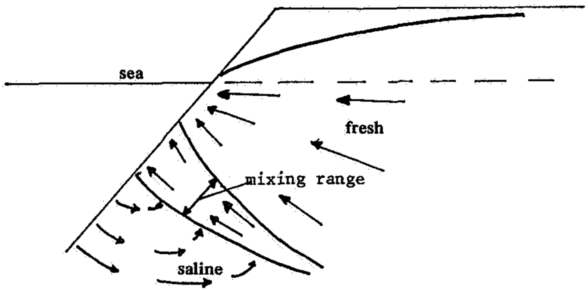


Fig. 5 Circular flow of the saline water.

groundwater is also the starting point adopted by Kohout and Klein (1967). With this circulation they explain the change in the mixing range as a result of a major recharge of the fresh groundwater. They give a quantitative analysis of results of measurements in the Biscayne Aquifer during and after a very wet period. During precipitation the mixing range moves towards the sea after which a new equilibrium is reached.

4. STEADY FLOW PROBLEMS IN WHICH DIFFUSION AND DISPERSION ARE ALLOWED TO BE NEGLECTED

After Badon Ghijben and Herzberg a number of other investigators have studied the problem of saltwater intrusion into an aquifer near the sea. The one-dimensional approximation according to Dupuit implies that the equipotential lines of the fresh groundwater are vertical. In consequence, the fresh water discharges into the sea at one point (fig. 6).

Glover (1959) constructs a two-dimensional network of streamlines and equipotential lines in the area near the location where the fresh water discharges into the sea, by means

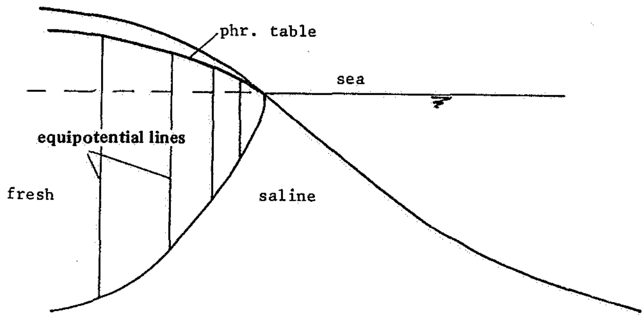


Fig. 6 One-dimensional model of saltwater intrusion.

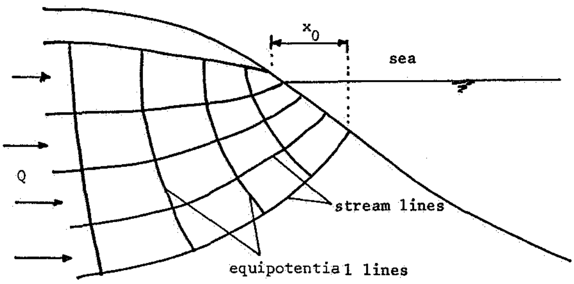


Fig. 7 Network of streamlines and equipotential lines.

of elementary complex functions. He assumes the saline water to be at rest and the ground to be homogeneous and isotropic. The steady flow of the fresh water is maintained by a constant discharge Q (fig. 7). Because of the two-dimensional approximation, it is no longer necessary to assume vertical equipotential lines. Hence it is possible to construct the network of streamlines and equipotential lines in such a way that the fresh water discharges into the sea over a certain distance x_0.

Van der Veer (1977[b]) consider the same problem. However, he does not assume a constant discharge as Glover does, but a constant recharge of the fresh groundwater (fig. 8).

The aquifer again is homogeneous and isotropic. To solve the problem Van der Veer uses the complex potential Ω. This complex potential is defined as

$$\Omega = \phi + i\psi$$

in which:

ϕ is the potential and
ψ is the stream function.

Van der Veer analytically derives an expression for the elevation of the phreatic table and the depth of the interface between fresh and saline groundwater, as well as an expression for the distance l_e over which the fresh water discharges into the sea. Van der Veer's as well as Glover's solution is two-dimensional (curved streamlines) and with the saline water at rest. Van der Veer compares his solution with the one-dimensional solution of Badon Ghijben and Herzberg. He concludes that their solution is a good approximation for locations at some distance from the sea, but that the one-dimensional solution clearly differs from the solution given by Van der Veer (1977[b]) when applied in the area near the sea. He gives (see 1977[a]) a solution for an area near the sea with flow of both fresh and saline groundwater (fig. 9). Here the fresh groundwater is not recharged with precipitation. He again uses the complex potential Ω to solve the flow problem for fresh as well

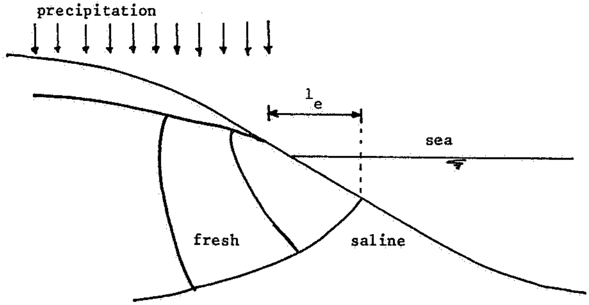


Fig. 8 Discharge into the sea of fresh groundwater with recharge.

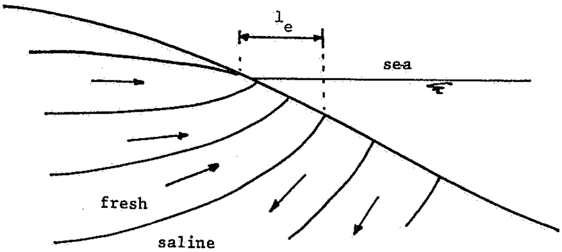


Fig. 9 Flow of fresh and saline groundwater in a coastal area.

as for saline groundwater. The solution only holds for a steady flow, and the interface between fresh and saline groundwater here again is assumed to be a definite boundary.

Strack (1975) considers an aquifer near the sea from which groundwater is extracted by means of a fully penetrating well at one point. The coastline is straight and there is a freshwater flow to the sea perpendicular to the coastline (Q_0). The fresh groundwater is not recharged by precipitation. He derives a second-order partial differential equation for the potential, in which he neglects the vertical component in relation to the horizontal components. Strack (1975) gives analytical solutions for four steady situations namely, for homogeneous phreatic water, for homogeneous confined water, for phreatic water with differences in density and for confined water with differences in density and he also gives examples for two different cases. For both cases the position of the interface fresh-saline groundwater is computed for a number of values of the water extraction rate. The two cases are presented in figures 10a and 10b.

In consequence of water extraction the fresh groundwater is recharged with surrounding groundwater. This groundwater flow has a component perpendicular to the plane of drawing in figures 10a and 10b. M is the point where the phreatic water table or

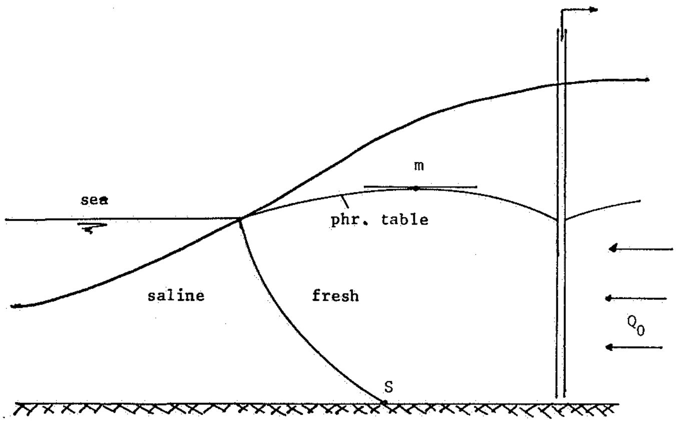


Fig. 10a Extraction in phreatic water.

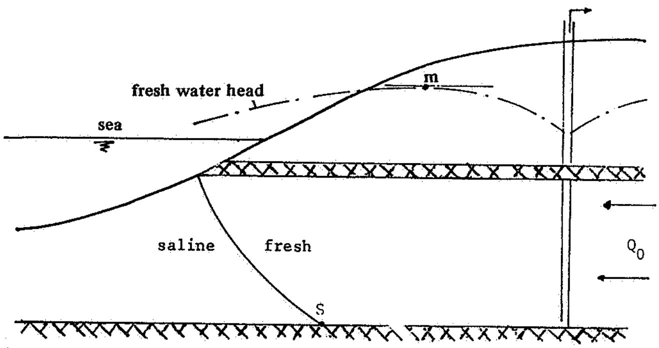


Fig. 10b Extraction in confined water.

the potential surface is horizontal. S is the toe of the saltwater wedge. With an increase in the extraction rate the horizontal distance between M and S will become smaller if M is initially located to the right of S. In the critical situation M is located exactly over S and the flow is unsteady. If extraction is further increased, the well will start to take saline water.

