3.4 A simple model of water-limited production

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3.4.1 Introduction

In the preceding sections the water balance of the soil, which determines the amount of water available for uptake by the plant, and the relation between water use and dry-matter production of crops have been treated. In this section that information will be used to present a calculation method for estimating crop yields under conditions where, at times during the growing season, water may be a limiting factor. The calculation procedure also yields information on the degree of water shortage for the crop during different periods. Such information gives an indication for the amount of irrigation water necessary to achieve potential production and for the timing of application of supplemental irrigation.

The calculation method is in principle identical to the one presented in Section 2.3, i.e. repetitive calculations are performed starting from a chosen point in time, where the state of the system can be described in quantitative terms. The state of the system, in this case, is not only defined by crop variables, but also by variables describing soil-moisture status. At the onset of the calculations this information must be available. The methodology is in first instance illustrated for a wheat crop growing in a semi-arid environment, for which detailed data on crop, site and weather conditions were available (Hochman, 1978).

3.4.2 Experimental details

The wheat crop was grown in an experiment to study the effects of water shortage during specific growth stages on crop performance and yield, as part of a research project on actual and potential production of semi-arid regions. The experiment was carried out in the central Negev desert of Israel (30° N, 34° E). The soil there is a uniform gray desert soil, of eolian origin, with a loamy sand texture. Field capacity, determined in the field, 48 hours after irrigation, is 0.225 $cm^3\ cm^{-3}$; wilting point, determined in the laboratory is 0.09 $cm^3\ cm^{-3}$; and total pore space 0.40 $cm^3\ cm^{-3}$.

Exercise 45

What is the maximum quantity of soil moisture, available for uptake by the crop (Equation 52)? If the soil is at wilting point at the end of the dry season and 25 mm of water is added, what is the wetting depth? How much water must be added to restore the potential rooting zone of 150 cm to field capacity?

Spring wheat, cv. Lachish, was sown on 30 October 1977; germination was completed in about 10 days. The crop was amply supplied with nutrients (nitrogen at 150 kg ha^{-1}, phosphorus at 90 kg ha^{-1} and potassium at 100 kg ha^{-1}). Soil-moisture measurements at emergence showed that the volumetric soil-moisture content in the deeper soil layers was 0.16 $cm^3\ cm^{-3}$, due to residual moisture from a previously irrigated crop. The top soil (10 cm) was wetted to field capacity just prior to sowing to ensure proper germination. In the control treatment, sufficient moisture was applied for optimum supply, i.e. the root zone was irrigated to field capacity whenever available moisture as determined by gravimetric sampling fell below 30% of its maximum value. Rooting depth was determined at two-weekly intervals by examining soil cores. In the treatment illustrated in this example, water stress was allowed to develop between maximum tillering and anthesis by withholding irrigation.

Meteorological data, i.e. daily minimum and maximum temperatures, wet and dry bulb temperatures and wind run were recorded at a standard meteorological station, about 2 km from the experimental field. Rainfall was recorded at the site. Daily total global radiation was obtained from a meteorological station about 9 km from the experimental area. These data were used to

Table 40. Relevant input data for the wheat experiment

No of 10 day period	T_a (°C)	P (mm)	I_e (mm)	F_{gs} (kg $ha^{-1}d^{-1}$)	ET_0 (mm d^{-1})
1	15.2	0	0	251	1.85
2	12.9	0.1	0	257	1.76
3	10.4	7.4	0	225	1.47
4	8.4	19.6	0	263	1.14
5	8.8	1.6	0	257	1.23
6	7.9	0.2	0	258	1.41
7	12.4	0	0	260	2.22
8	11.5	0	0	297	2.27
9	12.3	0	0	338	2.87
10	11.7	2.2	81	344	2.72
11	15.9	0.1	96	402	3.65
12	13.6	3.9	0	374	4.25
13	13.2	0	0	437	4.40
14	17.0	4.2	60	464	5.26
15	19.6	0	0	457	5.56

calculate ten-day averages of air temperature, rainfall and potential gross CO_2 assimilation (Table 40). Total evaporative demand of the atmosphere is calculated on the basis of weather data, applying Penman's equation (Section 3.1). These data too are given in Table 40.

3.4.3 The actual calculation procedure

The calculation procedure is illustrated in Table 41, the bracketed letters in the text referring to the various lines of the table. Because this example refers to a semi-arid region with perma-dry conditions, i.e. without a groundwater table affecting moisture content in the rooting zone, capillary rise is absent. The values of the hydraulic conductivity are so low that transport from the root zone to deeper layers may safely be neglected, hence unsaturated flow above the groundwater table (cf. (CR – D) in Section 3.2) is neglected.

Line a: the first line of Table 41 specifies the initial conditions, i.e. those existing at the onset of the calculations. For wheat, the moment of emergence is chosen as the starting point. This moment is assumed to coincide with the transition from the situation where the seedlings grow from the reserves contained in the seed, to one where current assimilation provides the substrate for growth. At emergence the rooting depth of the seedlings equals 100 mm (Line a, Column 6). The total amount of moisture in the rooted depth (W_r, Column 17) is 22.5 mm of water. Hence, the volumetric moisture content in the root zone is (Column 18):

$$SM_r = 22.5/100 = 0.225 \text{ cm}^3 \text{ cm}^{-3}$$

In Column 19, the matric head, S_r, in the rooting zone is calculated, applying Equation 27, solved for S:

$$S_r = \exp\left(\sqrt{(-\ln(SM_r/SM_0)/\gamma)}\right) \qquad (67)$$

exp stands here for e, the base of the natural logarithm, the value in brackets denoting the power; ln stands for natural logarithm.

