## 3.1 Potential evapotranspiration

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### *3.1.1 Introduction*

The atmosphere is the main source of carbon dioxide, which is needed by plants in the assimilation process. The rate of supply of carbon dioxide depends both on the concentration difference between the atmosphere and the active sites in the plant, and on the resistance to carbon dioxide transport, part of which is in the stomata in the leaves of the plant (Section 3.3).

The water status of a plant influences the rate of $CO_2$ supply, because stomatal opening, and hence the resistance, is affected by the water potential in the plant. The stomata of a plant start to close at some critical lower water content of the plant, resulting from a negative balance between the uptake of water through its roots and the amount of water lost through the stomata in the process of transpiration. This closure of the stomata continues until the balance of uptake and transpiration is restored, but now at a lower level. Hence, a direct but complicated interrelation exists between transpiration, assimilation and the water supply in the soil (Section 3.2).

In this section the transpiration process is examined under conditions of non-limiting supply of water to the plant, which is defined as potential transpiration. The analysis of Penman (1956) is followed. In that approach the radiation sources that provide the energy to evaporate the water are considered in combination with the turbulence of the air, to remove the water vapour. This procedure enables the estimation of potential evapotranspiration, using data obtained from standard meteorological stations.

### *3.1.2 Radiation*

The sun, which has a temperature of 6000 K, emits energy like any black body with a temperature above absolute zero. The solar constant, i.e. the mean energy received at the earth's mean distance from the sun, outside the earth's atmosphere, on a surface normal to the incident radiation, has a value of approximately 1.4 kJ $m^{-2}$ $s^{-1}$.

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**Exercise 13**
In the old system of units the solar constant was expressed in cal $cm^{-2}$ $min^{-1}$. What is the value of the solar constant in these units?

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In passing through the atmosphere, a variable part of the radiant energy is absorbed, scattered and reflected, due to the presence of ozone, water vapour, clouds and dust in the atmosphere. Total global radiation at the surface of the earth is measured at a limited number of meteorological stations. For other stations it may be possible to estimate the average total global radiation by means of an empirical relation, using the measured duration of bright sunshine (Black et al., 1954). The form of this relationship — the Angström formula – is:

$$R_I = R_A (a_A + b_A nN^{-1}) \quad (8)$$

where

$R_I$ is the radiation actually received ($J\ m^{-2}\ d^{-1}$).

$R_A$ is Angot's value, or the theoretical amount of radiation that would reach the earth's surface in the absence of an atmosphere ($J\ m^{-2}$) $d^{-1}$. Values of $R_A$ are tabulated as a function of day of the year and latitude in Table 14.

$nN^{-1}$ is the ratio of actual duration of bright sunshine (n) and the maximum possible length on a cloudless day (N), both in h. The actual duration of sunshine is recorded in most stations, whereas the maximum duration during the day is again tabulated as a function of date and latitude in Table15.

$a_A$ and $b_A$ are empirical constants.

The numerical value of these constants depends on the location or latitude (Glover & McCulloch, 1958) and the climate (Rietveld, 1978). Indicative values used by F.A.O. (Frere & Popov, 1979) are:

| | | |
|---|---|---|
| for cold and temperate zones | $a_A$ = 0.18 | and $b_A$ = 0.55 |
| for dry tropical zones | 0.25 | 0.45 |
| for humid tropical zones | 0.29 | 0.42 |

The Angström formula gives fair results for weekly or monthly averages.

Table 14. Angot's values ($10^7$ J $m^{-2}d^{-1}$) as a function of latitude and day of the year.