Another problem associated with a three-dimensional solution is reported by Kishi and Fukuo (1977). They consider an area with a very special geometrical constitution. The aquifer has the shape of a segment of a circle, and is bounded by aquicludes on both sides. The upper and the lower boundary of the layer also consist of aquicludes (fig. 11a

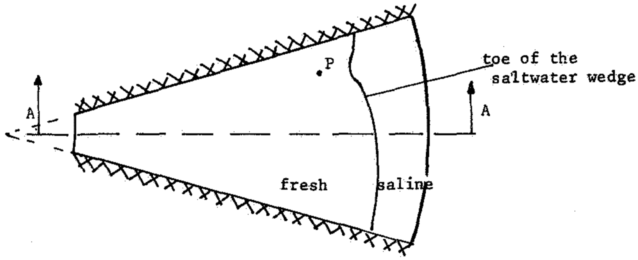


Fig. 11a View in plan of the situation considered by Kishi and Fukuo.

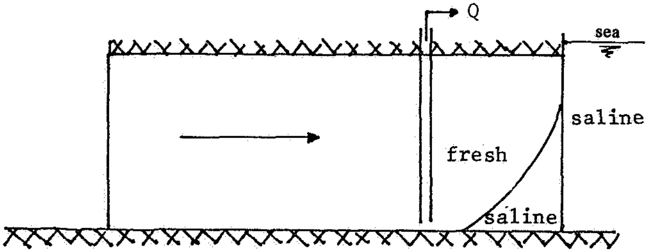
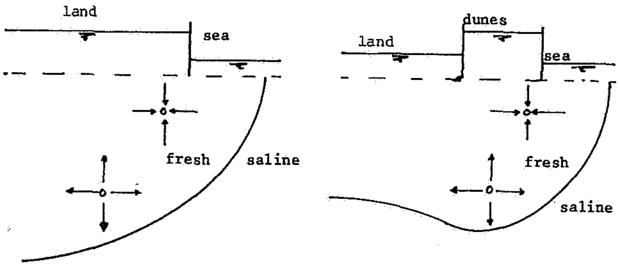


Fig. 11b Cross-section A-A.

and 11b). Steady-state analytical solutions are given for a number of locations of P and for different discharge rates Q. The solutions only hold for a homogeneous isotropic aquifer.

Starting from the complex potential $\Omega = + i\ \psi$ (see, for example, Van der Veer (1977b)) a range of groundwater flow problems can be solved with the help of the hodograph method. This method uses conformal mappings, which removes the problem that the interface fresh-saline groundwater is not known in advance. Verruyt, in his lecture notes, starts from conformal transformations as a method for the solution of groundwater flow problems and here a little attention is paid to flow problems with both fresh and saline groundwater. The use of the hodograph method is limited to steady, two-dimensional flow problems in a homogeneous aquifer. A further condition for the use of this method is that one of the two fluids has to be at rest. Strack (1973) has derived transformations with which sources and sinks can also be taken into account. He applies this to a number of examples, including the flow problems indicated in figures 12 and 13.

Henry (1959) has solved two groundwater flow problems by means of the hodograph method. In both cases the aquifer is bounded by an aquiclude at the upper and the lower boundary, and the fresh water flows from left to right. The difference between the two cases is that in figure 14a the aquifer is bounded vertically at point A and in figure 14b the aquifer is unbounded on the right.



Figs 12 and 13 Two of the examples given by Strack (1973).

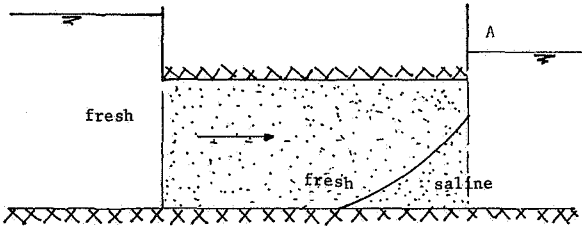


Fig. 14a Henry's problem with an aquifer bounded on the right.

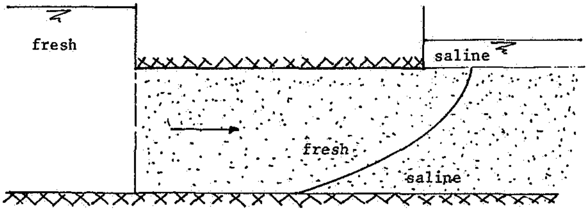


Fig. 14b Henry's problem (1959) with an aquifer unbounded in the right.

Four other flow problems are solved by Bear and Dagan (1964) by means of the hodograph method. The first problem (fig. 15) corresponds to Henry's second case. The second is given in figure 16. The aquifer is unbounded and there is a sink in the fresh water. In the other two problems (figs 17 and 18) the aquifer has one horizontal boundary. In figure 17 there is a sink just below the impervious layer. Fresh water is extracted from it and is replaced by flow from all sides to the sink. In figure 18 there is a drain under the impervious layer. Here the fresh water is replaced by water coming from one direction.

Verruyt (1969) gives a solution for a flow problem in which the lighter fluid is at rest.

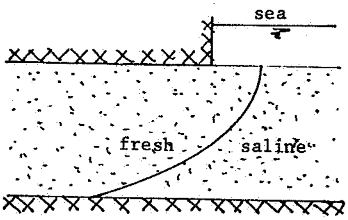


Fig. 15 Saltwater wedge in a confined aquifer.

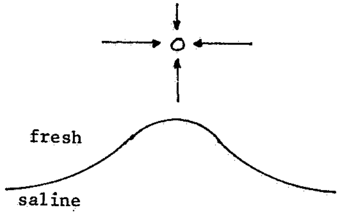


Fig. 16 Freshwater extraction from an unbounded aquifer.

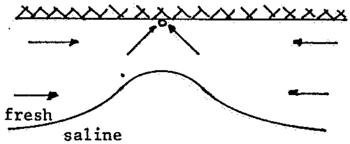


Fig. 17 Freshwater extraction with a sink.

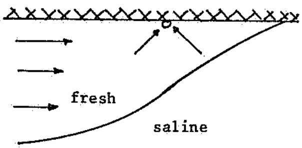


Fig. 18 Freshwater extraction with a drain and replacement of fresh water from one direction.

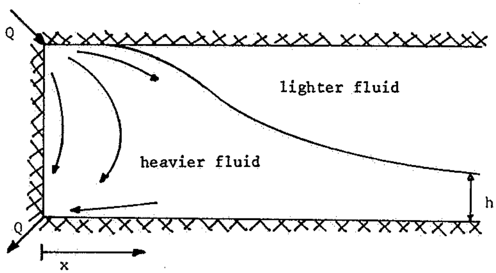


Fig. 19 Flow of the heavier fluid with a source and a sink.

The steady flow in the heavier fluid is caused by a source and a sink, both with a discharge Q (fig. 19).

h is the distance between the interface and the bottom of the aquifer for $x \to \infty$. It is shown by Verruyt (1969) that if Q approaches-infinity, h approaches zero, the results of the calculations with the hodograph method agree with the results of a Hele Shaw model.

Until now only phreatic water or confined water have been discussed. Van Dam (1976) gives a solution for a steady flow problem with fresh and saline semi-confined groundwater. The formulae are derived for a situation as shown in figure 20. The saline water is at rest and the vertical component of the flow in the aquifer is neglected with regard to the horizontal component of this flow. The seepage flow is vertical.

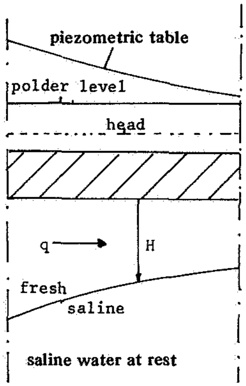


Fig. 20 Model for which Van Dam derives the formulae.

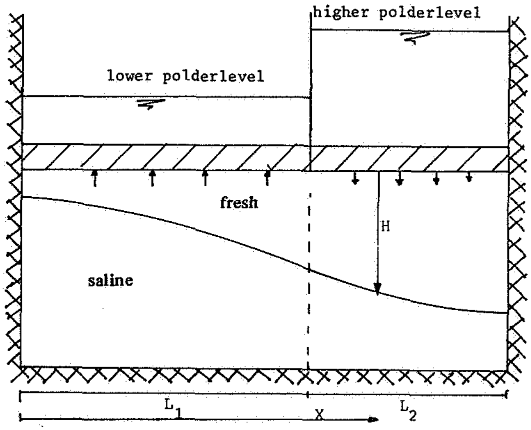


Fig. 21 The groundwater flow problem worked out as an example (Van Dam, 1976).

The starting points for the computation are Darcy's Law, the continuity equation and the Badon Ghijben- Herzberg principle. These lead to a differential equation for the depth H of the interface which is solved with the average value of H over an interval in which the change in H is small. For this purpose the x-axis is divided into a number of intervals. If the values of H and the freshwater discharge q are known at one of the two boundaries, the values of H and q at the other boundary of the interval can be calculated with the formulae derived by Van Dam (1976). These calculated values are the starting values for the next interval. If at both boundaries only one of the values of H and q is known, at one boundary the unknown boundary condition is estimated. With this all the intervals are calculated, and the calculated H and q at the other boundary are compared with the known boundary condition. For example, the boundary conditions for the problem in figure 21 are $x = 0$, $q = 0$ and $x = L_1 + L_2$, $q = 0$. In the last section Van Dam (1976) indicates how to solve a problem with a number of compartments with different levels.