For the soil in this experiment a value of 0.0189 for γ was adapted on the basis of measurements, hence:

$$S_r = \exp\left(\sqrt{(-\ln(0.225/0.40)/0.0189)}\right) = 250 \text{ cm}$$

From this, the average hydraulic conductivity in the root zone, k_r, can be calculated with Equation 31, given in Section 3.2:

Table 41. Calculation scheme for water-limited production

1		2	3	4	5	6	7	8	9	10
Period		T_a	TSUM	DVS	PGASS	RD	ET_0	P	I_e	IM
a	0					100				
b	1	15.2	152	0.14	15	220	1.85	0.0	0.	0.0
c	2	12.9	281	0.25	30.8	340	1.76	0.1	0.	0.01
d	3	10.4	385	0.35	47.3	460	1.47	7.4	0.	0.74
e	4	8.4	469	0.43	78.9	580	1.14	19.6	0.	1.96
f	5	8.8	557	0.51	123.4	700	1.23	1.6	0.	0.16
g	6	7.9	636	0.58	170.3	820	1.41	0.2	0.	0.02
h	7	12.4	760	0.69	205.4	940	2.22	0.0	0.	0.0
i	8	11.5	875	0.80	255.4	1060	2.27	0.0	0.	0.0
j	9	12.3	998	0.91	300.8	1180	2.87	0.0	0.	0.0
k	10a	11.7	1103	1.0	282.1	1288	2.72	2.2	81.	9.24
l	10b	11.7	1115	1.02	251.0	1288	2.72	0.0	0.	0.0
m	11	15.9	1274	1.27	293.5	1288	3.65	0.1	96.	9.61
n	12	13.6	1410	1.48	235.6	1288	4.25	3.9	0.	0.39
o	13	13.2	1542	1.68	244.7	1288	4.40	0.0	0.	0.0
p	14	17.0	1712	1.94	222.7	1288	5.26	4.2	60.	6.42
q	15	19.6	1750	2.0	187.0	1288	5.56	0.0	0.	0.0

Table 41. (continued)

1		11	12	13	14	15	16	17	18	19	20
Period		E_m	E_a	T_m	T	dM_r	dW_r	W_r	SM_r	S_r	k_r
a	0							22.5	0.225	250	0.01
b	1	1.74	1.74	0.11	0.11	1.91	0.06	23.1	0.105	4640	0.0002
c	2	1.55	0.59	0.21	0.17	1.91	1.16	34.7	0.102	4929	0.00015
d	3	1.16	0.43	0.31	0.22	1.91	2.00	54.7	0.119	2997	0.0003
e	4	0.80	0.36	0.34	0.34	1.91	3.17	86.4	0.149	1379	0.0009
f	5	0.64	0.39	0.59	0.59	1.91	1.09	97.3	0.139	1769	0.0006
g	6	0.48	0.27	0.92	0.92	1.91	0.74	104.7	0.128	2375	0.0004
h	7	0.47	0.24	1.75	1.75	1.91	−0.08	103.9	0.110	3884	0.0002
i	8	0.32	0.13	1.94	1.69	1.91	0.09	104.8	0.098	5578	0.0001
j	9	0.32	0.11	2.55	0.64	1.91	1.16	116.4	0.098	5578	0.0001
k	10a	0.30	0.10	2.22	0.57	1.91	10.48	210.7	0.163	976	0.0015
l	10b	0.30	0.20	1.99	1.99	0.0	−2.19	208.5	0.162	1024	0.001
m	11	0.40	0.27	2.67	2.67	0.0	6.67	275.2	0.214	319	0.007
n	12	0.47	0.44	2.69	2.69	0.0	−2.74	247.8	0.192	532	0.003
o	13	0.48	0.40	2.47	2.47	0.0	−2.87	219.1	0.170	831	0.002
p	14	0.58	0.42	2.51	2.51	0.0	3.49	254.0	0.197	453	0.004
q	15	0.61	0.52	2.29	2.29	0.0	−2.81	248.4	0.193	497	0.004

Table 41. (continued)

1		21	22	23	24	25	26	27	28	29	30
Period		W_{nr}	GASS	MRES	ASAG	DMI	FR	IWRT	WRT	FL	IWLV
a	0	222.5							50		
b	1	203.4	15.0	1.5	13.5	9.5	0.44	4.2	92	0.56	5.3
c	2	184.3	24.9	2.9	22.0	15.4	0.33	5.1	143	0.67	10.3
d	3	165.2	33.6	5.1	28.5	20.0	0.26	5.2	195	0.80	12.0
e	4	146.1	78.9	7.9	71.0	49.7	0.205	10.2	297	0.53	26.3
f	5	127.0	123.4	15.3	108.1	75.7	0.16	12.1	418	0.46	34.8
g	6	108.9	170.3	26.7	143.6	100.5	0.12	12.1	539	0.39	39.2
h	7	88.8	205.4	41.8	163.6	114.5	0.09	10.3	642	0.30	34.4
i	8	69.7	222.5	59.0	163.5	114.5	0.055	6.3	705	0.18	20.6
j	9	50.6	75.5	75.2	0.3	0.2	0.03	0.2	707	0.08	0.0
k	10a	33.4	73.0	69.1	3.9	2.7	0.01	0.0	707	0.02	0.0
l	10b	33.4	251.0	43.6	207.4	165.9	0.0	0.0	707	0.0	0.0
m	11	33.4	293.5	45.0	248.5	198.8	0.0	0.0	707	0.0	0.0
n	12	33.4	235.6	62.7	172.9	138.3	0.0	0.0	707	0.0	0.0
o	13	33.4	244.7	74.8	169.9	135.9	0.0	0.0	707	0.0	0.0
p	14	33.4	222.7	87.0	135.7	108.6	0.0	0.0	707	0.0	0.0
q	15	33.4	187.0	96.8	90.2	72.2	0.0	0.0	707	0.0	0.0

Table 41. (continued)