Date for northern latitudes

| Northern Latitude | 13 Jan. | 4 Feb. | 26 Feb. | 21 Mar. | 13 Apr. | 6 May | 29 May | 22 June | 15 July | 8 Aug. | 31 Aug. | 23 Sep. | 16 Oct. | 8 Nov. | 30 Nov. | 22 Dec. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 90 | — | — | — | — | 1.77 | 3.23 | 4.18 | 4.51 | 4.16 | 3.20 | 1.75 | — | — | — | — | — |
| 80 | — | — | 0.03 | 0.65 | 1.77 | 3.18 | 4.12 | 4.44 | 4.10 | 3.16 | 1.75 | 0.64 | 0.03 | — | — | — |
| 70 | — | 0.10 | 0.55 | 1.28 | 2.20 | 3.13 | 3.93 | 4.24 | 3.91 | 3.11 | 2.17 | 1.27 | 0.54 | 0.10 | — | — |
| 60 | 0.30 | 0.61 | 1.15 | 1.87 | 2.66 | 3.39 | 3.91 | 4.10 | 3.89 | 3.35 | 2.63 | 1.85 | 1.14 | 0.61 | 0.30 | 0.21 |
| 50 | 0.86 | 1.21 | 1.75 | 2.41 | 3.06 | 3.63 | 4.01 | 4.14 | 3.99 | 3.60 | 3.03 | 2.38 | 1.73 | 1.20 | 0.85 | 0.74 |
| 40 | 1.46 | 1.82 | 2.31 | 2.87 | 3.38 | 3.81 | 4.07 | 4.15 | 4.05 | 3.77 | 3.34 | 2.83 | 2.28 | 1.80 | 1.46 | 1.33 |
| 30 | 2.07 | 2.38 | 2.80 | 3.24 | 3.62 | 3.89 | 4.05 | 4.08 | 4.02 | 3.85 | 3.58 | 3.20 | 2.77 | 2.36 | 2.06 | 1.95 |
| 20 | 2.64 | 2.89 | 3.22 | 3.52 | 3.74 | 3.86 | 3.91 | 3.91 | 3.89 | 3.83 | 3.70 | 3.48 | 3.18 | 2.87 | 2.62 | 2.53 |
| 10 | 3.15 | 3.33 | 3.54 | 3.69 | 3.75 | 3.74 | 3.69 | 3.65 | 3.67 | 3.71 | 3.71 | 3.65 | 3.49 | 3.30 | 3.13 | 3.07 |
| 0 | 3.58 | 3.67 | 3.75 | 3.75 | 3.65 | 3.50 | 3.37 | 3.31 | 3.35 | 3.47 | 3.61 | 3.70 | 3.71 | 3.64 | 3.56 | 3.53 |
| −10 | 3.92 | 3.92 | 3.85 | 3.69 | 3.45 | 3.18 | 2.96 | 2.88 | 2.95 | 3.15 | 3.41 | 3.65 | 3.81 | 3.88 | 3.90 | 3.91 |
| −20 | 4.16 | 4.05 | 3.84 | 3.52 | 3.14 | 2.76 | 2.48 | 2.37 | 2.47 | 2.74 | 3.10 | 3.48 | 3.80 | 4.01 | 4.14 | 4.18 |
| −30 | 4.29 | 4.07 | 3.72 | 3.24 | 2.74 | 2.27 | 1.95 | 1.82 | 1.94 | 2.25 | 2.70 | 3.20 | 3.67 | 4.03 | 4.27 | 4.36 |
| −40 | 4.32 | 3.99 | 3.46 | 2.87 | 2.25 | 1.73 | 1.38 | 1.24 | 1.37 | 1.71 | 2.23 | 2.83 | 3.43 | 3.95 | 4.30 | 4.43 |
| −50 | 4.26 | 3.80 | 3.15 | 2.41 | 1.71 | 1.16 | 0.81 | 0.69 | 0.80 | 1.15 | 1.69 | 2.38 | 3.11 | 3.77 | 4.24 | 4.42 |
| −60 | 4.15 | 3.54 | 2.73 | 1.87 | 1.13 | 0.59 | 0.28 | 0.20 | 0.28 | 0.58 | 1.11 | 1.85 | 2.70 | 3.52 | 4.13 | 4.38 |
| −70 | 4.18 | 3.28 | 2.25 | 1.28 | 0.53 | 0.10 | — | — | — | 0.10 | 0.53 | 1.27 | 2.23 | 3.26 | 4.16 | 4.52 |
| −80 | 4.38 | 3.33 | 1.82 | 0.65 | 0.03 | — | — | — | — | — | 0.03 | 0.64 | 1.80 | 3.31 | 4.36 | 4.74 |
| −90 | 4.45 | 3.39 | 1.82 | — | — | — | — | — | — | — | — | — | 1.80 | 3.35 | 4.42 | 4.81 |

Table 15. Maximum duration of bright sunshine (hours) as a function of latitude and day of the year.

