Temporal and differential stabilizability and detectability

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1 Introduction

Feedback control design and stability analysis of non-linear systems along trajectories is often performed using the linearized dynamics about the trajectory [1]. If the trajectory is time-varying the linearized system is *time-varying*. If in addition the non-linear dynamics or the controls are non-smooth, i.e. in the case of bang-bang or *digital* control, the *structure* of the time-varying linearized system change. If no non-smoothness is present, changes of structure do not occur but may *almost* occur. For control system design this is vital information since this structure reveals the *temporal loss* of controllability and reconstructability of the linearized system [2]. They in turn may lead to *temporal instability* of the closed-loop system. Here *measures* of temporal stability of time-varying linear systems over arbitrary finite time intervals are developed, notably intervals where controllability or reconstructability is lost temporarily. Associated to this, measures of *temporal and differential stabilizability* and *temporal and differential detectability* are developed.

2 Temporal and differential stabilizability

Consider a time-varying linear system obtained by linearization about an optimal control and state trajectory $u^*(t),\ x^*(t),\ t \in (t_0, t_N)$ of a non-linear system [1]. The differential Kalman decomposition detects the time-instants $t_i,\ i = 1, 2, .., N-1$ where the structure of the time-varying system (almost) changes. It also identifies over which of the intervals $(t_i, t_{i+1}),\ \ i = 0, 1, .., N-1$ the system is (almost) temporal uncontrollable. This analysis requires consideration and definition of piecewise constant rank systems (PCR systems) [2]. Temporal uncontrollability over time intervals leads naturally to the definition of temporal stabilizability, ones stability of autonomous systems over finite time-intervals is defined [3].

Definition 1

An autonomous PCR system is called temporal stable over (t_i, t_{i+1}) if for any $x(t_i^+) \neq 0$, $\left\| x(t_{i+1}^-) \right\| / \left\| x(t_i^+) \right\| < 1$ □

Definition 2

Associate to Definition 1 the following *temporal stability measure*,

$$\rho(t_i, t_{i+1}) = \max_{x(t_i^+) \neq 0} \left(\left\| x(t_{i+1}^-) \right\|^2 / \left\| x(t_i^+) \right\|^2 \right) \geq 0 \qquad \square$$

Definition 3

Associate to Definition 1 and Definition 2 the following *temporal stabilizability measure* that applies to PCR systems considered over the intervals (t_s, t_{i+1}), $t_s \in (t_i, t_{i+1})$,

$$\rho_{\min}(t_s, t_{i+1}) = \max_{x(t_s) \neq 0} \left(\min_{u(t)|x(t_s)} \left(\left\| x(t_{i+1}^-) \right\|^2 / \left\| x(t_s) \right\|^2 \right) \right) \geq 0$$

where $u(t) \,|\, x(t_s)$ indicates a control law that depends on $x(t_s)$ □

Definition 4

A PCR system is called *temporal stabilizable over* $(t_s, t_{i+1}),\ t_s \in (t_i, t_{i+1})$ if $\rho_{\min}(t_s, t_{i+1}) < 1$ □

Theorem 1

$\rho_{\min}(t_s, t_{i+1}) = \left\| S(t_s) \right\|,\ t_s \in (t_i, t_{i+1})$ where $S(t_s)$ satisfies a Riccati differential equation associated with a standard finite horizon LQ problem over (t_i, t_{i+1}) with $Q(t_s) = 0,\ R(t_s) \downarrow 0,\ H = I$, $t_s \in (t_i, t_{i+1})$ □

From Theorem 1 it can be established that $-d\left\| S(t_s) \right\| / dt_s$ is a *measure* of *differential stabilizability*. If less than zero the system is differentially stabilizable at t_s. Dual results hold for detectability and differential detectability.

References

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