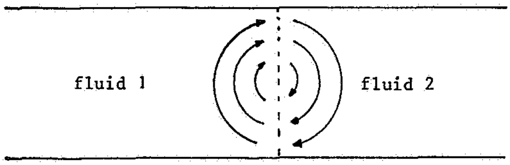


Fig. 22 The example worked out by De Josselin de Jong (1960).

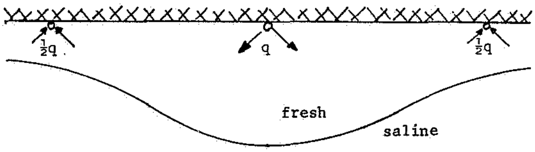


Fig. 23a Old situation after Haitjema (1977).

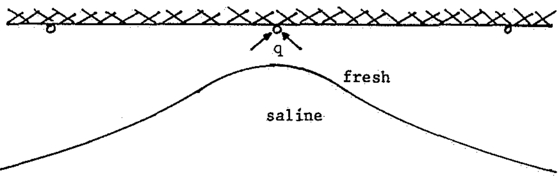


Fig. 23b New situation after Haitjema (1977).

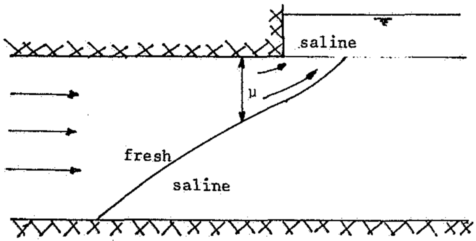


Fig. 24 Non-steady problem solved by Bear and Dagan (1964).

5. NON-STEADY FLOW PROBLEMS IN WHICH DIFFUSION AND DISPERSION ARE ALLOWED TO BE NEGLECTED

To predict how the groundwater flow will change in consequence of changing circumstances it is often desirable to know the displacement of the interface between saline and fresh water over a period of time. De Josselin de Jong (1960) has developed a theory with which it is possible to compute the displacement of the interface. He replaces the liquids of different density by one homogeneous liquid. Where there are changes in density in the original situation, he introduces singularities. These singularities can be vortices or a source and a sink at a zero distance. In the first part De Josselin de Jong (1969) derives formulae which the groundwater flow can be calculated. First, these formulae are derived for a liquid with a continuously changing density, then for a definite interface between fresh and saline water. In the second part De Josseling de Jong gives a worked example (fig. 22).

In this example liquid 1 is lighter than liquid 2. To begin with, the interface is vertical. The results of the calculation are compared with a Hele Shaw model and with an electric analogue model. In the electric model the discontinuity between the fresh and the saline water are represented by electric dipoles. This is a steady-state model. The computed values are good approximations of the measured values. De Josselin de Jong (1977) reviews the vortex theory. This article was written as an introduction to that of Haitjema (1977) in which a numerical model based on the vortex theory is presented. In both articles the singularities are assumed to be vortices. Haitjema replaces the liquid with a continuously changing density by a number of homogeneous liquids which a continuously changing density by a number of homogeneous liquids which are separated by definite interfaces. With the help of the vortex theory Haitjema computes the displacements of a number of points on the interfaces. For the next time step he determines by linear interpolation, those points of an interface which are located at the same horizontal position as the points of that interface in the previous time step. An example as shown in the figures 23a/23b is worked out by Haitjema. In this model the ground is assumed to be a homogeneous aquifer which is bounded only on the upper side by an impervious layer. The computation is given for two interfaces, namely fresh-brackish and brackish-saline. Finally Haitjema (1977) compared the results of the computation with a Hele Shaw model, from which it emerges that the computed values agree with the measured ones. Bear and Dagan (1964) use another method to compute the displacement of the interface between fresh and saline water. They start from a steady state as given in figure 24. The aquifer between the two aquicludes is homogeneous and isotropic. At the time $t = 0^+$ the freshwater discharge at the toe of the saltwater wedge changes abruptly.

The calculation method starts from an assumed rate of freshwater discharge as a function of time and of the distance μ. With this the gradient of the interface is calculated as a function of position, and finally the freshwater discharge as a function of time. Four problems are investigated, namely:

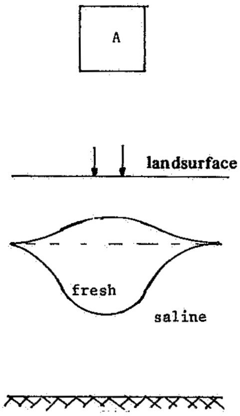


Fig. 25a Freshwater lens in an unbounded aquifer.

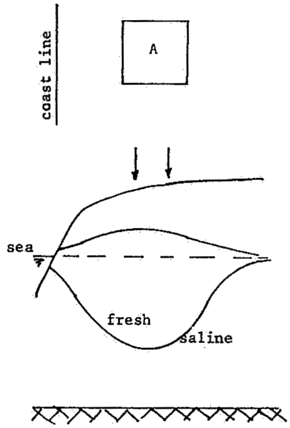


Fig. 25b Freshwater lens near the sea.

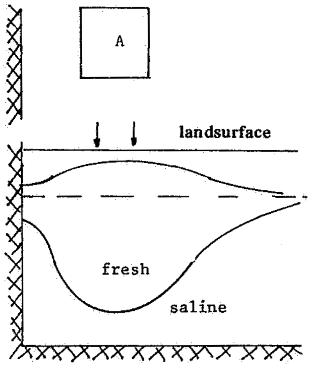


Fig. 25c Freshwater lens near an impervious well.

1. A sudden increase of the freshwater discharge towards the sea at the toe of the saltwater wedge.
2. A sudden decrease of the freshwater discharge towards the sea at the toe of the saltwater wedge.
3. Sudden stopping of the freshwater discharge towards the sea at the toe of the saltwater wedge.
4. Sudden reversal of the seaward freshwater discharge at the toe of the saltwater wedge.

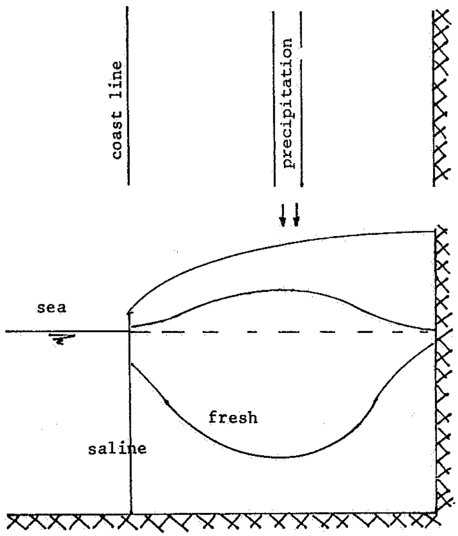


Fig. 26 Freshwater lens in an aquifer bounded by the sea on one side and by an impervious wall on the other.

The first two of these flow situations lead to a new equilibrium, while in the last two cases no new steady state can be obtained. The computed values of all cases agree with the measured values of a Hele Shaw model. Hantush (1968) gives, for a number of cases, the growth of the freshwater lens as a result of recharge of fresh water. He assumes that the displacements in the saline water are small enough to neglect the changes of the salt-water potential. Again the aquifer is homogeneous and isotropic and there is no mixing of the fresh and the saline water. The vertical velocity component of the fresh water is neglected with regard to the horizontal component. The cases for which he an analytical solution gives, are shown in figures 25a, 25b, 25c and 26.

In all these cases the groundwater is assumed to be phreatic. The fresh water is recharged by precipitation on an area A in the cases envisaged in figures 25a, 25b and 25c. This is shown above the respective cross-sections. Hantush also gives the results of calculations of the growth of a freshwater lens, when the fresh water is recharged by a source or an infiltration canal. Further Hantush calculates the growth of a freshwater lens in an aquifer bounded by the sea on one side and by an impervious wall on the other. In this case the fresh water is recharged by precipitation on an area L (fig. 26). Another non-steady groundwater flow problem is solved by Vappicha and Nagaraja (1976). They give a solution for a strip of land bounded by the sea on one side and by a freshwater reservoir on the other (fig. 27).

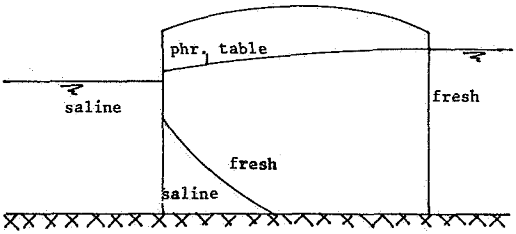


Fig. 27 Strip of land bounded by saline and fresh water.