1		31	32	33	34	35	36	37	38	39	40
Period		DWLV	WLV	FS	IWST	WST	FG	IWGR	WGR	LAI	TADW
a	0		50							0.1	50
b	1	0.0	103	0.0	0.0	0.0	0.0	0.0	0.0	0.21	103
c	2	0.6	200	0.0	0.0	0.0	0.0	0.0	0.0	0.40	206
d	3	1.7	303	0.14	2.8	28	0.0	0.0	0.0	0.61	354
e	4	0.0	566	0.265	13.2	160	0.0	0.0	0.0	1.1	721
f	5	0.0	914	0.38	28.8	448	0.0	0.0	0.0	1.8	1385
g	6	0.0	1306	0.49	49.2	940	0.0	0.0	0.0	2.6	2269
h	7	0.0	1650	0.61	69.8	1638	0.0	0.0	0.0	3.3	3311
i	8	6.4	1792	0.765	87.6	2514	0.0	0.0	0.0	3.6	4393
j	9	40.3	1389	0.61	0.0	2514	0.28	0.0	0.0	2.8	4395
k	10a	30.9	1111	0.19	0.5	2519	0.78	2.2	22	2.2	4420
l	10b	22.2	1089	0.0	0.0	2519	1.0	165.9	188	2.2	4586
m	11	21.8	871	0.0	0.0	2519	1.0	198.8	2176	1.7	6574
n	12	17.4	697	0.0	0.0	2519	1.0	138.3	3559	1.4	7957
o	13	13.9	558	0.0	0.0	2519	1.0	135.9	4918	1.1	9316
p	14	11.2	446	0.0	0.0	2519	1.0	108.6	6004	0.9	10402
q	15	8.9	428	0.0	0.0	2519	1.0	72.2	6148	0.8	10546

Table 41. (continued)

1		41	42	43
Period		TWD	TWDL	TDWD
a	0	100	100	
b	1	195	195	
c	2	349	343	6
d	3	549	526	23
e	4	1046	1023	23
f	5	1803	1780	23
g	6	2808	2785	23
h	7	3953	3930	23
i	8	5098	5011	87
j	9	5100	4610	490
k	10a	5127	4359	768
l	10b	5293	4503	790
m	11	7281	6273	1008
n	12	8664	7482	1182
o	13	10023	8702	1321
p	14	11309	9676	1433
q	15	11253	9802	1451

$$k_r = a \cdot (S_r)^{-1.4} \qquad (68)$$

The value of 'a' appeared to be equal to 22.6, hence:

$$k_r = 22.6 \times (250)^{-1.4} = 0.01 \text{ mm d}^{-1} \text{ (Column 20)}$$

The amount of moisture in the soil between the actual rooting depth and the potential rooting depth (in this case 1500 mm) is calculated as a separate state variable, W_{nr}, because this moisture becomes gradually available to the plants when the roots grow deeper. At emergence, the total amount in that zone equals 222.5 mm (Column 21), which is assumed to be evenly distributed.
At emergence, the dry weight of the root system, WRT, equals 50 kg ha^{-1} (Column 28) and that of the leaf blades, WLV, also 50 kg ha^{-1} (Column 32). From the latter value, the leaf area index, LAI, is calculated, assuming a constant specific leaf area of 20 m^2 kg^{-1}, hence:

$$LAI = 50 \times 20 \times 10^{-4} = 0.1 \text{ m}^2 \text{ m}^{-2} \text{ (Column 39)}$$

As in Section 2.3, four auxiliary state variables are calculated: the total amount of above-ground dry weight, TADW (Column 40), which is here equal to the weight of the leaf blades, the only above-ground organ present;

the total plant dry weight, TDW (Column 41), equal to the sum of leaf blades and roots (= 100 kg ha^{-1}). Finally the latter value is subdivided into live plant tissue, TDWL (Column 42) and dead plant tissue, TDWD (Column 43).

Line b: this line covers the first ten – day period of the growing period. Average air temperature during the period, T_a, equals 15.2 °C (Column 2). Integration over the ten – day period yields a temperature sum, TSUM, of 152 d °C (Column 3). The temperature sum is used to define the phenological development stage of the crop (Sections 2.2; 2.3). For spring wheat the temperature requirement between emergence and anthesis is 1100 d °C (van Keulen & Seligman, 1986). The development stage of the crop is thus calculated as the ratio between the accumulated temperature sum and 1100:

DVS = 152/1100 = 0.14 (Column 4)

Potential daily gross assimilation for the period, expressed in CH_2O is read from Table 40 as 251 kg ha^{-1} d^{-1}. The reduction factor for incomplete light interception, RA, is read from Table 42. For LAI = 0.1 the reduction factor equals 0.06. Potential gross assimilation in the absence of water stress is calculated as:

PGASS = 251 x 0.06 = 15 kg ha^{-1} d^{-1} (Column 5)

It is assumed on the basis of the experimental results that root extension proceeds at a rate of 12 mm d^{-1}, if the soil in which the roots grow is sufficiently wet. In this case the total potential rooting zone is wetted, thus that

Table 42. Reduction factor for gross Co_2 assimilation due to incomplete radiation interception ($f_h = 1 - e^{-0.6.LAI}$)

LAI	Reduction factor
0	0
0.25	0.14
0.5	0.26
1.0	0.45
1.5	0.59
2.0	0.70
2.5	0.78
3.0	0.84
3.5	0.88
4.0	0.91
5.0	0.95

condition is satisfied throughout. The rooted depth at the end of the ten – day period is 220 mm (Column 6).

Maximum evapotranspiration during the ten – day period, ET_0, is read from Table 40 and introduced in Column 7. For the present period it amounts to 1.85 mm d^{-1}. In the next columns the inputs of moisture into the system are defined. For this ten – day period, both precipitation (P, Column 8) and irrigation (I_e, Column 9) are absent, hence total infiltration (IM, Column 10) is also zero. Under the conditions prevailing in the area, loss of water by surface run – off has not been observed, hence in all cases total infiltration equals the sum of precipitation and irrigation. Maximum evaporation from the soil surface is derived from potential evapotranspiration, taking into account the reduction factor for the shading effect of the vegetation:

$$E_m = ET_0 \cdot (1 - RA) \qquad (69)$$

It is thus the complementary fraction of light interception by the vegetation. Hence:

$$E_m = 1.85 \times 0.94 = 1.74 \text{ mm d}^{-1} \text{ (Column 11)}$$

Actual evaporation is subsequently obtained from the average moisture content in the root zone. If the soil – moisture content is too low, the supply of water to the surface cannot meet the evaporative demand and actual evaporation falls short of the maximum. A detailed treatment of the process of soil evaporation is given by van Keulen (1975). In the present approach it is approximated by assuming a linear decline in evaporation rate between field capacity and air – dry soil, where evaporation ceases completely (Section 3.2). The moisture content of air – dry soil (SM_a), is estimated as one third of that at wilting point. For this soil it is 0.03 $cm^3\ cm^{-3}$. Hence:

$$E_a = E_m \cdot (SM_r - SM_a)/(SM_{fc} - SM_a) \qquad (70)$$

$$= 1.74 \times (0.225 - 0.03)/(0.225 - 0.03)$$
$$= 1.74 \times 1 = 1.74 \text{ mm d}^{-1}$$