Date for northern/southern latitudes

| Latitude (°N or °S) | 15 Jan. July | 15 Feb. Aug. | 15 Mar. Sep. | 15 Apr. Oct. | 15 May Nov. | 15 June Dec. | 15 July Jan. | 15 Aug. Feb. | 15 Sept. Mar. | 15 Oct. Apr. | 15 Nov. May | 15 Dec. June |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 12.1 | 12.1 | 12.1 | 12.1 | 12.1 | 12.1 | 12.1 | 12.1 | 12.1 | 12.1 | 12.1 | 12.1 |
| 10 | 11.7 | 11.8 | 12.0 | 12.4 | 12.5 | 12.7 | 12.7 | 12.5 | 12.2 | 11.9 | 11.7 | 11.5 |
| 20 | 11.1 | 11.5 | 12.0 | 12.6 | 13.1 | 13.3 | 13.2 | 12.9 | 12.3 | 11.7 | 11.3 | 11.0 |
| 30 | 10.4 | 11.2 | 11.9 | 12.9 | 13.6 | 14.0 | 13.9 | 13.3 | 12.4 | 11.5 | 10.8 | 10.3 |
| 40 | 9.9 | 10.7 | 11.8 | 13.3 | 14.3 | 14.9 | 14.7 | 13.9 | 12.5 | 11.2 | 10.1 | 9.5 |
| 50 | 9.2 | 10.2 | 11.6 | 13.8 | 15.3 | 16.2 | 15.9 | 14.6 | 12.6 | 10.9 | 9.3 | 8.3 |
| 60 | 7.5 | 9.2 | 11.6 | 14.6 | 17.0 | 18.5 | 18.0 | 15.9 | 12.9 | 10.3 | 7.2 | 6.4 |
| 70 | 3.1 | 7.4 | 11.4 | 16.2 | 18.5 | | | 18.7 | 13.4 | 9.2 | 4.7 | |
| 80 | | 1.9 | 10.6 | | | | | | 14.8 | 6.0 | | |

**Exercise 14**
Meteorological station Don Muang, Thailand (humid tropical zone)
Position: 13°55′ N, 100°36′ E

Mean monthly values of measured daily hours of sunshine (n):

| J | F | M | A | M | J | J | A | S | O | N | D |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 7.1 | 5.9 | 5.9 | 5.1 | 4.4 | 4.0 | 3.7 | 2.9 | 3.3 | 3.9 | 4.9 | 5.9 |

Calculate $R_I$. Use Equation 8, and Tables 14 and 15.

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Part of the solar radiation reaching the earth's surface is reflected. The term 'albedo' denotes that reflection fraction; its value is determined by a combination of surface properties (open water, dry soil, moist to wet soil, vegetation, etc.) and the inclination of the sun. Indicative mean albedo values are 0.05 for a water surface and 0.25 for a green crop surface completely covering the ground (Penman, 1963).
The atmosphere (including the clouds) and the earth's surface, which both have temperatures above the absolute zero, emit thermal radiation. The magnitude of this radiation is proportional to the temperature. As the atmosphere is partly transparent and the temperature of the earth's surface is higher than that of the atmosphere, the outgoing long-wave radiation component will be dominant. Empirical relations have been formulated to calculate this net outgoing long-wave radiation on the basis of air temperature, humidity and cloudiness. Penman (1956) used the following expression, which is derived from the Brunt-formula (Brunt, 1932):

$$R_B = \sigma (T_a + 273)^4 \cdot (0.56 - 0.079\, e_a^{0.5}) \cdot (0.1 + 0.9\, nN^{-1}) \qquad (9)$$

where

- $R_B$ is net outgoing long-wave radiation ($J\ m^{-2}\ d^{-1}$)
- $\sigma$ is the Stefan Boltzman constant ($4.9 \times 10^{-3}\ J\ m^{-2}\ d^{-1}\ K^{-4}$)
- $T_a$ is mean air temperature at screen height in °C. The factor 273 is added to convert to absolute temperature.
- $e_a$ is actual vapour pressure of the air at screen height (mbar).
- $nN^{-1}$ is the ratio of actual and maximum hours of sunshine (Equation 8)

Again, Equation 9 is not intended for periods shorter than a week.