The vertical component of the velocity is not neglected with regard to the horizontal component. A semi-steady state analysis is given. First, the general differential equation is derived, which is then solved numerically for three different cases. The first is the change of the interface as a result of a linear variation of the phreatic table with time. In the second case the freshwater discharge at the toe of the saltwater wedge changes linearly with time, and in the third case the freshwater discharge at the toe of the saltwater wedge changes suddenly. The question how the phreatic table and the freshwater discharge are varied is not considered by Vappacha et. al (1976). Finally the calculated results are compared with experimental results and are found to be in good agreement with them.

Collins (1976) gives an analytical solution for a number of flow problems. He uses a parameter for the seepage flow (β) and a parameter for the recharge of the fresh groundwater (σ). These parameters comprise geometric and geohydrologic quantities and time. Then a transformation is made from the (x, z) plane to the ($2 + \beta, \sigma - \mu$) plane, in which μ is a constant. There are two types of problems which can be solved with the help of this transformation. The first is the change of the freshwater lens as a result of precipitation. Collins gives three examples of this type, namely: an island in the ocean, an aquifer which bounded by a ditch and an impervious wall, and a coastal area (see figures 28a, 28b and 28c).

The precipitation is in these cases proportional to the distance H between the phreatic table and the reference plane, which distance is also the saltwater potential. The second type of flow problem is the change of the freshwater lens in a semi-confined aquifer. Four examples of this type are given. The figures 29a until 29d give a freshwater lens respectively beneath an island in the ocean, between two drains, in an aquifer bounded by a ditch and an impervious wall, and in a coastal area.

Bonnet and Sauty (1975) give an application of an implicit finite difference model. Little attention is paid to the deviation of the formulae. Bonnet et al. deal mainly with the application of the model on an area in Morocco. In this area the saltwater intrusion is studied for different possibilities of freshwater infiltration. The model employed is two-dimensional and only the horizontal groundwater flow is taken into account. Van der

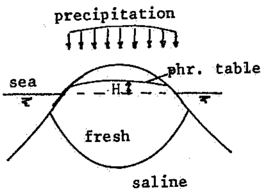


Fig. 28a Island in the ocean.

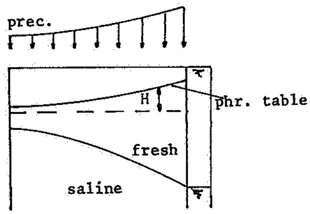


Fig. 28b Strip of land between a ditch and an impervious wall.

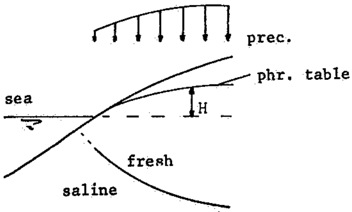


Fig. 28c Coastal area.

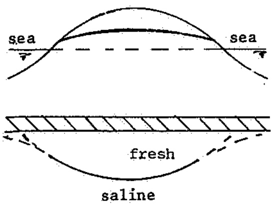


Fig. 29a Island in the ocean.

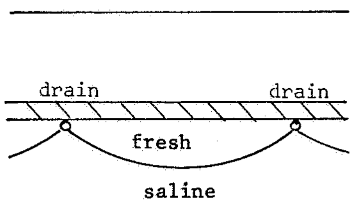


Fig. 29b Freshwater lens between two drains.

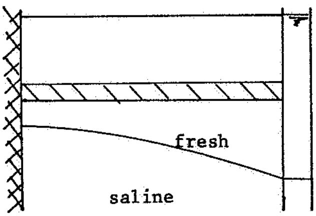


Fig. 29c Strip of land between a ditch and an impervious layer.

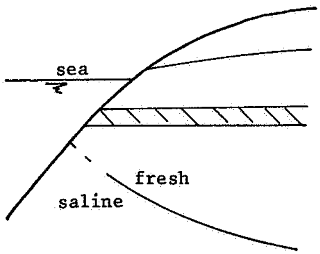


Fig. 29d Coastal area.

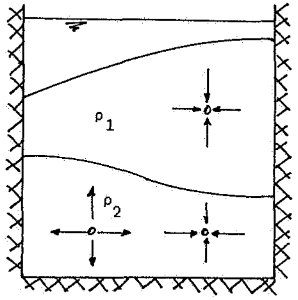


Fig. 30 Flow problem in a homogeneous aquifer.

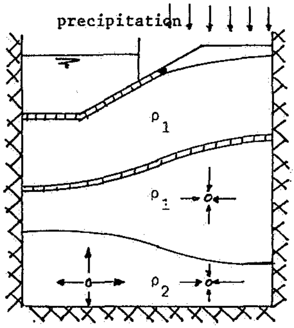


Fig. 31 Flow problem with semi-pervious layers and precipitation.

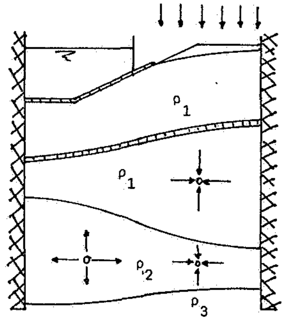


Fig. 32 Flow problem with semipervious layers.

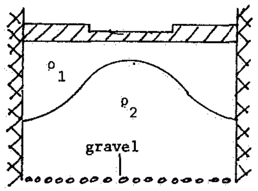


Fig. 33 Flow problem underneath a semi-pervious layer.

Veer (1976) gives a method for the solution of groundwater flow problems with a number of liquids with different densities. He starts from the complex potential Ω, with he finds an exact solution between the boundaries of the model by means of analytical functions with degrees of freedom. This theory is further discussed by Van der Veer (1978). The first part of his thesis contains the basis theory. Next, gives a summary of existing solution methods. In the third part he discusses his own method, the analytical function method. He devides the area into parts which are homogeneous and isotropic. The analytical function method uses functions with degrees of freedom, so that a flow pattern is generated which satisfies the boundary conditions at a number of points of the boundary. The time is taken into account by taking fixed boundary conditions over a time interval and then adjusting the boundary conditions for the next time step. Van der Veer discusses the boundary conditions for situations in which the boundaries are poten-

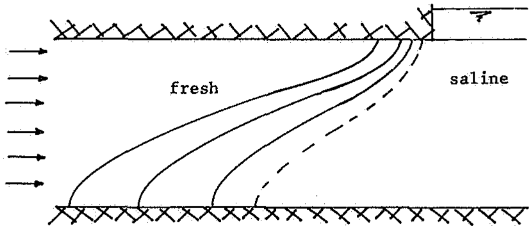


Fig. 34 Henry's problem.

tial lines, stream lines, semi-permeable boundaries, the phreatic table, the interface fresh-saline and inhomogeneities. He compares his method with the finite element method and with the finite difference method. Van der Veer (1978) gives four examples of computations in which there are differences in density. The example in figure 30 is a flow problem in one aquifer, in which there is a source and a sink in the saline water and a source in the fresh water. Figure 31 gives an example with semi-pervious layers and recharge. The example of figure 32 differs from the previous one because of the presence of a third fluid. The last example with regard to flow with differences in density is given in figure 33.

6. GROUNDWATER FLOW IN WHICH DIFFUSION AND DISPERSION ARE TAKEN INTO ACCOUNT

Pinder and Cooper (1970) describe a method with which the displacement of the saltwater wedge can be computed. In this method the results of diffusion and dispersion are considered. The starting points of the analysis are Darcy's Law, the continuity equation and the diffusion/dispersion equation. It starts with the calculation of the pressure distribution with the aid of a finite difference method. In this procedure the distribution of the chlorinity is taken from the previous time step. Next, the velocities with the displacements to match are calculated for a number of points and for a fixed length of time. The calculation is repeated until a sufficient number of time steps has been calculated. This method is applied to Henry's problem (fig. 34).

The steady state – dashed line – is reached from two starting positions, namely one on the left and one on the right of the equilibrium situation. In figure 34 only the curves for the variation of salinity with time are given from the starting position on the left of the steady state. The lines represent the 0.5 isohaline for different times. The method discussed by Pinder et al. (1970) can also applied to a case of irregular geometry and to an inhomogeneous aquifer.

Lee and Cheng (1974) solve Henry's problem by means of the finite element method. This solution is compared with the unsteady-state solution after Pinder et al (1970) and

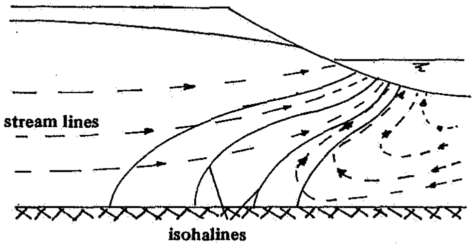


Fig. 35 Cross-section of the Biscayne aquifer.

the steady-state solution of Henry. The finite element method is used to solve a steady flow problem in the Biscayne aquifer. The aquifer is assumed to be homogeneous and isotropic (fig. 35). There is a circular flow of groundwater with a higher chlorinity underneath the coastline. For the same area Segol and Pinder (1976) carried out unsteady-state calculations with the finite element method. At the start of the calculation the flow is assumed to be in equilibrium. The displacement of the isohalines is due to a considerable rise of the groundwater table which occurs after a period of heavy rainfall. Segol and Pinder calculate the displacement of the 0.5 isohaline in time.