Potential transpiration could, in principle, be obtained from potential gross assimilation and the transpiration coefficient, as outlined in Section 3.3. For most field situations it appears that potential evapotranspiration calculated according to Penman is a reasonable approximation of potential transpiration for a closed crop surface, apparently because stomatal control by CO_2 is the most common behaviour in field crops. Therefore 'Penman' (Column 7) is used as the basis for potential transpiration. To calculate maximum transpiration for crops with incomplete soil cover, the same reduction factor, RA, is used as for gross assimilation. Hence:

$T_m = ET_0 \cdot RA = 1.85 \times 0.06 = 0.11 \text{ mm d}^{-1}$ (Column 13)

Actual transpiration is only equal to the maximum value, if soil moisture is adequate to supply sufficient water to the plant roots. As explained in Section 3.2, a crop–specific critical soil–moisture content exists, above which roots can freely take up water from the soil. The critical soil–moisture content is determined by both the physical properties of the soil and the evaporative demand of the atmosphere. It is approximated by:

$$SM_{cr} = (1 - p) \cdot (SM_{fc} - SM_w) + SM_w \qquad (71)$$

where

- SM_{cr} is critical soil – moisture content ($cm^3 \, cm^{-3}$)
- p is soil water depletion fraction (Table 20)
- SM_{fc} is soil – moisture content at field capacity ($cm^3 \, cm^{-3}$)
- SM_w is soil – moisture content at wilting point ($cm^3 \, cm^{-3}$)

For potential transpiration equal to 1.85 mm d^{-1}, the value of p equals 0.86, hence:

$$SM_{cr} = 0.14 \times (0.225 - 0.09) + 0.09 = 0.109 \, cm^3 \, cm^{-3}$$

As the actual moisture content is well above the critical value, actual transpiration (T, Column 14) equals maximum transpiration.

Another 'input' into the plant–soil system is the amount of moisture that becomes available to the vegetation as a result of vertical extension of the root system. In the present approach, this is approximated by assuming that the total amount of moisture in that part of the potential rooting zone, where the roots have not yet penetrated, is evenly distributed. The amount, added to the plant–soil system as a result of root growth, is then calculated as:

$$dM_r = W_{nr} \cdot R_r / D_{nr} \qquad (72)$$

where

- dM_r is amount of moisture added to the soil plant system by root growth (mm d^{-1})
- W_{nr} is total amount of moisture in the non – rooted part of the potential rooting zone (mm)
- R_r is growth rate of the roots (mm d^{-1})
- D_{nr} is thickness of the non – rooted part of the potential rooting zone (mm), hence: $D_{rm} - D_r$, with D_{rm} potential rooting depth and D_r the rooting depth (mm)

On the basis of Equation 72, the increase in available soil moisture by root growth is calculated as:

$dM_r = 222.5 \times 12/(1500 - 100) = 1.91 \text{ mm d}^{-1}$ (Column 15)

All processes influencing the water balance have been treated now, so that their effect on the rate of change in soil – moisture status of the root zone can be evaluated:

$$dW_r = IM + dM_r - E_a - T \qquad (73)$$

Hence:

$dW_r = (0 + 1.91 - 1.74 - 0.11) = 0.06 \text{ mm d}^{-1}$ (Column 16)

Total soil moisture in the root zone at the end of the ten – day period equals the value at the beginning (Column 17, line a) plus the rate of change multiplied by the length of the time interval:

$W_r = 22.5 + 0.06 \times 10 = 23.1$ mm (Column 17)

From the amount of moisture in the root zone, the average volumetric soil-moisture content (Column 18) is calculated, assuming it to be homogeneous. Hence:

$$SM_r = W_r/D_r \qquad (74)$$

where

SM_r is volumetric soil – moisture content in the root zone ($cm^3\ cm^{-3}$)
W_r is soil – moisture content in the root zone (mm)
D_r is rooting depth (mm)

For the present ten – day period:

$SM_r = 23.1/220 = 0.105 \text{ cm}^3 \text{ cm}^{-3}$

The values for the matric head in the root zone, S_r, and the average hydraulic conductivity, k_r, are calculated according to Equations 67 and 68. These variables are included here for completeness sake, but they are not used, because unsaturated flow above the groundwater table can be neglected in this situation. In the remainder of this example Columns 19 and 20 will therefore not be treated.

The total amount of moisture in the non–rooted zone of the profile at the end of the period, is obtained by subtracting dM_r times Δt (Column 15) from the amount at the beginning of the decade (Line a, Column 21):

$$W_{nr} = 222.5 - 1.91 \times 10 = 203.4 \text{ mm (Column 21)}$$

In Column 22 actual gross assimilation, AGASS, is introduced. For this ten-day period it equals potential gross assimilation (Column 5) as there is no reduction due to water shortage, i.e. actual transpiration is equal to the maximum value. Maintenance respiration, i.e. the amount of energy invested in maintaining the existing cells and their structures, is calculated from the total live dry weight of the crop and the relative maintenance respiration rate, R_m. During the pre–anthesis phase of the crop the latter value is set at 0.015 kg CH_2O per kg dry matter per day, hence:

$$MRES = 0.015 \times 100 = 1.5 \text{ kg ha}^{-1} \text{ d}^{-1} \text{ (Column 23)}$$