**Exercise 15**

$e_a$ is a measure of the amount of water vapour in the air. The unit mbar expresses this in the partial pressure of the water vapour. Conversion of this dimension into alternative ones may be obtained from the following expressions:

(a) 1 mbar = 100 Pa
where Pa is the abbreviation for Pascal, which is the SI unit for pressure.

(b) 1mbar = $T_k$ / 0.217 x $W_w$
where $W_w$ is the concentration of water vapour in the air in kg $m^{-3}$, and $T_k$ stands for the absolute temperature in K.

(c) 1 mbar = 0.75 mm Hg
where mm Hg stands for millimeters of mercury, which was used previously to express vapour pressure, among others in the original work of Penman (1948, 1956).

In Equation 9 the multiplication factor for $e_a$ equals 0.079 for $e_a$ expressed in mbar. What would be its value for $e_a$ expressed in kg $m^{-3}$ (20 °C), or in mm Hg?

**Exercise 16**

Meteorological station Don Muang mean monthly value of measured daily:

| | Temperature (°C) | Vapour pressure (mbar) |
|---|---|---|
| J | 26.0 | 22.5 |
| F | 27.4 | 25.6 |
| M | 28.9 | 27.9 |
| A | 29.8 | 29.8 |
| M | 29.3 | 30.2 |
| J | 28.7 | 29.2 |
| J | 28.2 | 28.7 |
| A | 28.0 | 30.2 |
| S | 28.2 | 29.8 |
| O | 28.1 | 29.3 |
| N | 27.4 | 27.0 |
| D | 25.6 | 22.7 |

Calculate the mean monthly outgoing longwave radiation ($R_B$)

Net radiation is given by:

$$R_N = R_I (1 - r_a) - R_B \qquad (10)$$

where

| | |
|---|---|
| $R_N$ | is net radiation ($J\ m^{-2}\ d^{-1}$) |
| $R_I$ | is incident shortwave radiation ($J\ m^{-2}\ d^{-1}$) |
| $r_a$ | is the albedo (dimensionless) |
| $R_B$ | is net outgoing longwave radiation ($J\ m^{-2}\ d^{-1}$) |

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**Exercise 17**
Calculate the mean monthly net radiation ($R_N$) for the Don Muang meteorological station for an open water surface.

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### *3.1.3 Evaporation*

Now the heat balance of a thin, extended water layer, that covers a black isolated floor is considered. The net radiation that is absorbed can be calculated from meteorological data (Equation 10) for an assumed albedo of 0.05. This net radiation ($R_N$) heats the water until the sensible heat loss to the surrounding air plus the heat loss due to the evaporation of water equals this net radiation, or:

$$R_N = H + LE \qquad (11)$$

where

| | |
|---|---|
| $R_N$ | is net radiation ($J\ m^{-2}\ d^{-1}$) |
| H | is sensible heat loss ($J\ m^{-2} d^{-1}$) |
| E | is the rate of water loss from the surface ($kg\ m^{-2}\ d^{-1}$) |
| L | is latent heat of vaporization of water ($2450 \times 10^3\ J\ kg^{-1}$) |
| LE | is the evaporative heat loss ($J\ m^{-2}\ d^{-1}$) |

The loss of sensible heat of a surface to its surroundings is proportional to the temperature difference, according to:

$$H = h_u (T_s - T_a) \qquad (12)$$

where

H is sensible heat loss ($J\ m^{-2}\ d^{-1}$)
$h_u$ is the sensible heat transfer coefficient ($J\ m^{-2}\ d^{-1}\ °C^{-1}$)
$T_s$ and
$T_a$ are the temperature of the evaporating surface and the temperature at standard screen height, respectively (°C)