Konikow (1977) describes a case-study of the Rocky Mountain Arsenal. As stated by Lee et al. (1974), Pinder et al. (1970) and by Segol et al (1976) the starting points are the differential equations of the groundwater flow and the dispersion. Konikow uses a method with finite differences to solve the equation of the groundwater flow. The dispersion equation is solved by means of characteristics. These equations of Konikow (1977) are not derived, but they are applied only to the area of interest.

Most of Konikow's work is devoted to the determination of geohydrological constants and to comparing the model with historical measurements.

REFERENCES

BADON GHIJBEN, W. and J. DRABBE (1889) – Nota in verband met de voorgenomen putboring nabij Amsterdam. (Notes on the probable results of the proposed well drilling near Amsterdam) Tijdschr. Kon. Instituut van Ingenieurs, pp. 8-12.

BEAR, J. (1972) – Dynamics of fluids in porous media. American Elsevier Environmental Science Series. New York.

BEAR, J. and G. DAGAN (1964) – Some exact solutions of interface problems by means of the hodograph method. Journal of Geophysical Research, Vol. 69, No: 8: 1563-1572.

BEAR, J. and G. DAGAN (1964) – Moving interface in coastal aquifers. Proceedings ASCE, Hydraulic Division, No. HY, 4, Vol. 90: 193-215.

BONNET, M. and J.P. SAUTY (1975) – Un modèle simplifié pour la simulation de nappes avec intrusion saline. Proceedings of the Bratislava Symposium, sept. 1975, IAHS Publication, No. 115.

COLLINS, M.A. (1976) – The extended Boussinesq Problem. Water Resources Research, Vol. 12, No. 1: 54-56.

DAM, J.C. VAN (1976) – Partial depletion of saline groundwater by seepage. Journal of Hydrology, Vol. 29 No. 3/4: 315-339.

DAM, J.C. VAN (1977) – Determination of horizontal and vertical groundwater flow from piezometric levels observed in groundwater in varied densities. Delft Progress Report, Vol. 3: 19-34.

GLOVER, R.E. (1959) – The pattern of freshwater flow in a coastal aquifer. Journal of Geophysical Research, Vol. 64, No. 4: 457-459.

HAITJEMA, H.M. (1977) – Numerical application of vortices to multiple fluid flow in porous media. Delft Progress Report, Vol. 2: 237-248.

HANTUSH, M.S. (1968) – Unsteady movement of fresh water in thick unconfined saline aquifers. Bulletin of the IASH, XIII, No. 2: 40-60.

HENRY, H.R. (1959) – Salt intrusion into fresh water aquifers. Journal of Geophysical Research, Vol. 64, No. 11: 1911-1919.

HERZBERG, (1901) – Die Wasserversorgung einiger Nordseebäder. Journal für Gasbeleuchting und Wasserversorgung XLIX, No. 44: 815-819 and No. 45: 842-844.

JOSSELIN DE JONG, G. DE (1960) – Singularity distributions for the analysis of multiple fluid flow through porous media. Journal of Geophysical Research, Vol. 65, No. 11: 3739-3758.

JOSSELIN DE JONG, G. DE (1977) – Review of vortex theory for multiple fluid flow. Delft Progress Report, Vol. 2: 225-236.

KISHI, Y. and Y. FUKUO (1977) – Studies on salinization of groundwater. I. Theoretical considerations of the three-dimensional salt water interface caused by the pumpage of confined groundwater in fanshaped alluvium. Journal of Hydrology, Vol. 35: 1-30.

KONIKOW, L.F. (1977) – Modeling chloride movement on the alluvial aquifer at the Rocky Mountain Arsenal, Colorado, U.S. Geological Survey, Water Supply paper 2044, Washington DC.

KOHOUT, F.A. and H. KLEIN (1967) – Effect of pulse recharge on the zone of diffusion in the Biscayne Aquifer. Extract of publication No. 72, Symposium of Haifa, 1967, IASH.

LEE, CHUNG-HIAN and RALPH TA-SHUN (1974) – On seawater encroachment in coastal aquifers. Water Resources Research, Vol. 10, No. 5: 1039-1043.

MEINARDI, C.R. (1973) – Het zoutvoorkomen in de ondergrond van de lage gedeelten van Nederland. H_2O 6/18: 454-460.

PINDER, G.F. and H.H. COOPER, jr. (1970) – A numerical technique for calculating the transient position of the saltwater front. Water Resources Research, Vol. 6, No. 3: 875-882.

SEGOL, G. and G.F. PINDER (1976) – Transient simulation of saltwater intrusion in southwestern Florida. Water Resources Research, Vol. 12, No. 1: 65-70.

STICHTING POSTAKADEMIALE VORMING GEZONDHEIDSTECHNIEK (1970-1971) – Cursus zout grondwater in Nederland. Deel I. Volume A en B, 1970/1971.

STRACK, O.D.L. (1976) – A single potential solution for regional interface problems in coastal aquifers. Water Resources Research, Vol. 12, No. 6: 1165-1174.

STRACK, O.D.L. (1973) – Many-valuedness encountered in groundwater flow. Ph.D.-thesis, Delft University of Technology.

TODD, D.K. (1974) – Saltwater intrusion and its control. Journal A.W.W.A., Vol. 56, No. 3: 180-187.

VAPPICHA, V.N. and S.H. NAGARAJA (1976) – An appropriate solution for the transient interface in an coastal aquifer. Journal of Hydrology, Vol. 31: 161-173.

VEER, P. VAN DER (1976) – Calculation method for two-dimensional groundwater flow. Delft Progress Report, Vol. 2: 35-49.

VEER, P. VAN DER (1977[a]) – The pattern of fresh and salt water flow in a coastal aquifer. Delft Progress Report, Vol. 2: 137-142.

VEER, P. VAN DER (1977[b]) – Analytical solution for steady interface flow in a coastal aquifer involving the phreatic surface with precipitation. Journal of Hydrology, Vol. 34, No 1/2: 1-11.

VEER, P. VAN DER (1977[c]) – Analytical solution for a two-fluid flow in a coastal aquifer involving a phreatic surface with precipitation. Journal of Hydrology, Vol. 35: 271-278.

VEER, P. VAN DER (1978) – Calculation methods for two-dimensional groundwater flow. Ph.D.-thesis, Delft University of Technology, 1978; also as: Rijkswaterstaat Communications No. 28, The Hague, 1978.

VERRUYT, A. (1969) – An interface problem with a source and a sink in the heavy fluid. Journal of Hydrology, Vol. 8: 197-206.

VERRUYT, A. The complex variable method for solving two-dimensional flow problems. Lecture notes on Geotechnics, Delft University of Technology.

VOLKER, A. (1961) – Source of brackish groundwater in pleistocene formations beneath the Dutch polderland. Economic Geology, Vol. 56: 1045-1057.

APPENDIX B*.

FRESH WATER – SALT WATER RELATIONSHIPS

J.C. VAN DAM

The following cases are dealt with:

1. confined groundwater;
2. phreatic groundwater with natural recharge;
3. semi-confined groundwater with invariable groundwater table above the semi-pervious toplayer.

Each of these cases is dealt with in:

A. one-dimensional situation,
B. radial-symmetric situation.

The following conditions hold for all cases:

a. homogeneous aquifer;
b. sharp interface;
c. the vertical velocity component in the aquifer is neglected;
d. the horizontal velocity component in the semi-pervious toplayer is neglected;
e. the saline groundwater is under static conditions.