The amount of assimilates available for increase in dry weight of the vegetation is the difference between actual gross assimilation and maintenance respiration:

$$ASAG = 15 - 1.5 = 13.5 \text{ kg ha}^{-1} \text{ d}^{-1} \text{ (Column 24)}$$

The rate of increase in dry weight of the vegetation is obtained from the amount of available assimilates, taking into account the losses associated with the conversion of primary photosynthates into structural plant material (growth respiration). For vegetative material of average composition, these losses amount to about 30% of the consumed carbohydrates (Penning de Vries, 1975). Thus:

$$DMI = 0.7 \times ASAG = 0.7 \times 13.5 = 9.5 \text{ kg ha}^{-1} \text{ d}^{-1} \text{ (Column 25)}$$

The assimilates are used to produce various plant organs at the same time (Sections 2.2 and 2.3). The partitioning of dry matter among the various organs is predominantly governed by the phenological stage of the crop. At the production level treated in this section, where at times production may be limited by moisture availability, the development of moisture stress in the plant may influence the partitioning pattern, according to the functional balance principle (Brouwer, 1963). The exact influence of water stress in the plant on the distribution of assimilates is difficult to quantify, however. Therefore, this effect has been neglected in the present approach.
The relation between phenological stage of the crop and the traction of assimilates diverted to the various organs is given in Table 43. The independent variable determining the fraction of assimilates partitioned to the root system

Table 43. Partitioning factors for dry matter to various plant organs as a function of development stage.

Development stage	f_r	f_l	f_s	f_g
0	0.50	0.50	0.0	0.0
0.1	0.42	0.58	0.0	0.0
0.2	0.33	0.67	0.0	0.0
0.3	0.26	0.60	0.14	0.0
0.4	0.20	0.52	0.28	0.0
0.5	0.15	0.43	0.42	0.0
0.6	0.10	0.34	0.56	0.0
0.7	0.07	0.23	0.70	0.0
0.8	0.04	0.12	0.84	0.0
0.9	0.02	0.06	0.42	0.50
1.0	0.0	0.0	0.0	1.0

(Column 26) is the development stage of the crop. For each ten – day period the relevant value is found as the average of the values at the end and at the beginning, as in Section 2.3. For the present period:

$$DVS = (0 + 0.14)/2 = 0.07$$

In Table 43 it is found that the fraction of dry matter partitioned to the root system at that value of DVS equals 0.44 (Column 26). The rate of increase in dry weight of the root system is thus:

$$IWRT = FR \cdot DMI = 0.44 \times 9.5 = 4.2 \text{ kg ha}^{-1} \text{ d}^{-1} \text{ (Column 27)}$$

The total dry weight of the root system at the end of the ten – day period is equal to its value at the beginning of the period (Line a, Column 28), augmented with the rate of increase times the length of the time interval, hence:

$$WRT = 50 + (4.2 \times 10) = 92 \text{ kg ha}^{-1} \text{ (Column 28)}$$

The fraction allocated to the leaf blades is also obtained from Table 43 at DVS is 0.07 and equals 0.56 (Column 29). The rate of increase in dry weight of the leaf blades follows from multiplication of this fraction with the rate of dry matter increase:

$$IWLV = FL \cdot DMI = 0.56 \times 9.5 = 5.3 \text{ kg ha}^{-1} \text{ d}^{-1} \text{ (Column 30)}$$

The weight of the leaf blades at the end of the period equals its value at the beginning plus the rate of increase multiplied by the length of the time interval:

$WLV = 50 + 5.3 \times 10 = 103$ kg ha^{-1} (Column 32)

In the early stages of crop development in spring wheat, only roots and leaf blades are produced (van Keulen & Seligman, 1986), thus the fraction allocated to the stems (FS, Column 33) is zero, and consequently the rate of increase in stem dry weight (IWST, Column 34) too. Total stem weight at the end of the period (WST, Column 35) is still zero. The crop is in its vegetative stage, hence no grain growth takes place (Columns 36, 37 and 38). The leaf area index at the end of the period is obtained from the dry weight of the leaf blades, taking into account the constant specific leaf area of 20 m^2 per kg dry matter and the surface area:

$LAI = 103 \times 20 \times 10^{-4} = 0.21$ m^2 m^{-2} (Column 39)

The total canopy variables are finally obtained from the weights of the various plant organs at the end of the period. Total above – ground dry weight is the sum of the weights of the above – ground organs, i.e. leaf blades, stems and grain:

$TADW = 103 + 0 + 0 = 103$ kg ha^{-1} (Column 40)

Total dry weight of the vegetation is equal to above ground dry weight, augmented with the weight of the root system:

$TDW = 103 + 92 = 195$ kg ha^{-1} (Column 41)

At this moment no dead tissue is present (Column 43), hence total live dry weight (Column 42) equals total dry weight. These calculations complete the treatment of the first ten – day period.

Line c: the calculations in this period are in principle equal to those in the previous one. Therefore, they will be treated in less detail.

Average daily air temperature during this ten – day period equals 12.9 °C (Column 2), so the accumulated temperature sum at the end of the period is 281 d °C (Column 3) and the associated development stage is equal to:

$DVS = 281 / 1100 = 0.25$ (Column 4)

Gross assimilation in the absence of water stress is obtained from the radiation determined value (Table 40) and the reduction factor for incomplete light interception, which is 0.12 at a LAI of 0.21 (Table 42). Hence:

$PGASS = 257 \times 0.12 = 30.8$ kg ha^{-1} d^{-1} (Column 5)

Root extension proceeds unhampered, thus the rooting depth at the end of the period equals 340 mm (Column 6). Potential evapotranspiration, obtained from Table 40, equals 1.76 mm d^{-1} (Column 7). Rainfall during the period is 0.1 mm. To convert this value into a daily rate, it must be divided by Δt. Hence P = 0.01 mm d^{-1} (Column 8). No irrigation was applied (Column 9), thus the rate of infiltration is equal to 0.01 mm d^{-1} (Column 10). Maximum soil evaporation is calculated on the basis of potential evapotranspiration, taking into account the reducing effect of partial shading by the vegetation:

$$E_m = ET_0 \cdot (1 - RA) = 1.76 \times 0.88 = 1.55 \text{ mm d}^{-1} \text{ (Column 11)}$$