The value of the sensible heat transfer coefficient, $h_u$, depends on the atmospheric turbulence and may be expressed as an empirically determined function of mean wind velocity at a defined height (Penman, 1948):

$$h_u = a_u (1 + b_u \bar{u}) \tag{13}$$

where
$\bar{u}$ is mean wind velocity ($m\ s^{-1}$)
$a_u$ and
$b_u$
are empirical constants.
For a smooth land surface and wind velocity measured at a standard height of 2 m, an indicative value for $a_u$ is $6.4 \times 10^5\ J\ m^{-2}\ d^{-1}\ °C^{-1}$ and for $b_u$, 0.54 $s\ m^{-1}$ (Frere & Popov, 1979).

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**Exercise 18**
Mean monthly value of measured daily wind velocity at Don Muang, measured at standard height (2 m) in $m\ s^{-1}$

| J | F | M | A | M | J | J | A | S | O | N | D |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.0 | 2.5 | 2.8 | 2.7 | 2.5 | 2.5 | 2.4 | 2.5 | 2.3 | 2.1 | 2.1 | 1.4 |

Calculate the mean monthly $h_u$ values.

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Analogous to the sensible heat, a water surface looses water vapour, in proportion to the vapour pressure difference between the surface and the surrounding air:

$$E = k_u (e_s - e_a) \tag{14}$$

where

E is the rate of water vapour loss in ($H_2O\ m^{-2}\ d^{-1}$)
$k_u$ is the vapour transfer coefficient ($kg\ m^{-2}\ d^{-1}\ mbar^{-1}$)
$e_s$ and
$e_a$ are the vapour pressure at the surface and at standard screen height,

respectively (mbar)

The air at the water surface is water vapour saturated. This saturated vapour pressure is related to the surface temperature $T_s$, a relation that may be approximated (Goudriaan, 1977) by:

$$e_s = 6.11 \times e^{(17.4T_s/(T_s + 239))} \tag{15}$$

where

$e_s$ is the saturated vapour pressure (mbar)
$T_s$ is the surface temperature (°C)

The exchange of sensible heat and that of evaporative heat are governed by the same physical processes of turbulence and diffusion. Therefore a relationship exists between the sensible heat transfer coefficient and the vapour transfer coefficient (Bowen's ratio), or

$$\gamma = h_u k_u^{-1} L^{-1} \tag{16}$$

where

$\gamma$ is the psychrometer constant. Its value is about 0.66

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**Exercise 19**
What is a psychrometer?
What is the dimension of $\gamma$?

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Substituting Equations 12, 14 and 15 in the balance Equation 11, yields:

$$R_N = h_u(T_s - T_a) + (h_u/\gamma) \cdot (e_s - e_a) \tag{17}$$

This equation contains two unknowns, the temperature of the water surface, $T_s$, and the saturated vapour pressure in the air at this surface, $e_s$. Both variables are related in the manner described in Equation 15. Hence, there are two equations with two unknowns, so that the surface temperature and the vapour pressure at the surface can be solved. This enables calculation of the evaporation of the water surface from Equation 14.

**Exercise 20**

- Calculate the evaporation rate at Don Muang in January for a thin extended water surface with an albedo of 0.05, using the data of Exercises 14, 17, and 18.
- Calculate for this purpose the sensible and evaporative heat loss for assumed water surface temperatures of 20, 21, 22, 23, and 24 °C. Plot the sum of both against these temperatures and draw in the same graph the net radiation. Read from the graph the equilibrium temperature, where the water surface neither gains nor looses heat. Calculate then the evaporation rate.

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The iterative procedure illustrated in Exercise 20 is rather cumbersome and can be avoided by linearizing the relation between temperature and saturated vapour pressure (Equation 15). This was first done by Penman (1948), who used the approximate relation:

$$(e_s - e_a) = \Delta(T_s - T_d) \tag{18}$$

where

$T_d$ is the dewpoint of the air, i.e. the temperature at which the vapour in the air would start to condense or, in other words, the temperature at which the actual vapour pressure in the air would be the saturated vapour pressure. The relation expressed in Equation 18 is depicted in Figure 22.

$\Delta$ is the slope of the saturation vapour pressure curve between the average air temperature and dewpoint.