List of symbols

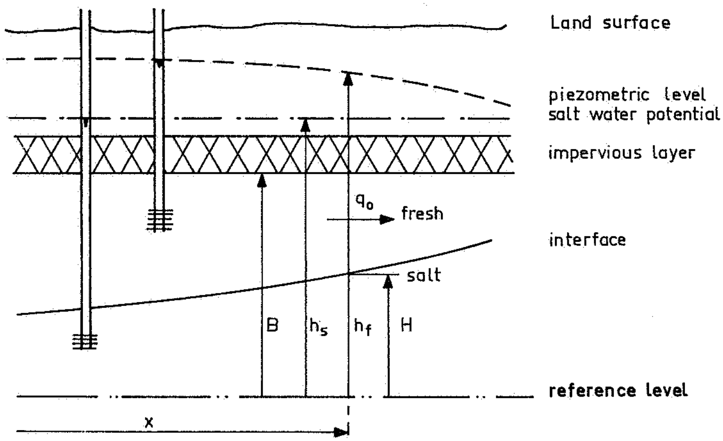
A = coefficient in solution 3.A ($A = \sqrt{y_1^2 + 2\,m}$)
B = position of top of aquifer with respect to reference level
C = integration constant
C_1 = integration constant
C_2 = integration constant
H = position of interface with respect to reference level
H_c = same as H but constant over a ring (solution 3.B)
Q = flow of fresh water in radial symmetric cases (B-cases)
Q_0 = same as Q but constant

* Appendix B is a part of the lecture notes on geohydrology, dealing with fresh water – salt water relationships in large scale flows.

b = coefficient in cases 3 ($b = \frac{1}{kc}$)
c = hydraulic resistance of semi-pervious layer
d = coefficient in solution 3.A ($d = \frac{g}{2b}$)
f = natural recharge
g = coefficient in cases 3 ($g = \frac{p - (1 + \alpha) h_s + \alpha B}{kc\alpha}$)
h_f = piezometric level of fresh water in the aquifer
h_{f_c} = same as h_f but constant over a ring (solution 3.B)
h_s = piezometric level of salt water in the aquifer
k = horizontal permeability of the aquifer
l = solution coefficient in solution 3.A
m = solution coefficient in solution 3.A
p = polder level
q = flow of fresh water in one-dimensional cases (A-cases)
q_0 = same as q but constant
r = horizontal coordinate in radial-symmetric cases (B-cases)
x = horizontal coordinate in one-dimensional cases (A-cases)
y = vertical coordinate: depth of interface with respect to top of aquifer in cases 3
y_1 = root in solution 3.A
y_2 = root in solution 3.A
y_3 = root in solution 3.A
z_3 = $y_3 - y_1$ in solution 3.A
α = relative difference in density between salt water and fresh water ($\alpha = \frac{\rho_s - \rho_f}{\rho_f}$)
α' = coefficient in solution 3.A (see elliptic functions)
β = integration constant
γ = integration constant
ϕ = coefficient in solution 3.A (see elliptic functions)
ρ_f = density of fresh water
ρ_s = density of salt water

1. CONFINED GROUNDWATER

A. *One-dimensional situation*



Darcy:

$$q = -k(B - H)\frac{dh_f}{dx}$$

Continuity:

$$q = q_0$$

Badon Ghijben-Herzberg:

$$h_f - h_s = \alpha(h_s - H) \rightarrow \frac{dh_f}{dx} = \alpha\frac{dH}{dx}$$

Combination gives:

$$k\,\alpha(B - H)\frac{dH}{dx} = q_0$$

Differential equation:

$$(B - H)\frac{dH}{dx} = \frac{q_0}{k\alpha}$$

Solution:

$$H = B \pm \sqrt{\frac{-2q_0}{k\alpha}\,x + C}$$

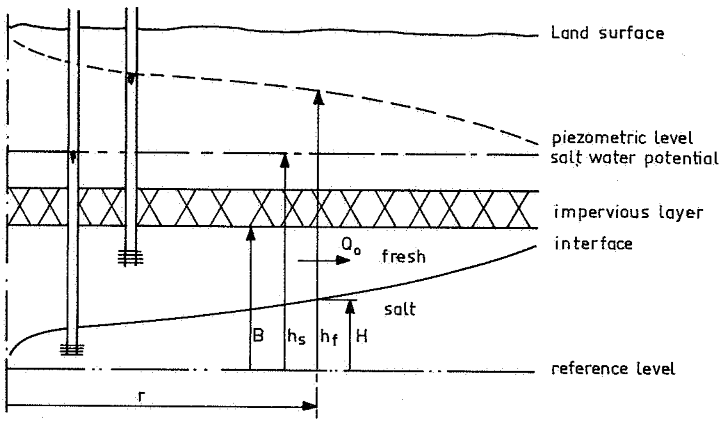
As $H < B$ we can use only:

$$H = B - \sqrt{\frac{-2q_0}{k\alpha}\,x + C}$$

$$h_f = \alpha\sqrt{\frac{-2q_0}{k\alpha}\,x + C} - \alpha B + (1+\alpha)\,h_s$$

$q = q_0$
C follows from boundary conditions

B. *Radial-symmetric situation*



Darcy:

$$Q = -2\pi r k(B - H)\frac{dh_f}{dr}$$

Continuity:

$$Q = Q_0$$

Badon Ghijben-Herzberg:

$$h_f - h_s = \alpha(h_s - H) \rightarrow \frac{dh_f}{dr} = -\alpha\frac{dH}{dr}$$

Combination gives:

$$2\pi r k \alpha (B - H) \frac{dH}{dr} = Q_0$$

Differential equation:

$$(B - H) \frac{dH}{dr} = \frac{Q_0}{2\pi r k \alpha}$$

Solution:

$$H = B \pm \sqrt{\frac{-Q_0}{\pi k \alpha} \ln r + C}$$

As H < B we can use only:

$$H = B - \sqrt{\frac{-Q_0}{\pi k \alpha} \ln r + C}$$

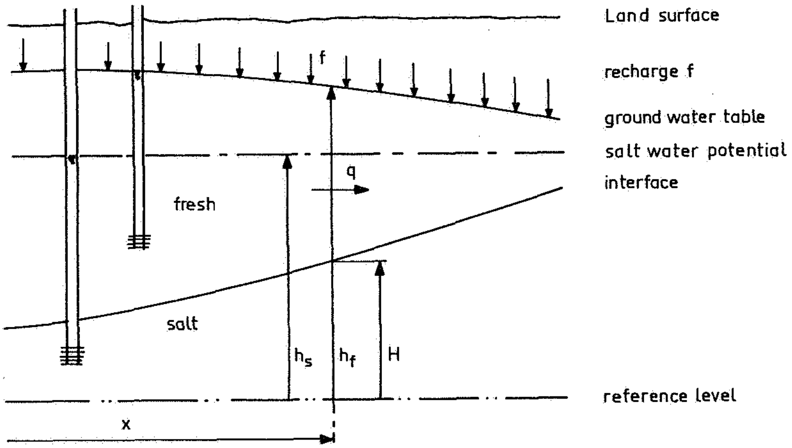
$$h_f = \alpha \sqrt{\frac{-Q_0}{\pi k \alpha} \ln r + C} - \alpha B + (1 + \alpha) h_s$$

$$Q = Q_0$$

C follows from boundary conditions.

2. PHREATIC GROUNDWATER

A. *One-dimensional situation*



Darcy:

$$q = -k(h_f - H)\frac{dh_f}{dx}$$

Continuity:

$$dq = f\,dx \rightarrow q = fx + C_1$$

Badon Ghijben-Herzberg:

$$h_f - h_s = \alpha(h_s - H) \rightarrow \frac{dh_f}{dx} = -\alpha\frac{dH}{dx}$$

Combination gives:

$$k\,\alpha\,(1+\alpha)(h_s - H)\frac{dH}{dx} = fx + C_1$$

Differential equation:

$$(h_s - H)\frac{dH}{dx} = \frac{fx + C_1}{k\alpha(1+\alpha)}$$

Solution:

$$H = h_s \pm \sqrt{-\frac{fx^2 + 2C_1\,x + C_2}{k\alpha(1+\alpha)}},$$

As $H < h_s$ we can use only:

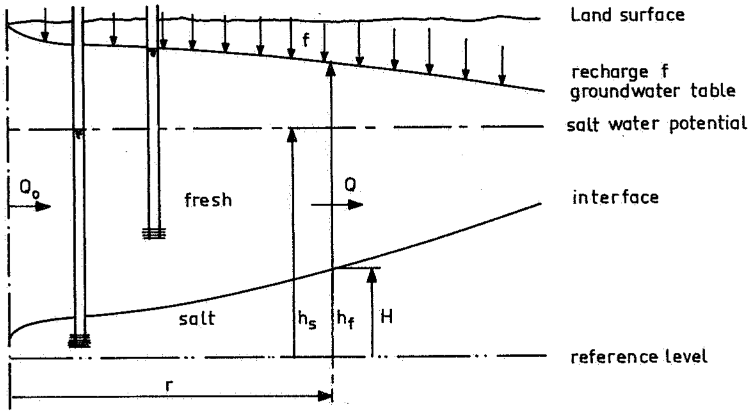
$$H = h_s - \sqrt{-\frac{fx^2 + 2C_1\,x + C_2}{k\alpha(1+\alpha)}},$$

$$h_f = h_s + \sqrt{-\alpha\frac{fx^2 + 2C_1\,x + C_2}{k(1+\alpha)}}$$

$$q = fx + C_1$$

C_1 and C_2 follow from boundary conditions.