The reduction factor for soil evaporation due to soil–moisture content follows from:

$$(SM_r - SM_a)/(SM_{fc} - SM_a) = (0.105 - 0.03)/(0.225 - 0.03) = 0.38$$

Actual soil evaporation equals E_m x 0.38, i.e. 0.59 mm d^{-1} (Column 12). Maximum transpiration is the complementary fraction of potential evapotranspiration and equals:

$$T_m = ET_0 \cdot RA = 1.76 \times 0.12 = 0.21 \text{ mm d}^{-1} \text{ (Column 13)}$$

For the determination of actual transpiration the critical soil–moisture content has to be calculated by Equation 71, substituting 0.86 for p (Table 20):

$$SM_{cr} = (0.14 \times 0.135) + 0.09 = 0.109 \text{ cm}^3 \text{ cm}^{-3}$$

Actual volumetric soil–moisture content is 0.105 cm^3 cm^{-3} (Line b, Column 18), which is below the critical value. The reduction in transpiration is calculated from:

$$T/T_m = (Sm_r - SM_w)/(SM_{cr} - SM_w) \qquad (75)$$

In this situation thus:

$$T = 0.21 \times (0.105 - 0.09)/(0.109 - 0.09) = 0.17 \text{ mm d}^{-1} \text{ (Column 14)}$$

To calculate the rate of water addition to the root zone as a result of root growth, Equation 72 is applied:

$$dM_r = 203.4 \times 12/(1500 - 220) = 1.91 \text{ mm d}^{-1} \text{ (Column 15)}$$

The rate of change in moisture content in the root zone follows from the balance:

$$dW_r = 0.01 + 1.91 - 0.59 - 0.17 = 1.16\,\mathrm{mm\,d^{-1}} \text{ (Column 16)}$$

Total soil moisture in the root zone at the end of the ten – day period equals:

$$W_r = 23.1 + (1.16 \times 10) = 34.7\,\mathrm{mm} \text{ (Column 17)}$$

The average volumetric moisture content at the end of the ten – day period is then total moisture content divided by the rooting depth:

$$SM_r = 34.7 / 340 = 0.102\,\mathrm{cm^3\,cm^{-3}} \text{ (Column 18)}$$

The moisture content in the non – rooted part of the profile equals:

$$W_{nr} = 203.4 - dM_r \cdot \Delta t = 203.4 - 19.1 = 184.3\,\mathrm{mm} \text{ (Column 21)}$$

Actual gross assimilation follows from the potential value, taking into account the reduction due to water shortage:

$$GASS = PGASS \cdot T/T_m = 30.8 \times 0.17/0.21 = 24.9\,\mathrm{kg\,ha^{-1}\,d^{-1}} \text{ (Column 22)}$$

The maintenance respiration rate is obtained from total live dry weight and the relative maintenance respiration rate:

$$MRES = 0.015 \times 195 = 2.9\,\mathrm{kg\,ha^{-1}\,d^{-1}} \text{ (Column 23)}$$

The amount of assimilates available for increase in dry matter is the balance between actual gross assimilation and maintenance respiration:

$$ASAG = 24.9 - 2.9 = 22.0\,\mathrm{kg\,ha^{-1}\,d^{-1}} \text{ (Column 24)}$$

The rate of dry – matter increase of the vegetation, taking into account growth respiration, equals:

$$DMI = 22.0 \times 0.7 = 15.4\,\mathrm{kg\,ha^{-1}\,d^{-1}} \text{ (Column 25)}$$

The fraction of dry – matter increase partitioned to the root system is obtained from Table 43 at a value of the development stage midway between the beginning and the end of the ten – day period, hence $(0.25 - 0.14)/2 + 0.14 = 0.195$. FR equals 0.33 at that value (Column 26) and the rate of increase in dry weight of the root system follows from:

$$IWRT = FR \cdot DMI = 0.33 \times 15.4 = 5.1\,\mathrm{kg\,ha^{-1}\,d^{-1}} \text{ (Column 27)}$$

and

$$WRT = 92 + 5.1 \times 10 = 143 \text{ kg ha}^{-1} \text{ (Column 28)}$$

The fraction allocated to the leaf blades also follows from Table 43 and equals 0.67 (Column 29), hence:

$$IWLV = 0.67 \times 15.4 = 10.3 \text{ kg ha}^{-1} \text{ d}^{-1} \text{ (Column 30)}$$

In the present approach it is assumed that leaf blades deteriorate when the vegetation suffers from water shortage, in an attempt to reduce transpirational losses. The rate of decline is proportional to the relative transpiration rate, T/T_m. At severe moisture stress, leaf blades are assumed to deteriorate at a maximum rate of 0.03 kg dry matter per kg leaf blade dry – matter present per day.

The rate of decline is thus calculated as:

$$DWLV = RDR \cdot WLV \qquad (76)$$

with:

$$RDR = 0.03 \cdot (1 - T/T_m) \qquad (77)$$

For the present ten – day period:

$$RDR = (1 - 0.17 / 0.21) \times 0.03 = 0.0057 \text{ d}^{-1}$$

and

$$DWLV = 0.0057 \times 103 = 0.6 \text{ kg ha}^{-1} \text{ d}^{-1} \text{ (Column 31)}$$

Leaf – blade dry weight at the end of the period equals:

$$WLV = 103 + (10.3 - 0.6) \times 10 = 200 \text{ kg ha}^{-1} \text{ (Column 32)}$$

At development stage 0.195, the fraction allocated to stem and grain is still zero, hence Columns 33 till 38 all contain zeros. The leaf area index at the end of the ten – day period follows from the dry weight of the leaf blades:

$$LAI = 200 \times 20 \times 10^{-4} = 0.40 \text{ m}^2 \text{ m}^{-2} \text{ (Column 39)}$$

Total above – ground dry matter equals the weight of the live leaf blades plus the weight of the dead leaves, hence:

$TADW = 200 + 6 = 206$ kg ha^{-1} (Column 40)

Total plant dry weight is obtained by adding root weight to the previous value:

$TDW = 206 + 143 = 349$ kg ha^{-1} (Column 41)

Total live dry weight equals 343 kg ha^{-1} (Column 42) and the amount of dead tissue is 6 kg ha^{-1} (Column 43). These calculations complete the treatment of the second ten – day period.