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**Exercise 21**

What is the dewpoint of air with a vapour pressure of 10, 15, 50 mbar?

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**Exercise 22**

What was the value of $\Delta$ in Don Muang in January?
Calculate the value of $\Delta$ at temperatures of 2, 6, 10 .....30, 34 and 38 °C. Plot the values of $\Delta$ versus temperature.

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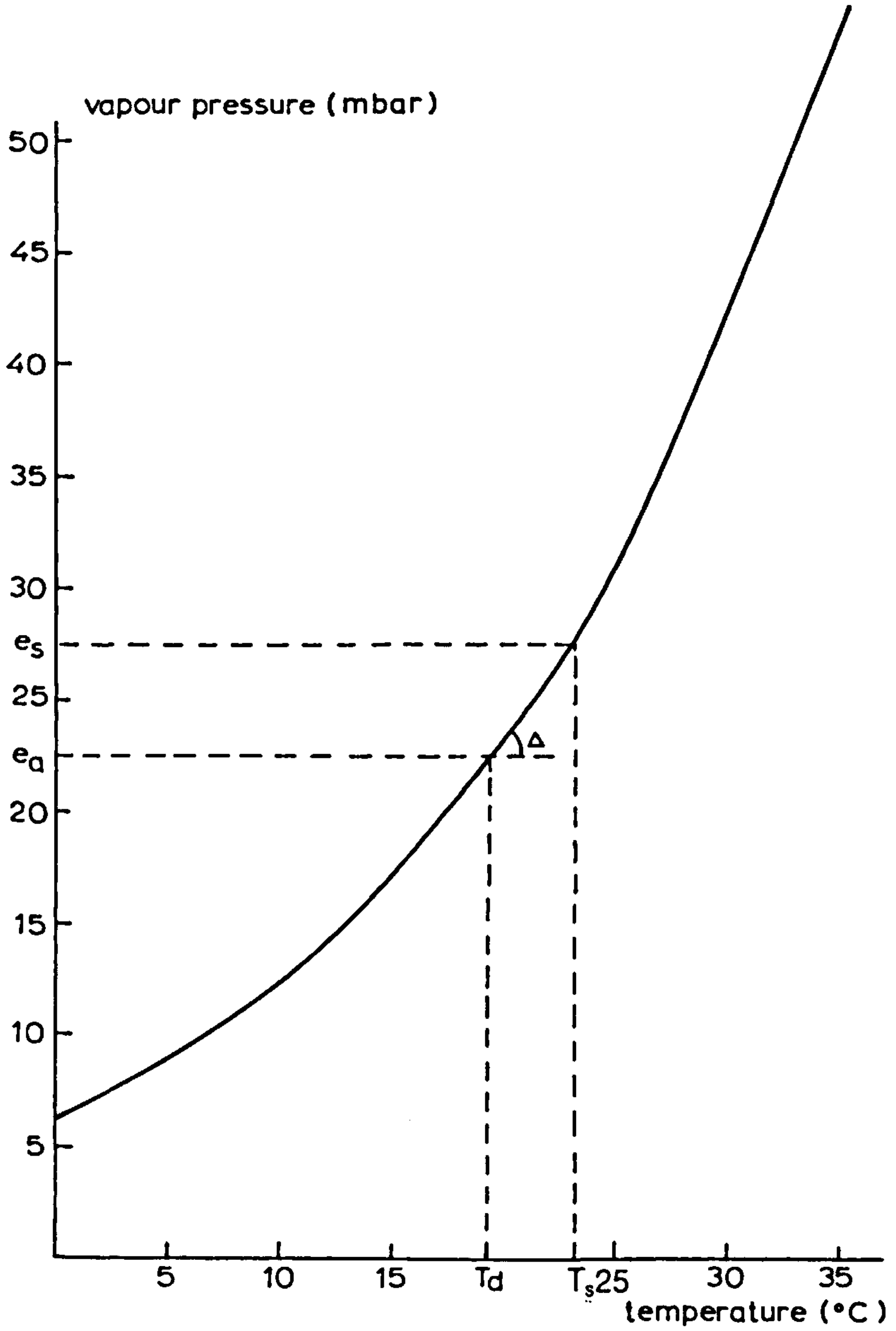


Figure 22. The relation between temperature and saturated vapour pressure. $T_a$ and $T_d$ represent air temperature and dewpoint, respectively; $e_s$ and $e_a$ are the saturated and actual vapour pressure in the atmosphere, respectively.