B. *Radial-symmetric situation*



Darcy:

$$Q = -2\pi rk(h_f - H)\frac{dh_f}{dr}$$

Continuity:

$$dQ = f\,2\pi r\,dr$$
$$\rightarrow Q = f\,\pi r^2 + Q_0$$

Badon Ghijben-Herzberg:

$$h_f - h_s = \alpha(h_s - H) \rightarrow \frac{dh_f}{dr} = -\alpha\frac{dH}{dr}$$

Combination gives:

$$2\,\pi rk\alpha(1+\alpha)(h_s - H)\frac{dH}{dr} = f\,\pi r^2 + Q_0$$

Differential equation:

$$(h_s - H)\frac{dH}{dr} = \frac{f\pi r^2 + Q_0}{2\pi rk\alpha(1+\alpha)}$$

Solution:

$$H = h_s \pm \sqrt{-\frac{\frac{1}{2} f \pi r^2 + Q_0 \ln r + C}{\pi k \alpha (1+\alpha)}}$$

As $H < h_s$ we can use only:

$$H = h_s - \sqrt{-\frac{\frac{1}{2} f \pi r^2 + Q_0 \ln r + C}{\pi k \alpha (1+\alpha)}}$$

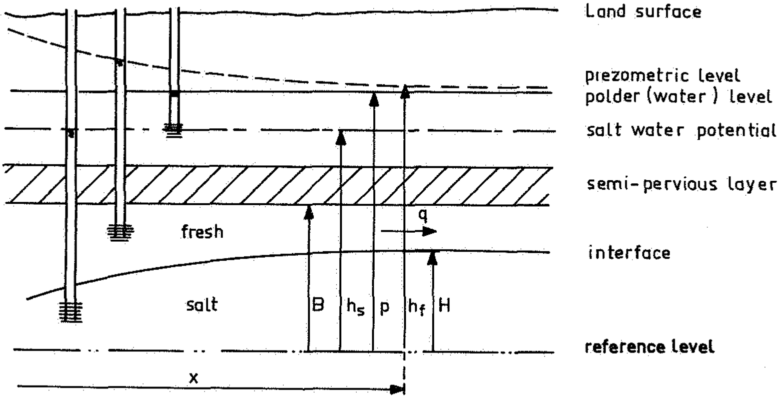
$$h_f = h_s + \sqrt{-\alpha \frac{\frac{1}{2} f \pi r^2 + Q_0 \ln r + C}{\pi k (1+\alpha)}}$$

$$Q = f \pi r^2 + Q_0$$

C follows from boundary conditions.

3. SEMI-CONFINED GROUNDWATER

A. *One-dimensional situation*



Darcy:

$$q = -k(B-H)\frac{dh_f}{dx} \rightarrow \frac{dq}{dx} = -k\left[(B-H)\frac{d^2h_f}{dx^2} - \frac{dH}{dx}\frac{dh_f}{dx}\right]$$

Continuity:

$$dq = \frac{p-h_f}{c}dx \rightarrow \frac{dq}{dx} = \frac{p-h_f}{c}$$

Combination gives:

$$-k\left[(B-H)\frac{d^2h_f}{dx^2}-\frac{dH}{dx}\frac{dh_f}{dx}\right]=\frac{p-h_f}{c} \tag{1}$$

Badon Ghijben-Herzberg:

$$h_f-h_s=\alpha(h_s-H)\rightarrow\frac{dh_f}{dx}=-\alpha\frac{dH}{dx} \tag{2}$$

$$\rightarrow\frac{d^2h_f}{dx^2}=-\alpha\frac{d^2H}{dx^2} \tag{3}$$

$$\text{(2) and (3) in (1)}\rightarrow+k\alpha\left[(B-H)\frac{d^2H}{dx^2}-\left(\frac{dH}{dx}\right)^2\right]=\frac{p-h_s-\alpha(h_s-H)}{c}$$

Differential equation:

$$(B-H)\frac{d^2H}{dx^2}-\left(\frac{dH}{dx}\right)^2-\frac{H}{kc}-\frac{p-(1+\alpha)h_s}{kc\alpha}=0$$

if:

$$b=\frac{1}{kc},\ g=-\frac{p-(1+\alpha)h_s+\alpha B}{kc\alpha}\ \text{and}\ y=B-H$$

we can write the equation as:

$$yy''+(y')^2-by+g=0$$

Solution (after Sikkema):

$$yy'=\pm\sqrt{\tfrac{2}{3}b}\sqrt{y^3-3dy^2+\beta}$$

with $d=\frac{g}{2b}$ and β as integration constant.

In the solution fourteen cases must be distinguished.

Case	Value g	Value β	Solution
G_0	any		$y = \frac{g}{b}$
G_1	$g>0$	$\beta<0$	formula 1
G_2	$g>0$	$\beta=0$	formula 2
G_3	$g>0$	$0<\beta<4d^3$	formula 3
G_4	$g>0$	$\beta=4d^3$	formula 4
G_5	$g>0$	$\beta>4d^3$	formula 1
G_6	$g=0$	$\beta=0$	as formula 2, but $d=0$
G_7	$g=0$	$\beta<0$	as formula 1, but A and $\sin^2\alpha'$ have other values
G_8	$g=0$	$\beta>0$	as formula 1, see G_7
G_9	$g<0$	$\beta<4d^3$	formula 1
G_{10}	$g<0$	$\beta=4d^3$	formula 5
G_{11}	$g<0$	$4d^3<\beta<0$	formula 3
G_{12}	$g<0$	$\beta=0$	formula 2
G_{13}	$g<0$	$\beta>0$	formula 1

List of formulae

1) $$[\sqrt{A}+\frac{y_1}{\sqrt{A}}]\ F(\phi/\alpha') - 2\sqrt{A}\,E(\phi/\alpha') + 2\sqrt{A}\,[\frac{\sin\phi\sqrt{1-\sin^2\alpha'\sin^2\phi}}{1+\cos\phi}]$$

$$= \pm\sqrt{\frac{2}{3}b}\,x + \gamma;$$

in which:

for G_1, G_5, G_9 and G_{13}: $A = \sqrt{y_1^2 + 2m}$; $\sin^2\alpha' = \dfrac{A-1-y_1}{2A}$;

for G_7 and G_8: $d = 0$; $A = |\sqrt[3]{\beta}\,|\sqrt{3}$; $\sin^2\alpha' = \frac{1}{2}+\frac{1}{4}\sqrt{3}\,\frac{\beta}{|\beta|}$;

for all: $y = A\,(\dfrac{1-\cos\phi}{1+\cos\phi}) + y_1$;

with:

l, m, y_1 from $(y - y_1)(y^2 + 2\,ly + m) = y^3 - 3\,dy^2 + \beta$, (G_1, G_5, G_9, G_{13});

γ: integration constant;

$F(\phi/\alpha')$ and $E(\phi/\alpha')$: elliptic integrals of first and second kind.

2) $$y = \frac{1}{6}bx^2 \pm \sqrt{\frac{2}{3}}b\gamma x + \gamma^2 + 3d;$$

with:

γ: integration constant;

3) a) for: $y_1 \leqslant y \leqslant y_2$:

$$-z_3\,E(\phi/\alpha') + y_3 \,.\, F(\phi/\alpha') = \pm\sqrt{\frac{bz_3}{6}}x + \gamma;$$

with:

$y_1 < y_2 < y_3;\ z_3 = y_3 - y_1;$

y_1, y_2, y_3 from $y^3 - 3dy^2 + \beta = 0$, (G_3, G_{11});

$$\sin\phi = \sqrt{\frac{y - y_1}{y_2 - y_1}};\ \sin^2\alpha' = \frac{y_2 - y_1}{y_3 - y_1}$$

3) b) for $y > y_3$

$$y_3\,F(\phi/\alpha') + z_3\,[\mathrm{tg}\,\phi\sqrt{1 - \sin^2\alpha'\sin^2\phi} - E(\phi/\alpha')] =$$

$$= \pm\sqrt{\frac{bz_3}{6}}\,x + \gamma;$$

with:

$y_1, y_2, y_3, z_3, \sin^2\alpha'$ as in 3) a);

$$\mathrm{tg}\,\phi = \sqrt{\frac{y - y_3}{y_3 - y_2}};$$

and both for a) and b): γ, $E(\phi/\alpha')$, $F(\phi/\alpha')$ as in formula 1).

4) (according to Nieuwaal and Nyhuis)

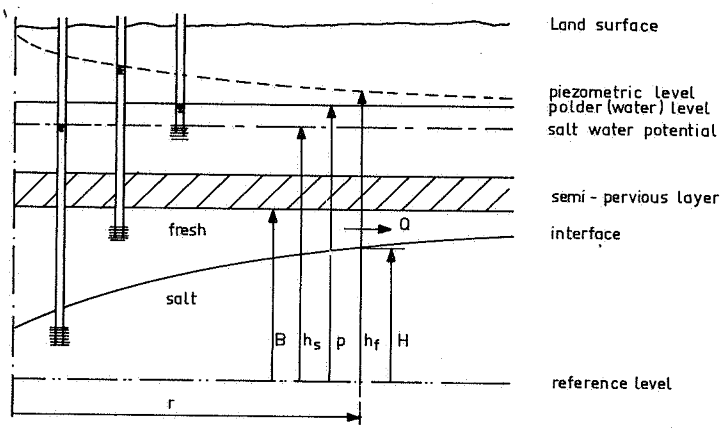
$$\sqrt{y+d}+\sqrt{\frac{d}{3}}\ln\left|\frac{\sqrt{y+d}-\sqrt{3d}}{\sqrt{y+d}+\sqrt{3d}}\right|=\pm\sqrt{\frac{b}{6}}\,x+\gamma;$$

with: γ: integration constant.