In the periods 3 to 9 no basically different processes take place, therefore these periods are not treated in detail here. From the 8th period onwards, a serious water shortage develops, because irrigation is purposely withheld in this treatment. The onset of moisture stress was somewhat delayed because of fairly heavy rainfall in the 4th period. The result of the water shortage is that growth practically ceases after the 8th period and that green leaf area declines drastically (Table 41).

Line k: this is the 10th ten – day period of the growing period (Column 1). Average air temperature during the period is 11.7 °C (Column 2). Temperature sum at the end of the period equals:

$TSUM = 998 + 117 = 1115$ d °C

and the associated development stage:

$DVS = 1115/1100 = 1.02$

That is a point beyond anthesis and, because the temperature relations for development are different before and after anthesis, this period is subdivided into two. The first part covers the period till anthesis, its duration being determined by the remainder of the required temperature sum:

$(1100 - 998)/11.7 = 9$ d

Hence, the temperature sum at the end of period 10a equals 1103 d °C and the development stage at the end of the period is 1.0, i.e. anthesis. Gross assimilation in the absence of moisture stress is calculated in the usual way and yields 254 kg ha^{-1} d^{-1} (Column 5). Root growth still proceeds and the rooting depth at the end of the period equals 1288 mm.

It should be noted here, that according to the partitioning factors shown in Table 43, the contribution to the roots is negligible during this period. However, if root extension proceeds, assimilates are necessary. It is likely therefore, that the moisture shortage, developing in this period, affected the distribution pattern to allow root growth. In the present schematized set – up, however, this effect is not taken into account, and the data of Table 43 are used indiscriminately. Potential evapotranspiration rate during the period equals 2.72 mm d^{-1}, showing that gradually time is moving towards a warmer and drier spring.

It was the intention to end the moisture stress at anthesis, therefore a heavy irrigation of 81 mm was applied during this period to restore the water content in the root zone to a value well above the critical level. As there was also a small rain shower, total infiltration amounted to 9.24 mm d^{-1} (Column 10).

For the calculation of the maximum rate of soil evaporation it is assumed that the dried leaves remain on the vegetation and act effectively in radiation interception. Thus, in calculating the shading effect, the maximum value of the leaf area index (LAI = 3.6) is applied. For this period:

$$E_m = ET_0 \cdot (1 - RA) = 2.72 \times 0.11 = 0.30 \text{ mm d}^{-1} \text{ (Column 11)}$$

Actual soil evaporation falls short of the maximum, because of the low moisture – content in the root zone, according to Equation 70:

$$E_a = E_m \cdot (0.098 - 0.03) / (0.225 - 0.03) = 0.10 \text{ mm d}^{-1} \text{ (Column 12)}$$

The maximum rate of transpiration is calculated in the usual way from the potential evapotranspiration rate and amounts to 2.22 mm d^{-1} (Column 13). The actual rate of transpiration is derived from the maximum value taking into account the moisture content in the root zone. The critical volumetric soil – moisture content follows from Equation 71 at 0.121 cm^3 cm^{-3}. The reduction in transpiration rate is obtained from Equation 75:

$$T = 2.22 \times (0.098 - 0.09)/(0.121 - 0.09) = 0.57 \text{ mm d}^{-1} \text{ (Column 14)}$$

The rate of increase of moisture in the root zone as a result of root growth amounts to 1.72 mm d^{-1} (Column 15). Total soil moisture in the root zone at the end of the period follows from realization of the calculated rates over the time interval of nine days and amounts to 210.7 mm. From this, the volumetric soil-moisture content in the root zone is calculated as 0.163 cm^3 cm^{-3} (Column 17). The actual assimilation rate amounts to 73 kg ha^{-1} d^{-1} after correcting for the transpiration deficit (Column 22). The rate of maintenance respiration is 69.1 kg ha^{-1} d^{-1}, calculated on the basis of total live dry weight of the vegetation (Column 23). The difference of 3.9 kg ha^{-1} d^{-1} (Column 24) allows for a rate of increase in dry matter of 2.7 kg ha^{-1} d^{-1} (Column 25). This amount is distributed among roots, leaf blades, stems and grain in accordance with the partitioning factors given in Table 43.

In this period, as in the preceding ten – day period, some of the assimilates are diverted to the grain, although anthesis has not yet been reached. This partitioning pattern has been chosen to account for the fact that during the last part of the pre-anthesis phase, a substantial part of the assimilates is stored in the stem as reserves, which after grain set are translocated to the growing grain (van Keulen & Seligman, 1986). In the present schematized calculation procedure these assimilates are designated grain weight directly. The remainder of this period follows the same rules as the ones that have been discussed in detail.

Line l: to follow through the ten-day subdivision, the one remaining day from the 10th period is treated in a separate line. After anthesis the development scale runs from 1 at anthesis to 2 at maturity, or in fact at the end of the grain filling period. The temperature requirement for that period is, for this and most other spring wheat varieties, 650 d °C (van Keulen & Seligman, 1986).
The development stage in the post-anthesis period is thus calculated as:

DVS = (TSUM − 1100)/650 + 1 (Column 4)

Extension growth of the root system is assumed to cease at anthesis, because no more assimilates are invested in growth of the root system (Column 26). Treatment of the Columns 6 till 22 is not essentially different from that detailed in the preceding lines, therefore the calculations are not repeated in detail at this point. A few remarks may serve to draw attention to specific points:

– the reduction factor for soil evaporation due to shading by the vegetation is kept constant at 0.11 (Column 11), corresponding to the maximum leaf area index of 3.6 $m^2\ m^{-2}$.
– after cessation of root extension, the 'input' of moisture into the root zone as a result of root growth remains zero (Column 15). Transport of moisture between the root zone and the non-rooted part of the profile, caused by developing potential gradients is neglected. Thus the soil moisture stored in the non-rooted zone remains constant (Column 21).
– the relative maintenance respiration rate is changed at anthesis from 0.015 $kg\ kg^{-1}\ d^{-1}$ to 0.01. The main reason for this change is that in general the nitrogen concentration in the vegetative material declines rapidly after anthesis, due to translocation of nutrients to the developing grain. Moreover, the dry matter accumulating in the grain requires less energy for maintenance. Hence, from anthesis onwards: MRES = 0.01 x TDWL
– the conversion efficiency of primary photosynthates into structural plant dry matter, set at 0.7 for vegetative material of average composition, is changed to 0.8 for grain dry matter. Again, the major reason for this change is the fact that the nitrogen concentration of the grain is lower than that of the vegetative material during the pre-anthesis phase.
– leaves have a limited life-time and even under optimum growing condi-

tions part of the earlier formed leaves deteriorate. This deterioration increases rapidly after anthesis, mainly because of the translocation of nitrogen and other essential growth substances to the developing grain. In the present model this process is taken into account by assuming a relative death rate of the leaf blades of 0.02 kg kg^{-1} d^{-1}.

Exercise 46
Calculate the weight of leaf blades 10 days after anthesis, starting from an initial value of 2000 kg ha^{-1}. Employ first the value of 0.02 kg kg^{-1} d^{-1} and a time step of one day. Apply subsequently the time step of the model.
What do you notice? What is the reason?

From Table 41 it follows, that under the prevailing environmental conditions, the post-anthesis phase lasts 43 days, the calculated grain yield amounts to 6.2 t ha^{-1} and total above-ground dry matter to 10.5 t ha^{-1}.

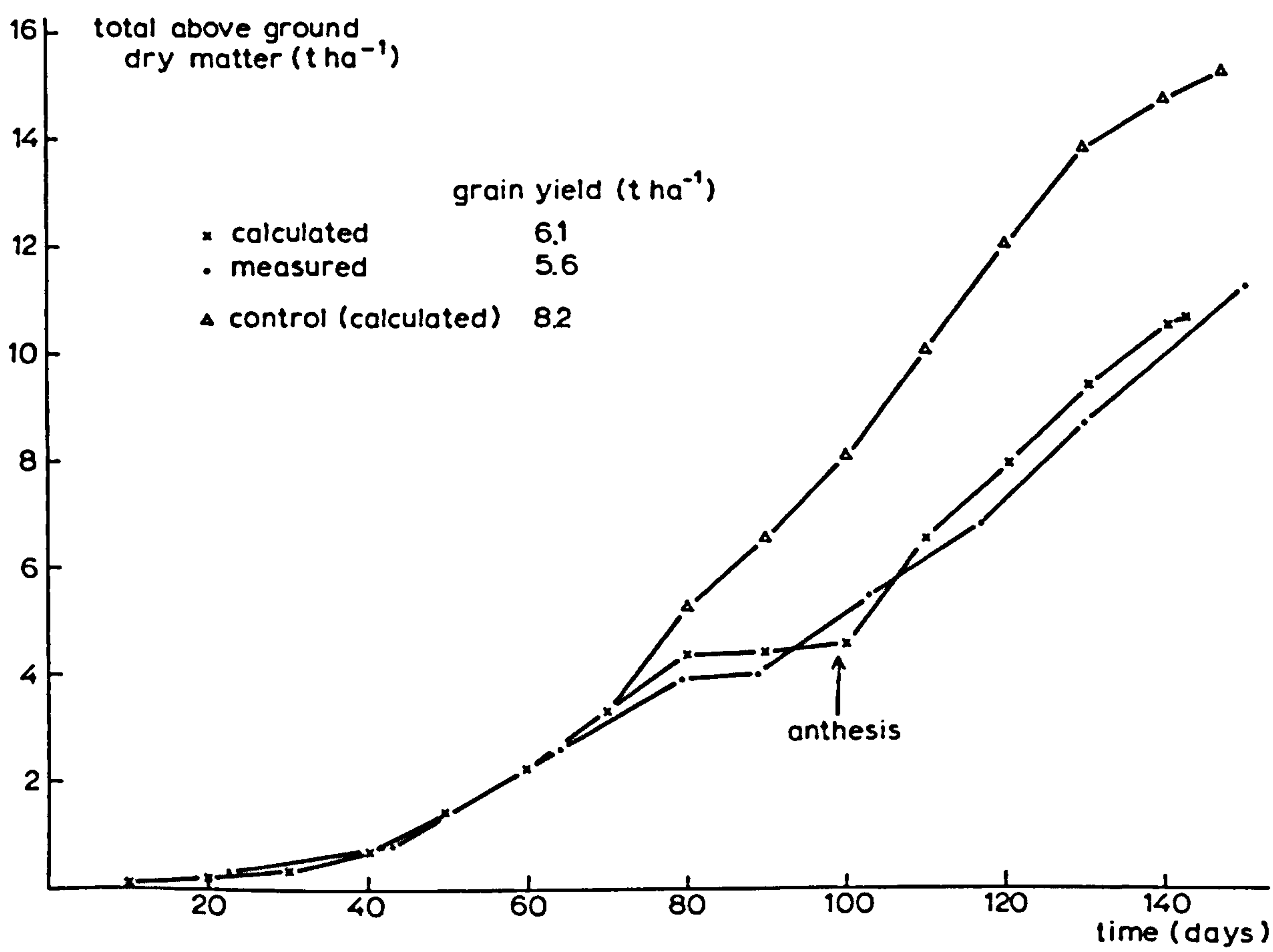

Figure 38. Comparison of measured and calculated dry-matter accumulation of wheat growing under temporary water shortage. The calculated dry-matter accumulation of the control is given for comparison.

Exercise 47
Calculate the transpiration coefficient for this example. Use both total dry matter and above – ground dry matter.

3.4.4 Comparison with the experiment

In Figure 38 the calculated growth curve is compared to the measured one, showing satisfactory agreement in terms of total dry – matter production. The calculated grain yield is however higher than the measured one, for which there is no obvious reason. It is clear from the graph, that the difference is equal to the difference in dry – matter accumulation after anthesis, provided that interpolation between the measured points on either side of the anthesis date is permitted.