Substituting Equation 18 into Equation 17 gives:

$$R_N = h_u (T_s - T_a) + (h_u \Delta/\gamma)(T_s - T_d) \quad (19)$$

In this equation the surface temperature, $T_s$, is the only unknown and can be made explicit:

$$T_s = T_a + \frac{\gamma}{\Delta + \gamma} R_N/h_u - \frac{\Delta}{\Delta + \gamma}(T_a - T_d) \quad (20)$$

**Exercise 23**
Verify that the psychrometric constant, $\gamma$, and the slope of the saturated vapour pressure curve, $\Delta$, have the same dimension and that $R_N/h_u$ has the dimension of °C.

**Exercise 24**
Under what conditions is the temperature of the surface equal to the temperature of the air? When is it certainly higher and when is it certainly lower?
Derive an equation for the relation between the wet bulb temperature and dewpoint for a psychrometer that is well – shielded from radiation.

Combining Equations 11, 12 and 20 results in:

$$LE = \frac{\Delta}{\Delta + \gamma} (R_N + h_u (T_a - T_d)) \tag{21}$$

This is the well – known Penman equation for the calculation of evaporation from a free water surface. $(T_a - T_d)$ in this equation may be replaced by $(e_d - e_a)/\Delta$ in which $e_d$ is the saturation vapour pressure at air temperature. Then the Penman equation can be written as:

$$LE = \frac{1}{\Delta + \gamma} (\Delta R_N + h_u(e_d - e_a)) \tag{22}$$

**Exercise 25**
– Calculate the evaporation rate from a free water surface at Don Muang for the months January, March, May, July, September and November.

### *3.1.4. Potential evapotranspiration*

Plant leaves are protected against water loss by a cuticular layer that is almost impermeable to water. However, there are numerous small stomata (see Section 3.3) in these leaves to enable the entrance of carbon dioxide. The walls of the substomatal cavities are wet, therefore the air inside the stomata is saturated with water vapour. In comparison to evaporation, this water vapour has to overcome an additional resistance to move out of the cavity to the surface of the leaves. Therefore the transpiration of an extended single leaf surface is smaller than that of an extended surface of water. On the other hand, the exchange surface is larger for a canopy with an LAI higher than one. Taking these effects into account, Monteith (1965) has modified the original equation for the evaporation of a free water surface for the description of canopy transpiration. It is beyond the scope of this book to go into the details of his theory, but the result is an equation that very closely resembles Equation 22:

$$LE = \frac{1}{((\Delta + \gamma) \cdot (1/h_u + 1/C_s)/1/h_u)} \cdot (\Delta R_N + h_u(e_d - e_a)) \qquad (23)$$

where

$C_s$ is conductance for water vapour, expressed in the same units as $h_u$.

In the case of a crop canopy, $C_s$ represents the conductance of a large number of leaves placed in parallel and is referred to as the surface conductance. For a well-watered crop, i.e. under conditions of potential transpiration, $C_s$ appears much larger than $h_u$. The influence of the correction factor in Equation 23 is therefore rather small. For practical purposes, therefore, the Penman equation in its original form appears to be a useful estimate of transpiration losses by crops. In the case of water shortage, the closure of stomata is reflected in a decrease of the canopy conductance $C_s$.

However, the difference in albedo between a water surface and a green crop surface, which is 0.05 for the first and roughly 0.25 for the latter, has to be taken into account. The water loss that is calculated in this way is referred to as the potential evapo-transpiration of a closed, short green crop surface well supplied with water. The prefix 'evapo' is used, because no distinction is made between water loss by transpiration from the leaves and that by evaporation from the wet soil surface under the crop.

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**Exercise 26**

- Calculate the evapo-transpiration at Don Muang for the months January, March, May, July, September and November.

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