5) $$\sqrt{y+d}-\frac{2}{3}\sqrt{-3d}\,\mathrm{arc\,tg}\sqrt{\frac{y+d}{-3d}}=\pm\sqrt{\frac{b}{6}}\,x+\gamma;$$

with: γ: integration constant.

B. *Radial-symmetric situation*



Darcy:

$$Q=-2\pi r k(B-H)\frac{dh_f}{dr}$$

$$\rightarrow\frac{dQ}{dr}=-2\pi k\left[r(B-H)\frac{d^2h_f}{dr^2}+r\left(-\frac{dH}{dr}\right)\frac{dh_f}{dr}+(B-H)\frac{dh_f}{dr}\right]$$

Continuity:

$$dQ=2\pi r\,dr\frac{p-h_f}{c}\rightarrow\frac{dQ}{dr}=2\pi r\frac{p-h_f}{c}$$

Combination gives:

$$-2\pi k\left[r(B-H)\frac{d^2h_f}{dr^2}-r\frac{dH}{dr}\frac{dh_f}{dr}+(B-H)\frac{dh_f}{dr}\right]=2\pi r\frac{p-h_f}{c} \tag{1}$$

Badon Ghijben-Herzberg:

$$h_f-h_s=\alpha(h_s-H)\rightarrow\frac{dh_f}{dr}=-\alpha\frac{dH}{dr} \tag{2}$$

$$\rightarrow\frac{d^2h_f}{dr^2}=-\alpha\frac{d^2H}{dr^2} \tag{3}$$

Differential equation: (2) and (3) in (1)

$$-(B-H)\frac{d^2H}{dr^2}+\left(\frac{dH}{dr}\right)^2-\frac{(B-H)}{r}\cdot\frac{dH}{dr}+\frac{H}{kc}+\frac{p-(1+\alpha)h_s}{kc\alpha}=0$$

if:

$$b=\frac{1}{kc},\ g=\frac{p-(1+\alpha)h_s+\alpha B}{kc\alpha}\ \text{and}\ y=B-H$$

we can write the equation as:

$$yy''+(y')^2+\frac{y}{r}y'-by+g=0$$

Approximative solution (after Van der Molen)

In this method the piezometric surface around the centre is approximated in successive rings. The solution is found over a ring at one side of which the conditions are known. From this the integration constant for that ring is found and the conditions at the other side of the ring can be calculated.

Darcy:

$$Q=-2\pi rk(B-H)\frac{dh_f}{dr}$$

Continuity:

$$dQ = 2\pi r \frac{p - h_f}{c} dr$$

$$\rightarrow Q = \pi r^2 \frac{p - h_{f_c}}{c} + C \text{ (approximation)}$$

Combination gives:

$$\pi r^2 \frac{p - h_{f_c}}{c} + C = -2\pi r k(B - H) \frac{dh_f}{dr}$$

$$2k(B - H) \frac{dh_f}{dr} + r \frac{p - h_{f_c}}{c} + \frac{C}{\pi r} = 0$$

Badon Ghijben-Herzberg:

$$h_f - h_s = \alpha(h_s - H) \rightarrow \frac{dh_f}{dr} = -\alpha \frac{dH}{dr}$$

also:

$$h_{f_c} - h_s = \alpha (h_s - H_c)$$

Differential equation:

$$-(B - H) \frac{dH}{dr} = -\frac{p - (1 + \alpha)h_s + \alpha H_c}{2\alpha kc} r - \frac{C}{2\pi r k\alpha}$$

Solution:

$$H = B \pm \sqrt{-\frac{p - (1 + \alpha)h_s + \alpha H_c}{2kc\alpha} r^2 - \frac{C}{\pi k\alpha} \ln r + C_1},$$

As $H < B$ we can use only:

$$H = B - \sqrt{-\frac{p - (1 + \alpha)h_s + \alpha H_c}{2kc\alpha} r^2 - \frac{C}{\pi k\alpha} \ln r + C_1},$$

$$h_f = \sqrt{-\frac{p - (1 + \alpha)h_s + \alpha H_c}{2kc} \alpha r^2 - \frac{\alpha C}{\pi k} \ln r + \alpha^2 C_1} - \alpha B + (1 + \alpha)h_s$$

$$Q = \frac{p - (1 + \alpha)h_s + \alpha H_c}{c} \pi r^2 + C$$

C and C_1 follow from boundary conditions for each step.

COMMITTEE FOR HYDROLOGICAL RESEARCH T.N.O.

Proceedings and Informations

No. 1. Proceedings of Technical Meetings 1–6 (with summaries in English), 1952.
1. Investigations into the Water Balance of the Rottegatspolder
2. The Water Supply for Crops I
3. Observations of Groundwater Levels
4. Investigations by Drain gauges in the Netherlands
5. The Water Supply for Crops II
6. The Problem of the increasing Salinity of Ground and Surface Water in the Netherlands

No. 2. Proceedings of Technical Meetings 7–10 and Report on the evaporation research in the Rottegatspolder 1947-1952 (with summaries in English), 1955.
7. The Study of Precipitation Data
8. Model Research on Groundwater Flows
9. Measurements and improvement Works in Basin of Brooks
10. Geo-electrical Research

No. 3. Proceedings of Technical Meetings 11–12 (with summaries in English) and Report on the Lysimeters in the Netherlands (in English), 1958.
11. The Water Supply of Sandy Soils
12. Quality Requirements for Surface Waters

No. 4. Evaporation Synposium and Report on the Lysimeters in the Netherlands II (with summaries in English), 1959.

No. 5. Proceedings of Technical Meetings 13–14 (with summaries in English), 1960.
13. Groundwater Levels and Groundwater Movement in the Sandy Areas of the Netherlands
14. Water in unsaturated Soil

No. 6. Proceeding of Technical Meeting 15 (with summaries in English and French), 1961.
The Regimen of the Rhine, the Ysselmeer and Zeeland Lake

No. 7. Proceeding of Technical Meeting 16 (with summaries in English), 1962.
The dry Year 1959

No. 8. Proceeding of Technical Meeting 17 (with summaries in English), 1963.
The Laws of Groundwater flow and their Application in Practice

No. 9. Proceeding of Technical Meeting 18 (with summaries in English), 1963.
Water Nuisance

No. 10. Steady Flow of Ground Water towards Wells, compiled by the Hydrologisch Colloquium (in English), 1964.

No. 11. Proceeding of Technical Meeting 19 (with summaries in French and German), 1964.
Geo-hydrological Cartography

No. 12. Proceeding of Technical Meeting 20 (in English), 1966.
Water Balance Studies

No. 13. Proceeding of Technical Meeting 21 (in English), 1966.
Recent Trends in Hydrograph Synthesis

No. 14. Proceeding of Technical Meeting 22, 1968.
Precipitation Data (II) and
Report on the Lysimeters in the Netherlands (III) (both with summaries in English)

No. 15. Proceedings and Informations no. 15 (in English), 1969.
Soil – Water – Plant

Proceeding of Technical Meeting 23, 1968.
Seepage
(Will not be published).

Proceeding of Technical Meeting 24, 1970.
Flow in the unsaturated Zone
(Will not be published).

No. 16. Proceeding of Technical Meeting 29 (with summaries in English), 1975.
Hydrological Investigations for Masterplan for the future Watersupply in the Netherlands

No. 17. Proceeding of Technical Meeting 25 (in English), 1973.
Automatic Processing of Hydrological Data

No. 18. Proceeding of Technical Meeting 26 (in English), 1974.
Hydraulic Research for Water Management

No. 19. Proceeding of Technical Meeting 27 (in English), 1974.
The Hydrological Investigation Programme in Salland (The Netherlands)

– Proceeding of Technical Meeting 28, 1973.
Water Quality of Dutch Rivers with respect to Water Management
(Will not be published).

No. 20. Proceeding of Technical Meeting 30 (in English), 1976.
Salt distribution in Estuaries

No. 21. Proceeding of Technical Meeting 31 (in English), 1976.
Groundwater Pollution

No. 22. Proceeding of Technical Meeting 32 (with summaries in English), 1976.
Systems Approach to the Management of Water Resources

No. 23. Proceeding of Technical Meeting 33 (in English), 1977.
Precipitation and Measurements of Precipitation

No. 24. Proceeding of Technical Meeting 34 (in English), 1978.
Urbanization and Water Management

No. 25. Proceeding of Technical Meeting 35 (in English), 1979.
The Relation between Water Quantity and Water Quality in Studies of Surface Waters

No. 26. Proceeding of Technical Meeting 36 (in English), 1980.
Research on possible Changes in the Distribution of saline Seepage in The Netherlands